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Chapter Contents

Unit & Dimension, Basic Maths and Vector

UNIT & DIMENSION, BASIC MATHS AND VECTOR

KEY CONCEPT

PHYSICAL QUANTITIES AND UNITS

Physical quantities :

All quantities that can be measured are called physical quantities. eg. time, length, mass, force, work done, etc. In physics we study about physical quantities and their inter relationship.

Measurement :

Measurement is the comparison of a quantity with a standard of the same physical quantity.

Classification :

Physical quantities can be classified on the following bases :

I. Based on their directional properties

1. **Scalars:** The physical quantities which have only magnitude but no direction are called *scalar quantities*. **Ex.** mass, density, volume, time, etc.
2. **Vectors :** The physical quantities which have both magnitude and direction and obey laws of vector algebra are called *vector quantities*. **Ex.** displacement, force, velocity, etc.

II. Based on their dependency

1. **Fundamental or base quantities :** A set of physical quantities which are completely independent of each other and all other physical quantities can be expressed in terms of these physical quantities is called Set of Fundamental Quantities.
2. **Derived quantities :** The quantities which can be expressed in terms of the fundamental quantities are known as *derived quantities*. **Ex.** Speed (= distance/time), volume, acceleration, force, pressure, etc.

Physical quantities can also be classified as dimensional and dimensionless quantities or constants and variables.

Ex. Classify the quantities displacement, mass, force, time, speed, velocity, acceleration, moment of inertia, pressure and work under the following categories :

(a) base and scalar (b) base and vector (c) derived and scalar (d) derived and vector

Ans. (a) mass, time (b) displacement (c) speed, pressure, work

(d) force, velocity, acceleration

Units of Physical Quantities

The chosen reference standard of measurement in multiples of which, a physical quantity is expressed is called the *unit* of that quantity. Four basic properties of units are :

1. They must be well defined.
2. They should be easily available and reproducible.
3. They should be invariable e.g. step as a unit of length is not invariable.
4. They should be accepted to all.

System of Units :

1. **FPS or British Engineering system :**

In this system length, mass and time are taken as fundamental quantities and their base units are foot (ft), pound (lb) and second (s) respectively.

2. **CGS or Gaussian system :**

   In this system the fundamental quantities are length, mass and time and their respective units are centimetre (cm), gram (g) and second (s).

3. **MKS system :**

   In this system also the fundamental quantities are length, mass and time but their fundamental units are metre (m), kilogram (kg) and second (s) respectively.

4. **International system (SI) of units :**

   This system is modification over the MKS system and so it is also known as *Rationalised MKS system. Besides* the three base units of MKS system four fundamental and two supplementary units are also included in this system.

Classification of Units : The units of physical quantities can be classified as follows :

1. **Fundamental or base units :**

   The units of fundamental quantities are called *base units*. In SI there are seven base units.

SI BASE QUANTITIES AND THEIR UNITS

S.No.	Physical quantity	SI unit	Symbol
1.	Length	metre	m
2.	Mass	kilogram	kg
3.	Time	second	s
4.	Temperature	Kelvin	K
5.	Electric current	ampere	A
6.	Luminous intensity	candela	cd
7.	Amount of substance	mole	mol

2. **Derived units :**

   The units of derived quantities or the units that can be expressed in terms of the base units are called *derived* units

Ex. Unit of speed $= \dfrac{\text{unit of distance}}{\text{unit of time}} = \dfrac{\text{metre}}{\text{second}} = \text{ms}^{-1}$

Some derived units are named in honour of great scientists.

• Unit of force – newton (N) • Unit of frequency – hertz (Hz) etc.

UNITS OF SOME PHYSICAL QUANTITIES IN DIFFERENT SYSTEMS

Type of Physical Quantity	Physical Quantity	CGS (Originated in France)	MKS (Originated in France)	FPS (Originated in Britain)
Fundamental	Length	cm	m	ft
	Mass	g	kg	lb
	Time	s	s	s
Derived	Force	dyne	newton(N)	poundal
	Work or Energy	erg	joule(J)	ft-poundal
	Power	erg/s	watt(W)	ft-poundal/s

Dimensions :

Dimensions of a physical quantity are the powers (or exponents) to which the base quantities are raised to represent that quantity. To make it clear, consider the physical quantity force. As we shall learn later, force is equal to mass times acceleration. Acceleration is change in velocity divided by time interval. Velocity is length divided by time interval. Thus,

force = mass × acceleration

$$= \text{mass} \times \frac{\text{velocity}}{\text{time}} = \text{mass} \times \frac{\text{length / time}}{\text{time}} = \text{mass} \times \text{length} \times (\text{time})^{-2}$$

Thus, the dimensions of force are 1 in mass, 1 in length and –2 in time. The dimensions in all other base quantities are zero.

1. Dimensional formula :

The physical quantity that is expressed in terms of the base quantities is enclosed in square brackets to remind that the equation is among the dimensions and not among the magnitudes. Thus above equation may be written as [force]= MLT^{-2}.

Such an expression for a physical quantity in terms of the base quantities is called the dimensional formula. Thus, the dimensional formula of force is MLT^{-2}. The two versions given below are equivalent and are used interchangeably.

(a) The dimensional formula of force is MLT^{-2}.

(b) The dimensions of force are 1 in mass, 1 in length and –2 in time.

The *dimensional formula* of any physical quantity is that expression which represents how and which of the base quantities are included in that quantity.

Ex. Dimensional formula of mass is $[M^1L^0\,T^0]$ and that of speed (= distance/time) is $[M^0L^1T^{-1}]$

2. Applications of dimensional analysis :

(i) To convert a physical quantity from one system of units to the other :

This is based on a fact that *magnitude of a physical quantity remains same whatever system is used for measurement* i.e. magnitude = numeric value (n) × unit (u) = constant or $n_1u_1 = n_2u_2$

So if a quantity is represented by $[M^aL^bT^c]$

Then $$\boxed{n_2 = n_1\left[\frac{u_1}{u_2}\right] = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c}$$

n_2 = numerical value in II system
n_1 = numerical value in I system
M_1 = unit of mass in I system
M_2 = unit of mass in II system
L_1 = unit of length in I system
L_2 = unit of length in II system
T_1 = unit of time in I system
T_2 = unit of time in II system

Ex. 1m = 100 cm= 3.28 ft = 39.4 inch
(SI) (CGS) (FPS)

Ex. The acceleration due to gravity is 9.8 m s^{-2}. Give its value in ft s^{-2}

Sol. As 1m = 3.2 ft $\quad \therefore 9.8\ m/s^2 = 9.8 \times 3.28\ ft/s^2 = 32.14\ ft/s^2 \approx 32\ ft/s^2$

Ex. Convert 1 newton (SI unit of force) into dyne (CGS unit of force)

Sol. The dimensional equation of force is $[F] = [M^1\,L^1\,T^{-2}]$

Therefore if n_1, u_1, and n_2, u_2 corresponds to SI & CGS units respectively, then

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^1\left[\frac{L_1}{L_2}\right]^1\left[\frac{T_1}{T_2}\right]^{-2} = 1\left[\frac{kg}{g}\right]\left[\frac{m}{cm}\right]\left[\frac{s}{s}\right]^{-2} = 1 \times 1000 \times 100 \times 1 = 10^5$$

$\therefore$ 1 newton = 10^5 dyne.

Q. The value of Gravitational constant G in MKS system is 6.67×10^{-11} N–m²/kg². What will be its value in CGS system ?

Ans. $6.67 \times 10^{-8}\ \text{cm}^3/\text{g s}^2$

(ii) ***To check the dimensional correctness of a given physical relation :***

If in a given relation, the terms on both the sides have the same dimensions, then the relation is dimensionally correct. This is known as the *principle of homogeneity of dimensions*.

Ex. Check the accuracy of the relation $T = 2\pi\sqrt{\dfrac{L}{g}}$ for a simple pendulum using dimensional analysis.

Sol. The dimensions of LHS = the dimension of T = $[M^0 L^0 T^1]$

The dimensions of RHS = $\left(\dfrac{\text{dimensions of length}}{\text{dimensions of acceleration}}\right)^{1/2}$ ($\because$ 2π is a dimensionless constant)

$$= \left(\frac{L}{LT^{-2}}\right)^{1/2} = (T^2)^{1/2} = [T] = [\,M^0 L^0 T^1]$$

Since the dimensions are same on both the sides, the relation is correct.

(iii) ***To derive relationship between different physical quantities :***

Using the same principle of homogeneity of dimensions new relations among physical quantities can be derived if the dependent quantities are known.

Ex. It is known that the time of revolution T of a satellite around the earth depends on the universal gravitational constant G, the mass of the earth M, and the radius of the circular orbit R. Obtain an expression for T using dimensional analysis.

Sol. We have $[T] = [G]^a [M]^b [R]^c$

$$[M]^0 [L]^0 [T]^1 = [M]^{-a} [L]^{3a} [T]^{-2a} \times [M]^b \times [L]^c = [M]^{b-a} [L]^{c+3a} [T]^{-2a}$$

Comparing the exponents

For [T] : $1 = -2a \Rightarrow a = -\dfrac{1}{2}$

For [M] : $0 = b - a \Rightarrow b = a = -\dfrac{1}{2}$

For [L] : $0 = c + 3a \Rightarrow c = -3a = \dfrac{3}{2}$

Putting the values we get $T = G^{-1/2} M^{-1/2} R^{3/2} = \sqrt{\dfrac{R^3}{GM}}$

So the actual expression is $T = 2\pi\sqrt{\dfrac{R^3}{GM}}$

Limitations of this method :

(i) In Mechanics the formula for a physical quantity depending on more than three physical quantities cannot be derived. It can only be checked.

(ii) This method can be used only if the dependency is of multiplication type. The formulae containing exponential, trigonometrical and logarithmic functions can't be derived using this method. Formulae containing more than one term which are added or subtracted like $s = ut + at^2/2$ also can't be derived.

(iii) The relation derived from this method gives no information about the dimensionless constants.

(iv) If dimensions are given, physical quantity may not be unique as many physical quantities have the same dimensions.

(v) It gives no information whether a physical quantity is a scalar or a vector.

Units and Dimensions of Physical Quantities

Quantity	Common Symbol	SI unit	Dimension
Displacement	s	METRE (m)	L
Mass	m, M	KILOGRAM (kg)	M
Time	t	SECOND (s)	T
Area	A	m^2	L^2
Volume	V	m^3	L^3
Density	ρ	kg/m^3	M/L^3
Velocity	v, u	m/s	L/T
Acceleration	a	m/s^2	L/T^2
Force	F	newton (N)	ML/T^2
Work	W	joule (J) (=N – m)	ML^2/T^2
Energy	E, U, K	joule (J)	ML^2/T^2
Power	P	watt (W) (=J/s)	ML^2/T^3
Momentum	p	kg-m/s	ML/T
Gravitational constant	G	$N\text{-}m^2/kg^2$	L^3/MT^2
Angle	θ, φ	radian	
Angular velocity	ω	radian/s	T^{-1}
Angular acceleration	α	$radian/s^2$	T^{-2}
Angular momentum	L	$kg\text{-}m^2/s$	ML^2/T
Moment of inertia	I	$kg\text{-}m^2$	ML^2
Torque	τ	N-m	ML^2/T^2
Angular frequency	ω	radian/s	T^{-1}
Frequency	ν	hertz (Hz)	T^{-1}
Period	T	s	T
Young's modulus	Y	N/m^2	M/LT^2
Bulk modulus	B	N/m^2	M/LT^2
Shear modulus	η	N/m^2	M/LT^2
Surface tension	S	N/m	M/T^2
Coefficient of viscosity	η	$N\text{-}s/m^2$	M/LT
Pressure	P, p	N/m^2, Pa	M/LT^2
Wavelength	λ	m	L
Intensity of wave	I	W/m^2	M/T^3
Temperature	T	KELVIN (K)	K
Specific heat capacity	c	J/kg–K	L^2/T^2K
Stefan's constant	σ	$W/m^2\text{–}K^4$	M/T^3K^4
Heat	Q	J	ML^2/T^2
Thermal conductivity	K	W/m–K	ML/T^3K

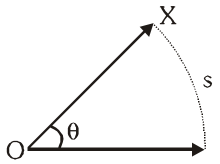
Basic Mathematics used in physics

Plane–angle

It is measure of change in direction.

If a line rotates in a plane about one of its ends, the other end sweeps an arc.

Angle (θ) between two orientation of the line is defined by ratio of the arc length(s) to length of the line(r) $\theta = \frac{s}{r}$ radian

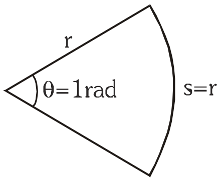


Angles measured in anticlockwise and clockwise directions are usually taken positive and negative respectively.

Angle is measured in radians (rad) or degrees. One radian is the angle subtended at the centre of a circle by an arc of the circle, whose length is equal to the radius of the circle.

π rad= 180° $\pi = 3.1415 = \frac{22}{7}$

1° = 60' (minute), 1' (minute) = 60'' (sec)

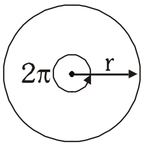


Example

Write expression for circumference of a circle of radius 'r'.

Solution

s = (Total angle about a point) r = 2πr



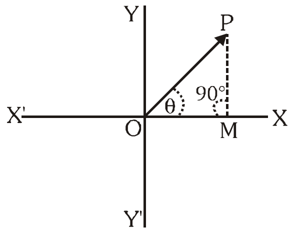
Trigonometrical ratios (or T–ratios)

Let two fixed lines XOX' and YOY' intersecting at right angles to each other at point O.

- Point O is called origin.
- Line XOX' is known as x–axis and YOY' as y–axis.
- Regions XOY, YOX', X'OY' and Y'OX are called I, II, III and IV quadrant respectively.

Consider a line OP making angle θ with OX as shown. Line PM is perpendicular drawn from P on OX. In the right angled triangle OPM, side OP is called hypotenuse, the side OM adjacent to angle θ is called base and the side PM opposite to angle θ is called the perpendicular.

Following ratios of the sides of a right angled triangle are known as trigonometrical ratios or T-ratio



$$\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{MP}{OP} \qquad \cos\theta = \frac{\text{base}}{\text{hypotenuse}} = \frac{OM}{OP} \qquad \tan\theta = \frac{\text{perpendicular}}{\text{base}} = \frac{MP}{OM}$$

$$\cot\theta = \frac{\text{base}}{\text{perpendicular}} = \frac{OM}{MP} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{base}} = \frac{OP}{OM} \qquad \text{cosec}\,\theta = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{OP}{MP}$$

$$\text{cosec}\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Some trigonometric identities

$\sin^2\theta + \cos^2\theta = 1$ $1 + \tan^2\theta = \sec^2\theta$ $1 + \cot^2\theta = \text{cosec}^2\theta$

Example

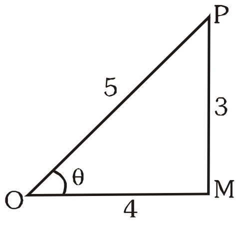
Given $\sin\theta = \frac{3}{5}$. Find all the other T–ratios, if θ lies in the first quadrant.

Solution

In Δ OMP , $\sin\theta = \frac{3}{5}$ So MP = 3 and OP = 5 $\because$ $OM = \sqrt{(5)^2-(3)^2} = \sqrt{25-9} = \sqrt{16} = 4$

Now $\cos\theta = \frac{OM}{OP} = \frac{4}{5}$ $\tan\theta = \frac{MP}{OM} = \frac{3}{4}$ $\cot\theta = \frac{OM}{MP} = \frac{4}{3}$

$\sec\theta = \frac{OP}{OM} = \frac{5}{4}$ $\text{cosec}\,\theta = \frac{OP}{MP} = \frac{5}{3}$



T–ratios of some commonly used angles

Angle (θ)	0 rad	$\frac{\pi}{6}$ rad	$\frac{\pi}{4}$ rad	$\frac{\pi}{3}$ rad	$\frac{\pi}{2}$ rad
	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

$\sin(90°-\theta)=\cos\theta$
$\cos(90°-\theta)=\sin\theta$
$\tan(90°-\theta)=\cot\theta$

$\sin(180°-\theta)=\sin\theta$
$\cos(180°-\theta)=-\cos\theta$
$\tan(180°-\theta)=-\tan\theta$

$\sin(360°-\theta)=-\sin\theta$
$\cos(360°-\theta)=\cos\theta$
$\tan(360°-\theta)=-\tan\theta$

$\sin(90°+\theta)=\cos\theta$
$\cos(90°+\theta)=-\sin\theta$
$\tan(90°+\theta)=-\cot\theta$

$\sin(180°+\theta)=-\sin\theta$
$\cos(180°+\theta)=-\cos\theta$
$\tan(180°+\theta)=\tan\theta$

$\sin(-\theta)=-\sin\theta$
$\cos(-\theta)=\cos\theta$
$\tan(-\theta)=-\tan\theta$

- When θ is very small we can use following approximations : $\cos\theta \approx 1$

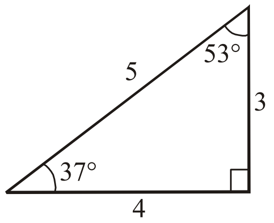
$$\left.\begin{aligned}\sin\theta \simeq \theta\\ \tan\theta \simeq \theta\end{aligned}\right\}\text{ If } \theta \text{ is in radians}$$

$\tan\theta \approx \sin\theta$.

- In the given right angled triangle we have very commonly used T-ratios

$\sin 37° = \frac{3}{5}$ $\cos 37° = \frac{4}{5}$ $\tan 37° = \frac{3}{4}$

$\sin 53° = \frac{4}{5}$ $\cos 53° = \frac{3}{5}$ $\tan 53° = \frac{4}{3}$



Example

Find the value of (i) cos (–60°) (ii) tan 210° (iii) sin 300° (iv) cos 120°

Solution

(i) $\cos(-60°) = \cos 60° = \frac{1}{2}$

(ii) $\tan 210° = \tan(180° + 30°) = \tan 30° = \frac{1}{\sqrt{3}}$

(iii) $\sin 300° = \sin(270° + 30°) = -\cos 30° = -\frac{\sqrt{3}}{2}$

(iv) $\cos 120° = \cos(180° - 60°) = -\cos 60° = -\frac{1}{2}$

VECTORS

Precise description of laws of physics and physical phenomena requires expressing them in form of mathematical equations. In doing so we encounter several physical quantities, some of them have only magnitude and other have direction in addition to magnitude. Quantities of the former kind are referred as scalars and the latter as vectors and mathematical operations with vectors are collectively known as vector analysis.

Vectors

A vector has both magnitude and sense of direction, and follows triangle law of vector addition. For example, displacement, velocity, and force are vectors.

Vector quantities are usually denoted by putting an arrow over the corresponding letter, as $\vec{A}$ or $\vec{a}$. Sometimes in print work (books) vector quantities are usually denoted by boldface letters as **A** or **a**.

Magnitude of a vector $\vec{A}$ is a positive scalar and written as $|\vec{A}|$ or A.

Geometrical Representation of Vectors.

A vector is represented by a directed straight line, having the magnitude and direction of the quantity represented by it.

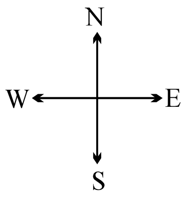
e.g. if we want to represent a force of 5 N acting 45° N of E

(i) We choose direction coordinates.

(ii) We choose a convenient scale like 1 cm ≡ 1N

(iii) We draw a line of length equal in magnitude and in the direction of vector to the chosen quantity.

(iv) We put arrow in the direction of vector.

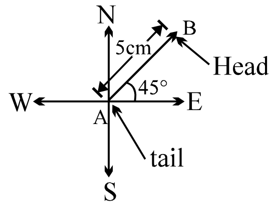


$\overrightarrow{AB}$

Magnitude of vector:

$|\overrightarrow{AB}| = 5N$

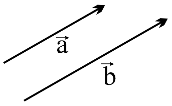
By definition magnitude of a vector quantity is scalar and <u>is always positive</u>.



1 cm ≡ 1 N

3. TERMINOLOGY OF VECTORS

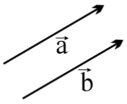
Parallel vector: If two vectors have same direction, they are parallel to each other. They may be located anywhere in the space.



Antiparallel vectors: When two vectors are in opposite direction they are said to be antiparallel vectors.

Equality of vectors: When two vectors have equal magnitude and are in same direction and represent the same physical quantity, they are equal.

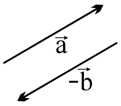
i.e. $\vec{a} = \vec{b}$



Thus when two parallel vectors have same magnitude they are equal. (Their initial point & terminal point may not be same)

Negative of a vector: When a vector have equal magnitude and is in opposite direction, it is said to be negative vector of the former.

i.e. $\vec{a} = -\vec{b}$ or $\vec{b} = -\vec{a}$

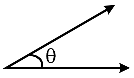


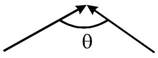
Thus when two antiparallel vectors have same magnitude they are negative of each other.

Remark : Vector shifting is allowed without change in their direction.

2. Angle Between two Vectors

It is the smaller angle formed when the initial points or the terminal points of the two vectors are brought together. It should be noted that $0^\circ \le \theta \le 180^\circ$.





3. Addition Of Vectors:

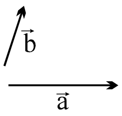
Parallelogram law of addition:

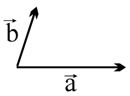
Steps:

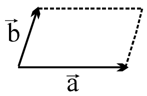
(i) Keep two vectors such that there tails coincide.

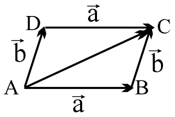
(ii) Draw parallel vectors to both of them considering both of them as sides of a parallelogram.

(iii) Then the diagonal drawn from the point where tails coincide represents the sum of two vectors, with its tail at point of coincidence of the two vectors.



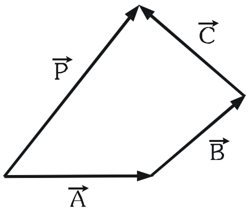
(i) 

(ii) 

(iii) 

$\overrightarrow{AC} = \vec{a} + \vec{b}$

Addition of more than two Vectors



The triangle law can be extended to define addition of more than two vectors. Accordingly, if vectors to be added are drawn in head to tail fashion, resultant is defined by a vector drawn from the tail of the first vector to the head of the last vector. This is also known as the ***polygon rule for vector addition.***

Operation of addition of three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ and their resultant $\vec{P}$ are shown in figure.

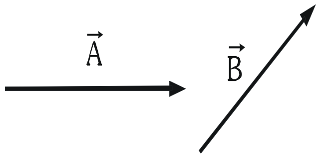
$\vec{A} + \vec{B} + \vec{C} = \vec{P}$

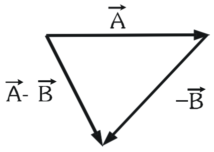
Here it is not necessary that three or more vectors and their resultant are coplanar. In fact, the vectors to be added and their resultant may be in different planes. However if all the vectors to be added are coplanar, their resultant must also be in the same plane containing the vectors.

Subtraction of Vectors

A vector opposite in direction but equal in magnitude to another vector $\vec{A}$ is known as negative vector of $\vec{A}$. It is written as $-\vec{A}$. Addition of a vector and its negative vector results a vector of zero magnitude, which is known as a null vector. A null vector is denoted by arrowed zero $(\vec{0})$.

The idea of negative vector explains operation of subtraction as addition of negative vector. Accordingly to subtract a vector from another consider vectors $\vec{A}$ and $\vec{B}$ shown in the figure. To subtract $\vec{B}$ from $\vec{A}$, the negative vector $-\vec{B}$ is added to $\vec{A}$ according to the triangle law as shown in figure-II.





If two vectors $\vec{a}$ & $\vec{b}$ are represented by $\overrightarrow{OA}$ & $\overrightarrow{OB}$ then their sum $\vec{a}+\vec{b}$ is a vector represented by $\overrightarrow{OC}$, where OC is the diagonal of the parallelogram OACB.

- $\vec{a} + \vec{b} = \vec{b} + \vec{a}$ (commutative)
- $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$ (associativity)
- $\vec{a} + \vec{0} = \vec{a} = \vec{0} + \vec{a}$
- $\vec{a} + (-\vec{a}) = \vec{0} = (-\vec{a}) + \vec{a}$
- $|\vec{a}+\vec{b}| \leq |\vec{a}| + |\vec{b}|$
- $|\vec{a}-\vec{b}| \geq ||\vec{a}| - |\vec{b}||$
- $\left|\vec{a} \pm \vec{b}\right| = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 \pm 2|\vec{a}||\vec{b}|\cos\theta}$ where θ is the angle between the vectors

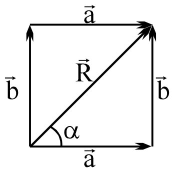
Some Important Results :

(1) If $\theta = 0° \Rightarrow \vec{a} \parallel \vec{b}$

then, $|\vec{R}| = |\vec{a}| + |\vec{b}|$ & $|\vec{R}|$ is maximum

(2) If $\theta = \pi \Rightarrow \vec{a}$ anti $\parallel \vec{b}$

then, $|\vec{R}| = \left||\vec{a}| - |\vec{b}|\right|$ & $|\vec{R}|$ is minimum



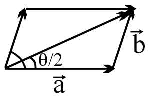
(3) If $\theta = \pi/2 \Rightarrow \vec{a} \perp \vec{b}$

$R = \sqrt{a^2 + b^2}$

& tan α = b/a (α is angle made by $\vec{R}$ with $\vec{a}$)

(4) $|\vec{a}| = |\vec{b}| = a$

$|\vec{R}| = 2a\cos\theta/2$ & $\alpha = \theta/2$



(5) If $|\vec{a}| = |\vec{b}| = a$ & $\theta = 120°$

then $|\vec{R}| = a$

4. Multiplication Of A Vector By A Scalar:

If $\vec{a}$ is a vector & m is a scalar, then m $\vec{a}$ is a vector parallel to $\vec{a}$ whose modulus is $|m|$ times that of $\vec{a}$.

This multiplication is called **SCALAR MULTIPLICATION**. If $\vec{a}$ and $\vec{b}$ are vectors & m, n are scalars, then:

$$m\ (\vec{a}) = (\vec{a})\ m = m\vec{a}$$

$$m\ (n\vec{a}) = n(m\vec{a}) = (mn)\vec{a}$$

$$(m+n)\ \vec{a} = m\vec{a} + n\vec{a}$$

$$m\ (a + \vec{b}) = m\vec{a} + m\vec{b}$$

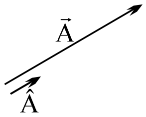
Resolution of a Vector into Components

Following laws of vector addition, a vector can be represented as a sum of two (in two-dimensional space) or three (in three-dimensional space) vectors each along predetermined directions. These directions are called axes and parts of the original vector along these axes are called components of the vector.

UNIT VECTOR:

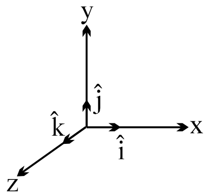
A unit vector is a vector of magnitude of 1, with no units. Its only purpose is to point, i.e. to describe a direction in space.

A unit vector in direction of vector $\vec{A}$ is represented as $\hat{A}$



& $\hat{A} = \dfrac{\vec{A}}{|\vec{A}|}$

or $\vec{A}$ can be expressed in terms of a unit vector in its direction i.e. $\vec{A} = |\vec{A}|\hat{A}$



Unit Vectors along three coordinates axes:–

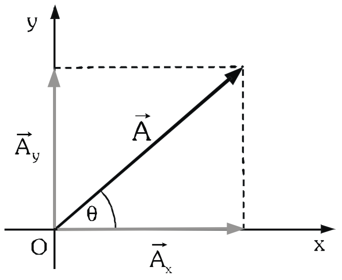
unit vector along x-axis is $\hat{i}$

unit vector along y-axis is $\hat{j}$

unit vector along z-axis is $\hat{k}$

Cartesian components in two dimensions

If a vector is resolved into its components along mutually perpendicular directions, the components are called Cartesian or rectangular components.



In figure is shown, a vector $\vec{A}$ resolved into its Cartesian components $\vec{A}_x$ and $\vec{A}_y$ along the x and y-axis. Magnitudes A_x and A_y of these components are given by the following equation.

$A_x = A\cos\theta$ and $A_y = A\sin\theta$

$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

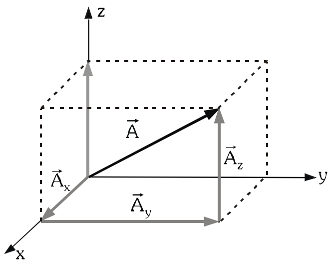
$$A = \sqrt{A_x^{\ 2} + A_y^{\ 2}}$$

Here $\hat{i}$ and $\hat{j}$ are the unit vectors for x and y coordinates respectively.

Mathematical operations e.g. addition, subtraction, differentiation and integration can be performed independently on these components. This is why in most of the problems use of Cartesian components becomes desirable.

Cartesian components in three dimensions

A vector $\vec{A}$ resolved into its three Cartesian components one along each of the directions x, y, and z-axis is shown in the figure.



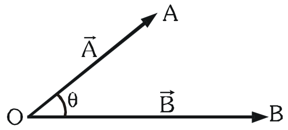
$$\vec{A} = \vec{A}_x + \vec{A}_y + \vec{A}_z = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Product of Vectors

In all physical situation, whose description involve product of two vectors, only two categories are observed. One category where product is also a vector involves multiplication of magnitudes of two vectors and sine of the angle between them, while the other category where product is a scalar involves multiplication of magnitudes of two vectors and cosine of the angle between them. Accordingly, we define two kinds of product operation. The former category is known as vector or cross product and the latter category as scalar or dot product.

Scalar or dot product of two vectors

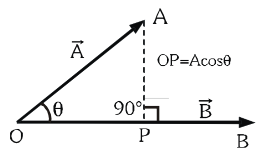


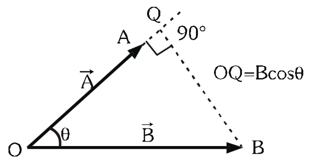
The scalar product of two vectors $\vec{A}$ and $\vec{B}$ equals to the product of their magnitudes and the cosine of the angle θ between them.

$$\vec{A}\cdot\vec{B} = AB\cos\theta = OA\cdot OB\cdot\cos\theta$$

The above equation can also be written in the following ways.

$$\vec{A}\cdot\vec{B} = (A\cos\theta)B = OP\cdot OB \qquad \vec{A}\cdot\vec{B} = A(B\cos\theta) = OA\cdot OQ$$

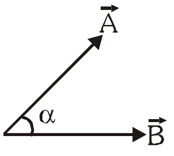


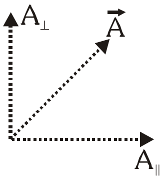


Above two equations and figures, suggest a scalar product as product of magnitude of the one vector and magnitude of the component of another vector in the direction of the former vector.

KEY POINTS

- Dot product of two vectors is commutative: $\vec{A}\cdot\vec{B}=\vec{B}\cdot\vec{A}$
- If two vectors are perpendicular, their dot product is zero. $\vec{A}\cdot\vec{B}=0$, if $\vec{A}\perp\vec{B}$
- Dot product of a vector by itself is known as self-product. $\vec{A}\cdot\vec{A}=A^2\Rightarrow A=\sqrt{\vec{A}\cdot\vec{A}}$
- The angle between the vectors $\theta=\cos^{-1}\left(\dfrac{\vec{A}\cdot\vec{B}}{AB}\right)$
- (a) Component of $\vec{A}$ in direction of $\vec{B}$





$$\vec{A}_{\parallel}=\left(|\vec{A}|\cos\theta\right)\hat{B}=|\vec{A}|\left(\frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}\right)\hat{B}=\left(\frac{\vec{A}.\vec{B}}{|\vec{B}|}\right)\hat{B}=\left(\vec{A}.\hat{B}\right)\hat{B}$$

(b) Component of $\vec{A}$ perpendicular to $\vec{B}$: $\vec{A}_{\perp}=\vec{A}-\vec{A}_{||}$

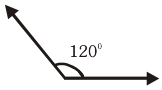
- Dot product of Cartesian unit vectors: $\hat{i}\cdot\hat{i}=\hat{j}\cdot\hat{j}=\hat{k}\cdot\hat{k}=1$

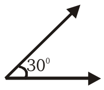
  $\hat{i}\cdot\hat{j}=\hat{j}\cdot\hat{k}=\hat{k}\cdot\hat{i}=0$
- If $\vec{A}=A_x\hat{i}+A_y\hat{j}+A_z\hat{k}$ and $\vec{B}=B_x\hat{i}+B_y\hat{j}+B_z\hat{k}$, their dot product is given by

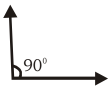
  $\vec{A}\cdot\vec{B}=A_xB_x+A_yB_y+A_zB_z$

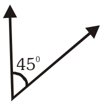
Solved Examples

1. Two displacement vectors of same magnitude are arranged in the following manner

(I)


(II)


(III)


(IV)


Magnitude of resultant is maximum for

(A) case I (B) case II (C) case III (D) case IV

Ans. **(B)**

Sol. Magnitude of Resultant of $\vec{A}$ and $\vec{B}$ = $\sqrt{A^2+B^2+2AB\cos\theta}$ which is maximum for $\theta=30^0$

2. Two vectors $\vec{P}$ and $\vec{Q}$ are added, the magnitude of resultant is 15 units. If $\vec{Q}$ is reversed and added to $\vec{P}$ resultant has a magnitude $\sqrt{113}$ units. The resultant of $\vec{P}$ and a vector perpendicular to $\vec{P}$ and equal in magnitude to $\vec{Q}$ has a magnitude

(A) 13 units (B) 17 units (C) 19 units (D) 20 units

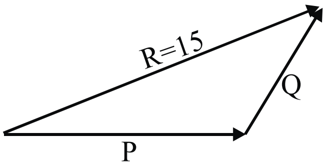
Ans. **(A)**

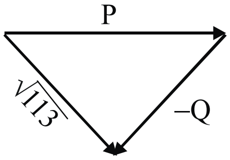
Sol. $P^2 + Q^2 + 2PQ\cos\theta = 225$ ...(i)

$P^2 + Q^2 - 2PQ\cos\theta = 113$ ...(ii)

By adding (i) & (ii) $2(P^2 + Q^2) = 338$

$P^2 + Q^2 = 169 \Rightarrow \sqrt{P^2 + Q^2} = 13$





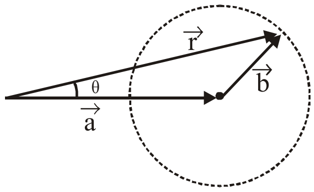
3. Three forces are acting on a body to make it in equilibrium, which set **can not** do it?

(A) 3 N, 3 N, 7 N (B) 2 N, 3 N, 6 N (C) 2 N, 1 N, 1 N (D) 8 N, 6 N, 1 N

Ans. (A, B, D)

Sol. They must form a triangle. $(a + b \geq c)$

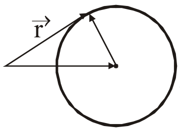
4. Keeping one vector constant, if direction of other to be added in the first vector is changed continuously, tip of the resultant vector describes a circle. In the following figure vector $\vec{a}$ is kept constant. When vector $\vec{b}$ added to $\vec{a}$ changes its direction, the tip of the resultant vector $\vec{r} = \vec{a} + \vec{b}$ describes circle of radius b with its center at the tip of vector $\vec{a}$. Maximum angle between vector $\vec{a}$ and the resultant $\vec{r} = \vec{a} + \vec{b}$ is



(A) $\tan^{-1}\left(\frac{b}{r}\right)$ (B) $\tan^{-1}\left(\frac{b}{\sqrt{a^2 - b^2}}\right)$ (C) $\cos^{-1}$ (r/a) (D) $\cos^{-1}$ (a/r)

Ans. (A,B,C)

Sol.



Angle between $\vec{r}$ and $\vec{b}$ is maximum when $\vec{r}$ is tangent to circle.

5. If $\vec{A} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{B} = 10\hat{i} + 5\hat{j} + 5\hat{k}$, if the magnitude of component of $(\vec{B} - \vec{A})$ along $\vec{A}$ is $4\sqrt{x}$. Then x will be.

Ans. 6

Sol. $r = \vec{B} - \vec{A} = 4(2\hat{i} + \hat{j} + \hat{k})$

$$r\cos\theta = \frac{\vec{r}.\vec{A}}{|A|} = \frac{4(4+1+1)}{\sqrt{6}} = 4\sqrt{6}$$

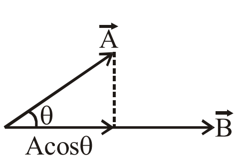
x = 6

6. The component of $\vec{A} = \hat{i} + \hat{j} + 5\hat{k}$ perpendicular to $\vec{B} = 3\hat{i} + 4\hat{j}$ is

(A) $-\frac{4}{25}\hat{i} + \frac{3}{25}\hat{j} + 5\hat{k}$ (B) $-\frac{8}{25}\hat{i} - \frac{6}{25}\hat{j} + 5\hat{k}$ (C) $\frac{4}{25}\hat{i} - \frac{3}{25}\hat{j} + 5\hat{k}$ (D) $+\frac{8}{25}\hat{i} - \frac{6}{25}\hat{j} + 5\hat{k}$

Ans. (C)

Sol.



$$\vec{A}_{\parallel} = A\cos\theta = A\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right)$$

$$= \frac{\vec{A}\cdot\vec{B}}{B} = \frac{3+4}{5} = \frac{7}{5}$$

$$\vec{A}_{\parallel} = \frac{7}{5}\left(\frac{3\hat{i}+4\hat{j}}{5}\right) = \frac{7}{25}\left(3\hat{i}+4\hat{j}\right)$$

$$\vec{A}_{\parallel} = \frac{21}{25}\hat{i} + \frac{28}{25}\hat{j}$$

$$\vec{A}_{\perp} = \left(\hat{i}+\hat{j}+5\hat{k}\right) - \left(\frac{21}{25}\hat{i}+\frac{28}{25}\hat{j}\right)$$

$$= \frac{4}{25}\hat{i} - \frac{3}{25}\hat{j} + 5\hat{k}$$

ALGEBRA : SOME USEFUL FORMULAE

Quadratic equation and its solution

An algebraic equation of second order (highest power of the variable is equal to 2) is called a quadratic equation. General quadratic equation is $ax^2 + bx + c = 0$. The general solution of the above quadratic equation or value of variable x is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\Rightarrow x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Example

Solve $2x^2 + 5x - 12 = 0$

Solution

By comparison with the standard quadratic equation $a = 2$, $b = 5$ and $c = -12$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4\times 2\times(-12)}}{2\times 2} = \frac{-5 \pm \sqrt{121}}{4} = \frac{-5 \pm 11}{4} = \frac{+6}{4}, \frac{-16}{4} \Rightarrow x = \frac{3}{2}, -4$$

Binomial approximation

In case, x is very small, then terms containing higher powers of x can be neglected. In such a case, $(1 + x)^n = 1 + nx$

Also $(1 + x)^{-n} = 1 - nx$ and $(1 - x)^n = 1 - nx$ and $(1 - x)^{-n} = 1 + nx$

Exponential Expansion

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \text{........ and } e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \text{........}$$

Componendo and dividendo theorem :

If $\frac{p}{q} = \frac{a}{b}$ then by componendo and dividendo theorem $\frac{p+q}{p-q} = \frac{a+b}{a-b}$

Determinant

$$D = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc, \text{ For example } \begin{vmatrix} -3 & 3 \\ -5 & 1 \end{vmatrix} = 12, \quad \begin{vmatrix} 2 & -4 \\ -3 & 3 \end{vmatrix} = -6$$

$$D = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$

Example

$$\begin{vmatrix} \overset{+}{5} & \overset{-}{4} & \overset{+}{3} \\ 2 & 1 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 5 \begin{vmatrix} 1 & 6 \\ 8 & 9 \end{vmatrix} - 4 \begin{vmatrix} 2 & 6 \\ 7 & 9 \end{vmatrix} + 3 \begin{vmatrix} 2 & 1 \\ 7 & 8 \end{vmatrix} = 5\,(9 - 48) - 4(18 - 42) + 3(16 - 7) = -72$$

Logarithm

$\log_e x = \ell n\, x$ (base e) $\log x = \log_{10} x$ (base 10)

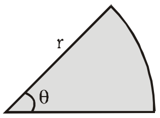
(a) Product formula $\log mn = \log m + \log n$ (b) Quotient formula $\log\frac{m}{n} = \log m - \log n$

(c) Power formula $\log m^n = n \log m$

GEOMETRY : SOME USEFUL FORMULAE

Formulae for determination of area :

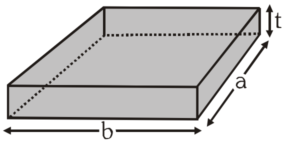
- Area of a square = $(\text{side})^2$
- Area of rectangle = length × breadth
- Area of a triangle = (1/2) × base × height
- Area of a trapezoid =(1/2) × (distance between parallel sides) × (sum of parallel sides)
- Area enclosed by a circle = πr^2 (where r = radius)
- Area of a sector a circle = $\frac{1}{2}\theta r^2$ (where r = is radius and θ is angle subtended at a centre)



- Area of ellipse = π ab (where a and b are semi major and semi minor axis respectively)
- Surface area of a sphere = $4\pi r^2$ (where r = radius)
- Area of a parallelogram = base × height
- Area of curved surface of cylinder = $2\pi r\ell$ (where r = radius and ℓ = length)

- Area of whole surface of cylinder = $2\pi r(r+\ell)$ (where ℓ = length)
- Surface area of a cube = $6(\text{side})^2$
- Total surface area of a cone = $\pi r^2 + \pi r\ell$ [where $\pi r\ell = \pi r\sqrt{r^2+h^2}$ = lateral area (h=height)]

Formulae for determination of volume :



- Volume of a rectangular slab = length × breadth × height = abt
- Volume of a cube = $(\text{side})^3$
- Volume of a sphere = $\frac{4}{3}\pi r^3$ (where r = radius)
- Volume of a cylinder = $\pi r^2\ell$ (where r = radius and ℓ = length)
- Volume of a cone = $\frac{1}{3}\pi r^2 h$ (where r = radius and h = height)

EXERCISE (S-1)

Units & Dimensions

1. A particle is in a unidirectional potential field where the potential energy (U) of a particle depends on the x-coordinate given by $U_x = k(1 - \cos ax)$ & k and 'a' are constants. Find the physical dimensions of 'a' & k.

UD0001

2. The equation for the speed of sound in a gas states that $v = \sqrt{\gamma k_B T / m}$. Speed v is measured in m/s, γ is a dimensionless constant, T is temperature in kelvin (K), and m is mass in kg. Find the SI unit for the Boltzmann constant, k_B?

UD0002

3. The time period (T) of a spring mass system depends upon mass (m) & spring constant (k) & length of the spring (ℓ) $\left[k = \frac{Force}{length}\right]$. Find the relation among T, m, ℓ & k using dimensional method.

UD0003

4. The distance moved by a particle in time t from centre of a ring under the influence of its gravity is given by $x = a \sin\omega t$, where a & ω are constants. If ω is found to depend on the radius of the ring (r), its mass (m) and universal gravitational constant (G). Using dimensional analysis find an expression for ω in terms of r, m and G.

UD0004

5. A satellite is orbiting around a planet. Its orbital velocity (v_0) is found to depend upon

(A) Radius of orbit (R)

(B) Mass of planet (M)

(C) Universal gravitation constant (G)

Using dimensional analysis find an expression relating orbital velocity (v_0) to the above physical quantities.

UD0005

6. Assume that the largest stone of mass 'm' that can be moved by a flowing river depends upon the velocity of flow v, the density d & the acceleration due to gravity g. If 'm' varies as the Kth power of the velocity of flow, then find the value of K.

UD0006

7. Given $\vec{F} = \frac{\vec{a}}{t}$ where symbols have their usual meaning. The dimensions of a is.

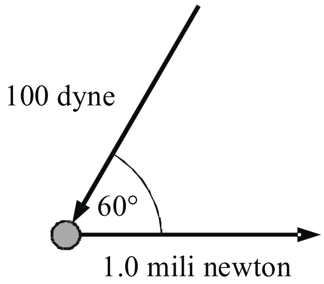
UD0007

Addition of vectors

8. A block is applied two forces of magnitude 5N each. One force is acting towards East and the other acting along 60° North of East. The resultant of the two forces (in N) is of magnitude :-

UD0008

9. Two forces act on a particle simultaneously as shown in the figure. Find net force in milli newton on the particle. [Dyne is the CGS unit of force]

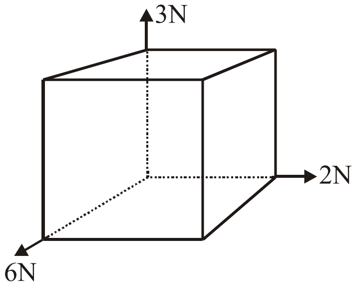


UD0009

10. The maximum and minimum magnitudes of the resultant of two forces are 35 N and 5 N respectively. Find the magnitude of resultant force when act orthogonally to each other.

UD0010

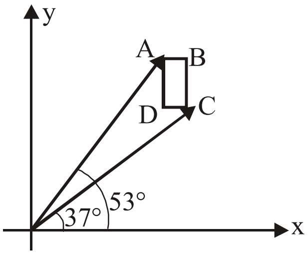
11. Three forces of magnitudes 2 N, 3 N and 6 N act at corners of a cube along three sides as shown in figure. Find the resultant of these forces in N.



UD0011

Resolution of vectors and unit vector

12. The farm house shown in figure has rectangular shape and has sides parallel to the chosen x and y axes. The position vector of corner A is 125 m at 53° and corner C is 100 m at 37° from x axis. Find the length of the fencing required in meter.

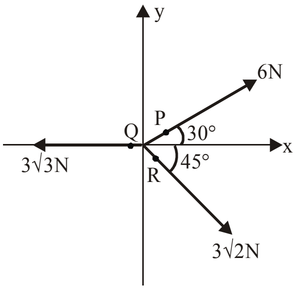


UD0012

13. Vector B has x, y and z components of 4.00, 6.00 and 3.00 units, respectively. Calculate the magnitude of B and the angles that B makes with the coordinates axes.

UD0013

14. Three ants P, Q and R are pulling a grain with forces of magnitude 6N, $3\sqrt{3}$N and $3\sqrt{2}$N as shown in the figure. Find the magnitude of resultant force (in N) acting on the grain.



UD0014

15. Three boys are pushing horizontally a box placed on horizontal table. One is pushing towards north with a $5\sqrt{3}$N force. The second is pushing towards east and third pushes with a force 10 N such that the box is in equilibrium. Find the magnitude of the force, second boy is applying in newton.

UD0015

Scalar product of vectors

16. Consider the two vectors : $\vec{L} = 1\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{l} = 4\hat{i}+5\hat{j}+6\hat{k}$. Find the value of the scalar α such that the vector $\vec{L}-\alpha\vec{l}$ is perpendicular to $\vec{L}$. **UD0016**

17. Find components of vector $\vec{a} = \hat{i}+\hat{j}+3\hat{k}$ in directions parallel to and perpendicular to vector $\vec{b} = \hat{i}+\hat{j}$.

UD0017

18. (a) Calculate $\vec{r} = \vec{a}-\vec{b}+\vec{c}$ where $\vec{a} = 5\hat{i}+4\hat{j}-6\hat{k}$, $\vec{b} = -2\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{c} = 4\hat{i}+3\hat{j}+2\hat{k}$.

(b) Calculate the angle between $\vec{r}$ and the z-axis.

(c) Find the angle between $\vec{a}$ and $\vec{b}$ **UD0018**

19. If the velocity of a particle is ($2\hat{i}+3\hat{j}-4\hat{k}$) and its acceleration is ($-\hat{i}+2\hat{j}+\hat{k}$) and angle between them is $\frac{n\pi}{4}$. The value of n is. **UD0019**

Method of approximation

20. Quito, a city in Ecuador and Kampala, a city situated in Uganda both lie on the Equator. The longitude of Quito is 82°30' W and that of Kampala is 37°30' E. What is the distance from Quito to Kampala.

(a) along the shortest surface path

(b) along a direct (through-the-Earth) path? (The radius of the Earth is 6.4×10^6 m) **UD0020**

21. Use the approximation $(1+x)^n \approx 1+nx, |x| << 1$, to find approximate value for

(a) $\sqrt{99}$ (b) $\frac{1}{1.01}$ **UD0021**

22. Use the small angle approximations to find approximate values for (A) sin 8° and (B) tan 5°

UD0022

EXERCISE (S-2)

1. The equation of state for a real gas at high temperature is given by $P = \frac{nRT}{V-b} - \frac{a}{T^{1/2}V(V+b)}$ where n, P, V & T are number of moles, pressure, volume & temperature respectively & R is the universal gas constant. Find the dimensions of constant *a* in the above equation.

UD0023

2. If Energy (E), velocity (v) and time (T) are fundamental units. What will be the dimension of surface tension?

UD0024

3. In system called the star system we have 1 star kilogram = 10^{20} kg. 1 starmeter = 10^8 m,
1 starsecond = 10^3s then calculate the value of 1 joule in this system.

UD0025

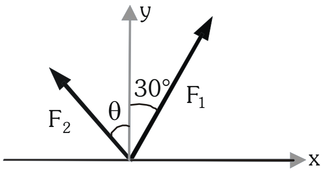
4. A vector $\vec{A}$ of length 10 units makes an angle of 60° with the vector $\vec{B}$ of length 6 units. Find the magnitude of the vector difference $\vec{A} - \vec{B}$ & the angle it makes with vector $\vec{A}$.

UD0026

5. A bird is at a point P (4, –1, –5) and sees two points P_1 (–1, –1, 0) and P_2 (3, –1, –3). At time t = 0, it starts flying with a constant speed of 10 m/s to be in line with points P_1 and P_2 in minimum possible time t. Find t, if all coordinates are in kilometers.

UD0027

6. In the figure, F_1 and F_2, the two unknown forces give a resultant force of $80\sqrt{3}$ N along the y–axis. It is required that F_2 must have minimum magnitude. Find the magnitudes of F_1 and F_2.



UD0028

7. A particle is displaced from A ≡ (2, 2, 4) to B ≡ (5, –3, –1). A constant force of 34 N acts in the direction of $\vec{AP}$, where P ≡ (10, 2, –11). (Coordinates are in m).

(i) Find the $\vec{F}$. (ii) Find the work done by the force to cause the displacement.

UD0029

EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

Units & Dimensions

1. The dimensional formula for which of the following pair is not the same

(A) impulse and momentum (B) torque and work

(C) stress and pressure (D) momentum and angular momentum

UD0030

2. Which of the following can be a set of fundamental quantities

(A) length, velocity, time (B) momentum, mass, velocity

(C) force, mass, velocity (D) momentum, time, frequency

UD0031

3. Which of the following functions of A and B may be performed if A and B possess different dimensions?

(A) $\frac{A}{B}$ (B) A + B (C) A – B (D) None

UD0032

4. The velocity v of a particle at time t is given by $v = at + \frac{b}{t+c}$, where a, b and c are constants. The dimensions of a, b and c are respectively :–

(A) LT^{-2}, L and T (B) L^2, T and LT^2 (C) LT^2, LT and L (D) L, LT and T^2

UD0033

5. If area (A), velocity (v), and density (ρ) are base units, then the dimensional formula of force can be represented as :-

(A) $Av\rho$ (B) $Av^2\rho$ (C) $Av\rho^2$ (D) $A^2v\rho$

UD0034

6. Density of wood is 0.5 g/ cc in the CGS system of units. The corresponding value in MKS units is :-

(A) 500 (B) 5 (C) 0.5 (D) 5000

UD0035

7. In a book, the answer for a particular question is expressed as $b = \frac{ma}{k}\left[\sqrt{1+\frac{2k\ell}{ma}}\right]$ here m represents mass, a represents acceleration, ℓ represents length. The unit of b should be :-

(A) m/s (B) m/s^2 (C) meter (D) /sec

UD0036

8. The frequency f of vibrations of a mass m suspended from a spring of spring constant k is given by $f=Cm^xk^y$, where C is a dimensionless constant. The values of x and y are, respectively

(A) $\frac{1}{2}, \frac{1}{2}$ (B) $-\frac{1}{2}, -\frac{1}{2}$ (C) $\frac{1}{2}, -\frac{1}{2}$ (D) $-\frac{1}{2}, \frac{1}{2}$

UD0037

9. If force, acceleration and time are taken as fundamental quantities, then the dimensions of length will be:

(A) FT^2 (B) $F^{-1}A^2T^{-1}$ (C) FA^2T (D) AT^2

UD0038

10. In a particular system the units of length, mass and time are chosen to be 10 cm, 10 g and 0.1 s respectively. The unit of force in this system will be equal to :-

(A) 0.1 N (B) 1 N (C) 10 N (D) 100 N

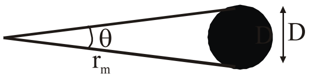
UD0039

11. The units of three physical quantities x, y and z are $gcm^2 s^{-5}$, gs^{-1} and cms^{-2} respectively. The relation among the units of x, y and z is :-

(A) $z = x^2y$ (B) $y^2 = xz$ (C) $x = yz^2$ (D) $x = y^2z$

UD0040

12. The angle subtended by the moon's diameter at a point on the earth is about 0.50°. Use this and the fact that the moon is about 384000 km away to find the approximate diameter of the moon.



(A) 192000 km (B) 3350 km (C) 1600 km (D) 1920 km

UD0041

13. **Statement 1 :** Method of dimensions cannot tell whether an equation is correct.

and

Statement 2 : A dimensionally incorrect equation may be correct.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

UD0042

14. The equation of the stationary wave is $y = 2A\sin\left(\frac{2\pi ct}{\lambda}\right)\cos\left(\frac{2\pi x}{\lambda}\right)$ which of the following statement is wrong?

(A) The unit of ct is same as that of λ. (B) The unit of x is same as that of λ.

(C) The unit of $\frac{2\pi c}{\lambda}$ is same as that of $\frac{2\pi x}{\lambda t}$ (D) The unit of $\frac{c}{\lambda}$ is same as that of $\frac{x}{\lambda}$

UD0043

15. Due to some unknown interaction, force F experienced by a particle is given by the following equation.

$$\vec{F} = -\frac{A}{r^3}\vec{r}$$

Where A is positive constant and r distance of the particle from origin of a coordinate system. Dimensions of constant A are :-

(A) ML^2T^{-2} (B) ML^3T^{-2} (C) ML^4T^{-2} (D) ML^0T^0

UD0044

Addition of vectors

16. Three concurrent forces of the same magnitude are in equilibrium. What is the angle between the force? Also name the triangle formed by the force as sides :-

(A) 60° equilateral triangle
(B) 120° equilateral triangle
(C) 120°, 30°, 30° an isosceles triangle
(D) 120° an obtuse angled triangle

UD0045

17. The resultant of two forces, one double the other in magnitude is perpendicular to the smaller of the two forces. The angle between two forces is :-

(A) 150° (B) 90° (C) 60° (D) 120°

UD0046

18. The resultant of two forces acting at an angle of 120° is 10 kg wt and is perpendicular to one of the forces. That force is :

(A) $10\sqrt{3}$ kg wt (B) $20\sqrt{3}$ kg wt (C) 10 kg wt (D) $\frac{10}{\sqrt{3}}$ kg wt

UD0047

19. If the resultant of two forces of magnitudes P and Q acting at a point at an angle of 60° is $\sqrt{7}$ Q, then P/Q is :-

(A) 1 (B) $\frac{3}{2}$ (C) 2 (D) 4

UD0048

20. There are two force vectors, one of 5N and other of 12N at what angle the two vectors be added to get resultant vector of 17N, 7N and 13N respectively.

(A) 0°, 180° and 90°
(B) 0°, 90° and 180°
(C) 0°, 90° and 90°
(D) 180°, 0° and 90°

UD0049

21. A body placed in free space, is simultaneously acted upon by three forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$. The body is in equilibrium and the forces $\vec{F}_1$ and $\vec{F}_2$ are known to be 36 N due north and 27 N due east respectively. Which of the following best describes the force $\vec{F}_3$?

(A) 36 N due south.
(B) 53 N due 60° south of east
(C) 45 N due 53° south of west
(D) 45 N due 37° north of west

UD0050

22. The ratio of maximum and minimum magnitudes of the resultant of two vectors $\vec{A}$ and $\vec{B}$ is 3 : 2. The relation between A and B is

(A) A = 5B (B) 5A = B (C) A = 3B (D) A = 4B

UD0051

23. Find the resultant of the following two vectors $\vec{A}$ and $\vec{B}$.

$\vec{A}$: 40 units due east and ; $\vec{B}$: 25 units 37° north of west

(A) 25 units 37° north of west
(B) 25 units 37° north of east
(C) 40 units 53° north of west
(D) 40 units 53° north of east

UD0052

24. Two vectors $\vec{a}$ and $\vec{b}$ add to give a resultant $\vec{c} = \vec{a} + \vec{b}$. In which of these cases angle between $\vec{a}$ and $\vec{b}$ is maximum: (a, b, c represent the magnitudes of respective vectors)

(A) c = a + b (B) $c^2 = a^2 + b^2$ (C) c = a – b (D) can not be determined

UD0053

25. If the angle between the unit vectors $\hat{a}$ and $\hat{b}$ is 60°, then $|\hat{a} - \hat{b}|$ is :-

(A) 0 (B) 1 (C) 2 (D) 4

UD0054

26. A man moves towards 3 m north then 4 m towards east and finally 5 m towards 37° south of west. His displacement from origin is :-

(A) $5\sqrt{2}$m (B) 0m (C) 1 m (D) 12 m

UD0055

Resolution of vectors and unit vector

27. The projection of a vector, $\vec{r} = 3\hat{i} + \hat{j} + 2\hat{k}$, on the x–y plane has magnitude :–

(A) 3 (B) 4 (C) $\sqrt{14}$ (D) $\sqrt{10}$

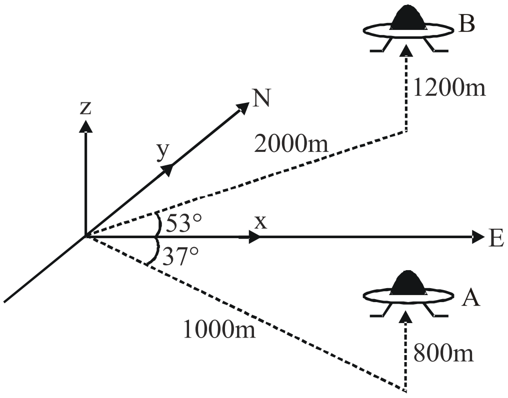
UD0056

28. A bird moves from point (1,–2) to (4, 2). If the speed of the bird is 10 m/s, then the velocity vector of the bird is

(A) $5(\hat{i} - 2\hat{j})$ (B) $5(4\hat{i} + 2\hat{j})$ (C) $0.6i + 0.8\hat{j}$ (D) $6\hat{i} + 8\hat{j}$

UD0057

29. Personnel at an air post control tower track a UFO. At 11:02 am it was located at position A and at 11:12 am is was located at position B. Displacement vector of UFO is :

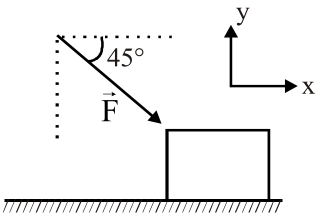


(A) $400\hat{i} + 2200\hat{j} + 400\hat{k}$ (B) $1200\hat{i} + 1000\hat{j} + 800\hat{k}$

(C) $2000\hat{i} + 2200\hat{j} + 2000\hat{k}$ (D) $400\hat{i} + 1000\hat{j} + 400\hat{k}$

UD0058

30. A person pushes a box kept on a horizontal surface with force of 100 N. In unit vector notation force $\vec{F}$ can be expressed as :



(A) $100\ (\hat{i} + \hat{j})$ (B) $100\,(\hat{i} - \hat{j})$ (C) $50\sqrt{2}\ (\hat{i} - \hat{j})$ (D) $50\sqrt{2}\ (\hat{i} + \hat{j})$

UD0059

31. For the given vector $\vec{A} = 3\hat{i} - 4\hat{j} + 10\hat{k}$, the ratio of magnitude of its component on the x-y plane and the component on z–axis is :-

(A) 2 (B) $\frac{1}{2}$ (C) 1 (D) None

UD0060

32. After firing, a bullet is found to move at an angle of 37° to horizontal. Its acceleration is 10 m/s^2 downwards. Find the component of acceleration in the direction of the velocity.

(A) – 6 m/s^2 (B) – 4 m/s^2 (C) – 8 m/s^2 (D) – 5 m/s^2

UD0061

Scalar product of vectors

33. In a methane (CH_4) molecule each hydrogen atom is at a corner of a regular tetrahedron with the carbon atom at the centre. In coordinates where one of theC–H bonds is in the direction of $\hat{i} + \hat{j} + \hat{k}$, an adjacent C–H bond in the $\hat{i} - \hat{j} - \hat{k}$ direction. Then angle between these two bonds :-

(A) $\cos^{-1}\left(-\frac{2}{3}\right)$ (B) $\cos^{-1}\left(\frac{2}{3}\right)$ (C) $\cos^{-1}\left(-\frac{1}{3}\right)$ (D) $\cos^{-1}\left(\frac{1}{3}\right)$

UD0062

34. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a} + 2\vec{b}$ and $5\vec{a} - 4\vec{b}$ are perpendicular to each other then the angle between $\vec{a}$ and $\vec{b}$ is :-

(A) 45° (B) 60° (C) $\cos^{-1}\left(\frac{1}{3}\right)$ (D) $\cos^{-1}\left(\frac{2}{7}\right)$

UD0063

35. The velocity of a particle is $\vec{v} = 6\hat{i} + 2\hat{j} - 2\hat{k}$. The component of the velocity of a particle parallel to vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ is :-

(A) $6\hat{i} + 2\hat{j} + 2\hat{k}$ (B) $2\hat{i} + 2\hat{j} + 2\hat{k}$ (C) $\hat{i} + \hat{j} + \hat{k}$ (D) $6\hat{i} + 2\hat{j} - 2\hat{k}$

UD0064

36. A particle moves from a position $3\hat{i} + 2\hat{j} - 6\hat{k}$ to a position $14\hat{i} + 13\hat{j} + 9\hat{k}$ in m and a uniform force of $4\hat{i} + \hat{j} + 3\hat{k}$ N acts on it. The work done by the force is :-

(A) 200 J (B) 100 J (C) 300 J (D) 500 J

UD0065

37. Which of the following is perpendicular to $\hat{i} - \hat{j} - \hat{k}$?

(A) $\hat{i} + \hat{j} + \hat{k}$ (B) $-\hat{i} + \hat{j} + \hat{k}$ (C) $\hat{i} + \hat{j} - \hat{k}$ (D) none of these

UD0066

MULTIPLE CORRECT TYPE QUESTIONS

38. Which of the following statements about the sum of the two vectors $\vec{A}$ and $\vec{B}$, is/are correct?

(A) $|\vec{A} + \vec{B}| \le A + B$ (B) $|\vec{A} + \vec{B}| \ge A + B$ (C) $|\vec{A} + \vec{B}| \ge |\vec{A} - \vec{B}|$ (D) $|\vec{A} + \vec{B}| \ge |A - B|$

UD0067

39. Priya says that the sum of two vectors by the parallelogram method is $\vec{R} = 5\hat{i}$. Subhangi says it is $\vec{R} = \hat{i} + 4\hat{j}$. Both used the parallelogram method, but one used the wrong diagonal. Which one of the vector pairs below contains the original two vectors?

(A) $\vec{A} = +3\hat{i} - 2\hat{j}$; $\vec{B} = -2\hat{i} + 2\hat{j}$
(B) $\vec{A} = -3\hat{i} - 2\hat{j}$; $\vec{B} = +2\hat{i} + 2\hat{j}$
(C) $\vec{A} = +3\hat{i} + 2\hat{j}$; $\vec{B} = +2\hat{i} - 2\hat{j}$
(D) $\vec{A} = +3\hat{i} + 2\hat{j}$; $\vec{B} = -2\hat{i} + 2\hat{j}$

UD0068

40. For the equation $x = AC \sin(Bt) + D e^{(BCt)}$, where x and t represent position and time respectively, which of the following is/are **CORRECT** :-

(A) Dimension of AC is LT^{-1}
(B) Dimension of B is T^{-1}
(C) Dimension of AC and D are same
(D) Dimension of C is T^{-1}

UD0069

COMPREHENSION TYPE QUESTIONS

Paragraph for Question no. 41 to 43

In a certain system of absolute units the acceleration produced by gravity in a body falling freely is denoted by 5, the kinetic energy of a 500 kg shot moving with velocity 400 metres per second is denoted by 2000 & its momentum by 100.

41. The unit of length is :-

(A) 15 m (B) 50 m (C) 25 m (D) 100 m

UD0070

42. The unit of time is :-

(A) 10 s (B) 20 s (C) 5 s (D) 15 s

UD0070

43. The unit of mass is :-

(A) 200 kg (B) 400 kg (C) 800 kg (D) 1200 kg

UD0070

Paragraph for Question Nos. 44 and 45

For any particle moving with some velocity $(\vec{v})$ & acceleration $(\vec{a})$, tangential acceleration & normal acceleration are defined as follows

Tangential acceleration – The component of acceleration in the direction of velocity.

Normal acceleration – The component of acceleration in the direction perpendicular to velocity.

If at a given instant, velocity & acceleration of a particle are given by.

$$\vec{v} = 4\hat{i} + 3\hat{j}$$

$$\vec{a} = 10\hat{i} + 15\hat{j} + 20\hat{k}$$

44. Find the tangential acceleration of the particle at the given instant :-

(A) $17(4\hat{i} + 3\hat{j})$ (B) $\frac{17}{5}(4\hat{i} + 3\hat{j})$ (C) $17(4\hat{i} - 3\hat{j})$ (D) $\frac{17}{5}(4\hat{i} - 3\hat{j})$

UD0071

45. Find the normal acceleration of the particle at the given instant :-

(A) $\frac{-9\hat{i} + 12\hat{j} + 50\hat{k}}{5}$ (B) $\frac{9\hat{i} - 12\hat{j} - 50\hat{k}}{5}$ (C) $\frac{-18\hat{i} + 24\hat{j} + 100\hat{k}}{5}$ (D) $\frac{18\hat{i} - 24\hat{j} - 100\hat{k}}{5}$

UD0071

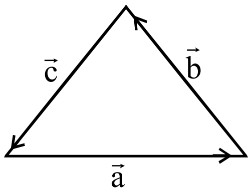
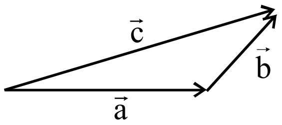
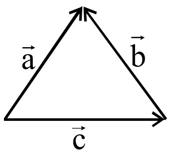
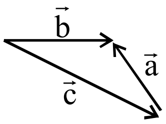
MATRIX MATCH TYPE QUESTION

46. Two particles A and B start from origin of a coordinate system towards point P (10, 20) and Q (20, 10) respectively with speed $5\sqrt{5}$ each. Both continue their motion for 10 s and then stop. There after particle B moves towards particle A with speed $2\sqrt{2}$ and after particle B meets particle A, they both return to origin following a straight line path with speed $5\sqrt{5}$. Match the items of column-I with suitable items of Column-II.

	Column-I		**Column-II**
(A)	Initial velocity vector of A	(P)	$\left(-5\hat{i}-10\hat{j}\right)$
(B)	Initial velocity of B	(Q)	$\left(5\hat{i}+10\hat{j}\right)$
(C)	Velocity vector of B while it moves towards A	(R)	$\left(10\hat{i}+5\hat{j}\right)$
(D)	Velocity vector of A and B while they return to origin	(S)	$\left(2\hat{i}-2\hat{j}\right)$
		(T)	$\left(-2\hat{i}+2\hat{j}\right)$

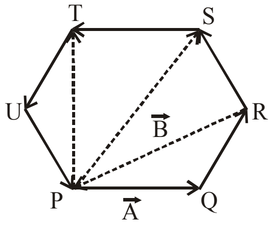
UD0072

47. Column-I show vector diagram relating three vectors $\vec{a}, \vec{b}$ and $\vec{c}$. Match the vector equation in column-II, with vector diagram in column-I :

	Column-I		**Column-II**
(A)		(P)	$\vec{a}-(\vec{b}+\vec{c})=0$
(B)		(Q)	$\vec{b}-\vec{c}=\vec{a}$
(C)		(R)	$\vec{a}+\vec{b}=-\vec{c}$
(D)		(S)	$\vec{a}+\vec{b}=\vec{c}$

UD0073

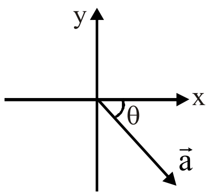
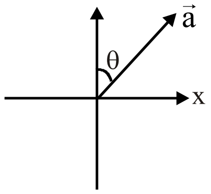
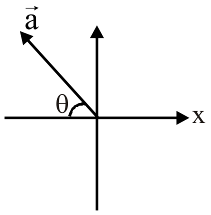
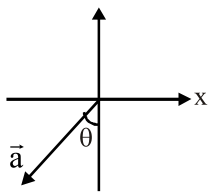
48. In a regular hexagon two vectors $\overrightarrow{PQ} = \vec{A}$, $\overrightarrow{RP} = \vec{B}$. Express other vector's in term of them :-



Column–I	Column–II
(A) $\overrightarrow{PS}$	(P) $-2\vec{B}-3\vec{A}$
(B) $\overrightarrow{PT}$	(Q) $-\vec{B}-\vec{A}$
(C) $\overrightarrow{RS}$	(R) $-\vec{B}-2\vec{A}$
(D) $\overrightarrow{TS}$	(S) $-2(\vec{B}+\vec{A})$
	(T) $\vec{A}$

UD0074

49. Show a vector $\vec{a}$ at angle θ as shown in the figure column-II. Show its unit vector representation.

Column–I	Column–II
(A) 	(P) $\vec{a} = a\sin\theta\,\hat{i} + a\cos\theta\,\hat{j}$
(B) 	(Q) $\vec{a} = -a\cos\theta\,\hat{i} + a\sin\theta\,\hat{j}$
(C) 	(R) $\vec{a} = -a\sin\theta\,\hat{i} - a\cos\theta\,\hat{j}$
(D) 	(S) $\vec{a} = a\cos\theta\,\hat{i} - a\sin\theta\,\hat{j}$

UD0075

EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

1. In a certain system of units, 1 unit of time is 5 sec, 1 unit of mass is 20 kg and 1 unit of length is 10 m. In this system, one unit of power will correspond to :-

(A) 16 watts (B) $\frac{1}{16}$ watts (C) 25 watts (D) none of these

UD0076

2. If the unit of length be doubled then the numerical value of the universal gravitation constant G will become (with respect to present value)

(A) Double (B) Half (C) 8 times (D) 1/8 times

UD0077

3. If in a system, the force of attraction between two point masses of 1 kg each situated 1 km apart is taken as a unit of force and is called notwen (newton written in reverse order) & if $G = 6.67\times 10^{-11}$ N–m^2 kg^{-2} in SI units then which of the following is true?

(A) 1 notwen = 6.67×10^{-11} newton (B) 1 newton = 6.67×10^{-17} notwen

(C) 1 notwen = 6.67×10^{-17} newton (D) 1 newton = 6.67×10^{-12} notwen

UD0078

4. In two different systems of units an acceleration is represented by the same number, while a velocity is represented by numbers in the ratio 1 : 3. The ratios of unit of length and time are respectively

(A) 3, 9 (B) 9, 3 (C) 1, 1 (D) None of these

UD0079

5. **Statement–1 :** Whenever the unit of measurement of a quantity is changed, its numerical value changes.

and

Statement–2 : Smaller the unit of measurement smaller is its numerical value.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

UD0080

6. Forces proportional to AB, BC and 2CA act along the sides of triangle ABC in order. Their resultant represented in magnitude and direction as

(A) CA (B) AC (C) BC (D) CB

UD0081

7. A man rows a boat with a speed of 18 km/hr in north–west direction. The shoreline makes an angle of 15° south of west. Obtain the component of the velocity of the boat along the shoreline.

(A) 9 km/hr (B) $18\frac{\sqrt{3}}{2}$ km/hr

(C) 18 cos (15°) km/hr (D) 18 cos (75°) km/hr

UD0082

8. **Statement 1** : Unit vector has a unit though its magnitude is one

and

Statement 2 : Unit vector is obtained by dividing a vector by its own magnitude.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

UD0083

9. A vector of magnitude 10 m in the direction 37° south of west has its initial point at (5 m, 2 m). If positive x–axis represents the east and positive y–axis the north, the coordinates of its terminal point are

(A) (–3 m, –4 m) (B) (3 m, 4 m) (C) (–4 m, 6 m) (D) (–4 m,–6 m)

UD0084

10. A plumber steps down 1 m out of his truck and walks 50 m east and then 25 m south, and then takes an elevator to the basement of the building 9 m below street level. If the east, the north and the upward direction are represented by the positive x, y and z–axes, which one of the following represents displacement (meters) of the plumber?

(A) $50\hat{i}-25\hat{j}-9\hat{k}$ (B) $50\hat{i}+25\hat{j}-9\hat{k}$

(C) $50\hat{i}-25\hat{j}-10\hat{k}$ (D) $50\hat{i}+25\hat{j}-10\hat{k}$

UD0085

11. A body moves in anticlockwise direction on a circular path in the x-y plane. The radius of the circular path is 5 m and its centre is at the origin. In a certain interval of time, displacement of the body is observed to be 6 m in the positive y-direction. Which of the following is true?

(A) Its initial position vector is $5\hat{i}$ m. (B) Its initial position vector is $(-3\hat{i}+4\hat{j})$ m.

(C) Its final position vector is $\left(4\hat{i}+3\hat{j}\right)$ m. (D) Its final position vector is $6\hat{j}$ m.

UD0086

12. A boy A is standing 20√3 m away in a direction 30° north of east from his friend B. Another boy C standing somewhere east of B can reach A, if he walks in a direction 60° north of east. In a Cartesian coordinate system with its x–axis towards the east, the position of C with respect to A is

(A) $\left(-20\hat{i}+-10\hat{j}\right)$ m (B) $\left(-10\hat{i}-10\sqrt{3}\hat{j}\right)$ m

(C) $\left(10\hat{i}+10\sqrt{3}\hat{j}\right)$ m (D) It depends on where we chose the origin.

UD0087

13. Find the component of $\vec{r}$ in the direction of $\vec{a}$:–

(A) $\frac{(\vec{r}.\vec{a})\vec{a}}{a^2}$ (B) $\frac{(\vec{r}.\vec{a})\vec{a}}{a}$ (C) $\frac{(\vec{r}\cdot\vec{a})\vec{r}}{r}$ (D) $\frac{(\vec{r}\cdot\vec{a})\vec{r}}{r^2}$

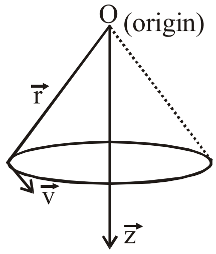
UD0088

14. Consider three vectors $\vec{A}=\hat{i}+\hat{j}-2\hat{k}$, $\vec{B}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{C}=2\hat{i}-3\hat{j}+4\hat{k}$. A vector $\vec{X}$ of the form $\alpha\vec{A}+\beta\vec{B}$ (α and β are numbers) is perpendicular to $\vec{C}$. The ratio of α and β is

(A) 1 : 1 (B) 2 : 1 (C) –1 : 1 (D) 3 : 1

UD0089

15. A string connected with bob is suspended by the point (O) such that it sweeps out conical surface in horizontal plane. Here $\vec{r}$ is the position vector of bob, $\vec{v}$ is its velocity and $\vec{z}$ is the axis of swept cone as shown. Select **INCORRECT** statement :-



(A) $\vec{r}.\vec{z}$ is always zero (B) $\vec{r}.\vec{v}$ is always zero

(C) $\vec{z}\cdot\vec{v}$ is always constant (D) $\vec{r}\cdot\vec{z}$ is always non zero constant

UD0090

16. x-component of a vector $\vec{A}$ is twice of its y-component and $\sqrt{2}$ times of its z-component. Find out the angle made by the vector from y-axis.

(A) $\cos^{-1}\left(\frac{2}{\sqrt{7}}\right)$ (B) $\cos^{-1}\left(\frac{1}{\sqrt{7}}\right)$ (C) $\cos^{-1}\left(\frac{1}{\sqrt{6}}\right)$ (D) $\cos^{-1}\left(\frac{2}{\sqrt{6}}\right)$

UD0091

17. Given the vectors $\vec{A}=2\hat{i}+3\hat{j}-\hat{k}$; $\vec{B}=3\hat{i}-2\hat{j}-2\hat{k}$ & $\vec{C}=p\hat{i}+p\hat{j}+2p\hat{k}$. Find the angle between $(\vec{A}-\vec{B})$ & $\vec{C}$

(A) $\theta=\cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (B) $\theta=\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (C) $\theta=\cos^{-1}\left(\frac{\sqrt{2}}{3}\right)$ (D) none of these

UD0092

MULTIPLE CORRECT TYPE QUESTIONS

18. Four forces acting on a particle keep it in equilibrium, then :-

(A) the force must be coplanar.

(B) the forces cannot be coplanar.

(C) the forces may or may not be coplanar.

(D) if three of these forces are coplanar, so must be the fourth. **UD0093**

19. A man is standing at point (x = 5 m, y = 0). Then he walks along straight line to (x = 0, y = 5m). A second man walks from the same initial position along the x-axis to the origin and then along the y-axis to (x = 0, y = 5m). Mark the **CORRECT** statement(s) :

(A) Displacement vector of first man and second man are equal

(B) Distance travelled by second man is greater

(C) Magnitude of displacement of second man is same that of first man but direction is different

(D) Magnitude of displacement is $\sqrt{50}$ m for 2nd man.

UD0094

20. The vector i + xj + 3k is rotated through an angle θ and doubled in magnitude, then it becomes 4i + (4x – 2) j + 2k. The values of x are

(A) $-\frac{2}{3}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) 2

UD0095

21. The value of $|\vec{A}+\vec{B}-\vec{C}+\vec{D}|$ can be zero if :–

(A) $|\vec{A}|=5, |\vec{B}|=3, |\vec{C}|=4; |\vec{D}|=13$

(B) $|\vec{A}|=2\sqrt{2}, |\vec{B}|=2, |\vec{C}|=2; |\vec{D}|=5$

(C) $|\vec{A}|=2\sqrt{2}, |\vec{B}|=2, |\vec{C}|=2; |\vec{D}|=10$

(D) $|\vec{A}|=5, |\vec{B}|=4, |\vec{C}|=3; |\vec{D}|=8$

UD0096

22. The four pairs of force vectors are given, which pairs of force vectors cannot be added to give a resultant vector of magnitude 10 N?

(A) 2N, 13 N (B) 5N, 16 N

(C) 7N, 8N (D) 100N, 105 N

UD0097

23. Select **CORRECT** statement(s) for three vectors $\vec{a}=-3\hat{i}+2\hat{j}-\hat{k}, \vec{b}=\hat{i}-3\hat{j}+5\hat{k}$ and $\vec{c}=2\hat{i}+\hat{j}-4\hat{k}$

(A) The above vectors can form triangle.

(B) Component of $\vec{a}$ along $\vec{c}$ is 3.

(C) $\vec{a}$ makes angle $\cos^{-1}\sqrt{\frac{2}{7}}$ with y-axis.

(D) A vector having magnitude twice the vector $\vec{a}$ and anti parallel to vector $\vec{b}$ is $\sqrt{\frac{2}{5}}\left(-2\hat{i}+6\hat{j}-10\hat{k}\right)$

UD0098

24. If a vector $\vec{P}$ makes an angle α, β, γ with x, y, z axis respectively then it can be represented as $\vec{P}=P\left[\cos\alpha\hat{i}+\cos\beta\hat{j}+\cos\gamma\hat{k}\right]$. Choose the **CORRECT** option(s) :-

(A) $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$ (B) $\vec{P}.\vec{P}=P^2$

(C) $\vec{P}.(\hat{i}-\hat{k})=P(\cos\alpha-\cos\gamma)$ (D) $\vec{P}.\hat{i}=\cos\alpha$

UD0099

COMPREHENSION TYPE QUESTIONS

Paragraph for Question no. 25 to 27

A boy lost in a jungle finds a note. In the note was written the following things.

Displacements

1. 300 m 53° South of East.
2. 400 m 37° North of East
3. 500 m North
4. 500 $\sqrt{2}$ m North-West
5. 500 m South

He starts walking at constant speed 2 m/s following these displacements in the given order.

25. How far and in which direction is he from the starting point after 5 min. and 50 s?

(A) 500 m due East
(B) 500 m due West
(C) 700 m due South-West
(D) 700 m due North-East

UD0100

26. How far and in which direction is he from the starting point after 10 minutes?

(A) 500 $\sqrt{2}$ m due North(B) 1200 m due North-East
(C) 500$\sqrt{2}$ m due North-East
(D) 900 m due 37° North of East

UD0100

27. How far and in which direction has he finally displaced after all the displacements in the note?

(A) 500$\sqrt{2}$ m due North-East
(B) 500 m due North
(C) 866 m due North-West
(D) 500$\sqrt{3}$ m due 60° North of West

UD0100

Paragraph for Question Nos. 28 to 30

A physical quantity is a physical property of a phenomenon, body, or substance, that can be quantified by measurement.

The magnitude of the components of a vector are to be considered dimensionally distinct. For example, rather than an undifferentiated length unit L, we may represent length in the x direction as L_x, and so forth. This requirement stems ultimately from the requirement that each component of a physically meaningful equation (scalar or vector) must be dimensionally consistent. As an example, suppose we wish to calculate the drift S of a swimmer crossing a river flowing with velocity V_x and of width D and he is swimming in direction perpendicular to the river flow with velocity V_y relative to river, assuming no use of directed lengths, the quantities of interest are then V_x, V_y both dimensioned as $\frac{L}{T}$, S the drift and D width of river both having dimension L. With these four quantities, we may conclude that the equation for the drift S may be written: $S \propto V_x^{\,a} V_y^{\,b} D^c$

Or dimensionally $L = \left(\frac{L}{T}\right)^{a+b} \times (L)^c$ from which we may deduce that $a+b+c=1$ and $a+b=0$, which leaves one of these exponents undetermined. If, however, we use directed length dimensions, then V_x will be dimensioned as $\frac{L_x}{T}$, V_y as $\frac{L_y}{T}$, S as L_x and D as L_y. The dimensional equation becomes : $L_x = \left(\frac{L_x}{T}\right)^a \left(\frac{L_y}{T}\right)^b \left(L_y\right)^c$ and we may solve completely as $a = 1$, $b = -1$ and $c = 1$. The increase in deductive power gained by the use of directed length dimensions is apparent.

28. Which of the following is not a physical quantity

(A) Height of a boy (B) Weight of a boy

(C) Fever of a boy (D) Speed of a running boy

UD0101

29. From the concept of directed dimension what is the formula for a range (R) of a cannon ball when it is fired with vertical velocity component V_y and a horizontal velocity component V_x assuming it is fired on a flat surface. [Range also depends upon acceleration due to gravity, g and k is numerical constant]

(A) $R = \frac{k(V_x V_y)}{g}$ (B) $R = \frac{k(V_x)^2}{g}$ (C) $R = \frac{k(V_x)^3}{V_y g}$ (D) $R = \frac{k(V_y)^3}{V_x g}$

UD0101

30. A conveyer belt of width D is moving along x-axis with velocity V. A man moving with velocity U on the belt in the direction perpendicular to the belt's velocity with respect to belt wants to cross the belt. The correct expression for the drift (S) suffered by man is given by (k is numerical constant)

(A) $S = k\frac{UD}{V}$ (B) $S = k\frac{VD}{U}$ (C) $S = k\frac{U^2D}{V^2}$ (D) $S = k\frac{V^2D}{U^2}$

UD0101

MATCHING LIST TYPE (4 × 4 × 4) MULTIPLE OPTION CORRECT (THREE COLUMNS AND FOUR ROWS)

Answer Q.31, Q.32 and Q.33 by appropriately matching the information given in the three columns of the following table.

L, M and T are units of length, Mass and Time respectively in a system of units.

	Coloumn–1		**Column-2**		**Column-3**
(I)	L = 10 m	(i)	M = 100 gm	(P)	T = 0.1 sec
(II)	L = 10 cm	(ii)	M = 10 kg	(Q)	T = 10 ms
(III)	L = 0.1 mm	(iii)	M = 10 gm	(R)	T = 10 sec
(IV)	L = 1 km	(iv)	M = 1 tonne	(S)	T = 0.01 sec

31. In which of the following combinations unit of force is 10^6 dyne.

(A) (IV) (i) (P) (B) (II) (iii) (S) (C) (III) (iv) (P) (D) (I) (ii) (Q)

UD0102

32. In which of the following system, unit of energy is 10^9 erg

(A) (III) (i) (S) (B) (IV) (iii) (R) (C) (II) (iv) (Q) (D) (I) (iii) (P)

UD0103

33. In which of the following system, unit for coefficient of viscosity is 100 poiseuille

(A) (III) (ii) (S) (B) (II) (i) (Q) (C) (III) (iii) (R) (D) (IV) (iv) (P)

UD0104

MATRIX MATCH TYPE QUESTION

34. In a new system of units known as RMP, length is measured in 'retem', mass is measured in 'marg' and time is measured in 'pal'.

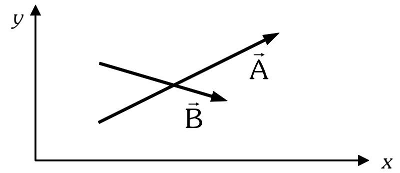
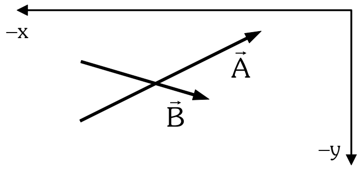
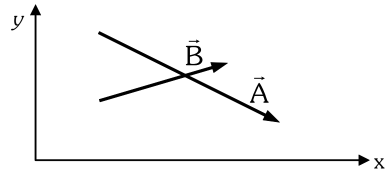
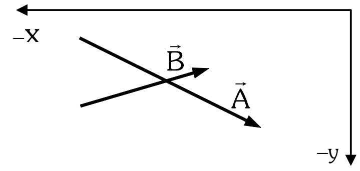
$$100 \text{ retem} = 1.0 \text{ meter}$$
$$1.0 \text{ marg} = 10^{-3} \text{ kilogram}$$
$$10 \text{ pal} = 1.0 \text{ second}$$

In the given table some unit conversion factors are given. Suggest suitable match.

Column-I	**Column-II**
(A) One SI unit of force	(P) 10^2 units of RMP
(B) One SI unit of potential energy	(Q) 10^3 units of RMP
(C) One SI unit of power	(R) 10^4 units of RMP
(D) One SI unit of momentum	(S) 10^5 units of RMP
	(T) 10^6 units of RMP

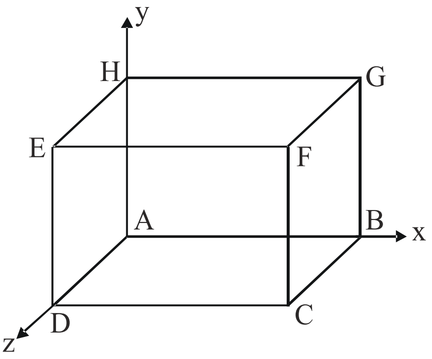
UD0105

35. Refer the following table, where in the first column four pairs of two vectors are shown and in the second column some possible outcomes of basic mathematical operation on these vectors are given. Suggest suitable match(s).

Column-I	**Column-II**
(A) 	(P) X-component of $\vec{A}+\vec{B}$ is positive
(B) 	(Q) Y-component of $\vec{A}+\vec{B}$ is negative
(C) 	(R) X-component of $\vec{A}-\vec{B}$ is positive
(D) 	(S) Y-component of $\vec{A}-\vec{B}$ is negative
	(T) $\vec{B}\cdot\vec{A}$ is positive

UD0106

36. Figure shows a cube of edge length *a*.



	Column-I		**Column-II**
(A)	The angle between AF and x-axis	(P)	60°
(B)	Angle between AF and DG	(Q)	$\cos^{-1}\frac{1}{3}$
(C)	Angle between AE and AG	(R)	$\cos^{-1}\frac{1}{\sqrt{3}}$
		(S)	$\cos^{-1}\sqrt{\frac{2}{3}}$

UD0107

EXERCISE (JM)

1. An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M. The piston and the cylinder have equal cross sectional area A. When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is P_0. The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency. **[JEE Main-2013]**

(1) $\frac{1}{2\pi}\frac{A\gamma P_0}{V_0 M}$ (2) $\frac{1}{2\pi}\frac{V_0 M P_0}{A^2\gamma}$ (3) $\frac{1}{2\pi}\sqrt{\frac{A^2\gamma P_0}{MV_0}}$ (4) $\frac{1}{2\pi}\sqrt{\frac{MV_0}{A\gamma P_0}}$

UD0108

EXERCISE (JA)

1. Match List I with List II and select the correct answer using the codes given below the lists :

	List I		List II
P.	Boltzmann constant	1.	$[ML^2T^{-1}]$
Q.	Coefficient of viscosity	2.	$[ML^{-1}T^{-1}]$
R.	Planck constant	3.	$[MLT^{-3}K^{-1}]$
S.	Thermal conductivity	4.	$[ML^2T^{-2}K^{-1}]$

[JEE Advanced-2013]

Codes :

	P	Q	R	S
(A)	3	1	2	4
(B)	3	2	1	4
(C)	4	2	1	3
(D)	4	1	2	3

UD0109

2. To find the distance d over which a signal can be seen clearly in foggy conditions, a railways engineer uses dimensional analysis and assumes that the distance depends on the mass density ρ of the fog, intensity (power/area) S of the light from the signal and its frequency f. The engineer finds that d is proportional to $S^{1/n}$. The value of n is. **[JEE Advanced-2014]**

UD0110

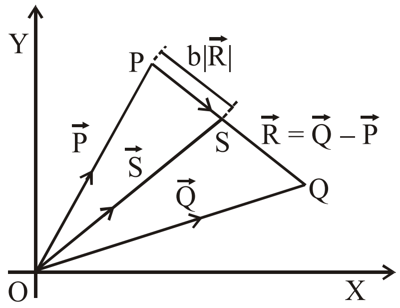
3. In terms of potential difference V, electric current I, permittivity ε_0, permeability μ_0 and speed of light c, the dimensionally correct equation(s) is(are) **[JEE Advanced-2015]**

(A) $\mu_0 I^2 = \varepsilon_0 V^2$ (B) $\varepsilon_0 I = \mu_0 V$ (C) $I = \varepsilon_0 cV$ (D) $\mu_0 cI = \varepsilon_0 V$

UD0111

4. Three vectors $\vec{P}, \vec{Q}$ and $\vec{R}$ are shown in the figure. Let S be any point on the vector $\vec{R}$. The distance between the points P and S is $b\left|\vec{R}\right|$. The general relation among vectors $\vec{P}, \vec{Q}$ and $\vec{S}$ is :

[JEE Advanced-2017]



(A) $\vec{S} = (1-b)\vec{P} + b^2\vec{Q}$ (B) $\vec{S} = (b-1)\vec{P} + b\vec{Q}$

(C) $\vec{S} = (1-b)\vec{P} + b\vec{Q}$ (D) $\vec{S} = (1-b^2)\vec{P} + b\vec{Q}$

UD0112

5. Two vectors $\vec{A}$ and $\vec{B}$ are defined as $\vec{A} = a\hat{i}$ and $\vec{B} = a(\cos\omega t\hat{i} + \sin\omega t\hat{j})$, whre a is a constant and $\omega = \pi/6$ rad s^{-1}. If $|\vec{A}+\vec{B}| = \sqrt{3}|\vec{A}-\vec{B}|$ at time t = τ for the first time, the value of τ, in seconds, is _____. **[JEE Advanced-2018]**

UD0113

PARAGRAPH "X"

In electromagnetic theory, the electric and magnetic phenomena are related to each other. Therefore, the dimensions of electric and magnetic quantities must also be related to each other. In the questions below, [E] and [B] stand for dimensions of electric and magnetic fields respectively, while $[\in_0]$ and $[\mu_0]$ stand for dimensions of the permittivity and permeability of free space respectively. [L] and [T] are dimensions of length and time respectively. All the quantities are given in SI units.

(There are two questions based on Paragraph "X", the question given below is one of them)

6. The relation between [E] and [B] is :- **[JEE Advanced-2018]**

(A) $[E] = [B][L][T]$ (B) $[E] = [B][L]^{-1}[T]$ (C) $[E] = [B][L][T]^{-1}$ (D) $[E] = [B][L]^{-1}[T]^{-1}$

UD0114

PARAGRAPH "X"

In electromagnetic theory, the electric and magnetic phenomena are related to each other. Therefore, the dimensions of electric and magnetic quantities must also be related to each other. In the questions below, [E] and [B] stand for dimensions of electric and magnetic fields respectively, while $[\in_0]$ and $[\mu_0]$ stand for dimensions of the permittivity and permeability of free space respectively. [L] and [T] are dimensions of length and time respectively. All the quantities are given in SI units.

(There are two questions based on Paragraph "X", the question given below is one of them)

7. The relation between $[\in_0]$ and $[\mu_0]$ is :- **[JEE Advanced-2018]**

(A) $[\mu_0] = [\in_0][L]^2[T]^{-2}$ (B) $[\mu_0] = [\in_0][L]^{-2}[T]^2$

(C) $[\mu_0] = [\in_0]^{-1}[L]^2[T]^{-2}$ (D) $[\mu_0] = [\in_0]^{-1}[L]^{-2}[T]^2$

UD0114

8. Let us consider a system of units in which mass and angular momentum are dimensionless. If length has dimension of L, which of the following statement (s) is/are correct ? **[JEE Advanced-2019]**

(1) The dimension of force is L^{-3}

(2) The dimension of energy is L^{-2}

(3) The dimension of power is L^{-5}

(4) The dimension of linear momentum is L^{-1} **UD0115**

ANSWER KEY

EXERCISE (S-1)

1. Ans. L^{-1}, ML^2T^{-2}

2. Ans. $kg.m^2.s^{-2}.K^{-1}$

3. Ans. $T = a\sqrt{\frac{m}{k}}$

4. Ans. $\omega = K\sqrt{\frac{GM}{r^3}}$

5. Ans. $v_0 = k\sqrt{\frac{GM}{R}}$

6. Ans. K = 6

7. Ans. $[MLT^{-1}]$

8. Ans. $\sqrt{75}$

9. Ans. 1

10. Ans. 25

11. Ans. 7

12. Ans. 90 m

13. Ans. $\cos\alpha = \left(\frac{4}{\sqrt{61}}\right)$, $\cos\beta = \left(\frac{6}{\sqrt{61}}\right)$ $\cos\gamma = \left(\frac{3}{\sqrt{61}}\right)$, $magnitude = \sqrt{61}$

14. Ans. 3

15. Ans. 5

16. Ans. $\frac{7}{16}$

17. Ans. $\hat{i} + \hat{j}$, $3\hat{k}$

18. Ans. (a) $11\hat{i} + 5\hat{j} - 7\hat{k}$, (b) $\cos^{-1}\left(\frac{-7}{\sqrt{195}}\right)$, (c) $\cos^{-1}\left(\frac{-20}{\sqrt{1309}}\right)$

19. Ans. 2

20. Ans. (a) $\frac{2\pi}{3} \times 6.4 \times 10^6 m$, (b) $\sqrt{3} \times 6.4 \times 10^6$ m

21. Ans. (a) 9.95, (b) 0.99

22. Ans. 0.14, 0.09

EXERCISE (S-2)

1. Ans. $ML^5T^{-2}K^{1/2}$

2. Ans. $[S] = Ev^{-2}T^{-2}$

3. Ans. 10^{-30} star joule

4. Ans. $2\sqrt{19}$, $\cos^{-1}\frac{7}{2\sqrt{19}}$ or $\tan^{-1}\frac{3\sqrt{3}}{7}$

5. Ans. 100 s

6. Ans. 120 N, $40\sqrt{3}$ N

7. Ans. $16\hat{i} - 30\hat{k}, 198J$

EXERCISE (O-1)

1. Ans. (D) **2. Ans. (C)** **3. Ans. (A)** **4. Ans. (A)** **5. Ans. (B)** **6. Ans. (A)**

7. Ans. (C) **8. Ans. (D)** **9. Ans. (D)** **10. Ans. (A)** **11. Ans. (C)** **12. Ans. (B)**

13. Ans. (C) **14. Ans. (D)** **15. Ans. (B)** **16. Ans. (B)** **17. Ans. (D)** **18. Ans. (D)**

19. Ans. (C) **20. Ans. (A)** **21. Ans. (C)** **22. Ans. (A)** **23. Ans. (B)** **24. Ans. (C)**

25. Ans. (B) **26. Ans. (B)** **27. Ans. (D)** **28. Ans. (D)** **29. Ans. (A)** **30. Ans. (C)**

31. Ans. (B) **32. Ans. (A)** **33. Ans. (C)** **34. Ans. (B)** **35. Ans. (B)** **36. Ans. (B)**

37. Ans. (D) **38. Ans. (A,D)** **39. Ans. (C,D)** **40. Ans. (B, C)** **41. Ans. (B)** **42. Ans. (C)**

43. Ans. (A) **44. Ans. (B)** **45. Ans. (C)**

46. Ans. (A) → (Q); (B) → (R); (C) → (T); (D) → (P)

47. Ans. (A)-R; (B)-S; (C)-P; (D)-Q

48. Ans. (A) → (S); (B) → (P); (C) → (R); (D) → (T)

49. Ans. (A)-S; (B)-P; (C)-Q; (D)-R

EXERCISE (O-2)

1. Ans. (A) **2. Ans. (D)** **3. Ans. (C)** **4. Ans. (B)** **5. Ans. (C)** **6. Ans. (A)**
7. Ans. (A) **8. Ans. (D)** **9. Ans. (A)** **10. Ans. (C)** **11. Ans. (C)** **12. Ans. (B)**
13. Ans. (A) **14. Ans. (A)** **15. Ans. (A)** **16. Ans. (B)** **17. Ans. (C)** **18. Ans. (C, D)**
19. Ans. (A,B,D) **20. Ans. (A, D)** **21. Ans. (B,D)** **22. Ans. (A,B)** **23. Ans. (A,C,D)**
24. Ans. (A, B, C) **25. Ans. (A)** **26. Ans. (C)** **27. Ans. (B)** **28. Ans. (C)**
29. Ans. (A) **30. Ans. (B)** **31. Ans. (B,C)** **32. Ans. (B,D)** **33. Ans. (B)**
34. Ans. (A) → (Q); (B) → (S); (C) → (R); (D) → (R)
35. Ans. (A) → (P,R,T); (B) → (P,R,T); (C) → (P,Q,R,S,T); (D) → (P,Q,R,S,T)
36. Ans. (A) R (B) Q (C) P

EXERCISE (JM)

1. Ans. (3)

EXERCISE (JA)

1. Ans. (C) **2. Ans. 3** **3. Ans. (A,C)** **4. Ans. (C)** **5. Ans. 2 [1.99, 2.01]**
6. Ans. (C) **7. Ans. (D)** **8. Ans. (1,2,4)**

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S. No.

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1	NEWTON'S LAWS OF MOTION & FRICTION	01-70
2	KINEMATICS-1D & CALCULUS	71-120
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PHYSICS

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Chapter 01
Contents

NEWTON'S LAWS OF MOTION & FRICTION

Important Notes

NEWTON'S LAWS OF MOTION & FRICTION

KEY CONCEPTS

FORCE

A push or pull that one object exerts on another.

FORCES IN NATURE

There are four fundamental forces in nature :

1. Gravitational force
2. Electromagnetic force
3. Strong nuclear force
4. Weak force

TYPES OF FORCES ON MACROSCOPIC OBJECTS

(a) Field Forces or Range Forces :

These are the forces in which contact between two objects is not necessary.

Ex. (i) Gravitational force between two bodies. (ii) Electrostatic force between two charges.

(b) Contact Forces :

Contact forces exist only as long as the objects are touching each another.

Ex. (i) Normal forces. (ii) Frictional force

(c) Attachment to Another Body :

Tension (T) in a string and spring force (F = kx) comes in this group.

NEWTON'S FIRST LAW OF MOTION (OR GALILEO'S LAW OF INERTIA)

Every body continues in its state of rest or uniform motion in a straight line unless compelled by an external force to change that state.

Definition of force from Newton's first law of motion *"Force is that push or pull which changes or tends to change the state of rest or of uniform motion in a straight line".*

Inertia : Inertia is the property of the body due to which body oppose the change of its state. Inertia of a body is measured by mass of the body. $\boxed{\text{inertia} \propto \text{mass}}$

TYPES OF INERTIA

Inertia of rest : It is the inability of a body to change, its state of rest by itself.

Examples :

- When we shake a branch of a mango tree, the mangoes fall down.
- When a bus or train starts suddenly the passengers sitting inside tends to fall backwards.
- The dust particles in a blanket fall off when it is beaten with a stick.
- When a stone hits a window pane, the glass is broken into a number of pieces whereas if the high speed bullet strikes the pane, it leaves a clean hole.

Inertia of motion : It is the inability of a body to change its state of uniform motion by itself.

Examples :

- When a bus or train stop suddenly, a passenger sitting inside tends to fall forward.
- A person jumping out of a speeding train may fall forward.
- A ball thrown upwards in a running train continues to move along with the train.

Inertia of direction : It is the inability of a body to change its direction of motion by itself.

Examples :

- When a car rounds a curve suddenly, the person sitting inside is thrown outwards.
- Rotating wheels of vehicle throw out mud, mudguard over the wheels stop this mud.
- A body released from a balloon rising up, continues to move in the direction of balloon.

NEWTON'S SECOND LAW OF MOTION

Rate of change of momentum of a body is directly proportional to the external force applied on it and the change in momentum takes place in the direction of force

$$\vec{F} \propto \frac{d\vec{p}}{dt} \quad \text{or} \quad \vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$$

if m = constant then $\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a}$

- Newton's Second Law Provides the Definition of the Concept of Force.
- **Definition of the 1 Newton (N) :–** If an object of mass one kilogram has an acceleration of $1\,ms^{-2}$ relative to an inertial reference frame, then the net force exerted on the object is one newton.

CONSEQUENCES OF NEWTON'S II LAW OF MOTION

- **Concept of inertial mass :** From Newton's II law of motion $a = \frac{F}{M}$

  i.e., the magnitude of acceleration produced by a given body is inversely proportional to mass i.e. greater the mass, smaller is the acceleration produced in the body. Thus, mass is the measure of inertia of the body. The mass given by above equation is therefore called the inertial mass.

- **An accelerated motion is the result of application of the force :**

  There may be two types of accelerated motion :

(i) When only the magnitude of velocity of the body changes : In this types of motion the force is applied along the direction of motion or opposite to the direction of motion.

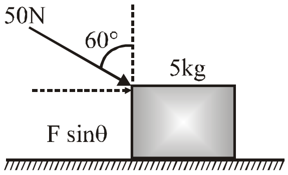
(ii) When only the direction of motion of the body changes : In this case the force is applied at right angles to the direction of motion of the body, e.g. uniform circular motion.

- **Acceleration produced in the body depends only on its mass and not on the final or initial velocity.**

Ex. A force $\vec{F} = (6\hat{i} - 8\hat{j} + 10\hat{k})\,N$ produces acceleration of 1 ms^{-2} in a body. Calculate the mass of the body.

Sol. $\because$ Acceleration $a = \frac{|\vec{F}|}{m}$ $\quad \therefore$ mass $m = \frac{|\vec{F}|}{a} = \frac{\sqrt{6^2 + 8^2 + 10^2}}{1} = 10\sqrt{2}$ kg

Ex. A force of 50 N acts in the direction as shown in figure. The block of mass 5kg, resting on a smooth horizontal surface. Find out the acceleration of the block.



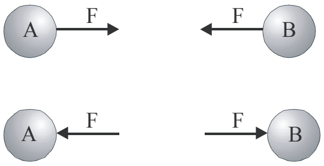
Sol. Horizontal component of the force = Fsinθ = 50sin60° = $\frac{50\sqrt{3}}{2}$N

acceleration of the block $a = \frac{F\sin\theta}{m} = \frac{50\sqrt{3}}{2} \times \frac{1}{5} = 5\sqrt{3}\ \text{m/s}^2$

NEWTON'S THIRD LAW OF MOTION

The first and second laws are statements about a single object, whereas the third law is a statement about two objects.

- According to this law, every action has equal and opposite reaction. Action and reaction act on different bodies and they are simultaneous. There can be no reaction without action.
- If an object A exerts a force F on an object B, then B exerts an equal and opposite force (–F) on A.



The forces between two objects A and B are equal and opposite, whether they are attractive or repulsive.

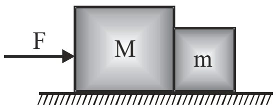
- Action and reaction never cancel each other, since they act on different bodies.

- **First law** : If no net force acts on a particle, then it is possible to select a set of reference frames, called inertial reference frames, observed from which the particle moves without any change in velocity.
- **Second law** : Observed from an inertial reference frame, the net force on a particle is proportional to the time rate of change of its linear momentum: $\frac{d(m\vec{v})}{dt}$
- **Third law** : Whenever a particle A exerts a force on another particle B, B simultaneously exerts a force on A with the same magnitude in the opposite direction.

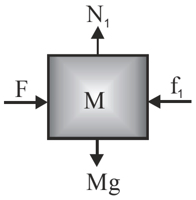
FREE BODY DIAGRAM

A diagram showing all external forces acting on an object is called "Free Body Diagram" (F.B.D.) In a specific problem, first we are required to choose a body and then we find the number of forces acting on it, and all the forces are drawn on the body, considering it as a point mass. The resulting diagram is known as free body diagram (FBD).

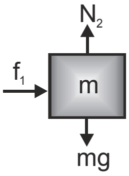
For example, if two bodies of masses m and M are in contact and a force F on M is applied from the left as shown in figure (a), the free body diagrams of M and m will be as shown in figure (b) and (c).



(a)



(b)



(c)

Important Point :

Two forces in Newton's third law never occur in the same free–body diagram. This is because a free–body diagram shows forces acting on a single object, and the action–reaction pair in Newton's third law always act on different objects.

MOTION OF BODIES IN CONTACT

Case I :

When two bodies of masses m_1 and m_2 are kept on the frictionless surface and a force F is applied on one body, then the force with which one body presses the other at the point of contact is called Force of Contact. These two bodies will move with same acceleration a.

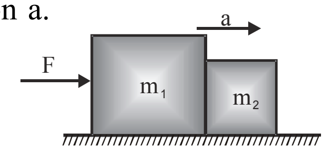


Fig.(1) : When the force F acts on mass m_1.

(i) When the force F acts on the body with mass m_1 as shown in fig. (1) $F = (m_1 + m_2)\ a$.

If the force exerted by m_2 on m_1 is f_1 (force of contact) then for body m_1 : $(F - f_1) = m_1\ a$

for body m_2 : $f_1 = m_2 a$

$\Rightarrow$ action of m_1 on m_2 : $f_1 = \dfrac{m_2 F}{m_1 + m_2}$

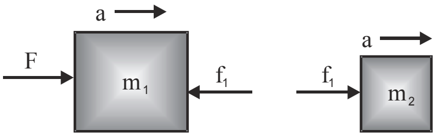


Fig. 1(a) : F.B.D. representation of action and reaction forces.

(ii) When the force F acts on the body with mass m_2 as shown in figure 2

$F = (m_1 + m_2)\ a$

for body with mass m_2

$F - f_2 = m_2\ a$

for body m_1, $f_2 = m_1 a$

$\Rightarrow$ action on m_1 , $f_2 = \dfrac{m_1 F}{m_1 + m_2}$

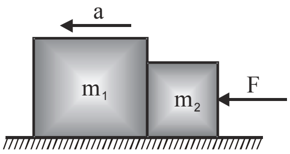


Fig. (2) : When the force F acts on mass m_2.

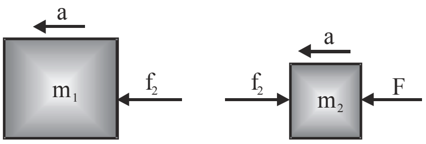
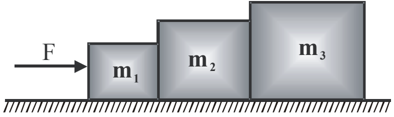


Fig. 2 (a) : F.B.D. representation of action and reaction forces.

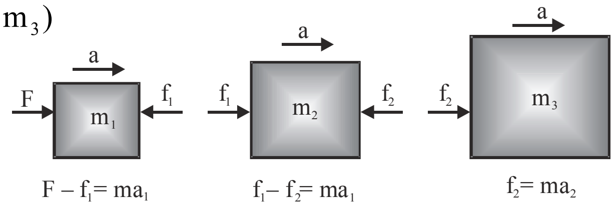
Case II :

Three bodies of masses m_1, m_2 and m_3 placed one after another and in contact with each other. Suppose a force F is applied horizontally on mass m_1



then $F = (m_1 + m_2 + m_3)\ a \Rightarrow a = \dfrac{F}{(m_1 + m_2 + m_3)}$

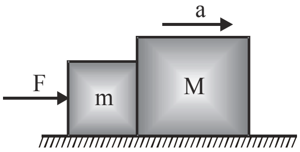
$$f_1 = \frac{(m_2 + m_3)F}{(m_1 + m_2 + m_3)} \text{ (action on both } m_2 \text{ and } m_3)$$



and $f_2 = \dfrac{m_3 F}{(m_1 + m_2 + m_3)}$ (action on m_3 alone) when the force F is applied on m_3, then

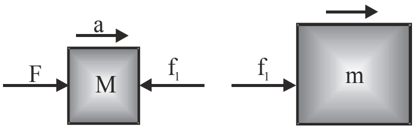
$$f_1 = \frac{m_1F}{(m_1+m_2+m_3)} \text{ (action on } m_1 \text{ alone) and } f_2 = \frac{(m_1+m_2)F}{(m_1+m_2+m_3)} \text{ (action on } m_1 \text{ and } m_2)$$

Ex. Two blocks of mass m = 2 kg and M = 5 kg are in contact on a frictionless table. A horizontal force F (= 35 N) is applied to m. Find the force of contact between the block, will the force of contact remain same if F is applied to M ?

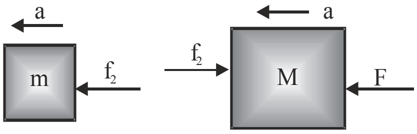


Sol. As the blocks are rigid under the action of a force F, both will move with same acceleration

$$a = \frac{F}{m+M} = \frac{35}{2+5} = 5 \text{ m/s}^2$$



force of contact $f_1 = Ma = 5 \times 5 = 25$ N

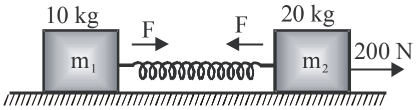


If the force is applied to M then its action on m will be

$f_2 = ma = 2 \times 5 = 10$ N.

From this problem it is clear that acceleration does not depends on the fact that whether the force is applied to m or M, but force of contact does.

Ex. Two masses 10 kg and 20 kg respectively are connected by a massless spring as shown in figure force of 200 N acts on the 20 kg mass. At the instant shown in figure the 10 kg mass has acceleration of 12 m/s², what is the acceleration of 20 kg mass?



Sol. Equation of motion for m_1 is $F = m_1a_1 = 10 \times 12 = 120$ N.

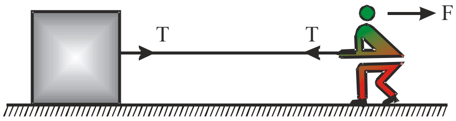
Force on 10 kg–mass is 120 N to the right. As action and reaction are equal and opposite, the reaction force F on 20 kg mass F = 120 N to the left.

∴ Equation of motion for m_2 is $200 - F = 20\,a_2$

$$\Rightarrow 200 - 120 = 20\,a_2 \qquad \Rightarrow 20\,a_2 = 80 \qquad \Rightarrow a_2 = \frac{80}{20} = 4 \text{ ms}^{-2}$$

SYSTEM OF MASSES TIED BY STRINGS

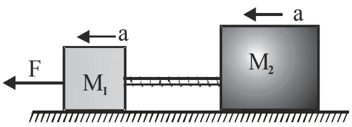
Tension in a String :



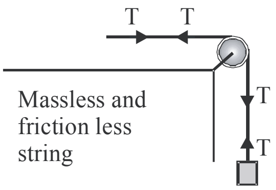
It is an intermolecular force between the atoms of a string, which acts or reacts when the string is streched.

Important points about the tension in a string :

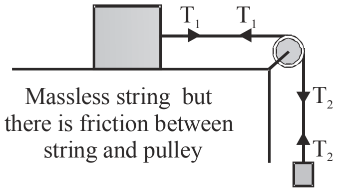
- Force of tension act on a body in the direction away from the point of contact or tied ends of the string.
- String is assumed to be inextensible so that the magnitude of accelerations of any number of masses connected through strings is always same.



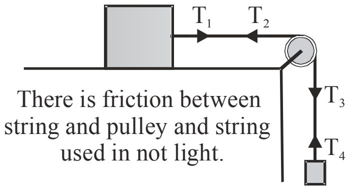
- If the string is extensible the acceleration of different masses connected through it will be different until the string can stretch.
- String is massless and frictionless so that tension throughout the string remains same.



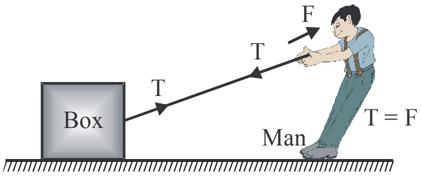
- If the string is massless but not frictionless, at every contact tension changes.



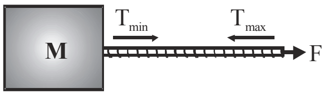
- If the string is not light, tension at each point will be different depending on the acceleration of the string.



- If a force is directly applied on a string as say man is pulling a tied string from the other end with some force the tension will be equal to the applied force irrespective of the motion of the pulling agent, irrespective of whether the box will move or not, man will move or not.

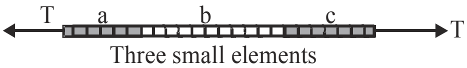


- String is assumed to be massless unless stated, hence tension in it everywhere remains the same and equal to applied force. However, if a string has a mass, tension at different points will be different being maximum (= applied force) at the end through which force is applied and minimum at the other end connected to a body.

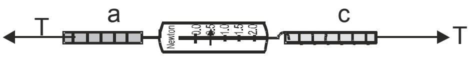


- In order to produce tension in a string two equal and opposite stretching forces must be applied. The tension thus produced is equal in magnitude to either applied force (i.e., T = F) and is directed inwards opposite to F. Here it must be noted that a string can never be compressed like a spring.

- If string is cut so that element b is replaced by a spring scale (the rest of the string being undisturbed), the scale reads the tension T.



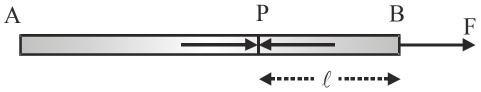
Three small elements of a string, a, b, c



- Every string can bear a maximum tension, i.e. if the tension in a string is continuously increased it will break if the tension is increased beyond a certain limit. The maximum tension which a string can bear without breaking is called "breaking strength". It is finite for a string and depends on its material and dimensions.

Ex. A uniform rope of length L is pulled by a constant force F. What is the tension in the rope at a distance ℓ from the end where it is applied?

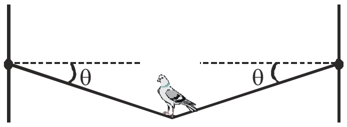
Sol. Let mass of rope is M and T be tension in the rope at point P, then. Acceleration of rope, $a = \frac{F}{M}$



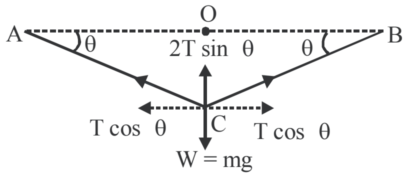
Equation of motion of part PB is F – T = $(m\ell)$ a

$$\Rightarrow T = F - (m\ell)\,a = F - \left(\frac{M}{L}\right)(\ell)\left(\frac{F}{M}\right) = \left[1 - \frac{\ell}{L}\right] F$$

Ex. A bird with mass m perches at the middle of a stretched string Show that the tension in the string is give by $T = \frac{mg}{2\sin\theta}$. Assume that each half of the string is straight.



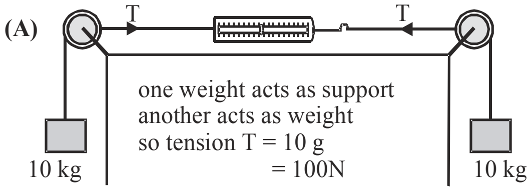
Sol. Initial position of wire = AOB. Final position of wire = ACB Let θ be the angle made by wire with horizontal, which is very small. Resolving tension T of string in horizontal and vertical directions, we note that the horizontal components cancel while vertical components add and balance the weight.



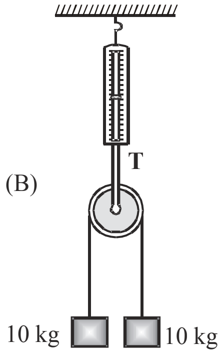
For equilibrium 2T sinθ = W = mg $\Rightarrow T = \frac{W}{2\sin\theta}$ = mg/2sinθ

Ex. The system shown in figure are in equilibrium. If the spring balance is calibrated in newtons, what does it record in each case? (g = 10 m/s²)

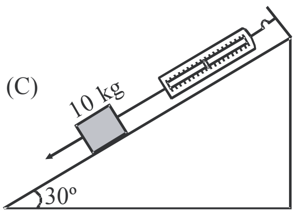
Sol.



one weight acts as support
another acts as weight
so tension T = 10 g
= 100N



T = 2×10×g = 2×10×10 = 200N



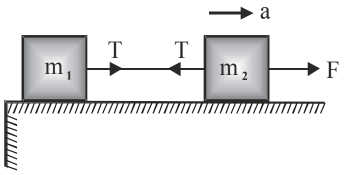
T = 10×10 sin30°
= 10 × 10 × ½ = 50 N

MOTION OF BODIES CONNECTED BY STRINGS

Case : I

Two bodies :

Let us consider the case of two bodies of masses m_1 and m_2 connected by a thread and placed on a smooth horizontal surface as shown in figure. A force F is applied on the body of mass m_2 in forward direction as shown. Our aim is to consider the acceleration of the system and the tension T in the thread. The forces acting separately on two bodies are also shown in the figure:

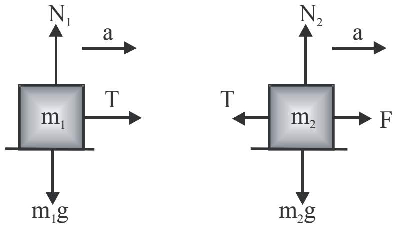


From figure $T = m_1a$

and $F - T = m_2a$

$\Rightarrow$ $F = (m_1 + m_2)a$

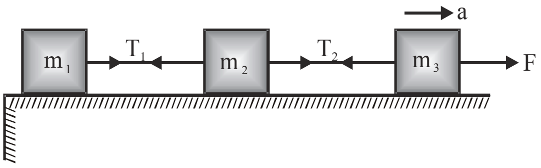
$\Rightarrow$ $a = \dfrac{F}{m_1+m_2}$ & $T = \dfrac{m_1F}{m_1+m_2}$



Case II:

Three bodies

In case of three bodies, the situation is shown in figure

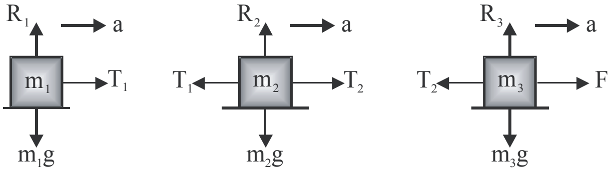


Acceleration $a = \dfrac{F}{m_1+m_2+m_3}$,

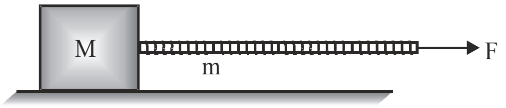
$$T_1 = m_1a = \frac{m_1F}{m_1+m_2+m_3}$$

$\because$ for block of mass m_3 $F - T_2 = m_3a$

$$\therefore\ T_2 = F - \frac{m_3F}{m_1+m_2+m_3} = \frac{(m_1+m_2)F}{m_1+m_2+m_3}$$



Ex. A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m as shown in fig. A horizontal force F is applied to one end of the rope. Find (i) The acceleration of the rope and block (ii) The force that the rope exerts on the block. (iii) Tension in the rope at its mid point.

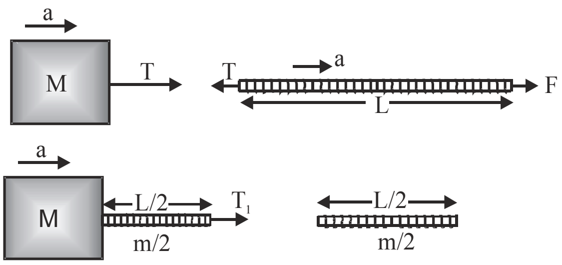


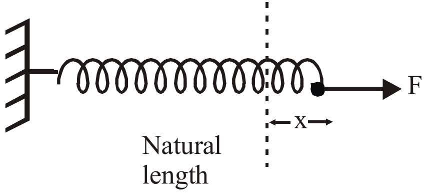
Sol. (i) Accelaration $a = \dfrac{F}{(m+M)}$

(ii)Force exerted by rope $T = Ma = \dfrac{M.F}{(m+M)}$

(iii) $T_1 = \left(\dfrac{m}{2}+M\right)\ a = \left(\dfrac{m+2M}{2}\right)\left(\dfrac{F}{m+M}\right)$

Tension in rope at midpiont $T_1 = \dfrac{(m+2M)F}{2(m+M)}$





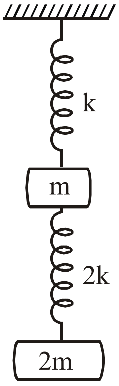
Spring Force (According to Hooke's law) :

In equilibrium $F = kx$

k is spring constant

Note : Spring force is non impulsive in nature.

Ex. If the lower spring is cut, find acceleration of the blocks, immediately after cutting the spring.

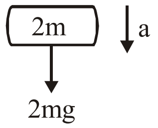


Sol. Intial stretches

$$x_{upper} = \frac{3mg}{k} \quad \& \quad x_{lower} = \frac{mg}{k}$$

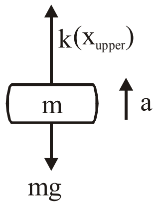
On cutting the lower spring, by virture of non-impulsive nature of spring the stretch in upper spring remains same. Thus,

Lower block :



$$2mg = 2ma \Rightarrow a = g$$

Upper block :



$$k\left(\frac{3mg}{k}\right) - mg = ma \Rightarrow a = 2\,g$$

FRAME OF REFERENCE

It is a conveniently chosen co–ordinate system which describes the position and motion of a body in space.

INERTIAL AND NON–INERTIAL FRAMES OF REFERENCE

Inertial frames of reference :

A reference frame which is either at rest or in uniform motion along the straight line. A non–accelerating frame of reference is called an inertial frame of reference.

- All the fundamental laws of physics have been formulated in respect of inertial frame of reference.
- All the fundamental laws of physics can be expressed as to have the same mathematical form in all the inertial frames of reference.
- The mechanical and optical experiments performed in an inertial frame in any direction will always yield the same results. It is called isotropic property of the inertial frame of reference.

Examples of inertial frames of reference :

- A frame of reference remaining fixed w.r.t. distant stars is an inertial frame of reference.
- A space–ship moving in outer space without spinning and with its engine cut–off is also inertial frame of reference.
- For practical purposes, a frame of reference fixed to the earth can be considered as an inertial frame. Strictly speaking, such a frame of reference is not an inertial frame of reference, because the motion of earth around the sun is accelerated motion due to its orbital and rotational motion. However, due to negligibly small effects of rotation and orbital motion, the motion of earth may be assumed to be uniform and hence a frame of reference fixed to it may be regarded as inertial frame of reference.

Non–inertial frame of reference :

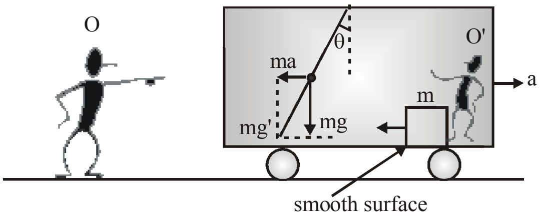
An accelerating frame of reference is called a non–inertial frame of reference.

Newton's laws of motion are not directly applicable in such frames, before application we must add pseudo force.

Note : A rotating frame of references is a non–inertial frame of reference, because it is also an accelerating one due to its centripetal acceleration.

PSEUDO FORCE

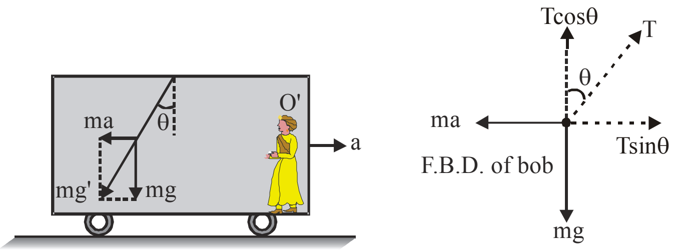
The force on a body due to acceleration of non–inertial frame is called fictitious or apparent or pseudo force and is given by $\vec{F} = -m\vec{a}_0$, where $\vec{a}_0$ is acceleration of non–inertial frame with respect to an inertial frame and m is mass of the particle or body. The direction of pseudo force must be opposite to the direction of acceleration of the non–inertial frame.



For observer O on ground train is moving with acceleration on "a" for observer O' in side the train there is pseudo force in opposite direction shown in figure.

- When we draw the free body diagram of a mass, with respect to an **inertial frame of reference** we apply only the real forces (forces which are actually acting on the mass).
- But when the free body diagram is drawn from a non–inertial frame of reference a pseudo force (in addition to all real forces) has to be applied to make the equation $\vec{F} = m\vec{a}$ to be valid in this frame also.

Ex. A pendulum of mass m is suspended from the ceiling of a train moving with an acceleration 'a' as shown in figure. Find the angle θ in equilibrium position.



Sol. Non–inertial frame of reference (Train)

F.B.D. of bob w.r.t. train. (real forces + pseudo force) : with respect to train, bob is in equilibrium

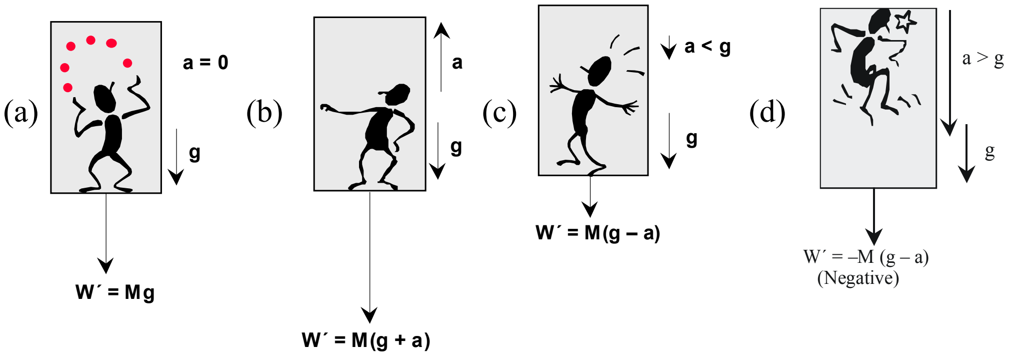
$\therefore\ \Sigma F_y = 0 \Rightarrow T\cos\theta = mg$ and

$\Sigma F_x = 0 \Rightarrow T\sin\theta = ma$

$$\Rightarrow\ \tan\theta = \frac{a}{g}\ \Rightarrow\ \theta = \tan^{-1}\left(\frac{a}{g}\right)$$

MOTION IN A LIFT

The weight of a body is simply the force exerted by earth on the body. If body is on an accelerated platform, the body experiences fictitious force, so the weight of the body appears changed and this new weight is called apparent weight. Let a man of weight W = Mg be standing in a lift. We consider the following cases :



Case (a) :

If the lift moving with constant velocity v upwards or downwards.

In this case there is no accelerated motion hence no pseudo force experienced by observer inside the lift.

So apparent weight W′ = Mg (Actual weight).

Case (b) :

If the lift is accelerated upward with constant acceleration a.

Then net forces acting on the man are (i) weight W = Mg downward (ii) fictitious force $F_0 = Ma$ downward.

So apparent weight $W' = W + F_0 = Mg + Ma = M(g + a)$

Case (c) :

If the lift is accelerated downward with acceleration a < g

Then fictitious force $F_0 = Ma$ acts upward while weight of man W = Mg always acts downward.

So apparent weight $W' = W + F_0 = Mg - Ma = M(g - a)$

Special Case : If a = g then W′ = 0 (condition of weightlessness).

Thus, in a freely falling lift the man will experience weightlessness.

Case (d) :

If lift accelerates downward with acceleration a > g :

Then as in Case c . Apparent weight W′ = M(g – a) is negative, i.e., the man will be accelerated upward and will stay at the ceiling of the lift.

Ex. A spring weighing machine inside a stationary lift reads 50 kg when a man stands on it. What would happen to the scale reading if the lift is moving upward with (i) constant velocity, and (ii) constant acceleration?

Sol. (i) In the case of constant velocity of lift, there is no fictitious force; therefore the apparent weight = actual weight. Hence the reading of machine is 50 kgwt.

(ii) In this case the acceleration is upward, the fictitious force ma acts downward,therefore apparent weight is more than actual weight i.e. W′= m (g + a).

Hence scale shows a reading $= m(g + a) = \dfrac{mg\left(1+\dfrac{a}{g}\right)}{g} = \left(50+\dfrac{50a}{g}\right)$ kgwt.

Ex. Two objects of equal mass rest on the opposite pans of an arm balance. Does the scale remain balanced when it is accelerated up or down in a lift?

Sol. Yes, since both masses experience equal fictitious forces in magnitude as well as direction.

Ex. A passenger on a large ship sailing in a quiet sea hangs a ball from the ceiling of her cabin by means of a long thread. Whenever the ship accelerates, she notes that the pendulum ball lags behind the point of suspension and so the pendulum no longer hangs vertically. How large is the ship's acceleration when the pendulum stands at an angle of 5° to the vertical?

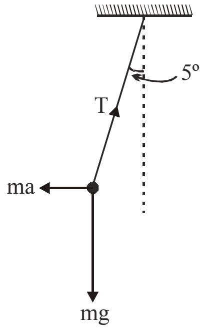
Sol. The ball is accelerated by the force T sin5°.

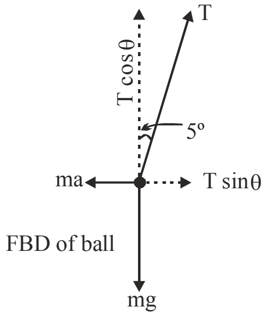
Therefore $T \sin 5° = ma$.

Vertical component $\Sigma F = 0$, so $T \cos 5° = mg$.

By solving $a = g \tan 5° = 0.0875\ g$

$= 0.86\ ms^{-2}$.





Ex. A 12 kg monkey climbs a light rope as shown in figure. The rope passes over a pulley and is attached to a 16 kg bunch of bananas. Mass and friction in the pulley are negligible so that the pulley's only effect is to reverse the direction of the rope. What is the maximum acceleration the monkey can have without lifting the bananas?

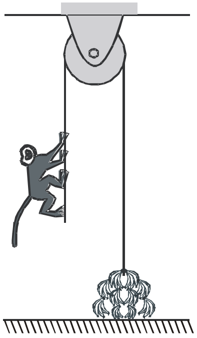
Sol. Effective weight of monkey $W_m = M_m(g + a)$

As per given condition $W_m = W_b$

$\Rightarrow \quad M_m (g + a) = M_b\, g$

$$\Rightarrow \quad a = \frac{(M_b - M_m)\ g}{M_m} = \left(\frac{16-12}{12}\right) \times 9.8$$

$$= \frac{9.8}{3} = 3.26\ \text{m/s}^2$$

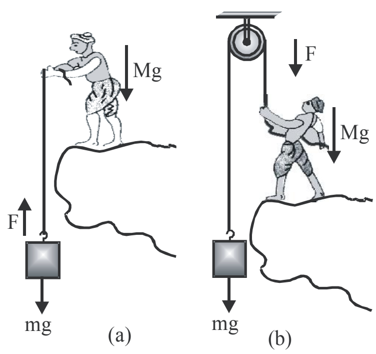


PULLEY SYSTEM

A single fixed pulley changes the direction of force only and in general, assumed to be massless and frictionless.

It is clear from example given below.

Ex. A block of mass 25 kg is raised by a 50 kg man in two different ways as shown in figure. What is the action on the floor by the man in the two cases ? If the floor yields to a normal force of 700 N, which mode should be the man adopt to lift the block without the floor yielding ?



Sol. Mass of the block, m = 25 kg ; mass of the man, M = 50 kg

Force applied to lift the block $F = mg = 25 \times 9.8 = 245$ N

Weight of the man, $M g = 50 \times 9.8 = 490$ N

(a) When the block is raised by the man by applying force F in upward direction, reaction equal and opposite to F will act on the floor in addition to the weight of the man.

$\therefore$ action on the floor $\quad Mg + F = 490 + 245 = 735$ N

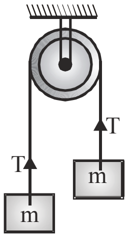
(b) When the block is raised by the mass applying force F over the rope (passed over the pulley) in downward direction, reaction equal and opposite to F will act on the floor,

$\therefore$ action on the floor $\quad Mg - F = 490 - 425 = 245$ N

floor yields to a normal force of 700 N, the mode (b) should be adopted by the man to lift block.

Some cases of pulley

I Case



$m_1 = m_2 = m$

Tension in the string $T = mg$

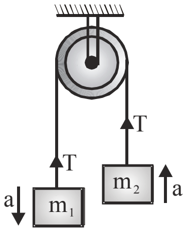
Acceleration 'a' = zero

Reaction at the suspension of the pulley

$R = 2T = 2$ mg.

$$\text{Acceleration} = \frac{\text{net pulling force}}{\text{total mass to be pulled}}$$

II Case



$m_1 > m_2$

now for mass m_1, $m_1g - T = m_1a$

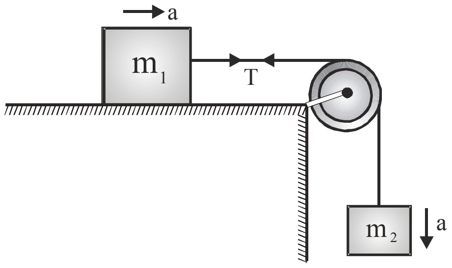
for mass m_2, $T - m_2g = m_2a$

$$a = \frac{(m_1 - m_2)}{(m_1 + m_2)}g \text{ and } T = \frac{2m_1m_2}{(m_1 + m_2)}g$$

$$\text{Tension} = \frac{2 \times \text{Product of masses}}{\text{Sum of two masses}}g$$

Reaction at the suspension of pulley $R = 2T = \dfrac{4m_1m_2\,g}{(m_1 + m_2)}$

III Case :



For mass m_1 : $T = m_1 a$

For mass m_2 : $m_2g - T = m_2 a$

$$\text{acceleration } a = \frac{m_2g}{(m_1 + m_2)} \text{ and } T = \frac{m_1m_2}{(m_1 + m_2)}g$$

IV Case :

$(m_1 > m_2)$

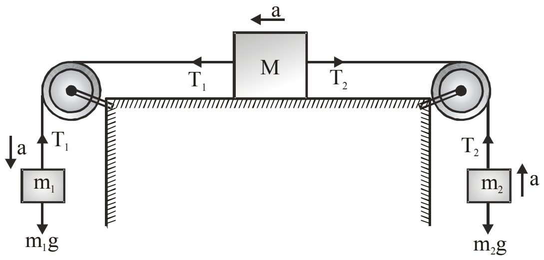
For m_1, $m_1g - T_1 = m_1a$

For m_2, $T_2 - m_2g = m_2a$

For M $T_1 - T_2 = Ma$

$$\Rightarrow a = \frac{(m_1 - m_2)}{(m_1 + m_2 + M)}g,$$

$$T_1 = \frac{(2m_2 + M)m_1g}{m_1 + m_2 + M}, \; T_2 = \frac{(2m_1 + M)m_2g}{m_1 + m_2 + M}$$



V Case :

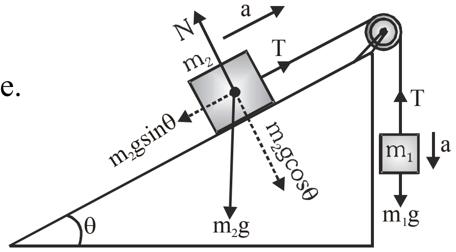
Mass suspended over a pulley from another on an inclined plane.

For mass m_1 : $m_1g - T = m_1 a$

For mass m_2 : $T - m_2g\sin\theta = m_2 a$

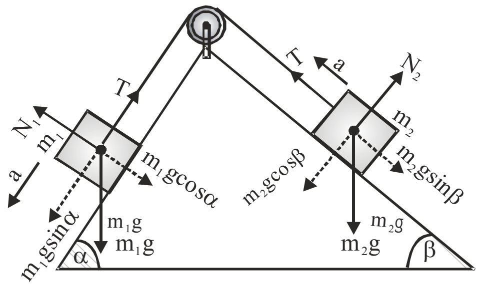
$$\text{acceleration } a = \frac{(m_1 - m_2\sin\theta)}{(m_1 + m_2)}g$$

$$T = \frac{m_1m_2(1 + \sin\theta)}{(m_1 + m_2)}g$$



VI Case :

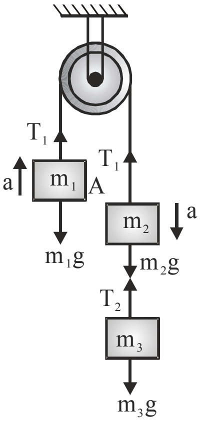
Masses m_1 and m_2 are connected by a string passing over a pulley ($m_1 > m_2$)



Acceleration $a = \frac{(m_1 \sin\alpha - m_2 \sin\beta)}{(m_1 + m_2)} g$

Tension $T = \frac{m_1 m_2 (\sin\alpha + \sin\beta)}{(m_1 + m_2)} g$

VII Case :



For mass m_1 : $T_1 - m_1 g = m_1 a$

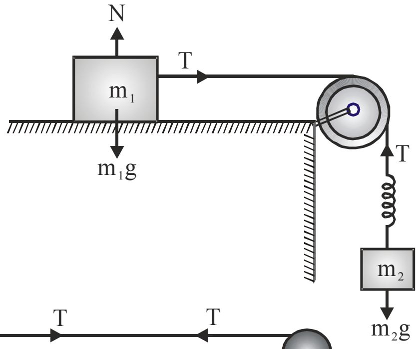
For mass m_2 : $m_2 g + T_2 - T_1 = m_2 a$

For mass m_3 : $m_3 g - T_2 = m_3 a$

Acceleration $a = \frac{(m_2 + m_3 - m_1)}{(m_1 + m_2 + m_3)} g$

we can calculate tensions T_1 and T_2 from above equations

VIII Case :



From case (iii)

tension $T = \frac{m_1 m_2}{(m_1 + m_2)} g$

If x is the extension in the spring,

then $T = kx$

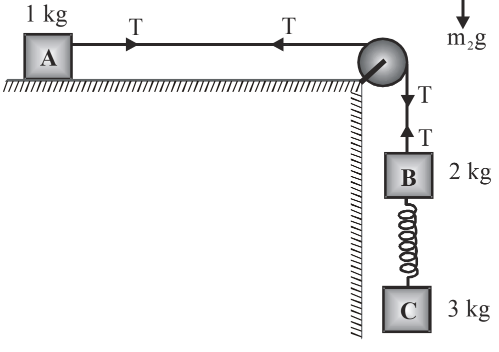
$x = \frac{T}{k} = \frac{m_1 m_2 g}{k(m_1 + m_2)}$

Ex. In the system shown in figure all surface are smooth, string is massless and inextensible. Find:

(a) acceleration of the system

(b) tension in the string and

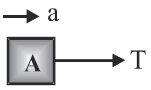
(c) extension in the spring if force constant of spring is $k = 50$ N/m (Take $g = 10$ m/s^2)



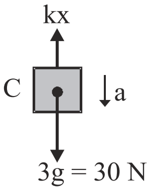
Sol. (a) In this case net pulling force $= m_c g + m_B g = 50$ N and total mass to be pulled is (1 + 2 + 3) kg = 6 kg.

$\therefore$ Acceleration of the system is $a = \frac{50}{6}$ ms^{-2}

(b) Free body diagram of 1 kg block gives $T = ma = (1)\left(\frac{50}{6}\right) N = \frac{50}{6} N$



(c) Free body diagram of 3 kg block gives



$30 - kx = ma$ but $ma = 3 \times \frac{50}{6} = 25N$

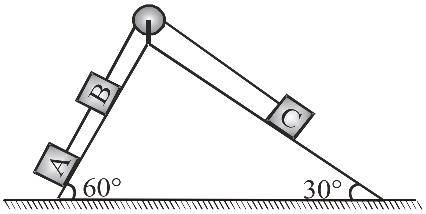
$x = \frac{30-25}{k} = \frac{5}{50} = 0.1$ m $= 10$ cm

Ex. In the adjacent figure, masses of A, B and C are 1 kg, 3 kg and 2 kg respectively.

Find : (a) the acceleration of the system and

(b) tensions in the string

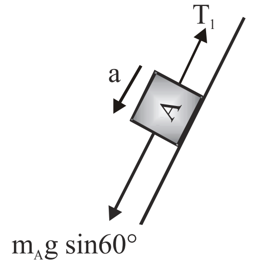
Neglect friction. ($g = 10\ ms^{-2}$)



Sol. (a) In this case net pulling force

$$= m_A g \sin 60° + m_B g \sin 60° - m_C g \sin 30°$$
$$= (m_A + m_B)\, g \sin 60° - m_C\, g \sin 30°$$
$$= (1 + 3) \times 10 \times \frac{\sqrt{3}}{2} - 2 \times 10 \times \frac{1}{2}$$
$$= 20\sqrt{3} - 10 = 20 \times 1.732 - 10 = 24.64 \text{ N}$$

Total mass being pulled $= 1 + 3 + 2 = 6$ kg

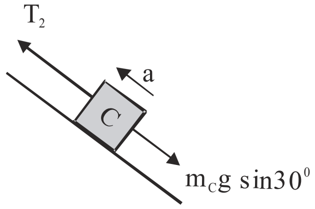


$\therefore$ Acceleration of the system $a = \frac{24.64}{6} = 4.1\ m/s^2$

(b) For the tension T_1 in the string between A and B, $m_A g \sin 60° - T_1 = (m_A)(a)$

$\therefore T_1 = m_A\, g \sin 60° - m_A\, a = m_A\, (g \sin 60° - a)$

$$\Rightarrow T_1 = (1)\left(10 \times \frac{\sqrt{3}}{2} - 4.1\right) = 4.56 \text{ N}$$



For the tension T_2 in the string between B and C.

$$T_2 - m_C\, g \sin 30° = m_C a$$

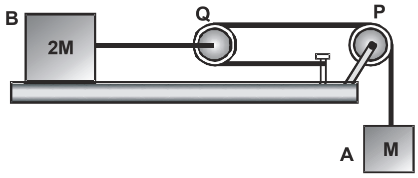
$$\Rightarrow T_2 = m_C\, (a + g \sin 30°) = 2\left[4.1 + 10\left(\frac{1}{2}\right)\right] = 18.2 \text{ N}$$

Ex.. Consider the situation shown in figure. Both the pulleys and the strings are light and all the surfaces are frictionless. Calculate

(a) the acceleration of mass M,

(b) tension in the string PQ and

(c) force exerted by the clamp on the pulley P.



Sol. As pulley Q is not fixed so if it moves a distance d the length of string between P and Q will changes by 2d (d from above and d from below), i.e., M will move 2d. This in turn implies that if a is the acceleration of M the acceleration of Q and of 2M will be $\frac{a}{2}$.

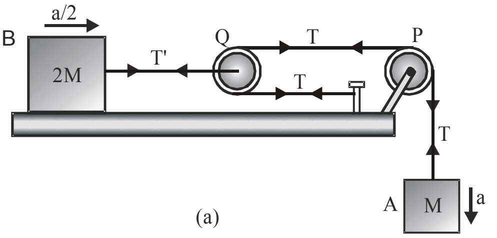
Now if we consider the motion of mass M, it is accelerated downward, so

$$T = M(g - a) \text{ ....(i)}$$

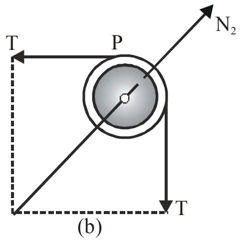
And for the motion of Q

$$2T - T' = 0 \times \frac{a}{2} = 0 \quad \Rightarrow T' = 2T \qquad \text{........(ii)}$$

And for the motion of mass 2M $\quad T' = 2M\left(\frac{a}{2}\right), \quad \Rightarrow \quad T' = Ma \qquad \text{.......(iii)}$



(a)



(b)

(a) From equation (ii) and (iii) as $T = \frac{1}{2}Ma$, so equation (i) reduces to

$$T = \frac{1}{2}\,Ma = M(g - a) \;\Rightarrow a = \frac{2}{3}\,g$$

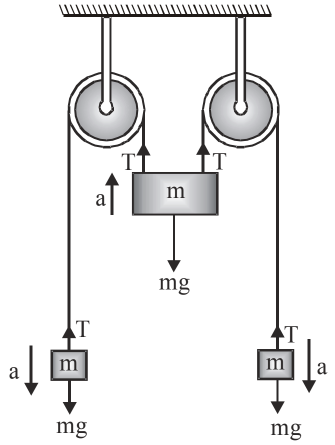
(b) So the acceleration of mass M is $\frac{2}{3}$ g while tension in the string PQ from equation (1) will be

$$T = M(g - \frac{2}{3}g) = \frac{1}{3}\,Mg$$

(c) Now from figure (b), it is clear that force on pulley by the clamp will be equal and opposite to the resultant of T and T at 90° to each other, i.e.,

$$(N_2) = \sqrt{T^2 + T^2} = \sqrt{2}\,T = \frac{\sqrt{2}}{3}\,Mg$$

Ex. Consider the double Atwood's machine as shown in the figure



(a) What is acceleration of the masses?

(b) What is the tension in each string?

Sol. (a) Here the system behaves as a rigid system, therefore every part of the system will move with same acceleration. Thus Applying newton's law

$mg - T = ma$ ...(1)

$2T - mg = ma$ ...(2)

Doubling the first equation and adding

$mg = 3\,ma \Rightarrow$ acceleration $a = \frac{1}{3}g$

(b) Tension in the string $T = m\,(g - a) = m\left(g - \frac{g}{3}\right) = \frac{2}{3}\,mg$

Ex. Consider the system of masses and pulleys shown in fig. with massless string and frictionless pulleys.

(a) Give the necessary relation between masses m_1 and m_2 such that system is in equilibrium and does not move.

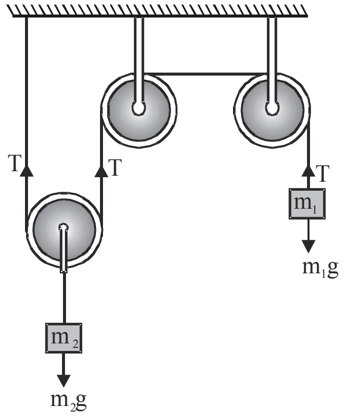
(b) If m_1 = 6 kg and m_2 = 8 kg, calculate the magnitude and direction of the acceleration of m_1.

Sol. (a) Applying newton's law $m_2g - 2T = 0$ (because there is no acceleration) and $T - m_1g = 0$

$\Rightarrow (m_2 - 2m_1)\, g = 0 \Rightarrow m_2 = 2m_1$

(b) If the upwards acceleration of m_1 is a,

then acceleration of m_2 is $\frac{a}{2}$ downwards



for mass m_2: $m_2g - 2T = m_2\left(\frac{a}{2}\right) \Rightarrow 2m_2g - 4T = m_2a$

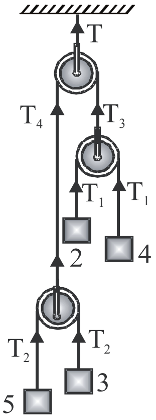
for mass m_1 : $T - m_1g = m_1a$

$$\Rightarrow \qquad a = \left(\frac{2m_2 - 4m_1}{m_2 + 4m_1}\right) g = \frac{2(8-12)}{8+24} g = -\frac{g}{4}$$

Negative sign shows that acceleration is opposite to considered direction i.e. it is downwards for m_1 and upwards for m_2.

Ex. In the given figure If $T_1 = 2T_2 = 50$ N then find the value of T.

Sol. As given in figure,



$$T_3 = 2T_1 = 2(2T_2) = 4T_2$$

and $\qquad T_4 = 2T_2$

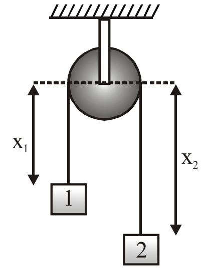
$\therefore \qquad T = T_3 + T_4 = 4T_2 + 2T_2 = 6T_2$

$$= 6 \times \frac{50}{2} = 150\text{N}$$

CONSTRAINT RELATIONS

These equations establish the relation between accelerations (or velocities) of different masses attached by string(s). Normally number of constraint equations are equal to number of strings in the system under consideration.

Ex. Find the relation between acceleration of 1 and 2.



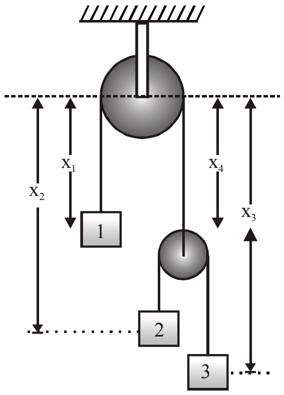
Sol. At any instant of time let x_1 and x_2 be the displacements of 1 and 2 from a fixed line. Then $x_1 + x_2$ = constant

Differentiating w.r.t. time, $v_1 + v_2 = 0$

Again differentiating w.r.t. time, $a_1 + a_2 = 0 \Rightarrow a_1 = -a_2$

So acceleration of 1 and 2 are equal but in opposite directions.

Ex. At certain moment of time, velocities of 1 and 2 both are 1 ms^{-1} upwards. Find the velocity of 3 at that moment.



Sol. $x_1 + x_4 = \ell_1$ (length of first string)

$x_2 - x_4 + x_3 - x_4 = \ell_2$ (length of second string)

$\Rightarrow\ v_1 + v_4 = 0\ \&\ v_2 + v_3 - 2v_4 = 0$

$\Rightarrow\ v_2 + v_3 + 2v_1 = 0$

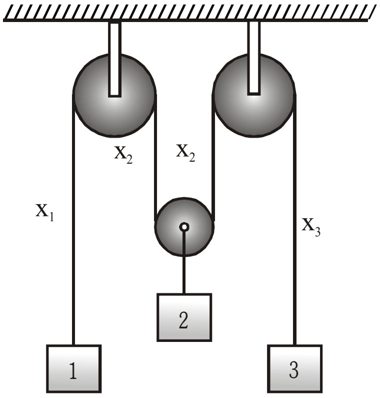
Taking upward direction as positive

$v_1 = v_2 = 1$

so $1 + v_3 + 2 \times 1 = 0 \Rightarrow v_3 = -3\ ms^{-1}$

i.e. velocity of block 3 is 3 ms^{-1} downwards.

Ex. Find the relation between acceleration of blocks a_1, a_2 and a_3.

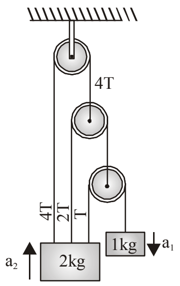


Sol. $x_1 + 2x_2 + x_3 = \ell$

$v_1 + 2v_2 + v_3 = 0$

$a_1 + 2a_2 + a_3 = 0$

Ex. Using contraint equation. Find the relation between a_1 and a_2.

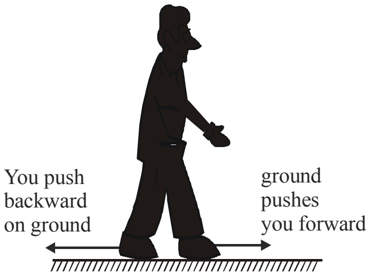


Sol. For this system $a_1T = a_2(4T+2T+T) \Rightarrow a_1=7a_2$

FRICTION

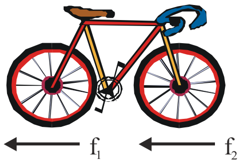
INTRODUCTION

Friction is the force of two surfaces in contact, or the force of a medium acting on a moving object. (i.e. air on aircraft.). Frictional forces may also exist between surfaces when there is no relative motion. Frictional forces arise due to molecular interactions. In some cases friction acts as a supporting force and in some cases it acts as opposing force.



- **Supporting :** Walking process can only take place because there is friction between the shoes and ground.
- **Opposing :** When a block slides over a surface the force of friction acts as an opposing force in the opposite direction of the motion
- **Both Supporting and Opposing :**
- **Pedaling :** When cyclist pedals the friction force on rear wheel acts as a supporting force and on front wheel as a opposing force.

Force of friction
when pedaling



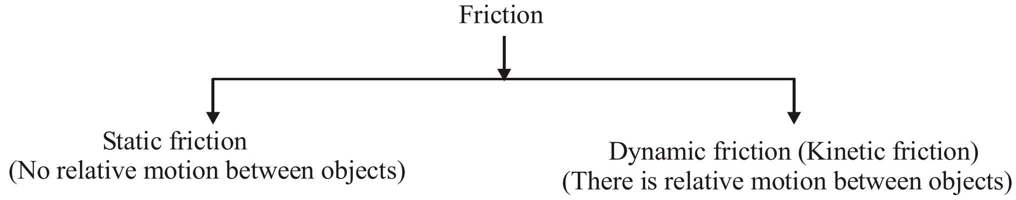
Force of friction
when not pedaling

- **Non–Pedaling :** When cyclist not pedals the friction force on rear wheel & front wheel act as a opposing force.

CAUSE OF FRICTION

- **Old View :** When two bodies are in contact with each other, the irregularities in the surface of one body set interlocked in the irregularites of another surface. This locking opposes the tendency of motion.
- **Modern View :** Friction is arises on account of strong atomic or molecular forces of attraction between the two surfaces at the point of actual contact.

TYPES OF FRICTION



STATIC FRICTION

- It is the frictional force which is effective before motion starts between two planes in contact with each other.
- It's nature is self adjusting.
- Numerical value of static friction is equal to external force which creates the tendency of motion of body.
- Maximum value of static friction is called limiting friction. $0 \le f_s \le \mu_s N$, $\vec{f}_s = -\vec{F}_{\text{applied}}$

LAWS OF LIMITING FRICTION

- The magnitude of the force of limiting friction (F) between any two bodies in contact is directly proportional to the normal reaction (N) between them $F \propto N$
- The direction of the force of limiting friction is always opposite to the direction in which one body is on the verge of moving over the other.
- The force of limiting friction is independent of the apporent contact area, as long as normal reaction between the two bodies in contact remains the same.
- Limiting friction between any two bodies in contact depends on the nature of material of the surfaces in contact and their roughness and smoothness.
- Its value is more than to other types of friction force.

DYNAMIC FRICTION

The friction opposing the relative motion between two bodies is called dynamic or kinetic friction

$$\vec{f}_k = -(\mu_k N)$$

- This is always slightly less than the limitng friction

COEFFICIENT OF FRICTION

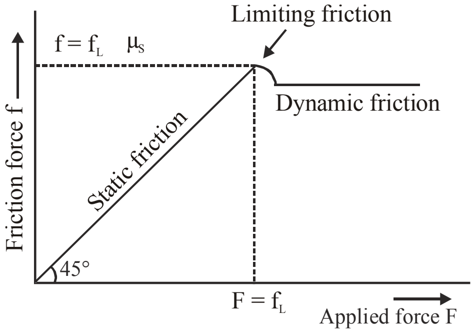
The frictional coefficient is a dimensionless scalar value which describes the ratio of the force of friction between two bodies and the force pressing them together.

- Static friction coefficient $\mu_s = \dfrac{F}{N}$
- Sliding friction coefficient $\mu_k = \dfrac{F_k}{N}$

The values of μ_s and μ_k depend on the nature of both the surfaces in contact.

GRAPH BETWEEN APPLIED FORCE AND FORCE OF FRICTION

If we slowly increase the force with which we are pulling the box, graph shows that the friction force increases with our force upto a certain critical value, f_L, the box suddenly begins to move, and as soon as it starts moving, a smaller force is required to maintain its motion as in motion friction is reduced. The friction value from 0 to f_L is known as static friction, which balances the external force on the body and prevent it from sliding. The value f_L is the maximum limit up to which the static friction acts is known as limiting friction, after which body starts sliding and friction reduces to kinetic friction.



- When two highly polished surfaces are pressed hard, then a situation similar to welding occurs. It is called **cold welding.**
- When two copper plates are highly polished and placed in contact with each other, then instead of decreasing, the force of friction increases. This arises due to the fact that for two highly polished surfaces in contact, the number of molecules coming in contact increases and as a result the cohesive/adhesive forces increases. This in turn, increases the force of friction.

Net contact force is the resultant of normal reaction and frictional force.

APPROXIMATE COEFFICIENTS OF FRICTION

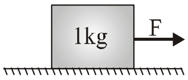
Materials	**Coefficient of static friction, μ_s**	**Coefficien of kinetic friction, μ_k**
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Copper on steel	0.53	0.36
Copper on cast iron	1.05	0.29
Brass on steel	0.51	0.44
Teflon on teflon	0.04	0.04
Rubber on concrete (dry)	1.0	0.8
Rubber on concrete (wet)	0.30	0.25

Ex. A block of mass 1 kg is at rest on a rough horizontal surface having coefficient of static friction 0.2 and kinetic friction 0.15, find the frictional forces if a horizontal force,

(a) F = 1N

(b) F = 1.96 N

(c) F = 2.5 N , is applied on a block



Sol. Maximum force of friction $f_{max} = 0.2 \times 1 \times 9.8\text{ N} = 1.96\text{ N}$

(a) for $F_{ext} = 1\text{ N}$, $F_{ext} < f_{max}$

So, body is in rest means static friction is present and hence $f_s = F_{ext} = 1\text{ N}$

(b) for $F_{ext} = 1.96\text{ N}$, $F_{ext} = f_{max} = 1.96\text{ N}$ so $f = 1.96\text{ N}$

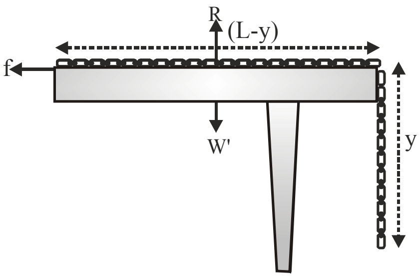
(c) for $F_{ext} = 2.5\text{ N}$, so $F_{ext} > f_{max.}$

now body is in moving condition

$\therefore\ f_{max.} = f_k = \mu_k N = \mu_k mg = 0.15 \times 1 \times 9.8 = 1.47\text{ N}$

Ex. Length of a chain is L and coefficient of static friction is μ. Calculate the maximum length of the chain which can be hang from the table without sliding.

Sol. Let y be the maximum length of the chain can be hold out side the table without sliding.



Length of chain on the table = (L – y)

Weight of part of the chain on table $W' = \frac{M}{L}(L-y)g$

Weight of hanging part of the chain $W = \frac{M}{L}yg$

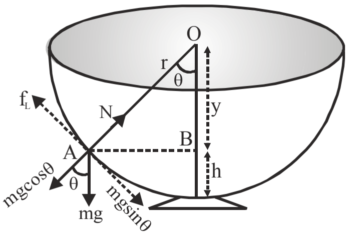
For equlibrium : limiting force of friction = weight of hanging part of the chain

$$\mu R = W \Rightarrow \quad \mu W' = W \quad \Rightarrow \mu\frac{M}{L}(L-y)g = \frac{M}{L}yg \Rightarrow \mu L - \mu y = y \Rightarrow y = \frac{\mu L}{1+\mu}$$

Ex. If the coefficient of friction between an insect and bowl is μ and the radius of the bowl is r, find the maximum height to which the insect can crawl up in the bowl.

Sol. The insect will crawl up the bowl till the component of its weight along the bowl is balanced by limiting frictional force. So, resolving weight perpendicular to the bowl and along the bowl,

$$N = mg\cos\theta\ ,\ f_L = mg\sin\theta \Rightarrow \tan\theta = \frac{f_L}{N} \Rightarrow \tan\theta = \mu\ \ [\because f_L = \mu N]$$



$$\Rightarrow \sqrt{\frac{(r^2-y^2)}{y}} = \mu \Rightarrow y = \frac{r}{\sqrt{1+\mu^2}}\quad \text{So } h = r - y = r\left[1-\frac{1}{\sqrt{1+\mu^2}}\right]$$

Ex. A body of mass M is kept on a rough horizontal surface (friction coefficient = μ). A person is trying to pull the body by applying a horizontal force F, but the body is not moving. What is the force by the surface on A.

Sol. Let f is the force of friction and N is the normal reaction,

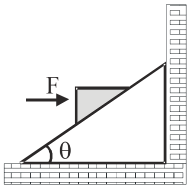
then the net force by the surface on the body is $F = \sqrt{N^2 + f^2}$

Let the applied force is F′ (varying), applied horizontally then $f \le \mu_s N$ (adjustable with f = F′).
Now if F′ is zero, f = 0 and $F_{min} = N = Mg$
and when F′ is increased to maximum value permissible for no motion. $f = \mu_s N$,

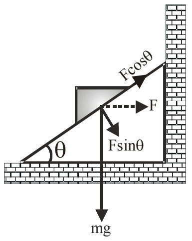
giving $F_{max} = \sqrt{N^2 + \mu_s^2 N^2} = Mg\sqrt{1+\mu_s^2}$

therefore we can write $\quad Mg \le F \le Mg\sqrt{1+\mu_s^2}$

Ex. A block rest on a rough inclined plane as shown in fig. A horizontal force F is applied to it (a) Find out the force of reaction, (b) Can the force of friction be zero if yes when? and (c) Assuming that friction is not zero find its magnitude and direction of its limiting value.

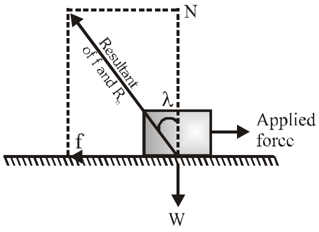


Sol. (a) N = mg cos θ + F sin θ
(b) Yes, if mg sin θ = F cos θ
(c) f = μR = μ (mg cos θ + F sin θ); up the plane if the body has tendency to slide down and down the plane if the body has tendency to move up.



ANGLE OF FRICTION

The angle of friction is the angle which the resultant of limiting friction f_S and normal reaction N makes with the normal reaction. It is represented by λ $\tan\lambda = \frac{f_S}{N} = \frac{\mu N}{N} = \mu$



- For smooth surface λ = 0

ANGLE OF REPOSE (θ)

If a body is placed on an inclined plane and if its angle of inclination is gradually increased, then at some angle of inclination θ the body will just on the point to slide down. The angle is called angle of repose (θ).

$F_S = mg\sin\theta$ and $N = mg\cos\theta$

so $\frac{F_S}{N} = \tan\theta \Rightarrow \mu = \tan\theta$

Relation between angle of friction (λ) and angle of repose (θ)

$\tan\lambda = \mu$ and $\mu = \tan\theta$, hence $\tan\lambda = \tan\theta \Rightarrow \theta = \lambda$

Thus, angle of repose = angle of friction

Ex. A block of mass 2 kg slides down an inclined plane which makes an angle of 30° with the horizontal. The coefficient of friction between the block and the surface is $\frac{\sqrt{3}}{2}$.

(i) What force must be applied to the block so that the block moves down the plane without acceleration ?

(ii) What force should be applied to the block so that it can move up without any acceleration ?

Sol. Make a 'free–body' diagram of the block. Take the force of friction opposite to the direction of motion.

(i) Project forces along and perpendicular to the plane

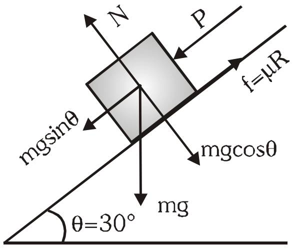
perpendicular to plane $N = mg\cos\theta$

along the plane $F + mg\sin\theta - f = 0$

(∵ there is no acceleration along the plane)

$F + mg\sin\theta - \mu N = 0 \Rightarrow F + mg\sin\theta = \mu\, mg\cos\theta$

$F = mg(\mu\cos\theta - \sin\theta) = 2 \times 9.8\left(\frac{\sqrt{3}}{2}\cos 30° - \sin 30°\right)$



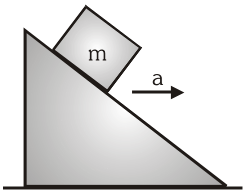
$= 19.6\left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2}\right) = 19.6\left(\frac{3}{4} - \frac{1}{2}\right) = 4.9\text{ N}$

(ii) This time the direction of F is reversed and that of the frictional force is also reversed.

$\therefore\ N = mg\cos\theta\ ;\ F = mg\sin\theta + f$

$\Rightarrow F = mg(\mu\cos\theta + \sin\theta) = 19.6\left(\frac{3}{4} + \frac{1}{2}\right) = 24.5\text{ N}$

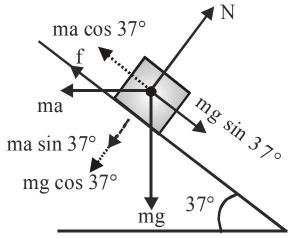
Ex. A block of mass 1 kg sits on an incline as shown in figure.



(a) What must be the frictional force between block and incline if the block is not to slide along the incline when the incline is accelerating to the right at 3 m/s^2 ?

(b) What is the least value μ_s can have for this to happen ?

Sol. $N = m(g\cos 37° + a\sin 37°) = 1(9.8 \times 0.8 + 3 \times 0.6) = 9.64$ N



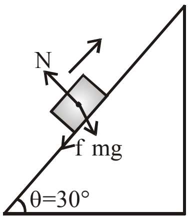
$mg\sin 37° = ma\cos 37° + f$

(a) $f = 1(9.8 \times 0.6 - 3 \times 0.8) = 3.48$

(b) $\because f = \mu N \qquad \therefore \mu = \dfrac{f}{N} = \dfrac{3.48}{9.64} = 0.36$

Ex. A body of mass 5×10^{-3} kg is launched up on a rough inclined plane making an angle of 30° with the horizontal. Obtain the coefficient of friction between the body and the plane if the time of ascent is half of the time of descent.

Sol. For upward motion: upward retardation $a_1 = \dfrac{\mu N + mg\sin\theta}{m}$

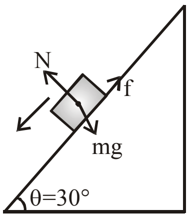


length of plane = s
time of ascent = t_1

$$a_1 = \mu g\cos 30° + g\sin 30° = (\sqrt{3}\mu + 1)\frac{g}{2}$$

$$\because s = \frac{1}{2}a_1 t_1^2 \quad \therefore t_1 = \sqrt{\frac{2s}{a_1}} = \sqrt{\frac{4s}{(\sqrt{3}\mu + 1)g}}$$

For downward motion: downward acceleration $a_2 = \dfrac{mg\sin\theta - \mu N}{m}$



time of descent = t_2

$$a_2 = g\sin 30° - g\cos 30° = (1 - \sqrt{3}\mu)\frac{g}{2}$$

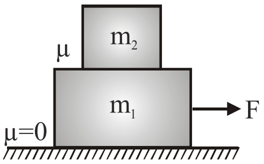
$$\Rightarrow t_2 = \sqrt{\frac{2s}{a_2}} = \sqrt{\frac{4s}{(1 - \sqrt{3}\mu)g}}$$

Now according to question $2t_1 = t_2$

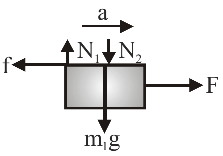
$$\Rightarrow 2\sqrt{\frac{4s}{(\sqrt{3}\mu + 1)g}} = \sqrt{\frac{4s}{(1 - \sqrt{3}\mu)g}}$$

$$\Rightarrow \frac{1 - \sqrt{3}\mu}{1 + \sqrt{3}\mu} = \frac{1}{4} \quad \Rightarrow \mu = \frac{\sqrt{3}}{5}$$

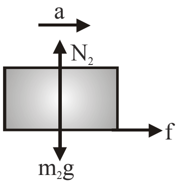
Ex. When force F applied on m_1 and there is no friction between m_1 and surface and the coefficient of friction between m_1 and m_2 is μ. What should be the minimum value of F so that there is no relative motion between m_1 and m_2



Sol. For m_1



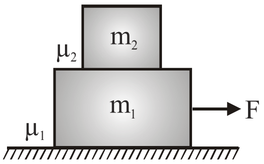
for m_2



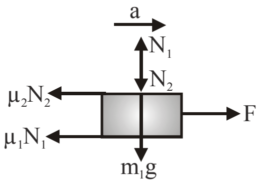
For system acceleration $a = \dfrac{F}{m_1 + m_2}$

For m_2 $f = m_2 a \Rightarrow \mu m_2 g = m_2\left(\dfrac{F}{m_1 + m_2}\right) \Rightarrow F_{min} = \mu(m_1 + m_2)\, g$

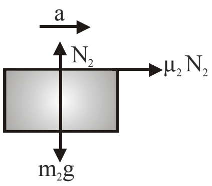
Ex. When force F applied on m_1 and the coefficient of friction between m_1 and surface. is μ_1 and the coefficient of friction between m_1 and m_2 is μ_2. What should be the minimum value of F so that there is no relative motion between m_1 and m_2 .



Sol. For m_1



for m_2

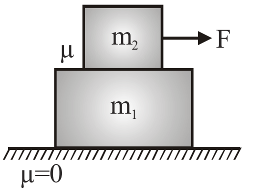


For system $a = \dfrac{F - \mu_1(m_1 + m_2)g}{m_1 + m_2}$

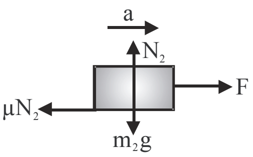
For m_2, $\mu_2\,(m_2 g) = m_2 a = m_2\left(\dfrac{F - \mu_1(m_1 + m_2)g}{m_1 + m_2}\right)$

$\Rightarrow \quad F_{min} = (m_1 + m_2)\,(\mu_1 + \mu_2)g$

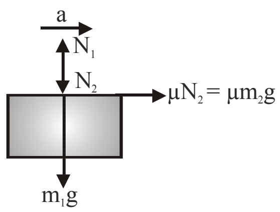
Ex. When force F applied on m_2 and there is no friction between m_1 and surface and the coefficient of friction between m_1 and m_2 is μ.. What should be the minimum value of F so that there is no relative motion between m_1 and m_2



Sol. For m_2



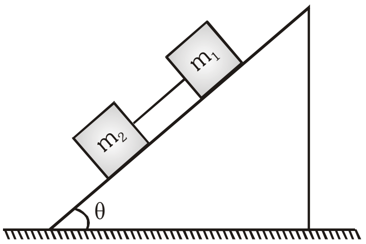
For m_1



for system : acceleration $= \dfrac{F}{m_1 + m_2}$

for m_1 : $\mu m_2 g = m_1 a = m_1\left(\dfrac{F}{m_1 m_2}\right)$, $F_{min} = (m_1 + m_2)\left(\dfrac{\mu m_2 g}{m_1}\right)$

Ex. Two blocks with masses $m_1 = 1$ kg and $m_2 = 2$ kg are connected by a string and side down a plane inclined at an angle $\theta = 45^0$ with the horizontal. The coefficient of sliding friction between m_1 and plane is $\mu_1 = 0.4$, and that between m_2 and plane is $\mu_2 = 0.2$. Calculate the common acceleration of the two blocks and the tension in the string.



Sol. As $\mu_2 < \mu_1$, block m_2 has greater acceleration than m_1 if we separately consider the motion of blocks. But they are connected so they move together as a system with common acceleration. So acceleration of the blocks :

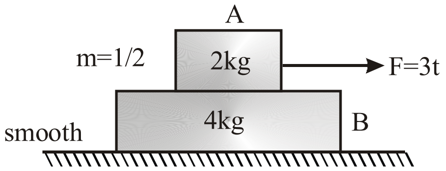
$$a = \frac{(m_1+m_2)g\sin\theta - \mu_1 m_1 g\cos\theta - \mu_2 m_2 g\cos\theta}{m_1+m_2}$$

$$= \frac{(1+2)(10)\left(\frac{1}{\sqrt{2}}\right) - 0.4\times1\times10\times\frac{1}{\sqrt{2}} - 0.2\times2\times10\times\frac{1}{\sqrt{2}}}{1+2} = \frac{22}{3\sqrt{2}}\ \text{ms}^{-2}$$

For block m_2 : $m_2 g\sin\theta - \mu_2 m_2 g\cos\theta - T = m_2 a \Rightarrow T = m_2 g\sin\theta - \mu_2 m_2 g\cos\theta - m_2 a$

$$= 2\times10\times\frac{1}{\sqrt{2}} - 0.2\times2\times10\times\frac{1}{\sqrt{2}} - 2\times\frac{22}{3\sqrt{2}} = \frac{2}{3\sqrt{2}}\text{N}$$

Ex. For shown situation draw a graph showing accelerations of A and B on y–axis and time on x–axis. ($g=10\text{ms}^{-2}$)



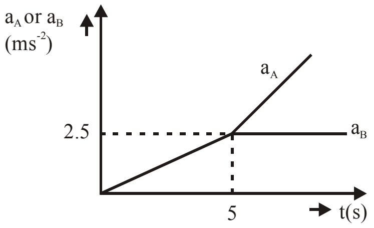
Sol. Limiting friction between A & B, $f_L = \mu m_A g = \left(\frac{1}{2}\right)(2)(10) = 10$ N

Block B moves due to friction only. So maximum acceleration of B,

$$a_{max} = \frac{f_L}{m_B} = \frac{10}{4} = 2.5\ \text{ms}^{-2}.$$

So both the blocks move together till the common acceleration becomes 2.5 ms^{-2}, after that acceleations of B will become constant while that of A will go on increasing. Slipping will starts between A & B at 2.5 ms^{-2}

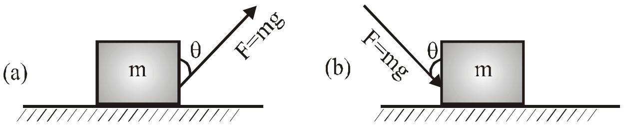
$$\Rightarrow 2.5 = \frac{F}{m_A+m_B} = \frac{3t}{6} \Rightarrow t = 5\text{ s}$$



Hence for $t \le 5$s, $a_A = a_B = \frac{F}{m_A+m_B} = \frac{3t}{6} = \frac{t}{2}$

and for $t > 5$ s $a_B = 2.5\ \text{ms}^{-2}$, $a_A = \frac{F-f_L}{m_A} = \frac{3t-10}{2} = \frac{3}{2}t - 5$

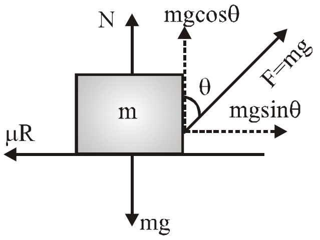
Ex. A block of mass m rests on a rough horizontal surface as shown in figure (a) and (b). Coefficient of friction between block and surface is μ. A force F = mg acting at an angle θ with the vertical side of the block. Find the condition for which block will move along the surface.



Sol. For (a) : normal reaction $N = mg - mg\cos\theta$, frictional force $= \mu N = \mu(mg - mg\cos\theta)$

Now block can be pulled when : Horizontal component of force $\geq$ frictional force

i.e. $mg\sin\theta \geq \mu(mg - mg\cos\theta)$



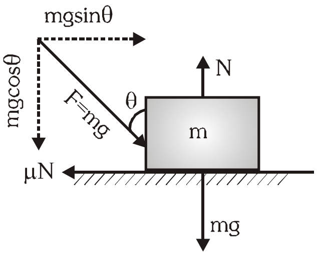
or $2\sin\frac{\theta}{2}\cos\frac{\theta}{2} \geq \mu(1-\cos\theta)$

or $2\sin\frac{\theta}{2}\cos\frac{\theta}{2} \geq 2\mu\sin^2\frac{\theta}{2}$ or $\cot\frac{\theta}{2} \geq \mu$

For (b) **:** Normal reaction $N = mg + mg\cos\theta = mg(1+\cos\theta)$

Hence, block can be pushed along the horizontal surface when horizontal component of force $\geq$ frictional force

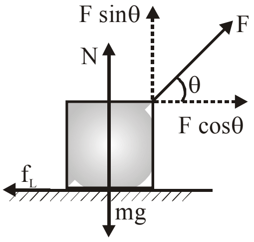
i.e. $mg\sin\theta \geq \mu\, mg(1+\cos\theta)$



or $2\sin\frac{\theta}{2}\cos\frac{\theta}{2} \geq \mu\times 2\cos^2\frac{\theta}{2} \Rightarrow \tan\frac{\theta}{2} \geq \mu$

Ex. A body of mass m rests on a horizontal floor with which it has a coefficient of static friction μ. It is desired to make the body move by applying the minimum possible force F. Find the magnitude of F and the direction in which it has to be applied.

Sol. Let the force F be applied at an angle θ with the horizontal as shown in figure.



For vertical equilibrium,

$R + F\sin\theta = mg \Rightarrow \quad N = mg - F\sin\theta$ (i)

for horizontal motion $F\cos\theta \geq f_L \Rightarrow F\cos\theta \geq \mu N$ [as $f_L = \mu N$] (ii)

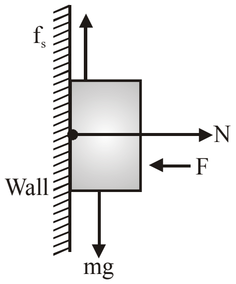
substituting value of R from equation (i) in (ii),

$$F\cos\theta \geq \mu(mg - F\sin\theta) \Rightarrow F \geq \frac{\mu mg}{(\cos\theta + \mu\sin\theta)} \qquad \text{(iii)}$$

For the force F to be minimum $(\cos\theta + \mu\sin\theta)$ must be maximum,

maximum value of $\cos\theta + \mu\sin\theta$ is $\sqrt{1+\mu^2}$ so that $F_{\min} = \frac{\mu mg}{\sqrt{1+\mu^2}}$ with $\theta = \tan^{-1}(\mu)$

Ex. A book of 1 kg is held against a wall by applying a perpendicular force F. If $\mu_S = 0.2$ then what is the minimum value of F ?



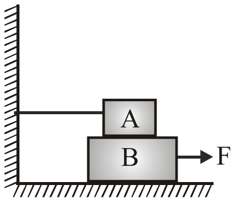
Sol. The situation is shown in fig. The forces acting on the book are–

For book to be at rest it is essential that $Mg = f_s$

But $f_{s\,max} = \mu_S N$ and $N = F$

$$\therefore\ Mg = \mu_S F \Rightarrow F = \frac{Mg}{\mu_s} = \frac{1\times 9.8}{0.2} = 49\ N$$

Ex. A is a 100 kg block and B is a 200 kg block. As shown in fig., the block A is attached to a string tied to a wall. The coefficient of friction between A and B is 0.2 and the coefficient of friction between B and floor is 0.3. Then calculate the minimum force required to move the block B. (take $g = 10\ m/s^2$).

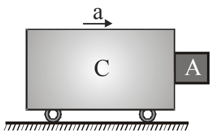


Sol. When B is tied to move, by applying a force F, then the frictional forces acting on the block B are f_1 and f_2 with limiting values, $f_1 = (\mu_S)_A m_A g$ and $f_2 = (\mu_S)_B\ (m_A + m_B)g$

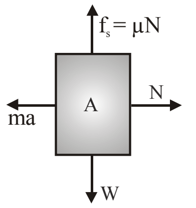
then minimum value of F should be (for just tending to move),

$F = f_1 + f_2 = 0.2 \times 100\ g + 0.3 \times 300\ g = 110\ g = 1100\ N$

Ex. Consider the figure shown here of a moving cart C. If the coefficient of friction between the block A and the cart is μ, then calculate the minimum acceleration a of the cart C so that the block A does not fall.



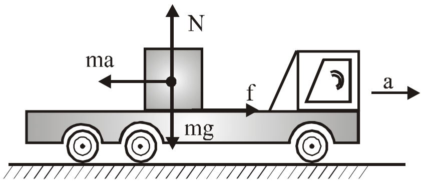
Sol. The forces acting on the block A (in block A's frame (i.e. non inertial frame) are :



For A to be at rest in block A's frame i.e. no fall,

we require $W = f_s$ $\Rightarrow mg = \mu(ma)$ Thus $a = \frac{g}{\mu}$

Ex. A block of mass 1kg lies on a horizontal surface in a truck, the coefficient of static friction between the block and the surface is 0.6, What is the force of friction on the block. If the acceleration of the truck is 5 m/s^2.



Sol. Fictitious force on the block $F = ma = 1 \times 5 = 5N$

While the limiting friction force

$$F = \mu_s N = \mu_s mg = 0.6 \times 1 \times 9.8 = 5.88\ N$$

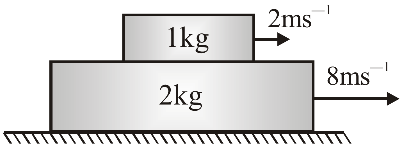
As required force F lesser than limiting friction force. The block will remain at rest in the truck and the force of friction will be equal to 5N and in the direction of acceleration of the truck.

Ex. Coefficient of friction between two blocks shown in figure is $\mu = 0.4$ The blocks are given velocities of $2ms^{-1}$ and $8\ ms^{-1}$ in the directions shown in figure. Find

(i) The time when relative motion between them will stop.

(ii) The common velocities of blocks upto that instant.

(iii) Displacement of blocks upto that instant ($g = 10\ ms^{-2}$)



Sol. (i) Frictional force between two blocks will oppose the relative motion. For 1 kg block friction support the motion & for 2 kg friction oppose the motion. Let common velocity be v then

for 1 kg $v = 2 + a_1 t$ where $a_1 = \dfrac{\mu(1g)}{1} = \dfrac{0.4\times10}{1} = 4\ ms^{-2}$

for 2 kg $v = 8 - a_2\ t$ where $a_2 = \dfrac{\mu(1g)}{2} = \dfrac{0.4\times10}{2} = 2\ ms^{-2} \Rightarrow 2 + 4t = 8 - 2\ t \Rightarrow 6t = 6 \Rightarrow t = 1s$

(ii) $v = 2 + 4\ t = 2 + 4 \times 1 = 6\ ms^{-1}$

(iii) Displacement of 1 kg block from rest

$$s = ut + \frac{1}{2}at^2 \Rightarrow s_1 = 2 \times 1 + \frac{1}{2} \times 4 \times 1^2 = 2 + 2 = 4m$$

Displacement of 2 kg block from rest

$$s = ut + \frac{1}{2}at^2 \Rightarrow s_2 = 8 \times 1 - \frac{1}{2} \times 2 \times 1^2 = 8 - 1 = 7\ m$$

Friction is a Necessary Evil :

Friction is a necessary evil. It means it has advantage as well as disadvantages. In other words, friction is not desirable but without friction, we cannot think of survival.

Disadvantages :

(i) A significant amount of energy of a moving object is wasted in the form of heat energy to overcome the force of friction.

(ii) The force of friction restricts the speed of moving vehicles like buses, trains, aeroplanes, rockets etc.

(iii) The efficiency of machines decreases due to the presence of force of friction.

(iv) The force of friction causes lot of wear and tear in the moving parts of a machine.

(v) Sometimes, a machine gets burnt due to the friction force between different moving parts.

Advantages :

(i) The force of friction helps us to move on the surface of earth. In the absence of friction, we cannot think of walking on the surface. That is why, we fall down while moving on a smooth surface.

(ii) The force of friction between the tip of a pen and the surface of paper helps us to write on the paper. It is not possible to write on the glazed paper as there is not force of friction.

(iii) The force of friction between the tyres of a vehicle and the road helps the vehicle to stop when brake is applied. In the absence of friction, the vehicle skid off the road when brake is applied.

(iv) moving belts remain on the rim of a wheel because of friction.

(v) The force of friction between a chalk and the black board helps us to write on the board.

Thus, we observe that inspect of various disadvantages of the friction, it is very difficult to part with it. So, friction is a necessary evil.

METHODS OF REDUCING FRICTION

As friction causes the wastage of energy so it becomes necessary to reduce the friction. Friction can be reduced by the following methods.

(i) Polishing the surface. We know, friction between rough surface is much more than between the polished surfaces. So we polish the surface to reduce the friction. The irregularities on the surface are filled with polish and hence the friction decreases.

(ii) Lubrication. To reduce friction, lubricants like oil or greese are used. When the oil or greese is put in between the two surfaces, the irregularities remain apart and do not interlock tightly. Thus, the surface can move over each other with less friction between them.

(iii) By providing the streamlined shape. When a body (e.g. bus, train, aeroplane etc.) moves with high speed, air resistance (friction) opposes its motion. The effect of air resistance on the motion of the objects (stated above) is decreased by providing them a streamlined shape.

EXERCISE (S-1)

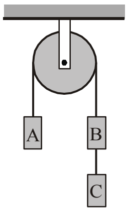
1. A force F applied to an object of mass m_1 produces an acceleration of 3.00 m/s^2. The same force applied to a second object of mass m_2 produces an acceleration of 1.00 m/s^2.

(i) What is the value of the ratio m_1/m_2 ?

(ii) If m_1 and m_2 are combined, find their acceleration under the action of the force F.

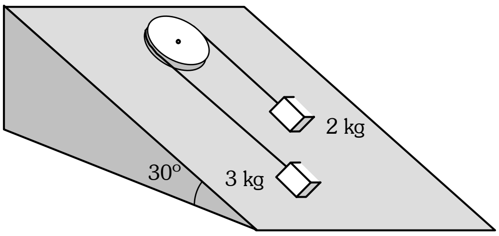
NL0001

2. In the system shown, the blocks A, B and C are of weight 4W, W and W respectively. The system set free. The tension in the string connecting the blocks B and C is



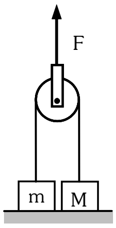
NL0002

3. Two blocks of masses 2.0 kg and 3.0 kg are connected by light inextensible string. The string passes over an ideal pulley pivoted to a fixed axel on a smooth incline plane as shown in the figure. When the blocks are released, find magnitude of their accelerations.



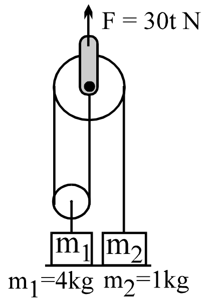
NL0003

4. In the system shown, pulley and strings are ideal. The vertically upward pull F is being increased gradually, find magnitude of F and acceleration of the 5 kg block at the moment the 10 kg block leaves the floor.



NL0004

5. Force F is applied on upper pulley. If $F = 30t\ N$ where t is time in second. Find the time when m_1 loses contact with floor.

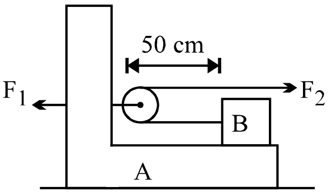


NL0005

6. A 40 kg boy climbs a rope that passes over an ideal pulley. The other end of the rope is attached to a 60 kg weight placed on the ground. What is the maximum upward acceleration the boy can have without lifting the weight? If he climbs the rope with upward acceleration 2 g, with what acceleration the weight will rise up?

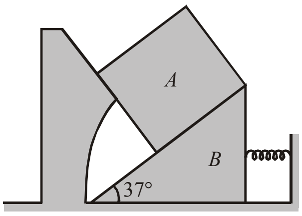
NL0006

7. A 1kg block B rests as shown on a bracket A of same mass. Constant forces $F_1 = 20\text{N}$ and $F_2 = 8\text{N}$ start to act at time t = 0 when the distance of block B from pulley is 50cm. Time when block B reaches the pulley is _________.



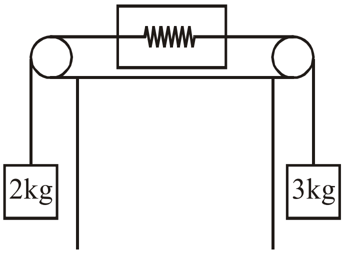
NL0007

8. In the figure shown, all surfaces are smooth and block A and wedge B have mass 10 kg and 20 kg respectively. Find normal reaction between block A & B, spring force and normal reaction of ground on block B. (g= 10 m/s^2).



NL0008

9. Find the reading of the massless spring balance in the given condition



NL0009

10. The system shown adjacent is in equilibrium. Find the acceleration of the blocks *A, B* & *C* all of equal masses *m* at the instant when

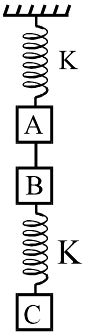
(Assume springs to be ideal)

(i) The spring between ceiling & *A* is cut.

(ii) The string (inextensible) between *A* & *B* is cut.

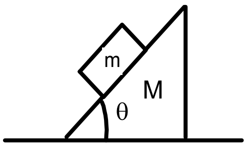
(iii) The spring between *B* & *C* is cut.

Also find the tension in the string when the system is at rest and in the above 3 cases.



NL0010

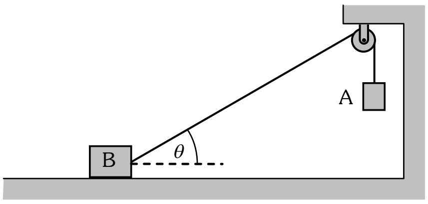
11. A block of mass *m* lies on wedge of mass *M* as shown in figure.



With what minimum acceleration must the wedge be moved towards right horizontally so that block m falls freely.

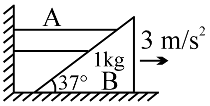
NL0011

12. The block A is moving downward with constant velocity v_0. Find the velocity of the block B, when the string makes an angle θ with the horizontal



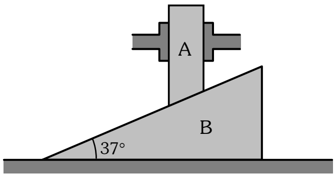
NL0012

13. Find force in newton which mass *A* exerts on mass *B* if *B* is moving towards right with 3 m/s^2. Also find mass of *A*. (All surfaces are smooth)



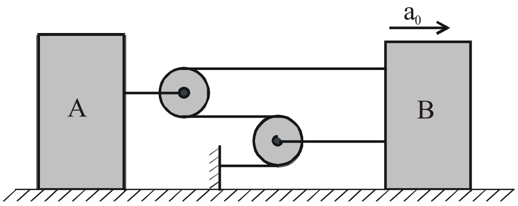
NL0013

14. Rod A can slide in vertical direction pushing the triangular wedge B towards right. The wedge is moving toward right with uniform acceleration a_B. Find acceleration of the rod A.



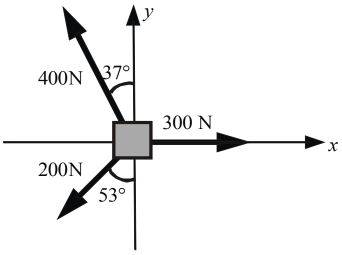
NL0014

15. Calculate the relative acceleration of *A* w.r.t. *B* if *B* is moving with acceleration a_0 towards right.



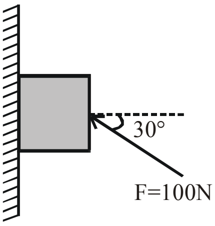
NL0015

16. A block is placed on a rough horizontal plane. Three horizontal forces are applied on the block as shown in the figure. If the block is in equilibrium, find the friction force acting on the block.



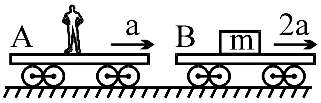
NL0016

17. A force of 100 N is applied on a block of mass 3 kg as shown in figure. The coefficient of friction between the wall and the surface of the block is 1/4. Calculate frictional force acting on the block.



NL0017

18. Two trolley A and B are moving with accelerations a and 2a respectively in the same direction. To an observer in trolley A, the magnitude of pseudo force acting on a block of mass m on the trolley B is



NL0018

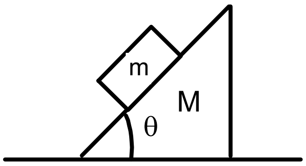
19. A thin rod of length 1 m is fixed in a vertical position inside a train, which is moving horizontally with constant acceleration 4 m/s^2. A bead can slide on the rod, and friction coefficient between them is 1/2. If the bead is released from rest at the top of the rod, find the time when it will reach at the bottom.[g = 10 m/s^2]

NL0019

20. A block of mass 1 kg is horizontally thrown with a velocity of 10 m/s on a stationary long plank of mass 2 kg whose surface has $\mu = 0.5$. Plank rests on frictionless surface. Find the time when block comes to rest w.r.t. plank.

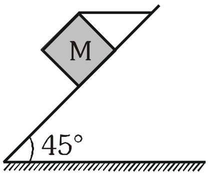
NL0020

21. A block of mass *m* lies on wedge of mass *M* as shown in figure. Find the minimum friction coefficient required between wedge *M* and ground so that it does not move while block *m* slips down on it.



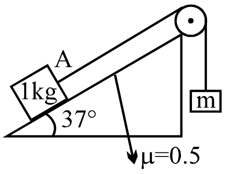
NL0021

22. A block of mass 15 kg is resting on a rough inclined plane as shown in figure.The block is tied up by a horizontal string which has a tension of 50 N. Calculate the minimum coefficient of friction between the block and inclined plane.



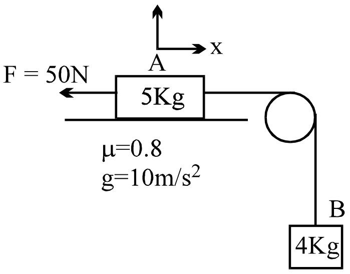
NL0022

23. In the figure, what should be mass m so that block *A* slides up with a constant velocity?



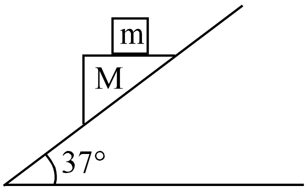
NL0023

24. Find the acceleration of the blocks and magnitude & direction of frictional force between block *A* and table, if block *A* is pulled towards left with a force of 50 N.



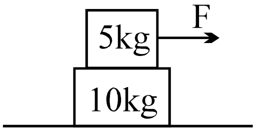
NL0024

25. Block *M* slides down on frictionless incline as shown. Find the minimum friction coefficient so that *m* does not slide with respect to *M*.



NL0025

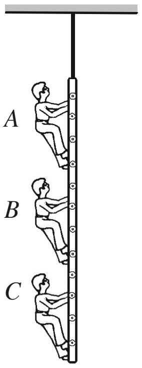
26. Coefficient of friction between 5 kg and 10 kg block is 0.5. If friction between them is 20 N. What is the value of force being applied on 5 kg. The floor is frictionless.



NL0026

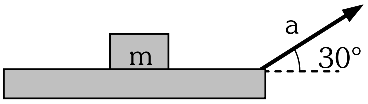
EXERCISE (S-2)

1. A ladder is hanging from ceiling as shown in figure. Three men *A, B* and *C* of masses 40 kg, 60 kg, and 50 kg are climbing the ladder. Man *A* is going up with retardation 2 m/s^2, *C* is going up with an acceleration of 1 m/s^2 and man *B* is going up with a constant speed of 0.5 m/s. Find the tension in the string supporting the ladder. [$g = 9.8\ m/s^2$]



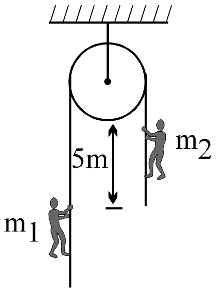
NL0027

2. A box of mass *m* is placed on a smooth horizontal platform as shown in the figure. The platform is made to move in direction 30° above the horizontal with acceleration *a* so that the contact force between the box and the platform becomes $3mg/2$. Find the magnitude of the acceleration.



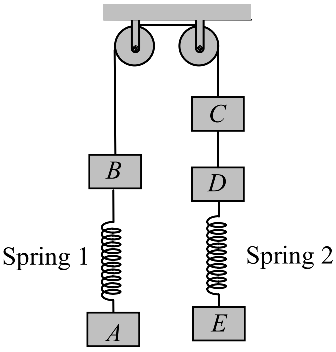
NL0028

3. Two men of masses m_1 and m_2 hold on the opposite ends of a rope passing over a frictionless pulley. The man m_1 climbs up the rope with an acceleration of 1.2 m/s^2 relative to the rope. The mann m_2 climbs up the rope with an acceleration of 2 m/s^2 relative to the rope. Find the tension in the rope if $m_1 = 40$ kg and $m_2 = 60$ kg. Also find the time after which they will be at same horizontal level if they start from rest and are initially separated by 5 m.



NL0029

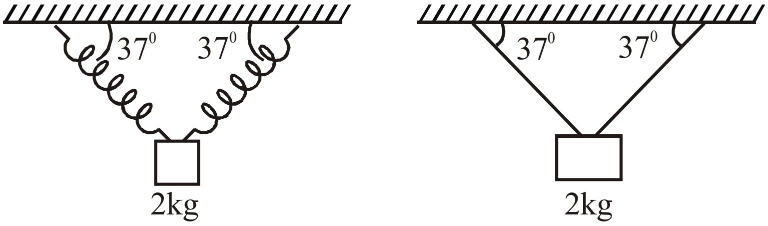
4. The system shown in the figure is initially in equilibrium. A is of mass $2m$ and B,C, D and E are of mass m. Certain actions are performed on the system. Every action has been taken individually when the system is intact. Find the direction and magnitude of acceleration of the blocks after each action of the following actions has been taken



(i) Spring 1 is cut
(ii) Spring 2 is cut
(iii) String between C and D is cut.
(iv) String between B and C is cut.

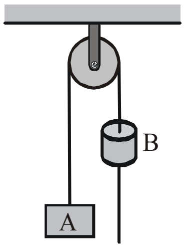
NL0030

5. The blocks are of mass 2 kg shown is in equilibrium. At $t = 0$ right spring in figure (i) and right string in figure (ii) breaks. Find the ratio of instantaneous acceleration of blocks?



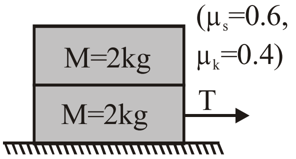
NL0031

6. A 2 kg block A is attached to one end of a light string that passes over an an ideal pulley and a 1 kg sleeve B slides down the other part of the string with an acceleration of 5 m/s^2 with respect to the string. Find the acceleration of the block, acceleration of sleeve and tension in the string. [$g = 10$ m/s^2]



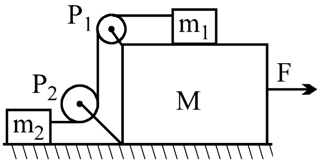
NL0032

7. The coefficient of static and kinetic friction between the two blocks and also between the lower block and the ground are $\mu_S = 0.6$ and $\mu_K = 0.4$. Find the value of tension *T* applied on the lower block at which the upper block begins to slip relative to lower block.



NL0033

8. In the figure masses m_1, m_2 and M are 20 kg, 5 kg and 50 kg respectively. The co-efficient of friction between M and ground is zero. The co-efficient of friction between m_1 and M and that between m_2 and ground is 0.3. The pulleys and the string are massless . The string is perfectly horizontal between P_1 and m_1 and also between P_2 and m_2 . The string is perfectly vertical between P_1 and P_2.An external horizontal force F is applied to the mass M. Take $g = 10\ m/s^2$.

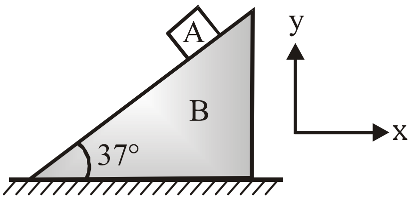


(i) Draw a free-body diagram for mass M, clearly showing all the forces.

(ii) Let the magnitude of the force of friction between m_1 and M be f_1 and that between m_2 and ground be f_2. For a particular F it is found that $f_1 = 2\ f_2$. Find f_1 and f_2 . Write down equations of motion of all the masses . Find F, tension in the string and accelerations of the masses.

NL0034

9. In the figure shown the acceleration of A is, $\vec{a}_A = (15\hat{i} + 15\hat{j})\ m/s^2$. If A is sliding on B then the acceleration of B is.

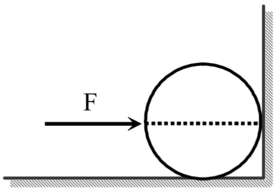


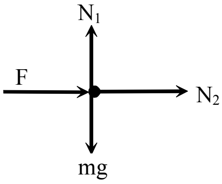
NL0035

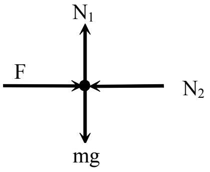
EXERCISE (O-1)

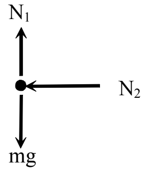
SINGLE CORRECT TYPE QUESTIONS

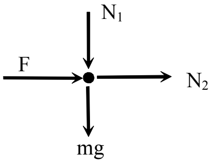
1. A ball of mass m kept at the corner as shown in the figure, is acted by a horizontal force F. The correct free body diagram of ball is



(A)


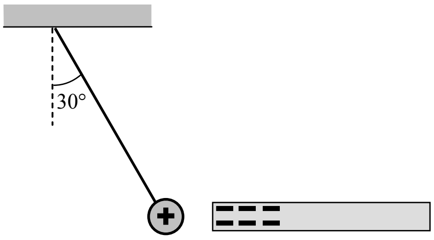
(B)


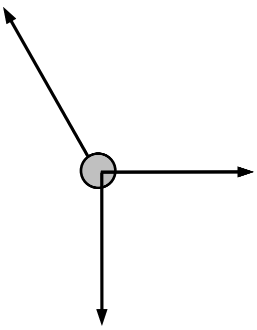
(C)


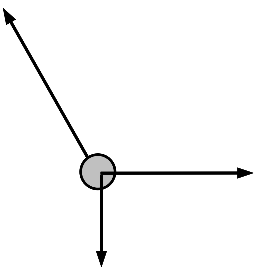
(D)


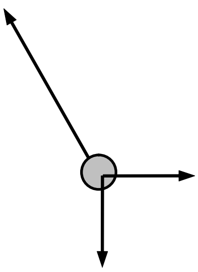
NL0036

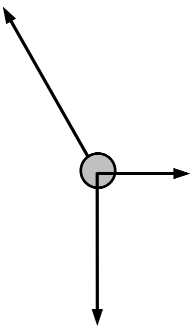
2. A small electrically charged sphere is suspended vertically from a thread. An oppositely charged rod is brought close to the sphere such that the sphere is in equilibrium displaced from the vertical by an angle of 30°. Which one of the following best represents the free body diagram for the sphere?



(A)


(B)


(C)


(D)


NL0037

3. Under what condition(s) will an object be in equilibrium ?

(A) Only if it is at rest

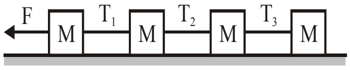
(B) Only if it is moving with constant velocity

(C) Only if it is moving with constant acceleration

(D) If it is either at rest or moving with constant velocity

NL0038

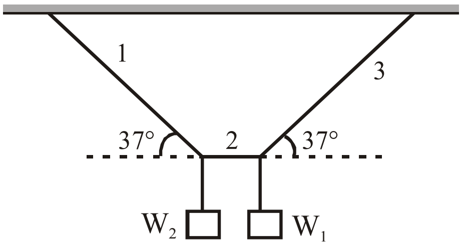
4. Four blocks of same mass connected by cords are pulled by force F on a smooth horizontal surface, as in figure. The tension T_1, T_2 and T_3 will be



(A) $T_1 = F/4$, $T_2 = 3F/2$, $T_3 = F/4$
(B) $T_1 = F/4$, $T_2 = F/2$, $T_3 = F/2$
(C) $T_1 = 3F/4$, $T_2 = F/2$, $T_3 = F/4$
(D) $T_1 = 3F/4$, $T_2 = F/2$, $T_3 = F/2$

NL0039

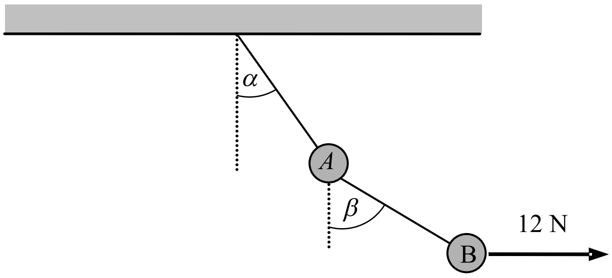
5. In a given figure system is in equilibrium. If $W_1 = 300$ N. Then W_2 is approximately equal to



(A) 500 N (B) 400 N (C) 670 N (D) 300 N

NL0040

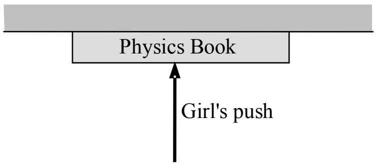
6. Two balls A and B weighing 7 N and 9 N are connected by a light cord. The system is suspended from a fixed support by connecting the ball A with another light cord. The ball B is pulled aside by a horizontal force 12 N and equilibrium is established. Angles α and β respectively are



(A) 30° and 60° (B) 60° and 30° (C) 37° and 53° (D) 53° and 37°

NL0041

7. A girl pushes her physics book up against the horizontal ceiling of her room as shown in the figure. The book weighs 20 N and she pushes upwards with a force of 25 N. The choices below list the magnitudes of the contact force F_{CB} between the ceiling and the book, and F_{BH} between the book and her hand. Select the correct pair.



(A) $F_{CB} = 20$ N and $F_{BH} = 25$ N
(B) $F_{CB} = 25$ N and $F_{BH} = 45$ N
(C) $F_{CB} = 5$ N and $F_{BH} = 25$ N
(D) $F_{CB} = 5$ N and $F_{BH} = 45$ N

NL0042

8. Two astronauts A and B connected with a rope stay stationary in free space relative to their spaceship. Mass of A is more than that of B and the rope is straight. Astronaut A starts pulling the rope but astronaut B does not. If you were the third astronaut in the spaceship, what do you observe?

(A) Astronaut B accelerates towards A and A remains stationery.

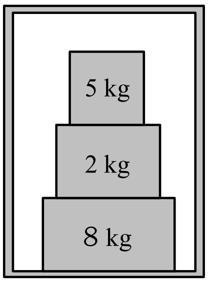
(B) Both accelerate towards each other with equal accelerations of equal modulus.

(C) Both accelerate towards each other but acceleration of B is greater than that of A.

(D) Both accelerate towards each other but acceleration of B is smaller than that of A.

NL0043

9. Three boxes are placed in a lift. When acceleration of the lift is 4 m/s^2, the net force on the 8 kg box is closest to



(A) 80 N (B) 48 N (C) 40 N (D) 32 N

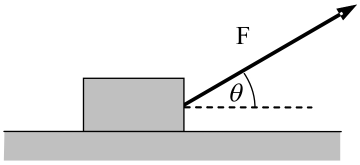
NL0044

10. A man is standing on a weighing machine with a block in his hand. The machine records w. When he takes the block upwards with some acceleration the machine records w_1. When he takes the block down with some acceleration, the machine records w_2.Then choose correct option

(A) $w_1 = w = w_2$ (B) $w_1 < w < w_2$ (C) $w_2 < w < w_1$ (D) $w_2 = w_1 > w$

NL0045

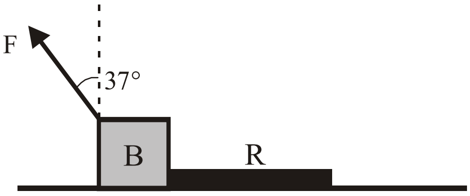
11. A block is being pulled by a force F on a long frictionless level floor. Magnitude of the force is gradually increases from zero until the block lifts off the floor. Immediately before the block leaves the floor, its acceleration is



(A) gcosθ (B) gcotθ (C) gsinθ (D) g tanθ

NL0046

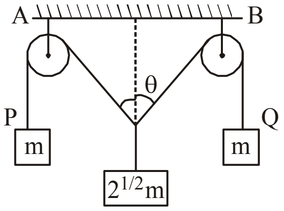
12. A block B is tied to one end of a uniform rope R as shown. The mass of block is 2 kg and that of rope is 1 kg. A force F = 15 N is applied at angle 37° with vertical. The tension at the mid-point of rope is



(A) 1.5 N (B) 2 N (C) 3N (D) 4.5 N

NL0047

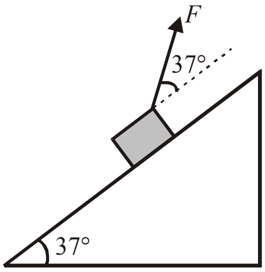
13. The pulleys and strings shown in the figure are smooth and of negligible mass. For the system to remain in equilibrium, the angle θ should be **[JEE (Scr) 2001]**



(A) 0° (B) 30° (C) 45° (D) 60°

NL0048

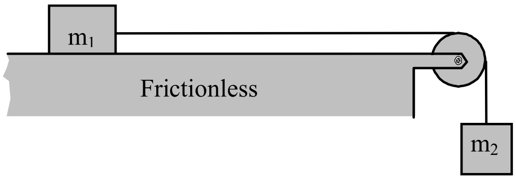
14. A block resting on a smooth inclined plane is acted upon by a force *F* as shown. If mass of block is 2 kg and *F* = 20 N and sin 37°= 3/5, the acceleration of block is



(A) 2 m/s^2 (B) 6 m/s^2 (C) 8 m/s^2 (D) zero

NL0049

15. In the arrangement shown, the blocks of unequal masses are held at rest. When released, acceleration of the blocks is

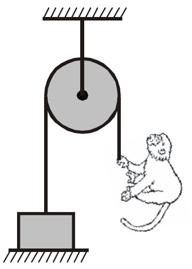


(A) g/2. (B) g.

(C) a value between zero and g. (D) a value that could be greater than g.

NL0050

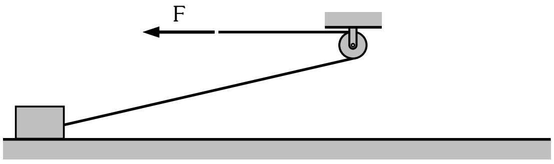
16. A monkey weighing 10 kg is climbing up a light rope and frictionless pulley attached to 15 kg mass at other end as in figure. In order to raise the 15 kg mass off the ground the monkey must climb-up



(A) with constant acceleration g/3.
(B) with an acceleration greater than g/2.
(C) with an acceleration greater than g/4.
(D) It is not possible because weight of monkey is lesser than the block.

NL0051

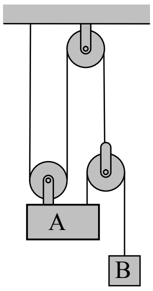
17. A heavy cart is pulled by a constant force F along a horizontal track with the help of a rope that passes over a fixed pulley, as shown in the figure. Assume the tension in the rope and the frictional forces on the cart remain constant and consider motion of the cart until it reaches vertically below the pulley. As the cart moves to the right, its acceleration



(A) decreases. (B) increases. (C) remains constant. (D) is zero

NL0052

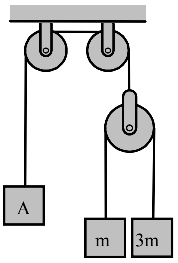
18. In arrangement shown the block A of mass 15 kg is supported in equilibrium by the block B. Mass of the block B is closest to



(A) 2 kg (B) 3 kg (C) 4 kg (D) 5 kg

NL0053

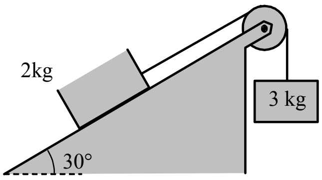
19. In the given figure, find mass of the block A, if it remains at rest, when the system is released from rest. Pulleys and strings are massless. [$g = 10\ m/s^2$]



(A) m (B) 2m (C) 2.5m (D) 3m

NL0054

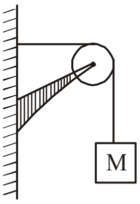
20. In the arrangement shown, the 2 kg block is held to keep the system at rest. The string and pulley are ideal. When the 2 kg block is set free, by what amount the tension in the string changes? [$g = 10\ m/s^2$]



(A) Increase of 12 N (B) Decrease of 12 N (C) Increase of 18 N (D) Decrease of 18 N

NL0055

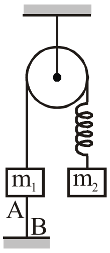
21. A string of negligible mass going over a clamped pulley of mass *m* supports a block of mass *M* as shown in the figure. The force on the pulley by the clamp is given **[JEE (Scr) 2001]**



(A) $\sqrt{2}\,Mg$ (B) $\sqrt{2}\,mg$ (C) $\sqrt{(M+m)^2+m^2}$ g (D) $\sqrt{(M+m)^2+M^2}$ g

NL0056

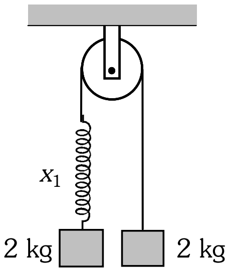
22. In a given figure two masses m_1 & m_2 ($m_2 > m_1$) are at rest in equilibrium position. Find the tension in string AB

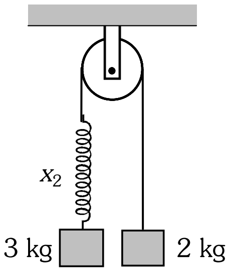


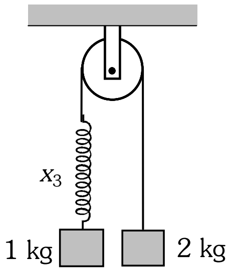
(A) m_1g (B) m_2g (C) $(m_1+m_2)g$ (D) $(m_2-m_1)g$

NL0057

23. Same spring is attached with 2 kg, 3 kg and 1 kg blocks in three different cases as shown. If x_1, x_2 and x_3 be the extensions in the spring in these three cases, when acceleration of both the blocks have same magnitude, then



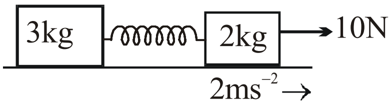




(A) $x_2 > x_3 > x_1$ (B) $x_2 > x_1 > x_3$ (C) $x_3 > x_1 > x_2$ (D) $x_1 > x_2 > x_3$

NL0058

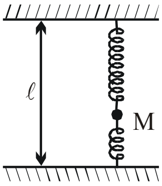
24. Find the acceleration of 3 kg mass when acceleration of 2 kg mass is 2 ms^{-2} as shown in figure.



(A) 3 ms^{-2} (B) 2 ms^{-2} (C) 0.5 ms^{-2} (D) zero

NL0059

25. A small ball of mass M is held in equilibrium with two identical springs as shown in the figure . Force constant of each spring is k and relaxed length of each spring is $\ell/2$. What is distance between the ball and roof?



(A) $\frac{\ell}{2}+\frac{Mg}{k}$ (B) $\frac{\ell}{2}-\frac{Mg}{k}$ (C) $\frac{\ell}{2}+\frac{Mg}{2k}$ (D) $\frac{\ell}{2}-\frac{Mg}{2k}$

NL0060

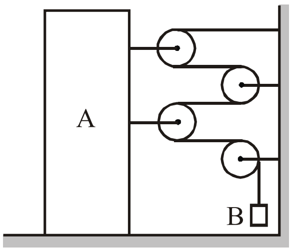
26. An elastic spring of relaxed length ℓ_0 and force constant k is cut into two parts of lengths ℓ_1 and ℓ_2. The force constants of these parts are respectively

(A) $\frac{k\ell_o}{\ell_1}$ and $\frac{k\ell_o}{\ell_2}$ (B) $\frac{k\ell_1}{\ell_0}$ and $\frac{k\ell_2}{\ell_o}$

(C) $\frac{k\ell_0}{\ell_2}$ and $\frac{k\ell_0}{\ell_1}$ (D) $\frac{k\ell_2}{\ell_0}$ and $\frac{k\ell_1}{\ell_0}$

NL0061

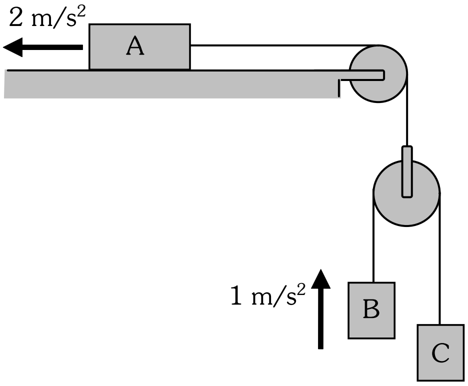
27. Block A is moving away from the wall at a speed v and acceleration a.



(A) Velocity of B is v with respect to A.
(B) Acceleration of B is a with respect to A.
(C) Acceleration of B is 4a with respect to A.
(D) Acceleration of B is $\sqrt{17}$a with respect to A.

NL0062

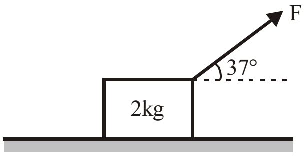
28. In the setup shown, find acceleration of the block C.



(A) 3 m/s² ↑ (B) 3 m/s² ↓ (C) 5 m/s² ↑ (D) 5 m/s² ↓

NL0063

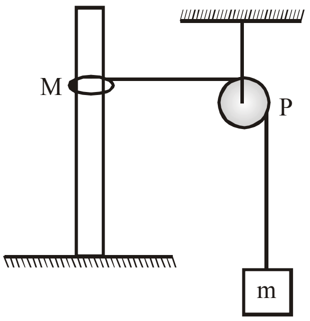
29. A block of mass 2 kg is kept on a rough horizontal floor and pulled with a force F. If the coefficient of friction is 0.5. then the minimum force required to move the block is :-



(A) 10 N (B) $\frac{100}{11}$N (C) $\frac{100}{8}$N (D) 20 N

NL0064

30. In the figure shown a ring of mass *M* and a block of mass *m* are in equilibrium. The string is light and pulley *P* does not offer any friction and coefficient of friction between pole and *M* is μ. The frictional force offered by the pole on *M* is



(A) *Mg* directed up

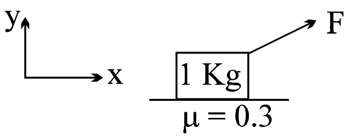
(B) μ*mg* directed up

(C) (*M* –*m*)g directed down

(D) μ*mg* directed down

NL0065

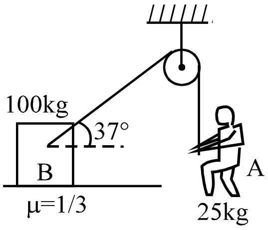
31. A force $\vec{F} = \hat{i} + 4\hat{j}$ acts on block shown. The force of friction acting on the block is :



(A) $-\hat{i}$ (B) $-1.8\hat{i}$ (C) $-2.4\hat{i}$ (D) $-3\hat{i}$

NL0066

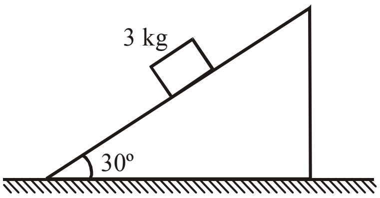
32. Block B of mass 100 kg rests on a rough surface of friction coefficient $\mu = 1/3$. A rope is tied to block B as shown in figure. The maximum acceleration with which boy A of 25 kg can climbs on rope without making block move is :



(A) $\frac{4g}{3}$ (B) $\frac{g}{3}$ (C) $\frac{g}{2}$ (D) $\frac{3g}{4}$

NL0067

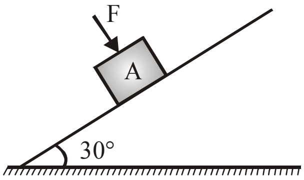
33. A block of mass 3 kg is at rest on a rough inclined plane as shown in the figure. The magnitude of net force exerted by the surface on the block will be ($g = 10$ m/s^2)



(A) 26 N (B) 19.5 N (C) 10 N (D) 30 N

NL0068

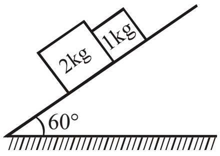
34. A block of mass $m = 2$ kg is resting on a rough inclined plane of inclination 30° as shown in figure. The coefficient of friction between the block and the plane is $\mu = 0.5$. What minimum force F should be applied perpendicular to the plane on the block, so that block does not slip on the plane ($g=10$m/s^2)



(A) zero (B) 6.24 N (C) 2.68 N (D) 4.34 N

NL0069

35. In the figure shown if friction coefficient of block 1kg and 2kg with inclined plane is μ_1=0.5 and $\mu_2 = 0.4$ respectively, then



(A) both block will move together.
(B) both block will move separately.
(C) there is a non zero contact force between two blocks.
(D) none of these

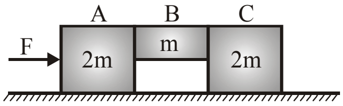
NL0070

36. A block is pushed with some velocity up a rough inclined plane. It stops after ascending few meters and then reverses its direction and returns back to point from where it started. If angle of inclination is 37° and the time to climb up is half of the time to return back then coefficient of friction is

(A) $\frac{9}{20}$ (B) $\frac{7}{5}$ (C) $\frac{7}{12}$ (D) $\frac{5}{7}$

NL0071

37. The system is pushed by a force *F* as shown in figure. All surfaces are smooth except between *B* and *C*. Friction coefficient between *B* and *C* is μ. Minimum value of *F* to prevent block *B* from downward slipping is :-

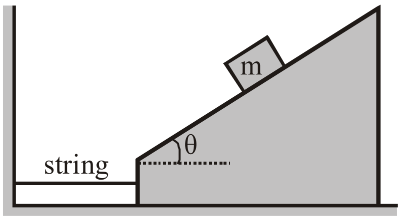


(A) $\left(\frac{3}{2\mu}\right)mg$ (B) $\left(\frac{5}{2\mu}\right)mg$ (C) $\left(\frac{5}{2}\right)\mu mg$ (D) $\left(\frac{3}{2}\right)\mu mg$

NL0072

MULTIPLE CORRECT TYPE QUESTIONS

38. Refer the system shown in the figure. Block is sliding down the wedge. All surfaces are frictionless. Find correct statement(s)

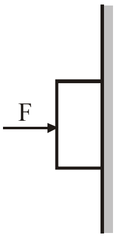


(A) Acceleration of block is $g\sin\theta$ (B) Acceleration block is $g\cos\theta$

(C) Tension in the string is $mg\cos^2\theta$ (D) Tension in the string is $mg\sin\theta.\cos\theta$

NL0073

39. A block of mass 1 kg is held at rest against a rough vertical surface by pushing by a force F horizontally. The coefficient of friction is 0.5. When



(A) $F = 40$ N, friction on the block is 20 N.

(B) $F = 30$ N, friction on the block is 10 N.

(C) $F = 20$ N, friction on the block is 10 N.

(D) Minimum value of force F to keep block at rest is 20 N.

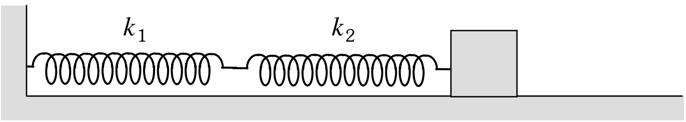
NL0074

40. A block is kept on a rough horizontal surface as shown. Its mass is 2 kg and coefficient of friction between block and surface (μ) = 0.5. A horizontal force F is acting on the block. When

(A) $F = 4$ N, acceleration is zero.
(B) $F = 4$ N, friction is 10 N and acceleration is 3 m/s^2.
(C) $F = 14$ N, acceleration is 2 m/s^2.
(D) $F = 14$ N, friction is 14 N.

NL0075

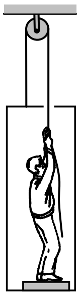
41. The mass in the figure can slide on a frictionless surface. When the mass is pulled out, spring 1 is stretched a distance x_1 and spring 2 is stretched a distance x_2. The spring constants are k_1 and k_2 respectively. Magnitude of spring force pulling back on the mass is



(A) k_1x_1 (B) k_2x_2 (C) $(k_1x_1 + k_2x_2)$ (D) $0.5\,(k_1 + k_2)\,(x_1 + x_2)$

NL0076

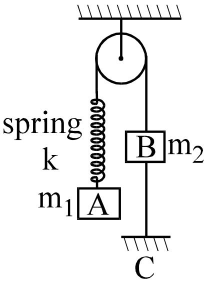
42. A carpenter of mass 50 kg is standing on a weighing machine placed in a lift of mass 20 kg. A light string is attached to the lift. The string passes over a smooth pulley and the other end is held by the carpenter as shown. When carpenter keeps the lift moving upward with constant velocity :- (g = 10 m/s^2)



(A) the reading of weighing machine is 15 kg
(B) the man applies a force of 350 N on the string
(C) net force on the man is 150 N
(D) Net force on the weighing machine is 150 N

NL0077

43. In the system shown in the figure $m_1 > m_2$. System is held at rest by thread BC. Just after the thread BC is burnt :



(A) initial acceleration of m_2 will be upwards

(B) magnitude of initial acceleration of both blocks will be equal to $\left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)$ g.

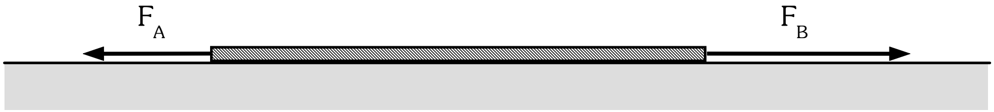
(C) initial acceleration of m_1 will be equal to zero
(D) magnitude of initial acceleration of two blocks will be non-zero and unequal.

NL0078

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 44 to 47

A uniform rope of mass (m) and length (L) placed on frictionless horizontal ground is being pulled by two forces F_A and F_B at its ends as shown in the figure. As a result, the rope accelerates toward the right.



44. Acceleration (A) of the rope is

(A) zero (B) $a=\dfrac{F_A+F_B}{m}$ (C) $a=\dfrac{F_A-F_B}{m}$ (D) $a=\dfrac{F_B-F_A}{m}$

NL0079

45. Tension (T) at the mid point of the rope is

(A) $T=F_B-F_A$ (B) $T=F_A+F_B$ (C) $T=\frac{1}{2}\left(F_B-F_A\right)$ (D) $T=\frac{1}{2}\left(F_A+F_B\right)$

NL0079

46. Expression (T_x) of tension at a point at distance x from the end A is

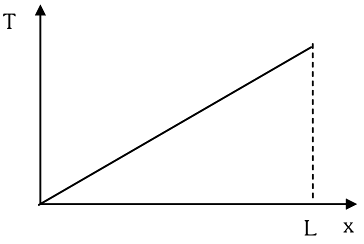
(A) $T_x=\left(\dfrac{F_B-F_A}{L}\right)x+F_A$ (B) $T_x=\left(\dfrac{F_B-F_A}{L}\right)x-F_A$

(C) $T_x=\left(\dfrac{F_B-F_A}{L}\right)x+F_B$ (D) $T_x=\left(\dfrac{F_B-F_A}{L}\right)x-F_B$

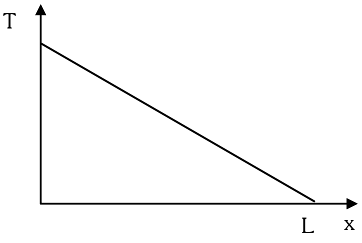
NL0079

47. Which of the following graph best represents variation in tension at a point on the rope with distance x of the point from the end A?

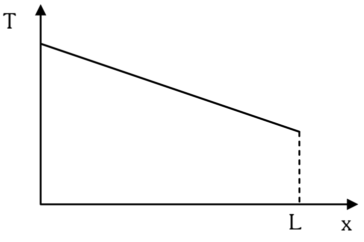
(A)



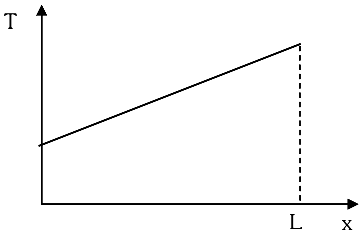
(B)



(C)



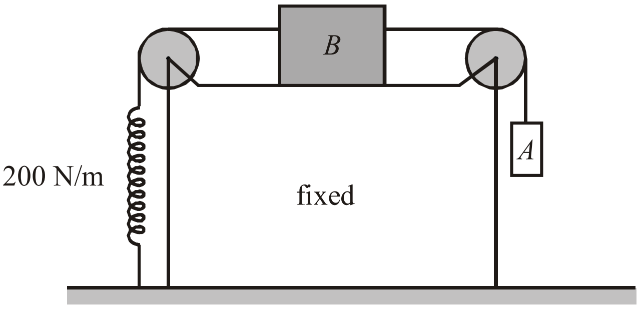
(D)



NL0079

Paragraph for Question No. 48 to 50

The figure shown blocks *A* and *B* are of mass 2 kg and 8 kg and they are connected through strings to a spring connected to ground. The blocks are in equilibrium. ($g = 10\text{m/s}^2$)



48. The elongation of the spring is
(A) 1 cm (B) 10 cm (C) 0.1 cm (D) 1m

NL0080

49. Now the block A is pulled downwards by a force gradually increasing to 20 N. The new elongation of spring is :-
(A) 2 cm (B) 4 cm (C) 20 cm (D) 40 cm

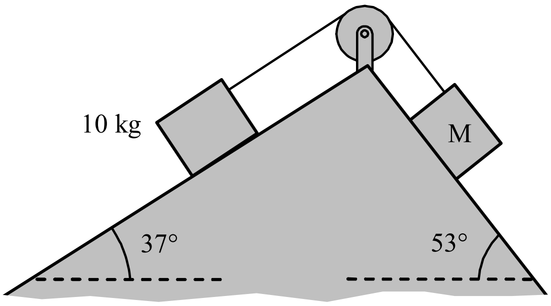
NL0080

50. Now the force on *A* is suddenly removed. The acceleration of block *B* becomes :-
(A) 1.0 m/s (B) 2.0 m/s^2 (C) 3.0 m/s^2 (D) 4.0 m/s^2

NL0080

Paragraph for Question No. 51 to 53

The blocks are on frictionless inclined ramp and connected by a massless cord. The cord passes over an ideal pulley. [g = 10 m/s^2]



51. When set free, the 10 kg block slides down the ramp with acceleration of 2 m/s^2. Mass M is closest to
(A) 13 kg (B) 8 kg (C) 5 kg (D) 4 kg

NL0081

52. When the 10 kg block slides down the ramp with acceleration of 2 m/s^2, tension in the cord is closest to :-
(A) 80 N (B) 60 N (C) 40 N (D) 30 N

NL0081

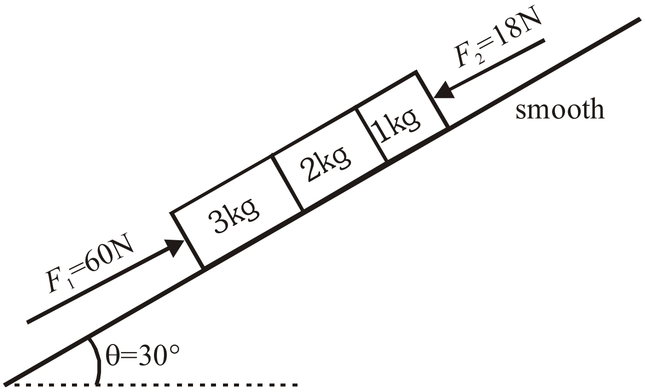
53. What value of M would keep the system at rest.

(A) 10 kg (B) 8 kg (C) 7.5 kg (D) 6 kg

NL0081

MATRIX MATCH TYPE QUESTION

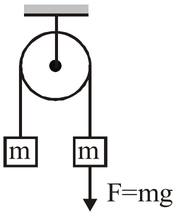
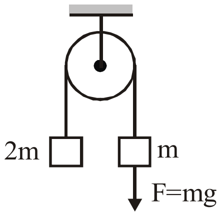
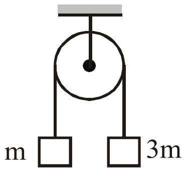
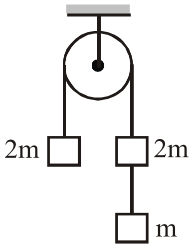
54. In the diagram shown in figure ($g = 10$ m/s^2)



	Column I		**Column II**
(A)	Acceleration of 2 kg block in m/s^2	(P)	8
(B)	Net force on 3 kg block in newton	(Q)	25
(C)	Normal reaction between 2 kg and 1 kg in newton	(R)	2
(D)	Normal reaction between 3 kg and 2 kg in newton	(S)	45
		(T)	None

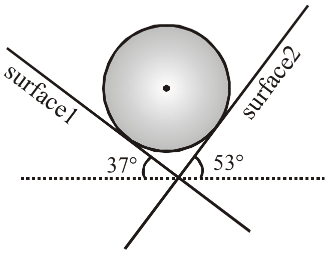
NL0082

55. Match the situations in column I to the accelerations of blocks in the column II (acceleration due to gravity is g and F is an additional force applied to one of the blocks ?

Column I		**Column II**	
(A)		(P)	$\frac{g}{5}$
(B)		(Q)	$\frac{g}{3}$
(C)		(R)	$\frac{g}{2}$
(D)		(S)	$\frac{2g}{3}$
		(T)	zero

NL0083

56. A sphere of mass 10 kg is placed in equilibrium in a V shaped groove plane made of two smooth surfaces 1 and 2 as shown in figure. (g = 10 ms^{-2})



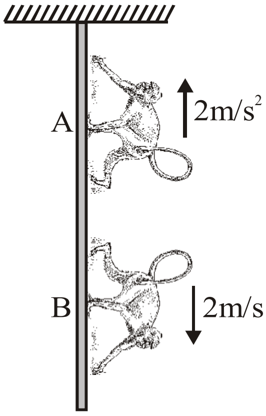
	Column I		**Column II**
(A)	Normal reaction by Surface 1	(P)	Zero
(B)	Normal reaction by surface 2	(Q)	60 N
(C)	Force on sphere by Earth	(R)	80 N
(D)	Net force on sphere	(S)	100 N
		(T)	120 N

NL0084

EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

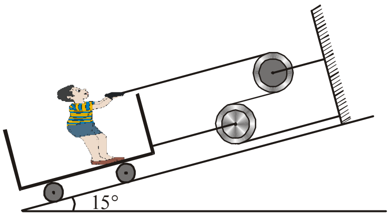
1. Two monkeys of masses 10 kg and 8 kg are moving along a vertical light rope, the former climbing up with an acceleration of 2 m/s^2, while the latter coming down with a uniform velocity of 2 m/s. Find tension in the rope at the fixed support.



(A) 180 N (B) 200 N (C) 80 N (D) 216 N

NL0085

2. A trolley is being pulled up an incline plane by a man sitting on it (as shown in figure). He applies a force of 250 N. If the combined mass of the man and trolley is 100 kg, the acceleration of the trolley will be [sin15° = 0.26]

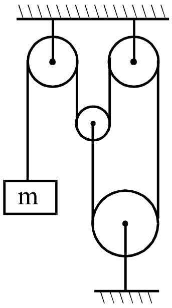


(A) 2.4 m/s^2 (B) 9.4 m/s^2

(C) 6.9 m/s^2 (D) 4.9 m/s^2

NL0086

3. If the string & all the pulleys are ideal, acceleration of mass *m* is :-



(A) $\frac{g}{2}$ (B) 0 (C) g (D) dependent on m

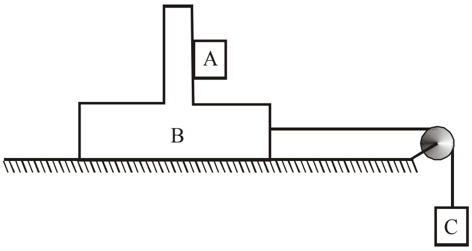
NL0087

4. The rear side of a truck is open and a box of mass 20 kg is placed on the truck 4 m away from the open end, $\mu = 0.15$ and $g = 10\ m/s^2$. The truck starts from rest with an acceleration of $2 m/s^2$ on a straight road. The distance moved by the truck when box starts fall down is :-

(A) 4 m (B) 8 m (C) 16 m (D) 32 m

NL0088

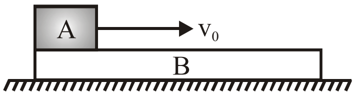
5. In the arrangement shown in the figure, mass of the block *B* and *A* is 2m and *m* respectively. Surface between *B* and floor is smooth. The block *B* is connected to the block *C* by means of a string-pulley system. If the whole system is released, then find the minimum value of mass of block *C* so that *A* remains stationary w.r.t. *B*. Coefficient of friction between *A* and *B* is μ.

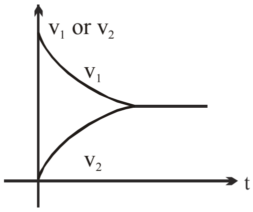


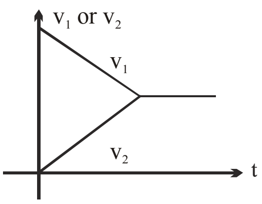
(A) $\frac{m}{\mu}$ (B) $\frac{2m+1}{\mu+1}$ (C) $\frac{3m}{\mu-1}$ (D) $\frac{6m}{\mu+1}$

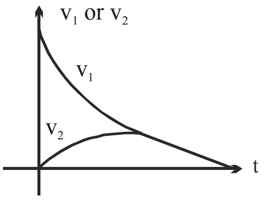
NL0089

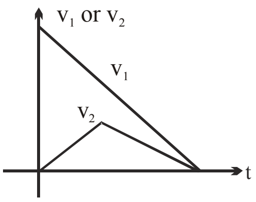
6. A block *A* is placed over a long rough plank *B* of same mass as shown in figure. The plank is placed over a smooth horizontal surface. At time t = 0, block *A* is given a velocity v_0 in horizontal direction. Let v_1 and v_2 be the velocities of *A* and *B* at time *t*. Then choose the correct graph between v_1 or v_2 and *t*.



(A)


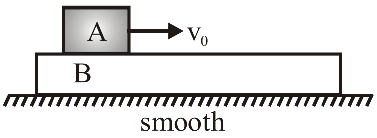
(B)


(C)


(D)


NL0090

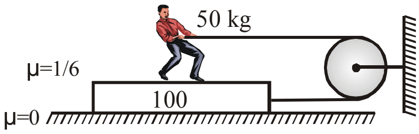
7. A block *A* of mass *m* is placed over a plank *B* of mass 2 m. Plank *B* is placed over a smooth horizontal surface. The coefficient of friction between *A* and *B* is 0.5. Block *A* is given a velocity v_0 towards right. Acceleration of *B* relative to *A* is :-



(A) $\frac{g}{2}$ (B) g (C) $\frac{3g}{4}$ (D) zero

NL0091

8. A man of mass 50 kg is pulling on a plank of mass 100 kg kept on a smooth floor as shown with force of 100 N. If both man & plank move together, find force of friction acting on man.

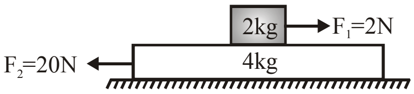


(A) $\frac{100}{3}$ N towards left (B) $\frac{100}{3}$ N towards right

(C) $\frac{250}{3}$ N towards left (D) $\frac{250}{3}$ N towards right

NL0092

9. In the arrangement shown in figure, coefficient of friction between the two blocks is $\mu = 1/2$. The force of friction acting between the two blocks is :-



(A) 8 N (B) 10 N (C) 6 N (D) 4 N

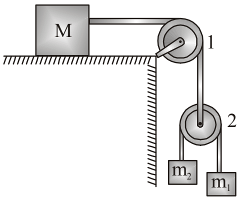
NL0093

10. A flexible chain of weight W hangs between two fixed points A & B which are at the same horizontal level. The inclination of the chain with the horizontal at both the points of support is θ. What is the tension of the chain at the mid point?

(A) $\frac{W}{2}.\ cosec\ \theta$ (B) $\frac{W}{2}.\ \tan\theta$ (C) $\frac{W}{2}\cot\theta$ (D) none

NL0094

11. In the arrangement shown in figure m_1 = 1kg, m_2 = 2kg. Pulleys are massless and strings are light. For what value of M the mass m_1 moves with constant velocity (Neglect friction)

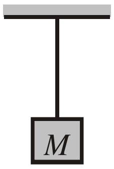


(A) 6 kg (B) 4 kg (C) 8 kg (D) 10 kg

NL0095

MULTIPLE CORRECT TYPE QUESTIONS

12. Consider a block suspended from a light string as shown in the figure.
Which of the following pairs of forces constitute Newton's third law pair?



(A) Force with which string pulls on the ceiling and the force with which string pulls on block

(B) Force with which string pulls on the block and weight of the block

(C) Force acting on block due to the earth and force the block exerts on the earth

(D) Force with which block pulls on string and force with which the string pulls on the block

NL0096

13. If a horizontal support exerts an upward force of 10 N on a block of weight 9.8 N placed on it, which of the following statements is/are correct. Assume acceleration due to gravity to be 9.8 m/s^2.

(A) The block exerts a force of 10 N on the support.

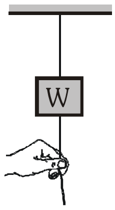
(B) The block exerts a force of 9.8 N on the support.

(C) The block has an upward acceleration.

(D) The block has a downward acceleration.

NL0097

14. A block of mass m is suspended from a fixed support with the help of a cord. Another identical cord is attached to the bottom of the block. Which of the following statement is /are true?



(A) If the lower cord is pulled suddenly, only the upper cord will break.

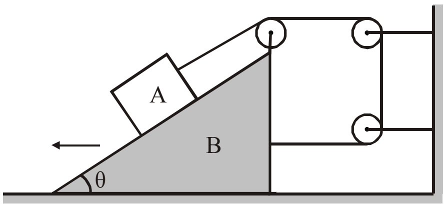
(B) If the lower cord is pulled suddenly, only the lower cord will break.

(C) If pull on the lower cord is increased gradually, only the lower cord will break.

(D) If pull on the lower cord is increased gradually, only the upper cord will break.

NL0098

15. A block A and wedge B connected through a string as shown. The wedge B is moving away from the wall with acceleration 2 m/s^2 horizontally and acceleration of block A is vertical upwards. Then



(A) Acceleration of A with respect to B is 4 m/s^2.

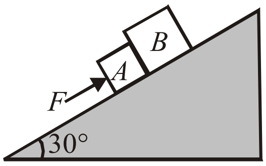
(B) Acceleration of A with respect to B is $2\sqrt{3}$ m/s^2.

(C) Angle θ is 60°.

(D) Acceleration of A is $2\sqrt{3}$ m/s^2.

NL0099

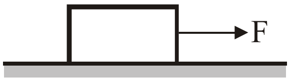
16. Two blocks *A* and *B* of mass 2 kg and 4 kg respectively are placed on a smooth inclined plane and 2 kg block is pushed by a force F acting parallel to the plane as shown. If N be the magnitude of contact force applied on B by A, which of the following is/are correct?



(A) if $F = 0$ N, $N = 10$ N
(B) if F=15 N, N=10 N
(C) If $F = 30$ N, $N = 20$ N
(D) if F=45 N, N=30 N

NL0100

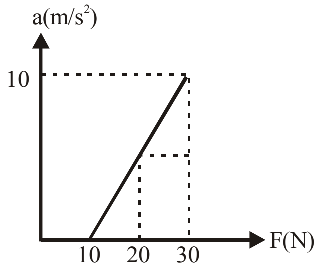
17. A block is kept on a rough surface and applied with a horizontal force as shown which is gradually increasing from zero. The coefficient of static and kinetic friction are $1/\sqrt{3}$ then



(A) When F is less than the limiting friction, angle made by net force on the block by the surface is less than 30° with vertical.
(B) When the block is just about to move, the angle made by net force by the surface on the block becomes equal to 30° with vertical.
(C) When the block starts to accelerate, the angle made by net force by the surface on the block becomes constant and equal to 30° vertical.
(D) The angle made by net force with vertical on the block by the surface, depends on the mass of the block.

NL0101

18. A block placed on a rough horizontal surface is pushed with a force F acting horizontally on the block. The magnitude of F is increased and acceleration produced is plotted in the graph shown.



(A) Mass of the block is 2 kg.
(B) Coefficient of friction between block and surface is 0.5.
(C) Limiting friction between block and surface is 10 N.
(D) When $F = 8$ N, friction between block and surface is 10 N.

NL0102

19. A block is placed over a plank. The coefficient of friction between the block and the plank is $\mu = 0.2$. Initially both are at rest, suddenly the plank starts moving with acceleration $a_0 = 4$ m/s^2. The displacement of the block in 1s is (g=10 m/s^2)

(A) 1 m relative to ground
(B) 1 m relative to plank
(C) zero relative to plank
(D) 2 m relative to ground

NL0103

20. A block is released from rest from a point on a rough inclined place of inclination 37°. The coefficient of friction is 0.5.

(A) The time taken to slide down 9 m on the plane is 3 s.

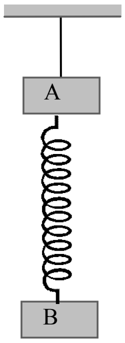
(B) The velocity of block after moving 4 m is 4 m/s.

(C) The block travels equal distances in equal intervals of time.

(D) The velocity of block increases linearly.

NL0104

21. In the given figure both the blocks have equal mass. When the thread is cut, which of the following statements give correct description immediately after the thread is cut?



(A) Relative to the block A, acceleration of block B is 2g upwards.

(B) Relative to the block B, acceleration of block A is 2g downwards.

(C) Relative to the ground, accelerations of the blocks A and B are both g downwards.

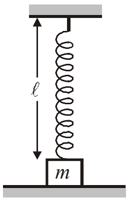
(D) Relative to the ground, accelerations of the blocks A and B are 2g downwards and zero respectively.

NL0105

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 22 to 24

A block of mass *m* is placed on a smooth horizontal floor is attached to one end of spring. The other end of the spring is attached to fixed support. When spring is vertical it is relaxed. Now the block is pulled towards right by a force *F*, which is being increased gradually. When the spring makes angle 53° with the vertical, block leaves the floor.



22. When blocks leaves the table, the normal force on it from table is

(A) mg (B) zero (C) $\frac{4mg}{3}$ (D) $\frac{3mg}{4}$

NL0106

23. Force constant of the spring is :-

(A) $\frac{5mg}{2\ell}$ (B) $\frac{15mg}{8\ell}$ (C) $\frac{5mg}{3\ell}$ (D) $\frac{5mg}{4\ell}$

NL0106

24. When the block leaves the table, the force F is :-

(A) $\frac{3mg}{4}$ (B) $\frac{4mg}{3}$ (C) $\frac{3mg}{5}$ (D) $\frac{4mg}{5}$

NL0106

Paragraph for Question No. 25 to 29

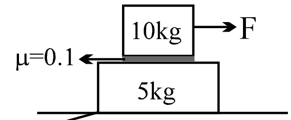


Fig. 1

25. When $F = 2\text{N}$, the frictional force between 5 kg block and ground is

(A) 2N (B) 0 (C) 8 N (D) 10 N

NL0107

26. When $F = 2\text{N}$, the frictional force between 10 kg block and 5 kg block is

(A) 2N (B) 15 N (C) 10 N (D) None

NL0107

27. The maximum F which will not cause motion of any of the blocks is

(A) 10 N (B) 15 N (C) data insufficient (D) None

NL0107

28. The maximum acceleration of 5 kg block is :-

(A) 1 m/s^2 (B) 3 m/s^2 (C) 0 (D) None

NL0107

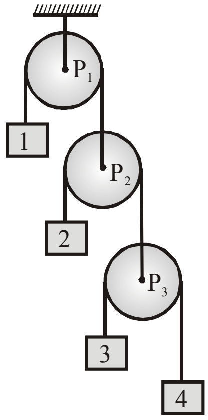
29. The acceleration of 10 kg block when $F = 30$ N is

(A) 2 m/s^2 (B) 3 m/s^2 (C) 1 m/s^2 (D) None

NL0107

MATRIX MATCH TYPE QUESTION

30. In the figure shown, acceleration of 1 is x (upwards). Acceleration of pulley P_3, w.r.t. pulley P_2 is y (downwards) and acceleration of 4 w.r.t. to pulley P_3 is z (upwards). Then



	Column I		**Column II**
(A)	Absolute acceleration of 2	(P)	(y–x) downwards
(B)	Absolute acceleration of 3	(Q)	(z–x–y) upwards
(C)	Absolute acceleration of 4	(R)	(x+y+z) downwards
		(S)	None

NL0108

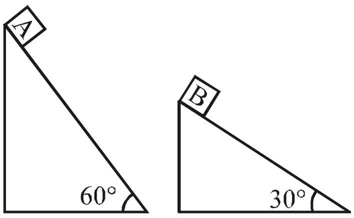
31. Velocity of three particles A, B and C varies with time t as, $\vec{v}_A = (2t\hat{i} + 6\hat{j})$ m/s; $\vec{v}_B = (3\hat{i} + 4\hat{j})$ m/s and $\vec{v}_C = (6\hat{i} - 4t\hat{j})$ m/s. Regarding the pseudo force match the following table

	Column I		**Column II**
(A)	On A as observed by B	(P)	Along positive x–direction
(B)	On B as observed by C	(Q)	Along negative x–direction
(C)	On A as observed by C	(R)	Along positive y–direction
(D)	On C as observed by A	(S)	Along negative y–direction
		(T)	Zero

NL0109

EXERCISE (JM)

1. Two fixed frictionless inclined planes making an angle 30° and 60° with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B ? **[AIEEE - 2010]**



(1) 4.9 ms^{-2} in vertical direction. (2) 4.9 ms^{-2} in horizontal direction
(3) 9.8 ms^{-2} in vertical direction (4) Zero

NL0110

2. The minimum force required to start pushing a body up a rough (frictional coefficient μ) inclined plane is F_1 while the minimum force needed to prevent it from sliding down is F_2. If the inclined plane makes an angle θ from the horizontal such that $\tan\theta = 2\mu$ then the ratio $\frac{F_1}{F_2}$ is :- **[AIEEE - 2011]**

(1) 4 (2) 1 (3) 2 (4) 3

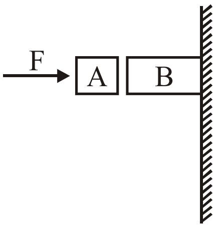
NL0111

3. A block of mass m is placed on a surface with a vertical cross section given by $y = \frac{x^3}{6}$. If the coefficient of friction is 0.5, the maximum height above the ground at which the block can be placed without slipping is :- **[JEE-Main-2014]**

(1) $\frac{1}{3}$m (2) $\frac{1}{2}$m (3) $\frac{1}{6}$m (4) $\frac{2}{3}$m

NL0112

4. Given in the figure are two blocks A and B of weight 20 N and 100 N, respectively. These are being pressed against a wall by a force F as shown. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall on block B is :- **[JEE-Main-2015]**



(1) 120 N (2) 150 N (3) 100 N (4) 80 N

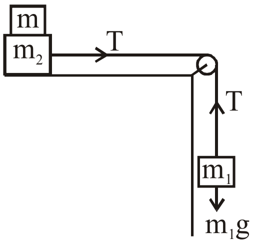
NL0113

5. A rocket is fired vertically from the earth with an acceleration of 2g, where g is the gravitational acceleration. On an inclined plane inside the rocket, making an angle θ with the horizontal, a point object of mass m is kept. The minimum coefficient of friction μ_{min} between the mass and the inclined surface such that the mass does not move is: **[JEE-Main Online-2016]**

(1) $2 \tan \theta$ (2) $3 \tan \theta$ (3) $\tan \theta$ (4) $\tan 2\theta$

NL0114

6. Two masses $m_1 = 5$kg and $m_2 = 10$kg, connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is :- **[JEE-Main-2018]**



(1) 27.3 kg (2) 43.3 kg (3) 10.3 kg (4) 18.3 kg

NL0115

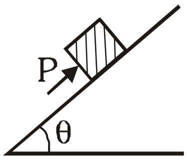
EXERCISE (JA)

1. A piece of wire is bent in the shape of a parabola $y = kx^2$ (y-axis vertical) with a bead of mass m on it. The bead can slide on the wire without friction. It stays at the lowest point of the parabola when the wire is at rest. The wire is now accelerated parallel to the x-axis with a constant acceleration a. The distance of the new equilibrium position of the bead, where the bead can stay at rest with respect to the wire, from the y-axis is **[IIT-JEE-2009]**

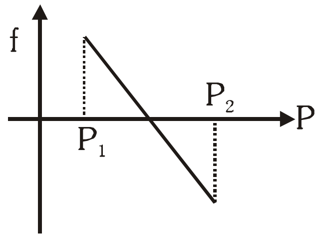
(A) $\frac{a}{gk}$ (B) $\frac{a}{2gk}$ (C) $\frac{2a}{gk}$ (D) $\frac{a}{4gk}$

NL0116

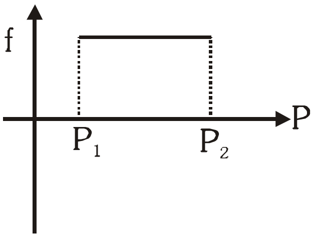
2. A block of mass m is on an inclined plane of angle θ. The coefficient of friction between the block and the plane is μ and tanθ >μ. The block is held stationary by applying a force P parallel to the plane. The direction of force pointing up the plane is taken to be positive. As P is varied from $P_1 = mg(\sin\theta - \mu\cos\theta)$ to $P_2 = mg(\sin\theta + \mu\cos\theta)$, the frictional force f versus P graph will look like **[IIT-JEE-2010]**



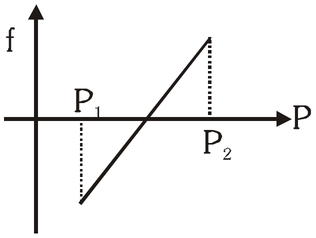
(A)



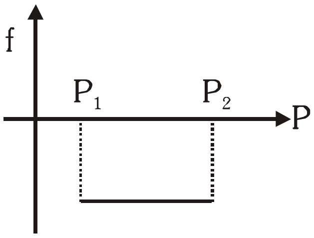
(B)



(C)



(D)

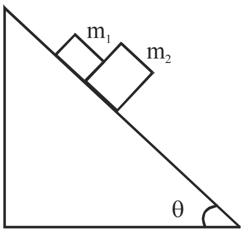


NL0117

3. A block is moving on an inclined plane making an angle 45° with the horizontal and the coefficient of friction is μ. The force required to just push it up the inclined plane is 3 times the force required to just prevent it from sliding down. If we define $N = 10\mu$, then N is **[IIT-JEE-2011]**

NL0118

4. A block of mass $m_1 = 1$ kg another mass $m_2 = 2$kg, are placed together (see figure) on an inclined plane with angle of inclination θ. Various values of θ are given in List I. The coefficient of friction between the block m_1 and the plane is always zero. The coefficient of static and dynamic friction between the block m_2 and the plane are equal to $\mu = 0.3$. In List II expressions for the friction on block m_2 are given. Match the correct expression of the friction in List II with the angles given in List I, and choose the correct option. The acceleration due to gravity is denoted by g.
[**useful information :** $\tan(5.5°) \approx 0.1$; $\tan(11.5°) \approx 0.2$; $\tan(16.5°) \approx 0.3$] **[IIT-JEE-2014]**



List–I	**List–II**
(P) $\theta = 5°$	(1) $m_2 g \sin\theta$
(Q) $\theta = 10°$	(2) $(m_1 + m_2) g \sin\theta$
(R) $\theta = 15°$	(3) $\mu m_2 g \cos\theta$
(S) $\theta = 20°$	(4) $\mu(m_1 + m_2) g \cos\theta$

Code :

(A) P-1, Q-1, R-1, S-3
(B) P-2. Q-2, R-2, S-3
(C) P-2,. Q-2, R-2, S-4
(D) P-2, Q-2, R-3, S-3

NL0119

ANSWER KEY

EXERCISE (S-1)

1. Ans. (i) $\frac{m_1}{m_2} = \frac{1}{3}$ (ii) $a = 3/4$ m/s^2 **2. Ans.** $\frac{4}{3}W$ **3. Ans.** $\frac{g}{10} m/s^2$ **4. Ans.** 200 N, 10 m/s^2

5. Ans. 2 sec **6. Ans.** $0.5g$, g **7. Ans.** 0.5 s **8. Ans.** 80 N, 48 N, 264 N **9. Ans. 24 N**

10. Ans. (i) $a_A = \frac{3g\downarrow}{2} = a_B$; $a_C = 0$; T = $mg/2$; **(ii)** $a_A = 2g\uparrow$, $a_B = 2g\downarrow$, $a_c = 0$, $T = 0;$

(iii) $a_A = a_B = g/2\uparrow$, $a_c = g\downarrow$, $T = \frac{3mg}{2}$;

11. Ans. $a = g \cot \theta$ **12. Ans.** $v_0/\cos\theta$ **13. Ans.** 5N, 16/31 kg **14. Ans.** $3a_B/4$

15. Ans. $\frac{a_0}{2}$ **16. Ans.** $\left(100\hat{i} - 200\hat{j}\right)$N **17. Ans.** 20 N vertically downward

18. Ans. (ma) **19. Ans.** 1/2 s **20. Ans.** 4/3 s **21. Ans.** $\mu_{\min} = \frac{m\sin\theta\cos\theta}{m\cos^2\theta + M}$

22. Ans. 0.5 **23. Ans.** 1 kg **24. Ans.** $10\hat{i}$ **25. Ans.** 3/4 **26. Ans.** 30 N

EXERCISE (S-2)

1. Ans. 1440 N **2. Ans.** g m/s^2 **3. Ans.** 556.8 N, 1.47 s

4. Ans. (i) $a_A = g\downarrow$, $a_B = \frac{2g}{3}\uparrow$, $a_c = \frac{2g}{3}\downarrow$, $a_D = \frac{2g}{3}\downarrow$, $a_E = 0$

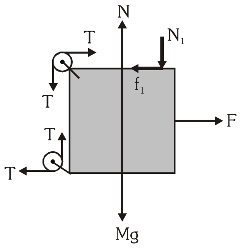
(ii) $a_A = 0$, $a_B = \frac{g}{3}\downarrow$, $a_C = \frac{g}{3}\uparrow$, $a_D = \frac{g}{3}\uparrow$, $a_E = g\downarrow$

(iii) $a_A = 0, a_B = g\downarrow, a_C = g\uparrow, a_D = 2g\downarrow, a_E = 0$

(iv) $a_A = 0, a_B = 3g\downarrow, a_C = \frac{3g}{2}\downarrow, a_D = \frac{3g}{2}\downarrow, a_E = 0$

5. Ans. $\frac{25}{24}$ **6. Ans.** 5 m/s^2 downwards, 0 m/s^2, 10 N **7. Ans.** 40 N

8. Ans. (i)



(ii) a = 3/5 m/s^2 , T = 18 N, F = 60N **9. Ans.** $-5\,\hat{i}$ m/s^2

EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

1. Ans. (B) 2. Ans. (D) 3. Ans. (D) 4. Ans. (C) 5. Ans. (D) 6. Ans. (C)
7. Ans. (C) 8. Ans. (C) 9. Ans. (D) 10. Ans. (C) 11. Ans. (B) 12. Ans. (A)
13. Ans. (C) 14. Ans. (A) 15. Ans. (C) 16. Ans. (B) 17. Ans. (A) 18. Ans. (B)
19. Ans. (D) 20. Ans. (B) 21. Ans. (D) 22. Ans. (D) 23. Ans. (B) 24. Ans. (B)
25. Ans. (C) 26. Ans. (A) 27. Ans. (D) 28.Ans. (A) 29. Ans. (B) 30. Ans. (A)
31. Ans. (A) 32. Ans. (B) 33. Ans. (D) 34. Ans. (C) 35. Ans. (B) 36. Ans. (A)
37. Ans. (B)

MULTIPLE CORRECT TYPE QUESTIONS

38. Ans. (A,D) 39. Ans. (B,C,D) 40. Ans. (A,C) 41. Ans. (A,B) 42. Ans. (A,B) 43. Ans. (A,C)

COMPREHENSION TYPE QUESTIONS

44. Ans. (D) 45. Ans. (D) 46. Ans. (A) 47. Ans. (D) 48. Ans. (B) 49. Ans. (C)
50. Ans. (B) 51. Ans. (D) 52. Ans. (C) 53. Ans. (C)

MATRIX MATCH TYPE QUESTION

54. Ans. (A) - (R); (B) - (T); (C) - (Q); (D) - (T) 55. Ans. (A) - (R); (B) - (T); (C) - (R); (D) - (P)
56. Ans. (A)-R; (B)-Q; (C)-S; (D)-P

EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

1. Ans. (B) 2. Ans. (D) 3. Ans. (C) 4. Ans. (C) 5. Ans. (C) 6. Ans. (B)
7. Ans. (C) 8. Ans. (A) 9. Ans. (A) 10. Ans. (C) 11. Ans. (C)

MULTIPLE CORRECT TYPE QUESTIONS

12. Ans. (C,D) 13. Ans. (A,C) 14. Ans. (B,D) 15. Ans. (A,C,D) 16. Ans. (B,C,D)
17. Ans. (A,B,C,D) 18. Ans. (A,B,C) 19. Ans. (A,B) 20. Ans. (A,B,D)
21. Ans. (A,B,D)

COMPREHENSION TYPE QUESTIONS

22. Ans. (B) 23. Ans. (A) 24. Ans. (B) 25. Ans. (A) 26. Ans. (A) 27. Ans. (A)
28. Ans. (C) 29. Ans. (A)

MATRIX MATCH TYPE QUESTION

30. Ans. (A) - (S) ; (B) - (R) ; (C) - (Q) 31. Ans. (A) - (T); (B) - (R); (C) - (R); (D) - (Q)

EXERCISE (JM)

1. Ans. (1) 2. Ans. (4) 3. Ans. (3) 4. Ans. (1) 5. Ans. (3) 6. Ans. (1)

EXERCISE (JA)

1. Ans. (B) 2. Ans. (A) 3. Ans. 5 4. Ans. (D)

Chapter Contents 02

Kinematics-1D & Calculus

Important Notes

KINEMATICS-1D & CALCULUS

KEY CONCEPT

Kinematics

Study of motion of objects without taking into account the factor which cause the motion (i.e. nature of force).

Motion

If a body changes its position with time, it is said to be moving else it is at rest. Motion is always relative to the observer.

Motion is a combined property of the object under study and the observer. There is no meaning of rest or motion without the viewer. In other words absolute motion or rest is meaningless.

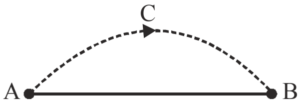
- To locate the position of a particle we need a reference frame. A commonly used reference frame is Cartesian coordinate system or simply coordinate system.

  The coordinates (x, y, z) of a particle specify the position of the particle with respect to origin of that frame. If all the three coordinates of the particle remain unchanged as time passes it means the particle is at rest w.r.t. this frame.
- The reference frame is chosen according to the problems.
- If the frame is not mentioned, then ground is taken as the reference frame.

DISTANCE AND DISPLACEMENT

Total length of path covered by the particle, in definite time interval is called distance.

Let a body moves from A to B via C. The length of path ACB is called the distance travelled by the body.



But overall, body is displaced from A to B. A vector from A to B, i.e. $\overrightarrow{AB}$ is its displacement vector or displacement that is the minimum distance and directed from initial position to final position.

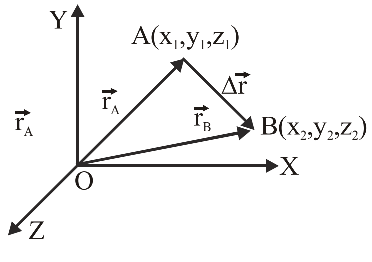
- Distance is a scalar while displacement is a vector.
- Distance depends on path while displacement is independent of path but depends only on final and initial position.
- For a moving body, distance can't have zero or negative values but displacement may be +ive, –ive or zero.
- For a moving/stationary object distance can't be decreasing.
- If motion is in straight line without change in direction then

  distance = |displacement| i.e. magnitude of displacement.
- Magnitude of displacement may be equal or less than distance but never greater than distance.

  distance $\geq$ |displacement|

Displacement in terms of position vector

Let a body is displaced from A (x_1, y_1, z_1) to B (x_2, y_2, z_2) then its displacement is given by vector $\vec{AB}$.



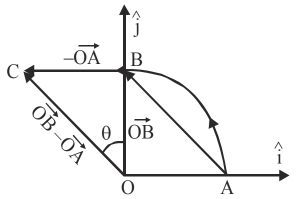
From ΔOAB $\vec{r}_A + \Delta\vec{r} = \vec{r}_B \Rightarrow \Delta\vec{r} = \vec{r}_B - \vec{r}_A$

$$\vec{r}_B = x_2\hat{i} + y_2\hat{j} + z_2\hat{k} \text{ and } \vec{r}_A = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$

$$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k} \Rightarrow \Delta\vec{r} = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}$$

Ex. A particle goes along a quadrant from A to B of a circle radius 10m as shown in fig. Find the direction and magnitude of displacement and distance along path AB.

Sol. Displacement $\vec{AB} = \vec{OB} - \vec{OA} = 10\hat{j} - 10\hat{i}$



$|\vec{AB}| = \sqrt{10^2 + 10^2} = 10\sqrt{2}$ m

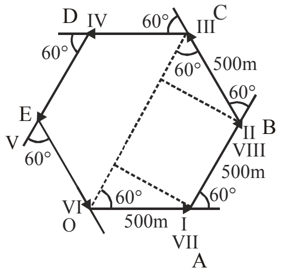
From ΔOBC $\tan\theta = \dfrac{OA}{OB} = \dfrac{10}{10} = 1 \Rightarrow \theta = 45°$

Angle between displacement vector $\vec{OC}$ and x–axis = 90° + 45° = 135°

Distance of path AB = $\dfrac{1}{4}$ (circumference) = $\dfrac{1}{4}(2\pi R)$ m = (5π) m

Ex. On an open ground a motorist follows a track that turns to his left by an angle of 60° after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of displacement with the total path length covered by the motorist in each case.

Sol. **At III turn**



Displacement $= \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{OC}$

= 500 cos 60° + 500 + 500 cos 60°

$= 500 \times \dfrac{1}{2} + 500 + 500 \times \dfrac{1}{2} = 1000$ m from O to C

Distance = 500 + 500 + 500 = 1500 m. So $\dfrac{\text{Displacement}}{\text{Distance}} = \dfrac{1000}{1500} = \dfrac{2}{3}$

At VI turn

∵ initial and final positions are same so displacement = 0 and distance = 500 × 6 = 3000 m

$\therefore \dfrac{\text{Displacement}}{\text{Distance}} = \dfrac{0}{3000} = 0$

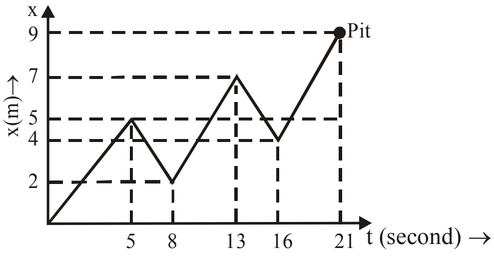
At VIII turn

$$\text{Displacement} = 2(500)\cos\left(\frac{60°}{2}\right) = 1000 \times \cos 30° = 1000 \times \frac{\sqrt{3}}{2} = 500\sqrt{3} \text{ m}$$

Distance = 500 × 8 = 4000 m

$\therefore \dfrac{\text{Displacement}}{\text{Distance}} = \dfrac{500\sqrt{3}}{4000} = \dfrac{\sqrt{3}}{8}$

Ex. A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward, followed again by 5 steps forward and 3 steps backward, and so on. Each step is 1m long and requires 1s. Plot the x–t graph of his motion. Determine graphically or otherwise how long the drunkard takes to fall in a pit 9m away from the start.



Sol. from x–t graph time taken $= 21$ s

or $(5m - 3m) + (5m - 3m) + 5m = 9m$

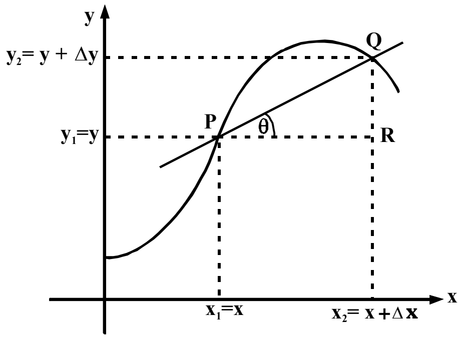
$\Rightarrow$ total steps $= 21 \Rightarrow$ time $= 21$ s

DERIVATIVE OF A FUNCTION

Average Rate of Change

Let a function y = f(x) be plotted by an arbitrary graph as shown in the figure. Average rate of change in y with respect to x in an interval $[x_1, x_2]$ is defined as

$$\text{Average rate of change} = \frac{\text{Change in y}}{\text{Change in x}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$



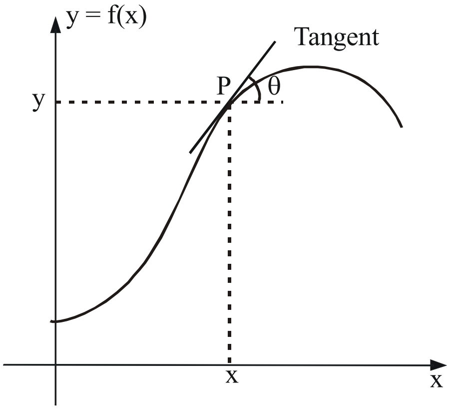
= slope of chord

If both the axes have equal scale

Average rate of change = slope of chord PQ = tanθ

Instantaneous Rate of Change : First derivative

It is defined as the rate of change in y with x at a particular value of x. It is measured graphically by the slope of the tangent drawn to the y-x graph at the point (x, y) and algebraically by the first derivative of the function y=f(x).



$$\text{Instantaneous rate of change} = \frac{dy}{dx} = \text{Slope of the tangent}$$

$$\text{If both the axes have equal scale then } \frac{dy}{dx} = \tan\theta$$

$$\text{Instantaneous rate of change} = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Derivatives of Commonly Used Functions.

- $y = \text{constant} \quad \Rightarrow \dfrac{dy}{dx} = 0$
- $y = x^n \quad \Rightarrow \dfrac{dy}{dx} = nx^{n-1}$
- $y = e^x \quad \Rightarrow \dfrac{dy}{dx} = e^x$
- $y = \ln x \quad \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{x}$
- $y = \sin x \quad \Rightarrow \dfrac{dy}{dx} = \cos x$
- $y = \cos x \quad \Rightarrow \dfrac{dy}{dx} = -\sin x$
- $y = \tan x \quad \Rightarrow \dfrac{dy}{dx} = \sec^2 x$
- $y = \cot x \quad \Rightarrow \dfrac{dy}{dx} = -\text{cosec}^2 x$
- $y = \text{cosec}\, x \quad \Rightarrow \dfrac{dy}{dx} = -\text{cosec}\, x \cot x$
- $y = \sec x \quad \Rightarrow \dfrac{dy}{dx} = \sec x \tan x$

Method of Differentiation.

If y = f(x), let us denote $\dfrac{dy}{dx} = f'(x)$

- Sum or Subtraction of two functions $y = f(x) \pm g(x) \Rightarrow \dfrac{dy}{dx} = f'(x) \pm g'(x)$
- Product of two functions

  $$y = f(x) \cdot g(x) \Rightarrow \frac{dy}{dx} = g(x)f'(x) + f(x)g'(x)$$

- Division of two functions.

  $$y = \frac{f(x)}{g(x)} \Rightarrow \frac{dy}{dx} = \frac{g(x)f'(x) - f(x)g'(x)}{\{g(x)\}^2}$$

- Chain Rule

  $$y = f\{g(x)\} \Rightarrow \frac{dy}{dx} = g'(x)f'\{g(x)\}$$

Ex. Find $\dfrac{dy}{dx}$, when (i) $y = \sqrt{x}$ (ii) $y = x^5 + x^4 + 7$ (iii) $y = x^2 + 4x^{-1/2} - 3x^{-2}$

Sol. (i) $\dfrac{dy}{dx} = \dfrac{d}{dx}(\sqrt{x}) = \dfrac{d}{dx}(x^{1/2}) = \dfrac{1}{2}x^{1/2-1} = \dfrac{1}{2}x^{-1/2} = \dfrac{1}{2\sqrt{x}}$

(ii) $\dfrac{dy}{dx} = \dfrac{d}{dx}(x^5 + x^4 + 7) = \dfrac{d}{dx}(x^5) + \dfrac{d}{dx}(x^4) + \dfrac{d}{dx}(7) = 5x^4 + 4x^3 + 0 = 5x^4 + 4x^3$

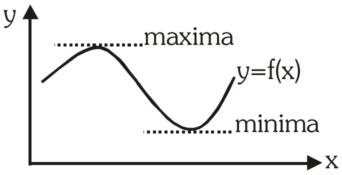
(iii) $\dfrac{dy}{dx} = \dfrac{d}{dx}(x^2 + 4x^{-1/2} - 3x^{-2}) = \dfrac{d}{dx}(x^2) + \dfrac{d}{dx}(4x^{-1/2}) - \dfrac{d}{dx}(3x^{-2})$

$$= \frac{d}{dx}(x^2) + 4\frac{d}{dx}(x^{-1/2}) - 3\frac{d}{dx}(x^{-2}) = 2x + 4\left(-\frac{1}{2}\right)x^{-3/2} - 3(-2)x^{-3} = 2x - 2x^{-3/2} + 6x^{-3}$$

Second Derivative and it's meaning

Second derivative of a function y = f(x) is defined as $\frac{d}{dx}\left[\frac{dy}{dx}\right]$. It is obtained by differentiating the function with respect to x two times successively. Geometrically it expresses rate of change in slope of graph of the function.

Maxima & Minima



Maxima & minima of a function y = f(x)

for maximum value $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2}$ = negative

for minimum value $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2}$ = positive

Ex. Find minimum value of $y = 25x^2 - 10x + 5$.

Sol. For maximum/minimum value $\frac{dy}{dx} = 0 \Rightarrow 50x - 10 = 0 \Rightarrow x = \frac{1}{5}$

Now at $x = \frac{1}{5}$, $\frac{d^2y}{dx^2} = 50$ which is positive. So $y_{min} = 25\left(\frac{1}{5}\right)^2 - 10\left(\frac{1}{5}\right) + 5 = 1 - 2 + 5 = 4$

INTEGRATION

- This operation enables us to find sum of infinite number of infinitely small quantities.

$$\lim_{\Delta x \to 0} \sum_{i=1}^{\infty} \Delta x_i = \int dx = x$$

- It is reverse operation of differentiation. If derivative, which is rate of change at a point, is given as a function $f(x) = \frac{dF(x)}{dx}$, operation of integration enables us to find original function F(x).

$$\int f(x)dx = F(x) + C$$

Here function f(x) is known as integrand, function F(x) as integral and C as constant of integration. Value of C is obtained by substituting initial, final or any other condition (known as boundary conditions) in the above equation.

This interpretation enables us to find integral of those functions whose derivative is known.

Integrand $f(x) = \frac{dF(x)}{dx}$	**Integral** $\int f(x)dx = F(x) + C$
k = Constant	kx+C
x^n	$\frac{x^{n+1}}{n+1} + C$ If $n \neq -1$
x^{-1}	$\ln x + C$
e^x	$e^x + C$
sin x	–cos x + C
cos x	sin x + C
f(ax+b)	$\frac{F(ax+b)}{a} + C$

Ex. Evaluate the following:

(i) $\int x^{-7}dx$ (ii) $\int x^{p/q}dx$

Sol. (i) $\int x^{-7}dx = \frac{x^{-7+1}}{-7+1} + c = -\frac{1}{6}x^{-6} + c$

(ii) $\int x^{\frac{p}{q}}dx = \frac{x^{\frac{p}{q}+1}}{\frac{p}{q}+1} + c = \frac{q}{p+q}x^{(p+q)/q} + c$

Ex. Evaluate $\int\left(x^2 - \cos x + \frac{1}{x}\right)dx$

Sol. $I = \int x^2dx - \int \cos x dx + \int \frac{1}{x}dx = \frac{x^{2+1}}{2+1} - \sin x + \log_e x + c = \frac{x^3}{3} - \sin x + \log_e x + c$

SPEED :

Speed is the rate of change of distance with respect to time.

Uniform speed :

An object is said to be moving with a uniform speed, if it covers equal distances in equal intervals of time, irrespective of duration of interval. The uniform speed is shown by straight line in distance–time graph.

For example, suppose a train travels 1000 m in 60 s. The train is said to be moving with uniform speed, if it travels 500 m. in 30s, 250 m in 15s, 125 m in 7.5s and so on.

Non Uniform speed :

An object is said to be moving with a variable speed if it covers equal distances in unequal intervals of time or unequal distances in equal intervals of time, irrespective of duration of interval.

For example, suppose a train travels first 1000 m in 60s next 1000 m in 120s and next 1000m in 50s, then the train is moving with variable speed.

Average speed :

Speed is distance travelled per unit time. Average speed of a trip $v_{av} = \dfrac{\text{Total travelled distance}}{\text{Total time taken}}$

If a particle travels a distance s in time t_1 to t_2, the average speed is $v_{av} = \dfrac{\Delta s}{\Delta t} = \dfrac{s}{t_2 - t_1}$

Instantaneous speed

The speed at a particular instant is defined as instantaneous speed (or speed) while average speed is defined for a time interval.

If Δt approaches zero, average speed becomes instantaneous speed. $v = \lim\limits_{\Delta t \to 0} \dfrac{\Delta s}{\Delta t} = \dfrac{ds}{dt}$

i.e. instantaneous speed is the time derivative of distance.

If a particle travels distances s_1, s_2, s_3 etc. with speeds v_1, v_2, v_3 etc. respectively, then total travelled distance

$$s = s_1 + s_2 + s_3 + \dots\dots\dots + s_n$$

Total time taken $t = \dfrac{s_1}{v_1} + \dfrac{s_2}{v_2} + \dfrac{s_3}{v_3} + \dots\dots + \dfrac{s_n}{v_n}$

No sign is needed for distance or speed.
They are always positive quantities.

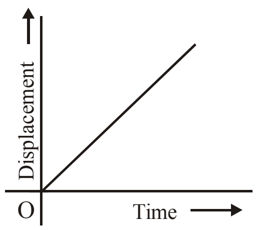
Average speed of the trip $= \dfrac{s_1 + s_2 + s_3 \dots\dots + s_n}{\left(\dfrac{s_1}{v_1} + \dfrac{s_2}{v_2} + \dots\dots + \dfrac{s_n}{v_n}\right)}$

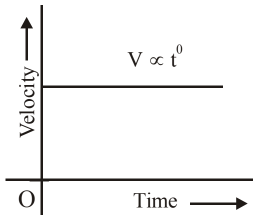
VELOCITY :

Velocity is the rate of change of displacement with respect to time.

Uniform Velocity :

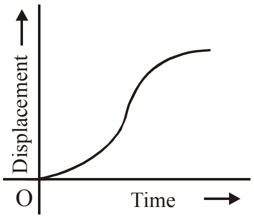
A body is said to move with uniform velocity, if it covers equal displacements in equal intervals of time, irrespective of duration of interval. When a body is moving with uniform velocity, then the magnitude and direction of the velocity of the body remains same at all points of its path.





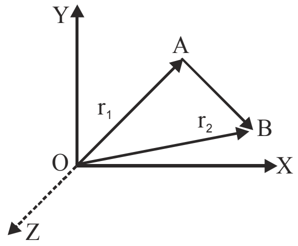
Non–uniform Velocity :

The particle is said to have non–uniform motion if it covers unequal displacements in equal intervals of time, irrespective of duration of interval. In this type of motion velocity does not remain constant.



Average Velocity

The average velocity of a particle in a time interval t_1 to t_2 is a defined as its displacement divided by the time interval. Let a particle is at a point A at time t_1 and B at time t_2. Position vectors of A and B are $\vec{r}_1$ and $\vec{r}_2$. The displacement in this time interval is the vector $\overrightarrow{AB} = (\vec{r}_2 - \vec{r}_1)$. The average velocity in this time interval is, $\vec{v}_{av} = \dfrac{\text{displacement vector}}{\text{time interval}}$

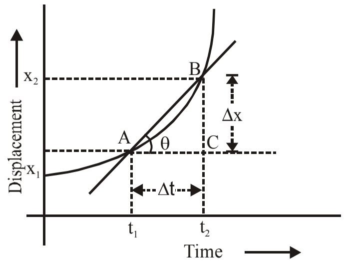


$$\vec{v}_{av} = \frac{\overrightarrow{AB}}{t_2 - t_1} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta\vec{r}}{\Delta t}$$

here $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \vec{r}_2 - \vec{r}_1$ = change in position vector.

Instantaneous velocity

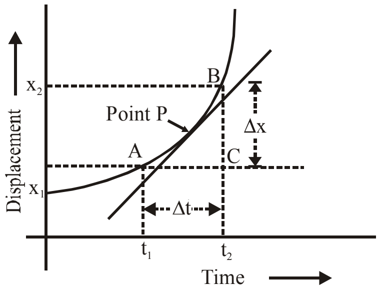
The velocity of the object at a given instant of time or at a given position during motion is called instantaneous velocity $\vec{v} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta\vec{r}}{\Delta t} = \dfrac{d\vec{r}}{dt}$



From fig., the average velocity between points A and B is

$$v_{av} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} = \text{slope of secant AB} = \tan\theta$$

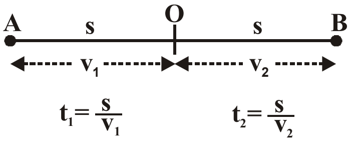
Average velocity is equal to slope of straight line joining two points on displacement time graph. If $\Delta t \to 0$, then average velocity becomes instantaneous velocity.



$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} = \text{slope of tangent at P} = \tan\alpha$$

- Magnitude of instantaneous velocity is the instantaneous speed.
  Note : When a particle moves with constant velocity, its magnitude of average velocity, its magnitude of instantaneous velocity and its speed all are equal.

Ex. If a particle travels the first half distance with speed v_1 and second half distance with speed v_2. Find its average speed during journey.

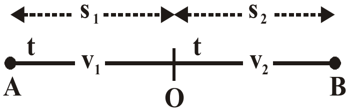


Sol. $$v_{avg.}=\frac{s+s}{t_1+t_2}=\frac{2s}{\frac{s}{v_1}+\frac{s}{v_2}}=\frac{2v_1v_2}{v_1+v_2}$$

Note :– Here $v_{avg.}$ is the harmonic mean of two speeds.

Ex. If a particle travels with speed v_1 during first half time interval and with v_2 speed during second half time interval. Find its average speed during its journey.

Sol. Total distance = $s_1 + s_2 = v_1t+v_2t = (v_1 + v_2)t$



Total time = t + t = 2t $$v_{avg.}=\frac{s_1+s_2}{t+t}=\frac{(v_1+v_2)t}{2t}=\frac{v_1+v_2}{2}$$

Note :– here v_{avg} is arithmatic mean of two speeds.

Ex. A car travels a distance A to B at a speed of 40 km/h and returns to A at a speed of 30 km/h.

(i) What is the average speed for the whole journey?

(ii) What is the average velocity?

Sol. (i) Let AB = s, time taken to go from A to B, $t_1=\frac{s}{40}$ h

and time taken to go from B to A, $t_2=\frac{s}{30}$ h

$\therefore$ total time taken = $t_1 + t_2 = \frac{s}{40}+\frac{s}{30}=\frac{(3+4)\ s}{120}=\frac{7s}{120}$ h

Total distance travelled = s + s = 2s

$$\therefore \text{ Average speed } =\frac{\text{total distance travelled}}{\text{total time taken}}=\frac{2s}{\frac{7s}{120}}=\frac{120\times 2}{7}=34.3 \text{ km/h.}$$

(ii) Total displacement = zero, since the car returns to the original position.

$$\text{Therefore, average velocity}=\frac{\text{total displacement}}{\text{time taken}}=\frac{0}{2t}=0$$

Ex. A man walks on a straight road from his home to a market 2.5 km away with a speed of 5 km/h. On reaching the market he instantly turns and walks back with a speed of 7.5 km/h. What is the (a) magnitude of average velocity and (b) average speed of the man, over the interval of time (i) 0 to 30 min. (ii) 0 to 50 min (iii) 0 to 40 min.

Sol. Time taken by man to go from his home to market, $t_1=\frac{\text{distance}}{\text{speed}}=\frac{2.5}{5}=\frac{1}{2}$ h

Time taken by man to go from market to his home, $t_2=\frac{2.5}{7.5}=\frac{1}{3}$ h

$\therefore$ Total time taken $= t_1 + t_2 = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}\,h = 50$ min.

(i) 0 to 30 min

$$\text{Average velocity} = \frac{\text{displacement}}{\text{time interval}} = \frac{2.5}{\frac{30}{60}} = 5 \text{ km/h towards market}$$

$$\text{Average speed} = \frac{\text{distance}}{\text{time interval}} = \frac{2.5}{\frac{30}{60}} = 5 \text{ km/h}$$

(ii) 0 to 50 min

Total displacement = zero so average velocity = 0

$$\text{So, average speed} = \frac{5}{50/60} = 6 \text{ km/h}$$

Total distance travelled = 2.5 + 2.5 = 5 km.

(iii) 0 to 40 min

Distance moved in 30 min (from home to market) = 2.5 km.

Distance moved in 10 min (from market to home) with speed 7.5 km/h $= 7.5 \times \frac{10}{60} = 1.25$ km

So, displacement = 2.5 – 1.25 = 1.25 km (towards market)

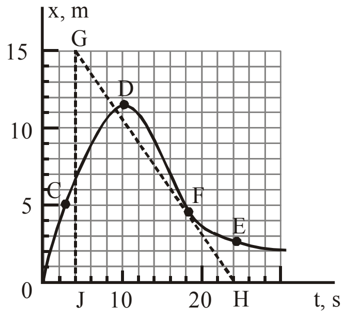
Distance travelled = 2.5 + 1.25 = 3.75 km

$$\text{Average velocity} = \frac{1.25}{\frac{40}{60}} = 1.875 \text{ km/h. (towards market)}$$

$$\text{Average speed} = \frac{3.75}{\frac{40}{60}} = 5.625 \text{ km/h.}$$

Note : Moving body with uniform speed may have variable velocity. e.g. in uniform circular motion speed is constant but velocity is non–uniform.

Ex. Refer to figure for the motion of an object along the x-axis.



What is the instantaneous velocity of the object at (a) F (b) D

Sol. **(a)** The tangent at F is the dashed line GH. Taking triangle GHJ,

$\Delta t = 24 - 4 = 20$ s $\quad \Delta x = 0 - 15 = -15$m

Hence slope at F is $v_F = \dfrac{\Delta x}{\Delta t} = \dfrac{-15\text{m}}{20\ \text{s}} = -0.75$ m/s

The negative sign tells us that the object is moving in the –x direction.

(b) At point D slope of curve is zero so $v = \dfrac{dx}{dt} = 0$.

ACCELERATION :

Acceleration

The acceleration is rate of change of velocity or change in velocity per unit time interval.

Velocity is a vector quantity hence a change in its magnitude or in direction or in both, will change the velocity .

Uniform acceleration :

An object is said to be moving with a uniform acceleration if its velocity changes by equal amounts in equal intervals of time, irrespective of duration of intervals.

Variable acceleration :

An object is said to be moving with a variable acceleration if its velocity changes by unequal amounts in equal intervals of time, irrespective of duration of intervals..

Average Acceleration :

When an object is moving with a variable acceleration, then the average acceleration of the object for the given motion is defined as the ratio of the total change in velocity of the object during motion to the total time taken

$$\text{Average Acceleration} = \frac{\text{Total change in velocity}}{\text{total time taken}}$$

Suppose the velocity of a particle is $\vec{v}_1$ at time t_1 and $\vec{v}_2$ at time t_2. Then $\vec{a}_{av} = \dfrac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \dfrac{\Delta \vec{v}}{\Delta t}$

Instantaneous Acceleration :

The acceleration of the object at a given instant of time or at a given point of motion, is called its instantaneous acceleration. Suppose the velocity of a particle at time $t_1 = t$ is $\vec{v}_1 = \vec{v}$ and becomes

$\vec{v}_2 = \vec{v} + \Delta\vec{v}$ at time $t_2 = t + \Delta t$, Then, $\vec{a}_{av} = \dfrac{\Delta \vec{v}}{\Delta t}$

If Δt approaches to zero then the rate of change of velocity will be instantaneous acceleration.

$$\text{Instantaneous acceleration } \vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right) = \frac{d^2\vec{r}}{dt^2}$$

Ex. An athlete takes 2 second to reach the maximum speed of 18 km/h from rest. What is the magnitude of his average acceleration.

Sol. Here, Initial velocity u = 0, $v = (v_{max}) = 18$ km/h $= 18 \times \dfrac{5}{18} = 5$ m/s, $t_1 = 0$s, $t_2 = 2$s.

$$a_{av} = \frac{v - u}{t_2 - t_1} = \frac{5.0}{2} = 2.5 \text{ m/s}^2$$

Ex. A car moving with a velocity of 20 ms^{-1} is brought to rest in 5 seconds by applying brakes. Calculate the retardation of the car.

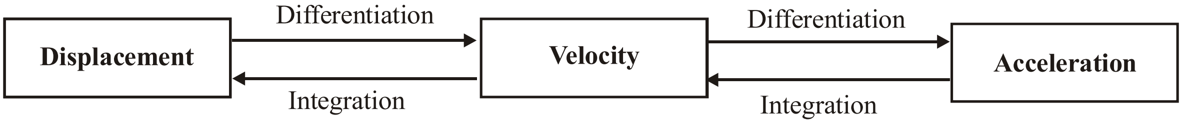
Sol. Here, $u = 20\text{ ms}^{-1}$, $v = 0$, $t = 5\text{s}$. acceleration $a = \frac{v-u}{t} = \frac{(0-20)}{5} = -4\text{m/s}^2$

–ve acceleration is known as retardation. Thus, retardation of the car = 4 ms^{-2}.

Use of Mathematical Tools in Solving Problems of One–Dimensional Motion

If displacement–time equation is given, we can get velocity–time equation with the help of differentiation. Again, we can get acceleration–time equation with the help of differentiation.

If acceleration–time equation is given, we can get velocity–time equation by integration. From velocity equation, we can get displacement–time equation by integration.



Ex. The velocity of any particle is related with its displacement as; $x = \sqrt{v+1}$, Calculate acceleration at $x = 5$ m.

Sol. $\therefore x = \sqrt{v+1}$ $\therefore x^2 = v + 1 \Rightarrow v = (x^2-1)$

Therefore $a = \frac{dv}{dt} = \frac{d}{dt}\left(x^2-1\right) = 2x\frac{dx}{dt} = 2x\ \ v = 2x\left(x^2-1\right)$

At $x = 5$ m, $a = 2 \times 5\ (25 - 1) = 240\text{ m/s}^2$

Ex. The velocity of a particle moving in the positive direction of x–axis varies as $v = \alpha\sqrt{x}$ where α is positive constant. Assuming that at the moment $t = 0$, the particle was located at $x = 0$ find, (i) the time dependance of the velocity and the acceleration of the particle and (ii) the average velocity of the particle averaged over the time that the particle takes to cover first s metres of the path.

Sol. (i) Given that $v = \alpha\sqrt{x}$

$$\Rightarrow \frac{dx}{dt} = \alpha\sqrt{x}\ \therefore \frac{dx}{\sqrt{x}} = \alpha\,dt \Rightarrow \int_0^x \frac{dx}{\sqrt{x}} = \int_0^t \alpha\,dt\ \ 2\sqrt{x} = \alpha t \Rightarrow x = \left(\alpha^2 t^2/4\right)$$

Velocity $\frac{dx}{dt} = \frac{1}{2}\alpha^2 t$ and Acceleration $\frac{d^2x}{dt^2} = \frac{1}{2}\alpha^2$

(ii) Time taken to cover first s metres $s = \frac{\alpha^2 t^2}{4} \Rightarrow t^2 = \frac{4s}{\alpha^2} \Rightarrow t = \frac{2\sqrt{s}}{\alpha}$;

$$\text{average velocity} = \frac{\text{total displacement}}{\text{total time}} = \frac{s\alpha}{2\sqrt{s}} = \frac{1}{2}\sqrt{s}\alpha$$

Ex. A particle moves in the plane xy with constant acceleration a directed along the negative y–axis. The equation of motion of the particle has the form $y = px - qx^2$ where p and q are positive constants. Find the velocity of the partcle at the origin of coordinates.

Sol. Given that $y = px - qx^2$

$$\therefore \frac{dy}{dt} = p\frac{dx}{dt} - q.2x\frac{dx}{dt} \text{ and } \frac{d^2y}{dt^2} = p\frac{d^2x}{dt^2} - 2qx\frac{d^2x}{dt^2} - 2q\left(\frac{dx}{dt}\right)^2 = (p-2qx)\frac{d^2x}{dt^2} - 2q\left(\frac{dx}{dt}\right)^2$$

$$\because \frac{d^2x}{dt^2} = 0 \text{ (no acceleration along x–axis) and } \frac{d^2y}{dt^2} = -a$$

$$\therefore v_x^2 = \frac{a}{2q} \Rightarrow v_x = \sqrt{\frac{a}{2q}} \text{ Further, } \left(\frac{dy}{dt}\right)_{x=0} = p\frac{dx}{dt} \Rightarrow v_y = p\sqrt{\left(\frac{a}{2q}\right)}$$

$$\text{Now } v = \sqrt{\left(v_x^2 + v_y^2\right)} = \sqrt{\left(\frac{a}{2q} + \frac{ap^2}{2q}\right)} \Rightarrow v = \sqrt{\left[\frac{a\left(p^2+1\right)}{2q}\right]}$$

Ex. A particle moves along a straight line path such that its magnitude of velocity is given by $v = (3t^2 - 6t)\ ms^{-1}$, where t is the time in seconds. If it is initially located at the origin O then determine the magnitude of particle's average velocity and average speed in time interval from $t = 0$ to $t = 4s$.

Sol. Average velocity $$= \frac{\int v dt}{\int dt} = \frac{\int_0^4 \left(3t^2 - 6t\right)dt}{\int_0^4 dt} = \frac{\left(t^3 - 3t^2\right)_0^4}{\left(t\right)_0^4} = 4\ ms^{-1}$$

$$\text{Average speed} = \frac{\int |v| dt}{\int dt} = \frac{\int_0^4 \left|3t^2 - 6t\right| dt}{\int_0^4 dt} = \frac{\int_0^2 \left(6t - 3t^2\right)dt + \int_2^4 \left(3t^2 - 6t\right)dt}{\int_0^4 dt}$$

$$= \frac{\left(3t^2 - t^3\right)_0^2 + \left(t^3 - 3t^2\right)_2^4}{\left(t\right)_0^4} = \frac{24}{4} = 6\ ms^{-1}$$

Ex. The coordinates a particle moving in a plane are given by $x = 3\cos 2t$ and $y = 4\sin 2t$.

(i) Find the equation of the path of the particle.

(ii) Find the angle between $\vec{r}$ and $\vec{v}$ at $t = \frac{\pi}{4}$.

(iii) Prove that acceleration of the particle is always directed towards a fixed point.

Sol. (i) Eliminating t from $x = 3\cos 2t$ & $y = 4\sin 2t$. We get $\left(\frac{x}{3}\right)^2 + \left(\frac{y}{4}\right)^2 = 1 \Rightarrow \frac{x^2}{9} + \frac{y^2}{16} = 1$

(ii) $\vec{r} = x\hat{i} + y\hat{j} = 3\cos 2t\,\hat{i} + 4\sin 2t\,\hat{j} \Rightarrow \vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = -6\sin 2t\,\hat{i} + 8\cos 2t\,\hat{j}$

At $t = \frac{\pi}{4}$, $\vec{r} = 4\hat{j}$, $\vec{v} = -6\hat{i}$

Angle between $\vec{r}$ and $\vec{v}$ $\cos\theta = \frac{\vec{r}.\vec{v}}{rv} = \frac{(4\hat{j}).(-6\hat{i})}{(4)(6)} = 0 \Rightarrow \theta = \frac{\pi}{2}$

(iii) $\vec{a} = \frac{d\vec{v}}{dt} = -12\cos 2t\,\hat{i} - 16\sin 2t\,\hat{j} = -4\,(3\cos 2t\,\hat{i} + 4\sin 2t\,\hat{j}) = -4\vec{r}$

So acceleation is always directed toward origin (a fixed point)

Ex. A particle moves in a straight line according to the relation $x = \frac{t^3}{3} - \frac{5t^2}{2} + 6t$.

Find the displacement and distance travelled by the particle upto t = 4 sec.

Sol. $v = \frac{dx}{dt} = t^2 - 5t + 6$

The particle turns when $v = 0 = t^2 - 5t + 6 \Rightarrow (t-2)(t-3) = 0$ i.e. t = 2 sec, 3 secs

Displacement $= x(4) - x(0) = \frac{64}{3} - \frac{80}{2} + 24 = \frac{16}{3}$ m

Distance $= |x(2) - x(0)| + |x(3) - x(2)| + |x(4) - x(3)|$

$$= \left[\frac{8}{3} - 10 + 12\right] + \left|\left(9 - \frac{45}{2} + 18 - \frac{14}{3}\right)\right| + \left|\frac{16}{3} - \frac{9}{2}\right| = \frac{2}{3} + \frac{1}{6} + \frac{5}{6} = \frac{5}{3}\text{ m}$$

Alter : distance $= \int_0^4 |v|\,dt$

Equations of motion (motion with constant acceleration)

If a particle moves with acceleration $\vec{a}$, then by definition $\vec{a} = \frac{d\vec{v}}{dt} \Rightarrow d\vec{v} = \vec{a}dt$. Let at starting (t = 0)

initial velocity of the particle $\vec{u}$ and at time t its final velocity $= \vec{v}$ then $\int_{\vec{u}}^{\vec{v}} d\vec{v} = \int_0^t \vec{a}dt$

If acceleration is constant

$$\int_{\vec{u}}^{\vec{v}} d\vec{v} = \vec{a}\int_0^t dt \Rightarrow [\vec{v}]_{\vec{u}}^{\vec{v}} = \vec{a}[t]_0^t \Rightarrow \vec{v} - \vec{u} = \vec{a}\,t \Rightarrow \vec{v} = \vec{u} + \vec{a}t \qquad ....(1)$$

Now by definition of velocity, equation (1) reduces to

$$\vec{v} = \frac{d\vec{s}}{dt} = \vec{u} + \vec{a}t \Rightarrow \int_0^{\vec{s}} d\vec{s} = \int_0^t (\vec{u} + \vec{a}t)\,dt \Rightarrow \vec{s} = \left[\vec{u}t + \frac{1}{2}\vec{a}t^2\right]_0^t \Rightarrow \vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2 \qquad ....(2)$$

Now substituting the value of t from equation (1) to equation (2)

$$s = u\frac{(v-u)}{a} + \frac{1}{2}a\left[\frac{v-u}{a}\right]^2 \Rightarrow 2as = 2uv - 2u^2 + v^2 + u^2 - 2uv \Rightarrow v^2 = u^2 + 2as \text{ ....(iii)}$$

vector form of equation (iii) $\boxed{v^2 = u^2 + 2\vec{a}.\vec{s}}$ ....(3)

These three equation are called equations of motion and are applicable only and only when acceleration is constant.

Distance travalled by the body in n^{th} second

$$s_{n^{th}} = s_n - s_{n-1} = un + \frac{1}{2}an^2 - u(n-1) - \frac{1}{2}a(n-1)^2 = un + \frac{1}{2}an^2 - un + u - \frac{1}{2}an^2 + an - \frac{a}{2}$$

vector form of equation (iv)

$$\boxed{s_{n^{th}} = u + \frac{a}{2}(2n-1)} \text{ ....(4)}$$

Ex. A driver takes 0.20 s to apply the brakes after he sees a need for it. This is called the reaction time of the driver. If he is driving a car at a speed of 54 km/h and the brakes cause a deceleration of 6.0m/s^2, find the distance travelled by the car after he sees the need to put the brakes on

Sol. Distance covered by the car during the application of brakes by driver –

$$s_1 = ut = \left(54 \times \frac{5}{18}\right)(0.2) = 15 \times 0.2 = 3.0 \text{ meter}$$

After applying the brakes; v = 0 u = 15 m/s, a = 6 m/s^2 s_2 = ?

Using $v^2 = u^2 - 2as \Rightarrow 0 = (15)^2 - 2 \times 6 \times s_2 \Rightarrow 12\, s_2 = 225 \Rightarrow s_2 = \frac{225}{12} = 18.75$ metre

Distance travelled by the car after driver sees the need for it $s = s_1 + s_2 = 3 + 18.75 = 21.75$ metre.

Ex. A passenger is standing d distance away from a bus. The bus begins to move with constant acceleration a. To catch the bus, the passenger runs at a constant speed u towards the bus. What must be the minimum speed of the passenger so that he may catch the bus?

Sol. Let the passenger catch the bus after time t.

The distance travelled by the bus, $s_1 = 0 + \frac{1}{2}at^2$ ....(1)

and the distance travelled by the passenger $s_2 = ut + 0$ ....(2)

Now the passenger will catch the bus if $d + s_1 = s_2$ ....(3)

$$\Rightarrow d + \frac{1}{2}at^2 = ut \Rightarrow \frac{1}{2}at^2 - ut + d = 0 \Rightarrow t = \frac{[u \pm \sqrt{u^2 - 2ad}]}{a}$$

So the passenger will catch the bus if t is real, i.e., $u^2 \geq 2\,ad \Rightarrow u \geq \sqrt{2ad}$

So the minimum speed of passenger for catching the bus is $\sqrt{2ad}$.

Vertical motion under gravity

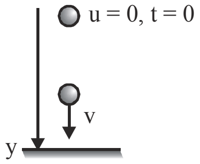
If air resistance is neglected and a body is freely moving along vertical line near the earth surface then an acceleration downward which is $9.8 m/s^2$ or $980 cm/s^2$ or $32 ft/s^2$ is experienced by the body.

Freely falling bodies from a height h above the ground

Taking initial position as origin and direction of motion (i.e. downward direction) positive y axis, as body is just released/dropped u = 0

acceleration along +Y axis a = g

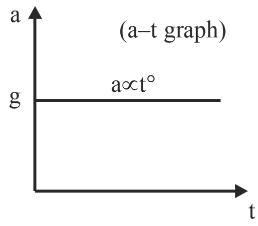
Use equations of motion to describe the motion, i.e.

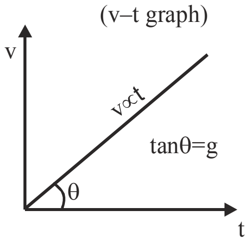


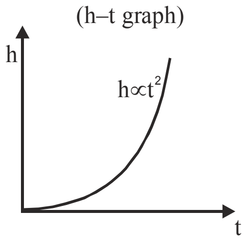
$$v = u + at,\ \ y = ut + \frac{1}{2}at^2,\ v^2 = u^2 + 2ay$$

Let the body acquires velocity v(downward) after falling a distance h in time t, then

$$v = gt \Rightarrow t = v/g \because h = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}},\ v^2 = 2gh \Rightarrow v = \sqrt{2gh}$$







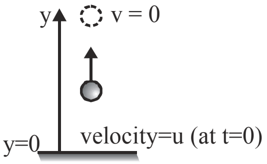
Body is projected vertically upward : With velocity u take initial position as origin and direction of motion

(i.e. vertically upward) as positive y–axis.

v = 0 at maximum height, at t = T,

a = – g (because directed downward)

Put the values in equation of motion



$$v = u + at \Rightarrow 0 = u - gT \Rightarrow u = gT$$

$$s = ut + \frac{1}{2}at^2 \qquad \Rightarrow h = ut - \frac{1}{2}gt^2$$

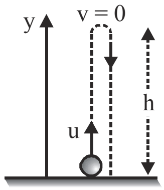
$$\Rightarrow h_{max} = uT - \frac{1}{2}gT^2 \qquad \Rightarrow h_{max} = (gT)T - \frac{1}{2}gT^2 = \frac{1}{2}gT^2$$

$$v^2 = u^2 + 2as \qquad \Rightarrow v^2 = u^2 - 2gh \Rightarrow 0 = u^2 - 2g\,h_{max} \Rightarrow u^2 = 2g\,h_{max} \Rightarrow u = \sqrt{2g\,h_{max}}$$

After attaining maximum height body turns and come back at ground. During complete flight acceleration is constant,

Time taken during up flight and down flight are equal

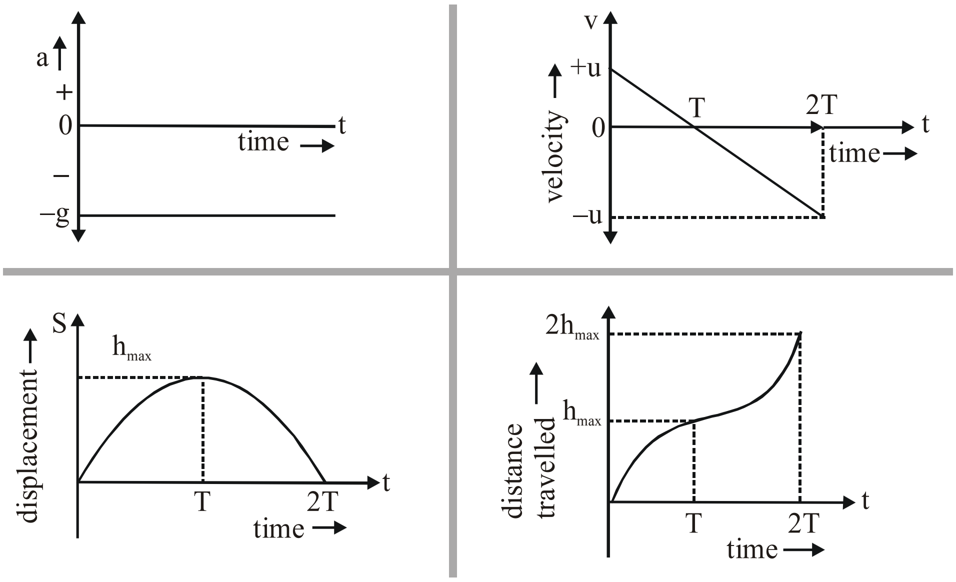
$$\text{Time for one side } T = \frac{u}{g} \text{ and total flight time} = 2T = \frac{2u}{g}$$



At each equal height from ground speed of body will be same either going up or coming down.

SOME RELATED GRAPHS FOR ABOVE MOTION'S

GRAPHS



Ex. A body is freely dropped from a height h above the ground. Find the ratio of distances fallen in first one second, first two seconds, first three seconds, also find the ratio of distances fallen in 1st second, in 2nd second, in 3rd second etc.

Sol. From second equation of motion, i.e. $h = \frac{1}{2}gt^2$ $(h = ut + \frac{1}{2}gt^2$ and $u = 0)$

$h_1 : h_2 : h_3 \ldots\ldots = \frac{1}{2}g(1)^2 : \frac{1}{2}g(2)^2 : \frac{1}{2}g(3)^2 = 1^2 : 2^2 : 3^2 \ldots\ldots.. = 1 : 4 : 9 : \ldots\ldots$.

Now from the of distance travelled in nth second

$$s_n = u + \frac{1}{2}a(2n-1) \text{ here } u = 0, a = g \Rightarrow s_n = \frac{1}{2}g(2n-1)$$

$$\Rightarrow s_1 : s_2 : s_3 \ldots\ldots.. = \frac{1}{2}g(2\times1-1) : \frac{1}{2}g(2\times2-1) : \frac{1}{2}g(2\times3-1) = 1 : 3 : 5 \ldots\ldots..$$

Ex. A rocket is fired vertically up from the ground with a resultant vertical acceleration of 10m/s^2. The fuel is finished in 1 minute and it continues to move up.

(a) What is the maximum height reached?

(b) After finishing fuel, calculate the time for which it continues its upwards motion. (Take $g = 10$ m/s^2)

Sol. (a) The distance travelled by the rocket during burning interval (1minute= 60s) in which resultant acceleration 10 m/s^2 is vertically upwards will be $h_1 = 0 \times 60 + (1/2) \times 10 \times 60^2 = 18000$ m = 18 km and velocity acquired by it will be $v = 0 + 10 \times 60 = 600$ m/s

Now after 1 minute the rocket moves vertically up with initial velocity of 600 m/s and acceleration due to

gravity opposes its motion. So, it will go to a height h_2 from this point, till its velocity becomes zero such that

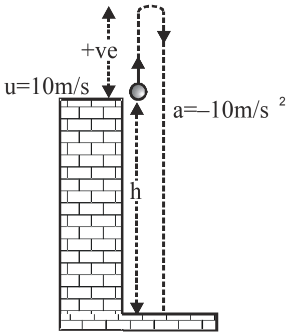
$0 = (600)^2 - 2gh_2 \Rightarrow h_2 = 18000$ m = 18 km [$g = 10$ ms^{-2}]

So the maximum height reached by the rocket from the ground, $H = h_1 + h_2 = 18 + 18 = 36$ km

(b) As after burning of fuel the initial velocity 600m/s and gravity opposes the motion of rocket, so from 1st equation of motion time taken by it till it velocity $v = 0$

$0 = 600 - gt \Rightarrow t = 60$ s

Ex. A ball is thrown upwards from the top of a tower 40 m high with a velocity of 10 m/s, find the time when it strikes the ground ($g = 10$ m/s^2)



Sol. In the problem $u = +10$ m/s, $a = -10$ m/s^2 and $s = -40$m (at the point where ball strikes the ground)

Substituting in $s = ut + \frac{1}{2}at^2$

$-40 = 10t - 5t^2 \Rightarrow 5t^2 - 10t - 40 = 0 \Rightarrow t^2 - 2t - 8 = 0$

Solving this we have $t = 4$ s and -2s. Taking the positive value $t = 4$s.

Ex. The acceleration of a particle moving in a straight line varies with its displacement as, $a = 2s$ velocity of the particle is zero at zero displacement. Find the corresponding velocity displacement equation.

Sol. $a = 2s \Rightarrow \frac{dv}{dt} = 2s \Rightarrow \frac{dv}{ds}.\frac{ds}{dt} = 2s \Rightarrow \frac{dv}{ds}.v = 2s$

$$\Rightarrow \int v\,dv = 2\int s\,ds \Rightarrow \left(\frac{v^2}{2}\right)_0^v = 2\left(\frac{s^2}{2}\right)_0^s$$

$$\Rightarrow \frac{v^2}{2} = s^2 \Rightarrow v = s\sqrt{2}$$

Ex. If a body travels half its total path in the last second of its fall from rest, find :

(a) The time and

(b) height of its fall. Explain the physically unacceptable solution of the quadratic time equation. ($g = 9.8$ m/s^2)

Sol. If the body falls a height h in time t, then

$h = \frac{1}{2}gt^2$ [$u = 0$ as the body starts from rest] ... (1)

Now, as the distance covered in $(t - 1)$ second is $h' = \frac{1}{2}g(t-1)^2$ ... (2)

So from Equations (1) and (2) distance travelled in the last second.

$h - h' = \frac{1}{2}gt^2 - \frac{1}{2}g(t-1)^2$ i.e., $h - h' = \frac{1}{2}g(2t - 1)$

But according to given problem as (h –h') $= \frac{h}{2}$

i.e., $\left(\frac{1}{2}\right)h = \left(\frac{1}{2}\right)g(2t-1)$ or $\left(\frac{1}{2}\right)gt^2 = g(2t-1)$ [as from equation (1) $h = \left(\frac{1}{2}\right)gt^2$]

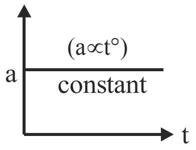
or $t^2 - 4t + 2 = 0$ or $t = [4 \pm \sqrt{(4^2 - 4\times 2)}]/2$ or $t = 2 \pm \sqrt{2}$ $\Rightarrow t = 0.59$ s or 3.41 s

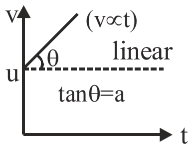
0.59 s is physically unacceptable as it gives the total time t taken by the body to reach ground lesser than one sec while according to the given problem time of motion must be greater than 1s.

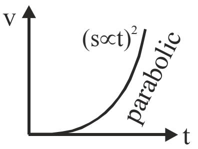
so $t = 3.41$s and $h = \frac{1}{2} \times (9.8) \times (3.41)^2 = 57$ m

Graphs based on 1–D

For constant acceleration, a/t, v/t and s/t curve from equations of motion are –







In case of constant acceleration motion in a straight line, scalar form of equations of motion can be applied and problem becomes fairly simple.

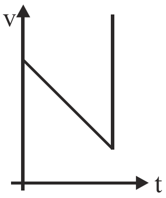
As $d\vec{v} = \vec{a}dt$ or $[\vec{v}]_{\vec{u}}^{\vec{v}} = \vec{v} - \vec{u} = \int_{t_1}^{t_2} \vec{a}dt$ = Area between curve and time axis from t_1 to t_2.

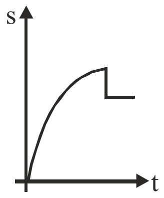
Area under the curve of a – t graph always gives the change in velocity.

Similarly $d\vec{s} = \int \vec{v}dt$ or $\vec{s} = \int_{t_1}^{t_2} \vec{v}dt$ = Area between curve and time axis from t_1 to t_2.

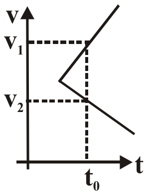
Here $\vec{s}$ is the displacement of particle in time interval t_1 to t_2, i.e. area under the curve of v/t graph always gives the displacement. If only magnitude of area is taken into account then sum of all area is the total distance travelled by the particle.

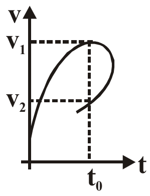
- Slopes of v–t or s–t graphs can never be infinite at any point, because infinite slope of v–t graph means infinite acceleration. Similarly, infinite slope of s–t graph means infinite velocity. Hence, the following graphs are not possible.

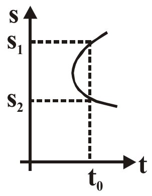




- At one time, two values of velocity or displacement are not possible Hence, the following graphs are not acceptable.







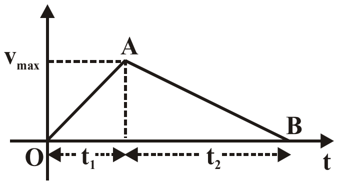
- The slope of velocity–time graph of uniform motion is zero.
- When a body is having uniform motion along a straight line in a given direction, the magnitude of the displacement of body is equal to the actual distance travelled by the body in the given time.
- The average and instantaneous velocity in a uniform motion are equal in magnitude.
- In a uniform motion along a straight line, the slope of position–time graph gives the velocity of the body.
- The position–time graph of a body moving along a straight line can never be a straight line parallel to position axis because it will indicate infinite velocity.
- The speed of a body can never be negative
- Medium effects the motion of a body falling freely under gravity due to thrust and viscous drag.

Ex. A car accelerates from rest at a constant rate α for some time, after which it decelerates at a constant rate β, to come to rest. If the total time elapsed is t evaluate (a) the maximum velocity attained and (b) the total distance travelled.

Sol. (a) Let the car accelerates for time t_1 and decelerates for time t_2 then

$$t = t_1 + t_2 \qquad \text{......(i)}$$

and corresponding velocity–time graph will be as shown in. fig.



From the graph α = slope of line AB $= \dfrac{v_{max}}{t_1} \Rightarrow t_1 = \dfrac{v_{max}}{\alpha}$

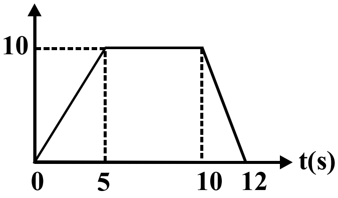
and β = – slope of line OB $= \dfrac{v_{max}}{t_2} \Rightarrow t_2 = \dfrac{v_{max}}{\beta}$

$$\Rightarrow \frac{v_{max}}{\alpha} + \frac{v_{max}}{\beta} = t \Rightarrow v_{max}\left(\frac{\alpha+\beta}{\alpha\beta}\right) = t \Rightarrow v_{max} = \frac{\alpha\beta t}{\alpha+\beta}$$

(b) Total distance = area under v–t graph $= \dfrac{1}{2} \times t \times v_{max} = \dfrac{1}{2} \times t \times \dfrac{\alpha\beta t}{\alpha+\beta} = \dfrac{1}{2}\left(\dfrac{\alpha\beta t^2}{\alpha+\beta}\right)$

Note: This problem can also be solved by using equations of motion (v = u + at, etc.).

Ex. Draw displacement time and acceleration – time graph for the given velocity–time graph



Sol. For $0 \le t \le 5 \; v \propto t \Rightarrow s \propto t^2$ and $a_1 =$ constant $\frac{10}{5} = 2 \text{ ms}^{-2}$

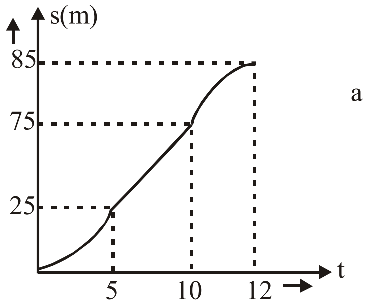
for whole interval s_1 = Area under the curve = $\frac{1}{2} \times 5 \times 10 = 25$ m

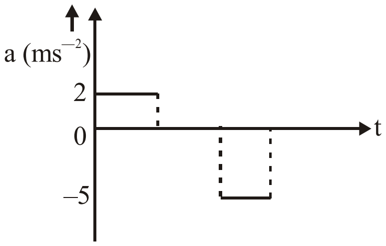
For $5 \le t \le 10$ $\quad v = 10 \text{ms}^{-1}$ $\quad \Rightarrow a = 0$

for whole interval s_2 = Area under the curve = $\frac{1}{2} \times 5 \times 10 = 50$ m

For $10 \le t \le 12$ v linearly decreases with time $\Rightarrow a_3 = -\frac{10}{2} = -5 \text{ ms}^{-1}$

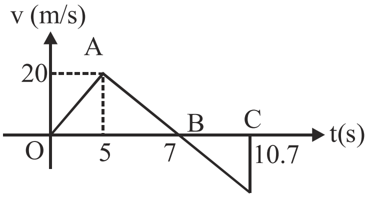
for whole interval s_3= Area under the curve = $\frac{1}{2} \times 2 \times 10 = 10$ m

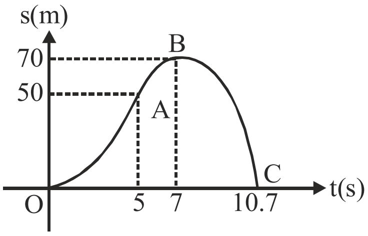




Ex. A rocket is fired upwards vertically with a net acceleration of 4 m/s^2 and initial velocity zero. After 5 seconds its fuel is finished and it decelerates with g. At the highest point its velocity becomes zero. Then it accelerates downwards with acceleration g and return back to ground. Plot velocity–time and displacement–time graphs for the complete journey. Take g = 10 m/s^2.

Sol.





In the graphs, $v_A = at_{OA} = (4)(5) = 20$ m/s $\qquad v_B = 0 = v_A - gt_{AB}$

$\therefore t_{AB} = \frac{v_A}{g} = \frac{20}{10} = 2\text{s}$ $\qquad \therefore t_{OAB} = (5 + 2)\text{s} = 7\text{s}$

Now, s_{OAB} = area under v–t graph between 0 to 7 s = $\frac{1}{2}(7)(20) = 70$ m

Now, $s_{OAB} = s_{BC} = \frac{1}{2} gt^2_{BC}$ $\qquad \therefore 70 = \frac{1}{2}(10) t^2_{BC}$

$\therefore \quad t_{BC} = \sqrt{14} = 3.7\text{s}$ $\qquad \therefore t_{OAB} = 7 + 3.7 = 10.7\text{s}$

Also s_{OA} = area under v–t graph between OA = $\frac{1}{2}(5)(20) = 50$ m

Ex. At the height of 500m, a particle A is thrown up with v = 75 ms^{-1} and particle B is released from rest. Draw, accelearation –time, velocity–time, speed–time and displacement–time graph of each particle.

For particle A :

Time of flight

$$-500 = +75\,t - \frac{1}{2} \times 10t^2$$

$\Rightarrow t^2 - 15t - 100 = 0$

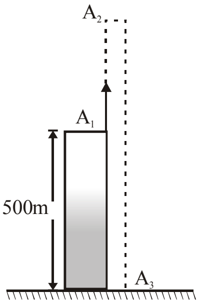
$\Rightarrow t = 20$ s

Time taken for A_1A_2

$= 75 - 10t \Rightarrow t = 7.5$ s

Velocity at A_3, $v = 75 - 10 \times 20 = -125$ ms^{-1}

$$\text{Height } A_2A_1 = \frac{1}{2}(10)(7.5)^2 = 281.25 \text{ m}$$



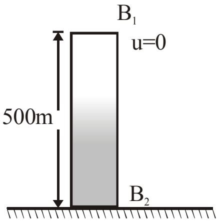
For Particle B

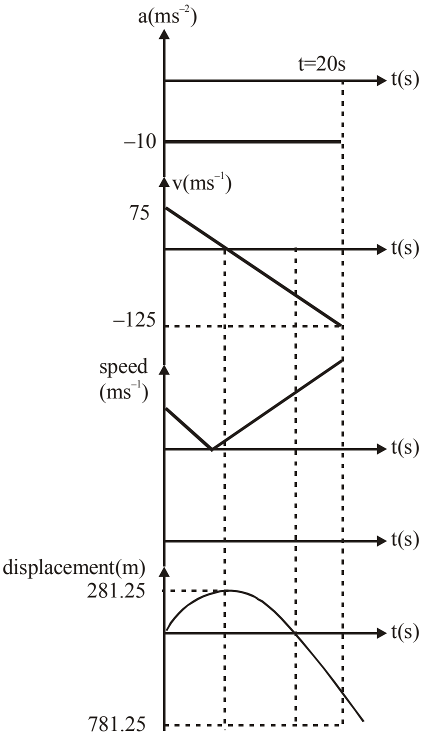
Time of flight

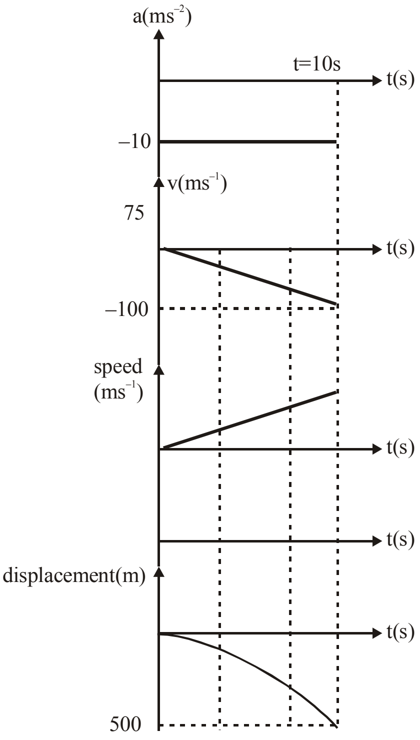
$$500 = \frac{1}{2}(10)t^2 \Rightarrow t = 10 \text{ s}$$

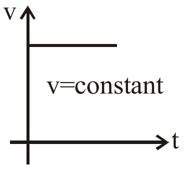
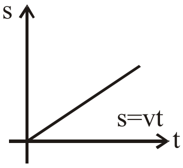
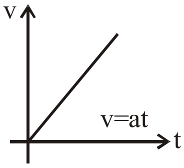
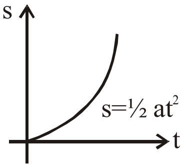
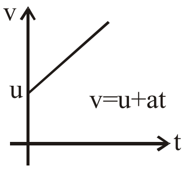
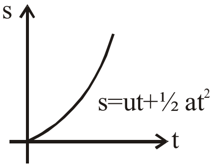
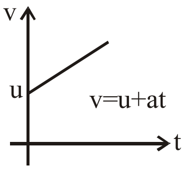
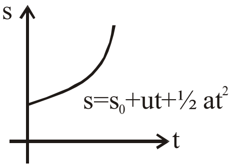
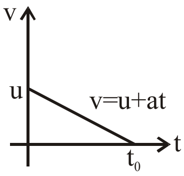
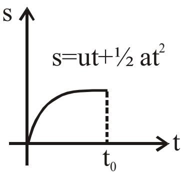
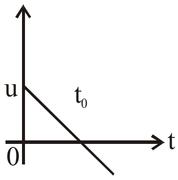
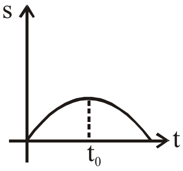
Velocity at B_2

$v = 0 - (10)(10) = -100$ ms^{-1}





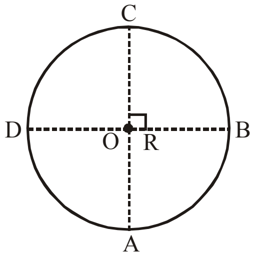


S.N.	Different Cases	v–t graph	s–t graph	Important Points
1.	Uniform motion			(i) Slope of s–t graph = v = constant (ii) In s–t graph s = 0 at t = 0
2.	Uniformly accelerated motion with u = 0 at t = 0			(i) u = 0, i.e. v = 0 at t = 0 (ii) u = 0, i.e., slope of s–t graph at t = u, should be zero (iii) a or slope of v – t graph is constant
3.	Uniformly accelerated with $u \neq 0$ at t = 0			(i) $u \neq 0$, i.e., v or slope motion of s – t graph at t = 0 is not zero (ii) v or slope of s – t graph gradually goes on increasing.
4.	Uniformly accelerated motion with $u \neq 0$ and $s = s_0$ at t = 0			(i) $s = s_0$ at t = 0
5.	Uniformly retarded motion till velocity becomes zero			(i) Slope of s – t graph at t = 0 gives u (ii) Slope of s – t graph at $t = t_0$ becomes zero (iii) In this case u can't be zero.
6.	Uniformly retarded then accelerated in opposite direction			(i) At time $t = t_0$, v = 0 or slope of s – t graph is zero (ii) In s – t graph slope or velocity first decreases then increases with opposite sign.

EXERCISE (S-1)

Definitions of kinematics variables

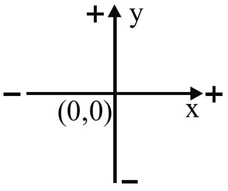
1. A particles starts from point A with constant speed v on a circle of radius R. Find magnitude of average velocity during its journey from :-



(a) A to B (b) A to C (c) A to D

KM0001

2. A particle is moving along x-axis. Initially it is located 5 m left of origin and it is moving away from the origin and slowing down. In this coordinate system, what are the signs of the initial velocity and acceleration.



v_0	a

KM0002

Motion with constant acceleration

3. A car accelerates with uniform rate from rest on a straight road. The distance travelled in the last second of a three second interval from the start is 15 m then find the distance travelled in first second in m.

KM0003

4. A particle moving in one-dimension with constant acceleration of 10 m/s^2 is observed to cover a distance of 100 m during a 4s interval. How far will the particle move in the next 4s?

KM0004

5. A particle starts from rest at t = 0 and x = 0 to move with a constant acceleration = +2 m/s^2, for 20 seconds. After that, it moves with –4 m/s^2 for the next 20 seconds. Finally, it moves with positive acceleration for 10 seconds until its velocity becomes zero.

(a) What is the value of the acceleration in the last phase of motion?

(b) What is the final x-coordinate of the particle?

(c) Find the total distance covered by the particle during the whole motion.

KM0005

6. A body moving with uniform acceleration has a velocity of –11 cm/s when its x coordinate is 3.00 cm. If its x coordinate 2 s later is – 5 cm, what is the magnitude in cm/s^2 of its acceleration?

KM0006

7. A driver travelling at speed 36 kmh^{-1} sees the light turn red at the intersection. If his reaction time is 0.6s, and then the car can deaccelerate at 4ms^{-2}. Find the stopping distance of the car.

KM0007

8. The window of the fourth floor of SANKALP building is 5 m high. A man looking out of the window sees an object moving up and down the height of window for 2 sec. Find the height that the object reaches from the top end of the window.

KM0008

9. A body is dropped from a height of 300 m. Exactly at the same instant another body is projected from the ground level vertically up with a velocity of 150 ms^{-1}. Find when they will meet.

KM0009

10. A stone is dropped from the top of a tall cliff, and 1s later a second stone is thrown vertically downward with a velocity of 20 ms^{-1}. How far below the top of the cliff will the second stone overtake the first?

KM0010

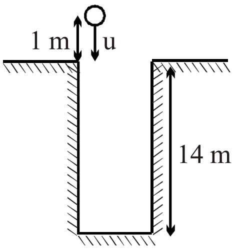
11. Speed of train is increasing linearly with time. The train passes a hut with speed 2 m/s and acquires a speed of 12 m/s after 10 s. What is the speed of the train in m/s, 5 s after passing the hut?

KM0011

12. Two particle A and B are moving in same direction on same straight line. A is ahead of B by 20m. A has constant speed 5 m/sec and B has initial speed 30 m/sec and retardation of 10 m/sec^2. Then if x (in m) is total distance travelled by B as it meets A for second time. Then value of x will be.

KM0012

13. A boy throws a ball with speed u in a well of depth 14 m as shown. On bounce with bottom of the well the speed of the ball gets halved. What should be the minimum value of u (in m/s) such that the ball may be able to reach his hand again? It is given that his hands are at 1 m height from top of the well while throwing and catching.



KM0013

14. From the top of a tower, a ball is thrown vertically upwards. When the ball reaches h below the tower, its speed is double of what it was at height h above the tower. Find the greatest height attained by the ball from the tower.

KM0014

15. A rocket is fired vertically upwards with initial velocity 40 m/s at the ground level. Its engines then fired and it is accelerated at 2 m/s^2 until it reaches an altitude of 1000 m. At that point the engines shut off and the rocket goes into free-fall. If the velocity (in m/s) just before it collides with the ground is 40α. Then fill the value of α. Disregard air resistance ($g = 10 m/s^2$).

KM0015

16. A balloon rises from rest on the ground with constant acceleration $\frac{g}{3}$. A stone is dropped when the balloon has rises to a height 60 metre. The time taken by the stone to reach the ground is.

KM0016

Motion with variable acceleration & calculus

17. The position x of a particle w.r.t. time t along x–axis is given by $x = 9t^2 - t^3$ where x is in metre and t in second. Find

(a) Maximum speed along +x direction

(b) Position of turning point

(c) Displacement in first ten seconds

(d) Distance travelled in first ten seconds **KM0017**

18. The momentum of a particle moving in straight line is given by $p = \ln t + \frac{1}{t}$ (in kg m/s) find the time $t > 0$ at which the net force acting on particle is 0 and it's momentum at that time. [Hint : $F = \frac{dp}{dt}$]

KM0018

19. The velocity of the particle is given as $v = 3t^3 + t - \frac{1}{t^2}$. Calculate the net force acting on the body at time $t = 2$ sec, if the mass of the body is 5 kg. **KM0019**

20. A wheel rotates so that the angle of rotation is proportional to the square of time. The first revolution was performed by the wheel for 8 sec. Find the angular velocity ω, 32 sec after the wheel started.
[**Hint:** Consider $\theta = kt^2$, find k]

KM0020

21. The charge flowing through a conductor beginning with time t = 0 is given by the formula $q = 2t^2 + 3t + 1$ (coulombs). Find the current $i = \frac{dq}{dt}$ at the end of the 5th second.

KM0021

22. The angle θ through which a pulley turns with time t is specified by the function $\theta = t^2 + 3t - 5$. Find the angular velocity $\omega = \frac{d\theta}{dt}$ at $t = 5$ sec. **KM0022**

23. The motion of a particle in a straight line is defined by the relation $x = t^4 - 12t^2 - 40$ where x is in meters and t is in sec. Determine the position x, velocity v and acceleration a of the particle at $t = 2$sec. **KM0023**

24. A point moves in a straight line so that its distance from the start in time t is equal to

$$s = \frac{1}{4}t^4 - 4t^3 + 16t^2.$$

(a) At what times was the point at its starting position?
(b) At what times is its velocity equal to zero? **KM0024**

25. A body whose mass is 3 kg performs rectilinear motion according to the formula $s = 1 + t + t^2$, where s is measured in centimetres & t in seconds. Determine the kinetic energy $\frac{1}{2}mv^2$ of the body in 5 sec after its start. **KM0025**

26. A force of 40N is responsible for the motion of a body governed by the equation $s = 2t + 2t^2$ where s is in meters and t in sec. What is the momentum of the body at t = 2 sec?
[**Hint**: Find acc. then $m = F/a$ & $p = mv$] **KM0026**

27. The angle rotated by a disc is given by $\theta = \frac{2}{3}t^3 - \frac{25}{2}t^2 + 77t + 5$, where θ is in rad and t in seconds.

(a) Find the times at which the angular velocity of the disc is zero.
(b) Its angular acceleration at these times.

KM0027

28. The acceleration of a particle starting from rest vary with respect to time is given by $a = (2t - 6)$, where t is in seconds. Find the time (in seconds) at which velocity of particle in negative direction is maximum.

KM0028

29. Acceleration of a particle is defined as $a=\left(75V^2-30V+3\right)$ (m/s^2). If the constant speed achieved by the particle is given by V_C, then find the value of $10V_C$.

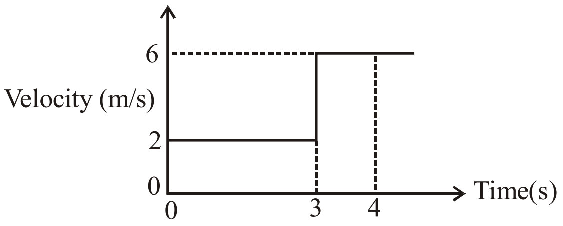
KM0029

30. Position vector of a particle is given by $\vec{r}=3t^3\hat{i}+4t\hat{j}+t^2\hat{k}$. Find avg acceleration of particle from t = 1 to t = 2sec.

KM0030

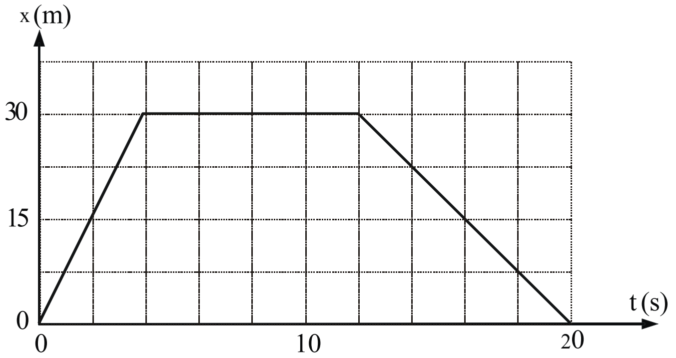
Question based on graph

31. In the following graph variation with time (t), in velocity (v) of a particle moving rectilinearly is shown. What is average velocity in m/s of the particle in time interval from 0 s to 4 s?



KM0031

32. The graph illustrates motion of a bucket being lowered into a well from the top at the instant t = 0, down to the water level, filled with water and drawn up again. Here 'x' is the depth. Find the average speed of the bucket in m/s during whole operation.



KM0032

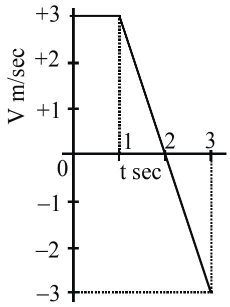
33. A particle moves along a straight line, x. At time t = 0, its position is at x = 0. The velocity, V, of the object changes as a function of time t, as indicated in the figure; t is in seconds, V in m/sec and x in meters.

(a) What is x at t = 3 sec?

(b) What is the instantaneous acceleration (in m/sec^2) at t = 2 sec?

(c) What is the average velocity (in m/sec) between t = 0 and t = 3 sec?

(d) What is the average speed (in m/sec) between t = 1 and t = 3 sec?

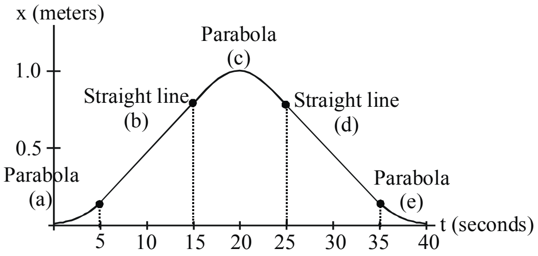


KM0033

34. The figure below is a displacement vs time plot for the motion of an object, answer questions (i) & (ii) with the letter of appropriate section of the graph.

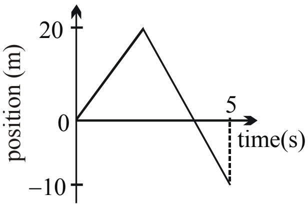
(i) Which section represents motion in the forward direction with positive acceleration?

(ii) Which section represents uniform motion backwards (–x direction)?

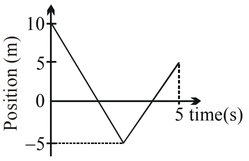


KM0034

35. **(a)** The diagram shows the position-time graph for a particle moving in a straight line. Find the average velocity for the interval from t = 0 to t = 5.



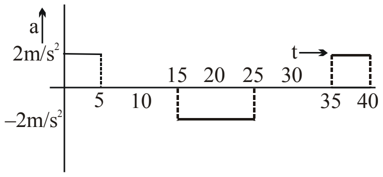
(b) The diagram shows the position-time graph for a particle moving in a straight line. Find the average speed for the interval from t = 0 to t = 5.



KM0035

36. Figure shows a graph of acceleration of a particle moving on the x-axis. Plot the following graphs if the particle is at origin and at rest at t = 0.

(i) velocity-time graph (ii) displacement-time graph (iii) distance-time graph.



KM0036

EXERCISE (S-2)

1. At a distance L = 400m from the traffic light, brakes are applied to a locomotive moving at a velocity v = 54 km/hr. Determine the position of the locomotive relative to the traffic light 1 minute after the application of the brakes if its acceleration is -0.3m/sec^2. **KM0037**

2. A particle goes from A to B with a speed of 40km/h and B to C with a speed of 60km/h. If AB = 6BC, the average speed in km/h between A and C is.

KM0038

3. A flower pot falls off a window sill and falls past the window below. It takes 0.30 s to pass a window 3.45 m high. How far is the top of the window below upper window sill?

KM0039

4. A juggler performs in a room whose ceiling is 3 m above the level of his hands. He throws a ball vertically upward so that it just reaches the ceiling.

(a) With what initial velocity does he throw the ball ?

(b) What time is required for the ball to reach the ceiling ?

He throws a second ball upward with the same initial velocity, at the instant that the first ball is at the ceiling.

(c) How long after the second ball is thrown do the two ball pass each other ?

(d) When the balls pass each other, how far are they above the juggler's hands ?

KM0040

5. A train, travelling at 20 km/hr is approaching a platform. A bird is sitting on a pole on the platform. When the train is at a distance of 2 km from pole, brakes are applied which produce a uniform deceleration in it to stop it at pole. At that instant the bird flies towards the train at 60 km/hr and after touching the nearest point on the train flies back to the pole and then flies towards the train and continues repeating itself. Calculate how much distance will the bird have flown before the train stops?

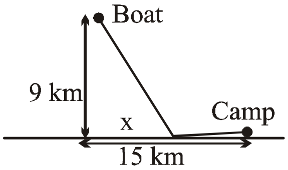
KM0041

6. A helicopter takes off along the vertical with an acceleration of 3m/sec^2 & zero initial velocity. In a certain time, the pilot switches off the engine. At the point of take off, the sound dies away in 30 sec. Determine the velocity of the helicopter at the moment when its engine is switched off, assuming the velocity of sound is 320 m/sec. **KM0042**

7. A fishing boat is anchored 9 km away from the nearest point on shore. A messenger must be sent from the fishing boat to a camp, 15 km from the point on shore closest to the boat. If the messenger can walk at a speed of 5km per hour and can row at 4 km per hour.

(i) Form an expression relating time taken to reach the camp t with distance x on shore where he lands.

(ii) At what point on shore must he land in order to reach the camp in the shortest possible time?



KM0043

8. Two body move from the same point along a straight line. The first body moves with velocity $v = (3t^2 - 6t)$ m/s, the second with velocity $v = (10t + 20)$ m/s. At what instant and at what distance from the initial point will they meet.

KM0044

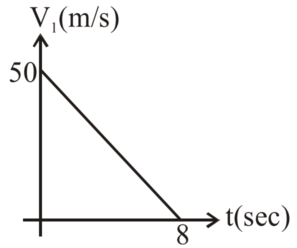
9. Velocity of a car depends on its distance ℓ from a fixed pole on a straight road as $v = 2\sqrt{\ell}$, where ℓ is in meters and v in m/s. Find acceleration (in m/s^2) when $\ell = 8$m.

KM0045

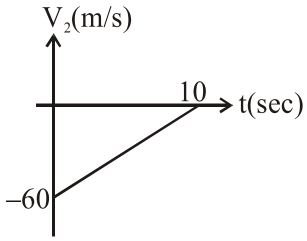
10. A particle is moving with uniform acceleration along x-axis with initial velocity along positive x. At $t = \frac{3\sqrt{2}}{\sqrt{2}-1}$ s the magnitude of displacement becomes $\frac{1}{3}$ the total distance travelled. By this time the x coordinate of particle is still positive. The instant (in sec) at which displacement becomes zero is

KM0046

11. Two trains are moving in opposite direction on same track. When their separation was 600 m their drivers notice the mistake and start slowing down to avoid collision. Graphs of their velocities as function of time is as shown. If separation between the drivers when first train stops is x then find the value of $\frac{x}{16}$.



(First train)



(Second train)

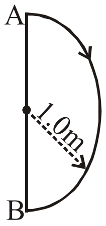
KM0047

EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

Definitions of kinematics variables

1. In 1.0 sec. a particle goes from point A to point B moving in a semicircle of radius 1.0 m. The magnitude of average velocity is : **[JEE '99]**



(A) 3.14 m/sec (B) 2.0 m/sec (C) 1.0 m/sec (D) zero

KM0048

2. An object is tossed vertically into the air with an initial velocity of 8 m/s. Using the sign convention upwards as positive, how does the vertical component of the acceleration a_y of the object (after leaving the hand) vary during the flight of the object?

(A) On the way up $a_y > 0$, on the way down $a_y > 0$

(B) On the way up $a_y < 0$, on the way down $a_y > 0$

(C) On the way up $a_y > 0$, on the way down $a_y < 0$

(D) On the way up $a_y < 0$, on the way down $a_y < 0$

KM0049

Motion with constant acceleration

3. A body starts from rest and is uniformly accelerated for 30 s. The distance travelled in the first 10 s is x_1, next 10 s is x_2 and the last 10 s is x_3. Then $x_1 : x_2 : x_3$ is the same as :-

(A) 1 : 2 : 4 (B) 1 : 2 : 5 (C) 1 : 3 : 5 (D) 1 : 3 : 9

KM0050

4. If a body starts from rest and travels 120 cm in the 6th second, with constant acceleration then what is the acceleration :

(A) 0.20 m/s^2 (B) 0.027 m/s^2 (C) 0.218 m/s^2 (D) 0.03 m/s^2

KM0051

5. A particle travels 10m in first 5 sec and 10m in next 3 sec. Assuming constant acceleration what is the distance travelled in next 2 sec.

(A) 8.3 m (B) 9.3 m (C) 10.3 m (D) None of above

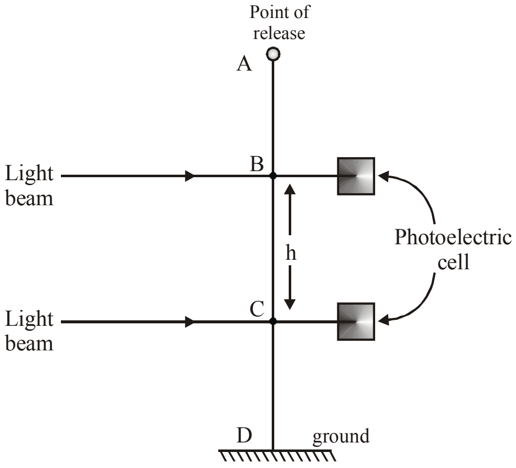
KM0052

6. The engine of a motorcycle can produce a maximum acceleration 5 m/s^2. Its brakes can produce a maximum retardation 10 m/s^2. If motorcyclist start from point A and reach at point B. What is the minimum time in which it can cover if distance between A and B is 1.5 km. (Given : that motorcycle comes to rest at B)

(A) 30 sec (B) 15 sec (C) 10 sec (D) 5 sec

KM0053

7. The acceleration of free fall at a planet is determined by timing the fall of a steel ball photo –electrically. The ball passes B and C at times t_1 and t_2 after release from A. The acceleration of free fall is given by



(A) $\frac{2h}{t_2-t_1}$ (B) $\frac{h}{t_2^2-t_1^2}$ (C) $\frac{2h}{t_2^2-t_1^2}$ (D) $\frac{2h}{t_2^2+t_1^2}$

KM0054

8. A particle has an initial velocity of 9 m/s due east and a constant acceleration of 2 m/s^2 due west. The distance covered by the particle in the fifth second of its motion is :-

(A) 0 (B) 0.5 m (C) 2 m (D) none of these

KM0055

9. A physics teacher finds a scrap of paper on which one of his students has written the following equation : $0^2 - 5^2 = 2 \times (-9.8) \times x$; of which of the following problem would this equation be part of the correct solution?

(A) Find the speed of an object 5 seconds after it was dropped from rest.

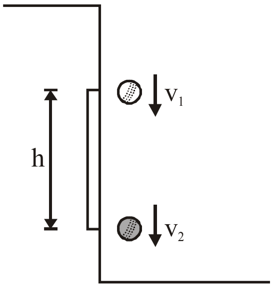
(B) Find the distance of an object has fallen 5 seconds after it was released from rest on Earth.

(C) Find the height from which a ball when released will strike the ground with a speed of 5 m/s.

(D) Find the maximum height to which a ball will rise if it is thrown upward with an initial speed of 5 m/s.

KM0056

10. A ball dropped from the top of a building passes past a window of height h in time t. If its speeds at the top and the bottom edges of the window are denoted by v_1 and v_2 respectively, which of the following set of equations are correct?



(A) $v_2-v_1 = gt$ and $(v_2-v_1)t = h$ (B) $v_2-v_1 = gt$ and $(v_2+v_1)t = 2h$

(C) $v_2+v_1 = gt$ and $(v_2-v_1)t = h$ (D) None of the above.

KM0057

11. A body falls freely from rest. It covers as much distance in the last second of its motion as covered in the first three seconds. The body has fallen for a time of :

(A) 3 s (B) 5 s (C) 7 s (D) 9 s

KM0058

12. A ball is thrown vertically upward with initial velocity 30 m/sec. What will be its position vector at time t = 5 sec taking origin at the point of projection, vertical up as positive y-axis and horizontal as x-axis :-

(A) (0, 25) (B) (0, 20) (C) (0, 45) (D) (0, 5)

KM0059

13. A particle is thrown upwards from ground. It experiences a constant resistance force which can produce retardation 2 m/s^2. The ratio of time of ascent to the time of descent is [g = 10 m/s^2]

(A) 1 : 1 (B) $\sqrt{\frac{2}{3}}$ (C) $\frac{2}{3}$ (D) $\sqrt{\frac{3}{2}}$

KM0060

14. A body of mass 'm' is travelling with a velocity 'u'. When a constant retarding force 'F' is applied, it comes to rest after travelling a distance 's_1'. If the initial velocity is '2u', with the same force 'F', the distance travelled before it comes to rest is 's_2'. Then

(A) $s_2 = 2s_1$ (B) $s_2 = \frac{s_1}{2}$ (C) $s_2 = s_1$ (D) $s_2 = 4\ s_1$

KM0061

15. A ball is thrown vertically upward with initial velocity 30 m/sec. What will be its position vector at time t = 5 sec, taking origin at 45 m above the point of projection, vertical up as positive y-axis and horizontal as x-axis :-

(A) (0, –25) (B) (0, –20) (C) (0, –45) (D) (0, –5)

KM0062

Motion with variable acceleration & calculus

16. If $s = 2t^3 + 3t^2 + 2t + 8$ then the time at which acceleration is zero, is :-

(A) $t = \frac{1}{2}$ (B) t = 2 (C) $t = \frac{1}{2\sqrt{2}}$ (D) Never

KM0063

17. Velocity of a particle varies with time as v = 4t. The displacement of particle between t = 2 to t = 4 sec, is :-

(A) 12 m (B) 36 m (C) 24 m (D) 6 m

KM0064

18. A point mass moves with velocity $v = (5t-t^2)$ ms^{-1} in a straight line. Find the distance travelled (i.e. $\int v\,dt$) in fourth second.

(A) $\frac{31}{6}$ m (B) $\frac{29}{6}$ m (C) $\frac{37}{6}$ m (D) None of these

KM0065

19. A particle is projected with velocity v_0 along x–axis. The deceleration on the particle is proportional to the square of the distance from the origin i.e., $a = -\alpha x^2$. The distance at which the particle stops is:–

(A) $\sqrt{\frac{3v_0}{2\alpha}}$ (B) $\left(\frac{3v_0}{2\alpha}\right)^{\frac{1}{3}}$ (C) $\sqrt{\frac{3v_0^2}{2\alpha}}$ (D) $\left(\frac{3v_0^2}{2\alpha}\right)^{\frac{1}{3}}$

KM0066

20. The acceleration vector along x–axis of a particle having initial speed v_0 changes with distance as $a = \sqrt{x}$. The distance covered by the particle, when its speed becomes twice that of initial speed is:-

(A) $\left(\frac{9}{4}v_0\right)^{\frac{4}{3}}$ (B) $\left(\frac{3}{2}v_0\right)^{\frac{4}{3}}$ (C) $\left(\frac{2}{3}v_0\right)^{\frac{4}{3}}$ (D) $2v_0$ **KM0067**

21. For a particle moving in a straight line the position of the particle at time (t) is given by $x = \frac{t^3}{6} - t^2 - 9t + 18$ m. What is the velocity of the particle when its acceleration is zero :-

(A) 18 m/s (B) –9 m/s (C) –11 m/s (D) 6 m/s

KM0068

22. A particle moves along a straight line such that at time t its displacement from a fixed point O on the line is $3t^2 - 2$. The velocity of the particle when t = 2 is:

(A) 8 ms^{-1} (B) 4 ms^{-1} (C) 12 ms^{-1} (D) 0

KM0069

23. Temperature of a body varies with time as $T = (T_0 + \alpha t^2 + \beta \sin t)K$, where T_0 is the temperature in Kelvin at t = 0 sec. & $\alpha = 2/\pi$. K/s^2 & $\beta = -4$ K, then rate of change of temperature at t = πsec. is

(A) 8 K (B) 8^0K (C) 8K/sec (D) 8^0K/sec

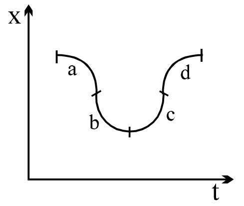
KM0070

24. The velocity of a particle moving on the x-axis is given by $v = x^2 + x$ where v is in m/s and x is in m. Find its acceleration in m/s^2 when passing through the point x = 2m

(A) 0 (B) 5 (C) 11 (D) 30 **KM0071**

Question based on graph

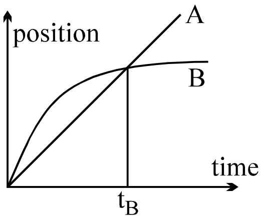
25. The graph shown is a plot of position versus time. For which labeled region is the velocity positive and the acceleration negative?



(A) a (B) b (C) c (D) d

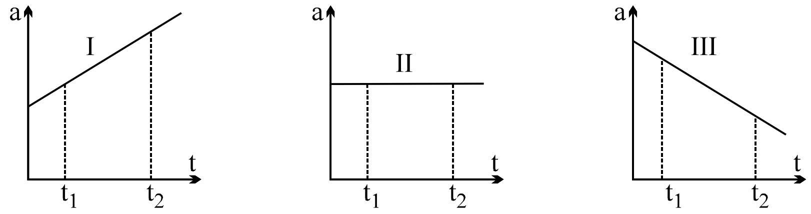
KM0072

26. The graph shows position as a function of time for two trains running on parallel tracks. Which statement is true?



(A) At time t_B, both trains have the same velocity.
(B) Both trains have the same velocity at some time after t_B.
(C) Both trains have the same velocity at some time before t_B.
(D) Somewhere on the graph, both trains have the same acceleration. **KM0073**

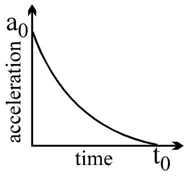
27. Each of the three graphs represents acceleration versus time for an object that already has a positive velocity at time t_1. Which graphs show an object whose speed is increasing for the entire time interval between t_1 and t_2?

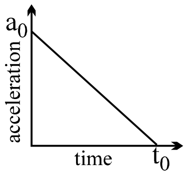


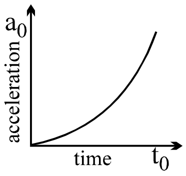
(A) graph I, only
(B) graphs I and II, only
(C) graphs I and III, only
(D) graphs I, II, and III

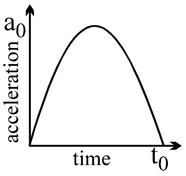
KM0074

28. Acceleration versus time graphs for four objects are shown below. All axes have the same scale. Which object had the greatest change in velocity during the interval?

(A)


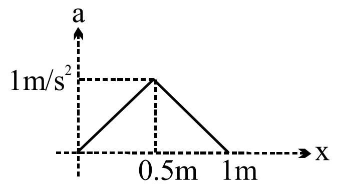
(B)


(C)


(D)


KM0075

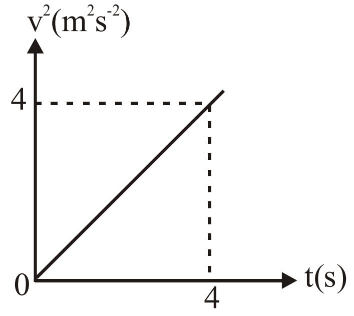
29. A body initially at rest, starts moving along x-axis in such a way so that its acceleration vs displacement plot is as shown in figure. The maximum velocity of particle is :-



(A) 1 m/s
(B) 6 m/s
(C) 2 m/s
(D) none

KM0076

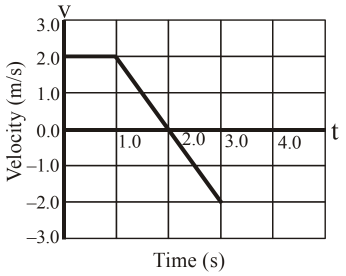
30. A particle is moving along a straight line such that square of its velocity varies with time as shown in the figure. What is the acceleration of the particle at t = 4 s?



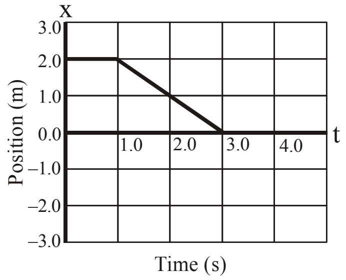
(A) 4 m/s^2
(B) 1/4 m/s^2
(C) 1/2 m/s^2
(D) 0

KM0077

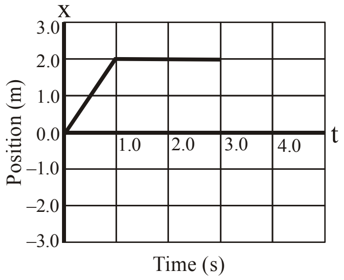
31. The graph below shows the velocity of a particle moving in a straight line. At t = 0, the particle is located at x = 0. Which of the following graphs shows the position of the particle with respect to time, x(t) ?



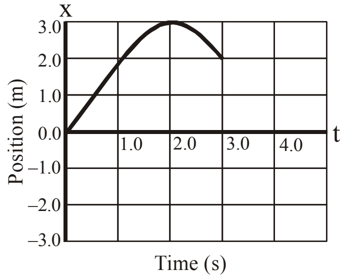
(A)



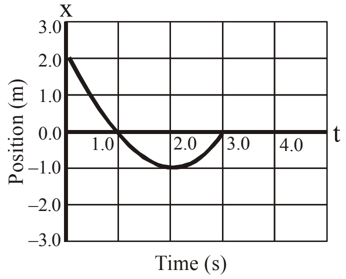
(B)



(C)

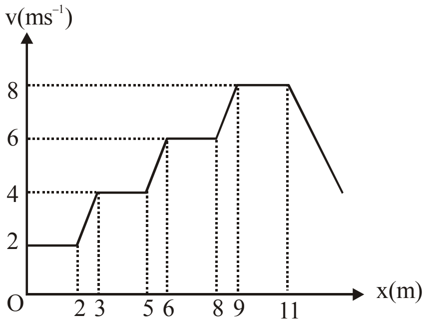


(D)



KM0078

32. The velocity of a particle that moves in the positive x-direction varies with its position as shown in figure. The acceleration of the particle when x = 5.5 m is-



(A) 0 (B) 5 ms^{-2} (C) 10 ms^{-2} (D) 20 ms^{-2}

KM0079

COMPREHENSION TYPE QUESTIONS

Paragraph for Question no. 33 and 34

A particle is moving in a straight line along positive y-axis. Its displacement from origin at any time t is given by $y = 5t^2 - 10t + 5$ where y is in meters and t is in seconds.

33. The velocity at $t = 2s$ will be :

(A) 20 ms^{-1} (B) 10 ms^{-1} (C) 5 ms^{-1} (D) 15 ms^{-1}

KM0080

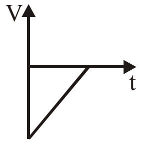
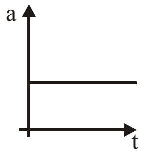
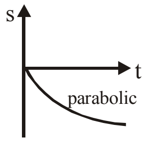
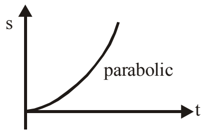
34. Displacement of particle when its velocity is zero, is

(A) 2.5 m (B) 1.25 m (C) 5m (D) 0m

KM0080

MATRIX MATCH TYPE QUESTION

35. v, a, s and t denote velocity, acceleration, displacement and time respectively. Match the columns :-

Column-I	**Column-II**
(A) 	(P) Velocity of the particle is in positive direction, acceleration in negative direction
(B) 	(Q) Both velocity and acceleration of the particle are in negative directions.
(C) 	(R) Velocity of the particle is in negative direction and acceleration in positive direction
(D) 	(S) Velocity and acceleration both in positive direction
	(T) Acceleration is constant

KM0081

EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

1. A parachutist jumps out of an airplane and accelerates with gravity for 6 seconds. He then pulls the parachute cord and after a 4 s deceleration period, descends at 10 m/s for 60 seconds, reaching the ground. From what height did the parachutist jump? Assume acceleration due to gravity to be 10 m/s^2 throughout the motion.

(A) 840 m (B) 920 m (C) 980 m (D) 1020 m

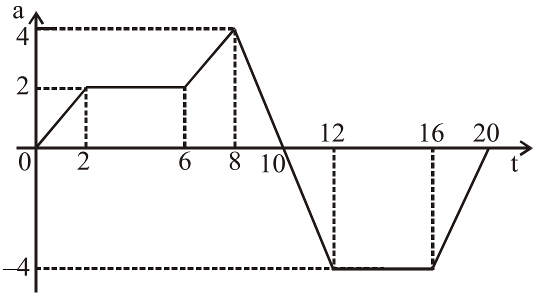
KM0082

2. A train moving with a speed of 60 km/hr is slowed down uniformly to 30 km/hr for repair purposes during running. After this it was accelerated uniformly to reach to its original speed. If the distance covered during constant retardation be 2 km and that covered during constant acceleration be 1 km, find the time lost in the above journey

(A) 1 min (B) 2 min (C) 4 min (D) 5 min

KM0083

3. If initial velocity of particle is 2 m/s, the maximum velocity of particle from t = 0 to t = 20 sec is :



(A) 20 m/s (B) 18 m/s (C) 22 m/s (D) 24 m/s

KM0084

ASSERTION & REASON

These questions contains, Statement-1 (assertion) and Statement-2 (reason).

(A) Statement-1 is true, Statement-2 is true ; Statement-2 is correct explanation for Statement-1.

(B) Statement-1 is true, Statement-2 is true ; Statement-2 is NOT a correct explanation for statement-1.

(C) Statement-1 is true, Statement-2 is false.

(D) Statement-1 is false, Statement-2 is true.

(E) Both Statement-1 and Statement-2 are false.

4. **Statement I :** When velocity of a particle is zero then acceleration of particle is zero.

and

Statement II : Acceleration is equal to rate of change of velocity. **KM0085**

5. **Statement-I :** A particle moves in a straight line with constant acceleration. The average velocity of this particle cannot be zero in any time interval.

and

Statement-II : For a particle moving in straight line with constant acceleration, the average velocity in a time interval is $\frac{u+v}{2}$, where u and v are initial and final velocity of the particle of the given time interval. **KM0086**

6. A particle moves in a straight line, according to the law $x = 4a\left[t + a\sin\left(\frac{t}{a}\right)\right]$, where x is its position in meters, t in sec. & a is some constants, then the velocity is zero at :-

(A) $x = 4a^2\pi$ meters (B) $t = \pi$ sec. (C) $t = 0$ sec (D) none

KM0087

7. A particle moving on the x-axis with constant acceleration has displacements of 6 m from t = 4 s to t = 7 s and 3 m from t = 5 s to t = 8 s. The distance covered from t = 6 s to t = 9 s is

(A) 1.75 m (B) 2.25 m (C) 3.0 m (D) 0

KM0088

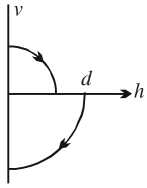
8. A point moves in a straight line so that its displacement is x m at time t sec, given by $x^2 = t^2 + 1$. Its acceleration in m/s^2 at time t sec is :

(A) $\frac{1}{x}$ (B) $\frac{1}{x} - \frac{1}{x^2}$ (C) $-\frac{t}{x^2}$ (D) $\frac{1}{x^3}$

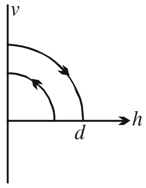
KM0089

9. A ball is dropped vertically from a height *d* above the ground, hits the ground and bounces up vertically to a height *d*/2. Neglecting subsequent motion and air resistances, its velocity *v* varies with the height *h* above the ground as :- **[IIT-JEE'2000 (Scr)]**

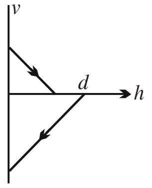
(A)



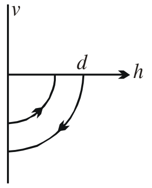
(B)



(C)



(D)



KM0090

MULTIPLE CORRECT TYPE QUESTIONS

10. A particle moving along a straight line with uniform acceleration has velocities 7m/s at A and 17 m/s at C. B is the mid point of AC. Then

(A) The velocity at B is 12 m/s.

(B) The average velocity between A and B is 10 m/s.

(C) The ratio of the time to go from A to B to that from B to C is 3 : 2.

(D) The average velocity between B and C is 15 m/s. **KM0091**

11. A particle moves along the X–axis as $x = u(t-2s) + a(t-2s)^2$

(A) The initial velocity of the particle is u

(B) The acceleration of the particle is a

(C) The acceleration of the particle is 2a

(D) At t = 2s particle is at the origin. **KM0092**

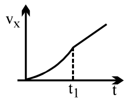
12. The position of a particle with time is given by

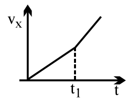
$(x, y) = (8t^2, 3)$ for $t \le t_1$

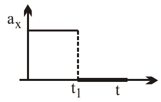
$= (8t\,t_1, 3)$ for $t > t_1$

Choose the **CORRECT** alternative.

(A) Particle moves along a straight line parallel to x axis.

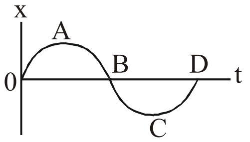
(B)


(C)


(D)


KM0093

13. A particle has a rectilinear motion and the figure gives its displacement as a function of time. Which of the following statements are true with respect to the motion



(A) in the motion between O and A the velocity is positive and acceleration is negative

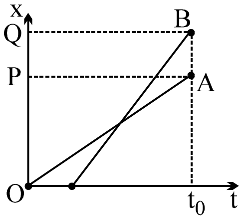
(B) between A and B the velocity and acceleration are positive

(C) between B and C the velocity is negative and acceleration is positive

(D) between C and D the acceleration is positive

KM0094

14. The position–time (x–t) graphs for two children A and B returning from their school O to their homes P and Q respectively along straight line path (taken as x axis) are shown in figure below. Choose the **CORRECT** statement (s):



(A) A lives closer to the school than B

(B) A starts from the school earlier than B

(C) A and B have equal average velocities from 0 to t_0.

(D) B overtakes A on the way

KM0095

15. A ball is dropped from a building. Somewhere down it crosses a window of length 4 m in 0.5 sec. Speed of ball at top of window is v_1 and at bottom v_2, then choose the **CORRECT** option(s) $(g = 10\ m/s^2)$:-

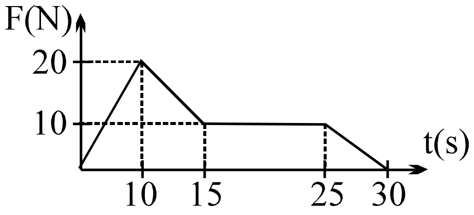
(A) $v_2 - v_1 = 5$ m/s (B) $v_2 + v_1 = 16$ m/s (C) $\frac{v_2}{v_1} = 9$ (D) $\frac{v_2}{v_1} = \frac{21}{11}$

KM0096

COMPREHENSION TYPE QUESTIONS

Paragraph for Question Nos. 16 to 19

In figure shown, the graph shows the variation of a unidirectional force F acting on a body of mass 10kg (in gravity free space), with time t. The velocity of the body at t = 0 is zero. (Area under F-t curve gives change in momentum)



16. The velocity of the body at t = 30 s is

(A) 30 m/s (B) 20 m/s (C) 40 m/s (D) none

KM0097

17. The power of the force at t = 12 s is (Power = force × velocity)

(A) 225.0 W (B) 217.6 W (C) 226.7 W (D) none

KM0097

18. The average acceleration of the body from t = 0 to t = 15 s is :-

(A) 1.25 m/s^2 (B) 4/7 m/s^2 (C) 5/6 m/s^2 (D) 7/6 m/s^2

KM0097

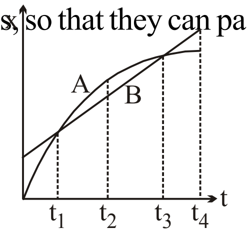
19. The change in momentum of the body between the time t = 10 s to 15 s is :-

(A) 100 kg.m/s (B) 75 kg.m/s (C) 125 kg.m/s (D) none

KM0097

Paragraph for Question nos. 20 to 23

The graph given shows the **POSITION** of two cars, A and B, as a function of time. The cars move along the x-axis on parallel but separate tracks, so that they can pass each other's position without colliding.



20. At which instant in time is car-A overtaking the car-B ?

(A) t_1 (B) t_2 (C) t_3 (D) t_4

KM0098

21. At time t_3, which car is moving faster?

(A) car A (B) car B (C) same speed (D) None of these

KM0098

22. At which instant do the two cars have the same velocity ?

(A) t_1 (B) t_2 (C) t_3 (D) t_4 **KM0098**

23. Which one of the following best describes the motion of car A as shown on the graphs?

(A) speeding up (B) constant velocity

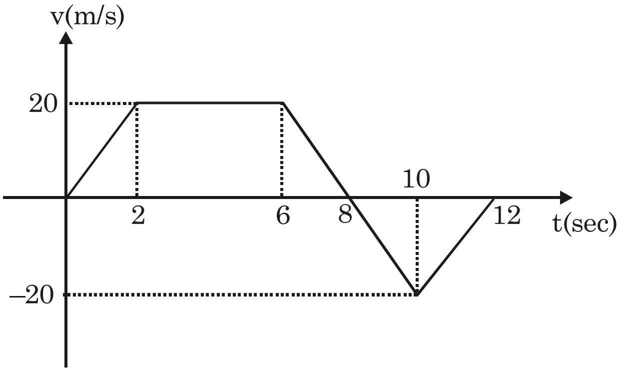
(C) slowing down (D) first speeding up, then slowing down

KM0098

MATCHING LIST TYPE (4 × 4 × 4) SINGLE OPTION CORRECT (THREE COLUMNS AND FOUR ROWS)

Answer Q.24, Q.25 and Q.26 by appropriately matching the information given in the three columns of the following table.

The velocity-time graph of an object moving along a straight line is given below.



	Column-I Time interval		Column-II Average velocity		Column-III Average acceleration
(I)	0 to 2 sec	(i)	10 m/s	(P)	zero
(II)	2 to 6 sec	(ii)	$-\frac{10}{3}\text{m/s}$	(Q)	$-2\ \text{m/s}^2$
(III)	0 to 10 sec	(iii)	15 m/s	(R)	$-\frac{10}{3}\text{m/s}^2$
(IV)	6 to 12 sec	(iv)	20 m/s	(S)	$-4\ \text{m/s}^2$

24. Which of the following combination is correctly matched :-

(A) (III) (iv) (S) (B) (III) (i) (R) (C) (III) (i) (S) (D) (III) (i) (Q)

KM0099

25. Which of the following combination is correctly matched :-

(A) (II) (iv) (P) (B) (I) (i) (Q) (C) (II) (iv) (Q) (D) (I) (i) (S)

KM0099

26. Which of the following combination is correctly matched :-

(A) (IV) (ii) (S) (B) (IV) (iii) (R) (C) (IV) (i) (S) (D) (IV) (ii) (R)

KM0099

MATRIX MATCH TYPE QUESTION

27. A balloon starts rising up from ground with constant net acceleration of $10 m/s^2$. After 2 s a particle drops from the balloon. After further 2 s match the following : ($g = 10\ m/s^2$)

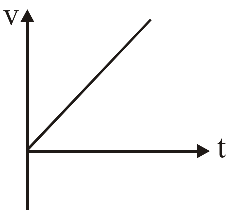
	Column-I		Column-II
(A)	Height of particle from ground	(P)	Zero
(B)	Speed of particle	(Q)	10 SI units
(C)	Displacement of Particle	(R)	40 SI units
(D)	Acceleration of particle	(S)	20 SI units

KM0100

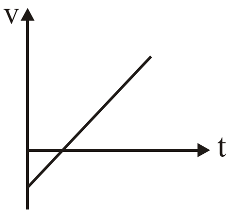
28. In the first column of the given table, some velocity-time (v-t) graphs and in the second column some position-time (x-t) graphs are shown. Sugest suitable match or matches.

Column-I

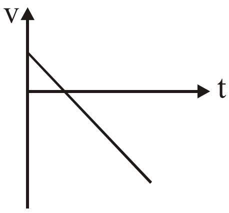
(A)



(B)



(C)



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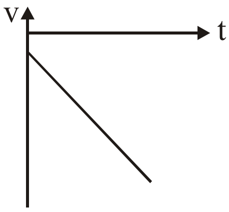
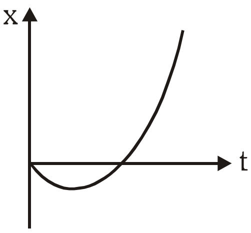
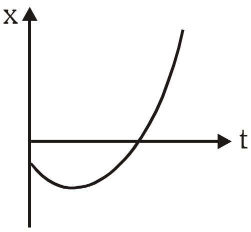


Column-II

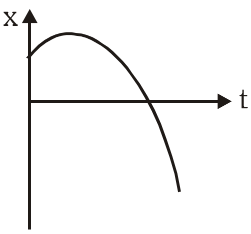
(P)



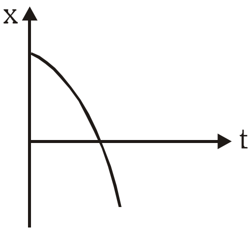
(Q)



(R)



(S)



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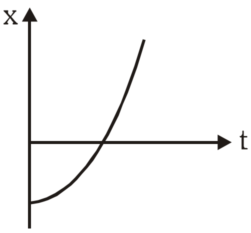


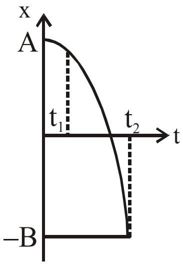
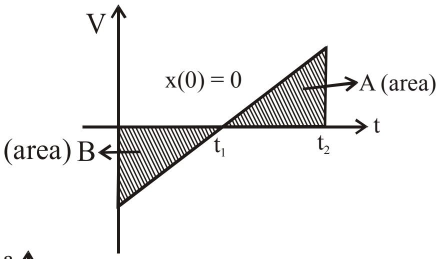
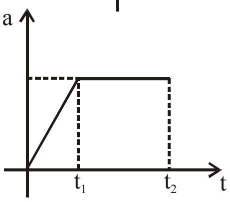
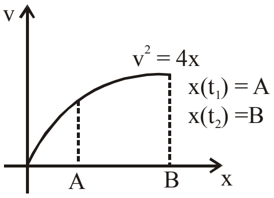
KM0101

29. **Match the column :-**

Column-I : Shows graph of One Dimension motion of a particle. Symbols have their usual meaning such as $x(0)$ = initial position, $x(t_1)$ = position at $t = t_1$, $v(0)$ = initial velocity.

Column-II : Shows physical quantities. Displacement and distance are asked for $0 < t < t_2$, and average values are asked for $0 < t < t_2$

V_0, A & B are positive constant

Column-I	Column-II
(A) 	(P) |Displacement| = Distance
(B) 	(Q) |Instantaneous velocity| = |Instantaneous speed|
(C)  $x(0) = 0$, $x(t_1) = A$, $x(t_2) = B$, $v(0) = v_0$	(R) |Average velocity| ≤ Average speed
(D) 	(S) Instantaneous acceleration = Average acceleration
	(T) Displacement = A – B and Distance = A + B

KM0102

EXERCISE (J-M)

1. An object, moving with a speed of 6.25 m/s, is decelerated at a rate given by

$$\frac{dv}{dt} = -2.5\sqrt{v}$$

where v is the instantaneous speed. The time taken by the object, to come to rest, would be :-

[AIEEE-2011]

(1) 4 s (2) 8 s (3) 1 s (4) 2 s

KM0103

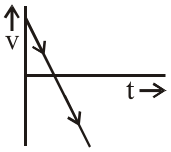
2. From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle, to hit the ground, is n times that taken by it to reach the highest point of its path. The relation between H, u and n is : **[JEE-Main-2014]**

(1) $2gH = nu^2(n-2)$ (2) $gH = (n-2)u^2$ (3) $2gH = n^2u^2$ (4) $gH = (n-2)^2u^2$

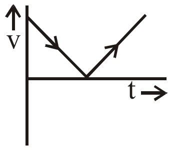
KM0104

3. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity vs time? **[JEE-Main-2017]**

(1)



(2)



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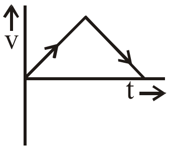

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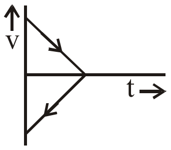
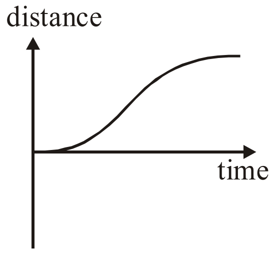


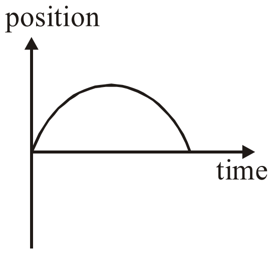
KM0105

4. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up. **[JEE-Main-2018]**

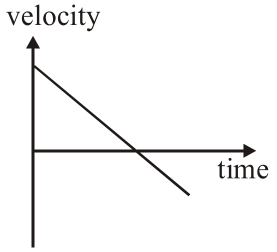
(1)



(2)



(3)



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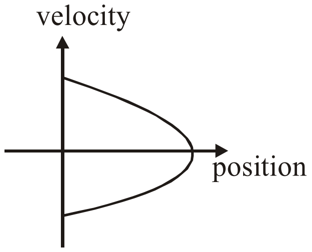


KM0106

EXERCISE (J-A)

1. Consider an expanding sphere of instantaneous radius R whose total mass remains constant. The expansion is such that the *instantaneous* density ρ remains uniform throughout the volume. The rate of fractional change in density $\left(\frac{1}{\rho}\frac{d\rho}{dt}\right)$ is constant. The velocity v of any point on the surface of the expanding sphere is proportional to : **[JEE Advanced-2017]**

(A) R^3 (B) $\frac{1}{R}$ (C) R (D) $R^{2/3}$

KM0107

ANSWER KEY

EXERCISE (S-1)

1. Ans. (a) $2\sqrt{2}\frac{v}{\pi}$, (b) $\frac{2v}{\pi}$, (c) $\frac{2\sqrt{2}v}{3\pi}$

2. Ans.

v_0	a
—	+

Because particle is slowing down so velocity & acceleration are in opposite direction.

3. Ans. 3

4. Ans. 260 m

5. Ans. (a) 4 m/s^2, (b) 200, (c) 1000 m

6. Ans. 7

7. Ans. 18.5 m

8. Ans. Zero

9. Ans. 2 sec. after body is dropped

10. Ans. $\frac{45}{4}$m

11. Ans. 7

12. Ans. 50

13. Ans. 30

14. Ans. 5h/3

15. Ans. 4

16. Ans. 6

17. Ans. (a) 27 m/s, (b) 108 m, (c) –100 m, (d) 316 m

18. Ans. 1 kg m/sec.

19. Ans. 186.25 N

20. Ans. 2π rad/sec.

21. Ans. 23 amp

22. Ans. 13 rad/s

23. Ans. –72, –16, 24

24. Ans. (a) 0, 8 sec (b) 0, 4, 8 sec

25. Ans. 1.815×10^5 ergs.

26. Ans. 100 kgm/s

27. Ans. (a) 7, $\frac{11}{2}$ (b) 3, –3

28. Ans. 3

29. Ans. 2

30. Ans. $\left(27\hat{i}+2\hat{k}\right)m/s^2$

31. Ans. 3

32. Ans. 3

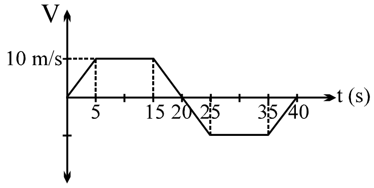
33. Ans. (a) 3 m; (b) –3 m/s^2; (c) 1m/s; (d) 3/2 m/s

34. Ans. (i) section (a) as slope = $v = \frac{dx}{dt}$ is positive and increasing.

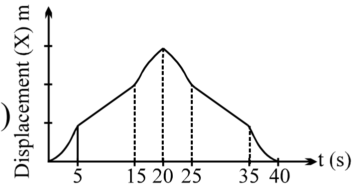
(ii) section (d) as slope = $v = \frac{dx}{dt}$ is negative and constant.

35. Ans. (a) $-2\ ms^{-1}$ (b) 5 m/s

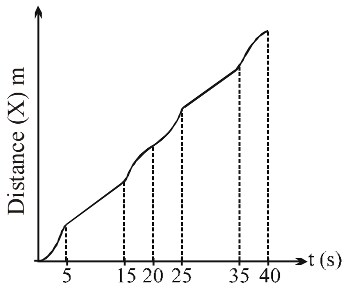
36. Ans. (i)



(ii)



(iii)



EXERCISE (S-2)

1. Ans. 25 m **2. Ans.** 42 km/hr **3. Ans.** 5 m

4. Ans. (a) $\sqrt{60}\,\text{m / sec}$ (b) $\sqrt{\frac{3}{5}}$ sec (c) $\frac{3}{\sqrt{60}}$ sec (d) $\frac{9}{4}$ m **5. Ans.** 12 km

6. Ans. 80 m/sec **7. Ans.** (i) $t=\frac{\sqrt{x^2+(9)^2}}{4}+\frac{15-x}{5}$ (ii) 3 km from the camp.

8. Ans. 10 sec ; 700m **9. Ans.** 2 **10. Ans.** 012 **11. Ans.** 7

EXERCISE (O-1)

1. Ans. (B) 2. Ans. (D) 3. Ans. (C) 4. Ans. (C) 5. Ans. (A) 6. Ans. (A)
7. Ans. (C) 8. Ans. (B) 9. Ans. (D) 10. Ans. (B) 11. Ans. (B) 12. Ans. (A)
13. Ans. (B) 14. Ans. (D) 15. Ans. (B) 16. Ans. (D) 17. Ans. (C) 18. Ans. (A)
19. Ans. (D) 20. Ans. (B) 21. Ans. (C) 22. Ans. (C) 23. Ans. (C) 24. Ans. (D)
25. Ans. (D) 26. Ans. (C) 27. Ans. (D) 28. Ans. (D) 29. Ans. (A) 30. Ans. (B)
31. Ans. (C) 32. Ans. (C) 33. Ans. (B) 34. Ans. (D)
35. Ans. (A) → (R,T) ; (B) → (T) ; (C) → (R, T) ; (D) → (S, T)

EXERCISE (O-2)

1. Ans. (B) 2. Ans. (A) 3. Ans. (C) 4. Ans. (D) 5. Ans. (D) 6. Ans. (A)
7. Ans. (B) 8. Ans. (D) 9. Ans. (A) 10. Ans. (B,C,D) 11. Ans. (C,D)
12. Ans. (A,D) 13. Ans. (A,C,D) 14. Ans. (A,B,D) 15. Ans. (A,B,D)
16. Ans. (A) 17. Ans. (B) 18. Ans. (D) 19. Ans. (B) 20. Ans. (A) 21. Ans. (B)
22. Ans. (B) 23. Ans. (C) 24. Ans. (D) 25. Ans. (A) 26. Ans. (D)
27. Ans. (A) - (R); (B) - (P); (C) - (S); (D) - (Q)
28. Ans. (A) → (T); (B) → (P,Q); (C) → (R); (D) → (S)
29. Ans. (A) → (P,Q,R) ; (B) → (Q,R,S,T) ; (C) → (P,Q,R) ; (D) → (P,Q,R,S)

EXERCISE (JM)

1. Ans. (4) 2. Ans. (1) 3. Ans. (1) 4. Ans. (1)

EXERCISE (JA)

1. Ans. (C)

Chapter Contents 04

KINEMATICS-2D

Important Notes

KINEMATICS-2D

KEY CONCEPT

MOTION IN TWO AND THREE DIMENSIONS :

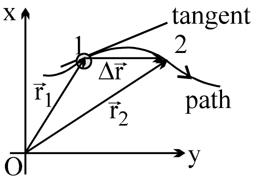
When a particle is moving in space then its motion can be broken up in three co-ordinate axis (x, y& z). The motion in these three directions is governed only by velocity & acceleration in that particular direction and is totally independent of the velocities and acceleration in other directions. Lets say a particle is moving in space

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Gives position of particle in space.

VELOCITY

Using the language of calculus, we may write $\vec{v}$ as the derivative



$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left(x\hat{i} + y\hat{j} + z\hat{k}\right) = \left(\frac{dx}{dt}\right)\hat{i} + \left(\frac{dy}{dt}\right)\hat{j} + \left(\frac{dz}{dt}\right)\hat{k}$$

where the scalar components of $\vec{v}$ are

$$v_x = \frac{dx}{dt},\ v_y = \frac{dy}{dt},\ v_z = \frac{dz}{dt}$$

Differentiating $\vec{r}$ w.r.t. time gives us velocity vector of particle at that time.

ACCELRATION

Similarly, if we differentiate $\vec{V}$ w.r.t. time we get acceleration of particle $\vec{a} = \frac{d\vec{V}}{dt}$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$$

where the scalar components of $\vec{a}$ are

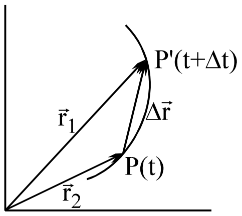
$$a_x = \frac{dv_x}{dt},\ a_y = \frac{dv_y}{dt},\ a_z = \frac{dv_z}{dt}$$

Now, collecting equations of motion relating to x & y axes separately

x-axis	y-axis
$V_x = \frac{dx}{dt}$	$V_y = \frac{dy}{dt}$
$a_x = \frac{dV_x}{dt}$	$a_y = \frac{dV_y}{dt}$

Thus we can see that motion in plane is composed of two straight line motions. **These motions are completely independent of each other**. Only thing connecting them is fact that they are occuring simultaneously.

Velocity is along tangent of path



The direction of the instantaneous velocity $\vec{v}$ of a particle is always tangent to the particle's path at the particle position. $\vec{V} = \lim\limits_{\Delta t\to 0}\dfrac{\Delta\vec{r}}{\Delta t}$

The result is the same in three dimensions:

Ex. A particle with velocity $\vec{v}_0 = -2\hat{i} + 4\hat{j}$ (in meters per second) at t = 0 undergoes a constant acceleration $\vec{a}$ of magnitude a = 3 m/s^2 at an angle θ = 127° from the positive direction of the x axis. What is the particle's velocity $\vec{v}$ at t = 5 sec, in unit vector notation ?

Sol. We know that $v = v_0 + at$

now $v_x = v_{0x} + a_x t$ and $v_y = v_{0y} + a_y t$

$a_x = a\cos\theta = (3\ \text{m/s}^2)(\cos 127°) = -1.80\ \text{m/s}^2$

$a_y = a\sin\theta = (3\ \text{m/s}^2)(\sin 127°) = +2.40\ \text{m/s}^2$

at time t = 5 sec

$v_x = -2\ \text{m/s} + (-1.80\ \text{m/s}^2)(5\ \text{sec}) = -11\ \text{m/s}$

$v_y = 4\ \text{m/s} + (2.40\ \text{m/s}^2)(5\ \text{sec}) = 16\ \text{m/s}$

Thus, at t = 5 sec,

$\vec{v} = (-11 m/s)\hat{i} + (16 m/s)\hat{j}$ **Ans.**

Ex. A particle moves in the x–y plane according to the law x = at; y = at(1–αt) where a and α are positive constants and t is time. Find the velocity and acceleration vector. Also find the moment t_0 at which the velocity vector forms angle of 90° with acceleration vector.

Sol. $V_x = a$; $V_y = a - 2a\alpha t \Rightarrow \vec{V} = a\hat{i} + (a - 2a\alpha t)\hat{j}$

$a_x = 0$; $a_y = -2a\alpha \Rightarrow \vec{a} = -2a\alpha\hat{j}$

for 90°, $\vec{V}\cdot\vec{a} = 0$

$-2a\alpha(a - 2a\alpha t_0) = 0$

$1 - 2\alpha t_0 = 0 \Rightarrow t_0 = 1/(2\alpha)$ sec.

PROJECTILE MOTION

We next consider a special case of two-dimensional motion: A particle moves in a vertical plane with some initial velocity $\vec{v}_0$ but its acceleration is always the freefall acceleration $\vec{g}$, which is downward. Such a particle is called a projectile (meaning that it is projected or launched) and its motion is called **projectile motion**.

Assumptions:–

Particle remains close to earth's surface, so acceleration due to gravity remains constant.

Air resistance is neglected.

Distance that projectile travels is small so that earth can be treated as plane surface.

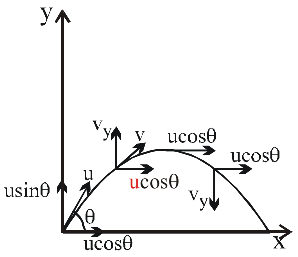
Two straight line motions:–

Our goal here is to analyse projectile motion using the tools for two dimensional motion. This feature allows us to break up a problem involving two dimensional motion into two separate and easier one-dimensional problems,

(a) The horizontal motion is motion with uniform velocity (no effect of gravity)

(b) The vertical motion is motion of uniform acceleration, or freely falling bodies.

Note: In projectile motion, the horizontal motion and the vertical motion are independent of each other, that is either motion does not affects the other.



Treating as two straight line motions:–

The horizontal Motion(x axis):

Because there is no accelration in the horizontal direction, the horizontal component v_x of the projectile velocity remains unchanged from its initial value v_{0x} throughout the motion,

The vertical motion(y axis):

The vertical motion is the motion we discussed for a particle in free fall.

As is illustrated in figure, the vertical component behaves just as for a ball thrown vertically upward. It is directed upward initially and its magnitude steadily decreasing to zero, at the maximum height of the path. The vertical velocity component then reverses direction, and its magnitude becomes larger with time.

x-axis	**y-axis**
Initial velocity(u_x) = ucosθ	Initial velocity(u_y) = usinθ
acceleration(a_x) = 0	acceleration(a_y) = –g
Thus, velocity after time t	Thus, velocity after time t
v_x = ucosθ	v_y = usinθ – gt
Displacement after time t	Displacement after time t
x = ucosθt	y = usinθt – $gt^2/2$

Resultant velocity

$$(\vec{V}_R) = (u\cos\theta)\hat{i} + (u\sin\theta - gt)\hat{j}$$

$$|\vec{V}_R| = \sqrt{u^2\cos^2\theta + (u\sin\theta - gt)^2}$$

$$\& \quad \tan\alpha = \frac{u\sin\theta - gt}{u\cos\theta}$$

where α is angle that velocity vector makes with horizontal. Also known as direction or angle of motion

Time of flight(T)

$$T = \frac{2u\sin\theta}{g}$$

Considering vertical motion

$s_y = 0$; $u_y = u\sin\theta$; $a_y = -g$

$$0 = u\sin\theta T - gT^2/2 \Rightarrow T = \frac{2u\sin\theta}{g}$$

Maximum Height(H)

$$H = \frac{u^2\sin^2\theta}{2g}$$

Vertical velocity at maximum height $v_y = 0$

$$0 = u^2\sin^2\theta - 2gH \Rightarrow H = \frac{u^2\sin^2\theta}{2g}$$

Horizontal Range(R)

$$R = \frac{u^2\sin 2\theta}{g} = \frac{2u_x u_y}{g}$$

Total time $T = \dfrac{2u\sin\theta}{g}$

Velocity in horizontal direction $u_x = u\cos\theta$

Total displacement in horizontal direction $R = u\cos\theta T$

$$R = \frac{u^2\sin 2\theta}{g}$$

Note:– *For complementry angles i.e. $\theta + \alpha = 90°$, the range is same for same projection speed but maximum height and time of flight are different.*

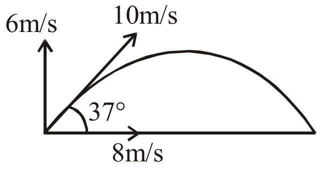
Ex. A body is thrown with initial velocity 10m/sec. at an angle 37° from horizontal. Find (Take : $g = 10m/s^2$)

(i) Time of flight (ii) Maximum height.

(iii) Range (iv) Position vector at t = 1 sec.

Ans. (i) 1.2 sec, (ii) 1.8 m, (iii) 9.6 m, (iv) $(8\hat{i} + \hat{j})$

Sol. (i) Time of flight $T = \dfrac{2u\sin\theta}{g} = \dfrac{2\times 10\times\frac{3}{5}}{10} = \dfrac{6}{5} = 1.2$ sec



(ii) Maximum height $H = \dfrac{u^2\sin^2\theta}{2g} = \dfrac{100\left(\frac{9}{25}\right)}{2\times 10} = \dfrac{9}{5} = 1.8$ m

(iii) $R = \dfrac{u^2\sin 2\theta}{g} = \dfrac{100\times 2\times\frac{3}{5}\times\frac{4}{5}}{10} = \dfrac{240}{25} = 9.6$m

(iv) $x = 8 \times 1 = 8$ m

$$y = 6 \times 1 - \frac{1}{2} \times 10 \times (1)^2 = 1\text{m}$$

$$\vec{r} = 8\hat{i} + \hat{j}$$

Maximum Range

$$R = \frac{u^2 \sin 2\theta}{g}$$

for $\theta = 45°$, R is maximum

$$R_{max} = \frac{u^2}{g}$$

Ex. A person can throw a ball vertically upto maximum height of 20 mt. How far can he throw the ball.

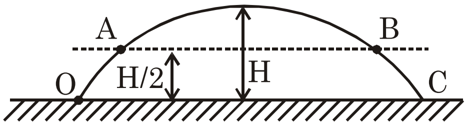
Sol. $H_{max} = \frac{u^2}{2g}$

$\therefore$ u = 20 m/s

$$R_{max} = \frac{u^2}{g} = 40 \text{ m}$$

Ex. A particle is projected with a speed u at an angle θ with horizontal. Find the average velocity of projectile for the period during which it crosses half of maximum height.

Sol.



avg. velocity is a vector

Let t_1 is time taken by particle to travel from A to B

First we will find vertical component.

For motion along AB

$$(\vec{V}_y)_{avg} = \frac{\text{Total Displacement along y direction}}{\text{Total time}} = \frac{0}{t_1} = 0$$

For Horizontal component

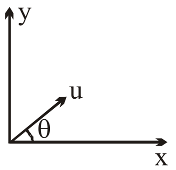
$$(\vec{V}_x)_{avg} = \frac{\text{Total Displacement along x direction}}{\text{Total time}}$$

$$(\vec{V}_x)_{avg} = \frac{(u\cos\theta)t_1}{t_1} = u\cos\theta$$

So average velocity for motion along AB is ucosθ

EQUATION OF TRAJECTORY

Lets say point of projection is our origin and horizontal direction is x-axis and vertically upwards is positive y-axis.



We know $x = u\cos\theta t$

$$\therefore \quad t = \frac{x}{u\cos\theta} \qquad .....(1)$$

also $y = u\sin\theta t - \frac{1}{2}gt^2 \qquad .....(2)$

Putting value of 't' from eq. (1) in eq. (2), we get

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

Ex. A particle is projected with a velocity 10 m/s at an angle 37° to the horizontal. Find the location at which the particle is at a height 1m from point of projection. ($g = 10\ m/s^2$)

Ans. 1.6 m, 8 m.

Sol. $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

for $y = 1$; $\theta = 37°$; $u = 10$ m/s

$$1 = \frac{3}{4}x - \frac{10x^2}{2\times100\times\left(\frac{16}{25}\right)}$$

$$1 = \frac{3}{4}x - \frac{5}{64}x^2$$

$5x^2 - 48x + 64 = 0$

$5x^2 - 40x - 8x + 64 = 0$

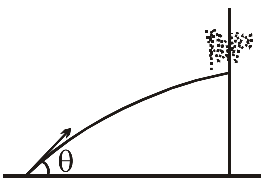
$x = 8m, 1.6m$

Ex. We have a hose pipe which disposes water at the speed of 10 ms^{-1}. The safe distance from a building on fire, on ground is 5 m. How high can this water go? (take : $g = 10\ ms^{-2}$)

Sol. Here we must understand that taking range of projectile as 10m and making projectile hit the building when it is at maximum height is wrong. By doing this we are not acheiving maximum y for given x = 5m. This just makes highest point of path to like on x = 5, But there may be other path for which y will be maximum for given x. This problem will be solved by using equation of trajectory by putting x = 5m and maximising y by varying θ.

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

Putting x = 5m we get



$$y = 5\tan\theta - \frac{10\times25\sec^2\theta}{2\times100}$$

$5\tan^2\theta - 20\tan\theta + (4y + 5) = 0$

for real roots discriminant must be positive.

$400 - 4 \times 5\,(4y + 5) > 0$

Solving $3.75 \geq y$

hence maximum y = 3.75 m

If we have taken range as 10 m then angle of projection will be θ = 45° corsponding maximum hight H = 2.5m which is smaller than our answer.

Ex. A projectile is fired horizontaly with a velocity of 98 m/s from the top of a hill 490 m high. Find
(i) Time to reach ground
(ii) The horizontal distance from foot of hill to ground.
(iii) The speed with which it hits the ground.

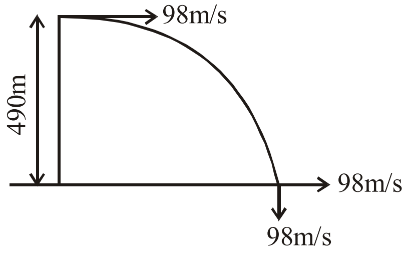
Ans. (i) 10 sec, (ii) 980 m, (iii) $98\sqrt{2}$ m/se

Sol. $u_y = 0; u_x = 98$ m/s
on y-axis

$$s = ut + \frac{1}{2}at^2$$

$$-490 = 0 - \frac{1}{2} \times 9.8 \times t^2$$

$$t = \sqrt{\frac{490\times 2}{9.8}} = 10 \text{ sec}$$



on x-axis for horizontal displacement,
$R = u_x t = 98 \times 10 = 980$ m
$v_x = 98$ m/s
$v_y = 0 - 9.8 \times 10 = -98$ m/s
So speed = $98\sqrt{2}$ m/s

Ex. A ball is thrown from the top of a tower with an initial velocity of 10m/s at an angle 37° above the horizontal, hits the ground at a distance 16m from the base of tower. Calculate height of tower. [$g = 10$ m/s^2]

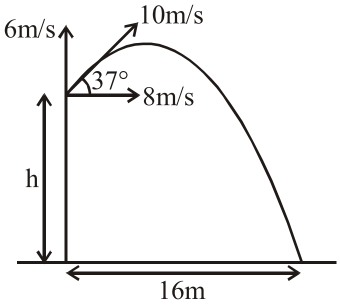
Ans. 8 m

Sol. Time to reach ground is $= \frac{16}{8} = 2$ sec
on y-axis (for height of tower)

$$-h = 6\times 2 - \frac{1}{2}\times 10\times (2)^2$$

$$-h = 12 - 22$$

$$h = 8 \text{ m}$$



Ex. Prithvi missile is fired to destroy an enemy military base situated on same horizontal level, situated 99 km away. The missile rises vertically for 1 km & then for remainder of flight, it followes parbolic path like a free body under earth's gravity, at an angle of 45°. Calculate its velocity at beginning of parabolic path. ($g = 10$ ms^{-2})

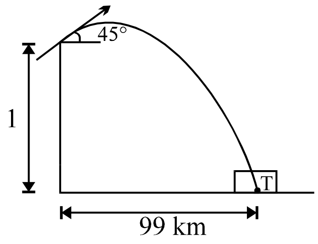
Sol. for horizontal motion time t

$$t = \frac{99\times 10^3}{u\cos 45°}$$

for vertical

$$-1\times 10^3 = u\sin 45°\, t - \frac{1}{2}\times 10 \times t^2$$

$$1\times 10^3 + \frac{u\sin 45°}{u\sin 45°}\times 99\times 10^3 = \frac{10}{2}\times\frac{(99\times 10^3)^2\times 2}{u^2}$$

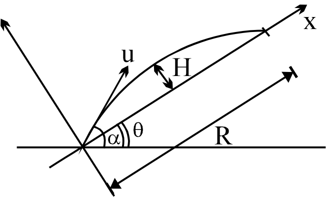


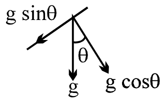
$$u^2 = \frac{(99\times10^3)^2\times10}{100\times10^3}$$

$$u = 99\times10^3\sqrt{\frac{1}{10^4}} = 990\ \text{ms}^{-1}$$

PROJECTION ON INCLINED PLANE

There is an inclined plane making an angle θ with horizontal. A particle is projected at an angle α from horizontal.





x-axis	y-axis
$u_x = u\cos(\alpha-\theta)$	$u_y = u\sin(\alpha-\theta)$
$a_x = -g\sin\theta$	$a_y = -g\cos\theta$
vel. at any time t	vel. at any time t
$v_x = u\cos(\alpha-\theta) - g\sin\theta\, t$	$v_y = u\sin(\alpha-\theta) - g\cos\theta\, t$

Time of flight

Displacement in y direction $s_y = 0$

$$0 = u\sin(\alpha-\theta)\,T - \frac{1}{2}g\cos\theta\,T^2$$

$$T = \frac{2u\sin(\alpha-\theta)}{g\cos\theta}$$

Maximum distance of particle from inclined plane

Point where $v_y = 0$ is max.-distance from inlcine plane

$$(0)^2 = u^2\sin^2(\alpha-\theta) - 2g\cos\theta\,H$$

$$H = \frac{u^2\sin^2(\alpha-\theta)}{2g\cos\theta}$$

Range along the inclined plane

$$s_x = u_xT + \frac{1}{2}a_xT^2$$

$$R = \frac{u\cos(\alpha-\theta)\times2u\sin(\alpha-\theta)}{g\cos\theta} - \frac{g\sin\theta\times2\times2u^2\sin^2(\alpha-\theta)}{2g^2\cos^2\theta}$$

$$= \frac{2u^2\sin(\alpha-\theta)[\cos(\alpha-\theta)\cos\theta-\sin\theta\sin(\alpha-\theta)]}{g\cos^2\theta}$$

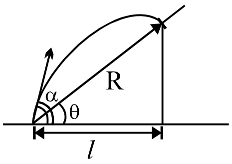
$$R = \frac{2u^2 \sin(\alpha - \theta)\cos\alpha}{g\cos^2\theta}$$

$$R = \frac{u^2[\sin(2\alpha - \theta) - \sin\theta]}{g\cos^2\theta}$$

Alternate Method

$l = u \cos \alpha\, T$

$$R = \frac{l}{\cos\theta}$$



$$R = \frac{u\cos\alpha}{\cos\theta} \times \frac{2u\sin(\alpha - \theta)}{g\cos\theta}$$

$$R = \frac{2u^2 \sin(\alpha - \theta)\cos\alpha}{g\cos^2\theta}$$

Note : Presence of incline plane does not affect the path of projectile in any way.

Maximum Range :

$$R = \frac{u^2[\sin(2\alpha - \theta) - \sin\theta]}{g\cos^2\theta}$$

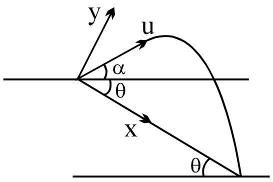
For max. range $2\alpha - \theta = \frac{\pi}{2} \Rightarrow \alpha = \frac{\pi}{4} + \frac{\theta}{2}$

so $R_{max} = \frac{u^2}{g(1 + \sin\theta)}$

Projection from top of incline plane:

Incline plane is at an angle θ with horizontal and a particle is projected at an angle α from horizontal.

In all formulae replace θ with –θ



$$H = \frac{u^2 \sin^2(\alpha + \theta)}{2g\cos\theta}$$

$$T = \frac{2u\sin(\alpha + \theta)}{g\cos\theta}$$

$$R = \frac{2u^2 \sin(\alpha + \theta)\cos\alpha}{g\cos^2\theta}$$

$R_{max} = \frac{u^2}{g(1 - \sin\theta)}$ and $\alpha = \frac{\pi}{4} - \frac{\theta}{2}$

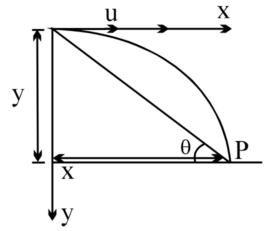
Note : If a particle strikes the incline plane ⊥ then its comp. of velocity along incline must be zero.

Ex. A particle is projected horizontally with a speed u from the top of a plane inclined at an angle θ with the horizontal. How far along the plane, from the point of projection will particle strike the plane?

Sol.

x-axis	**y-axis**
$u_x = u$	$u_y = 0$
$a_x = 0$	$a_y = g$
$x = ut$	$y = \frac{gt^2}{2}$

$$\Rightarrow \; y = \frac{gx^2}{2u^2}$$



$$\text{also } \frac{y}{x} = \tan\theta \Rightarrow x\tan\theta = \frac{gx^2}{2u^2}$$

$$x = 0, \; \frac{2u^2\tan\theta}{g}$$

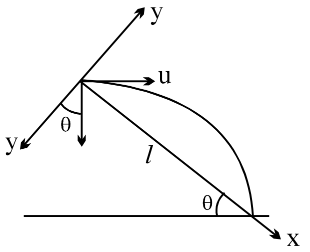
$$x = \frac{2u^2\tan\theta}{g}$$

$$\Rightarrow y = \frac{2u^2\tan^2\theta}{g}$$

$$\text{dist. } l = \sqrt{x^2+y^2}$$

$$l = \frac{2u^2\tan\theta\sec\theta}{g}$$

Alternate Method.

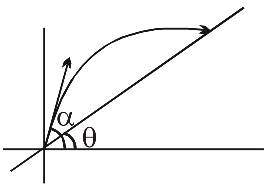


$$R = \frac{2u^2\sin(\alpha+\theta)\cos\alpha}{g\cos^2\theta} \quad \text{as } \alpha = 0$$

$R = 2\,u^2 \tan\theta \sec\theta$

Ex. A particle is projected up an inclined plane. Plane is inclined at an angle θ with horizontal and particle is projected at an angle α with horizontal. If particle strikes the plane horizontally prove that $\tan\alpha = 2\tan\theta$

Sol. We know time of flight



$$T = \frac{2u\sin(\alpha+\theta)}{g\cos\theta}$$

considering vertical motion

$u = v\sin\alpha$

$a = -g$

$v = 0$

$$\therefore\; T = \frac{u\sin\alpha}{g} = \frac{2u\sin(\alpha-\theta)}{g\cos\theta}$$

$\sin\alpha\cos\theta = 2\sin\alpha\;\cos\theta - 2\cos\alpha\sin\theta$

$2\cos\alpha\sin\theta = \sin\alpha\cos\theta$

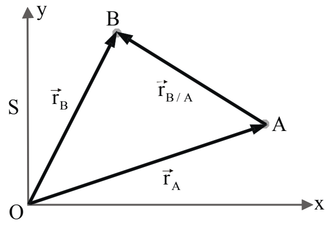
$2\tan\theta = \tan\alpha$

Relative Motion

Motion of a body can only be observed, when it changes its position with respect to some other body. In this sense, motion is a relative concept. To analyze motion of a body say A, therefore we have to fix our reference frame to some other body say B. The result obtained is motion of body A relative to body B.

Relative position, Relative Velocity and Relative Acceleration

Let two bodies represented by particles A and B at positions defined by position vectors $\vec{r}_A$ and $\vec{r}_B$, moving with velocities $\vec{v}_A$ and $\vec{v}_B$ and accelerations $\vec{a}_A$ and $\vec{a}_B$ with respect to a reference frame S. For analyzing motion of terrestrial bodies the reference frame S is fixed with the ground.



The vectors $\vec{r}_{B/A}$ denotes position vector of B relative to A. Following triangle law of vector addition, we have

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A} \qquad ...(i)$$

First derivatives of $\vec{r}_A$ and $\vec{r}_B$ with respect to time equals to velocity of particle A and velocity of particle B relative to frame S and first derivative of $\vec{r}_{B/A}$ with respect to time defines velocity of B relative to A.

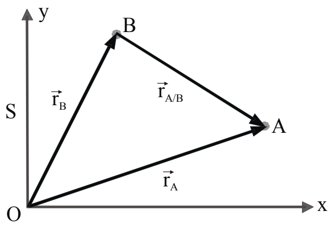
$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} \qquad ...(ii)$$

Second derivatives of $\vec{r}_A$ and $\vec{r}_B$ with respect to time equals to acceleration of particle A and acceleration of particle B relative to frame S and second derivative of $\vec{r}_{B/A}$ with respect to time defines acceleration of B relative to A.

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} \qquad ...(iii)$$

In similar fashion motion of particle A relative to particle B can be analyzed with the help of adjoining figure. You can observe in the figure that position vector of A relative to B is directed from B to A and therefore

$$\vec{r}_{B/A} = -\vec{r}_{A/B}, \ \vec{v}_{B/A} = -\vec{v}_{A/B} \text{ and } \vec{a}_{B/A} = -\vec{a}_{A/B}.$$

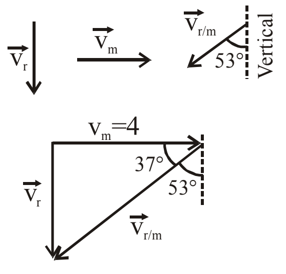


The above equations elucidate that how a body A appears moving to another body B is opposite to how body B appears moving to body A.

Ex. A man when standstill observes the rain falling vertically and when he walks at 4 km/h he has to hold his umbrella at an angle of 53° from the vertical. Find velocity of the raindrops.

Sol. Assigning usual symbols $\vec{v}_m$, $\vec{v}_r$ and $\vec{v}_{r/m}$ to velocity of man, velocity of rain and velocity of rain relative to man, we can express their relationship by the following eq. $\vec{v}_r = \vec{v}_m + \vec{v}_{r/m}$

The above equation suggests that a standstill man observes velocity $\vec{v}_r$ of rain relative to the ground and while he is moving with velocity $\vec{v}_m$, he observes velocity of rain relative to himself $\vec{v}_{r/m}$. It is a common intuitive fact that umbrella must be held against $\vec{v}_{r/m}$ for optimum protection from rain. According to these facts, directions of the velocity vectors are shown in the adjoining figure.



The addition of velocity vectors is represented according to the above equation. From the figure we have $v_r = v_m \tan 37° = 3$ km/h **Ans.**

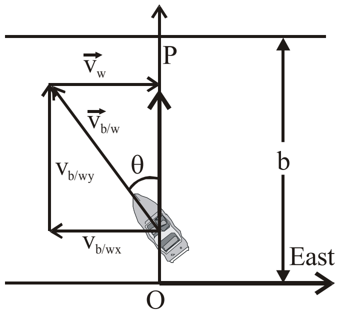
Ex. A boat can be rowed at 5 m/s on still water. It is used to cross a 200 m wide river from south bank to the north bank. The river current has uniform velocity of 3 m/s due east.

(a) In which direction must it be steered to cross the river perpendicular to current?

(b) How long will it take to cross the river in a direction perpendicular to the river flow?

(c) In which direction must the boat be steered to cross the river in minimum time? How far will it drift?

Sol. (a) Velocity of a boat on still water is its capacity to move on water surface and equals to its velocity relative to water.



$\vec{v}_{b/w}$ = Velocity of boat relative to water = Velocity of boat on still water

On flowing water, the water carries the boat along with it. Thus velocity $\vec{v}_b$ of the boat relative to the ground equals to vector sum of $\vec{v}_{b/w}$ and $\vec{v}_w$. The boat crosses the river with the velocity $\vec{v}_b$.

$$\vec{v}_b = \vec{v}_{b/w} + \vec{v}_w$$

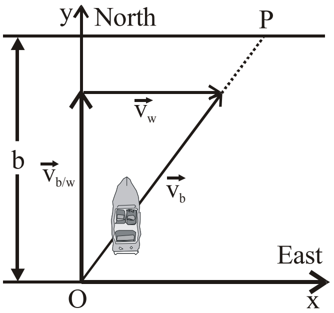
(b) To cross the river perpendicular to current the boat must be steered in a direction so that one of the components of its velocity ($\vec{v}_{b/w}$) relative to water becomes equal and opposite to water flow velocity $\vec{v}_w$ to neutralize its effect. It is possible only when velocity of boat relative to water is grater than water flow velocity. In the adjoining figure it is shown that the boat starts from the point O and moves along the line OP (y-axis) due north relative to ground with velocity $\vec{v}_b$. To achieve this it is steered at an angle θ with the y-axis.

$$v_{b/w}\sin\theta = v_w \rightarrow 5\sin\theta = 3 \Rightarrow \theta = 37° \textbf{ Ans.}$$

(c) The boat will cover river width b with velocity

$$v_b = v_{b/wy} = v_{b/w}\sin 37° = 4 \text{ m/s in time t, which is given by}$$

$$t = b/v_b \rightarrow t = 50\text{s} \textbf{ Ans.}$$



(d) To cross the river in minimum time, the component perpendicular to current of its velocity relative to ground must be kept to maximum value. It is achieved by steering the boat always perpendicular to current as shown in the adjoining figure. The boat starts from O at the south bank and reaches point P on the north bank. Time t taken by the boat is given by

$$t = b/v_{b/w} \rightarrow t = 40\text{s} \textbf{ Ans.}$$

Drift is the displacement along the river current measured from the starting point. Thus, it is given by the following equation. We denote it by x_d.

$$x_d = v_{bx}t$$

Substituting $v_{bx} = v_w = 3$ m/s, from the figure, we have

x_d = 120 m **Ans.**

EXERCISE (S-1)

General 2-D motion

1. The vertical height y and horizontal distance x of a projectile on a certain planet are given by x = (3t)m, y = $(4t - 6t^2)$ m where t is in seconds. Find the speed of projection (in m/s).

KM0108

2. The position of a particle is given by

$$r = 3.0t\,\hat{i} - 2.0t^2\,\hat{j} + 4.0\,k \text{ m}$$

where t is in seconds and the coefficients have the proper units for r to be in metres.

(a) Find the v and a of the particle?

(b) What is the magnitude and direction of velocity of the particle at t = 2.0 s ?

KM0109

3. A particle moves in xy plane such that $v_x = 50 - 16\,t$ and $y = 100 - 4t^2$ where v_x is in m/s and y is in m. It is also known that x = 0 when t = 0. Determine (i) Acceleration of particle (ii) Velocity of particle when y = 0.

KM0110

4. The position of a particle is given by $x = 7 + 3t^3$ m and $y = 13 + 5t - 9t^2$ m, where x and y are the position coordinates, and t is the time in s. Find the speed (magnitude of the velocity) when the x component of the acceleration is 36 m/s^2.

KM0111

Projectile motion

5. A particle is projected with a speed of 10 m/s at an angle 37° with the vertical. Find (i) time of flight (ii) maximum height above ground (iii) horizontal range.

KM0112

6. A particle is thrown with a speed 60 ms^{-1} at an angle 60° to the horizontal. When the particle makes an angle 30° with the horizontal in downward direction, it's speed at that instant is v. What is the value of v^2 in SI units ?

KM0113

7. A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high above the ground can the cricketer throw the same ball ?

KM0114

8. A particle is projected upwards with a velocity of 100 m/s at an angle of 60° with the vertical. Find the time when the particle will move perpendicular to its initial direction, taking $g = 10$ m/s^2.

KM0115

9. A particle is projected in *x*-*y* plane with *y*-axis along vertical, the point of projection is origin. The equation of a path is $y = \sqrt{3}x - \frac{gx^2}{2}$. Find angle of projection and speed of projection.

KM0116

10. (a) Show that for a projectile the angle between the velocity and the x-axis as a function of time is given by

$$\theta(t) = \tan^{-1}\left(\frac{v_{0y} - gt}{v_{ox}}\right)$$

(b) Show that the projection angle θ_0 of a projectile launched from the origin is given by

$$\theta_0 = \tan^{-1}\left(\frac{4h_m}{R}\right)$$

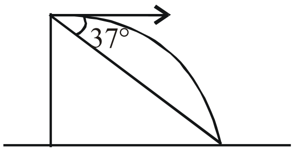
where the symbols have their usual meaning.

KM0117

11. A particle is projected in the *x*-*y* plane with *y*-axis along vertical. Two second after projection the velocity of the particle makes an angle 45° with the *X*-axis. Four second after projection, it moves horizontally. Find the velocity of projection.

KM0118

12. A ball is thrown horizontally from a cliff such that it strikes ground after 5 s. The line of sight from the point of projection to the point of landing makes an angle of 37° with the horizontal. What is the initial velocity of projection?



KM0119

13. A Bomber flying upward at an angle of 53° with the vertical releases a bomb at an altitude of 800 m. The bomb strikes the ground 20 s after its release. Find: [Given sin 53° = 0.8; $g = 10\ m/s^2$]

(i) The velocity of the bomber at the time of release of the bomb .

(ii) The maximum height attained by the bomb .

(iii) The horizontal distance travelled by the bomb before it strikes the ground

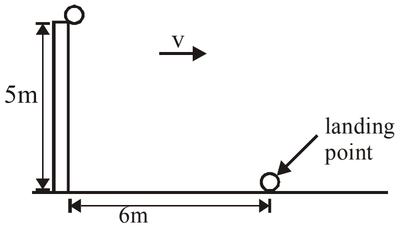
(iv) The velocity (magnitude & direction) of the bomb just when it strikes the ground.

KM0120

14. A ball is projected at an angle of 30° above the horizontal from the top of a tower and strikes the ground in 5 s at an angle of 45° with the horizontal. Find the height of the tower and the speed with which it was projected.

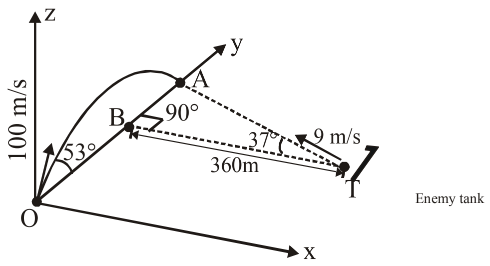
KM0121

15. A ball is dropped from rest from a tower of height 5m. As a result of the wind it lands at a distance 6m from the bottom of the tower as shown. Assuming no air resistance but that the wind gives the ball a constant horizontal velocity v. Find value of v in m/s.



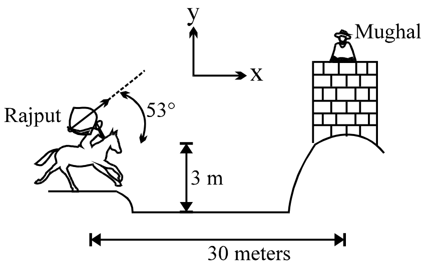
KM0122

16. A tank is initially at a perpendicular distance BT = 360 m from the plane of firing as shown. The enemy tank is moving with a speed of 9 m/s in direction TA as shown in figure. A gun can fire shell in y-z plane only with a speed 100 m/s at an angle of 53° such that the shell lands at points A. If tank started at t = 0 then time interval (in sec) after which shell is to be fired to hit the tank is



KM0123

17. A Rajput soldier, sits on a horse next to a river. Across the river there is a hill and atop the hill is a fortress. He sees a Mughal, sitting on the fortress's top wall. There is a full moon, so he angrily shoots an arrow at an angle 53° relative to the horizontal. The arrow hits Mughal after a 2 second flight. The horizontal distance from Rajput to Mughal is 30 meters. The arrow is 3 meters above the river when Rajput shoots it.



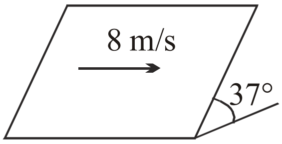
(a) What is the original velocity , $\vec{V}$, of the arrow when Rajput shoots it?

(b) What is Mughal elevation above the river?

(c) What is the flight direction of the arrow the instant before it strikes Mughal, i.e. what is the angle, θ, between its direction and the horizontal when it skins into Mughal's tender flesh?

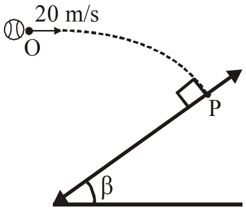
KM0124

18. A ball is projected on smooth inclined plane in direction perpendicular to line of greatest slope with velocity of 8 m/s. Find it's speed after 1 s.



KM0125

19. A ball is thrown horizontally from a point O with speed 20 m/s as shown. Ball strikes the incline plane along the normal to it after two seconds. Find value of *x*, if $\beta = \pi/x$ (where β is the angle of incline in degree).



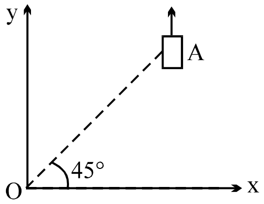
KM0126

Relative motion

20. A person decided to walk on an escalator which is moving at constant rate (speed). When he moves at the rate 1 step/sec, then he reaches top in 20 steps. Next day he goes 2 steps / sec. and reaches top in 32 steps. If speed of escalator is n steps / sec. Find the value of n.

KM0127

21. On a frictionless horizontal surface, assumed to be the x-y plane, a small trolley. A is moving along a straight line parallel to the y-axis (see figure) with a constant velocity of $(\sqrt{3}-1)$ m/s. At a particular instant, when the line OA makes an angle of 45° with the x-axis, a ball is thrown along the surface from the origin O. Its velocity makes an angle ϕ with the x-axis and it hits the trolley.

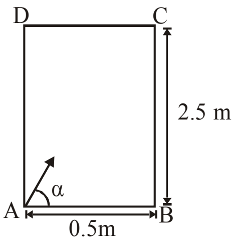


(a) The motion of the ball is observed from the frame of trolley. Calculate the angle θ made by the velocity vector of the ball with the x-axis in this frame.

(b) Find the speed of the ball with respect to the surface, if $\phi = \frac{4\theta}{3}$.

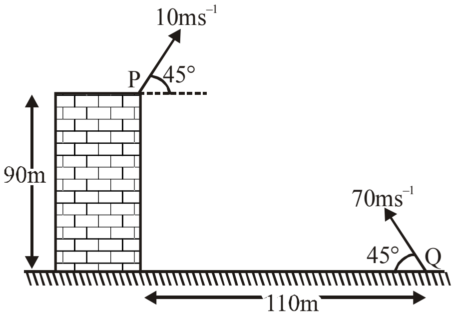
KM0128

22. A cuboidal elevator cabin is shown in the figure. A ball is thrown from point A on the floor of cabin when the elevator is falling under gravity. The plane of motion is ABCD and the angle of projection of the ball with AB, relative to elevator, if the ball collides with point C, is α. Then find the value of tanα.



KM0129

23. Two particles P and Q are launched simultaneously as shown in figure. Find the minimum distance between particles in meters.



KM0130

24. A man crosses a river by a boat. If he crosses the river in minimum time he takes 10 minutes with a drift 120 m. If he crosses the river taking shortest path, he takes 12.5 minutes. Assuming $v_{b/r} > v_r$, find (i) width of the river, (ii) velocity of the boat with respect to water ($v_{b/r}$) (iii) speed of the current (v_r)

KM0131

25. Rain is falling vertically with a speed of 20 m/s relative to air. A person is running in the rain with a velocity of 5 m/s and a wind is also blowing with a speed of 15 m/s (both towards east). Find the angle with the vertical at which the person should hold his umbrella for best protection from rain.

KM0132

26. A glass wind-screen of adjustable inclination is mounted on a car. The car moves horizontally with a speed of 6 m/s. At what angle α with the vertical should the wind screen be adjusted so that the rain drops falling vertically with 2 m/s strike the wind screen perpendicularly?

KM0133

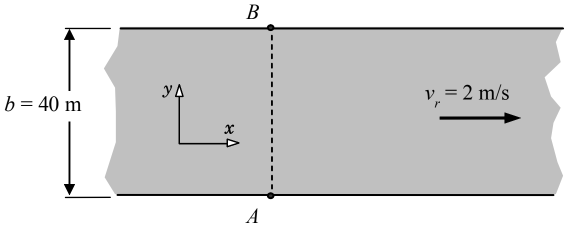
27. Boat moves with velocity 5m/s on still water. It is steered perpendicular to the river current.

(a) Will it reach point B or somewhere else on the other bank ?

(b) How long will it take to cross the river ?

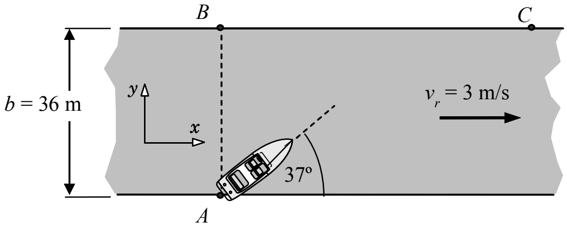
(c) How far down stream, will it reach the other bank ?

(d) Does it take minimum time in this way ?



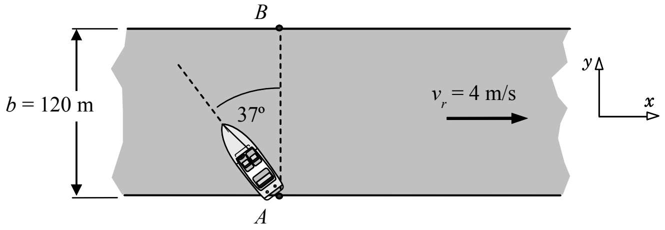
KM0134

28. Velocity of the boat with respect to river is 10 m/s. From point A it is steered in the direction shown to reach point C. Find the time of the trip and distance between B and C.



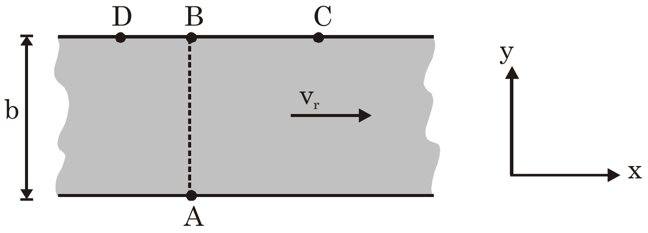
KM0135

29. Velocity of the boat with respect to river is 10 m/s. From point A it is steered in the direction shown. Where will it reach on the opposite bank?



KM0136

30. Drift is distance along a river a boat covers in crossing the river. If the boat reaches point C, distance BC is called downstream drift and if the boat reaches point D, distance BD is called upstream drift. To cross a river without drift, what should be relation between v_{br} and v_r. If a boat crosses a river without drift, in which direction must it be steered.



KM0137

EXERCISE (S-2)

1. A particle travels so that its acceleration is given by $\vec{a} = 5\cos t\,\hat{i} - 3\sin t\,\hat{j}$. If the particle is located at (–3, 2) at time t = 0 and is moving with a velocity given by $(-3\hat{i} + 2\hat{j})$. Find

(a) the velocity at time t and

(b) the position vector of the particle at time (t > 0).

KM0138

2. A man can throw a stone with initial speed of 10 m/s. Find the maximum horizontal distance to which he can throw the stone in a room of height *h* for :

(i) $h = 2$ m &

(ii) $h = 4$ m

KM0139

3. Two inclined planes *OA* and *OB* having inclinations 30° and 60° respectively, intersect each other at *O* as shown in figure. A particle is projected from point *P* with velocity $u = 10\sqrt{3}$ m/s along a direction perpendicular to plane *OA*. If the particle strikes the plane *OB* perpendicularly at *Q*. Calculate

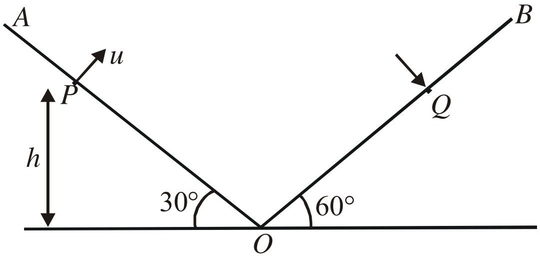
(i) Time of flight.

(ii) Velocity with which particle strikes the plane *OB*.

(iii) Vertical height of *P* from *O*.

(iv) Maximum height from *O* attained by the particle.

(v) Distance *PQ*.

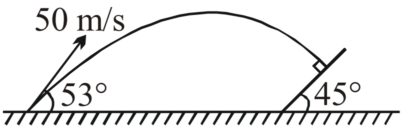


KM0140

4. A projectile is thrown with velocity of 50 m/s towards an inclined plane from ground such that it strikes the inclined plane perpendicularly. The angle of projection of the projectile is 53° with the horizontal and the inclined plane is inclined at an angle of 45° to the horizontal.

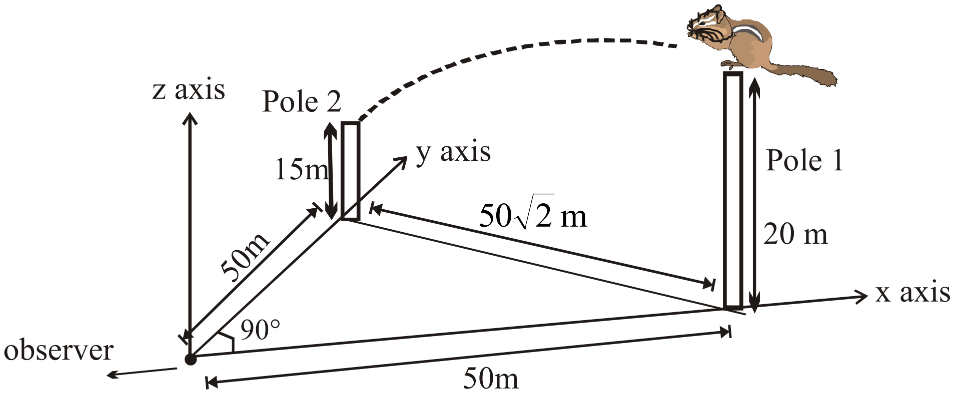
(i) Find the time of flight.

(ii) Find the distance between the point of projection and the foot of inclined plane.



KM0141

5. A small squirrel jumps from pole 1 to pole 2 in horizontal direction. Squirrels is observed by a very small observer at origin. What is average velocity vector of squirrel ? If average velocity vector is expressed as $v_x\hat{i}+v_y\hat{j}+v_z\hat{k}$, express your answer as sum of magnitudes of its components $|v_x|+|v_y|+|v_z|$ in unit m/s.



KM0142

6. Two particles start simultaneously from a point and move along line *OP* and *OQ*, one with a uniform velocity 10 m/s and other from rest with a constant acceleration 5 m/s^2 respectively. Line *OP* makes an angle 37° with the line *OQ*. Find the time at which they appear to each other moving at minimum speed?

KM0143

7. Two guns, situated at the top of a hill of height 10 m, fire one shot each with the same speed $5\sqrt{3}$ m/s at some interval of time. One gun fires horizontally and other fires at an angle of 60° up the horizontal. The shots collide in air at a point *P*. Find

(i) the time interval between the firings, and (ii) the coordinates of the point *P*.

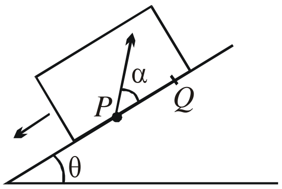
(Take origin of the coordinates system at the foot of the hill vertically below the muzzle and trajectories in *X*-*Y* plane)

KM0144

8. A hunter is riding an elephant of height 4 m moving in straight line with uniform velocity of 2 m/s. A deer starts running with uniform velocity from a point $4\sqrt{5}$ m away in front of the elephant along a line perpendicular to velocity of the elephant. If hunter can throw his spear with a speed of 10 m/s, relative to the elephant, at what angle θ to it's direction of motion must he throw his spear horizontally for a successful hit. Find also the speed of the deer.

KM0145

9. A large heavy box is sliding without friction down a smooth plane of inclination θ. From a point *P* on the bottom of the box, a particle is projected inside the box. The initial speed of the particle with respect to box is *u* and the direction of projection makes an angle α with the bottom as shown in figure. **[JEE]**



(i) Find the distance along the bottom of the box between the point of projection P and the point Q where the particle lands. (Assume that the particle does not hit any other surface of the box. Neglect air resistance).

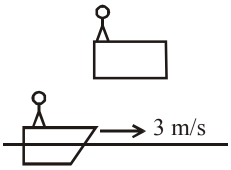
(ii) If the horizontal displacement of the particle as seen by an observer on the ground is zero, find the speed of the box with respect to the ground at the instant when the particle was projected.

KM0146

10. A particle is projected from ground towards a vertical wall 80 m away at an angle of 37° with horizontal with initial velocity of 50 m/s. After its collision with wall & then once with ground find at what distance in meter from wall will it strike the ground. The component of velocity normal to the surface becomes half after collision with each surface.

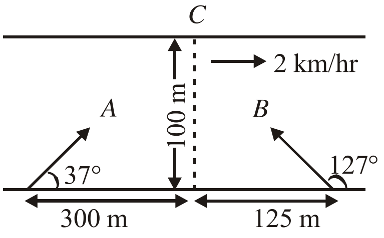
KM0147

11. You are standing on the chambal Bridge watching the boats in the river. You see a motorboat pass directly below you, traveling perpendicular to the bridge at a speed of 3 m/s. A person on the boat throws a baseball at an initial speed of v_0 and at an angle of 37° from the vertical (Note: both v_0 and the angle are with respect to the boat). Find the value of v_0 (in m/s) necessary for the ball to travel straight up towards you.



KM0148

12. Two swimmers start a race. One who reaches the point C first on the other bank wins the race. Boy A makes his strokes in a direction of 37° to the river flow with velocity 5 km/hr relative to water. Boy B makes his strokes in a direction of 127° to the river flow with same relative velocity. River is flowing with speed of 2 km/hr and is 100 m wide. Who will win the race? Compute the time taken by A and B to reach the point C if the speeds of A and B on the ground are 8 km/hr and 6 km/hr respectively.

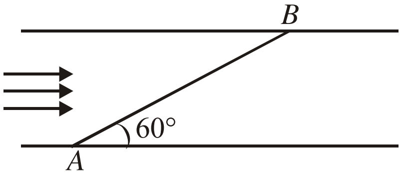


KM0149

13. A swimmer starts to swim from point A to cross a river. He wants to reach point B on the opposite side of the river. The line AB makes an angle 60° with the river flow as shown. The velocity of the swimmer in still water is same as that of the water

(i) In what direction should he try to direct his velocity? Calculate angle between his velocity and river velocity.

(ii) Find the ratio of the time taken to cross the river in this situation to the minimum time in which he can cross this river.



KM0150

14. Hailstones falling vertically with speed of 10 m/s hit with respect to himself the windscreen of a moving car and rebound elastically. Find the velocity of the car if the driver find the hailstones rebound vertically after striking. Windscreen makes an angle 30° with the horizontal.

KM0151

15. A motor boat set out at 11 a.m. from a position $-6i - 2j$ and travels at a steady speed of magnitude $\sqrt{53}$ on a direct course to intercept a ship. The ship maintains a steady velocity vector $3i + 4j$ and at 12 noon is at a position $3i - j$. Find

(a) the velocity vector of the motor boat,

(b) the time of interception and

(c) the position vector of point of interception. If distances are measured in kilometres and speeds in kilometres per hour.

KM0152

EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

General 2-D motion

1. A particle is projected from a horizontal plane (*x*-*z* plane) such that its velocity vector at time t is given by $\vec{V} = a\hat{i} + (b - ct)\hat{j}$. Its range on the horizontal plane is given by :-

(A) $\frac{ba}{c}$ (B) $\frac{2ba}{c}$ (C) $\frac{3ba}{c}$ (D) None

KM0153

2. A body of mass 5 kg starts from the origin with an initial velocity $\vec{u} = \left(30\hat{i} + 40\hat{j}\right)\text{ms}^{-1}$. If a constant force $\left(-6\hat{i} - 5\hat{j}\right)\text{N}$ acts on the body, the time in which the y component of the velocity becomes zero, is :-

(A) 5s (B) 20 s (C) 40 s (D) 80 s

KM0154

Projectile motion

3. A boy throws a ball from shoulder height at an initial velocity of 30 m/s. Spending 4.8 s in air, the ball is caught by another boy as the same shoulder-height level. What is the angle of projection?

(A) 37° (B) 30° (C) 53° (D) 60°

KM0155

4. A particle is projected from the ground with velocity *u* at angle θ with horizontal. The horizontal range, maximum height and time of flight are *R*, *H* and *T* respectively. They are given by

$$R = \frac{u^2 \sin 2\theta}{g} \qquad H = \frac{u^2 \sin^2\theta}{2g} \qquad \text{and} \qquad T = \frac{2u\sin\theta}{g}$$

Now keeping *u* fixed, θ is varied from 30° to 60°, then

(A) *R* will first increase then decrease, *H* will increase and *T* will decrease

(B) *R* will first increase then decrease while *H* and *T* both will increase

(C) *R* will decrease while *H* and *T* both will increase

(D) *R* will increase while *H* and *T* both will also increase

KM0156

5. Suppose a player hits several baseballs. Which baseball will be in the air for the longest time?

(A) The one with the farthest range.

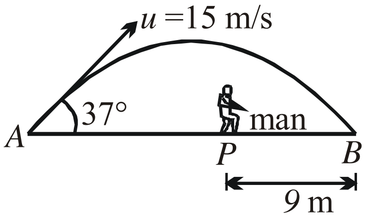
(B) The one which reaches maximum height.

(C) The one with the greatest initial velocity.

(D) The one leaving the bat at 45° with respect to the ground.

KM0157

6. A ball is hit by a batsman at an angle of 37° as shown in figure. The man standing at *P* should run at what minimum velocity so that he catches the ball before it strikes the ground. Assume that height of man is negligible in comparison to maximum height of projectile.



(A) 3 m/s (B) 5 m/s (C) 9 m/s (D) 12 m/s

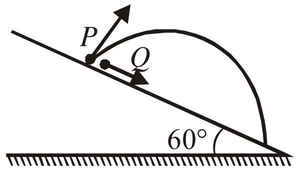
KM0158

7. A ball is projected horizontally. After 3 s from projection its velocity becomes 1.25 times of the velocity of projection. Its velocity of projection is :-

(A) 10 m/s (B) 20 m/s (C) 30 m/s (D) 40 m/s

KM0159

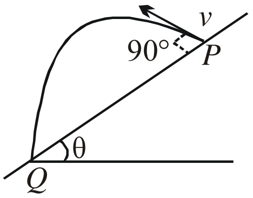
8. A particle *P* is projected from a point on the surface of smooth inclined plane (see figure). Simultaneously another particle *Q* is released on the smooth inclined plane from the same position. *P* and *Q* collide after t = 4 s. The speed of projection of *P* is :-



(A) 5 m/s (B) 10 m/s (C) 15 m/s (D) 20 m/s

KM0160

9. In the given figure, if time taken by the projectile to reach *Q* is *T*, than *PQ* =



(A) $Tv \sin\theta$ (B) $Tv \cos\theta$ (C) $Tv \sec\theta$ (D) $Tv \tan\theta$

KM0161

10. A particle is projected up the incline such that its component of velocity along the incline is 10m/s. Time of flight is 2 second and maximum perpendicular distance during the motion from the incline is 5 m. Then velocity of projection will be :-

(A) 10 m/s (B) $10\sqrt{2}$ m/s (C) $5\sqrt{5}$ m/s (D) none of these

KM0162

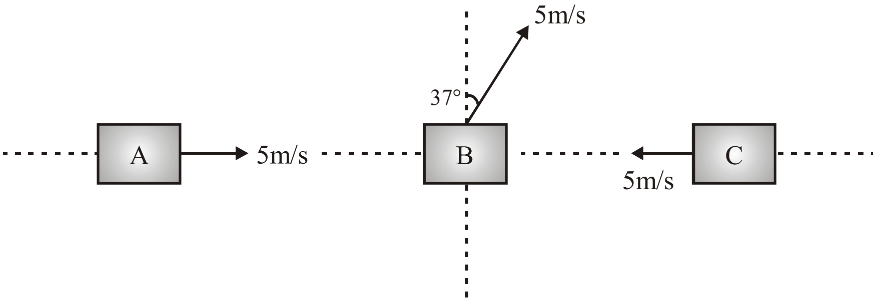
11. A particle A is projected with speed v_A from a point making an angle 60° with the horizontal. At the same instant, a second particle B is thrown vertically upward from a point directly below the maximum height point of parabolic path of A with velocity v_B. If the two particles collide then the ratio of v_A/v_B should be :-

(A) 1 (B) $\frac{2}{\sqrt{3}}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\sqrt{3}$

KM0163

Relative motion

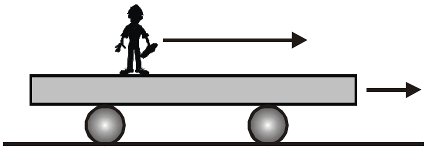
12. Consider the motion of three bodies as shown for an observer on B, what is the magnitude of relative velocity of A with respect to C ?



(A) 10 m/s (B) 0 (C) $\sqrt{20}$ m/s (D) 8 m/s

KM0164

13. A trolley is moving horizontally with a constant velocity of v with respect to earth. A man starts running from one end of the trolley with a velocity 1.5 v with respect to trolley. After reaching the opposite end, the man returns back and continues running with a velocity of 1.5 v w.r.t. the trolley in the backward direction. If the length of the trolley is L then the displacement of the man with respect to earth during the process will be :-



(A) 2.5 L (B) 1.5 L (C) $\frac{5L}{3}$ (D) $\frac{4L}{3}$

KM0165

14. An elevator car (lift) is moving upward with uniform acceleration of 2 m/s^2. At the instant, when its velocity is 2 m/s upwards a ball is thrown upward from its floor. The ball strikes back the floor 2 s after its projection. Find the velocity of projection of the ball relative to the lift.

(A) 10 m/s ↑ (B) 10 m/s ↓ (C) 12 m/s ↑ (D) 12 m/s ↓

KM0166

15. A flag is mounted on a car moving due North with velocity of 20 km/hr. Strong winds are blowing due East with velocity of 20 km/hr. The flag will point in direction :-

(A) East (B) North-East (C) South-East (D) South-West

KM0167

16. Three ships *A*, *B* & *C* are in motion. Ship *A* moves relative to *B* is with speed *v* towards North-East. Ship B moves relative to *C* with speed *v* towards the North-West. Then relative to *A*, *C* will be moving towards :-

(A) North (B) South (C) East (D) West

KM0168

17. Wind is blowing in the north direction at speed of 2 m/s which causes the rain to fall at some angle with the vertical. With what velocity should a cyclist drive so that the rain appears vertical to him :

(A) 2 m/s south (B) 2 m/s north (C) 4 m/s west (D) 4 m/s south

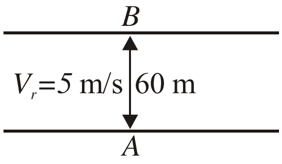
KM0169

18. A boat having a speed of 5 km/hr in still water, crosses a river of width 1 km along the shortest possible path in 15 minutes. The speed of the river in Km/hr.

(A) 1 (B) 3 (C) 4 (D) $\sqrt{41}$

KM0170

19. A man is crossing a river flowing with velocity of 5 m/s. He reaches a point directly across the river at a distance of 60 m in 5 sec. His velocity in still water should be :-



(A) 12 m/s (B) 13 m/s (C) 5 m/s (D) 10 m/s

KM0171

20. A motor boat is to reach at a point 30° upstream (w.r.t. normal) on other side of a river flowing with velocity 5m/s. The angle 30° is measured from a direction perpendicular to river flow. Velocity of motorboat with respect to water is $5\sqrt{3}$ m/s. The driver should steer the boat at an angle

(A) 120° with respect to stream direction.

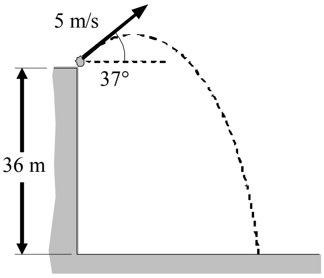
(B) 30° with respect to the perpendicular to the bank.

(C) 30° with respect to the line of destination from starting point.

(D) None of these.

KM0172

21. A ball is thrown from the top of 36 m high tower with velocity 5 m/s at an angle 37° above the horizontal as shown. Its horizontal range on the ground is closest to [g = 10 m/s^2]



(A) 12 m (B) 18 m (C) 24 m (D) 30 m

KM0173

MULTIPLE CORRECT TYPE QUESTIONS

22. A particle moves in the *xy*-plane and at time *t* is at the point $(t^2,\ t^3 - 2t)$. Then

(A) At $t = 2/3$ s, directions of velocity and acceleration are perpendicular

(B) At $t = 0$, directions of velocity and acceleration are perpendicular

(C) At $t = \sqrt{2/3}$ s, particle is moving parallel to *x*-axis

(D) Acceleration of the particle when it is at point (4, 4) is $\left(2\hat{i} + 12\hat{j}\right) m/s$

KM0174

23. A particle moves in the *x-y* plane with a constant acceleration *g* in the negative *y*-direction. Its equation of motion is $y = ax - bx^2$, where *a* and *b* are constants. Which of the following is/are correct?

(A) The *x*-component of its velocity is constant.

(B) At the origin, the *y*-component of its velocity is $a\sqrt{\frac{g}{2b}}$.

(C) At the origin, its velocity makes an angle $\tan^{-1}(a)$ with the *x*-axis.

(D) The particle moves exactly like a projectile.

KM0175

24. Two particles *A* and B projected along different directions from the same point *P* on the ground with the same velocity of 70 m/s in the same vertical plane . They hit the ground at the same point *Q* such that *PQ* = 480 m. Then [g = 9.8 m/s^2]

(A) Ratio of their times of flight is 4 : 5

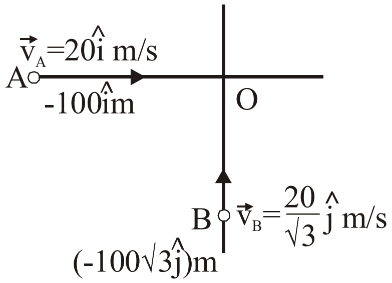
(B) Ratio of their maximum heights is 9 : 16

(C) Ratio of their minimum speeds during flights is 4 : 3

(D) The bisector of the angle between their directions of projection makes 45° with horizontal

KM0176

25. Positions of two vehicles A and B with reference to origin O and their velocities are as shown.



(A) they will collide

(B) distance of closest approach is 100 m.

(C) their relative speed is $\frac{40}{\sqrt{3}}$ m/s

(D) their relative velocity is $\frac{20}{\sqrt{3}}$ m/s

KM0177

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 26 to 28

Two projectiles are thrown simultaneously in the same vertical plane from the same point. If their velocities of projection are v_1 and v_2 at angles θ_1 and θ_2 respectively from the horizontal, then answer the following questions

26. The trajectory of particle 1 with respect to particle 2 will be

(A) a parabola

(B) a straight line

(C) a vertical straight line

(D) a horizontal straight line

KM0178

27. If $v_1\cos\theta_1 = v_2\cos\theta_2$, then choose the ***incorrect*** statement

(A) One particle will remain exactly below or above the other particle

(B) The trajectory of one with respect to other during the flight will be a vertical straight line

(C) Both will have the same range

(D) Both will attain same maximum height

KM0178

28. If $v_1\sin\theta_1 = v_2\sin\theta_2$, then choose the ***correct*** statement

(A) The time of flight of both the particles will be same

(B) The maximum height attained by the particles will be same

(C) The trajectory of one with respect to another during the flight will be a horizontal straight line

(D) None of these

KM0178

Paragraph for Question Nos. 29 to 31

A particle leaves the origin with initial velocity $\vec{v}_0 = 11\hat{i} + 14\hat{j}\ \text{m/s}$. It undergoes a constant acceleration given by $\vec{a} = -\frac{22}{5}\hat{i} + \frac{2}{15}\hat{j}$ m/s^2.

29. When does the particle cross the y axis ?

(A) 2 sec (B) 4 sec (C) 5 sec (D) 7 sec

KM0179

30. At the instant when particle crosses y-axis, direction in which particle is moving is :-

(A) At angle 37° from +x-axis towards +y-axis
(B) At angle 37° from –x-axis towards +y-axis
(C) At angle 53° from +x-axis towards +y-axis
(D) At angle 53° from –x-axis towards +y-axis

KM0179

31. How far is it from the origin, at that time ?

(A) 70 m (B) 71.67 m (C) 125 m (D) 15 m

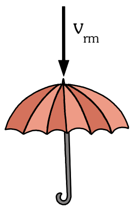
KM0179

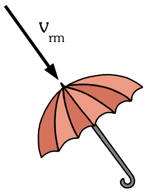
Paragraph for Question no. 32 to 35: Rain and man

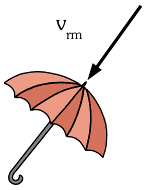
By the term velocity of rain, we mean velocity with which raindrops fall relative to the ground. In absence of wind, raindrops fall vertically and in presence of wind raindrops fall obliquely. Moreover raindrops acquire a constant terminal velocity due air resistance very quickly as they fall toward the earth. A moving man relative to himself observes an altered velocity of raindrops, which is known as velocity of rain relative to the man. It is given by the following equation.

$$\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$$

A standstill man relative to himself observes rain falling with velocity, which is equal to velocity of the raindrops relative to the ground. To protect himself a man should hold his umbrella against velocity of raindrops relative to himself as shown in the following figure.







32. Rain is falling vertically with velocity 80 cm/s.

(a) How should you hold your umbrella?
(b) You start walking towards the east with velocity 60 cm/s. How should you hold your umbrella?
(c) You are walking towards the west with velocity 60 cm/s. How should you hold your umbrella?
(d) You are walking towards the north with velocity 60 cm/s. How should you hold your umbrella?
(e) You are walking towards the south with velocity 80 cm/s. How should you hold your umbrella?

KM0180

33. When you are standstill in rain, you have to hold your umbrella vertically to protect yourself.

(a) When you walk with velocity 90 cm/s, you have to hold your umbrella at 53° above the horizontal. What is velocity of the raindrops relative to the ground and relative to you?

(b) If you walk with speed 160 cm/s, how should you hold your umbrella?

KM0180

34. A man walks in rain at 72 cm/s due east and observes the rain falling vertically. When he stops, rain appears to strike his back at 37° from the vertical. Find velocity of raindrops relative to the ground.

KM0180

35. When you walk in rain at 75 cm/s, you have to hold your umbrella vertically and when you double your speed in the same direction, you have to hold your umbrella at 53° above the horizontal. What is the rain velocity?

Paragraph for Question no. 36 to 38 : Flag in wind

When you are standstill holding a flag, the flag flutters in the direction of wind. When you start running the direction of fluttering of the flag changes in to the direction of the wind relative to you. In all case a flag flutters in the direction of the wind relative to the flag.

36. When you are standstill holding a flag the flag flutters in the north and when you run at 8 m/s due east, the flag flutters in direction 37° north of west. Find the wind velocity.

KM0181

37. Wind is blowing uniformly due north everywhere with velocity 12 m/s. A car mounted with a flag starts running towards east. After 9 s from start the flag flutters in 53° north of west and after 16 s from the start the flag flutters in 37° north of west.

(a) Find velocity of the car 9 s after it starts.

(b) Find velocity of the car 16 s after it starts.

(c) If the car maintains uniform acceleration, find acceleration of the car.

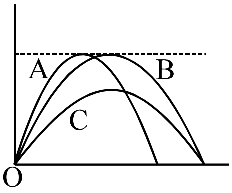
KM0181

38. Holding a flag, when you run at 8 m/s due east, the flag flutters in the north and when you run at 2 m/s due south, the flag flutters in the northeast. If the wind velocity is uniform and remain constant, find the wind velocity.

KM0181

MATRIX MATCH TYPE QUESTION

39. Trajectories are shown in figure for three kicked footballs. Initial vertical & horizontal velocity components are u_y and u_x respectively. Ignoring air resistance, choose the correct statement from **Column-II** for the value of variable in **Column-I**.



	Column-I		**Column-II**
(A)	time of flight	(P)	greatest for A only
(B)	u_y/u_x	(Q)	greatest for C only
(C)	u_x	(R)	equal for A and B
(D)	$u_x u_y$	(S)	equal for B and C

KM0182

40. Trajectory of particle in a projectile motion is given as $y = x - \frac{x^2}{80}$. Here, x and y are in meters. For this projectile motion, match the following with $g = 10\ m/s^2$.

	Column–I		**Column–II**
(A)	Angle of projection (in degrees)	(P)	20
(B)	Angle of velocity with horizontal after 4s (in degrees)	(Q)	80
(C)	Maximum height (in metres)	(R)	45
(D)	Horizontal range (in metres)	(S)	30
		(T)	60

KM0183

EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

1. A particle is moving in x-y plane. At certain instant, the components of its velocity and acceleration are as follows ; $V_x = 3$m/s, $V_y = 4$m/s, $a_x = 2$ m/s^2 and $a_y = 1$ m/s^2. The rate of change of speed at this moment is :-

(A) $\sqrt{10}\ \text{m/s}^2$ (B) 4 m/s^2 (C) 10 m/s^2 (D) 2 m/s^2

KM0184

2. A particle leaves the origin with an initial velocity $\vec{v} = \left(3\hat{i} + 4\hat{j}\right) ms^{-1}$ and a constant acceleration $\vec{a} = \left(-\hat{i} - 0.5\hat{j}\right) ms^{-2}$. When the particle reaches its maximum x–coordinate, what is the y–coordinate?

(A) $\frac{27}{4}$m (B) $\frac{37}{4}$m (C) $\frac{29}{4}$m (D) $\frac{39}{4}$m

KM0185

3. The position vector of a particle is determined by $\vec{r} = 3t^2\hat{i} + 4t^2\hat{j} + 7\hat{k}$. The distance travelled in first 10 sec is :-

(A) 100 m (B) 150 m (C) 500 m (D) 300 m

KM0186

4. A point moves in x–y plane according to the law x = 4sin6t and y = 4(1 – cos6t). The distance traversed by the particle in 4 seconds is (x and y are in metres)

(A) 96 m (B) 48 m (C) 24 m (D) 108 m

KM0187

5. A particle moves in the x-y plane. It x and y coordinates vary with time t according to equations $x = t^2 + 2t$ and $y = 2t$. Possible shape of path followed by the particle is

(A) Straight line (B) Circle

(C) Parabola (D) More information is required to decide.

KM0188

6. Particle is dropped from the height of 20 m from horizontal ground. A constant force acts on the particle in horizontal direction due to which horizontal acceleration of the particle becomes 6 m/s^2. Find the horizontal displacement of the particle till it reaches ground.

(A) 6 m (B) 10 m (C) 12 m (D) 24 m

KM0189

7. A projectile is fired with a velocity u making an angle θ with the horizontal. What is the magnitude of change in velocity when it is at the highest point ?

(A) u cos θ (B) u (C) u sin θ (D) u cos θ – u

KM0190

8. A particle is projected at an angle of 45° from a point lying 2 m from the foot of a wall. It just touches the top of the wall and falls on the ground 4m from it. The height of the wall is

(A) 3/4 m (B) 2/3 m (C) 4/3m (D) 1/3 m

KM0191

9. A ball was thrown by a boy A at angle 60° with horizontal at height 1m from ground. Boy B is running in the plane of motion of ball and catches the ball at height 1m from ground. He finds the ball falling vertically. If the boy is running at a speed 20 km/hr. Then the velocity of projection of ball is-

(A) 20 km/hr (B) 30 km/hr (C) 40 km/hr (D) 50 km/hr

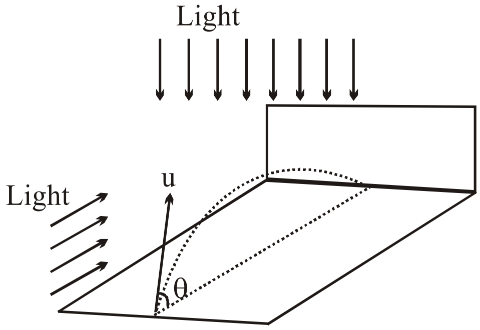
KM0192

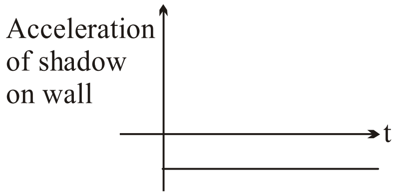
10. A light body is projected with a velocity $(10\hat{i}+20\hat{j}+20\hat{k})$ ms^{-1}. Wind blows along X-axis with an acceleration of 2.5 ms^{-2}. If Y-axis is vertical then the speed of particle after 2 second will be (g = 10 ms^{-2})

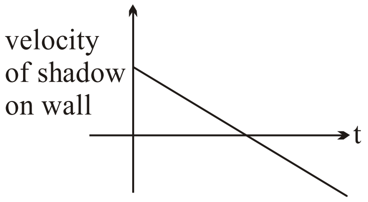
(A) 25 ms^{-1} (B) $10\sqrt{5}$ ms^{-1} (C) 30 ms^{-1} (D) None of these

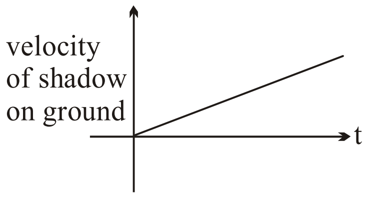
KM0193

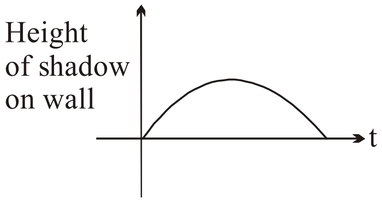
11. A projectile is projected as shown in figure. A proper light arrangement makes a shadow on the wall as well as on the floor ? Which of the following graphs is incorrect.



(A)


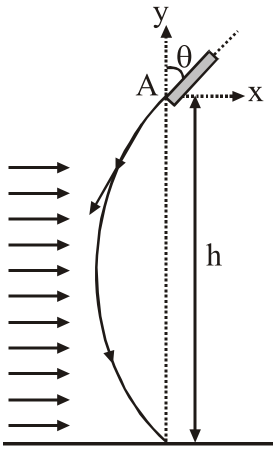
(B)


(C)


(D)


KM0194

12. A particle is ejected from the tube at A with a velocity v at an angle θ with the vertical y–axis. A strong horizontal wind gives the particle a constant horizontal acceleration a in the x–directions. If the particle strikes the ground at a point directly under its released position and the downward y–acceleration is taken as g then



(A) $h = \dfrac{2v^2 \sin\theta\cos\theta}{a}$

(B) $h = \dfrac{2v^2 \sin\theta\cos\theta}{g}$

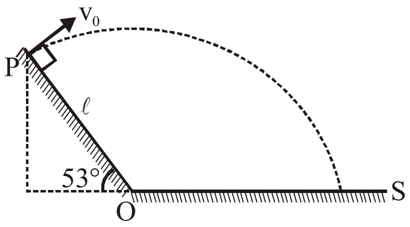
(C) $h = \dfrac{2v^2}{g}\sin\theta\left(\cos\theta + \dfrac{a}{g}\sin\theta\right)$

(D) $h = \dfrac{2v^2}{a}\sin\theta\left(\cos\theta + \dfrac{g}{a}\sin\theta\right)$

KM0195

13. A stone is projected from point P on the inclined plane with velocity $v_0 = 10$ m/s directed perpendicular to the plane. The time taken by the stone to strike the horizontal ground S is

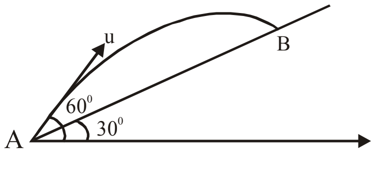
(Given PO = ℓ = 10 meter)



(A) 1.5 sec (B) 1.4 sec (C) 2 sec (D) 2.3 sec

KM0196

14. Time taken by the projectile to reach from A to B is t. Then the distance AB is equal to :-



(A) $\dfrac{ut}{\sqrt{3}}$ (B) $\dfrac{\sqrt{3}ut}{2}$ (C) $\sqrt{3}$ ut (D) 2 ut

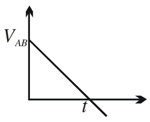
KM0197

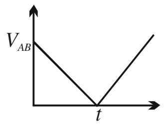
15. A particle is projected from a point *P* (2 m, 0 m, 0 m) with a velocity 10 m/s making an angle 45° with the horizontal. The plane of projectile motion passes through a horizontal line *PQ* which makes an angle of 37° with positive x–axis and *xy* plane is horizontal. The coordinates of the point where the particle will strike the line *PQ* is ($g = 10$ m/s^2)

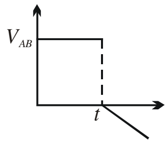
(A) (10 m, 6 m, 0 m) (B) (8 m, 6 m, 0 m) (C) (10 m, 8 m, 0 m) (D) (6 m, 10 m, 0 m)

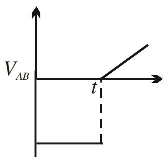
KM0198

16. A body A is thrown vertically upwards with such a velocity that it reaches a maximum height of *h*. Simultaneously another body *B* is dropped from height *h*. It strikes the ground and does not rebound. The velocity of *A* relative to *B* v/s time graph is best represented by (upward direction is positive)

(A)


(B)


(C)


(D)


KM0199

17. An object moves to the East across a frictionless surface with constant speed. A person then applies a constant force to the North on the object. What is the resulting path that the object takes?

(A) A straight line path partly Eastward, partly Northward

(B) A straight line path totally to the North

(C) A parabolic path opening toward the North

(D) A parabolic path opening toward the East

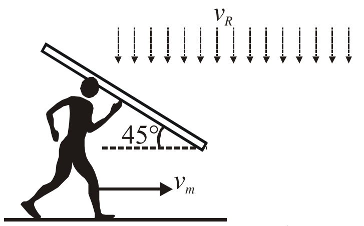
KM0200

18. A particle is thrown from a stationary platform with velocity v at an angle of 60° with the horizontal. The range obtained is R. If the platform moves horizontally in the direction of target with velocity v, the range will increase to :

(A) $\frac{3R}{2}$ (B) $\frac{5R}{2}$ (C) 2 R (D) 3 R

KM0201

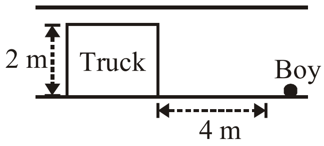
19. On a particular day rain drops are falling vertically at a speed of 5 m/s. A man holding a plastic board is running to escape from rain as shown. The lower end of board is at a height half that of man and the board makes 45° with horizontal. The maximum speed of man so that his feet does not get wet, is



(A) 5 m/s (B) $5\sqrt{2}$ m/s (C) $5/\sqrt{2}$ m/s (D) zero

KM0202

20. A 2 m wide truck is moving with a uniform speed of 8 m/s along a straight horizontal road. A pedestrian starts crossing the road at an instant when the truck is 4 m away from him. The minimum constant velocity with which he should run to avoid an accident is :-



(A) $1.6\sqrt{5}$ m/s (B) $1.2\sqrt{5}$ m/s (C) $1.2\sqrt{7}$ m/s (D) $1.6\sqrt{7}$ m/s

KM0203

21. Two trucks are moving on parallel tracks. A person on one truck projects a ball vertically upward then path of the ball as seen by four observers: from the ground, from the second truck moving with same velocity as that first truck, from the second truck moving with speed greater than first one in same direction and from the second truck moving with speed less than the first truck in same direction are:

(A) Parabola, Parabola, Parabola and Parabola

(B) Straight line, Straight line, Parabola and Parabola

(C) Parabola, Straight line, Parabola and Parabola

(D) None of these

KM0204

22. Man A sitting in a car moving at 54 km/hr observes a man B in front of the car crossing perpendicularly the road of width 15 m in three seconds. Then the velocity of man B will be

(A) $5\sqrt{10}$ towards the car (B) $5\sqrt{10}$ away from the car

(C) 5 m/s perpendicular to the road (D) None

KM0205

23. A swimmer swims in still water at a speed = 5 km/hr. He enters a 200 m wide river, having river flow speed = 4 km/hr at point *A* and proceeds to swim at an angle of 127° with the river flow direction. Another point *B* is located directly across *A* on the other side. The swimmer lands on the other bank at a point *C*, from which he walks the distance *CB* with a speed = 3 km/hr. The total time in which he reaches from *A* to *B* is

(A) 5 minutes (B) 4 minutes (C) 3 minutes (D) None

KM0206

24. A man wishes to swim across a river 400 m wide flowing with a speed of 3m/s, such that he reaches the point just infront on the other bank in time not greater than 100s. The angle made by the direction he swims and river flow direction is :-

(A) 90° (B) 127° (C) 150° (D) 143°

KM0207

25. An observer on ground sees a boat cross a river of width 800 m perpendicular to its stream in 200seconds. He also finds a man on a raft floating at speed of 3 m/s with river. The distance travelled by boat as seen by man on the raft in crossing the river is-

(A) 800 m (B) 1000m (C) 1200m (D) 1600m

KM0208

26. A boatman moves his boat with a velocity 'v' (relative to water) in river and finds to his surprise that velocity of river 'u' (with respect to ground) is more than 'v'. He has to reach a point directly opposite to the starting point on another bank by travelling minimum possible distance. Then

(A) he must steer the boat (with velocity v) at certain angle with river flow so that he can reach the opposite point on other bank directly.

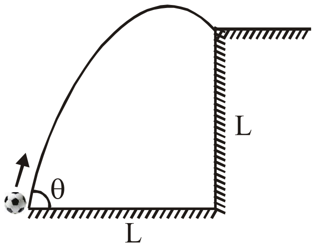
(B) his velocity 'v' must be towards directly opposite point, So, that he can travel rest of distance by walking on other bank to reach the directly opposite point.

(C) boatman should maintain velocity v of boat at certain angle greater than 90° with direction of river flow to minimize drifting and then walk rest of distance on other bank.

(D) boat velocity 'v' should be at an angle less than 90° with direction of river flow to minimize the drift and then walk to the point.

KM0209

27. A ball is thrown at an angle θ up to the top of a cliff of height L, from a point at a distance L from the base, as shown in figure. Assuming that one of the following quantities is the initial speed required to make the ball hit right at the edge of the cliff, which one is it :-



(A) $\sqrt{\frac{gL}{2(\tan\theta-1)}}$ (B) $\frac{1}{\cos\theta}\sqrt{\frac{gL}{2(\tan\theta-1)}}$ (C) $\frac{1}{\cos\theta}\sqrt{\frac{gL}{2(\tan\theta+1)}}$ (D) $\sqrt{\frac{gL\tan\theta}{2(\tan\theta+1)}}$

KM0210

28. A particle is projected with a velocity of $\sqrt{20}$ m/s such that it strikes on the same level as the point of projection at a distance of $\sqrt{3}$ m. Which of the following options is/are incorrect (mass = 1kg) :

(A) The maximum height reached by the projectile can be 0.25 m.

(B) The minimum velocity during its motion can be $\sqrt{5}$ m/s

(C) The time taken for the flight can be $\sqrt{3/5}$ s.

(D) Minimum kinetic energy during its motion can be 6 J.

KM0211

MULTIPLE CORRECT TYPE QUESTION

29. A particle is moving with a position vector, $\vec{r} = \left[a_0 \sin(2\pi t)\hat{i} + a_0 \cos(2\pi t)\hat{j}\right]$. Then-

(A) Magnitude of displacement of the particle between time t = 4 sec and t = 6 sec is zero

(B) Distance travelled by the particle in 1 sec is $2\pi a_0$

(C) The speed of particle in the whole motion is constant and equal to $2\pi a_0$.

(D) None of these

KM0212

30. A point mass is moving in the x-y plane. Its acceleration is a constant vector perpendicular to the x-axis. Which of the following do/does not change with time?

(A) only y-component of its velocity vector

(B) only x-component of its velocity vector

(C) only y-component of its acceleration vector

(D) only x-component of its acceleration vector

KM0213

31. A ball is thrown from ground such that it just crosses two poles of equal height kept 80 m apart. The maximum height attained by the ball is 80 m. When the ball passes the first pole, its velocity makes 45° with horizontal. The correct alternatives is/are :- (g = 10 m/s^2)

(A) Time interval between the two poles is 4 s.

(B) Height of the pole is 60 m.

(C) Range of the ball is 160 m.

(D) Angle of projection is $\tan^{-1}(2)$ with horizontal.

KM0214

32. Position vector of a particle is expressed as function of time by equation $\vec{r} = 2t^2 + (3t-1)\hat{j} + 5\hat{k}$, where *r* is in meters and *t* is in seconds.

(A) It always moves in a plane that is parallel to the *x-y* plane.

(B) At the instant $t = 0$ s, it is observed at point (0 m, –1 m, 5 m), moving with velocity 3 m/s in the positive *y*-direction.

(C) Its acceleration vector is uniform.

(D) It is an example of three dimensional motion.

KM0215

33. A projectile is thrown with speed u into air from a point on the horizontal ground at an angle θ with horizontal. If the air exerts a constant horizontal resistive force on the projectile then select correct alternative(s).

(A) At the farthest point, the velocity is horizontal.

(B) The time for ascent equals the time for descent.

(C) The path of the projectile may be parabolic.

(D) The path of the projectile may be a straight line.

KM0216

34. A block is thrown horizontally with a velocity of 2 m/s (relative to ground) on a belt, which is moving with velocity 4 m/s in opposite direction of the initial velocity of block. If the block stops slipping on the belt after 4 s it was dropped then choose the correct statement(s) :-

(A) Displacement with respect to ground is zero after 2.66 s and magnitude of displacement with respect to ground is 12 m after 4 s.

(B) Magnitude of displacement with respect to ground in 4 s is 4 m.

(C) Magnitude of displacement with respect to belt in 4 s is 12 m.

(D) Displacement with respect to ground is zero in 8/3 s.

KM0217

35. A man on a rectilinearly moving cart, facing the direction of motion, throws a ball straight up with respect to himself

(A) The ball will always return to him.

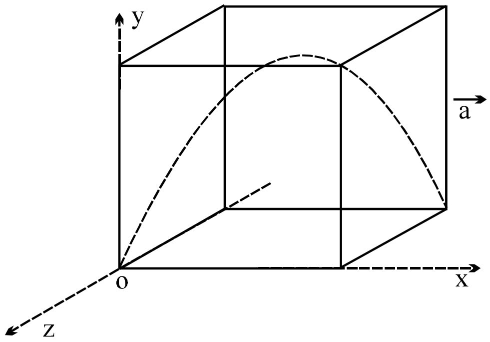
(B) The ball will never return to him.

(C) The ball will return to him if the cart moves with constant velocity.

(D) The ball will fall behind him if the cart moves with some positive acceleration.

KM0218

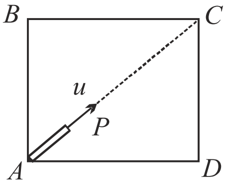
36. A cubical box dimension L = 5/4 metre starts moving with an acceleration $\vec{a} = 0.5\ \text{m/s}^2\,\hat{i}$ from the state of rest. At the same time, a stone is thrown from the origin with velocity $\vec{V} = v_1\hat{i} + v_2\hat{j} - v_3\hat{k}$ with respect to earth. Acceleration due to gravity $\vec{g} = 10\text{m/s}^2(-\hat{j})$. The stone just touches the roof of box and finally falls at the diagonally opposite point then :



(A) $v_1 = \frac{3}{2}$ (B) $v_2 = 5$ (C) $v_3 = \frac{5}{4}$ (D) $v_3 = \frac{5}{2}$

KM0219

37. A large rectangular box moves vertically downward with an acceleration a. A toy gun fixed at *A* and aimed towards *C* fires a particle *P*.



(A) *P* will hit *C* if $a = g$

(B) *P* will hit the roof *BC*, if $a > g$

(C) *P* will hit the wall *CD* if $a < g$

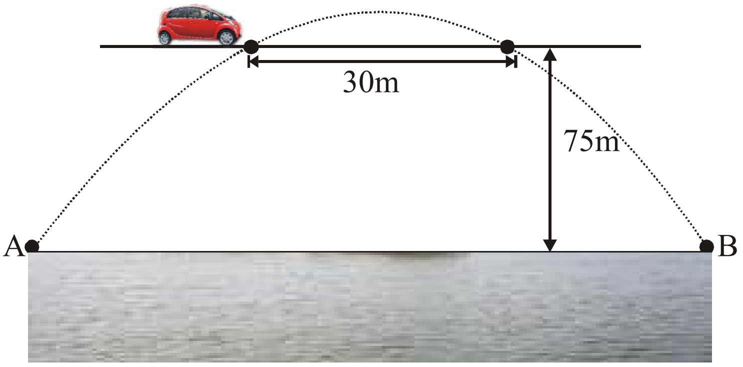
(D) May be either (A), (B) or (C), depending on the speed of projection of *P*

KM0220

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 38 to 40

In the figure shown there is a long horizontal bridge over a river 75 m high from water surface. A strong man throws a stone in the parallel plane of the bridge. A observer in a car travelling on the bridge finds the stone going pass by the car while ascending and also while descending between two points on the road 30 m away. The car is travelling at a speed of 15 m/s. The stone is thrown from the bank of river just at the same level of water.



38. What is the angle the velocity makes with the bridge when it goes past the car while ascending ?

(A) 30° (B) 45° (C) $\tan^{-1}\left(\frac{2}{3}\right)$ (D) $\tan^{-1}\left(\frac{3}{2}\right)$ **KM0221**

39. Horizontal distance AB travelled by stone is :-

(A) 0 m (B) 75 m (C) 120 m (D) 240 m

KM0221

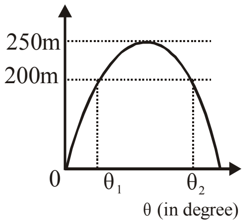
40. What is the distance between car and stone at the instant when particle reaches at point B ?

(A) 0 m (B) 75 m (C) 120 m (D) 240 m

KM0221

Paragraph for Question 41 & 42

From the ground level, a ball is to be shot with a certain speed. Graph shows the range (R) of the particle versus the angle of projection from horizontal (θ).

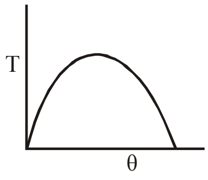


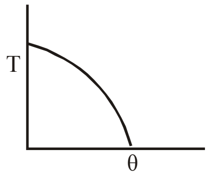
41. Values of θ_1 and θ_2 are

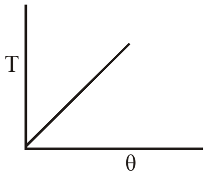
(A) 53° and 37° (B) 26.5° and 63.5° (C) 18.5° and 71.5 ° (D) 15° and 75°

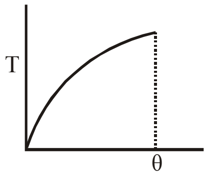
KM0222

42. The corresponding time of flight vs θ graph is :-

(A)


(B)


(C)


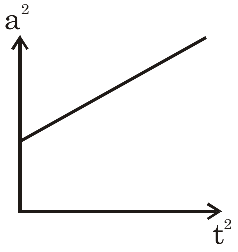
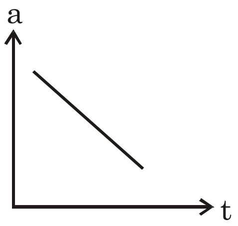
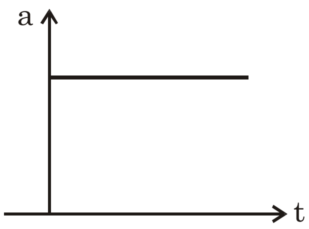
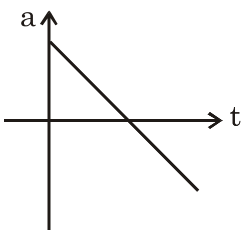
(D)


KM0222

MATCHING LIST TYPE (4 × 4 × 4) SINGLE OPTION CORRECT (THREE COLUMNS AND FOUR ROWS)

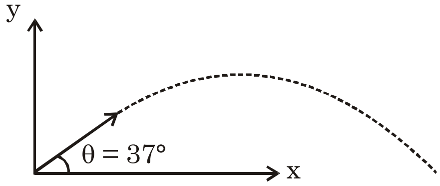
Answer Q.43, Q.44 and Q.45 by appropriately matching the information given in the three columns of the following table.

Match the following

Column–I **Time of flight (in sec)**	**Column-II** **Range (in m) Along the ground/ along the inclined plane**	**Column-III** **Graph**
(I) 2	(i) 3840	(P) 
(II) 1	(ii) $20\sqrt{2}$	(Q) 
(III) 10	(iii) 7	(R) 
(IV) 32	(iv) 3600	(S) 

43. A particle is projected with initial speed $u = \frac{25}{3}$ m/s as shown, here acceleration vector is given as

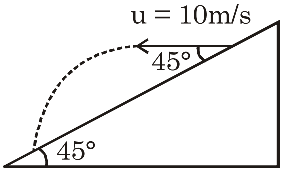
$a_x = 2t\hat{i}\ \text{m/s}^2$; $a_y = -10\hat{j}\ \text{m/s}^2$



(A) (II) (iv) (P) (B) (II) (iii) (P) (C) (III) (iii) (S) (D) (II) (iii) (S)

KM0223

44. A particle is projected from a large–fixed incline plane as shown. Here $\vec{a} = g$ (Vertically downward) take g = 10 m/s^2.



(A) (II) (iv) (P) (B) (IV) (ii) (S) (C) (I) (ii) (R) (D) (IV) (ii) (P)

KM0223

45. In ground to ground projection a particle is projected at 53° from horizontal. At t = 25 sec after projection, its velocity vecotor becomes perpendicular to its initial veocity vector.

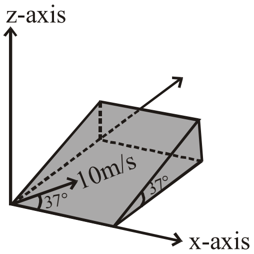
(Given $\vec{a} = g \downarrow = 10\text{m}/\text{s}^2$)

(A) (IV) (i) (R) (B) (IV) (ii) (S) (C) (II) (ii) (P) (D) (I) (ii) (Q)

KM0223

MATRIX MATCH TYPE QUESTION

46. A small ball is projected along the surface of a smooth inclined plane with speed 10m/s along the direction shown at t = 0. The point of projection is origin, z-axis is along vertical. The acceleration due to gravity is 10 m/s^2. Column-I lists values of certain parameters related to motion of ball and column-II lists different time instants. Match appropriately.



	Column-I		Column-II
(A)	Distance from x-axis is 2.25m	(P)	0.5 s
(B)	Speed is minimum	(Q)	1.0 s
(C)	Velocity makes angle 37° with x-axis	(R)	1.5 s
		(S)	2.0 s

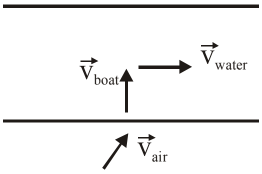
KM0224

47.

Column-I	Column-II
(A) Time for a boat to cross a river of width ℓ by the shortest distance ($\vec{v}$-velocity of boat with respect to water; $\vec{u}$-velocity of water) $\lvert\vec{v}\rvert > \lvert\vec{u}\rvert$	(P) $\frac{\ell}{\lvert\vec{v}+\vec{u}\rvert}$
(B) Time for two particles moving with velocities $\vec{v}$ and $\vec{u}$ in opposite directions to meet each other. (initial separation of particles is ℓ)	(Q) $\frac{\ell}{\sqrt{v^2-u^2}}$
(C) Time for a boat to cross a river of width ℓ in the shortest time ($\vec{v}$-velocity of boat with respect to water; $\vec{u}$-velocity of water)	(R) $\frac{\ell}{\lvert\vec{v}\rvert+\lvert\vec{u}\rvert}$
(D) Time for a boat to travel a distance ℓ downstream ($\vec{v}$-velocity of boat with respect to water; $\vec{u}$-velocity of water)	(S) $\frac{\ell}{\lvert\vec{v}\rvert}$
	(T) $\frac{\ell}{\sqrt{u^2+v^2}}$

KM0225

48. A boat is being rowed in a river. Air is also blowing. Direction of velocity vectors of boat, water and air in ground frame are as shown in diagram.



Column-I	Column-II (possible directions)
(A) Direction in which boat is being steered	(P) ↖
(B) Direction in which a flag on the boat may flutter	(Q) →
(C) Direction of velocity of water relative to boat	(R) ↑
(D) Direction of velocity of air relative to a piece of wood floating on river.	(S) ↘

KM0226

EXERCISE (JM)

1. A particle is moving with velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is : **[AIEEE - 2010]**

(1) $y^2 = x^2$ + constant (2) $y = x^2$ + constant (3) $y^2 = x$ + constant (4) xy = constant

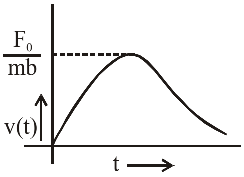
KM0227

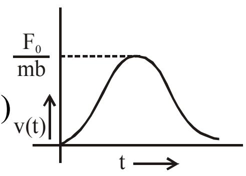
2. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is :- **[AIEEE - 2011]**

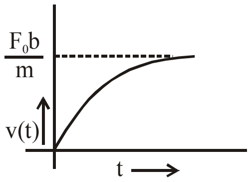
(1) $\frac{\pi}{2}\frac{v^4}{g^2}$ (2) $\pi\frac{v^2}{g^2}$ (3) $\pi\frac{v^2}{g}$ (4) $\pi\frac{v^4}{g^2}$

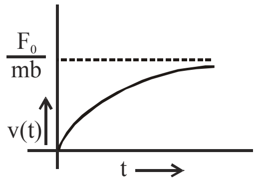
KM0228

3. A particle of mass m is at rest at the origin at time t = 0. It is subjected to a force $F(t) = F_0e^{-bt}$ in the x direction. Its speed v(t) is depicted by which of the following curves ? **[AIEEE - 2012]**

(1)


(2)


(3)


(4)


KM0229

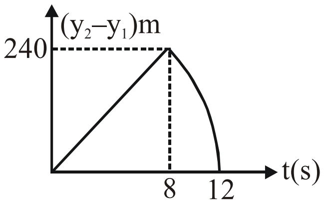
4. A projectile is given an initial velocity of $(\hat{i} + 2\hat{j})$ m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If g = 10 m/s^2, the equation of its trajectory is : **[AIEEE - 2013]**

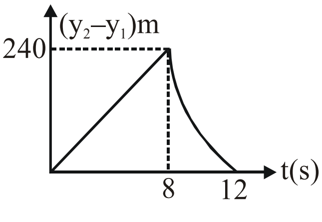
(1) $y = x - 5x^2$ (2) $y = 2x - 5x^2$ (3) $4y = 2x - 5x^2$ (4) $4y = 2x - 25x^2$

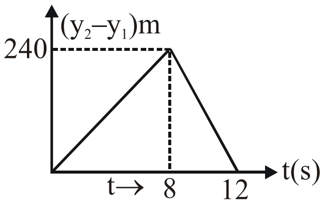
KM0230

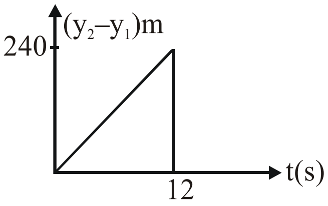
5. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first ?

(Assume stones do not rebound after hitting the ground and neglect air resistance, take g = 10 m/s^2) *(The figure are schematic and not drawn to scale)* **[JEE Main-2015]**

(1)


(2)


(3)


(4)


KM0231

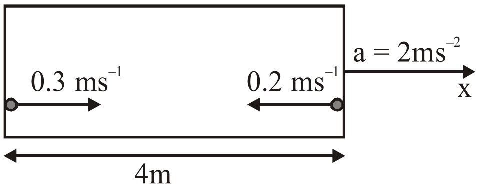
EXERCISE (JA)

1. A train is moving along a straight line with a constant acceleration 'a'. A boy standing in the train throws a ball forward with a speed of 10 m/s, at an angle of 60° to the horizontal. The boy has to move forward by 1.15 m inside the train to catch the ball back at the initial height. The acceleration of the train, in m/s^2, is

[IIT-JEE 2011]

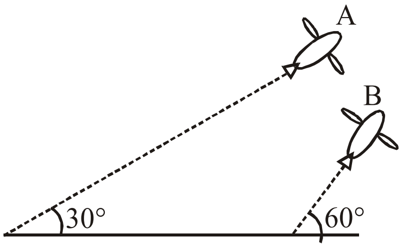
KM0232

2. A rocket is moving in a gravity free space with a constant acceleration of 2 ms^{-2} along + x direction (see figure). The length of a chamber inside the rocket is 4m. A ball is thrown from the left end of the chamber in + x direction with a speed of 0.3 ms^{-1} relative to the rocket. At the same time, another ball is thrown in –x direction with a speed of 0.2 ms^{-1} from its right end relative to the rocket. The time in seconds when the two balls hit each other is **[JEE Advanced 2014]**



KM0233

3. Airplanes A and B are flying with constant velocity in the same vertical plane at angles 30° and 60° with respect to the horizontal respectively as shown in figure. The speed of A is $100\sqrt{3}$ ms^{-1}. At time t = 0 s, an observer in A finds B at a distance of 500 m. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at t = t_0, A just escapes being hit by B, t_0 in seconds is **[JEE Advanced-2014]**



KM0234

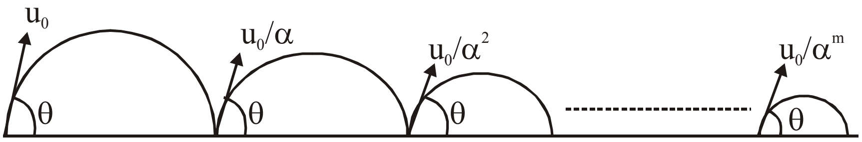
4. A ball is projected from the ground at an angle of 45° with the horizontal surface. It reaches a maximum height of 120 m and returns to the ground. Upon hitting the ground for the first time, it loses half of its kinetic energy. Immediately after the bounce, the velocity of the ball makes an angle of 30° with the horizontal surface. The maximum height it reaches after the bounce, in metres, is................

[JEE Advanced-2018]

KM0235

5. A ball is thrown from ground at an angle θ with horizontal and with an initial speed u_0. For the resulting projectile motion, the magnitude of average velocity of the ball up to the point when it hits the ground for the first time is V_1. After hitting the ground, ball rebounds at the same angle θ but with a reduced speed of u_0/α. Its motion continues for a long time as shown in figure. If the magnitude of average velocity of the ball for entire duration of motion is 0.8 V_1, the value of α is______

[JEE Advanced-2019]



KM0236

6. Starting at time t = 0 from the origin with speed 1 ms^{-1}, a particle follows a two-dimensional trajectory in the x-y plane so that its coordinates are related by the equation $y = \frac{x^2}{2}$. The x and y components of its acceleration are denoted by a_x and a_y, respectively. Then **[JEE Advanced-2020]**

(A) $a_x = 1\ ms^{-2}$ implies that when the particle is at the origin, $a_y = 1\ ms^{-2}$

(B) $a_x = 0$ implies $a_y = 1\ ms^{-2}$ at all times

(C) at t = 0, the particle's velocity points in the x-direction

(D) $a_x = 0$ implies that at t = 1 s, the angle between the particle's velocity and the x axis is 45°

KM0254

ANSWER KEY

EXERCISE (S-1)

1. Ans. 5 **2. Ans.** (a) $v(t) = \left(3.0\hat{i} - 4.0t\,\hat{j}\right)\ \hat{a}\left(t\right) = -4.0\,\hat{j}$ (b) 8.54 ms^{-1}, 70° with x-axis

3. Ans. (i) $\vec{a} = \left(-16\hat{i} - 8\hat{j}\right) m/s^2$ (ii) $\vec{v} = \left(-30\hat{i} - 40\hat{j}\right) m/s$ **4. Ans.** $\sqrt{2257}$ m/s

5. Ans. (i) 1.6 sec (ii) 3.2 m (iii) 9.6 m **6. Ans.** 1200 **7. Ans.** 50 m

8. Ans. 20 s **9. Ans.** 60°, 2 m/s. **11. Ans.** $20\sqrt{5}$ m/s **12. Ans.** $\frac{100}{3}$ m/s

13. Ans. (i) 100 m/s (ii) 980 m (iii) 1600 m (iv) $\left(80\hat{i} - 140\hat{j}\right)$

14. Ans. $u = 50(\sqrt{3} - 1)$ m/s, $H = 125\left(2 - \sqrt{3}\right)$ m **15. Ans.** 6 **16. Ans.** 34

17. Ans. (a) With $\hat{i}$ to right and $\hat{j}$ up $\vec{V} = (15\hat{i} + 20\hat{j})$ m/s; (b) 23 meters; (c) It is horizontal. $\theta = 0$]

18. Ans. 10 m/s **19. Ans.** 4 **20. Ans.** 3

21. Ans. (a) 45°, (b) 2 m/sec **22. Ans.** 5 **23. Ans.** 6

24. Ans. 200 m, 20 m/min, 12 m/min **25. Ans.** $\tan^{-1}(1/2)$ **26. Ans.** $\tan^{-1}(1/3)$

27. Ans. (a) Somewhere down stream (b) 8 s (c) 16 m (d) Yes **28. Ans.** 6 s, 66 m

29. Ans. 30 m upstream **30. Ans.** $v_{br} > v_r,\ \theta = \sin^{-1}\left(\frac{v_r}{v_{br}}\right)$ upstream of line AB

EXERCISE (S-2)

1. Ans. (a) $\vec{v} = (5\sin t - 3)\hat{i} + (3\cos t - 1)\hat{j}$, (b) $(2 - 5\cos t - 3t)\hat{i} + (2 + 3\sin t - t)\hat{j}$

2. Ans. (i) $4\sqrt{6}$ m, (ii) 10 m **3. Ans.** (i) 2 s (ii) 10 m/s (iii) 5m (iv) 16.25 m (v) 20 m

4. Ans. (i) 7 s, (ii) 175 m **5. Ans.** 0105 **6. Ans.** t = 1.6 s

7. Ans. (i) 1 s, (ii) ($5\sqrt{3}$ m, 5 m) **8. Ans.** $\theta = 37°$, v = 6 m/s

9. Ans. (i) $\frac{u^2 \sin 2\alpha}{g\cos\theta}$ (ii) $v = \frac{u\cos(\alpha + \theta)}{\cos\theta}$ **10. Ans.** 140 **11. Ans.** 5

12. Ans. B, $t_A = 165$ s, $t_B = 150$ s **13. Ans**. (i) 120° (ii) $2/\sqrt{3}$

14. Ans. $10\sqrt{3}$ m/s

15. Ans. (a) 7i + 2j; (b) 12.30p.m, (c) 9/2i + j)

EXERCISE (O-1)

1. Ans. (B) **2. Ans. (C)** **3. Ans. (C)** **4. Ans. (B)** **5. Ans. (B)** **6. Ans. (B)**

7. Ans. (D) **8. Ans. (B)** **9. Ans. (D)** **10. Ans. (B)** **11. Ans. (B)** **12. Ans. (A)**

13. Ans. (D) **14. Ans. (C)** **15. Ans. (C)** **16. Ans. (B)** **17. Ans. (B)** **18. Ans. (B)**

19. Ans. (B) **20. Ans. (C)** **21. Ans. (A)** **22. Ans. (A,B,C,D)**

23. Ans. (A,B,C,D) **24. Ans. (B,C,D)** **25. Ans. (B,C)** **26. Ans. (B)** **27. Ans. (C,D)**

28. Ans. (A,B,C) **29. Ans. (C)** **30. Ans. (D)** **31. Ans. (B)**

32. Ans. (a) Vertically (b) 53° above east (c) 53° above west (d) 53° above north (e) 45° above south

33. Ans. (a) 120 cm/s vertically150 cm/s 53° above horizontal (b) 37° above the horizontal.

34. Ans. 120 cm/s **35. Ans.** 125 cm/s at 37° from the vertical **36. Ans.** 6 m/s due north

37. Ans. (a) 9 m/s (b) 16 m/s (c) 1 m/s^2 **38. Ans.** 10 m/s, 37° north of east

39. Ans. (A) R; (B) P; (C) Q; (D) S **40. Ans. (A) → (R); (B) → (R); (C) → (P); (D) → (Q)**

EXERCISE (O-2)

1. Ans. (D) **2. Ans. (D)** **3. Ans. (C)** **4. Ans. (A)** **5. Ans. (C)** **6. Ans. (C)**

7. Ans. (C) **8. Ans. (C)** **9. Ans. (C)** **10. Ans. (A)** **11. Ans. (C)** **12. Ans. (D)**

13. Ans. (C) **14. Ans. (A)** **15. Ans. (A)** **16. Ans. (C)** **17. Ans. (C)** **18. Ans. (D)**

19. Ans. (A) **20. Ans. (A)** **21. Ans. (C)** **22. Ans. (B)** **23. Ans. (B)** **24. Ans. (B)**

25. Ans. (B) **26. Ans. (C)** **27. Ans. (B)** **28. Ans. (D)** **29. Ans. (A,B,C)**

30. Ans. (B,C,D) **31. Ans. (A, B,C,D)** **32. Ans. (A,B,C)**

33. Ans. (B,C,D) **34. Ans. (B,C,D)** **35. Ans. (C,D)** **36. Ans. (A,B,C)**

37. Ans. (A,B) **38. Ans. (C)** **39. Ans. (C)** **40. Ans. (B)** **41. Ans. (B)**

42. Ans. (D) **43. Ans. (B)** **44. Ans. (C)** **45. Ans. (A)**

46. Ans. (A) - (P,R) ; (B) - (Q) ; (C) - (S)

47. Ans. (A) →(P,Q); (B) →(R); (C) →(S); (D) →(P,R)

48. Ans. (A)-P; (B)-Q, S; (C)-S; (D)-P,R

EXERCISE (JM)

1. Ans. (1) **2. Ans. (4)** **3. Ans. (4)** **4. Ans. (2)** **5. Ans. (1)**

EXERCISE (JA)

1. Ans. 5 **2. Ans. 8 or 2** **3. Ans. 5** **4. Ans. 30 [29.60, 30.40]** **5. Ans. 4.00**

6. Ans. (A,B,C,D)

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Chapter Contents

01 CIRCULAR MOTION

Important Notes

CIRCULAR MOTION

KEY CONCEPT

Cross product (Vector Product) of two vectors :

Vector product of two vectors $\vec{A}$ and $\vec{B}$, also called the cross product, is denoted by $\vec{A}\times\vec{B}$. As the name suggests, the vector product is itself a vector. We will use this product to describe torque and angular momentum and extensively to describe magnetic fields forces.

Vector product of two vectors $\vec{A}$ and $\vec{B}$ which are at an angle ϕ is defined as

$$\vec{C} = \vec{A} \times \vec{B},$$

then $$C = AB \sin\phi \ \hat{n}$$

where $\hat{n}$ is an unit vector perpendicular to plane containing vector $\vec{A}$ and $\vec{B}$. Direction of $\hat{n}$ is given by right hand rule as described below.

Note: the vector product of two parallel or antiparallel vectors is always zero. In particular, the vector product of any vector with itself is zero.

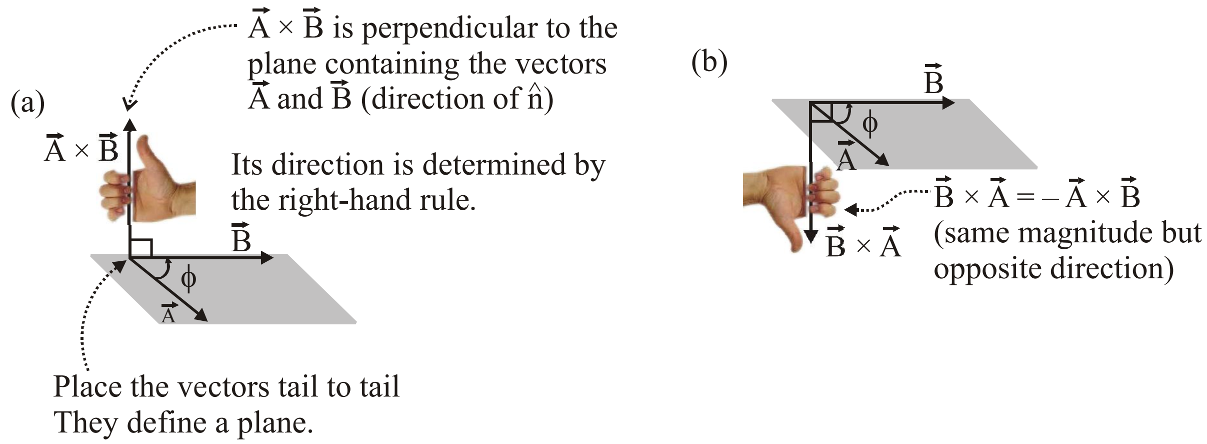


Figure : (a) The vector product $\vec{A}\times\vec{B}$. determined by the right-hand rule. (b) $\vec{B}\times\vec{A} = -\vec{A}\times\vec{B}$

Note: The vector product is not commutative! In fact, for any two vectors $\vec{A}$ and $\vec{B}$,

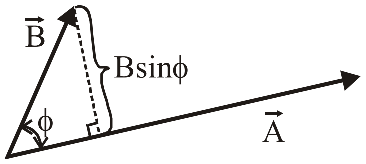
$$\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$$

(Magnitude of A × $\vec{B}$) equals A (B sin ϕ).

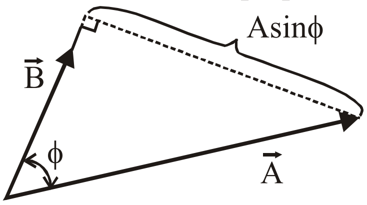
(Magnitude of $\vec{A}$) times (Component of $\vec{B}$ perpendicular to $\vec{A}$)

(Magnitude of $\vec{A}\times\vec{B}$) also equals B(A sin ϕ).

(Magnitude of $\vec{B}$) times (Component of $\vec{A}$ perpendicular to $\vec{B}$)



(a)



(b)

Figure: Calculating the magnitude AB sin ϕ of the vector product of two vector, $\vec{A}\times\vec{B}$.

Calculating the Vector Product Using Components

$$\hat{i}\times\hat{i}=\hat{j}\times\hat{j}=\hat{k}\times\hat{k}=0$$

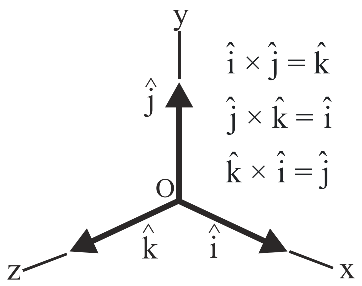
Using the right-hand rule, we find

$$\hat{i}\times\hat{j}=-\hat{j}\times\hat{i}=\hat{k};$$

$$\hat{j}\times\hat{k}=-\hat{k}\times\hat{j}=\hat{i};$$

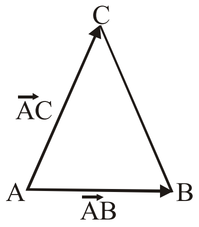
$$\hat{k}\times\hat{i}=-\hat{i}\times\hat{k}=\hat{j}$$

A right-handed coordinate system



Ex. Find the area of the triangle formed by the points having position vectors $\vec{A}=\hat{i}-\hat{j}-3\hat{k}$, $\vec{B}=4\hat{i}-3\hat{j}+\hat{k}$ and $\vec{C}=3\hat{i}-\hat{j}+2\hat{k}$

Sol.



Let ABC is the triangle formed by the points A, B and C. Then

$$\overrightarrow{AB}=\vec{B}-\vec{A}=\left(4\hat{i}-3\hat{j}+\hat{k}\right)-\left(\hat{i}-\hat{j}-3\hat{k}\right)=3\hat{i}-2\hat{j}+4\hat{k}$$

$$\overrightarrow{AC}=\vec{C}-\vec{A}=\left(3\hat{i}-\hat{j}+2\hat{k}\right)-\left(\hat{i}-\hat{j}-3\hat{k}\right)=2\hat{i}+5\hat{k}$$

Now, $\overrightarrow{AB}\times\overrightarrow{AC}=\left(3\hat{i}-2\hat{j}+4\hat{k}\right)\times\left(2\hat{i}+5\hat{k}\right)$

$$=\hat{i}\left(-10-0\right)+\hat{j}\left(8-15\right)+\hat{k}\left(0+4\right)=-10\hat{i}-7\hat{j}+4\hat{k}$$

Ex. Find unit vector perpendicular to the plane determined by the points P(1, –1, 2), Q(2, 0, –1) and R(0, 2, 1).

Sol. $\vec{P}Q$ = (Position vector of Q) – (Position vector of P)

$$= \left(2\hat{i}-\hat{k}\right)-\left(\hat{i}-\hat{j}+2\hat{k}\right)=\hat{i}+\hat{j}-3\hat{k}$$

Similarly, $\vec{P}R=\left(2\hat{j}+\hat{k}\right)-\left(\hat{i}-\hat{j}+2\hat{k}\right)=-\hat{i}+3\hat{j}-\hat{k}$

$\therefore\ \left|\vec{P}Q\times\vec{P}R\right|=\sqrt{8^2+4^2+4^2}=4\sqrt{6}$

$\therefore$ Required unit vectors

$$=\pm\frac{1}{4\sqrt{6}}\left(8\hat{i}+4\hat{j}+4\hat{k}\right)=\pm\frac{1}{\sqrt{6}}\left(2\hat{i}+\hat{j}+9\hat{k}\right)$$

Definition of Circular Motion

When a particle moves in a plane such that its distance from a fixed (or moving) point remains constant then its motion is called as circular motion with respect to that fixed point.

That fixed point is called centre and the distance is called radius of circular path.

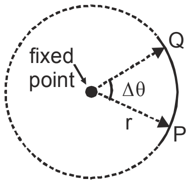
Radius Vector

The vector joining the centre of the circle and the center of the particle performing circular motion is called radius vector. It has constant magnitude and variable direction.

KINEMATICS OF CIRCULAR MOTION

Angular Displacement

- Angle traced by position vector of a particle moving w.r.t. some fixed point is called angular displacement.



$\Delta\theta$ = angular displacement

$$\text{Angle}=\frac{\text{Arc}}{\text{Radius}}\quad\text{or}\quad\Delta\theta=\frac{\text{Arc PQ}}{r}$$

- Small Angular displacement $d\vec{\theta}$ is a vector quantity, but large angular displacement θ is scalar quantity.

$$d\vec{\theta}_1+d\vec{\theta}_2=d\vec{\theta}_2+d\vec{\theta}_1\ \text{But}\ \vec{\theta}_1+\vec{\theta}_2\neq\vec{\theta}_2+\vec{\theta}_1$$

- Its direction is perpendicular to plane of rotation and given by right hand screw rule.
- It is dimensionless and has S.I. unit "Radian" while other units are degree or revolution.

2π radian = 360° =1 revolution

Frequency (n) : Number of revolutions describes by particle per second is its frequency. Its unit is revolutions per second (r.p.s.) or revolutions per minute (r.p.m.)

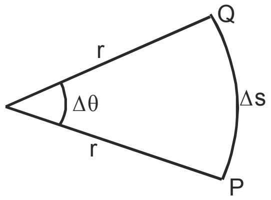
Time Period (T) : It is time taken by particle to complete one revolution. $T=\frac{1}{n}$

Angular Velocity (ω) : It is defined as the rate of change of angular displacement of moving particle

$$\omega=\frac{\text{Angle traced}}{\text{Time taken}}=\lim_{\Delta t\to 0}\frac{\Delta\theta}{\Delta t}=\frac{d\theta}{dt}$$

Relation between linear and Angular velocity

$$\text{Angle} = \frac{\text{Arc}}{\text{Radius}} \text{ or } \frac{\Delta s}{r} \qquad \Delta\theta = \frac{\Delta s}{r} \text{ or } \Delta s = r\Delta\theta$$



$$\therefore\ \frac{\Delta s}{\Delta t} = \frac{r\Delta\theta}{\Delta t} \text{ if } \Delta t \to 0 \text{ then } \frac{ds}{dt} = r\frac{d\theta}{dt} \qquad \boxed{v = \omega r}$$

$$\boxed{\vec{v} = \vec{\omega} \times \vec{r}}$$ (direction of $\vec{v}$ is according to right hand thumb rule)

Average Angular Velocity (ω_{av})

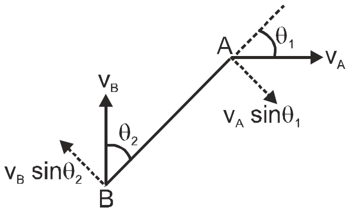
$$\omega_{av} = \frac{\text{total angle of rotation}}{\text{total time taken}} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T} = 2\pi n$$

where θ_1 and θ_2 are angular position of the particle at instant t_1 and t_2.

Instantaneous Angular Velocity

The angular velocity at some particular instant $\omega = \lim\limits_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$ or $\vec{\omega} = \frac{d\vec{\theta}}{dt}$

Relative Angular Velocity



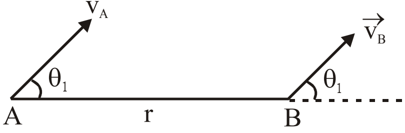
- Relative angular velocity of a particle 'A' w.r.t. other moving particle B is the angular velocity of the position vector of A w.r.t. B. That means it is the rate at which position vector of 'A' w.r.t. B rotates at that instant

$$\omega_{AB} = \frac{(v_{AB})_\perp}{r_{AB}} = \frac{\text{Relative velocity of A w.r.t. B perpendicular to line AB}}{\text{seperation between A and B}}$$

here $(v_{AB})_\perp = v_A \sin\theta_1 + v_B \sin\theta_2$ $\qquad \therefore\ \omega_{AB} = \frac{v_A \sin\theta_1 + v_B \sin\theta_2}{r}$

- It is an axial vector quantity.
- Its direction is same as that of angular displacement i.e. perpendicular to the plane of rotation and along the axis according to right hand screw rule.
- Its unit is radian/second.

If particles A and B are moving with a velocity $\vec{v}_A$ and $\vec{v}_B$ and separated by a distance r at a given instant then

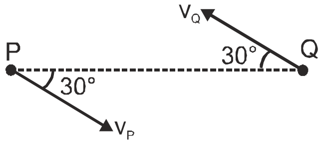


(i) $\frac{dr}{dt} = v_B\cos\theta_2 - v_A\cos\theta_1$ (ii) $\frac{d\theta_{AB}}{dt} = w_{AB} = \frac{v_B \sin\theta_2 - v_A \sin\theta_1}{r}$

Ex. A particle revolving in a circular path completes first one third of circumference in 2 s, while next one third in 1 s. Calculate the average angular velocity of particle.

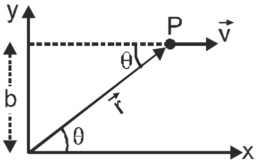
Sol. $\theta_1 = \frac{2\pi}{3}$ and $\theta_2 = \frac{2\pi}{3}$ total time T = 2 + 1 = 3 s $\therefore \langle \omega_{av} \rangle = \frac{\theta_1 + \theta_2}{T} = \frac{\frac{2\pi}{3} + \frac{2\pi}{3}}{3} = \frac{\frac{4\pi}{3}}{3} = \frac{4\pi}{9}$ rad/s

Ex. Two moving particles P and Q are 10 m apart at any instant. Velocity of P is 8 m/s at 30°, from line joining the P and Q and velocity of Q is 6m/s at 30°. Calculate the angular velocity of P w.r.t. Q

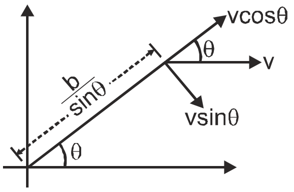


Sol. $\omega_{PQ} = \frac{8\sin 30^\circ - (-6\sin 30^\circ)}{10} = 0.7$ rad/s.

Ex. A particle moving parallel to x–axis as shown in fig. such that at all instant the y–axis component of its position vector is constant and is equal to 'b'. Find the angular velocity of the particle about the origin when its radius vector makes angle θ from the x-axis.



Sol. $\therefore \ \omega_{PO} = \frac{v\sin\theta}{\frac{b}{\sin\theta}} = \frac{v}{b}\sin^2\theta$



Ex. The angular velocity of a particle is given by $\omega = 1.5\,t - 3t^2 + 2$, Find the time when its angular acceleration becomes zero.

Sol. $\alpha = \frac{d\omega}{dt} = 1.5 - 6\,t = 0 \ \Rightarrow \ t = 0.25$ s.

Ex. A disc starts from rest and on the application of a torque, it gains an angular acceleration given by $\alpha = 3t - t^2$. Calculate the angular velocity after 2 s.

Sol. $\frac{d\omega}{dt} = 3t - t^2 \ \Rightarrow \int_0^{\omega} d\omega = \int_0^{t} (3t - t^2)dt \ \Rightarrow \omega = \frac{3t^2}{2} - \frac{t^3}{3} \Rightarrow$ at t =2 s, $\quad \omega = \frac{10}{3}$ rad/s

Angular Acceleration (α)

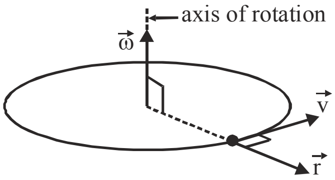
- Rate of change of angular velocity is called angular acceleration. $\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$ or $\vec{\alpha} = \frac{d\vec{\omega}}{dt}$
- Its an axial vector quantity. It direction is along the axis according to right hand screw rule.
- Unit → rad/s^2

Relation between Angular and Linear Acceleration

$\vec{v} = \vec{\omega} \times \vec{r}$ ($\vec{v}$ is a tangential vector, $\vec{\omega}$ is a axial vector and $\vec{r}$ is a radial vector.)

These three vectors are mutually perpendicular. but $\vec{a} = \dfrac{d\vec{v}}{dt} = \dfrac{d}{dt}(\vec{\omega} \times \vec{r}) = \dfrac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \dfrac{d\vec{r}}{dt}$

or $\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}$ ($\dfrac{d\vec{\omega}}{dt} = \vec{\alpha}$ and $\dfrac{d\vec{r}}{dt} = \vec{v}$) $\Rightarrow \vec{a} = \vec{a}_T + \vec{a}_C$



($\vec{a}_T = \vec{\alpha} \times \vec{r}$ is tangential acceleration &

$\vec{a}_C = \vec{\omega} \times \vec{v}$ is centripetal acceleration)

$\vec{a} = \vec{a}_T + \vec{a}_C$ ($\vec{a}_T$ and $\vec{a}_C$ are two component of net linear acceleration)

Tangential Acceleration

$\vec{a}_T = \vec{\alpha} \times \vec{r}$, its direction is parallel to velocity. $\vec{v} = \vec{\omega} \times \vec{r}$ and $\vec{a}_T = \vec{\alpha} \times \vec{r}$ as $\vec{\omega}$ and $\vec{\alpha}$ both are parallel and along the axis so that $\vec{v}$ and $\vec{a}_T$ are also parallel and along the tangential direction.

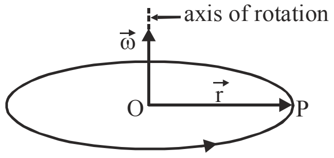
Magnitude of tangential acceleration in case of circulation motion.

$a_T = \alpha\, r \sin 90° = \alpha r$ ($\vec{\alpha}$ is axial, $\vec{r}$ is radial so that $\vec{\alpha} \perp \vec{r}$)

As $\vec{a}_T$ is along the direction of motion (in the direction of $\vec{v}$) so that $\vec{a}_T$ is responsible for change in speed of the particle. Its magnitude is rate of change of speed of the particle. If particle is moving on a circular path with constant speed then tangential acceleration is zero.

Centripetal acceleration

$\vec{a}_C = \vec{\omega} \times \vec{v} \Rightarrow \vec{a}_C = \vec{\omega} \times (\vec{\omega} \times \vec{r})$ ($\because \vec{v} = \vec{\omega} \times \vec{r}$)



Let $\vec{r}$ is in $\hat{i}$ direction and $\vec{\omega}$ is in $\hat{j}$ direction then direction of

$\vec{a}_C$ is along $\hat{j} \times (\hat{j} \times \hat{i}) = \hat{j} \times (-\hat{k}) = -\hat{i}$

opposite direction of $\vec{r}$ i.e., from P to O and it is

centripetal direction.Magnitude of centripetal acceleration, $a_C = \omega v = \dfrac{v^2}{r} = \omega^2 r$, $\vec{a}_C = \dfrac{v^2}{r}(-\hat{r})$

- Centripetal acceleration is always perpendicular to the velocity or displacement at each point.

Net Linear Acceleration :

$\vec{a} = \vec{a}_T + \vec{a}_C$ and $\vec{a}_T \perp \vec{a}_C$ so that $|\vec{a}| = \sqrt{a_T^2 + a_C^2}$

About uniform circular motion :–

- Position vector $(\vec{r})$ is always perpendicular to the velocity vector $(\vec{v})$ i.e. $\vec{r} . \vec{v} = 0$
- velocity vector is always perpendicular to the acceleration. $\vec{v} . \vec{a} = 0$
- $\because |\vec{v}|$ = constant so tangential acc. $a_t = 0$ $\therefore$ $f_t = 0$
- Important difference between the projectile motion and uniform circular motion :
  In projectile motion, both the magnitude and the direction of acceleration (g) remain constant, while in uniform circular motion the magnitude remains constant but the direction continuously changes.

DYNAMICS OF CIRCULAR MOTION

If a particle moves with constant speed in a circle, motion is called uniform circular. In uniform circular motion a resultant non-zero force acts on the particle. The acceleration is due to the change in direction of the velocity vector. In uniform circular motion tangential acceleration (a_t) is zero. The acceleration of the particle is towards the centre and its magnitude is $\frac{v^2}{r}$. Here, v is the speed of the particle and r the radius of the circle. The direction of the resultant force F is therefore towards centre and its magnitude is $F = \frac{mv^2}{r} = mr\omega^2 (\text{as } v = r\omega)$

Here, ω is the angular speed of the particle. This force F is called the centripetal force. Thus, a centripetal force of magnitude $\frac{mv^2}{r}$ is needed to keep the particle moving in a circle with constant speed. This force is provided by some external source such as friction, magnetic force, coulomb force, gravitation, tension, etc.

Circular Turning of Roads

When vehicles go through turnings, they travel along a nearly circular arc. There must be some force which will produce the required centripetal acceleration. If the vehicles travel in a horizontal circular path, this resultant force is also horizontal. The necessary centripetal force is being provided to the vehicles by following three ways:

- By friction only.
- By banking of roads only.
- By friction and banking of roads both.

In real life the necessary centripetal force is provided by friction and banking of roads both.

- **By Friction only**

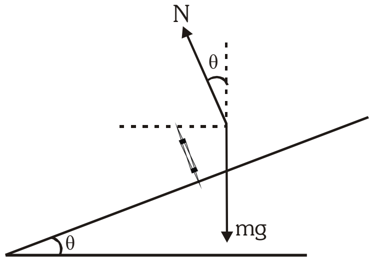
Suppose a car of mass m is moving at a speed v in a horizontal circular arc of radius r. In this case, the necessary centripetal force to the car will be provided by force of friction f acting towards centre.

Thus, $f = \frac{mv^2}{r}$ $\quad \because f_{max} = \mu N = \mu mg$

Therefore, for a safe turn without sliding $\frac{mv^2}{r} \le f_{max} \Rightarrow \frac{mv^2}{r} \le \mu mg \Rightarrow v \le \sqrt{\mu rg}$

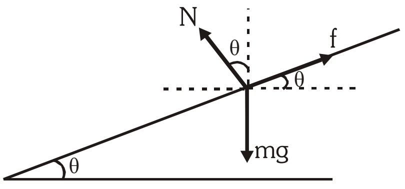
- **By Banking of Roads only**

Friction is not always reliable at circular turns if high speeds and sharp turns are involved. To avoid dependence on friction, the roads are banked at the turn so that the outer part of the road is some what lifted compared to the inner part.



$$N \sin\theta = \frac{mv^2}{r} \text{ and } N\cos\theta = mg \Rightarrow \tan\theta = \frac{v^2}{rg} \Rightarrow v = \sqrt{rg\tan\theta}$$

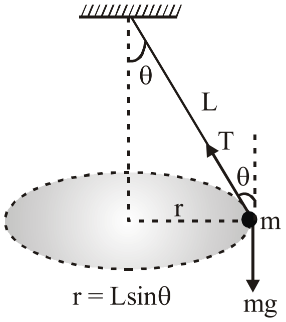
- **Friction and Banking of Road both**



If a vehicle is moving on a circular road which is rough and banked also, then three forces may act on the vehicle, of these the first force, i.e., weight (mg) is fixed both in magnitude and direction. The direction of second force, i.e., normal reaction N is also fixed (perpendicular to road) while the direction of the third force, i.e., friction f can be either inwards or outwards while its magnitude can be varied upto a maximum limit ($f_{max} = \mu N$). So, the magnitude of normal reaction N and direction plus magnitude of friction f are so adjusted that the resultant of the three forces mentioned above is $\frac{mv^2}{r}$ towards the centre.

Conical Pendulum

If a small particle of mass m tied to a string is whirled in a horizontal circle, as shown in figure. The arrangement is called the 'conical pendulum'. In case of conical pendulum the vertical component of tension balances the weight while its horizontal component provides the necessary centripetal force. Thus,,



$$T\sin\theta = \frac{mv^2}{r} \text{ and } T\cos\theta = mg \Rightarrow v = \sqrt{rg\tan\theta}$$

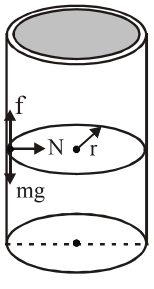
$$\therefore \text{ Angular speed } \omega = \frac{v}{r} = \sqrt{\frac{g\tan\theta}{r}}$$

So, the time period of pendulum is $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{r}{g\tan\theta}} = 2\pi\sqrt{\frac{L\cos\theta}{g}}$

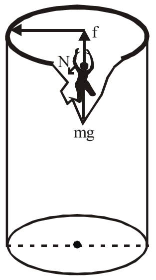
'Death Well' or Rotor

In case of 'death well' a person drives a motorcycle on a vertical surface of a large wooden well while in case of a rotor at a certain angular speed of rotor a person hangs resting against the wall without any support from the bottom. In death well walls are at rest and person revolves while in case of rotor person is at rest and the walls rotates.

In both cases friction balances the weight of person while reaction provides the centripetal force for circular motion,



Death well



Rotor

i.e., $f = mg$ and $N = \frac{mv^2}{r} = mr\omega^2$

Ex. Find the maximum speed at which a car can turn round a curve of 30 m radius on a level road if the coefficient of friction between the tyres and the road is 0.4 [acceleration due to gravity = 10 m/s^2]

Sol. Here centripetal force is provided by friction so

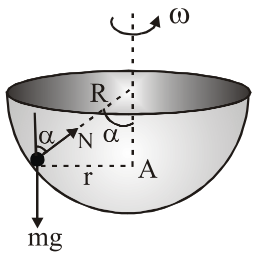
$$\frac{mv^2}{r} \leq \mu mg \Rightarrow v_{max} = \sqrt{\mu rg} = \sqrt{120} \approx 11 \text{ ms}^{-1}$$

Ex. For traffic moving at 60 km/hr, if the radius of the curve is 0.1 km, what is the correct angle of banking of the road ? (g = 10 m/s^2)

Sol. In case of banking $\tan\theta = \frac{v^2}{rg}$ Here v= 60 km/hr = $60 \times \frac{5}{18}$ ms^{-1} = $\frac{50}{3}$ ms^{-1} r = 0.1 km = 100 m

So $\tan\theta = \frac{50/3 \times 50/3}{100 \times 10} = \frac{5}{18} \Rightarrow \theta = \tan^{-1}\left(\frac{5}{18}\right)$

Ex. A hemispherical bowl of radius R is rotating about its axis of symmetry which is kept vertical. A small ball kept in the bowl rotates with the bowl without slipping on its surface. If the surface of the bowl is smooth and the angle made by the radius through the ball with the vertical is α. Find the angular speed at which the bowl is rotating.

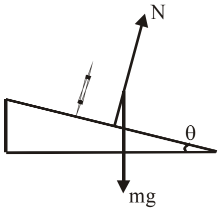


Sol. $N\cos\alpha = mg$ and $N\sin\alpha = mr\omega^2$ but $r = R\sin\alpha$

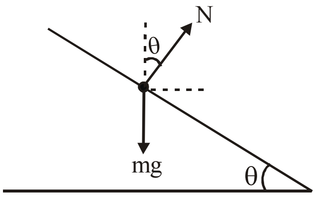
$\Rightarrow N\sin\alpha = mR\sin\alpha\,\omega^2 \Rightarrow N = mR\omega^2$

$\Rightarrow (mR\omega^2)\cos\alpha = mg \Rightarrow \omega = \sqrt{\frac{g}{R\cos\alpha}}$

Ex. A car is moving along a banked road laid out as a circle of radius r. (a) What should be the banking angle θ so that the car travelling at speed v needs no frictional force from the tyres to negotiate the turn? (b) The coefficients of friction between tyres and road are $\mu_s = 0.9$ and $\mu_k = 0.8$. At what maximum speed can a car enter the curve without sliding towards the top edge of the banked turn?

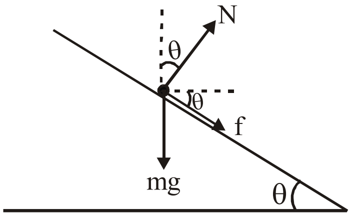


Sol. (a) $N\sin\theta = \frac{mv^2}{r}$ and $N\cos\theta = mg \Rightarrow \tan\theta = \frac{v^2}{rg}$



Note : In above case friction does not play any role in negotiating the turn.

(b) If the driver moves faster than the speed mentioned above, a friction force must act parallel to the road, inward towards centre of the turn.



$\Rightarrow F\cos\theta + N\sin\theta = \frac{mv^2}{r}$ and $N\cos\theta = mg + f\sin\theta$

For maximum speed of $f = \mu N$

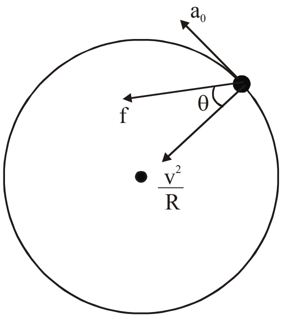
$\Rightarrow N(\mu\cos\theta + \sin\theta) = \frac{mv^2}{r}$ and $N(\cos\theta - \mu\sin\theta) = mg$

$\Rightarrow \frac{v^2}{rg} = \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} \Rightarrow v = \sqrt{\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)rg}$

Ex. A car starts from rest with a consant tangential acceleration a_0 in a circular path of radius r. At time t, the car skids, find the value of coefficient of friction.

Sol. The tangential and centripetal acceleration is provided only by the frictional force.

Thus, $f\sin\theta = ma_0$, $f\cos\theta = \dfrac{mv^2}{r} = \dfrac{m(a_0t)^2}{r}$



$$\Rightarrow f = m\sqrt{a_0^2 + \frac{(a_0t)^4}{r^2}} = ma_0\sqrt{1+\frac{a_0^2t^4}{r^2}} = f_{max}$$

$$\mu mg = ma_0\sqrt{1+\frac{a_0^2t^4}{r^2}} \Rightarrow \mu = \frac{a_0}{g}\sqrt{1+\frac{a_0^2t^4}{r^2}}$$

Centrifugal force :-

Centrifugal force is a pseudo force which an observer needs to consider while making observations in a rotating frame. This force is non physical and arise from kinematics and not due to physical interactions. Centrifugal force is directed away from axis of rotation of rotating frame and its value is $m\omega^2r$, where ω is angular speed of rotating frame where observer has kept himself fixed and r is distance of object of mass m from axis of rotation.

EXERCISE (S-1)

Kinematics of circular motion

1. A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the magnitude and direction of acceleration of the stone ?

CR0001

2. A particle is revolving in a circle of radius 1m with an angular speed of 12 rad/s. At $t = 0$, it was subjected to a constant angular acceleration α and its angular speed increased to $(480/\pi)$ rotation per minute (rpm) in 2 sec. Particle then continues to move with attained speed. Calculate

(i) angular acceleration of the particle,

(ii) tangential velocity of the particle as a function of time.

(iii) acceleration of the particle at $t = 0.5$ second and at $t = 3$ second

(iv) angular displacement at $t = 3$ second.

CR0002

3. A particle moves in a circle of radius 1.0 cm at a speed given by $v = 2.0\,t$ where v is in cm/s and t in seconds.

(a) Find the radial acceleration of the particle at $t = 1$s.

(b) Find the tangential acceleration at $t = 1$s.

(c) Find the magnitude of the acceleration at $t = 1$ s.

CR0003

4. A particle is travelling in a circular path of radius 4m. At a certain instant the particle is moving at 20m/s and its acceleration is at an angle of 37° from the direction to the centre of the circle as seen from the particle

(i) At what rate is the speed of the particle increasing?

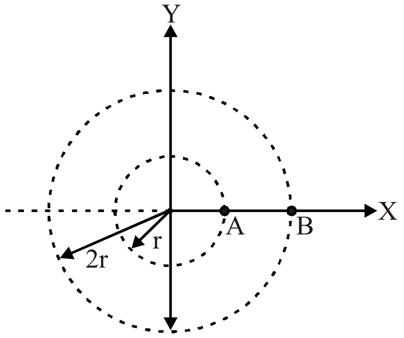
(ii) What is the magnitude of the acceleration?

CR0004

5. A disk rotates about its central axis starting from rest and accelerates with constant angular acceleration. At one instant it is rotating at 12 rad/s and after 80 radian of more angular displacement, its angular speed becomes 28 rad/s. How much time (seconds) does the disk takes to complete the mentioned angular displacement of 80 radians.

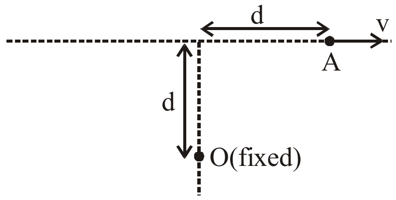
CR0005

6. Two particles A and B are moving in a horizontal plane anticlockwise on two different concentric circles with different constant angular velocities 2ω and ω respectively. Find the relative velocity (in m/s) of B w.r.t. A after time t = π/ω. They both start at the position as shown in figure (Take ω = 3rad/sec, r = 2m)



CR0006

7. Find angular velocity of A with respect to O at the instant shown in the figure.



CR0007

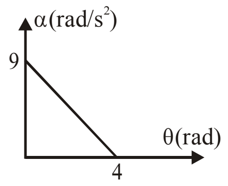
8. A stone is thrown horizontally with the velocity 15m/s. Determine the tangential and normal accelerations of the stone in 1 second after it begins to move.

CR0008

9. A particle moves in the x-y plane with the velocity $\vec{v} = a\hat{i} + bt\hat{j}$. At the instant $t = a\sqrt{3}/b$ the magnitude of tangential, normal and total acceleration are _____, _______, & _________.

CR0009

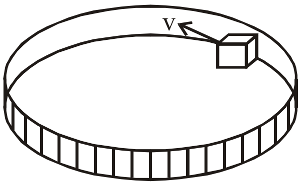
10. A particle starts moving in a non-uniform circular motion, has angular acceleration as shown in figure. The angular velocity at the end of 4 radian is given by ω rad/s then find the value of ω.



CR0010

Dynamics of circular motion

11. A block of mass m moves with speed v against a smooth, fixed vertical circular groove of radius r kept on smooth horizontal surface.



Find :

(i) normal reaction of the floor on the block.

(ii) normal reaction of the vertical wall on the block.

CR0011

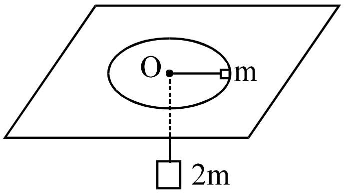
12. A cyclist speeding at 18 km/h on a level road takes a sharp circular turn of radius 3 m without reducing the speed. The co~efficient of static- friction between the tyres and the road is 0.1. Will the cyclist slip while taking the turn?

CR0012

13. A stone of mass 0.25 kg tied to the end of a string is whirled round in a circle of radius 1.5 m with a speed of 40 rev/min in a horizontal plane. What is the tension in the string ? What is the maximum speed with which the stone can be whirled around if the string can withstand a maximum tension of 200 N ?

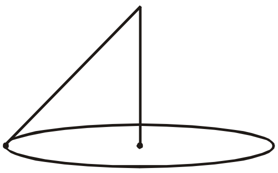
CR0013

14. A mass m rotating freely in a horizontal circle of radius 1 m on a frictionless smooth table supports a stationary mass 2m, attached to the other end of the string passing through smooth hole O in table, hanging vertically. Find the angular velocity of rotation.



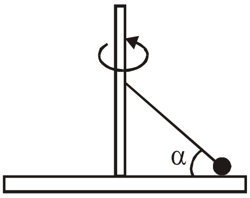
CR0014

15. Consider a conical pendulum having bob of mass m is suspended from a ceiling through a string of length L. The bob moves in a horizontal circle of radius r. Find (a) the angular speed of the bob and (b) the tension in the string.



CR0015

16. A circular platform rotates around a vertical axis with angular velocity $\omega = 10$ rad/s. On the platform is a ball of mass 1 kg, attached to the long axis of the platform by a thin rod of length 10 cm ($\alpha = 30°$). Find normal force exerted by the ball on the platform (in newton). Friction is absent.



CR0016

17. An aircraft executes a horizontal loop at a speed of 720 km/h with its wings banked at 15°. What is the radius of the loop ?

CR0017

18. A block of mass m = 20 kg is kept at a distance R = 1m from central axis of rotation of a round turn table (A table whose surface can rotate about central axis). Table starts from rest and rotates with constant angular acceleration, $\alpha = 3$ rad/sec^2. The friction coefficient between block and table is $\mu = 0.5$. At time $t = \frac{x}{3}$ sec from starting of motion (i.e. t = 0 sec) the block is just about to slip. Find the value of x.

CR0018

EXERCISE (S-2)

1. A stone is launched upward at 45° with speed v_0. A bee follows the trajectory of the stone at a constant speed equal to the initial speed of the stone.

(i) Find the radius of curvature at the top point of the trajectory.

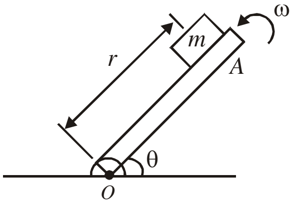
(ii) What is the acceleration of the bee at the top point of the trajectory? For the stone, neglect the air resistance.

CR0019

2. A particle is moving along a circular path of radius R in such a way that at any instant magnitude of radial acceleration & tangential acceleration are equal. If at t = 0 velocity of particle is V_0. Find the speed of the particle after time $t = \frac{R}{2V_0}$

CR0020

3. The member OA rotates in vertical plane about a horizontal axis through O with a constant counter clockwise velocity ω =3 rad/s. As it passes the position θ =0, a small mass m is placed upon it at a radial distance r = 0.5m. If the mass is observed to slip at θ =37^0, find the coefficient of friction between the mass & the member.



CR0021

4. Two blocks of mass m_1=10kg and m_2=5kg connected to each other by a massless inextensible string of length 0.3m are placed along a diameter of a turn table. The coefficient of friction between the table and m_1 is 0.5 while there is no friction between m_2 and the table. The table is rotating with an angular velocity of 10rad/sec about a vertical axis passing through its centre. The masses are placed along the diameter of the table on either side of the centre O such that m_1 is at a distance of 0.124 m from O. The masses are observed to be at rest with respect to an observer on the turn table.

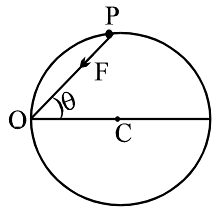
(i) Calculate the frictional force on m_1

(ii) What should be the minimum angular speed of the turn table so that the masses will slip from this position.

(iii) How should the masses be placed with the string remaining taut, so that there is no frictional force acting on the mass m_1.

CR0022

5. A particle P is moving on a circle under the action of only one force acting always towards fixed point O on the circumference. Find ratio of $\frac{d^2\theta}{dt^2}$ & $\left(\frac{d\theta}{dt}\right)^2$.



CR0023

6. A thin circular loop of radius R rotates about its vertical diameter with an angular frequency ω. Show that a small bead on the wire loop remains at its lowermost point for $\omega \le \sqrt{g/R}$. What is the angle made by the radius vector joining the centre to the bead with the vertical downward direction for $\omega = \sqrt{2g/R}$? Neglect friction.

CR0024

EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

Cross product of vectors

1. The area of parallelogram represented by the vectors $\vec{A}=2\hat{i}+3\hat{j}$ and $\vec{B}=\hat{i}+4\hat{j}$ is :-

(A) 14 unit (B) 7.5 unit (C) 10 unit (D) 5 unit

CR0025

2. What is the angle between $(\vec{P}+\vec{Q})$ and $(\vec{P}\times\vec{Q})$?

(A) 0 (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{4}$ (D) π

CR0026

3. The value of n so that vectors $2\hat{i}+3\hat{j}-2\hat{k}$, $5\hat{i}+n\hat{j}+\hat{k}$ and $-\hat{i}+2\hat{j}+3\hat{k}$ may be coplanar, will be :-

(A) 18 (B) 28 (C) 9 (D) 36

CR0027

4. If $\vec{a}$ and $\vec{b}$ are two vectors then the value of $(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})$ is :-

(A) $2(\vec{b}\times\vec{a})$ (B) $-2(\vec{b}\times\vec{a})$ (C) $\vec{b}\times\vec{a}$ (D) $\vec{a}\times\vec{b}$

CR0028

5. If $|\vec{a}\,.\,\vec{b}|=\sqrt{3}\,|\vec{a}\times\vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is :-

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

CR0029

Kinematics of circular motion

6. A body is executing circular motion in the vertical plane containing directions. If the direction of velocity $(\vec{v})$ at the top most point is towards west, what is the direction of angular velocity $(\vec{\omega})$?

(A) east (B) west (C) north (D) south

CR0030

7. If the magnitude of velocity in the previous question is decreasing with time, what is the direction of angular acceleration $(\vec{\alpha})$?

(A) east (B) west (C) north (D) south

CR0031

8. A mass is revolving in a circle which lies in a plane of paper. The direction of angular acceleration can be :-

(A) perpendicular to radius and velocity (B) towards the radius

(C) tangential (D) at right angle to angular velocity

CR0032

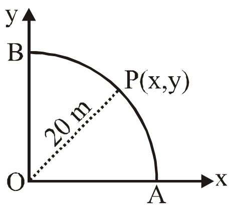
9. The second's hand of a watch has length 6 cm. Speed of end point and magnitude of difference of velocities at two perpendicular positions will be

(A) 2π & 0 mm/s (B) $2\sqrt{2}\,\pi$ & 4.44 mm/s

(C) $2\sqrt{2}\,\pi$ & 2π mm/s (D) 2π & $2\sqrt{2}\,\pi$ mm/s

CR0033

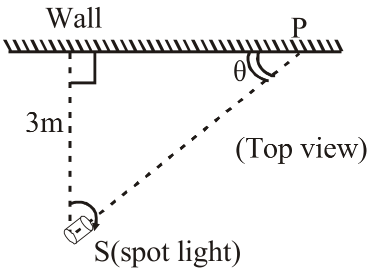
10. A point P moves in counter clockwise direction on a circular path as shown in the figure. The movement of 'P' is such that it sweeps out a length $s = t^2 + 5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of 'P' when $t = 5\sqrt{\frac{3}{10}}$ seconds is nearly :



(A) 2 m/s^2 (B) 1.5 m/s^2 (C) 2.5 m/s^2 (D) 3 m/s^2

CR0034

11. A spot light S rotates in a horizontal plane with a constant angular velocity of 0.1 rad/s. The spot of light P moves along the wall at a distance 3m. What is the velocity of the spot P when $\theta = 45°$?



(A) 0.6 m/s (B) 0.5 m/s (C) 0.4 m/s (D) 0.3 m/s

CR0035

12. Which of the following statements is false for a particle moving in a circle with a constant angular speed ? **[AIEEE - 2004]**

(A) The velocity vector is tangent to the circle

(B) The acceleration vector is tangent to the circle

(C) the acceleration vector points to the centre of the circle

(D) The velocity and acceleration vectors are perpendicular to each other

CR0036

Dynamics of circular motion

13. A particle is moving in a circle :

(A) the resultant force on the particle must be towards the centre

(B) the cross product of the tangential acceleration and the angular velocity will be zero

(C) the direction of the angular acceleration and the angular velocity must be the same

(D) the resultant force may be towards the centre

CR0037

14. A particle of mass m is tied to a light string and rotated with a speed v along a circular path of radius r. If T = tension in the string and mg = gravitational force on the particle then the actual forces acting on the particle are :

(A) mg and T only

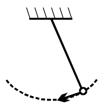
(B) mg, T and an additional force of $\frac{mv^2}{r}$ directed inwards.

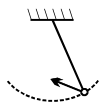
(C) mg, T and an additional force of $\frac{mv^2}{r}$ directed outwards.

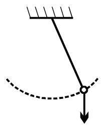
(D) only a force $\frac{mv^2}{r}$ directed outwards.

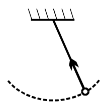
CR0038

15. Which vector in the figures best represents the acceleration of a pendulum mass at the intermediate point in its swing?

(A) 

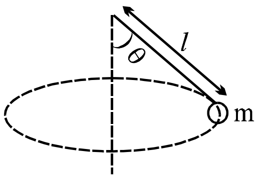
(B) 

(C) 

(D) 

CR0039

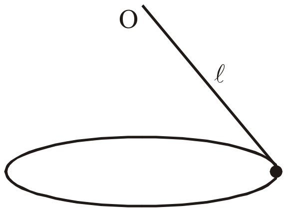
16. A conical pendulum is moving in a circle with angular velocity ω as shown. If tension in the string is T, which of following equations are correct ?

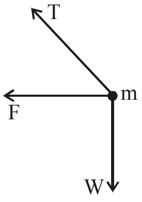


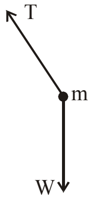
(A) $T = m\omega^2 l$ (B) $T \sin\theta = m\omega^2 l$ (C) $T = mg\cos\theta$ (D) $T = m\omega^2 l \sin\theta$

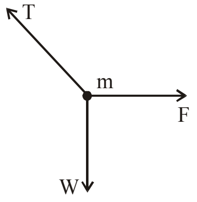
CR0040

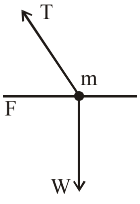
17. A point mass m is suspended from a light thread of length ℓ, fixed at O, is whirled in a horizontal circle at constant speed as shown. From your point of view, stationary with respect to the mass, the forces on the mass are :



(A) 

(B) 

(C) 

(D) 

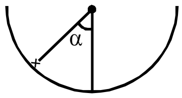
CR0041

18. A long horizontal rod has a bead which can slide along its length and is initially placed at a distance L from one end A of the rod. The rod is set in angular motion about A with a constant angular acceleration, α. If the coefficient of friction between the rod and bead is μ, and gravity is neglected, then the time after which the bead starts slipping is :– **[IIT-JEE 2000]**

(A) $\sqrt{\frac{\mu}{\alpha}}$ (B) $\frac{\mu}{\sqrt{\alpha}}$ (C) $\frac{1}{\sqrt{\mu\alpha}}$ (D) infinitesimal

CR0042

19. A insect crawls up a hemispherical surface very slowly (see the figure). The coefficient of friction between the surface and the insect is 1/3. If the line joining the centre of the hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α is given **[IIT-JEE 2001]**



(A) $\cot\alpha = 3$ (B) $\tan\alpha = 3$ (C) $\sec\alpha = 3$ (D) $\operatorname{cosec}\alpha = 3$

CR0043

20. The maximum velocity (in ms^{-1}) with which a car driver must traverse a flat curve of radius 150 m and coefficient of friction 0.6 to avoid skidding is - **[AIEEE - 2002]**

(A) 60 (B) 30 (C) 15 (D) 25

CR0044

21. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane, it follows that- **[AIEEE - 2004]**

(A) Its velocity is constant (B) Its acceleration is constant

(C) Its kinetic energy is constant (D) It moves in a straight line

CR0045

MULTIPLE CORRECT TYPE QUESTIONS

22. A car runs around a curve of radius 10 m at a constant speed of 10 ms^{-1}. Consider the time interval for which car covers a curve of 120° arc :-

(A) Resultant change in velocity of car is $10\sqrt{3}$ ms^{-1}

(B) Instantaneous acceleration of car is 10 ms^{-2}

(C) Average acceleration of car is $\frac{5}{24}$ ms^{-2}

(D) Instantaneous and average acceleration are same for the given period of motion.

CR0046

23. A car is moving with constant speed on a rough banked road.

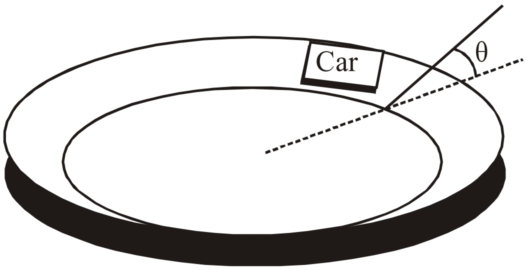
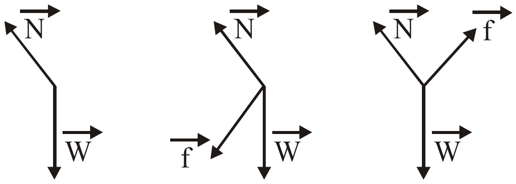


Figure shows the free body diagram of car in three situation A, B & C respectively:-



(A) (B) (C)

(A) Car in A has more speed than car in C

(B) Car in A has less speed than car in B

(C) FBD for car in A is not possible

(D) If $\mu > \tan\theta$ the FBD for car C is not possible

CR0047

24. A heavy particle is tied to the end A of a string of length 1.6 m. Its other end O is fixed. It revolves as a conical pendulum with the string making 60° with the vertical. Then

(A) its period of revolution is $\frac{4\pi}{7}$ sec.

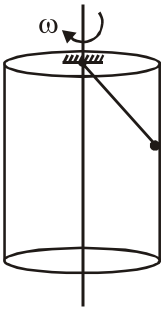
(B) the tension in the string is double the weight of the particle

(C) the velocity of the particle = $2.8\sqrt{3}$ m/s

(D) the centripetal acceleration of the particle is $9.8\sqrt{3}$ m/s^2.

CR0048

25. In the shown figure inside a fixed hollow cylinder with vertical axis a pendulum is rotating in a conical path with its axis same as that of the cylinder with uniform angular velocity. Radius of cylinder is 30 cm, length of string is 50 cm and mass of bob is 400 gm. The bob makes contact with the inner frictionless wall of the cylinder while moving :-

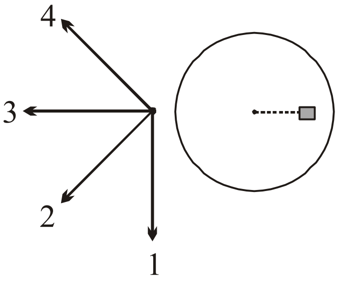


(A) The minimum value of angular velocity of the bob so that it does not leave contact is 5 rad/s
(B) Tension in the string is 5N for all values of angular velocity
(C) For angular velocity of 10 rad/s the bob pushes the cylinder with a force of 9N
(D) For angular velocity of 10 rad/s, tension in the string is 20N

CR0049

MATRIX MATCH TYPE QUESTION

26. A block is placed on a horizontal table which can rotate about its axis. The block is placed at a certain distance from centre as shown in figure. Table rotates such that particle does not slide. Select possible direction of net acceleration of block at the instant shown in figure.



	Column-I		Column-II
(A)	When rotation is clockwise with constant ω	(P)	1
(B)	When rotation is clock wise with decreasing ω	(Q)	2
(C)	When rotation is clockwise with increasing ω	(R)	3
(D)	Just after clockwise rotation begins from rest	(S)	4

CR0050

EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

1. A ring of radius r and mass per unit length m rotates with an angular velocity ω in free space. The tension in the ring is :

(A) zero (B) $\frac{1}{2}m\omega^2r^2$ (C) $m\omega^2r^2$ (D) $mr\omega^2$

CR0051

2. A uniform rod of mass m and length ℓ rotates in a horizontal plane with an angular velocity ω about a vertical axis passing through one end. The tension in the rod at a distance x from the axis is :

(A) $\frac{1}{2}m\omega^2x$ (B) $\frac{1}{2}m\omega^2\frac{x^2}{\ell}$ (C) $\frac{1}{2}m\omega^2\ell\left(1-\frac{x}{\ell}\right)$ (D) $\frac{1}{2}\frac{m\omega^2}{\ell}[\ell^2-x^2]$

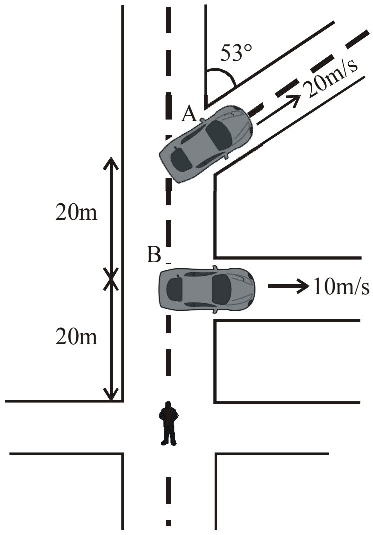
CR0052

3. The magnitude of displacement of a particle moving in a circle of radius a with constant angular speed ω varies with time t as :-

(A) 2 a sinωt (B) 2a sin$\frac{\omega t}{2}$ (C) 2a cos ωt (D) 2a cos $\frac{\omega t}{2}$

CR0053

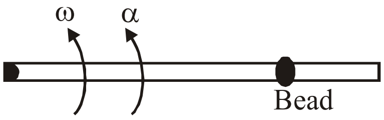
4. A traffic policeman standing at the intersection sees 2 cars A & B turning at angles 53° & 90° respectively (as shown in the figure). Their velocities are $V_A = 20$ m/s, $V_B = 10$ m/s. Then, which car appears to be moving faster to the traffic policeman :-



(A) A (B) B (C) Both equally fast (D) Insufficient info.

CR0054

5. A bead is constrained to move on rod in gravity free space as shown in figure. The rod is rotating with angular velocity ω and angular acceleration α about its end. If μ is coefficient of friction. Mark the correct option. (Rod rotates in the plane of paper.)



(A) If $\mu = \frac{\omega^2}{\alpha}$ friction on bead is static in nature

(B) If $\mu > \frac{\omega^2}{\alpha}$ friction on bead is kinetic in nature

(C) If $\mu < \frac{\omega^2}{\alpha}$ friction is static

(D) If bead does not slide relative to rod. Friction will not exist between bead and rod.

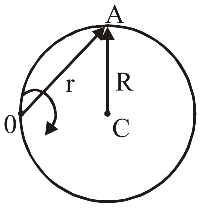
CR0055

6. A particle is moving in a circular path. The acceleration and momentum of the particle at a certain moment are $\vec{a} = (4\hat{i} + 3\hat{j})$ m/s^2 and $\vec{p} = (8\hat{i} - 6\hat{j})$kg–m/s. The motion of the particle is

(A) uniform circular motion
(B) accelerated circular motion
(C) de–accelerated circular motion
(D) we can not say anything with $\vec{a}$ and $\vec{p}$ only

CR0056

7. A particle A moves along a circle of radius R=50 cm so that its radius vector r relative to the point O (figure) rotates with the constant angular velocity ω=0.40 rad/s. Then modulus of the velocity of the particle, and the modulus of its total acceleration will be



(A) v= 0.4 m/s, a = 0.4 m/s^2
(B) v = 0.32 m/s, a = 0.32 m/s^2
(C) v = 0.32 m/s, a = 0.4 m/s^2
(D) v = 0.4 m/s, a = 0.32 m/s^2

CR0057

MULTIPLE CORRECT TYPE QUESTIONS

8. For a curved track of radius R, banked at angle θ (Take $v_0 = \sqrt{Rg\tan\theta}$)

(A) a vehicle moving with a speed v_0 is able to negotiate the curve without calling friction into play at all

(B) a vehicle moving with any speed $v > v_0$ is always able to negotiate the curve, with friction called into play

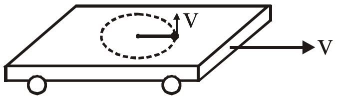
(C) a vehicle moving with any speed $v < v_0$ must have the force of friction into play

(D) the minimum value of the angle of banking for a vehicle parked on the banked road can stay there without slipping, is given by $\theta = \tan^{-1}\mu_0$ (μ_0 = coefficient of static friction)

CR0058

9. On a train moving along east with a constant speed v, a boy revolves a bob with string of length ℓ on smooth surface of a train, with equal constant speed v relative to train. Mark the correct option(s).

(A) Maximum speed of bob is 2 v in ground frame.



(B) Tension in string connecting bob is $\frac{4mv^2}{\ell}$ at an instant.

(C) Tension in string is $\frac{mv^2}{\ell}$ at all the moments.

(D) Minimum speed of bob is zero in ground frame.

CR0059

10. Let $\vec{v}(t)$ be the velocity of a particle at time t. Then :

(A) $|d\vec{v}(t)/dt|$ and $d|\vec{v}(t)|/dt$ are always equal

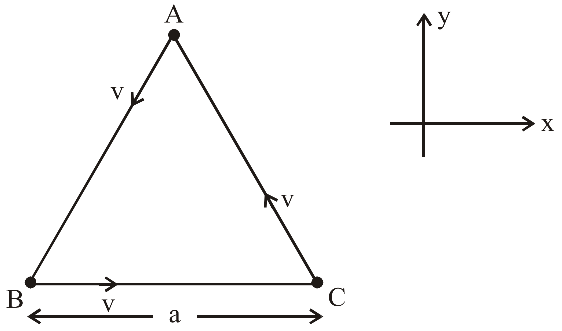
(B) $|d\vec{v}(t)/dt|$ and $d|\vec{v}(t)|/dt$ may be equal

(C) $d|\vec{v}(t)|/dt$ can be zero while $|d\vec{v}(t)/dt|$ is not zero

(D) $d|\vec{v}(t)|/dt \neq 0$ implies $|d\vec{v}(t)/dt| \neq 0$

CR0060

11. Three particles A, B, C are located at the corners of an equilateral triangle as shown in figure. Each of the particle is moving with velocity v. Then at the instant shown, the relative angular velocity of



(A) A wrt B is $\frac{v\cos 30°}{a}$ in z-direction

(B) B wrt C is $\frac{v\cos 30°}{a}$ in z-direction

(C) A wrt C is $\frac{v\cos 30°}{a}$ in z-direction

(D) B wrt A is $\frac{v\cos 30°}{a}$ in –z direction

CR0061

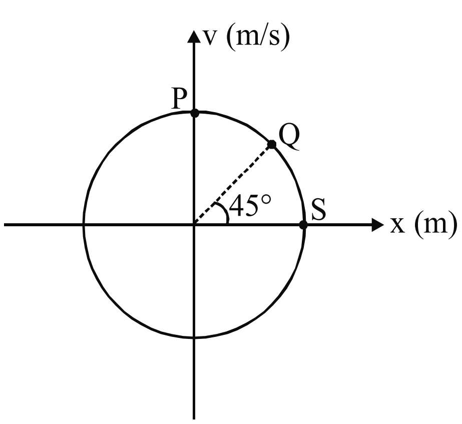
12. A particle is in motion on the x-axis. The variation of its velocity with position is as shown. The graph is circle and its equation is $x^2 + v^2 = 1$, where x is in m and v in m/s. The **CORRECT** statement(s) is/are :-

(A) When x is positive, acceleration is negative.

(B) When x is negative, acceleration is positive.

(C) At Q, acceleration has magnitude $\frac{1}{\sqrt{2}}$ m/s^2

(D) At S, acceleration is infinite.



CR0062

13. An ant travels along a long rod with a constant velocity $\vec{u}$ relative to the rod starting from the origin. The rod is kept initially along the positive x-axis. At t = 0, the rod also starts rotating with an angular velocity ω (anticlockwise) in x-y plane about origin. Then

(A) the position of the ant at any time t is $\vec{r} = ut[\cos\omega t\hat{i} + \sin\omega t\hat{j}]$

(B) the speed of the ant at any time t is $u\sqrt{1+\omega^2t^2}$

(C) the magnitude of the tangential acceleration of the ant at any time t is $\frac{\omega^2 tu}{\sqrt{1+\omega^2t^2}}$

(D) the speed of the ant at any time t is $\sqrt{1+2\omega^2t^2u}$

CR0063

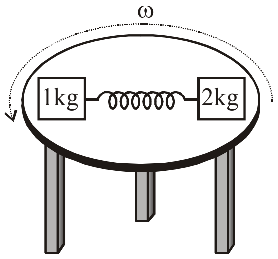
14. On a circular turn table rotating about its center horizontally with uniform angular velocity ω rad/s placed two blocks of mass 1 kg and 2 kg, on a diameter symmetrically about center. Their separation is 1m and friction is sufficient to avoid slipping. The spring between them as shown is stretched and applied force of 5N. If f_1 and f_2 are values of friction on 1 kg & 2kg block respectively:-

(A) For ω = 2 rad/s, f_1 = 3N & f_2 = 1N

(B) For ω = 3 rad/s, f_1 = 0.5 N & f_2 = 4N

(C) For ω = $\sqrt{10}$ rad/s, f_1 = 0 & f_2 = 5N

(D) For ω = $\sqrt{10}$ rad/s, f_1 = 0 & f_2 = 0N



CR0064

15. Two particles move on a circular path (one just inside and the other just outside) with angular velocities ω and 5ω starting from the same point. Then

(A) they cross each other at regular intervals of time $\frac{2\pi}{4\omega}$ when their angular velocities are oppositely directed.

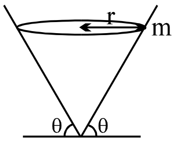
(B) they cross each other at points on the path subtending an angle of 60°at the centre if their angular velocities are oppositely directed.

(C) they cross at intervals of time $\frac{\pi}{3\omega}$ if their angular velocities are oppositely directed.

(D) they cross each other at points on the path subtending 90° at the centre if their angular velocities are in the same sense.

CR0065

16. A ball of mass 'm' is rotating in a circle of radius 'r' with speed v inside a smooth cone as shown in figure. Let N be the normal reaction on the ball by the cone, then choose the correct option.



(A) $N = mg\cos\theta$

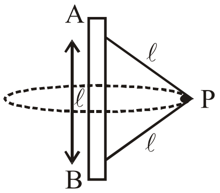
(B) $g\sin\theta = \frac{v^2}{r}\cos\theta$

(C) $N\sin\theta - \frac{mv^2}{r} = 0$

(D) none of these

CR0066

17. A particle P of mass m is attached to a vertical axis by two strings AP and BP of length l each. The separation AB=l. P rotates around the axis with an angular velocity ω. The tensions in the two strings are T_1 and T_2



(A) $T_1=T_2$

(B) $T_1+T_2=m\omega^2 l$

(C) $T_1-T_2=2mg$

(D) BP will remain taut only if $\omega \geq \sqrt{\frac{2g}{\ell}}$

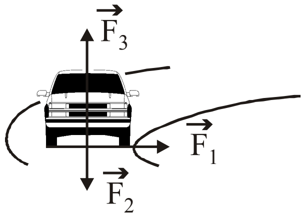
CR0067

MATRIX MATCH TYPE QUESTION

18. Column-I shows certain situations and column-2 shows information about forces.

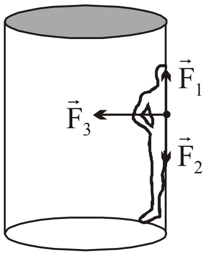
Column - I **Column - II**

Situation

(A) 

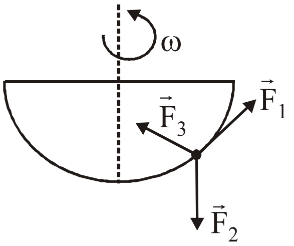
Front view of a car rounding a curve with constant speed.

(P) $\vec{F}_1 + \vec{F}_2 + \vec{F}_3$ is centripetal force.

(B) 

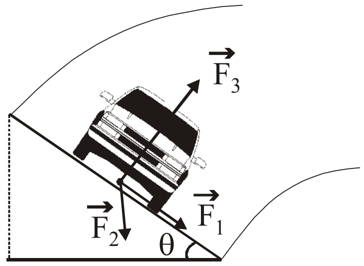
Passengers in a rotor not sliding relative to rotor wall cylindrical rotor is rotating with constant angular velocity about its symmetry axis.

(Q) $\vec{F}_1$ is static friction.

(C) 

Particle kept on rough surface of a bowl, no relative motion of particle in bowl, bowl has constant angular velocity

(R) $\vec{F}_1$ can be in direction opposite to that shown in figure.

(D) 

Car moving on a banked road with constant speed, no sideways skidding

(S) $\vec{F}_1 + \vec{F}_2 = \vec{0}$

(T) $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{0}$

CR0068

EXERCISE (J-M)

1. For a pacticle in uniform circular motion, the acceleration $\vec{a}$ at a point P(R, θ) on the circle of radius R is (Here θ is measured from the x-axis). **[AIEEE - 2010]**

(1) $\frac{v^2}{R}\hat{i} + \frac{v^2}{R}\hat{j}$

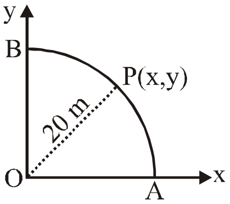
(2) $-\frac{v^2}{R}\cos\theta\,\hat{i} + \frac{v^2}{R}\sin\theta\,\hat{j}$

(3) $-\frac{v^2}{R}\sin\theta\,\hat{i} + \frac{v^2}{R}\cos\theta\,\hat{j}$

(4) $-\frac{v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\,\hat{j}$

CR0069

2. A point P moves in counter clockwise direction on a circular path as shown in the figure. The movement of 'P' is such that it sweeps out a length $s = t^3 + 5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of 'P' when t = 2s is nearly : **[AIEEE - 2010]**



(1) 14 m/s^2 (2) 13 m/s^2 (3) 12 m/s^2 (4) 7.2 m/s^2

CR0070

3. Two cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2, respectively. Their speeds are such that they make complete circles in the same time t. The ratio of their centripetal acceleration is :- **[AIEEE - 2012]**

(1) 1 : 1 (2) $m_1 r_1 : m_2 r_2$ (3) $m_1 : m_2$ (4) $r_1 : r_2$

CR0071

4. Concrete mixture is made by mixing cement, stone and sand in a rotating cylindrical drum. If the drum rotates too fast, the ingredients remain stuck to the wall of the drum and proper mixing of ingredients does not take place. The maximum rotational speed of the drum in revolutions per minute(rpm) to ensure proper mixing is close to : (Take the radius of the drum to be 1.25 m and its axle to be horizontal): **[JEE Main (Online) - 2016]**

(1) 8.0 (2) 0.4 (3) 1.3 (4) 27.0

CR0072

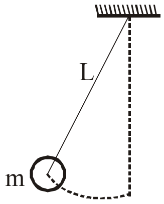
5. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the nth power of R. If the period of rotation of the particle is T, then, **[JEE Main-2018]**

(1) $T \propto R^{\frac{n}{2}+1}$ (2) $T \propto R^{(n+1)/2}$ (3) $T \propto R^{n/2}$ (4) $T \propto R^{3/2}$ for any n

CR0073

EXERCISE (J-A)

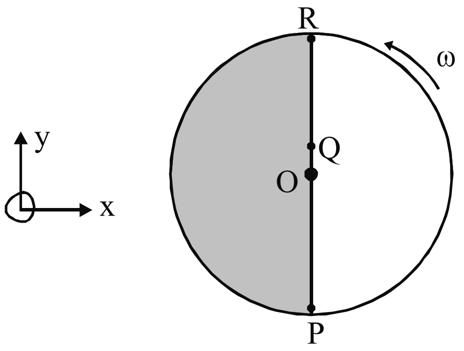
1. A ball of mass (m) 0.5 kg is attached to the end of a string having length (L) 0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N. The maximum possible value of angular velocity of ball (in radian/s) is **[IIT-JEE-2011]**



(A) 9 (B) 18 (C) 27 (D) 36

CR0074

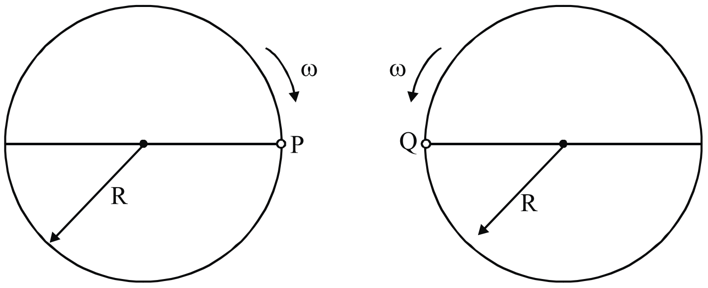
2. Consider a disc rotating in the horizontal plane with a constant angular speed ω about its centre O. The disc has a shaded region on one side of the diameter and an unshaded region on the other side as shown in the figure. When the disc is in the orientation as shown, two pebbles P and Q are simultaneously projected at an angle towards R. The velocity of projection is in the y-z plane and is same for both pebbles with respect to the disc. Assume that (i) they land back on the disc before the disc has completed $\frac{1}{8}$ rotation, (ii) their range is less than half the disc radius, and (iii) ω remains constant throughout. Then **[IIT-JEE-2012]**

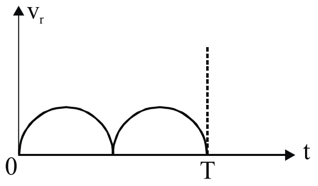


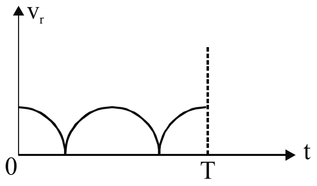
(A) P lands in the shaded region and Q in the unshaded region.
(B) P lands in the unshaded region and Q in the shaded region.
(C) Both P and Q land in the unshaded region.
(D) Both P and Q land in the shaded region.

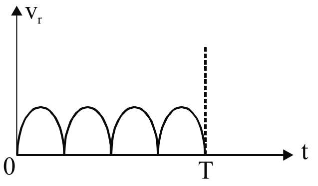
CR0075

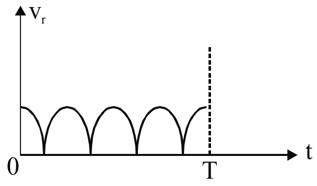
3. Two identical discs of same radius R are rotating about their axes in opposite directions with the same constant angular speed ω. The discs are in the same horizontal plane. At time t =0, the points P and Q are facing each other as shown in the figure. The relative speed between the two points P and Q is v_r. In one time period (T) of rotation of the discs, v_r as a function of time is best represented by **[IIT-JEE 2012]**



(A)


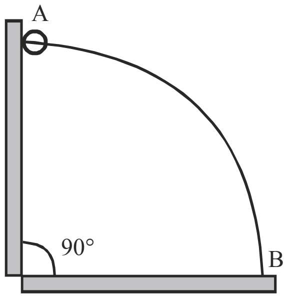
(B)


(C)


(D)


CR0076

4. A wire, which passes through the hole in a small bead, is bent in the form of quarter of a circle. The wire is fixed vertically on ground as shown in the figure. The bead is released from near the top of the wire and it slides along the wire without friction. As the bead moves from A to B, the force it applies on the wire is :- **[IIT-JEE Advanced 2014]**



(A) Always radially outwards

(B) Always radially inwards

(C) Radially outwards initially and radially inwards later.

(D) Radially inwards initially and radially outwards later.

CR0077

Paragraph for Question No. 5 and 6

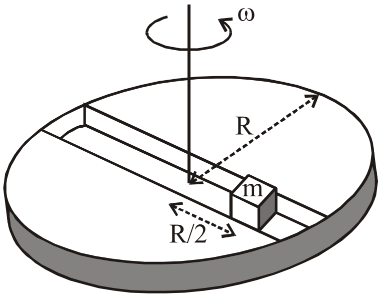
A frame of reference that is accelerated with respect to an inertial frame of reference is called a non-inertial frame of reference. A coordinate system fixed on a circular disc rotating about a fixed axis with a constant angular velocity ω is an example of a non-inertial frame of reference. The relationship between the force $\vec{F}_{rot}$ experienced by a particle of mass m moving on the rotating disc and the force $\vec{F}_{in}$ experienced by the particle in an inertial frame of reference is

$$\vec{F}_{rot} = \vec{F}_{in} + 2m(\vec{\upsilon}_{rot} \times \vec{\omega}) + m(\vec{\omega} \times \vec{r}) \times \vec{\omega},$$

where $\vec{\upsilon}_{rot}$ is the velocity of the particle in the rotating frame of reference and $\vec{r}$ is the position vector of the particle with respect to the centre of the disc.

Now consider a smooth slot along a diameter of a disc of radius R rotating counter-clockwise with a constant angular speed ω about its vertical axis through its center. We assign a coordinate system with the origin at the centre of the disc, the x-axis along the slot, the y-axis perpendicular to the slot and the z-axis along the rotation axis $(\vec{\omega} = \omega\hat{k})$. A small block of mass m is gently placed in the slot at $\vec{r} = (R/2)\hat{i}$ at t = 0 and is constrained to move only along the slot.

[IIT-JEE Advanced-2016]



5. The distance r of the block at time t is :

(A) $\frac{R}{4}(e^{2\omega t} + e^{-2\omega t})$ (B) $\frac{R}{2}\cos 2\omega t$ (C) $\frac{R}{2}\cos \omega t$ (D) $\frac{R}{4}(e^{\omega t} + e^{-\omega t})$

CR0078

6. The net reaction of the disc on the block is :

(A) $-m\omega^2 R \cos \omega t \hat{j} - mg\hat{k}$ (B) $m\omega^2 R \sin \omega t \hat{j} - mg\hat{k}$

(C) $\frac{1}{2}m\omega^2 R(e^{\omega t} - e^{-\omega t})\hat{j} + mg\hat{k}$ (D) $\frac{1}{2}m\omega^2 R(e^{2\omega t} - e^{-2\omega t})\hat{j} + mg\hat{k}$

CR0078

ANSWER KEY

EXERCISE (S-1)

1. Ans. 9.9 ms^{-2}, in radial direction towards the centre at all points.

2. Ans. (i) 2 rad/s^2 (ii) 12+2t for $t \leq 2s$, 16 for $t \geq 2s$ (iii) $a = 169.01\ m/s^2$ (iv) 44 rad

3. Ans. (a) 4 cm/s^2 (b) 2 cm/s^2 (c) $\sqrt{20}\ cm/s^2$ **4. Ans.** (i) 75m/s^2, (ii) 125m/s^2

5. Ans. 4 **6. Ans. 024** **7. Ans.** $\frac{v}{2d}$ **8. Ans.** $a_t = \frac{20}{\sqrt{13}}$, $a_n = \frac{30}{\sqrt{13}}$

9. Ans. $\sqrt{3}b/2, b/2, b$ **10. Ans. 6** **11. Ans.** (i) mg, (ii) $\frac{mv^2}{r}$

12. Ans. Yes, $a_c = 8$, $\mu g = 1$ **13. Ans.** T = 6.6 N, $v_{max} = 35\ ms^{-1}$

14. Ans. $\sqrt{2g}$ rad/s **15. Ans.** (a) $\sqrt{\frac{g}{\sqrt{L^2 - r^2}}}$ (b) $\frac{mgL}{\sqrt{L^2 - r^2}}$

16. Ans. 5 **17. Ans. 15 km** **18. Ans. 2**

EXERCISE (S-2)

1. Ans. (i) $\frac{V_0^2}{2g}$, (ii) 2g **2. Ans.** $2V_0$ **3. Ans.** $\mu = \frac{3}{16}$

4. Ans. (i) 36N, (ii) 11.66rad/sec, (iii) 0.1m, 0.2m **5. Ans. 2 tanθ** **6. Ans.** θ = 60°

EXERCISE (O-1)

1. Ans. (D) **2. Ans. (B)** **3. Ans. (A)** **4. Ans. (A)** **5. Ans. (A)** **6. Ans. (D)**

7. Ans. (C) **8. Ans. (A)** **9. Ans. (D)** **10. Ans. (C)** **11. Ans. (A)** **12. Ans. (B)**

13. Ans. (D) **14. Ans. (A)** **15. Ans. (B)** **16. Ans. (A)** **17. Ans. (C)** **18. Ans. (A)**

19. Ans. (A) **20. Ans. (B)** **21. Ans. (C)**

22. Ans. (A, B) **23. Ans. (A, B)** **24. Ans. (A,B,C,D)** **25. Ans. (A, B, C)**

26. Ans. (A)-R; (B)-S; (C)-Q; (D)-P

EXERCISE (O-2)

1. Ans. (C) **2. Ans. (D)** **3. Ans. (B)** **4. Ans. (B)**

5. Ans. (A) **6. Ans. (B)** **7. Ans. (D)**

8. Ans. (A, C) **9. Ans. (A,C,D)** **10. Ans. (B,C,D)** **11. Ans. (A,B,C)**

12. Ans. (A, B, C) **13. Ans. (A, B, C)** **14. Ans. (A, B, C)** **15. Ans. (B, C, D)**

16. Ans. (B, C) **17. Ans. (B, C, D)**

18. Ans. (A) P,Q (B) P,Q,S (C) P,Q,R (D) P,Q,R

EXERCISE (J-M)

1. Ans. (4) **2. Ans. (1)** **3. Ans. (4)** **4. Ans. (4)**

5. Ans. (2)

EXERCISE (J-A)

1. Ans. (D) **2. Ans. (C OR D)** **3. Ans. (A)** **4. Ans. (D)** **5. Ans. (D)**

6. Ans. (C)

Important Notes

Chapter Contents

02 WORK, POWER & ENERGY

Important Notes

WORK, POWER & ENERGY

KEY CONCEPT

WORK

- Whenever a force acting on a body, displaces it in its direction, work is said to be done by the force.
- Work done by a force is equal to scalar product of force applied and displacement of the point of application, $W = \vec{F}.\vec{d}$
- Work is a scalar quantity.

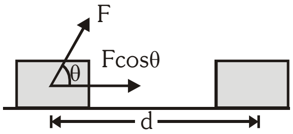
Work done by a constant force :

If the direction and magnitude of a force applied on a body is constant, the force is said to be constant. Work done by a constant force, W = Force × component of displacement along force

= displacement × component of force along displacement.

The work done will, be $W = (F\cos\theta)\,d$

$= F\,(d\cos\theta)$



In vector form, $W = \vec{F}\cdot\vec{d}$

Note : The force of gravity is the example of constant force, hence work done by it is the example of work done by a constant force.

Work done by a variable force

If the force applying on a body is changing its direction or magnitude or both, the force is said to be variable. Suppose a variable force causes displacement in a body from position P_1 to position P_2. To calculate the work done by the force the path from P_1 to P_2 can be divided into infinitesimal element, each element is so small that during displacement of body through it, the force is supposed to be constant. If $d\vec{r}$ be small displacement of point of application and $\vec{F}$ be the force applied on the body, the work done by force is $dW = \vec{F}\cdot d\vec{r}$

The total work done in displacing body from P_1 to P_2 is given by $\int dW = \int_{P_1}^{P_2} \vec{F}.d\vec{r} \Rightarrow W = \int_{P_1}^{P_2} \vec{F}\cdot d\vec{r}$

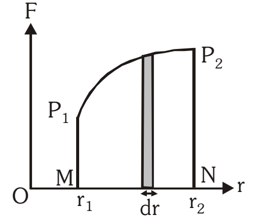
If $\vec{r}_1$ and $\vec{r}_2$ be the position vectors of the points P_1 and P_2 respectively, the total work done $W = \int_{r_1}^{r_2} \vec{F}.d\vec{r}$

Note : When we consider a block attached to a spring, the force on the block is k times the elongation of the spring, where k is spring constant. As the elongation changes with the motion of the block, therefore the force is variable. This is an example of work done by variable force.

Here : $W_s = \int_{x_i}^{x_f} -k\ x\ dx = \frac{1}{2}k\ \left(x_i^2 - x_f^2\right)$

Calculation of work done from force displacement graph :

Suppose a body, whose initial position is r_1, is acted upon by a variable force $\vec{F}$ and consequently the body acquires its final position r_2. From position r to r + dr or for small displacement dr, the work done will be $\vec{F}.d\vec{r}$ whose value will be the area of the shaded strip of width dr. The work done on the body in displacing it from position r_1 to r_2 will be equal to the sum of areas of all such strips



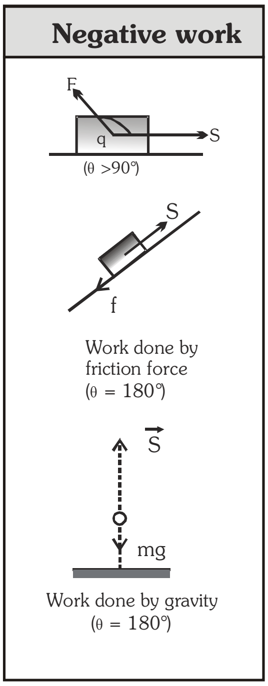
Thus, total work done, $W = \sum_{r_1}^{r_2} dW = \sum_{r_1}^{r_2} F.dr$ = Area of P_1P_2NM

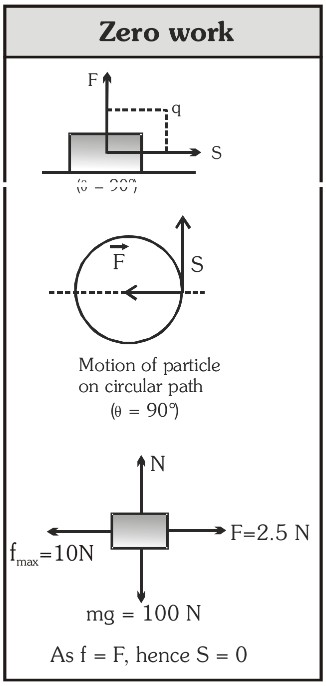
The area of the graph between curve and displacement axis is equal to the work done.

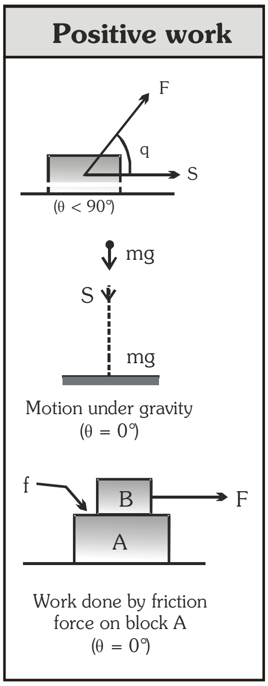
Note : To calculate the work done by graphical method, for the sake of simplicity, here we have assumed the direction of force and displacement as same, but if they are not in same direction, the graph must be plotted between F cos θ and r.

Nature of work done

Although work done is a scalar quantity, yet its value may be positive, negative or even zero







UNITS :

SI Unit : joule (J).

joule : One joule of work is said to be done when a force of one newton displaces a body by one meter in the direction of force.

1 joule = 1 newton × 1 meter = 1 kgm^2s^{-2}

erg : One erg of work is said to be done when a force of one dyne displaces a body by one cm. in the direction of force.

1 erg = 1 dyne × 1 cm. = 1 gm. cm^2 s^{-2}

Other Units :
(a) 1 joule = 10^7 erg
(b) 1 erg = 10^{-7} joule
(c) 1 eV = 1.6×10^{-19} joule
(d) 1 joule = 6.25×10^{18} eV
(e) 1 MeV = 1.6×10^{-13} J
(f) 1 J = 6.25×10^{12} MeV
(g) 1 kilo watt hour (kWh) = 3.6×10^6 joule

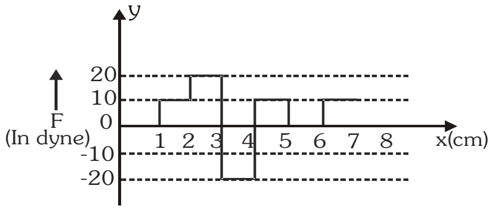
DIMENSIONS :

[Work] = [Force] [Displacement]= $[MLT^{-2}][L] = [ML^2T^{-2}]$

Ex. A position dependent force $F = 7 - 2x + 3x^2$ acts on a small body of mass 2kg and displaces it from x = 0 to x = 5 m. Calculate the work done in joule.

Sol. $W = \int_{x_1}^{x_2} F dx = \int_0^5 (7 - 2x + 3x^2) dx = \left[7x - \frac{2x^2}{2} + \frac{3x^3}{3}\right]_0^5 = 135J$

Ex. For the force displacement diagram shown in adjoining diagram. Calculate the work done by the force in displacing the body from x = 1 cm to x = 5 cm.



Sol. Work = Area under the curve and displacement axis = 10 + 20 – 20 + 10 = 20 erg

Ex. Calculate work done to move a body of mass 10 kg along a smooth inclined plane ($\theta = 30°$) with constant velocity through a distance of 10 m.

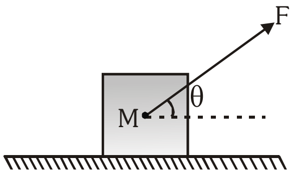
Sol. Here the motion is not accelerated, the resultant force parallel to the plane must be zero. So

$F - Mg \sin 30° = 0 \Rightarrow F = Mg \sin 30°$ & d=10m

$W = Fd\cos\theta = (Mg\sin 30°)d \cos 0° = 10 \times 10 \times \frac{1}{2} \times 10 \times 1 = 500$ J

Ex. Calculate work done in pulling an object with a constant force F as shown in figure.
(given that the ground is rough with coefficient of friction μ)

Sol. From the figure $F \sin\theta + N = Mg$



$\therefore$ $N = Mg - F \sin\theta$

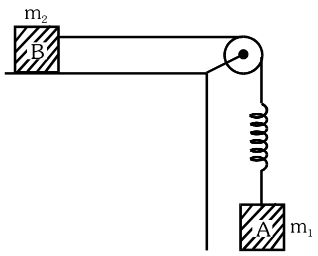
$F \cos\theta = f = \mu N = \mu[Mg - F \sin\theta]$

$F (\cos\theta + \mu\sin\theta) = \mu Mg$

$\therefore$ $F = \frac{\mu Mgd}{\cos\theta + \mu\sin\theta}$ = force required to pull an object

Work done in pulling an object $W = Fd = \frac{\mu Mgd}{\cos\theta + \mu\sin\theta}$

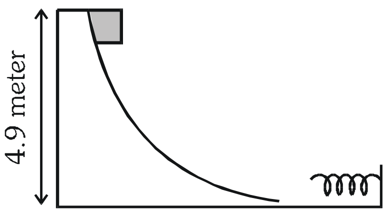
Ex. If the two blocks moves with a constant uniform speed then find coefficient of friction between the surface of the block B and the table. The spring is massless and the pulley is frictionless.



Sol. **For block B** : $T = f = \mu m_2 g$ and **For block A** : $T = m_1 g$

By solving above equations $\mu = \frac{m_1}{m_2}$

Ex. Figure shows a smooth curved track terminating in a smooth horizontal part, A spring of force constant 400 N/m is attached at one end to a wedge fixed rigidly with a horizontal part. A 40 g mass is released from rest at a height of 4.9 m on the curved track. Find the maximum compression of the spring.



Sol. From the law of conservation of mechanical energy $\Rightarrow mgh = \frac{1}{2}kx^2$

$$\Rightarrow \quad x = \sqrt{\frac{2mgh}{k}} = \sqrt{\frac{2 \times (0.04) \times 9.8 \times 4.9}{400}} = 9.8 \text{ cm.}$$

ENERGY

- The energy of a body is defined as the capacity of doing work.
- There are various form of energy

  (i) mechanical energy (ii) chemical energy (iii) electrical energy (iv) sound energy
  (v) light energy etc (vi) magnetic energy (vii) nuclear energy
- Energy of an isolated system always remain constant it can neither be created nor it can be destroyed however it may be converted from one form to another

  Examples

  Electrical energy $\xrightarrow{\text{Motor}}$ Mechanical energy

  Mechanical energy $\xrightarrow{\text{Generator}}$ Electrical energy

  Light energy $\xrightarrow{\text{Photocell}}$ Electrical energy

  Electrical energy $\xrightarrow{\text{Heater}}$ Heat energy

  Electrical energy $\xrightarrow{\text{Radio / Speaker}}$ Sound energy

  Nuclear energy $\xrightarrow{\text{Nuclear Reactor}}$ Electrical energy

  Chemical energy $\xrightarrow{\text{Cell}}$ Electrical energy

  Electrical energy $\xrightarrow[\text{Chargeable}]{\text{Secondary Cell}}$ Chemical energy
- Energy is a scalar quantity
- Unit : Its unit is same as that of work or torque. In MKS : joule, watt second ; In CGS : erg

  Note : 1 eV = 1.6×10^{-19} joule; 1 kWh = 3.6×10^{6} joule; 10^{7} erg = 1 joule.
- Dimension $[M^1L^2T^{-2}]$
- According to Einstein's mass energy equivalence principle mass and energy are inter convertible i.e. they can be changed into each other. Energy equivalent of mass m is, $E = mc^2$ where, m : mass of the particle, c : velocity of light , E : equivalent energy corresponding to mass m.
- In mechanics we are concerned with mechanical energy only which is of two type :

  (i) kinetic energy (ii) potential energy

Kinetic energy

- The energy possessed by a body by virtue of its motion is called kinetic energy.
- If a body of mass m is moving with velocity v, its kinetic energy KE = 1/2 mv^2.
- Kinetic energy is always positive.
- If linear momentum of body is p, the kinetic energy for translatory motion is KE= $\frac{p^2}{2m}=\frac{1}{2}mv^2$.

Ex. In a ballistics demonstration, a police officer fires a bullet of mass 50.0 g with speed 200 ms^{-1} on soft plywood of thickness 2.00 cm. The bullet emerges with only 10% of its initial kinetic energy. What is the emergent speed of the bullet?

Sol. Initial kinetic energy, $K_i=\frac{1}{2}\times\frac{50}{1000}\times200\times200J=1000J$

Final kinetic energy, $K_f=\frac{10}{100}\times1000J=100J$

If v_f is emergent speed of the bullet, then $\frac{1}{2}\times\frac{50}{1000}\times v_f^2=100$

$\Rightarrow v_f^2 = 4000 \Rightarrow v_f = 63.2\ ms^{-1}$.

Note that the speed is reduced by approximately 68% and not 90%.

Work Energy Theorem

Work done by all the forces (conservative or non conservative, external or internal) acting on a particle or an object is equal to the change in it's kinetic energy. So work done by all the forces = change in kinetic energy

$$W = \Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Ex. A particle of mass m moves with velocity $v = a\sqrt{x}$ where a is a constant. Find the total work done by all the forces during a displacement from x = 0 to x = d.

Sol. Work done by all forces $= W = \Delta KE = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

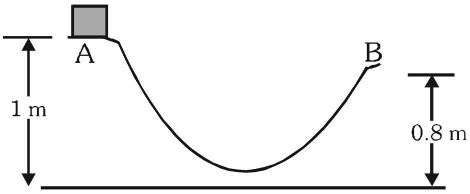
Here $v_1 = a\sqrt{0} = 0$, $v_2 = a\sqrt{d}$, So $W = \frac{1}{2}ma^2d - 0 = \frac{1}{2}ma^2d$

Ex. The displacement x of a body of mass 1 kg on horizontal smooth surface as a function of time t is given by $x = \frac{t^3}{3}$. Find the work done by the external agent for the first one second.

Sol. $\because x = \frac{t^3}{3}\quad \therefore v = \frac{dx}{dt} = t^2$, Velocity at t = 0, u = 0 and at t = 1s v = 1 m/s

Using work energy theorem : $W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2}1(1)^2 = 0.5$ J

Ex. A block of mass 1kg is placed at the point A of a rough track shown in figure. If it slightly pushed towards right it stops at the point B of the track. Calculate the workdone by the frictional force on the block during its transit from A to B.



Sol. $W_c + W_{nc} + W_{ext} = \Delta K$

$mg\,(1 - 0.8) + W_{nc} + 0 = 0 \Rightarrow W_{nc} = -1 \times 9.8 \times 0.2 = -1.96$ J

POWER

When we purchase a car or jeep we are interested in the horsepower of its engine. We know that usually an engine with large horsepower is most effective in accelerating the automobile.

In many cases it is useful to know not just the total amount of work being done, but how fast the work is done. We define power as the rate at which work is being done.

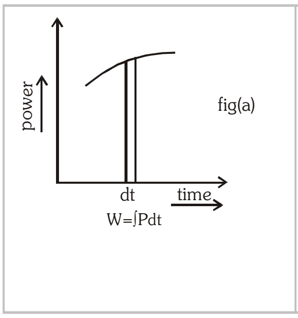
$$\text{Average Power} = \frac{\text{Work done}}{\text{Time taken to do work}} = \frac{\text{Total change in kinetic energy}}{\text{Total change in time}}$$

If ΔW is the amount of work done in the time interval Δt. Then $P = \frac{\Delta W}{\Delta t} = \frac{W_2 - W_1}{t_2 - t_1}$

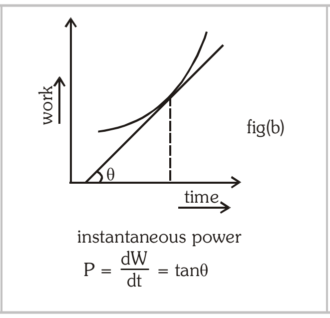
When work is measured in joules and t is in seconds, the unit for power is the joule per second, which is called watt. For motors and engines, power is usually measured in horsepower, where horsepower is 1 hp = 746 W. The definition of power is applicable to all types of work like mechanical, electrical, thermal. Instanteneous power $P = \frac{dW}{dt} = \frac{\vec{F}.d\vec{r}}{dt} = \vec{F}.\vec{v}$

Where v is the instantaneous velocity of the particle and dot product is used as only that component of force will contribute to power which is acting in the direction of instantaneous velocity.

- Power is a scalar quantity with dimension $M^1L^2T^{-3}$
- SI unit of power is J/s or watt
- 1 horsepower = 746 watt

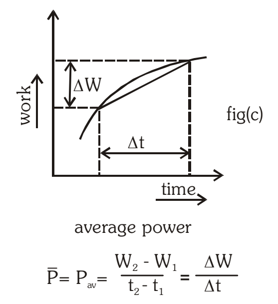


$W = \int Pdt$



instantaneous power

$P = \frac{dW}{dt} = \tan\theta$



average power

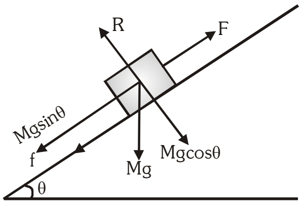
$\bar{P} = P_{av} = \frac{W_2 - W_1}{t_2 - t_1} = \frac{\Delta W}{\Delta t}$

- Area under power–time graph gives the work done. $W = \int Pdt$ (See Fig. a)
- The slope of tangent at a point on work time graph gives instantaneous power (See Fig. b)
- The slope of a straight line joining two points on work time graph gives average power between two points (See Fig. c)

- For a system of varying mass $F = \frac{d}{dt}(mv) = m\frac{dv}{dt} + v\frac{dm}{dt}$

- Ifv = constant then $F = v\frac{dm}{dt}$ then $P = \vec{F}.\vec{v} = v^2\frac{dm}{dt}$

Ex. A truck pulls a mass of 1200 kg at constant speed of 10m/s on a level road. The tension in coupling is 1000 N. What is the power spent on the mass. Find tension when truck moves up a road with inclination 1 in 6.



Sol. Force applied by truck f = 1000 N

Power spent in pulling the mass $P = fv = 1000 \times 10 = 10^4$ W

Here $\sin\theta = 1/6$, the required force for truck to move up is

$F = f + Mg\sin\theta$

$F = 1000\text{ N} + 1200 \times 9.8 \times \frac{1}{6} = 2960\text{ N}$

CONSERVATIVE FORCE

- A force is said to be conservative if the work done by or against the force is independent of path and depends only on initial and final positions
- It does not depend on the nature of path followed between the initial and final positions.

Examples of Conservative Force

All central forces are conservative like gravitational, electrostatic, elastic force, restoring force due to spring etc.

- In presence of conservative forces mechanical energy remains constant.
- Work done along a closed path or in a cyclic process is zero. i.e. $\oint \vec{F}.d\vec{r} = 0$

CENTRAL FORCE

The force whose line of action always passes through a fixed point (which is known as centre of force) and magnitude of force depends only on the distance from this point is known as **central force.**

$$\vec{F} = F(r)\hat{r}$$

All forces following inverse square law are called central forces.

$\vec{F} = \frac{k}{r^2}\hat{r}$ is central force like Gravitational force and Coulomb force.

- All central forces are conservative forces
- Central forces are function of position

NON CONSERVATIVE FORCE

A force is said to be non–conservative if work done by or against the force in moving a body depends upon the path between the initial and final positions.

Work done in a closed path is not zero in a non–conservative force field.

The frictional forces are non–conservative forces. This is because the work done against friction depends on the length of the path along which a body is moved. It does not depend on the initial and final positions. The work done by frictional force in a round trip is not zero.

Examples of non–conservative force

The velocity–dependent forces such as air resistance, viscous force etc. are non–conservative forces.

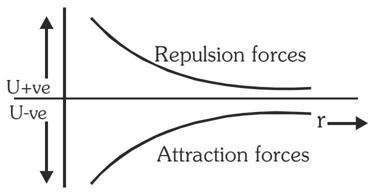
Conservative Forces	Non–conservative Forces
• Work done does not depend upon path.	• Work done depends upon path.
• Work done in a round trip is zero.	• Work done in a round trip is not zero.
• Central forces, spring forces etc. are conservative forces	• Force are velocity–dependent & retarding in nature e.g. friction, viscous force etc.
• When only a conservative force acts within a system, the kinetic energy and potential energy can change. However, their sum, the mechanical energy of the system, doesnot change.	• Work done against a non–conservative force may be dissipated as heat energy.
• Work done is completely recoverable.	• Work done is not completely recoverable.

Potential energy

- The energy which a body has by virtue of its position or configuration in a conservative force field.
- Potential energy is a relative quantity.
- Potential energy is defined only for conservative force field.
- Potential energy of a body at any position in a conservative force field is defined as the workdone by an external agent against the action of conservative force in order to shift it from reference point. (PE = 0) to the present position.
- Potential energy of a body in a conservative force field is equal to the work done by the body in moving from its present position to reference position.
- At reference position, the potential energy of the body is zero or the body has lost the capacity of doing work.
- Relationship between conservative force field and potential energy :

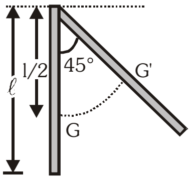
$$\vec{F} = -\nabla U = -\text{grad}(U) = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}$$

- If force varies only with one dimension (along x-axis) then $F = -\frac{dU}{dx} \Rightarrow U = -\int_{x_1}^{x_2} F dx$



- Potential energy may be positive or negative
  i) Potential energy is positive, if force field is repulsive in nature
  ii) Potential energy is negative, if force field is attractive in nature
- If r ↑ (separation between body and force centre), U ↑, force field is attractive or vice–versa.
- If r ↑, U ↓, force field is repulsive in nature.

Ex. A meter scale of mass m initially vertical is displaced at 45^0 keeping the upper end fixed. Find out the change in potential energy.



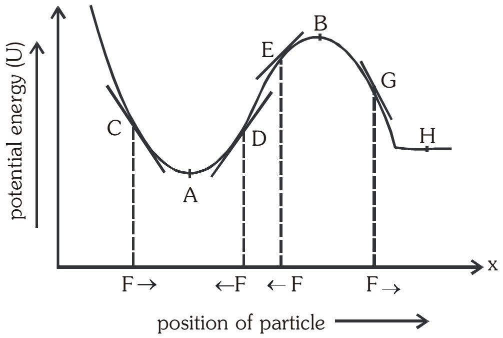
Sol. $\Delta U = mg\,\Delta h_{cm} = mg\frac{\ell}{2}(1-\cos\theta) = mg \times \frac{1}{2}\ (1-\cos 45^0) = \frac{mg}{2}\left(1-\frac{1}{\sqrt{2}}\right)$

Ex. A uniform rod of length 4m and mass 20kg is lying horizontal on the ground. Calculate the work done in keeping it vertical with one of its ends touching the ground.

Sol. As the rod is kept in vertical position the shift in the centre of gravity is equal to the half the length

$= \ell/2$ Work done $W = mgh = mg\,\frac{\ell}{2} = (20)(9.8)\left(\frac{4}{2}\right) = 392$ J

POTENTIAL ENERGY CURVE AND EQUILIBRIUM



It is a curve which shows change in potential energy

with postion of a particle.

Stable Equilibrium :

When a particle is slightly displaced from equilibrium position and it tends to come back towards equilibrium
then it is said to be in stable equilibrium

At point **C :** slope $\frac{dU}{dx}$ is negative so F is positive

At point **D :** slope $\frac{dU}{dx}$ is positive so F is negative

At point **A :** it is the point of stable equilibrium.

At **A** $U = U_{min}$, $\frac{dU}{dx} = 0$ and $\frac{d^2U}{dx^2}$ = positive

Unstable equilibrium :

When a particle is slightly displaced from equilbrium and it tends to move away from equilibrium position then it is said to be in unstable equilbirum

At point **E** : slope $\frac{dU}{dx}$ is positive so F is negative

At point **G** : slope $\frac{dU}{dx}$ is negative so F is postive

At point **B** : it is the point of unstable equilibrium.

At **B** $U = U_{max}$, $\frac{dU}{dx} = 0$ and $\frac{d^2U}{dx^2} =$ negative

Neutral equilibrium :

When a particle is slightly displaced from equilibrium position and no force acts on it then equilbirum is said to be neutral equilibrium

Point **H** is at neutral equilibrium $\Rightarrow U =$ constant ; $\frac{dU}{dx} = 0,\ \frac{d^2U}{dx^2} = 0$

Ex. The potential energy for a conservative force system is given by $U = ax^2 - bx$. Where a and b are constants find out

(a) The expression of force

(b) Equilibrium position

(c) Potential energy at equilibrium.

Sol. (a) For conservative force $F = -\frac{dU}{dx} = -(2ax - b) = -2ax + b$

(b) At equilibrium $F = 0 \Rightarrow -2ax + b = 0 \Rightarrow x = \frac{b}{2a}$ (c) $U = a\left(\frac{b}{2a}\right)^2 - b\left(\frac{b}{2a}\right) = \frac{b^2}{4a} - \frac{b^2}{2a} = -\frac{b^2}{4a}$

Law of conservation of Mechanical energy

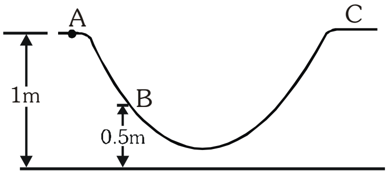
Total mechanical (kinetic + potential) energy of a system remains constant if only conservative forces are acting on the system of particles and the work done by all other forces is zero.

From work energy theorem $W = \Delta KE$

For internal conservative forces $W_{int} = -\Delta U$

So $W = W_{ext} + W_{int} = 0 + W_{int} = -\Delta U \Rightarrow -\Delta U = \Delta KE \Rightarrow (KE + U)) = 0 \Rightarrow KE + U = (\text{constant})$

Ex. A particle is placed at the point A of a frictionless track ABC. It is pushed slightly towards right. Find its speed when it reaches the point B. [Take $g=10\ m/s^2$]



Sol. $mg(1 - 0.5) = \frac{1}{2} \times m \times v^2 \Rightarrow v^2 = (2)(10)(0.5) \Rightarrow v = \sqrt{10}$ m/s

Ex. Determine the average force necessary to stop a bullet of mass 20 g and speed 250 ms^{-1} as it penetrates wood to a distance of 12 cm.

Sol. If F newton be the retarding force, then the work done by force $W = F \times s = F \times 0.12$ joule

Loss of kinetic energy $= \frac{1}{2} \times \frac{20}{1000} \times (250)^2 = 625$ joule

(This kinetic energy is consumed in stopping the bullet and is converted into heat energy)

Applying work–energy theorem, $F \times 0.12 = 625 \Rightarrow F = \frac{625}{0.12}\ N = 5.2 \times 10^3 N$

It is interesting to note that the retarding force is nearly 30,000 times the weight of the bullet.

Ex. A body of mass 8 kg moves under the influence of a force. The position of the body and time are related as $x = \frac{1}{2}t^2$ where x is in meter and t in sec. Find the work done by the force in first two seconds.

Sol. Work done = change in kinetic energy $= \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 = \frac{1}{2}m\left(\frac{2t}{2}\right)^2 = \frac{1}{2}\times 8\times\left[\frac{2\times 2}{2}\right]^2 = 16J$

Ex. A body falls on the surface of the earth from a height of 20 cm. If after colliding with the earth, its mechanical energy is lost by 75%, then determine height upto the body would reach.

Sol. $\frac{1}{4}mgh = mgh' \quad \therefore\ h' = \frac{h}{4} = \frac{1}{4}\times 20 = 5\text{ cm}$

Ex. Calculate the stopping distance for a vehicle of mass m moving with speed v along level road. (μ is the coefficient of friction between tyres and the road)

Sol. When the vehicle of mass m is moving with velocity v, the kinetic energy of the vehicle $K = \frac{1}{2}mv^2$

and if S is the stopping distance, work done by the friction

$W = FS\cos\theta = \mu mgS\cos 180^\circ = -\mu mgS$

So by Work–Energy theorem, $W = \Delta K = K_f - K_i \Rightarrow -\mu mgS = 0 - 1/2\ mv^2 \Rightarrow S = \frac{v^2}{2\mu g}$

Ex. A particle of mass m is moving in a horizontal circle of radius r, under a centripetal force equal to $(-k/r^2)$, where k is constant. Calculate the total energy of the particle.

Sol. As the particle is moving in a circle, so $\frac{mv^2}{r} = \frac{k}{r^2}$. Now $K.E = \frac{1}{2}mv^2 = \frac{k}{2r}$

As $U = -\int_\infty^r F.dr = \int_\infty^r +\left(\frac{k}{r^2}\right)dr = -\frac{k}{r}$. So total energy = U + K.E. $= -\frac{k}{r} + \frac{k}{2r} = -\frac{k}{2r}$

Negative energy means that particle is in bound state.

Ex. A man throws the bricks to the height of 12 m where they reach with a speed of 12 m/sec. If he throws the bricks such that they just reach this height, what percentage of energy will he save?

Sol. In first case, $W_1 = \frac{1}{2}m(v_1)^2 + mgh = \frac{1}{2}m(12)^2 + m\times 10\times 12 = 72\text{ m} + 120\text{ m} = 192m$

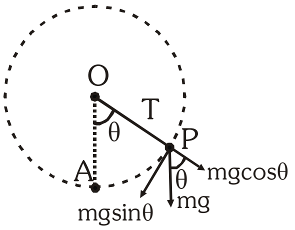
and in second case, $W_2 = mgh = 120\text{ m}$

The percentage of energy saved $= \frac{192m - 120m}{192m}\times 100 = 38\%$

CIRCULAR MOTION IN VERTICAL PLANE

Suppose a particle of mass m is attached to an inextensible light string of length R. The particle is moving in a vertical circle of radius R about a fixed point O. It is imparted a velocity u in horizontal direction at lowest point A. Let v be its velocity at point B of the circle as shown in figure. Here, $h = R(1 - \cos\theta)$ ...(i)

From conservation of mechanical energy



$$\frac{1}{2}m(u^2 - v^2) = mgh \Rightarrow v^2 = u^2 - 2gh \qquad ....(ii)$$

The necessary centripetal force is provided by the resultant of tension

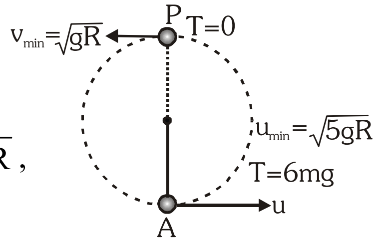
T and mg cosθ $T - mg\cos\theta = \frac{mv^2}{R}$ ...(iii)

Since speed of the particle decreases with height, hence tension is maximum at the bottom, where $\cos\theta = 1$ (as $\theta = 0^0$)

$\Rightarrow T_{max} = \frac{mv^2}{R} + mg$; $T_{min} = \frac{mv^2}{R} - mg$ at the top. Here, v' = speed of the particle at the top.

Condition of Looping the Loop $\left(u \geq \sqrt{5gR}\right)$

The particle will complete the circle if the string does not slack even at the highest point $(\theta = \pi)$. Thus, tension in the string should be greater than or equal to zero ($T \geq 0$) at $\theta = \pi$. In critical case substituting T = 0 and $\theta = \pi$ in Eq. (iii), we get $mg = \frac{mv_{min}^2}{R} \Rightarrow v_{min} = \sqrt{gR}$ (at highest point)



Substituting $\theta = \pi$ in Eq. (i), Therefore, from Eq. (ii)

$$u_{min}^2 = v_{min}^2 + 2gh = gR + 2g(2R) = 5gR \Rightarrow u_{min} = \sqrt{5gR}$$

Thus, if $u \geq \sqrt{5gR}$, the particle will complete the circle. At $u = \sqrt{5gR}$, velocity at highest point is $v = \sqrt{gR}$ and tension in the string is zero.

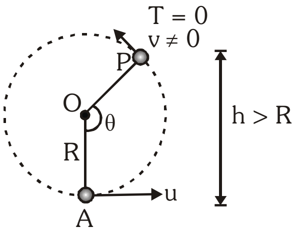
Substituting $\theta = 0^0$ and $v = \sqrt{5gR}$ in Eq. (iii), we get T = 6 mg or in the critical condition tension in the string at lowest position is 6 mg. This is shown in figure. If $u < \sqrt{5gR}$, following two cases are possible.

Condition of Leaving the Circle $\left(\sqrt{2gR} < u < \sqrt{5gR}\right)$

If $u < \sqrt{5gR}$, the tension in the string will become zero before reaching the highest point. From Eq. (iii), tension in the string becomes zero (T = 0) where, $\cos\theta = \frac{-v^2}{Rg} \Rightarrow \cos\theta = \frac{2gh - u^2}{Rg}$

Substituting, this value of $\cos\theta$ in Eq. (i), we get $\frac{2gh - u^2}{Rg} = 1 - \frac{h}{R} \Rightarrow h = \frac{u^2 + Rg}{3g} = h_1$ (say) or we can say that at height h_1 tension in the string becomes zero. Further, if $u < \sqrt{5gR}$, velocity of the particle becomes zero when $0 = u^2 - 2gh \Rightarrow h = \frac{u^2}{2g} = h_2$(say)...(v) i.e., at height h_2 velocity of particle becomes zero.

Now, the particle will leave the circle if tension in the string becomes zero but velocity is not zero. or $T = 0$ but $v \neq 0$. This is possible only when $h_1 < h_2$



$$\Rightarrow \frac{u^2 + Rg}{3g} < \frac{u^2}{2g} \Rightarrow 2u^2 + 2Rg < 3u^2 \Rightarrow u^2 > 2Rg \Rightarrow u > \sqrt{2Rg}$$

Therefore, if $\sqrt{2gR} < u < \sqrt{5gR}$, the particle leaves the circle.

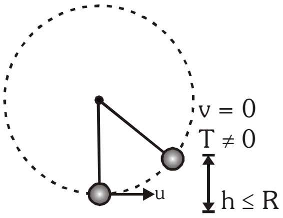
From Eq. (iv), we can see that $h > R$ if $u^2 > 2gR$. Thus, the particle, will leave the circle when $h > R$ or $90^0 < \theta < 180^0$. This situation is shown in the figure

$$\sqrt{2gR} < u < \sqrt{5gR} \text{ or } 90^0 < \theta < 180^0$$

Note : That after leaving the circle, the particle will follow a parabolic path.

Condition of Oscillation $\left(0 < u \le \sqrt{2gR}\right)$

The particle will oscillate if velocity of the particle becomes zero but tension in the string is not zero or $v = 0$, but $T \neq 0$. This is possible when $h_2 < h_1$



$$\Rightarrow \frac{u^2}{2g} < \frac{u^2 + Rg}{3g} \Rightarrow 3u^2 < 2u^2 + 2Rg \Rightarrow u^2 < 2Rg \Rightarrow u < \sqrt{2Rg}$$

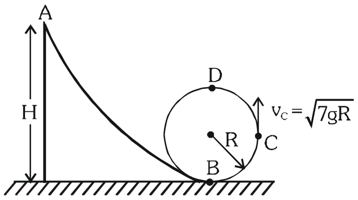
Moreover, if $h_1 = h_2$, $u = \sqrt{2Rg}$ and tension and velocity both becomes zero simultaneously. Further, from Eq. (iv), we can see that $h \le R$ if $u \le \sqrt{2Rg}$.

Thus, for $0 < u \le \sqrt{2gR}$, particle oscillates in lower half of the circle ($0^0 < \theta \le 90°$)

This situation is shown in the figure. $0 < u \le \sqrt{2gR}$ or $0° < \theta \le 90°$

Ex. Calculate following for shown situation :–

(a) Speed at D (b) Normal reaction at D (c) Height H



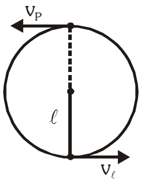
Sol. (a) $v_D^2 = v_C^2 - 2gR = 5gR \Rightarrow v_D = \sqrt{5gR}$

(b) $mg + N_D = \frac{mv_D^2}{R} \Rightarrow N_D = \frac{m(5gR)}{R} - mg = 4mg$

(c) by energy conservation between point A & C

$$mgH = \frac{1}{2}mv_C^2 + mgR = \frac{1}{2}m(5gR) + mg2R = \frac{9}{2}mgR \Rightarrow H = \frac{9}{2}R$$

Ex. A stone of mass 1 kg tied to a light string of length $\ell = \frac{10}{3}$ m is whirling in a circular path in vertical plane. If the ratio of the maximum to minimum tension in the string is 4, find the speed of the stone at the lowest and highest points

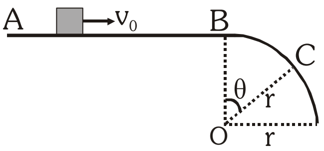


Sol. $$\because \frac{T_{max}}{T_{min}} = 4 \therefore \frac{\frac{mv_\ell^2}{\ell}+mg}{\frac{mv_p^2}{\ell}-mg} = 4 \Rightarrow \frac{v_\ell^2+g\ell}{v_p^2-g\ell} = 4$$

We know $v_\ell^2 = v_p^2 + 4g\ell \Rightarrow \frac{v_P^2+5g\ell}{v_P^2-g\ell} = 4 \Rightarrow 3{v_P}^2 = 9g\ell$

$$\Rightarrow {v_P}^2 = \sqrt{3g\ell} = \sqrt{3\times10\times\frac{10}{3}} = 10\ \text{ms}^{-1} \Rightarrow v_\ell = \sqrt{7g\ell} = \sqrt{7\times10\times\frac{10}{3}} = 15.2\ \text{ms}^{-1}$$

Ex. A small block slides with velocity $0.5\sqrt{gr}$ on the horizontal frictionless surface as shown in the figure. The block leaves the surface at point C. Calculate angle θ in the figure.

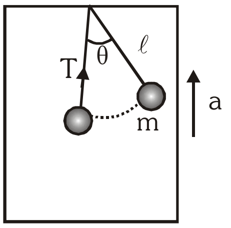


Sol. As block leaves the surface at C so at C, normal reaction = 0 $\Rightarrow mg\cos\theta = \frac{mv_C^2}{r}$

By energy conservation at point B & C $\frac{1}{2}m{v_C}^2 - \frac{1}{2}m{v_0}^2 = mgr(1-\cos\theta)$

$$\Rightarrow \frac{1}{2}m(rg\cos\theta) - \frac{1}{2}m\left(0.5\sqrt{gr}\right)^2 = mgr\left(1-\cos\theta\right) \Rightarrow \cos\theta = \frac{3}{4} \Rightarrow \theta = \cos^{-1}\left(\frac{3}{4}\right)$$

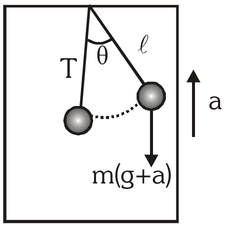
Ex. A particle of mass m is attached to the ceiling of a cabin with an inextensible light string of length ℓ. The cabin is moving upward with an acceleration 'a'. The particle is taken to a position such that the string makes an angle θ with vertical. When string becomes vertical, find the tension in the string.



Sol. In a frame associated with cabin work done on the particle when it comes in the vertical position = $mg\ell(1-\cos\theta)+ma(1-\cos\theta)$

By work energy theorem,

$$\frac{mv^2}{2} = \left(mg\ell + ma\ell\right)\left(1-\cos\theta\right) \Rightarrow \frac{v^2}{2} = (g+a)\,l\,(1-\cos\theta)$$



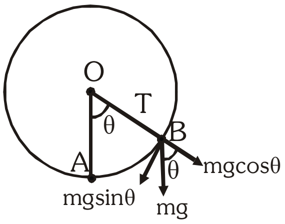
At vertical position, $T = (mg + ma) = \frac{mv^2}{\ell}$

$\Rightarrow T = (mg + ma) + 2m(g+a)(1-\cos\theta) = mg\,(g+a)(3-2\cos\theta)$

Ex. A heavy particle hanging from a fixed point by a light inextensible string of length ℓ, is projected horizontally with speed $\sqrt{(g\ell)}$. Find the speed of the particle and the inclination of the string to the vertical at the instant of the motion when the tension in the string is equal to the weight of the particle.

Sol. Let tension in the string becomes equal to the weight of the particle when particle reaches the point B and deflection of the string from vertical is θ. Resolving mg along the string and perpendicular to the string, we get net radial force on the particle at B i.e.

$$F_R = T - mg\cos\theta \qquad ....(i)$$



If v_B be the speed of the particle at B, then

$$F_R = \frac{mv_B^2}{\ell} \qquad ....(ii)$$

From (i) and (ii), we get, $T - mg\cos\theta = \frac{mv_B^2}{\ell}$ ....(iii)

Since at B, T = mg $\Rightarrow mg\left(1-\cos\theta\right) = \frac{mv_B^2}{\ell} \Rightarrow v_B^2 = g\ell\left(1-\cos\theta\right)$ ...(iv)

Applying conservation of mechanical energy of the particle at point A and B, we have

$$\frac{1}{2}mv_A^{\,2} = mg\ell\,(1-\cos\theta) + \frac{1}{2}mv_B^2;\ \text{where } v_A = \sqrt{g\ell} \text{ and } v_B = \sqrt{g\ell\left(1-\cos\theta\right)}$$

$$\Rightarrow g\ell = 2g\ell\left(1-\cos\theta\right) + g\ell\,(1-\cos\theta) \Rightarrow \cos\theta = \frac{2}{3} \Rightarrow \theta = \cos^{-1}\left(\frac{2}{3}\right)$$

Putting the value of $\cos\theta$ in equation (iv), we get : $v = \sqrt{\frac{g\ell}{3}}$

EXERCISE (S-1)

Work

1. The sign of work done by a force on a body is important to understand. State carefully if the following quantities are positive or negative: **(NCERT)**

(a) work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket.

(b) work done by gravitational force in the above case,

(c) work done by friction on a body sliding down an inclined plane,

(d) work done by an applied force on a body moving on a rough horizontal plane with uniform velocity,

(e) work done by the resistive force of air on a vibrating pendulum in bringing it to rest.

WE0001

2. A body constrained to move along the z-axis of a coordinate system is subject to a constant force F given by **(NCERT)**

$$F = -\hat{i} + 2\hat{j} + 3\hat{k}N$$

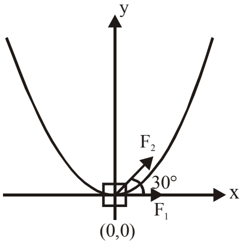
where $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along the x-, y- and z-axis of the system respectively. What is the work done by this force in moving the body a distance of 4 m along the z-axis ?

WE0002

3. A point mass of 0.5 kg is moving along x-axis as $x = t^2 + 2t$, where, x is in meters and t is in seconds. Find the work done (in J) by all the forces acting on the body during the time interval [0, 2s].

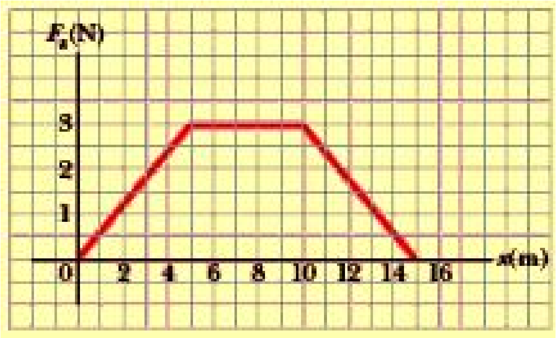
WE0003

4. A sleeve of mass 2 kg at origin can move on wire of parabolic shape $x^2 = 4y$. Two forces $F_1 = 6N$ and $F_2 = 8N$ are applied on the sleeve. F_1 is constant and is in x-direction. F_2 is constant in direction and magnitude. Body is displaced from origin to x = 4, then net work done by F_1 and F_2 is



WE0004

5. A particle is subject to a force F_x that varies with position as in figure. Find the work done by the force on the body as it moves (a) from x = 0 to x = 5.00 m, (b) from x = 5.00 m to x = 10.0 m, and (c) from x = 10.0 m to x = 15.0 m. (d) What is the total work done by the force over the distance x = 0 to x = 15.0 m?



WE0005

6. A spring, which is initially in its unstretched condition, is first stretched by a length x and then again by a further length x. The work done in the first case is W_1 and in the second case is W_2. Find $\frac{W_2}{W_1}$.

WE0006

Kinetic energy, Work energy theorem, Power

7. A body of mass 2 kg initially at rest moves under the action of an applied horizontal force of 7 N on a table with coefficient of kinetic friction = 0.1. Compute the **(NCERT)**

(a) work done by the applied force in 10 s,

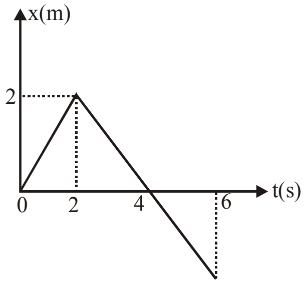
(b) work done by friction in 10 s,

(c) work done by the net force on the body in 10 s,

(d) change in kinetic energy of the body in 10 s, and interpret your results.

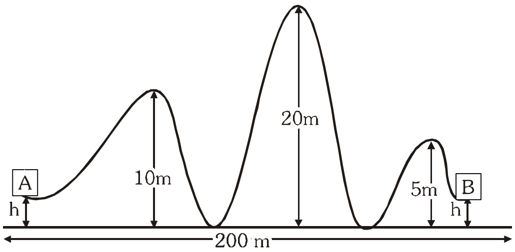
WE0007

8. Position-time graph of a particle of mass 2 kg is shown in figure. Total work done on the particle from t = 0 to t = 4s is



WE0008

9. A point object of mass 2 kg is moved from point A to point B very slowly on a curved path by applying a tangential force on a curved path as shown in figure. Then find the work done by external force in moving the body. Given that $\mu_s = 0.3$, $\mu_k = 0.1$. [$g = 10\ m/s^2$]



WE0009

10. A 4 kg particle moves along the X-axis. It's position x varies with time according to $x(t) = t + 2t^3$, where x is in m and t is in seconds. Compute:

(i) The kinetic energy at time t.

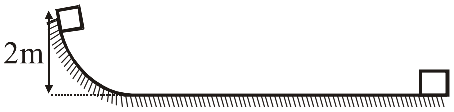
(ii) The force acting on the particle at time t.

(iii) The power delivered to the particle at time t.

(iv) The work done on the particle from t = 0 to t = 2 seconds.

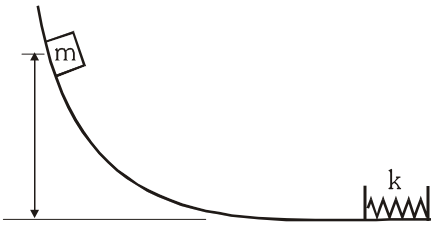
WE0010

11. A block is released from rest from top of a rough curved track as shown in figure. It comes to rest at some point on the horizontal part. If its mass is 200 gm, calculate negative of work by friction in joules.



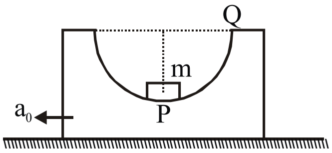
WE0011

12. A mass m slides from rest at height h down a smooth curved surface which becomes horizontal at zero height (see figure). A spring is fixed horizontally on the level part of the surface. The spring constant is k N/m. When the mass encounters the spring it compresses it by an amount x = h/10. If m = 1 kg, h = 5m then find k /100.



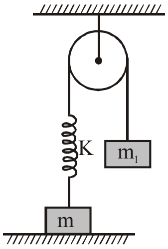
WE0012

13. A small block of mass m is lying at rest at point P of a wedge having a smooth semi circular track of radius R. The minimum value of horizontal acceleration a_0 (in m/s^2) of wedge so that mass can just reach the point Q, is



WE0013

14. Initially spring is relaxed when m_1 is released from rest. Calculate for what minimum value of m_1 the block of mass m will just leave the contact with surface ? Give answer in terms of $\frac{m}{m_1}$.

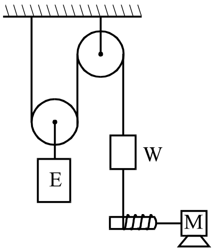


WE0014

15. The elevator E has a mass of 3000 kg when fully loaded and is connected as shown to a counterweight W of mass 1000 kg. Determine the power in kilowatts delivered by the motor

(a) when the elevator is moving down at a constant speed of 3 m/s,

(b) when it has an upward velocity of 3 m/s and a deceleration of 0.5 m/s^2.



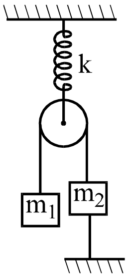
WE0015

16. Power applied to a particle varies with time as $P = (3t^2 - 2t + 1)$ watt, where t is in second. Find the change in its kinetic energy between time t = 2 s and t = 4 s.

WE0016

Conservative & non conservation forces, Potential energy, Conservation of energy

17. In the figure shown, pulley and spring are ideal. Find the potential energy stored in the spring $(m_1 > m_2)$.



WE0017

18. The potential energy (in joules) function of a particle in a region of space is given as :

$$U = (2x^2 + 3y^3 + 2z)$$

Here x, y and z are in metres. Find the magnitude of x component of force (in newton) acting on the particle at point P (1m, 2m, 3m).

WE0018

19. The P.E. of a particle oscillating on x-axis is given as $U = 20 + (x - 2)^2$ here U is in Joules & x is in meters. Total mechanical energy of particle is 36 J

(i) Find the mean position (ii) Find the max. K.E. of the particle

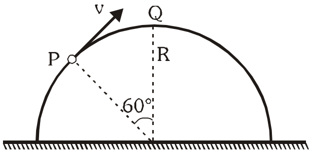
WE0019

20. The potential energy of a 2 kg particle moving along the x axis is given by $U(x) = (4.0\,J/m^2)x^2 + (1.0\,J/m^4)x^4$. When the particle is at x = 1.0m, find its acceleration. [only conservative forces are acting]

WE0020

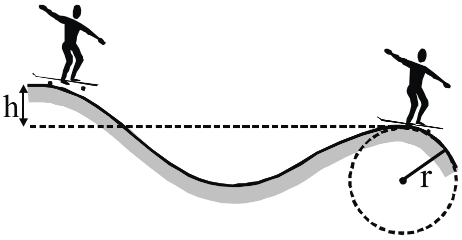
Potential energy diagram, Equilibrium, Vertical circular motion

21. A particle is given a certain veloicty v at point P as shown on a hemispherical smooth surface. Find the value of v (in m/s), such that when particle reaches Q, the normal reaction of surface becomes equal to particle's weight. [R = 1.6 m, g = 10 m/s²]



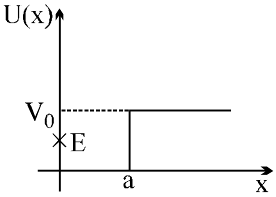
WE0021

22. A skier starts from rest at the top of a hill. The skier coasts down the hill and up a second hill, as the drawing illustrates. The crest of the second hill is circular, with a radius of r = 36 m. Neglect friction and air resistance. What must be the height h (in m) of the first hill so that the skier just loses contact with the snow at the crest of the second hill?



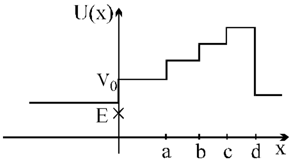
WE0022

23. The given graph is a potential energy function in one dimension. The total energy of particle is indicated by cross on the ordinate axis. The graph of figure-1 is given as an example.From the figure-1, it can be interpreted that for the given total energy indicated by cross on the ordinate axis the particle cannot be found in the Region : x > a. Now, for the following potential functions in one dimensions, specify the regions, in which the particle cannot be found for the energy marked as E on graphs. Give your answer in the blocks shown.

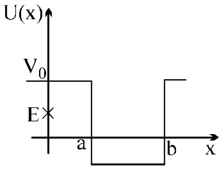


(Figure -1)

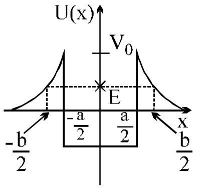
(Graph-1)



(Graph-2)



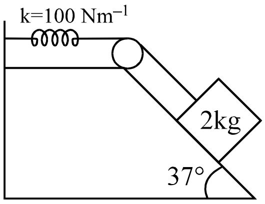
(Graph-3)



WE0023

24. (a) A 2 kg block situated on a smooth fixed incline is connected to a spring of negligible mass, with spring constant k = 100 Nm^{-1}, via a frictionless pulley. The block is released from rest when the spring is unstretched. How far does the block move down the incline before coming (momentarily) to rest? What is its acceleration at its lowest point?

(b) The experiment is repeated on a rough incline. If the block is observed to move 0.20 m down along the incline before it comes to instantaneous rest, calculate the coefficient of kinetic friciton.

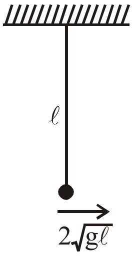


WE0024

25. A ball is attached to a horizontal cord of length L whose other end is fixed, (a) If the ball is released, what will be its speed at the lowest point of its path ? (b) A peg is located a distance h directly below the point of attachment of the cord. If h = 0.75 L, what will be the speed of the ball when it reaches the top of its circular path about the peg ?

WE0025

26. One end of a string of length $\ell = \frac{14}{9}$ m is fixed and a mass of 1 kg is tied to the other end. The ball is given a velocity $2\sqrt{g\ell}$ at the bottom most point as shown in figure. The string is cut when the ball becomes horizontal. Find the distance (in m) travelled till it stop for the 1st time (Take $\pi = \frac{22}{7}$).



WE0026

EXERCISE (S-2)

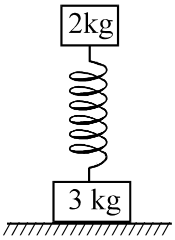
1. A ring of mass m can slide over a smooth vertical rod. The ring is connected to a spring of force constant $K = \frac{4mg}{R}$ where 2R is the natural length of the spring. The other end of the spring is fixed to the ground at a horizontal distance 2R from the base of the rod. The mass is released at a height of 1.5R from ground

(i) calculate the work done by the spring.

(ii) calculate the velocity of the ring as it reaches the ground.

WE0027

2. The ends of spring are attached to blocks of mass 3kg and 2kg. The 3kg block rests on a horizontal surface and the 2kg block which is vertically above it is in equilibrium producing a compression of 1cm of the spring. The 2kg mass must be compressed further by at least _______, so that when it is released, the 3 kg block may be lifted off the ground.



WE0028

3. A uniform rod of mass *m* length *L* is sliding along its length on a horizontal table whose top is partly smooth & rest rough with friction coefficient μ. If the rod after moving through smooth part, enters the rough with velocity v_0.

(i) What will be the magnitude of the friction force when its x length (< L) lies in the rough part during sliding.

(ii) Determine the minimum velocity v_0 with which it must enter so that it lies completely in rough region before coming to rest.

(iii) If the velocity is double the minimum velocity as calculated in part (a) then what distance does its front end A would have travelled in rough region before rod comes to rest.

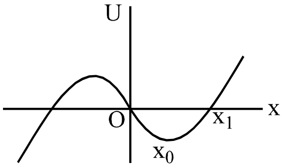
WE0029

4. A particle is confined to move along the +x axis under the action of a force F(x) that is derivable from the potential $U(x) = ax^3 - bx$.

(i) Find the expression for F(x)

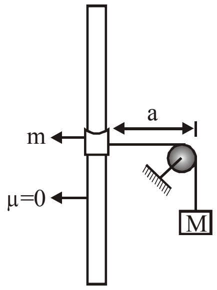
(ii) When the total energy of the particle is zero, the particle can be trapped with in the interval x = 0 to $x = x_1$. For this case find the values of x_1.

(iii) Determine the maximum kinetic energy that the trapped particle has in its motion. Express all answers in terms a and b. At what value of x will the kinetic energy be maximum ?



WE0030

5. A ring of mass m slides on a smooth vertical rod. A light string is attached to the ring and is passing over a smooth peg distant a from the rod, and at the other end of the string is a mass M (M>m). The ring is held on a level with the peg and released. Show that it first comes to rest after falling a distance $\frac{2mMa}{M^2 - m^2}$.



WE0031

6. A 650-kg elevator starts from rest. It moves upward for 3.00 s with constant acceleration until it reaches its cruising speed of 1.75 m/s.

(i) What is the average power of the elevator motor during this period?

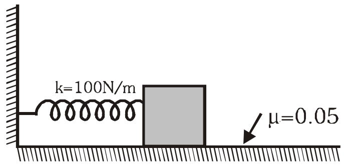
(ii) How does this power compare with its power when it moves at its cruising speed?

WE0032

7. A particle is moving with kinetic energy E, straight up an inclined plane with angle α, the coefficient of friction being μ. The work done against friction before the particle comes down to rest is

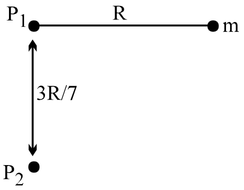
WE0033

8. A block of mass 1 kg is attached to a spring with a force constant 100 N/m and rests on a rough horizontal ground as shown in the figure. Initial displacement of block from natural length is 50 cm. The total distance covered by the block if coefficient of friction between block & ground is 0.05. [$g = 10 m/s^2$]



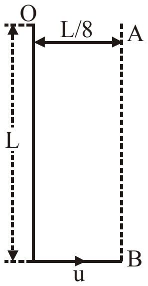
WE0034

9. A simple pendulum consists of a bob of mass m and a string of length R suspended from a peg P_1 on the wall. A second peg P_2 is fixed vertically below the first one at a distance $3R/7$ from it. The pendulum is drawn aside such that the string is horizontal and released. Calculate the maximum height (with respect to the lowest point) to which it rises



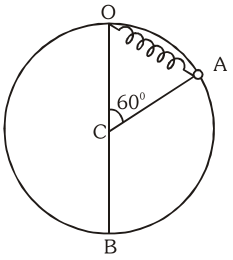
WE0035

10. A particle is suspended vertically from a point O by an inextensible massless string of length L. A vertical line AB is at a distance $\frac{L}{8}$ from O as shown in figure. The object is given a horizontal velocity u. At some point, its motion ceases to be circular and eventually the object passes through the line AB. At the instant of crossing AB, its velocity is horizontal. Find u.



WE0036

11. A particle of mass 5 kg is free to slide on a smooth ring of radius r = 20 cm fixed in a vertical plane. The particle is attached to one end of a spring whose other end is fixed to the top point O of the ring. Initially the particle is at rest at a point A of the ring such that $\angle$ OCA = 60^0, C being the centre of the ring. The natural length of the spring is also equal to r = 20 cm. After the particle is released and slides down the ring the contact force between the particle & the ring becomes zero when it reaches the lowest position B. Determine the force constant of the spring.



WE0037

EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

Work

1. A force of magnitude of 30 N acting along $\hat{i}+\hat{j}+\hat{k}$, displaces a particle from point (2, 4, 1) to (3, 5, 2). The work done during this displacement is :-

(A) 90 J (B) 30 J (C) $30\sqrt{3}$ J (D) $30/\sqrt{3}$ J.

WE0038

2. A spring of force constant 800 N/m has an extension of 5 cm. The work done in extending it from 5cm to 15 cm is- **[AIEEE - 2002]**

(A) 16 J (B) 8 J (C) 32 J (D) 24 J

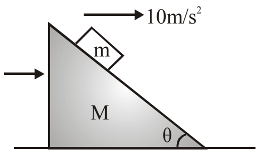
WE0039

3. A rope is used to lower vertically a block of mass M by a distance x with a constant downward acceleration g/2. The work done by the rope on the block is :

(A) Mgx (B) ½ Mgx^2 (C) – ½ Mgx (D) Mgx^2

WE0040

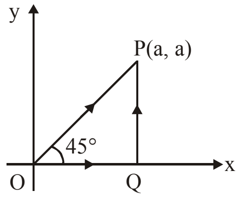
4. In the figure shown all the surfaces are frictionless, and mass of the block, m = 1 kg. The block and wedge are held initially at rest. Now wedge is given a horizontal acceleration of 10 m/s^2 by applying a force on the wedge, so that the block does not slip on the wedge. Then work done by the normal force in ground frame on the block in $\sqrt{3}$ seconds is :



(A) 30J (B) 60 J (C) 150 J (D) $100\sqrt{3}$ J

WE0041

5. A particle is moved from (0, 0) to (a, a) under a force $\vec{F}=(3\hat{i}+4\hat{j})$ from two paths. Path 1 is OP and path 2 is OQP. Let W_1 and W_2 be the work done by this force in these two paths. Then :

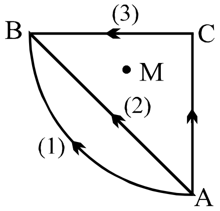


(A) $W_1 = W_2$ (B) $W_1 = 2W_2$ (C) $W_2 = 2W_1$ (D) $W_2 = 4W_1$

WE0042

6. In a region of only gravitational field of mass 'M' a particle is shifted from A to B via three different paths in the figure. The work done in different paths are W_1, W_2, W_3 respectively then

[IIT-JEE (Scr.)'2003]



(A) $W_1 = W_2 = W_3$ (B) $W_1 = W_2 > W_3$ (C) $W_1 > W_2 > W_3$ (D) $W_1 < W_2 < W_3$

WE0043

Kinetic energy, Work energy theorem, Power

7. A body of mass 0.5 kg travels in a straight line with velocity $v = ax^{3/2}$ where $a = 5\ \text{m}^{-1/2}\text{s}^{-1}$. The work done by the net force during its displacement from $x = 0$ to $x = 2$ m is

(A) 1.5 J (B) 50 J (C) 10 J (D) 100 J

WE0044

8. A man who is running has half the kinetic energy of the boy of half his mass. The man speeds up by 1 m/s and then has the same kinetic energy as the boy. The original speed of the man was

(A) $\sqrt{2}$ m/s (B) $(\sqrt{2} - 1)$ m/s (C) 2 m/s (D) $(\sqrt{2} + 1)$ m/s

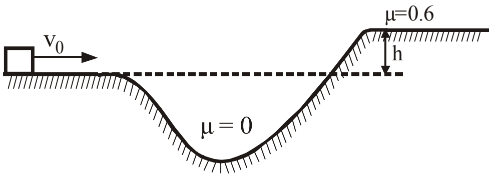
WE0045

9. A mass of 5 kg is moving along a circular path of radius 1 m. If the mass moves with 300 revolutions per minute, its kinetic energy would be

(A) $250\ \pi^2$ joule (B) $100\pi^2$ joule (C) $5\pi^2$ joule (D) 0 joule

WE0046

10. In the figure, a block slides along a track from one level to a higher level, by moving through an intermediate valley. The track is frictionless untill the block reaches the higher level. There a frictional force stops the block in a distance d. The block's initial speed v_0 is 6 m/s, the height difference h is 1.1m and the coefficient of kinetic friction μ is 0.6. The value of d is



(A) 1.17 m (B) 1.71 m (C) 7.11 m (D) 11.7 m

WE0047

11. In a shotput event an athlete throws the shotput of mass 10 kg with an initial speed of 1m/s at 45° from a height 1.5 m above ground. Assuming air resistance to be negligible and acceleration due to gravity to be 10 m/s^2, the kinetic energy of the shotput when it just reaches the ground will be

(A) 2.5 J (B) 5.0 J (C) 52.5 J (D) 155.0 J

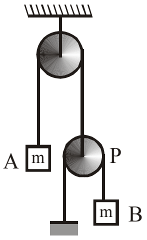
WE0048

12. A particle moves on a rough horizontal ground with some initial velocity say v_0. If 3/4 of its kinetic energy is lost in friction in time t_0 then coefficient of friction between the particle and the ground is :-

(A) $\frac{v_0}{2gt_0}$ (B) $\frac{v_0}{4gt_0}$ (C) $\frac{3v_0}{4gt_0}$ (D) $\frac{v_0}{gt_0}$

WE0049

13. In the figure shown, the system is released from rest. Find the velocity of block A when block B has fallen a distance 'ℓ'. Assume all pulleys to be massless and frictionless.



(A) $\sqrt{\frac{g\ell}{5}}$ (B) $\sqrt{g\ell}$ (C) $\sqrt{5g\ell}$ (D) None of these

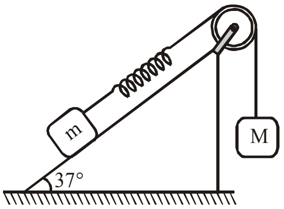
WE0050

14. A block of mass m is hung vertically from an elastic thread of force constant mg/a. Initially the thread was at its natural length and the block is allowed to fall freely. The kinetic energy of the block when it passes through the equilibrium position will be :

(A) mga (B) mga/2 (C) zero (D) 2mga

WE0051

15. A block of mass m is attached with a massless spring of force constant k. The block is placed over a rough inclined surface for which the coefficient of friction is $\mu = \frac{3}{4}$. The minimum value of M required to move the block up the plane is : (Neglect mass of string and pulley and friction in pulley)



(A) $\frac{3}{5}$ m (B) $\frac{4}{5}$ m (C) 2 m (D) $\frac{3}{2}$ m

WE0052

16. A body is moved along a straight line by a machine delivering a constant power. The distance moved by the body in time t is proportional to- **[AIEEE - 2003]**

(A) $t^{3/4}$ (B) $t^{3/2}$ (C) $t^{1/4}$ (D) $t^{1/2}$

WE0053

17. A particle moves in a straight line with retardation proportional to its displacement. Its loss of kinetic energy for any displacement x is proportional to- **[AIEEE - 2004]**

(A) x^2 (B) e^x (C) x (D) $\log_e x$

WE0054

18. A body of mass m accelerates uniformly from rest to v_1 in time t_1. The instantaneous power delivered to the body as a function of time t is- **[AIEEE - 2004]**

(A) $\frac{mv_1t}{t_1}$ (B) $\frac{mv_1^2t}{t_1^2}$ (C) $\frac{mv_1t^2}{t_1}$ (D) $\frac{mv_1^2t}{t_1}$

WE0055

19. Assume the aerodynamic drag force on a car is proportional to its speed. If the power output from the engine is doubled, then the maximum speed of the car.

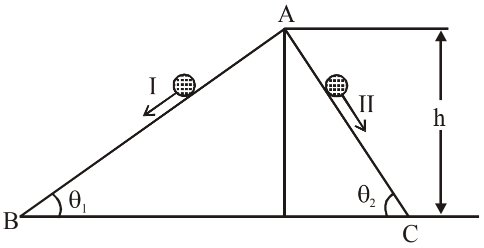
(A) is unchanged (B) increases by a factor of $\sqrt{2}$

(C) is also doubled (D) increases by a factor of four.

WE0056

Conservative & non conservative forces, Potential energy, Conservation of energy

20. Two inclined frictionless tracks, one gradual and the other steep meet at A from where two stones are allowed to slide down from rest, one on each track as shown in figure. Which of the following statement is correct?



(A) Both the stones reach the bottom at the same time but not with the same speed.

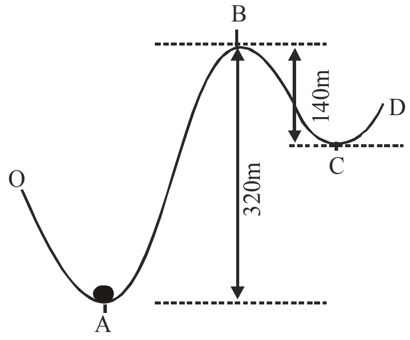
(B) Both the stones reach the bottom with the same speed and stone I reaches the bottom earlier than stone II.

(C) Both the stones reach the bottom with the same speed and stone II reaches the bottom earlier than stone I.

(D) Both the stones reach the bottom at different times and with different speeds.

WE0057

21. Track OABCD (as shown is figure) is smooth and fixed in vertical plane. What minimum speed has to be given to a particle lying at point A, so that it can reach point C?

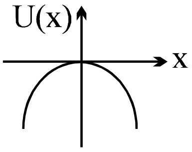


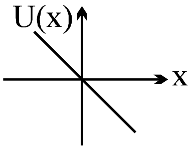
(A) 60 m/s (B) 100 m/s (C) 70 m/s (D) 80 m/s

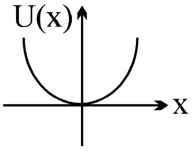
WE0058

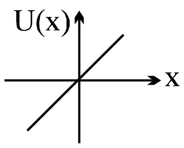
22. A particle is placed at the origin and a force F = kx is acting on it (where k is a positive constant). If U(0) = 0, the graph of U(x) versus x will be (where U is the potential energy function)

[IIT-JEE' 2004(Scr)]

(A)


(B)


(C)


(D)


WE0059

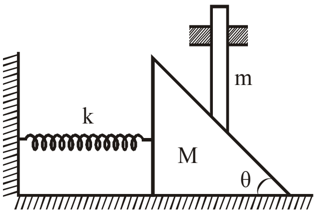
23. When a conservative force does positive work on a body

(A) the potential energy increases (B) the potential energy decreases

(C) total energy increases (D) total energy decreases

WE0060

24. A wedge of mass M fitted with a spring of stiffness 'k' is kept on a smooth horizontal surface. A rod of mass m is kept on the wedge as shown in the figure. System is in equilibrium. Assuming that all surfaces are smooth, the potential energy stored in the spring is:

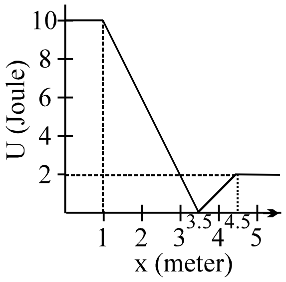


(A) $\dfrac{mg^2\tan^2\theta}{2k}$ (B) $\dfrac{m^2g\tan^2\theta}{2k}$ (C) $\dfrac{m^2g^2\tan^2\theta}{2k}$ (D) $\dfrac{m^2g^2\tan^2\theta}{k}$

WE0061

Potential energy diagram, Equilibrium, Vertical circular motion

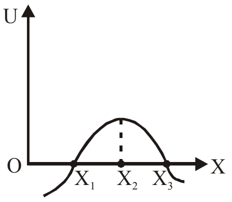
25. A body with mass 2 kg moves in one direction in the presence of a force which is described by the potential energy graph. If the body is released from rest at x = 2m, then its speed when it crosses x = 5 m is



(A) zero (B) 1 ms^{-1} (C) 2 ms^{-1} (D) 3 ms^{-1}

WE0062

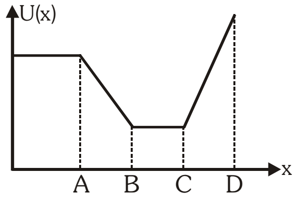
26. In the figure shown the potential energy (U) of a particle is plotted against its position 'x' from origin. Then which of the following statement is correct. A particle at :



(A) x_1 is in stable equilibrium
(B) x_2 is in stable equilibrium
(C) x_3 is in stable equilibrium
(D) None of these

WE0063

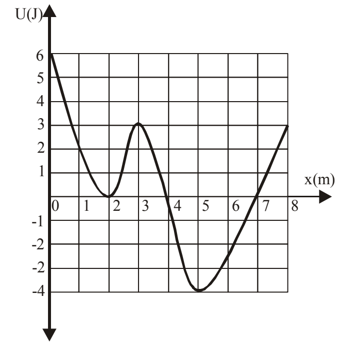
27. As a particle moves along the x–axis it is acted upon by a conservative force. The potential energy is shown below as a function of the coordinate x of the particle. Rank the labelled regions according to the magnitude of the force, least to greatest.



(A) AB, BC, CD (B) AB, CD, BC (C) BC, CD, AB (D) BC, AB, CD

WE0064

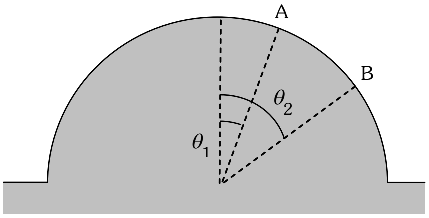
28. Potential energy curve U of a particle as function of the position of a particle is shown. The particle has total mechanical energy E of 3.0 joules.



(A) It can never be present at x= 0 m.
(B) It can never be present at x=5 m
(C) At x=2 its kinetic energy is 0 J
(D) At x =1 its kinetic energy 3 J

WE0065

29. A small block slides down from rest at point A on the surface of a smooth circular cylinder, as shown. At point B, the block falls off (leaves) the cylinder. The equation relating the angles θ_1 and θ_2 is given by



(A) $\sin\theta_2 = \frac{2}{3}\sin\theta_1$ (B) $\sin\theta_2 = \frac{3}{2}\sin\theta_1$ (C) $\cos\theta_2 = \frac{2}{3}\cos\theta_1$ (D) $\cos\theta_2 = \frac{3}{2}\cos\theta_1$

WE0066

MULTIPLE CORRECT TYPE QUESTIONS

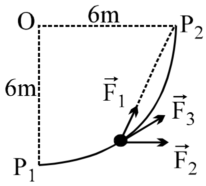
30. Which of the following statements is TRUE for a system comprising of two bodies in contact exerting frictional force on each other :
(A) total work done by static friction on whole system is always zero.
(B) work done by static friction on a body is always zero
(C) work done by kinetic friction on a body is always negative
(D) total work done by internal kinetic friction on whole system is always negative

WE0067

31. A particle of mass *m* is at rest in a train moving with constant velocity with respect to ground. Now the particle is accelerated by a constant force F_0 acting along the direction of motion of train for time t_0. A girl in the train and a boy on the ground measure the work done by this force. Which of the following are **INCORRECT**?
(A) Both will measure the same work
(B) Boy will measure higher value than the girl
(C) Girl will measure higher value than the boy
(D) Data are insufficient for the measurement of work done by the force F_0

WE0068

32. A smooth track in the form of a quarter circle of radius 6 m lies in the vertical plane. A particle moves from P_1 to P_2 under the action of forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$. Force $\vec{F}_1$ is always toward P_2 and is always 20N in magnitude. Force $\vec{F}_2$ always acts horizontally and is always 30 N in magnitude. Force $\vec{F}_3$ always acts tangentially to the track and is of magnitude 15 N. Select the correct alternative(s)



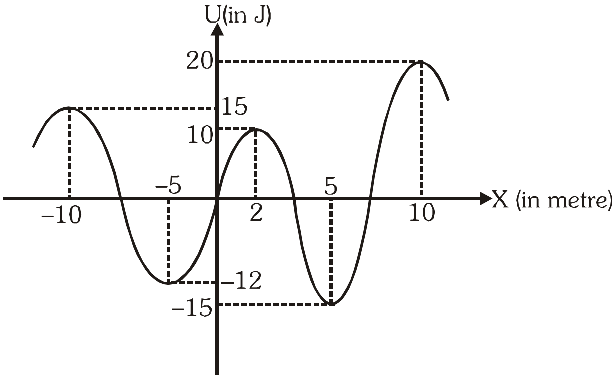
(A) work done by $\vec{F}_1$ is 120 J
(B) work done by $\vec{F}_2$ is 180 J
(C) work done by $\vec{F}_3$ is 45 π
(D) $\vec{F}_1$ is conservative in nature

WE0069

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 33 to 35

In the figure the variation of potential energy of a particle of mass m = 2kg is represented w.r.t. its x-coordinate. The particle moves under the effect of this conservative force along the x-axis.



33. If the particle is released at the origin then :
(A) It will move towards positive x-axis.
(B) It will move towards negative x-axis.
(C) It will remain stationary at the origin.
(D) Its subsequent motion cannot be decided due to lack of information.

WE0070

34. If the particle is released at $x = 2 + \Delta$ where $\Delta \to 0$ (it is positive) then its maximum speed in subsequent motion will be-
(A) $\sqrt{22}$ m/s (B) $\sqrt{25}$ m/s (C) $\sqrt{24}$ m/s (D) $\sqrt{23}$ m/s

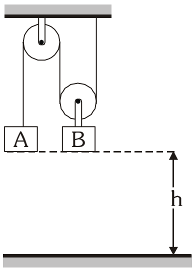
WE0070

35. x = –5 m and x = 10 m positions of the particle are respectively of-
(A) Neutral and stable equilibrium
(B) Neutral and unstable equilibrium
(C) Unstable and stable equilibrium
(D) Stable and unstable equilibrium.

WE0070

MATRIX MATCH TYPE QUESTIONS

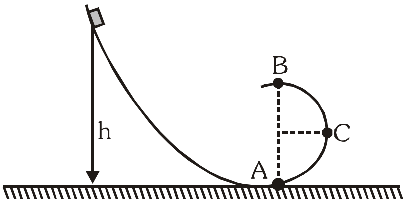
36. In the figure shown are two blocks A and B of same mass connected with pulley and string to each other. Initially both of them are at a height of h = 0.5 m from ground. After they are released they move in either direction and one of them strike the ground. For, the interval from releasing to when one of them strike, some physical quantities are in column I and their modulus values in SI units are in column II.



	Column I		**Column II**
(A)	Velocity of A immediately before any one of them strike ground.	(P)	1
(B)	Velocity of B immediately before any one of them strike ground.	(Q)	2
(C)	Ratio of work done by gravity on A to workdone by gravity on B.	(R)	3
(D)	Acceleration of block A before any one of them strike ground.	(S)	4
		(T)	5

WE0071

37. A block of mass m is released from top of a smooth track as shown in the figure. The end part of the track is a circle in vertical plane of radius R. N is normal reaction of the track at any point of the track. Match the entries of column I with entries of column–II.



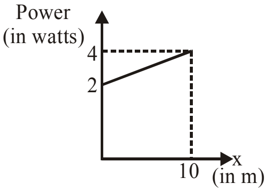
	Column I		**Column II**
(A)	$h = \frac{5}{2} R$	(P)	Net force on the block at C is mg
(B)	$h = \frac{9}{2} R$	(Q)	$N_A - N_B = 6\ mg$
(C)	h = R	(R)	Block leaves contact before B
(D)	h = 2R	(S)	Block will keep contact with the track is region between A & B.
		(T)	$N_C > mg$

WE0072

EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

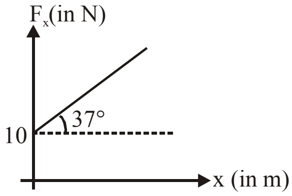
1. A particle 'A' of mass $\frac{10}{7}$ kg is moving in the positive x–direction. Its initial position is x = 0 & initial velocity is 1 m/s. The velocity at x = 10m is : (use the graph given)

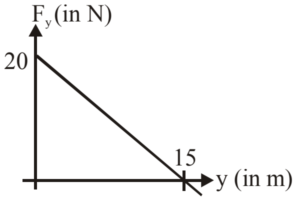


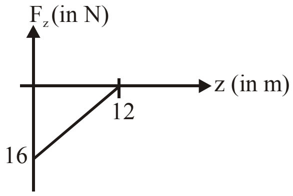
(A) 4 m/s (B) 2 m/s (C) $3\sqrt{2}\ m/s$ (D) 100/3 m/s

WE0073

2. The components of a force acting on a particle are varying according to the graphs shown. To reach at point B (8, 20, 0) from point A(0, 5, 12) the particle moves on paths parallel to x-axis then y-axis and then z-axis, then work done by this force is :-







(A) 192 J (B) 58 J (C) 250 J (D) 125 J

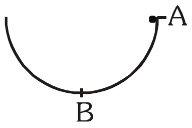
WE0074

3. A light spring of length 20 cm and force constant 2 N/cm is placed vertically on a table. A small block of mass 1 kg falls on it. The length h from the surface of the table at which the block will have the maximum velocity is :

(A) 20 cm (B) 15 cm (C) 10 cm (D) 5cm

WE0075

4. A small ball can move in a vertical plane along a semi–circle of radius r without friction. At what speed is the ball to launch from point A so that its acceleration is 3g at point B ?



(A) $(3gr)^{1/2}$ (B) $(2gr)^{1/2}$ (C) $(gr)^{1/2}$ (D) $2(gr)^{1/2}$

WE0076

MULTIPLE CORRECT TYPE QUESTIONS

5. A particle of mass 2 kg is projected with an initial speed u = 10 m/sec at an angle θ = 30° with the horizontal

(A) Total work done on the particle during the first half of the total time of flight of the particle is (–25) J.

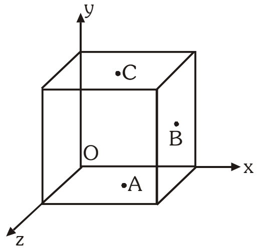
(B) Total work done on the particle during the total time of flight of the particle is 0 J.

(C) Average power delivered to the particle during the first half of the flight is (–50) watt.

(D) The radius of curvature of the trajectory of the particle at the highest point of the projectile is 7.5m.

WE0077

6. A particle is shifted from A to B and then from B to C where A, B and C are the midpoints of the corresponding faces of a cube of side 2m. If a force $\vec{F} = \left(3\hat{i} + 4\hat{j} - 5\hat{k}\right)N$ is continuously acting on the particle, then select correct alternative :-



(A) work done from A to B is 7 J

(B) work done from B to C is 1 J

(C) work done A to C is 8 J

(D) $\vec{F}$ is a conservative force

WE0078

7. Which of the following is/are conservative force(s)?

(A) $\vec{F} = 2r^3\hat{r}$

(B) $\vec{F} = -\frac{5}{r}\hat{r}$

(C) $\vec{F} = \frac{3(x\hat{i} + y\hat{j})}{(x^2+y^2)^{3/2}}$

(D) $\vec{F} = \frac{3(y\hat{i} + x\hat{j})}{(x^2+y^2)^{3/2}}$

WE0079

8. If one of the forces acting on a particle is conservative then :

(A) Work done by this force is zero when the particle moves exactly once around any closed path.

(B) Work done by this force equals the change in the kinetic energy of the particle.

(C) It obeys Newton's second law.

(D) Work done by this force depends on the end points of the motion, not on the path in between.

WE0080

9. A particle of mass m = 1 kg lying on x–axis experiences a force given by law F=x(3x–2) Newton, where x is the x–coordinate of the particle in meters. The points on x–axis where the particle is in equilibrium are :

(A) x = 0

(B) x = 1/3

(C) x = 2/3

(D) x = 1

WE0081

10. A particle is given a velocity at the bottom most position in a smooth spherical shell of radius 2m. It just complete a vertical circular motion. Then

(A) acceleration of particle when the velocity of the particle is in vertically upward direction is $g\sqrt{10}$

(B) acceleration of particle when the velocity of the particle is in vertically downward direction is $g\sqrt{10}$

(C) acceleration of the particle at the top most point of path is g

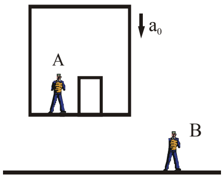
(D) acceleration of the particle at the bottom most point is 5 g

WE0082

COMPREHENSION TYPE QUESTIONS

Comprehension for Question no. 11 to 14

A block of mass m is kept in an elevator which starts moving downward with an acceleration a_0 as shown in figure. The block is observed by two observers A and B for a time interval t_0.



11. The observer B finds that the work done by gravity is

(A) $\frac{1}{2}mg^2t_0^2$ (B) $-\frac{1}{2}mg^2t_0^2$ (C) $\frac{1}{2}mga_0t_0^2$ (D) $-\frac{1}{2}mga_0t_0^2$

WE0083

12. The observer B finds that work done by normal reaction N is :-

(A) zero (B) $-Na_0t_0^2$ (C) $\frac{Na_0t^2}{2}$ (D) None of these

WE0083

13. According to observer B, the net work done on the block is

(A) $-\frac{1}{2}ma_0t_0^2$ (B) $\frac{1}{2}ma_0^2t_0^2$ (C) $\frac{1}{2}mga_0t_0^2$ (D) $-\frac{1}{2}mga_0t_0^2$

WE0083

14 According to the observer A

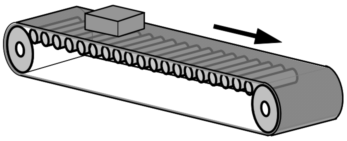
(A) the work done by gravity is zero (B) the work done by normal reaction is zero

(C) the work done by pseudo force is zero (D) all the above

WE0083

Paragraph for Question 15 & 16

An object of mass M is gently placed on a horizontal conveyor belt, which is moving with uniform velocity v_o as shown in the figure. The coefficient of static friction is μ_s, the coefficient of kinetic friction is μ_k, and the acceleration of gravity is g. Initially the object slips for a while but finally moves without slipping together with the belt.



15. How far the conveyor belt moves while the object is slipping?

(A) $\frac{v_o^2}{\mu_k g}$ (B) $\frac{v_o^2}{2\mu_k g}$ (C) $\frac{v_o^2}{\mu_s g}$ (D) $\frac{v_o^2}{2\mu_s g}$

WE0084

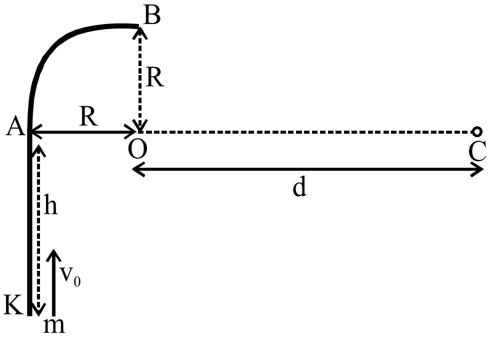
16. Work done on the object by friction force relative to the reference frame moving with the conveyer belt is

(A) $\frac{1}{2}Mv_o^2$ (B) $-\frac{1}{2}Mv_o^2$ (C) Mv_o^2 (D) zero

WE0084

Paragraph for Question Nos.17 to 19

A point like object of mass m starts from point K as shown in the figure. It slides inside along the full length of the smooth track of radius R, and then moves freely and travels to point C. [The track is kept in vertical plane]



17. Determine the vertical initial velocity of the pointlike object.

(A) $v_0 = \sqrt{2g(h+R)+\frac{d^2g}{R}}$ (B) $v_0 = \sqrt{2g(h-R)+\frac{d^2g}{2R}}$

(C) $v_0 = \sqrt{2g(h+R)+\frac{d^2g}{2R}}$ (D) $v_0 = \sqrt{2g(h+R)+\frac{dg}{2R}}$

WE0085

18. What is the minimum possible distance OC = d, necessary for the object to slide along the entire length of the track ?

(A) $d_{min} = R\sqrt{2}$ (B) $d_{min} = \sqrt{3}R$ (C) $d_{min} = \frac{R}{\sqrt{2}}$ (D) $d_{min} = \frac{R}{\sqrt{3}}$

WE0085

19. Find the normal force exerted by the track at point A.

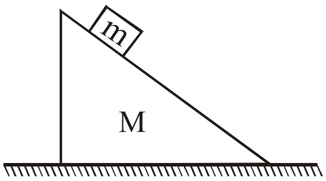
(A) $F_A = mg\left(\frac{d^2}{2R^2} - 2\right)$ (B) $F_A = mg\left(\frac{d^2}{2R^2} + 2\right)$

(C) $F_A = mg\left(\frac{3d^2}{R^2} + 2\right)$ (D) $F_A = mg\left(\frac{d}{3R^2} + 2\right)$

WE0085

MATRIX MATCH TYPE QUESTIONS

20. A block of mass m lies on wedge of mass M. The wedge in turn lies on smooth horizontal surface. Friction is absent everywhere. The wedge block system is released from rest. All situation given in column–I are to be estimated in duration the block undergoes a vertical displacement 'h' starting from rest (assume the block to be still on the wedge, g is acceleration due to gravity).



	Column I		**Column II**
(A)	Work done by normal reaction acting on the block is	(p)	Positive
(B)	Work done by normal reaction (exerted by block) acting on wedge is	(q)	Negative
(C)	The sum of work done by normal reaction on block and work done by normal reaction (exerted by block) on wedge is	(r)	Zero
(D)	Net work done by all forces on block is	(s)	Less than mgh in magnitude

WE0086

EXERCISE (JM)

1. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^{6}}$, where a and b are constant and x is the distance between the atoms. if the dissociation energy of the molecule is $D = [U(x = \infty) - U_{\text{at equilibrium}}]$, D is : **[AIEEE-2010]**

(1) $\frac{b^2}{6a}$ (2) $\frac{b^2}{2a}$ (3) $\frac{b^2}{12a}$ (4) $\frac{b^2}{4a}$

WE0087

2. At time t = 0s particle starts moving along the x-axis. If its kinetic energy increases uniformly with time 't', the net force acting on it must be proportional to :- **[AIEEE-2011]**

(1) $\sqrt{t}$ (2) constant (3) t (4) $\frac{1}{\sqrt{t}}$

WE0088

3. **This question has Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.** **[AIEEE-2012]**

If two springs S_1 and S_2 of force constants k_1 and k_2, respectively, are stretched by the same force, it is found that more work is done on spring S_1 than on spring S_2.

Statement-1: If stretched by the same amount, work done on S_1, will be more than that on S_2

Statement-2 : $k_1 < k_2$

(1) Statement-1 is true, Statement-2 is true and Statement-2 is not the correct explanation of Statement-1.

(2) Statement-1 is false, Statement-2 is true

(3) Statement-1 is true, Statement-2 is false

(4) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation of statement-1.

WE0089

4. When a rubber-band is stretched by a distance x, it exerts a restoring force of magnitude $F = ax + bx^2$ where a and b are constants. The work done in stretching the unstretched rubber-band by L is:- **[JEE-Main-2014]**

(1) $\frac{aL^2}{2}+\frac{bL^3}{3}$ (2) $\frac{1}{2}\left(\frac{aL^2}{2}+\frac{bL^3}{3}\right)$ (3) $aL^2 + bL^3$ (4) $\frac{1}{2}(aL^2+bL^3)$

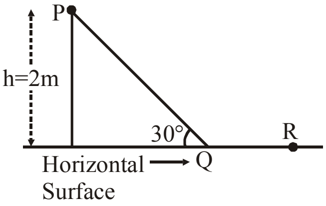
WE0090

5. A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20% efficiency rate. Take $g = 9.8\ ms^{-2}$:- **[JEE-Main-2016]**

(1) 12.89×10^{-3} kg (2) 2.45×10^{-3} kg (3) 6.45×10^{-3} kg (4) 9.89×10^{-3} kg

WE0091

6. A point particle of mass, moves along the uniformly rough track PQR as shown in the figure. The coefficient of friction, between the particle and the rough track equals μ. The particle is released, from rest, from the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and PR, of the track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR. The values of the coefficient of friction μ and the distance x(=QR) are, respecitvely close to :- **[JEE-Main-2016]**



(1) 0.29 and 6.5 m (2) 0.2 and 6.5 m (3) 0.2 and 3.5 m (4) 0.29 and 3.5 m

WE0092

7. A body of mass $m = 10^{-2}$ kg is moving in a medium and experiences a frictional force $F = -kv^2$. Its intial speed is $v_0 = 10\ ms^{-1}$. If, after 10 s, its energy is $\frac{1}{8}mv_0^2$, the value of k will be :- **[JEE-Main-2017]**

(1) 10^{-4} kg m^{-1} (2) 10^{-1} kg m^{-1} s^{-1} (3) 10^{-3} kg m^{-1} (4) 10^{-3} kg s^{-1}

WE0093

8. A time dependent force F = 6t acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 sec. will be : **[JEE-Main-2017]**

(1) 9 J (2) 18 J (3) 4.5 J (4) 22 J

WE0094

9. A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$. Its total energy is :- **[JEE-Main-2018]**

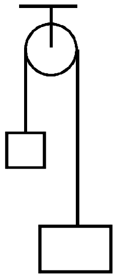
(1) $\frac{k}{2a^2}$ (2) Zero (3) $-\frac{3}{2}\frac{k}{a^2}$ (4) $-\frac{k}{4a^2}$

WE0095

EXERCISE (JA)

1. A light inextensible string that goes over a smooth fixed pulley as shown in the figure connects two blocks of masses 0.36 kg and 0.72 kg. Taking $g = 10\ m/s^2$, find the work done (in **joules**) by the string on the block of mass 0.36 kg during the first second after the system is released from rest.

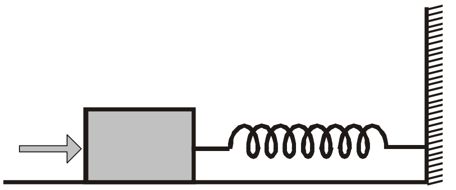
[IIT-JEE-2009]



WE0096

2. A block of mass 0.18 kg is attached to a spring of force-constant 2 N/m. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is un-stretched. An impulse is given to the block as shown in the figure. The block slides a distance of 0.06 m and comes to rest for the first time. The initial velocity of the block in m/s is V= N/10. Then N is

[IIT-JEE-2011]



WE0097

3. The work done on a particle of mass m by a force, $K\left[\frac{x}{\left(x^2+y^2\right)^{3/2}}\hat{i}+\frac{y}{\left(x^2+y^2\right)^{3/2}}\hat{j}\right]$ (K being a constant of appropriate dimensions), when the particle is taken from the point (a, 0) to the point (0, a) along a circular path of radius a about the origin in the x-y plane is :- **[JEE-Advance-2013]**

(A) $\frac{2K\pi}{a}$ (B) $\frac{K\pi}{a}$ (C) $\frac{K\pi}{2a}$ (D) 0

WE0098

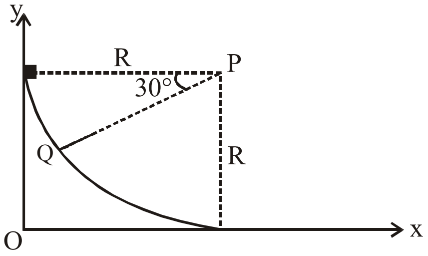
4. A particle of mass 0.2 kg is moving in one dimension under a force that delivers a constant power 0.5W to the particle. If the initial speed (in ms^{-1}) of the particle is zero, the speed (in ms^{-1}) after 5 s is.

[JEE-Advance-2013]

WE0099

Paragraph for Questions 5 and 6

A small block of mass 1 kg is released from rest at the top of a rough track. The track is a circular arc of radius 40 m. The block slides along the track without toppling and a frictional force acts on it in the direction opposite to the instantaneous velocity. The work done in overcoming the friction up to the point Q, as shown in the figure below, is 150 J. (Take the acceleration due to gravity, $g = 10 \text{ m s}^{-2}$) **[JEE-Advance-2013]**



5. The magnitude of the normal reaction that acts on the block at the point Q is

(A) 7.5 N (B) 8.6 N (C) 11.5 N (D) 22.5 N

WE0100

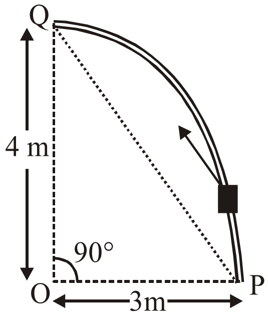
6. The speed of the block when it reaches the point Q is

(A) 5 ms^{-1} (B) 10 ms^{-1} (C) $10\sqrt{3} \text{ms}^{-1}$ (D) 20 ms^{-1}

WE0100

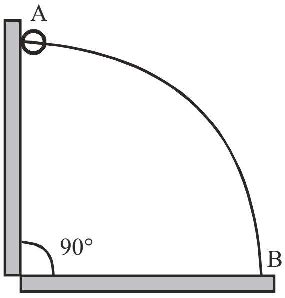
7. Consider an elliptically shaped rail PQ in the vertical plane with OP = 3 m and OQ = 4m. A block of mass 1 kg is pulled along the rail from P to Q with a force of 18 N, which is always parallel to line PQ (see the figure given). Assuming no frictional losses, the kinetic energy of the block when it reaches Q is (n × 10) Joules. The value of n is (take acceleration due to gravity = 10 ms^{-2})

[JEE-Advance-2014]



WE0101

8. A wire, which passes through the hole in a small bead, is bent in the form of quarter of a circle. The wire is fixed vertically on ground as shown in the figure. The bead is released from near the top of the wire and it slides along the wire without friction. As the bead moves from A to B, the force it applies on the wire is :- **[JEE Advanced-2014]**



(A) Always radially outwards

(B) Always radially inwards

(C) Radially outwards initially and radially inwards later.

(D) Radially inwards initially and radially outwards later.

WE0102

9. A particle of mass m is initially at rest at the origin. It is subjected to a force and starts moving along the x-axis. Its kinetic energy K changes with time as $dK/dt = \gamma t$, where γ is a positive constant of appropriate dimensions. Which of the following statements is (are) true ? **[JEE Advanced-2018]**

(A) The force applied on the particle is constant

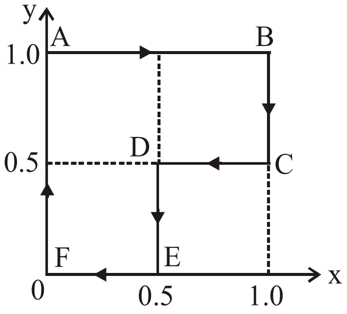
(B) The speed of the particle is proportional to time

(C) The distance of the particle from the origin increses linerarly with time

(D) The force is conservative

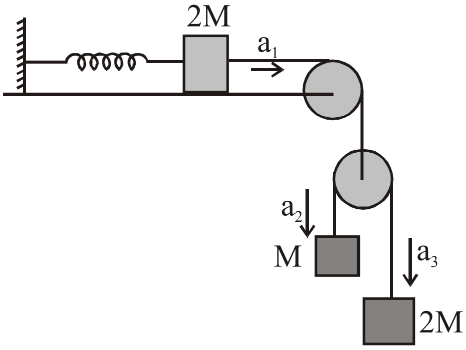
WE0103

10. A particle is moved along a path AB-BC-CD-DE-EF-FA, as shown in figure, in presence of a force $\vec{F} = \left(\alpha y\hat{i} + 2\alpha x\hat{j}\right)$ N, where x and y are in meter and $\alpha = -1$ N/m^{-1}. The work done on the particle by this force $\vec{F}$ will be ____ Joule. **[JEE Advanced-2019]**



WE0104

11. A block of mass 2M is attached to a massless spring with spring-constant k. This block is connected to two other blocks of masses M and 2M using two massless pulleys and strings. The accelerations of the blocks are a_1, a_2 and a_3 as shown in figure. The system is released from rest with the spring in its unstretched state. The maximum extension of the spring is x_0. Which of the following option(s) is/are correct ? [g is the acceleration due to gravity. Neglect friction] **[JEE Advanced-2019]**



(1) $x_0 = \frac{4Mg}{k}$

(2) When spring achieves an extension of $\frac{x_0}{2}$ for the first time, the speed of the block connected to the spring is $3g\sqrt{\frac{M}{5k}}$

(3) $a_2 - a_1 = a_1 - a_3$

(4) At an extension of $\frac{x_0}{4}$ of the spring, the magnitude of acceleration of the block connected to the spring is $\frac{3g}{10}$

WE0105

ANSWER KEY

EXERCISE (S-1)

1. Ans. (a) +ve (b) –ve (c) –ve (d) +ve (e) –ve **2. Ans.** 12 J **3. Ans.** 8
4. Ans. 67.7 J **5. Ans.** (a) 7.5 J; (b) 15 J; (c) 7.5 J; (d) 30 J **6. Ans.** 3
7. Ans. (a) 875 J; (b) –250 J; (c) 625 J ; (d) 625 J; **8. Ans.** 0 **9. Ans.** 400 J
10. Ans. (i) $2 + 24t^2 + 72t^4$ J, (ii) 48 t N, (iii) $48t + 288t^3$ W, (iv) 1248 J **11. Ans.** 4
12. Ans. 4 **13. Ans.** 10 **14. Ans.** $m_1 = m/2$
15. Ans. (a) –30 kW, 19.5 kW **16. Ans.** 46 J **18. Ans.** 4
19. Ans. (i) x =2, (ii) 16 J **20. Ans.** $-6\ m/s^2$ **21. Ans.** 4 **22. Ans.** 018
23. Ans. Graph-1 : For all x, Graph-2 : x < a & x > b, Graph-3 : $-\frac{b}{2} < x < -\frac{a}{2}$ & $\frac{a}{2} < x < \frac{b}{2}$
24. Ans. (a) s = 0.24 m, a = 6 m/s², (b) $\mu = 1/8$ **25. Ans.** $\sqrt{2gL}, \sqrt{gL}$ **26. Ans.** 4

EXERCISE (S-2)

1. Ans. (i) mgR/2, (ii) $2\sqrt{gR}$ **2. Ans.** 2.5cm
3. Ans. (i) $f = \frac{\mu m}{\ell}xg$; (ii) $\sqrt{\mu g\ell}$; (iii) $\frac{5\ell}{2}$ **4. Ans.** $F = -3ax^2 + b$, $x = \sqrt{\frac{b}{a}}$, $K_{max} = \frac{2b}{3}\sqrt{\frac{b}{3a}}$
6. Ans. (i) 6×10^3 W (ii) 1×10^4 W **7. Ans.** $\frac{E\mu\cos\alpha}{\sin\alpha + \mu\cos\alpha}$ **8. Ans.** 25 m
9. Ans. 27R/28 **10. Ans.** $u = \sqrt{gL\left(2 + \frac{3\sqrt{3}}{2}\right)}$ **11. Ans.** 500 N/m

EXERCISE (O-1)

1. Ans. (C) **2. Ans.** (B) **3. Ans.** (C) **4. Ans.** (C) **5. Ans.** (A)
6. Ans. (A) **7. Ans.** (B) **8. Ans.** (D) **9. Ans.** (A) **10. Ans.** (A)
11. Ans. (D) **12. Ans.** (A) **13. Ans.** (A) **14. Ans.** (B) **15. Ans.** (A)
16. Ans. (B) **17. Ans.** (A) **18. Ans.** (B) **19. Ans.** (B) **20. Ans.** (C)
21. Ans. (D) **22. Ans.** (A) **23. Ans.** (B) **24. Ans.** (C) **25. Ans.** (C)
26. Ans. (D) **27. Ans.** (D) **28. Ans.** (A) **29. Ans.** (C) **30. Ans.** (A,D)
31. Ans. (A,C) **32. Ans.** (B,C,D) **33. Ans.** (B) **34. Ans.** (B) **35. Ans.** (D)
36. Ans. (A) Q; (B) P; (C) Q; (D) S **37. Ans.** (A) Q,S,T; (B) Q,S,T; (C) P; (D) R,T

EXERCISE (O-2)

1. Ans. (A) **2. Ans.** (C) **3. Ans.** (B) **4. Ans.** (C) **5. Ans.** (A,B,C,D)
6. Ans. (ABCD) **7. Ans.** (A,B,C) **8. Ans.** (A,C,D) **9. Ans.** (A,C) **10. Ans.** (A,B,C,D)
11. Ans. (C) **12. Ans.** (D) **13. Ans.** (B) **14. Ans.** (D) **15. Ans.** (A)
16. Ans. (B) **17. Ans.** (C) **18. Ans.** (A) **19. Ans.** (B)
20. Ans. (A) Q,S; (B) P,S; (C) R, S; (D) P,S

EXERCISE (J-M)

1. Ans. (4) **2. Ans.** (4) **3. Ans.** (2) **4. Ans.** (1) **5. Ans.** (1)
6. Ans. (4) **7. Ans.** (1) **8. Ans.** (3) **9. Ans.** (2)

EXERCISE (J-A)

1. Ans. 8 **2. Ans.** 4 **3. Ans.** (D) **4. Ans.** 5 **5. Ans.** (A) **6. Ans.** (B)
7. Ans. 5 **8. Ans.** (D) **9. Ans.** (A,B,D) **10. Ans.** 0.75 **11. Ans.** (3)

Chapter Contents

03

CENTER OF MASS, MOMENTUM & COLLISION

Important Notes

CENTER OF MASS, MOMENTUM & COLLISION

KEY CONCEPTS

CENTRE OF MASS :

For a system of particles centre of mass is that point at which its total mass is supposed to be concentrated. The centre of mass of an object is a point that represents the entire body and moves in the same way as a point mass having mass equal to that of the object, when subjected to the same external forces that act on the object.

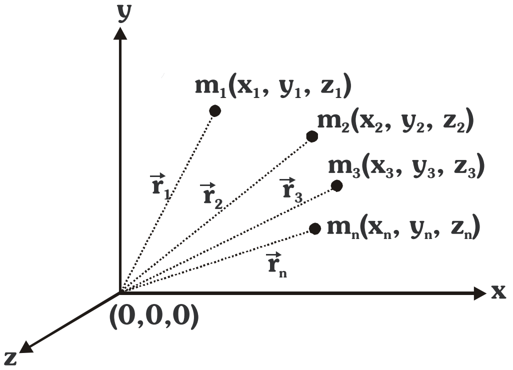
Centre of mass of system of discrete particles

Total mass of the body : $M = m_1 + m_2 + ... + m_n$

Then $$\vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + ...}{m_1 + m_2 + m_3 + ...} = \frac{1}{M}\Sigma m_i\vec{r}_i$$

co-ordinates of centre of mass :

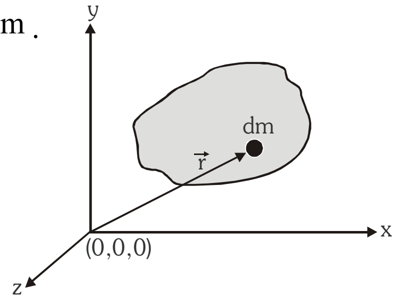
$$x_{cm} = \frac{1}{M}\Sigma m_i x_i,\ y_{cm} = \frac{1}{M}\Sigma m_i y_i \text{ and } z_{cm} = \frac{1}{M}\Sigma m_i z_i$$



Centre of mass of continuous distribution of particles

If the system has continuous distribution of mass, treating the mass element dm at position $\vec{r}$ as a point mass and replacing summation by integration. $\vec{R}_{CM} = \frac{1}{M}\int \vec{r}dm$.

So that $$x_{cm} = \frac{1}{M}\int x\,dm,\ y_{cm} = \frac{1}{M}\int y\,dm \text{ and } z_{cm} = \frac{1}{M}\int z\,dm$$



If co-ordinates of particles of mass $m_1, m_2, ...$ are
$(x_1, y_1, z_1), (x_2, y_2, z_2)....$
then position vector of their centre of mass is

$$\vec{R}_{CM} = x_{cm}\hat{i} + y_{cm}\hat{j} + z_{cm}\hat{k}$$

$$= \frac{m_1(x_1\hat{i} + y_1\hat{j} + z_1\hat{k}) + m_2(x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) + m_3\left(x_3\hat{i} + y_3\hat{j} + z_3\hat{k}\right) + ...}{m_1 + m_2 + m_3 + ...}$$

$$= \frac{(m_1x_1 + m_2x_2 + ...)\hat{i} + (m_1y_1 + m_2y_2...)\hat{j} + (m_1z_1 + m_2z_2 + ..)\hat{k}}{m_1 + m_2 + m_3 + ..}$$

So, $$x_{cm} = \left(\frac{m_1x_1 + m_2x_2 + ...}{m_1 + m_2 + m_3 + ...}\right),\ y_{cm} = \left(\frac{m_1y_1 + m_2y_2 + ...}{m_1 + m_2 + ...}\right),\ z_{cm} = \left(\frac{m_1z_1 + m_2z_2 + ...}{m_1 + m_2 + ...}\right)$$

The centre of mass after removal of a part of a body

If a portion of a body is taken out, the remaining portion may be considered as,
Original mass (M) – mass of the removed part (m) = {original mass (M)} + {– mass of the removed part (m)}

The formula changes to : $$x_{CM} = \frac{Mx - mx'}{M - m};\ y_{CM} = \frac{My - my'}{M - m};\ z_{CM} = \frac{Mz - mz'}{M - m}$$

Where x', y' and z' represent the coordinates of the centre of mass of the removed part.

MOTION OF CENTRE OF MASS

As for a system of particles, position of centre of mass is $\vec{R}_{CM} = \dfrac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + ...}{m_1 + m_2 + m_3 + ...}$

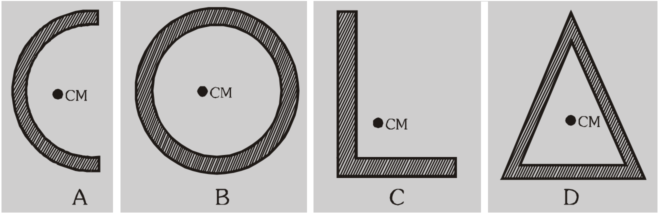
So $\dfrac{d}{dt}\left(\vec{R}_{CM}\right) = \dfrac{m_1\dfrac{d\vec{r}_1}{dt} + m_2\dfrac{d\vec{r}_2}{dt} + m_3\dfrac{d\vec{r}_3}{dt} + ...}{m_1 + m_2 + m_3 + ...} \Rightarrow$

Similarly acceleration $\vec{a}_{CM} = \dfrac{d}{dt}\left(\vec{v}_{CM}\right) = \dfrac{m_1\vec{a}_1 + m_2\vec{a}_2 + ...}{m_1 + m_2 + ...}$

We can write $M\vec{v}_{CM} = m_1\vec{v}_1 + m_2\vec{v}_2 + ... = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + ...$ $[\because \vec{p} = m\vec{v}]$

$M\vec{v}_{CM} = \vec{p}_{CM}$ $[\because \Sigma\vec{p}_i = \vec{p}_{CM}]$

IMPORTANT POINTS



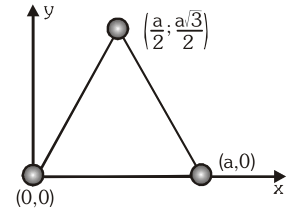
- There may or may not be any mass present physically at centre of mass (See Figure A, B, C)
- Centre of mass may be inside or outside of the body (See figure A, B, C)
- Position of centre of mass depends on the shape of the body. (See figure A, B, C)
- For a given shape it depends on the distribution of mass of within the body and is closer to massive part. (See figure A,C)
- For symmetrical bodies having homogeneous distribution of mass it coincides with centre of symmetry of geometrical centre. (See figure B,D).
- If we know the centre of mass of parts of the system and their masses, we can get the combined centre of mass by treating the parts as point particles placed at their respective centre of masses.
- It is independent of the co-ordinate system, e.g., the centre of mass of a ring is at its centre whatever be the co-ordinate system.
- If the origin of co-ordinate system is at centre of mass, i.e., $\vec{R}_{CM} = 0$, then by definition.

$\dfrac{1}{M}\Sigma m_i\vec{r}_i = 0 \Rightarrow \Sigma m_i\vec{r}_i = 0$

The sum of the moments of the masses of a system about its centre of mass is always zero.

Ex. Three bodies of equal masses are placed at (0, 0), (a, 0) and at $\left(\frac{a}{2}, \frac{a\sqrt{3}}{2}\right)$.

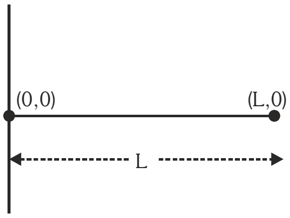
Find out the co-ordinates of centre of mass.



Sol. $$x_{CM} = \frac{0\times m + a\times m + \frac{a}{2}\times m}{m+m+m} = \frac{a}{2},\ y_{CM} = \frac{0\times m + 0\times m + \frac{a\sqrt{3}}{2}\times m}{m+m+m} = \frac{a\sqrt{3}}{6}$$

Ex. Calculate the position of the centre of mass of a system consisting of two particles of masses m_1 and m_2 separated by a distance L apart, from m_1.

Sol. Treating the line joining the two particles as x axis



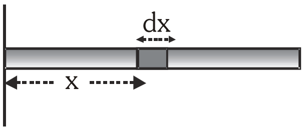
$$x_{CM} = \frac{m_1\times 0 + m_2\times L}{m_1+m_2} = \frac{m_2 L}{m_1+m_2},\ y_{CM} = 0 \qquad z_{CM} = 0$$

Ex. If the linear density of a rod of length L varies as λ = A + Bx, compute position of its centre of mass.

Sol. Let the x–axis be along the length of the rod and origin at one of its end as shown in figure. As rod is along x–axis, for all points on it y and z will be zero so, $y_{CM} = 0$ and $z_{CM} = 0$ i.e., centre of mass will be on the rod.

Now consider an element of rod of length dx at a distance x from the origin, mass of this element $dm = \lambda dx = (A+Bx)dx$

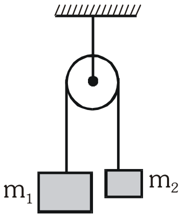
$$\text{so, } x_{CM} = \frac{\int_0^L x\,dm}{\int_0^L dm} = \frac{\int_0^L x(A+Bx)dx}{\int_0^L (A+Bx)dx} = \frac{\frac{AL^2}{2}+\frac{BL^3}{3}}{AL+\frac{BL^2}{2}} = \frac{L(3A+2BL)}{3(2A+BL)}$$



Note : (i) If the rod is of uniform density then λ = A = constant & B = 0 then $X_{CM} = L/2$

(ii) If the density of rod varies linearly with x, then λ = Bx and A = 0 then $X_{CM} = 2L/3$

Ex. Two bodies of masses m_1 and m_2 ($<m_1$) are connected to the ends of a massless cord and allowed to move as shown in. The pulley is both massless and frictionless. Calculate the acceleration of the centre of mass.



Sol. If $\vec{a}$ is the acceleration of m_1,then $-\vec{a}$ is the acceleration of m_2 then

acceleration of each body $a = \frac{\text{Net force}}{\text{Net mass}} = \left(\frac{m_1-m_2}{m_1+m_2}\right)g$

$$\vec{a}_{cm} = \frac{m_1\vec{a} + m_2(-\vec{a})}{m_1+m_2} = \left(\frac{m_1-m_2}{m_1+m_2}\right)\vec{a}$$

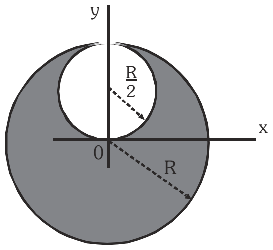
But $\vec{a} = \left(\frac{m_1-m_2}{m_1+m_2}\right)\vec{g}$ so $\vec{a}_{cm} = \left(\frac{m_1-m_2}{m_1+m_2}\right)^2 \vec{g}$

Ex. A circle of radius R is cut from a uniform thin sheet of metal. A circular hole of radius $\frac{R}{2}$ is now cut out of the circle, with the hole tangent to the rim. Find the distance of centre of mass from the centre of the original uncut circle to the centre of mass.

Sol. We treat the hole as a 'negative mass' object that is combined with the original uncut circle. (When the two are added together, the hole region then has zero mass). By symmetry, the CM lies along the +y–axis in figure, so $x_{CM} = 0$. With the origin at the centre of the original circle whose mass is assumed to be m.

Mass of original uncut circle $m_1 = m$ & (0, 0)

Mass of hole of negative mass : $m_2 = \frac{m}{4}$; Location of CM $\left(0, \frac{R}{2}\right)$



$$\text{Thus } y_{CM} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{m(0) + \left(-\frac{m}{4}\right)\frac{R}{2}}{m + \left(-\frac{m}{4}\right)} = \frac{R}{6}$$

So the centre of mass is at the point $\left(0, -\frac{R}{6}\right)$

Ex. Two particles of mass 1 kg and 0.5 kg are moving in the same direction with speeds of 2m/s and 6m/s, respectively, on a smooth horizontal surface. Find the speed of centre of mass of the system.

Sol. Velocity of centre of mass of the system $\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$ Since the particles m_1 and m_2 are moving in same direction, $m_1\vec{v}_1$ and $m_2\vec{v}_2$ are parallel. $\Rightarrow$ $|m_1\vec{v}_1 + m_2\vec{v}_2| = m_1 v_1 + m_2 v_2$

$$\text{Therefore, } v_{cm} = \frac{|m_1\vec{v}_1 + m_2\vec{v}_2|}{m_1 + m_2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{(1)(2) + \left(\frac{1}{2}\right)(6)}{\left(1 + \frac{1}{2}\right)} = 3.33 \text{ ms}^{-1}$$

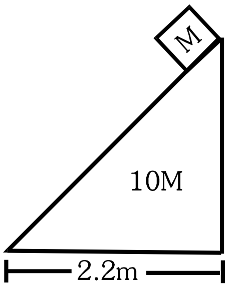
Ex. Two particles of masses 2 kg and 4 kg are approaching towards each other with acceleration 1 m/s^2 and 2 m/s^2, respectively, on a smooth horizontal surface. Find the acceleration of centre of mass of the system.

Sol. The acceleration of centre of mass of the system $\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}$ $\Rightarrow$ $a_{cm} = \frac{|m_1\vec{a}_1 + m_2\vec{a}_2|}{m_1 + m_2}$

Since $\vec{a}_1$ and $\vec{a}_2$ are anti–parallel, so $a_{cm} = \frac{|m_1 a_1 - m_2 a_2|}{m_1 + m_2} = \frac{|(2)(1) - (4)(2)|}{2 + 4} = 1 \text{ ms}^{-2}$

Since $m_2 a_2 > m_1 a_1$ so the direction of acceleration of centre of mass will be directed in the direction of a_2.

Ex. A block of mass M is placed on the top of a bigger block of mass 10 M as shown in figure. All the surfaces are frictionless. The system is released from rest. Find the distance moved by the bigger block at the instant the smaller block reaches the ground.



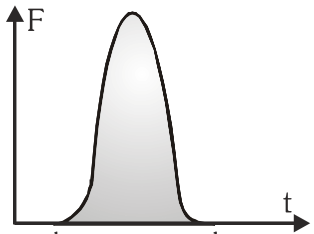
Sol. If the bigger block moves towards right by a distance (X), the smaller block will move towards left by a distance (2.2 –X) (taking the two blocks together as the system). The horizontal position of CM remains same $\Rightarrow$ M (2.2 – X) = 10 MX $\Rightarrow$ X = 0.2 m.

MOMENTUM

The total quantity of motion possessed by a moving body is known as the momentum of the body. It is the product of the mass and velocity of a body i.e. momentum $\vec{p} = m\vec{v}$

IMPULSE

When a large force act for an extremely short duration, neither the magnitude of the force nor the time for which it acts is important. In such a case, the total effect of force is measured. The total effect of force is called impulse (measure of the action of force). This type of force is generally variable in magnitude and is sometimes called impulsive force.



impulse = area under curve

If a large force act on a body or particle for a small time then

Impulse = product of force with time.

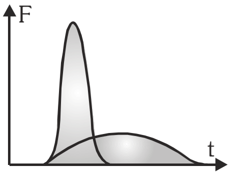
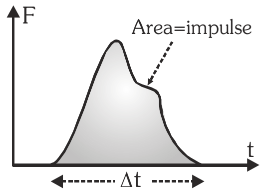
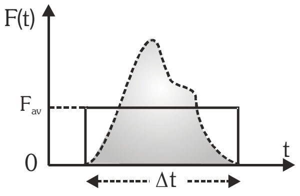
Suppose a force $\vec{F}$ acts for a short time dt then impulse = $\vec{F}dt$

For a finite interval of time from t_1 to t_2 then the impulse = $\int_{t_1}^{t_2} \vec{F}dt$

If constant force $\vec{F}$ acts for an interval Δt then Impulse = $\vec{F}\Delta t$

Impulse – Momentum theorem :

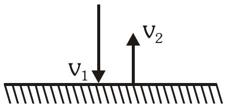
Impulse of a force is equal to the change of momentum $\boxed{\vec{F}\Delta t = \Delta\vec{p}}$

 A large force acting for a short time and a small force acting for a long time may result in the same impulse, as indicated by the equal areas under the force-versus time curves.	 The force F(t) involved in the collision between a ball and bat. The interaction occurs during the time Δt. The impulse is equal in magnitude to the shaded area under the curve.	 The constant force F_{av} acting for the same time interval Δt produces the same impulse as in figure since the areas under the two graphs are the same.

Ex. A ball of mass 50 g is dropped from a height h = 10 m. It rebounds losing 75 percent of its total mechanical energy. If it remains in contact with the ground for 0.01s, find the impulse of the impact force.

Sol. Impulse = change in momentum = $m(v_1+v_2)$

Here $v_1 = \sqrt{2gh}$ and for v_2, $\frac{1}{2}mv_2^2 = \frac{1}{2}mv_1^2\left(1-\frac{75}{100}\right) \Rightarrow v_2 = \frac{v_1}{2}$



So impulse = $m\left(v_1+\frac{v_1}{2}\right) = \frac{3mv_1}{2} = \frac{3}{2}\,m \times \sqrt{2gh} = \frac{3}{2} \times 50 \times 10^{-3} \times \sqrt{2\times9.8\times10} = 1.05$ N–s

LAW OF CONSERVATION OF LINEAR MOMENTUM

According to Newton's Second law of motion the rate of change of momentum is equal to the applied force.

$$\vec{F} = \frac{d\vec{p}}{dt} \quad \text{if } \vec{F} = \vec{0} \text{ then } \frac{d\vec{p}}{dt} = \vec{0} \text{ i.e. } \vec{p} = \text{constant}$$

This leads to the law of conservation of momentum which is" In the absence of external forces, the total momentum of the system is conserved."

IMPORTANT POINTS

- For an isolated system, the initial momentum of the system is equal to the final momentum of the system. If the system consists of n bodies having momentum

  $\vec{p}_1, \vec{p}_2, \vec{p}_3, \ldots\ldots\ldots\vec{p}_n$, then $\vec{p}_1+\vec{p}_2+\vec{p}_3+\ldots\ldots\ldots+\vec{p}_n$ = constant
- As linear momentum depends on frame of reference. Observers in different frames would find different values of linear momentum of a given system but each would agree that his own value of linear momentum does not change with time provided. But the system should be isolated and closed, i.e., law of conservation of linear momentum is independent of frame of reference though linear momentum depends on frame of reference.
- Conservation of linear momentum is equivalent to Newton's III law of motion for a system of two particles in absence of external force by law of conservation of linear momentum.

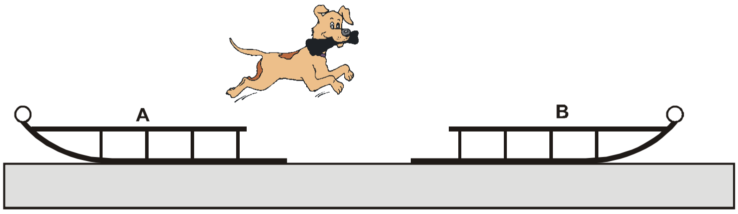
  $\Rightarrow \quad \vec{p}_1+\vec{p}_2$ = constant $\qquad$ i.e. $m_1\vec{v}_1+m_2\vec{v}_2$ = constant

  Differentiating above with respect to time $m_1\frac{d\vec{v}_1}{dt}+m_2\frac{d\vec{v}_2}{dt}=\vec{0}$ [as m is constant]

  $\Rightarrow m_1\vec{a}_1+m_2\vec{a}_2=\vec{0}$ $\left[\because \frac{d\vec{v}_1}{dt}=\vec{a}\right]$ $\Rightarrow \vec{F}_1+\vec{F}_2=\vec{0}$ $\quad[\because \vec{F}=m\vec{a}]$ $\quad\Rightarrow \vec{F}_1=-\vec{F}_2$

  i.e., for every action there is equal and opposite reaction which is Newton's III law of motion.
- This law is universal, i.e., it applies to body macroscopic as well as microscopic systems.

Ex. Two 22.7 kg ice sleds A and B are placed a short distance apart, one directly behind the other, as shown in figure. A 3.63 kg dog, standing on one sled, jumps across to the other and immediately back to the first. Both jumps are made at a speed of 3.05 ms^{-1} relative to the ice. Find the final speeds of the two sleds.



Sol. Total momentum imparted to B $p_B = 2 \times 3.63 \times 3.05$ kg ms^{-1}.

Velocity of $B = \dfrac{p_B}{m_B} = \dfrac{2\times3.63\times3.05}{22.7} = 0.975$ ms^{-1}.

Velocity of A when the dog jumps away from $A = \dfrac{p_A}{m_A} = \dfrac{3.63\times3.05}{22.7} = 0.4877$ ms^{-1}.

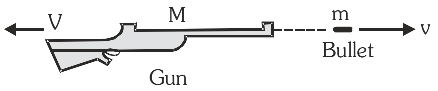
When the dog comes back to A, Velocity of $A = \dfrac{22.7\times0.4877+3.63\times3.05}{22.7+3.63} = 0.841$ ms^{-1}.

APPLICATIONS OF CONSERVATION OF LINEAR MOMENTUM

Firing a Bullet from a Gun :

- If the bullet is the system, the force exerted by trigger will be external and so the linear momentum of the bullet will change from 0 to mv. This is not the violation of law of conservation of linear momentum as linear momentum is conserved only in absence of external force .
- If the bullet and gun is the system, the force exerted by trigger will be internal so.

  Total momentum of the system $\vec{p}_s = \vec{p}_B + \vec{p}_G$ = constant.



Now as initially both bullet and gun are at rest so $\vec{p}_B + \vec{p}_G = \vec{0}$ From this it is evident that :

- $\vec{p}_G = -\vec{p}_B$, i.e., if bullet acquires forward momentum, the gun will acquire equal and opposite (backward) momentum.
- As $\vec{p} = m\vec{v},\ m\vec{v} + M\vec{V} = \vec{0}$, i.e, $\vec{V} = -\dfrac{m}{M}\vec{v}$ i.e, if the bullet moves forward, gun 'recoils' or 'kicks' backward. Heavier the gun lesser will be the recoil velocity V.
- Kinetic energy $K = \dfrac{p^2}{2m}$ and $|\vec{p}_B| = |\vec{p}_G| = p$ Kinetic energy of gun $K_G = \dfrac{p^2}{2M}$,

  Kinetic energy of bullet $K_B = \dfrac{p^2}{2m}$ $\therefore \dfrac{K_G}{K_B} = \dfrac{m}{M} < 1$ ($\because$ M $\gg$ m) Thus kinetic energy of gun is smaller than bullet i.e., kinetic energy of bullet and gun will not be equal.
- Initial kinetic energy of the system is zero as both are at rest initially.

  Final kinetic energy of the system $[(1/2)(mv^2 + MV^2)] > 0$.

  So, here kinetic energy of the system is not constant but increases. If PE is assumed to be constant then Mechanical energy = (kinetic energy + potential energy) will also increase. However, energy is always conserved. Here chemical energy of gun powder is converted into KE.

Ex. A bullet of mass 100g is fired by a gun of 10kg with a speed 2000 m/s. Find recoil velocity of gun.

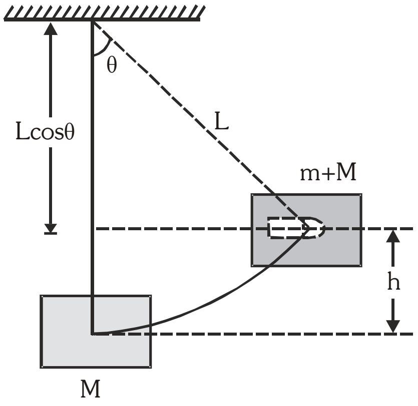
Sol. According to conservation of momentum mv + MV = 0.

Velocity of gun $V = -\dfrac{mv}{M} = -\dfrac{0.1\times2000}{10} = -20\text{m/s}$

Block Bullet System :

(a) When bullet remains in the block

Conserving momentum of bullet and block $mv + 0 = (M+m)\,V$



Velocity of block $V = \dfrac{mv}{M+m}$ ...(i)

By conservation of mechanical energy

$$\frac{1}{2}(M+m)V^2 = (M+m)gh \Rightarrow V = \sqrt{2gh} \quad \text{....(ii)}$$

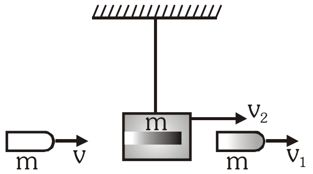
From eq^n. (i) and eq^n. (ii) $\dfrac{mv}{M+m} = \sqrt{2gh}$;

Speed of bullet $v = \dfrac{(M+m)\sqrt{2gh}}{m}$,

Maximum height gained by block $h = \dfrac{V^2}{2g} = \dfrac{m^2v^2}{2g(M+m)^2}$

$h = L - L\cos\theta \qquad \therefore \cos\theta = 1 - \dfrac{h}{L} \Rightarrow \theta = \cos^{-1}\left(1 - \dfrac{h}{L}\right)$

(b) If bullet moves out of the block



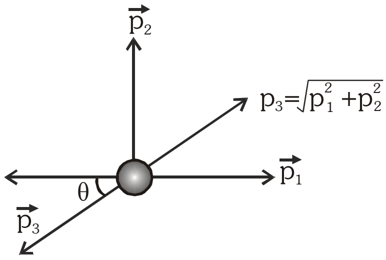
Conserving momentum $mv + 0 = mv_1 + Mv_2$

$m(v - v_1) = Mv_2$ ....(i)

Conserving energy $\dfrac{1}{2}Mv_2^2 = Mgh \Rightarrow v_2 = \sqrt{2gh}$ ....(ii)

From eq^n. (i) & eq^n. (ii) $m(v - v_1) = M\sqrt{2gh} \Rightarrow h = \dfrac{m^2(v-v_1)^2}{2gM^2}$

Explosion of a Bomb at rest



Conserving momentum

$\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = \vec{0} \Rightarrow \vec{p}_3 = -(\vec{p}_1 + \vec{p}_2) \Rightarrow p_3 = \sqrt{p_1^2 + p_2^2}$ as $\vec{p}_1 \perp \vec{p}_2$

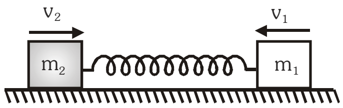
Angle made by $\vec{p}_3$ from $\vec{p}_1 = \pi + \theta$

Angle made by $\vec{p}_3$ from $\vec{p}_2 = \dfrac{\pi}{2} + \theta$

Energy released in explosion $= K_f - K_i = \dfrac{p_1^2}{2m_1} + \dfrac{p_2^2}{2m_2} + \dfrac{p_3^2}{2m_3} - 0 = \dfrac{p_1^2}{2m_1} + \dfrac{p_2^2}{2m_2} + \dfrac{p_3^2}{2m_3}$

Motion of Two Masses Connected to a Spring

Consider two blocks, resting on a frictionless surface and connected by a massless spring as shown in figure. If the spring is stretched (or compressed) and then released from rest,



Then $F_{ext}=0$ so $\vec{p}_s = \vec{p}_1 + \vec{p}_2$ = constant

However, initially both the blocks were at rest so, $\vec{p}_1 + \vec{p}_2 = \vec{0}$

It is clear that :

- $\vec{p}_2 = -\vec{p}_1$, i.e., at any instant the two blocks will have momentum equal in magnitude but opposite in direction (Though they have different values of momentum at different positions).
- As momentum $\vec{p} = m\vec{v}$, $m_1\vec{v}_1 + m_2\vec{v}_2 = \vec{0} \Rightarrow \vec{v}_2 = -\left(\frac{m_1}{m_2}\right)\vec{v}_1$

  The two blocks always move in opposite directions with lighter block moving faster.
- Kinetic energy $KE = \frac{p^2}{2m}$ and $|\vec{p}_1| = |\vec{p}_2|$, $\frac{KE_1}{KE_2} = \frac{m_2}{m_1}$ or the kinetic energy of two blocks will not be equal but in the inverse ratio of their masses and so lighter block will have greater kinetic energy.
- Initially kinetic energy of the blocks is zero (as both are at rest) but after some time kinetic energy of the blocks is not zero (as both are in motion). So, kinetic energy is not constant but changes. Here during motion of blocks KE is converted into elastic potential energy of the spring and vice–versa but total mechanical energy of the system remain constant.

  Kinetic energy + Potential energy = Mechanical Energy = Constant

Note – If $\vec{F}$ is the average of the time varying force during collision and Δt is the duration of collision then impulse $\vec{I} = \vec{F}\Delta t$.

Conservation of Linear Momentum During Impact :

If two bodies of masses m_1 and m_2 collide in air, the total external force acting on the system of bodies ($m_1 + m_2$) is equal to $\vec{F}_1 + m_1\vec{g} + \vec{F}_2 + m_2\vec{g} \Rightarrow F_{total} = m_1\vec{g} + m_2\vec{g} + \vec{F}_1 + \vec{F}_2$

During collision the impact forces $\vec{F}_1$ and $\vec{F}_2$ are equal in magnitude and opposite in direction.

According to Newton's 3rd law of motion, $\vec{F}_1 + \vec{F}_2 = \vec{0} \Rightarrow \vec{F}_{net} = m_1\vec{g} + m_2\vec{g}$

So Impulse = $\vec{F}_{net}\Delta t = \left(m_1\vec{g} + m_2\vec{g}\right)\Delta t$

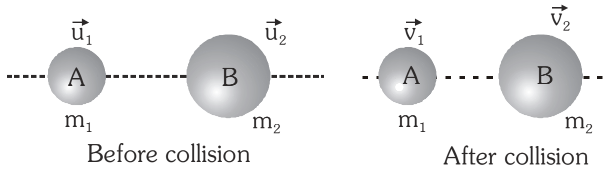
Since Δt is a very small time interval, the impulse $F(\Delta t)$ will be negligibly small. As impulse is equal to change in momentum of the system, a negligible impulse means negligible change of momentum. Let the change of momentum of 1 & 2 be $\Delta\vec{p}_1 \,\&\, \Delta\vec{p}_2$, respectively then the total change in momentum of the system $\Rightarrow \Delta\vec{p} = \Delta\vec{p}_1 + \Delta p_2 = \vec{F}_{net}.dt \approx \vec{0} \Rightarrow \Delta\left(\vec{p}_1 + \vec{p}_2\right) = 0 \Rightarrow \vec{p}_1 + \vec{p}_2 = \text{constant}.$

Therefore, the net or total momentum of the colliding bodies remains practically unchanged along the line of action (impact) during the collision. In other words, the momentum of the system remains constant or conserved during the period of impact. Therefore, we can conveniently equate the net momentum of the colliding bodies at the beginning and at the end of the collision (or just before and just after the impact).

Note : Remember that the impact force F is not an external force for the system of colliding bodies. If no external force acts on the system, its momentum remains constant for all the times including the time of collision. Even if some external forces like gravitation and friction (known as non–impulsive forces in general) are present, we can conserved the momentum of the system during the impact, because the finite external forces cannot change the momentum of the system significantly in very short time. Therefore, the change in position of the system during infinitesimal time of impact can also be neglected.

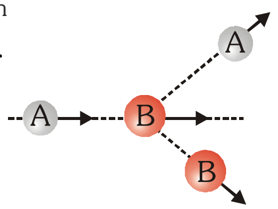
- Types of collision according to the direction of collision :

(a) **Head on collision** : Direction of velocities of bodies is similar to the direction of collision.



Before collision After collision

(b) **Oblique collision** : Direction of velocities of bodies is not similar to the direction of collision.



- Types of collision according to the conservation law of kinetic energy :

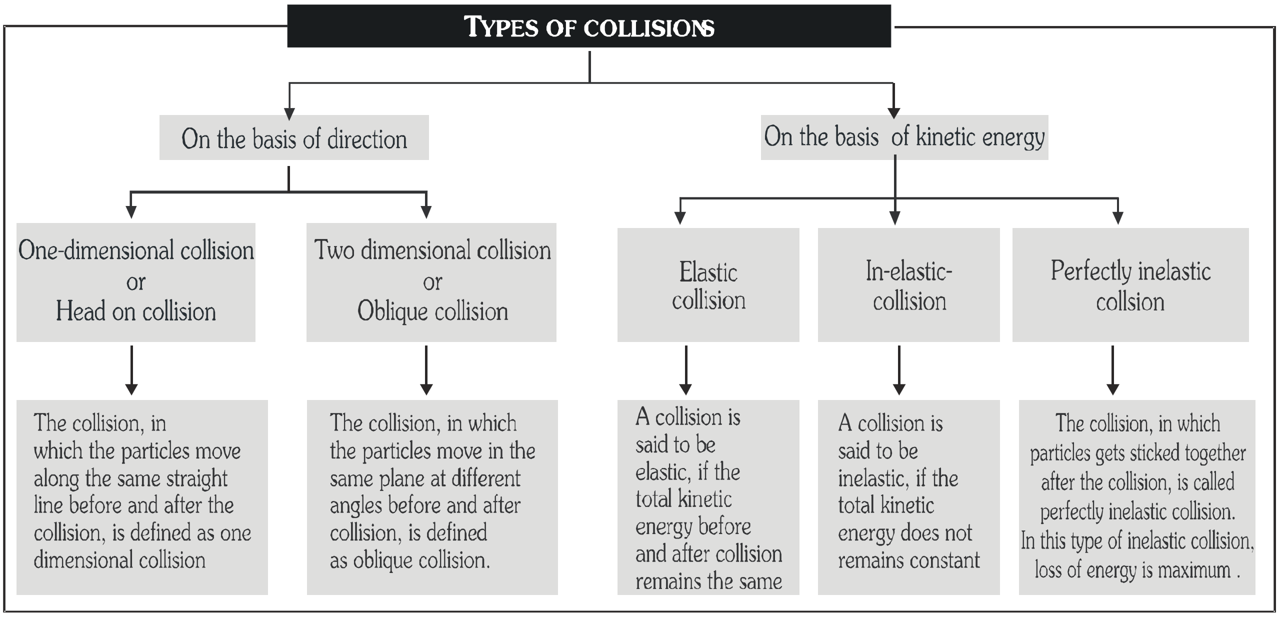
(a) **Elastic collision** : $KE_{\text{before collision}} = KE_{\text{after collision}}$

(b) **Inelastic collision** : kinetic energy is not conserved.

Some energy is lost in collision $KE_{\text{before collision}} > KE_{\text{after collision}}$

(c) **Perfect inelastic collision :** Two bodies stick together after the collision.

momentum remains conserved in all types of collisions.



The collision, in which the particles move along the same straight line before and after the collision, is defined as one dimensional collision

The collision, in which the particles move in the same plane at different angles before and after collision, is defined as oblique collision.

A collision is said to be elastic, if the total kinetic energy before and after collision remains the same

A collision is said to be inelastic, if the total kinetic energy does not remains constant

The collision, in which particles gets sticked together after the collision, is called perfectly inelastic collision. In this type of inelastic collision, loss of energy is maximum .

Coefficient of restitution (e)

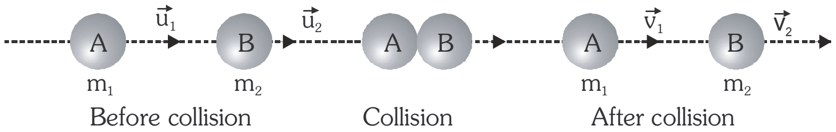
The coefficient of restitution is defined as the ratio of the impulses of recovery and deformation of either body.

$$\boxed{e = \frac{\text{impulse of recovery}}{\text{impulse of deformation}}} \qquad \boxed{e = \frac{\text{velocity of separation along line of impact}}{\text{velocity of approach along line of impact}}}$$

Value of e is 1 for elastic collision, 0 for perfectly inelastic collision and $0 < e < 1$ for inelastic collision.

HEAD ON ELASTIC COLLISION

The elastic collision in which the colliding bodies move along the same straight line path before and after the collision.



Assuming initial direction of motion to be positive and $u_1 > u_2$ (so that collision may take place) and applying law of conservation of linear momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \Rightarrow m_1(u_1 - v_1) = m_2(v_2 - u_2) \qquad ...(i)$$

For elastic collision, kinetic energy before collision must be equal to kinetic energy after collision, i.e.,

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \Rightarrow m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2) \qquad ...\text{(ii)}$$

Dividing equation (ii) by (i) $u_1 + v_1 = v_2 + u_2 \Rightarrow (u_1 - u_2) = (v_2 - v_1)$ ...(iii)

In 1–D elastic collision 'velocity of approach' before collision is equal to the 'velocity of recession' after collision, no matter what the masses of the colliding particles be.

This law is called **Newton's law for elastic collision**

Now if we multiply equation (iii) by m_2 and subtracting it from (i)

$$(m_1 - m_2)\, u_1 + 2m_2u_2 = (m_1 + m_2)v_1 \Rightarrow v_1 = \frac{m_1 - m_2}{m_1 + m_2}u_1 + \frac{2m_2}{m_1 + m_2}u_2 \qquad ...\text{(iv)}$$

Similarly multiplying equation (iii) by m_1 and adding it to equation (i)

$$2m_1u_1 + (m_2 - m_1)u_2 = (m_2 + m_1)v_2 \Rightarrow v_2 = \frac{2m_1}{m_1 + m_2}u_1 + \frac{m_2 - m_1}{m_1 + m_2}u_2 \qquad ...\text{(v)}$$

IMPORTANT POINTS

- **If the two bodies are of equal masses :** $m_1 = m_2 = m$, $v_1 = u_2$ and $v_2 = u_1$

  Thus, if two bodies of equal masses undergo elastic collision in one dimension, then after the collision, the bodies will exchange their velocities.

- **If two bodies are of equal masses and second body is at rest.**

  $m_1 = m_2$ and initial velocity of second body $u_2 = 0$, $v_1 = 0$, $v_2 = u_1$

  When body A collides against body B of equal mass at rest, the body A comes to rest and the body B moves on with the velocity of the body A. In this case transfer of energy is hundred percent

  e.g.. Billiard's Ball, Nuclear moderation.

- **If the mass of a body is negligible as compared to other.**

  If $m_1 >> m_2$ and $u_2 = 0$ then $v_1 = u_1$, $v_2 = 2u_1$

  When a heavy body A collides against a light body B at rest, the body A should keep on moving with same velocity and the body B will move with velocity double that of A.

  If $m_2 >> m_1$ and $u_2 = 0$ then $v_2 = 0$, $v_1 = -u_1$

  When light body A collides against a heavy body B at rest, the body A should start moving with same velocity just in opposite direction while the body B should practically remains at rest.

Ex. Two ball of mass 5kg each is moving in opposite directions with equal speed 5m/s. collides head on with each other. Find out the final velocities of the balls if collision is elastic.

Sol. Here $m_1 = m_2 = 5\text{kg}$, $u_1 = 5$ m/s, $u_2 = -5$ m/s

In such type of condition velocity get interchange so $v_2 = u_1 = 5$ m/s & $v_1 = u_2 = -5$ m/s

Ex. A ball of 0.1 kg makes an elastic head on collision with a ball of unknown mass that is initially at rest. If the 0.1kg ball rebounds at one third of its original speed. What is the mass of other ball ?

Sol. Here $m_1 = 0.1$ kg, $m_2 = ?$, $u_2 = 0$, $u_1 = u$, $v_1 = -u/3$

As $$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u + \frac{2m_2u_2}{m_1 + m_2} \Rightarrow -\frac{u}{3} = \left(\frac{0.1 - m_2}{0.1 + m_2}\right)u \Rightarrow m_2 = 0.2 \text{ kg}$$

HEAD ON INELASTIC COLLISION OF TWO PARTICLES

Let the coefficient of restitution for collision is e

(i) Momentum is conserved $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ ...(i)

(ii) Kinetic energy is not conserved.

(iii) According to Newton's law $\dfrac{v_2 - v_1}{u_2 - u_1} = -e$ ...(ii)

By solving eq. (i) and (ii)

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 + \left(\frac{(1+e)m_2}{m_1 + m_2}\right)u_2\ ,\ v_2 = \left(\frac{m_2 - em_1}{m_1 + m_2}\right)u_2 + \left(\frac{(1+e)m_1}{m_1 + m_2}\right)u_1$$

PERFECT INELASTIC COLLISION

In case of inelastic collision, after collision two bodies move with same velocity (or stick together).

If two particles of masses m_1 and m_2, moving with velocity u_1 and u_2 ($u_2 < u_1$) respectively along the same line collide 'head on ' and after collision they have same common velocity v, then by conservation of linear momentum,

$$m_1u_1 + m_2u_2 = m_1v + m_2v \Rightarrow v = \frac{m_1u_1 + m_2u_2}{(m_1 + m_2)} \quad ...(i)$$

Kinetic energy of the system before collision is $KE_i = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$

And after collision is $KE_f = \frac{1}{2}(m_1+m_2)v^2$

Loss in KE during collision

$$\Delta KE = KE_i - KE_f = \left[\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right] - \frac{1}{2}(m_1 + m_2)v^2 \quad ...(ii)$$

Substituting the value of v from eq. (i),

$$\Delta KE = \frac{1}{2}[(m_1u_1^2 + m_2u_2^2) - \frac{(m_1u_1 + m_2u_2)^2}{(m_1 + m_2)}]$$

$$\Rightarrow \Delta KE = \frac{1}{2}\left[\frac{m_1m_2(u_1^2 + u_2^2 - 2u_1u_2)}{(m_1 + m_2)}\right] \Rightarrow \boxed{\Delta KE = \frac{1}{2}\frac{m_1m_2}{(m_1 + m_2)}(u_1 - u_2)^2}$$

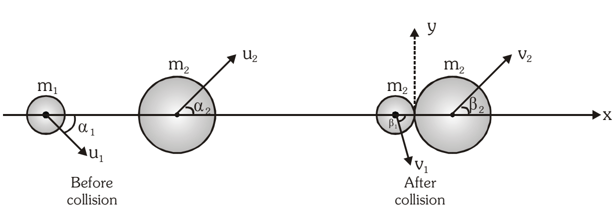
If the target is initially at rest $u_2 = 0$ and $u_1 = u$

$$\Delta KE = \frac{m_1m_2}{2(m_1 + m_2)}u^2,\quad \frac{\Delta KE}{KE_i} = \frac{m_2}{(m_1 + m_2)}\quad [\because KE_i = \frac{1}{2}m_1u_1^2]$$

Now if target is massive, i.e., $m_2 >> m_1$ then $\dfrac{\Delta KE}{KE_i} \approx 1$ so percentage loss in KE = 100%

i.e., if a light moving body strikes a heavy target at rest and sticks to it, practically all its KE is lost.

Oblique Collision



In oblique impact the relative velocity of approach of the bodies doesn't coincide with the line of impact. Conserving the momentum of system in directions along normal (x axis in our case) and tangential (y axis in our case)

$m_1u_1\cos\alpha_1 + m_2u_2\cos\alpha_2 = m_1v_1\cos\beta_1 + m_2v_2\cos\beta_2$ and $m_2u_2\sin\alpha_2 - m_1u_1\sin\alpha_1 = m_2v_2\sin\beta_2 - m_1v_1\sin\beta_1$

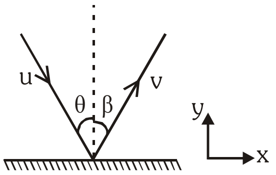
Since no force is acting on m_1 and m_2 along the tangent (i.e. y–axis) the individual momentum of m_1 and m_2 remains conserved. $m_1u_1\sin\alpha_1 = m_1v_1\sin\beta_1$ & $m_2u_2\sin\alpha_2 = m_2v_2\sin\beta_2$

By using Newton's experimental law along the line of impact $e = \dfrac{v_2\cos\beta_2 - v_1\cos\beta_1}{u_1\cos\alpha_1 - u_2\cos\alpha_2}$

Oblique Impact on a Fixed Plane

Let a small ball collides with a smooth horizontal floor with a speed u at an angle θ to the vertical as shown in the figure. Just after the collision, let the ball leaves the floor with a speed v at angle β to vertical.

It is quite clear that the line of action is perpendicular to the floor. Therefore, the impact takes place along the (normal) vertical. Now we can use Newton's experimental law as



$$e = \frac{\text{velocity of separation}}{\text{velocity of approach}}$$

$\Rightarrow$ e [velocity of approach] = velocity of separation

$\Rightarrow$ $e[u\cos\theta\,(-\hat{j})] = -[v\cos\beta\,(+\hat{j})] \Rightarrow v\cos\beta = e\,u\cos\theta$ ...(i)

Since impulsive force N acts on the body along the normal, we cannot conserve its momentum. Since along horizontal the component of N is zero, therefore we can conserve the horizontal momentum of the body.

Momentum $(p_x)_{\text{body}} = \text{constant} \Rightarrow (p_x)_{\text{initial}} = (p_x)_{\text{final}}$

$\Rightarrow$ $m\,u\sin\theta = mv\sin\beta \Rightarrow v\sin\beta = u\sin\theta$ ..(ii)

Squaring equations(i) and (ii) and adding, $v^2\cos^2\beta + v^2\sin^2\beta = e^2u^2\cos^2\theta + u^2\sin^2\theta$

$\Rightarrow$ $v^2 = u^2[e^2\cos^2\theta + \sin^2\theta]$ $\Rightarrow v = u\sqrt{\sin^2\theta + e^2\cos^2\theta}$

Dividing equation (i) by (ii)

$$\Rightarrow \frac{v\cos\beta}{v\sin\beta} = \frac{eu\cos\theta}{u\sin\theta} \Rightarrow \cot\beta = e\cot\theta \Rightarrow \beta = \cot^{-1}(e\cot\theta)$$

Impulse of the blow = change of momentum of the body

$= \{(mv\sin\beta)\,\hat{i} + (mv\cos\beta)\,\hat{j}\} - \{(mu\sin\theta)\,\hat{i} - (mu\cos\theta)\,\hat{j}\}$

$= (mv\sin\beta - mu\sin\theta)\,\hat{i} + (mv\cos\beta + mu\cos\theta)\,\hat{j}$

Since $v\sin\beta = u\sin\theta \Rightarrow$ Impulse $= m(v\cos\beta + u\cos\theta)\,\hat{j}$

Putting $v\cos\beta = eu\cos\theta$ from eq. (i),

Impulse $= m(1+e)\,u\cos\theta\hat{j}$ $\quad\therefore$ Magnitude of the impulse $= m(1+e)\,u\cos\theta$

Change in Kinetic energy: $\Delta K.E. = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

Putting the value of v we obtain

$$\Delta KE = \frac{1}{2}m\left[\left[\sqrt{u\left(\sin^2\theta + e^2\cos^2\theta\right)}\right]^2 - u^2\right] = \frac{1}{2}mu^2\,[\sin^2\theta + e^2\cos^2\theta - 1]$$

$$= -\frac{1}{2}mu^2\,[\cos^2\theta - e^2\cos^2\theta] = -\frac{1}{2}(1-e^2)\,mu^2\cos^2\theta$$

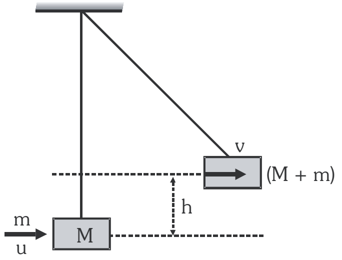
Negative sign indicates the loss of kinetic energy

IMPORTANT POINTS

- Momentum remains conserved in all types of collisions.
- Total energy remains conserved in all types of collisions.
- Only conservative forces works in elastic collisions.
- In inelastic collisions all the forces are not conservative.

Ex. A simple pendulum of length 1m has a wooden bob of mass 1kg. It is struck by a bullet of mass 10^{-2}kg moving with a speed of 2×10^2 m/s. The bullet gets embedded into the bob. Obtain the height to which the bob rises before swinging back.

Sol. Applying principle of conservation of linear momentum



$$mu = (M+m)v \Rightarrow 10^{-2} \times (2 \times 10^2) = (1 + .01)v \Rightarrow v = \frac{2}{1.01}$$

KE_i of the block with bullet in it, is converted into P.E. as it rises through a height h

$$\frac{1}{2}(M+m)v^2 = (M+m)gh \Rightarrow v^2 = 2gh \Rightarrow h = \frac{v^2}{2g} = \left(\frac{2}{1.01}\right)^2 \times \frac{1}{2 \times 9.8} = 0.2\text{ m}$$

Ex. A body falling on the ground from a height of 10m, rebounds to a height 2.5m calculate

(i) The percentage loss in K.E.

(ii) Ratio of the velocities of the body just before and just after the collision.

Sol. Let v_1 and v_2 be the velocity of the body just before and just after the collision

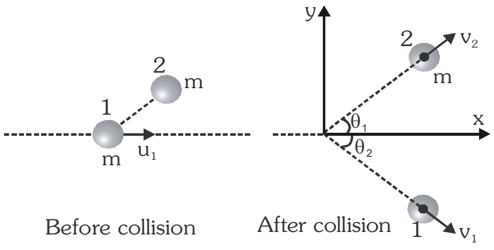
$$KE_1 = \frac{1}{2}mv_1^2 = mgh_1 \text{...(i) and } KE_2 = \frac{1}{2}mv_2^2 = mgh_2 \text{ ...(ii)}$$

$$\Rightarrow \quad \frac{v_1^2}{v_2^2} = \frac{h_1}{h_2} = \frac{10}{2.5} = 4 \Rightarrow \frac{v_1}{v_2} = 2$$

$$\text{Percentage loss in KE} = \frac{mg(h_1 - h_2)}{mgh_1} \times 100 = \frac{10 - 2.5}{10} \times 100 = 75\%$$

Ex. A body strikes obliquely with another identical stationary rest body elastically. Prove that they will move perpendicular to each other after collision.

Sol.



Conservation of linear momentum in x–direction gives

$mu_1 = mv_1\cos\theta_1 + mv_2\cos\theta_2 \Rightarrow u_1 = v_1\cos\theta_1 + v_2\cos\theta_2$ ... (i)

Conservation of linear momentum in y–direction gives

$0 = mv_1\sin\theta_1 - mv_2\sin\theta_2 \Rightarrow 0 = v_1\sin\theta_1 - v_2\sin\theta_2$ ...(ii)

Conservation of kinetic energy

$$\frac{1}{2}mu_1^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 \Rightarrow u_1^2 = v_1^2 + v_2^2 \quad ...(iii)$$

$(i)^2 + (ii)^2$

$\Rightarrow u_1^2 + 0 = v_1^2\cos^2\theta_1 + v_2^2\cos^2\theta_2 + 2v_1v_2\cos\theta_1\cos\theta_2 + v_1^2\sin^2\theta_1 + v_2^2\sin^2\theta_2 - 2v_1v_2\sin\theta_1\sin\theta_2$

$\Rightarrow u_1^2 = v_1^2(\cos^2\theta_1 + \sin^2\theta_1) + v_2^2(\cos^2\theta_2 + \sin^2\theta_2) + 2v_1v_2(\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2)$

$\Rightarrow u_1^2 = v_1^2 + v_2^2 + 2v_1v_2\cos(\theta_1 + \theta_2)$ $\{\because u_1^2 = v_1^2 + v_2^2\}$

$\Rightarrow \cos(\theta_1 + \theta_2) = 0 \Rightarrow \theta_1 + \theta_2 = 90°$

Ex. A steel ball is dropped on a smooth horizontal plane from certain height h. Assuming coefficient of restitution of impact as e, find the average speed of the ball till it stops.

Sol. Since the ball falls through a height h, just before the first impact its speed v will be given as $v = \sqrt{2gh}$. Let its speed the v_1 just after the first impact.Then, Newton's experimental formula yields,

$$\frac{0 - v_1}{v} = e \Rightarrow v_1 = ev$$

Similarly, its speed just before 2nd impact, $v_1 = ev = e\sqrt{2gh}$

Speed just after n^{th} impact, $v_n = e^n v = e^n\sqrt{2gh}$

The maximum height attained after 1^{st} impact $= h_1 = \frac{v^2}{2g} = (e\sqrt{2gh})^2 = e^2h$. Similarly, the maximum height attained after 2nd impact, $h_2 = e^4h$. Hence, the maximum height attained after n^{th} impact $= e^{2n}h$

The ball experiences infinite impacts till it becomes stationary. ⇒ The total distance covered,

$d = h + 2h_1 + 2h_2 + ... d = h + 2e^2h + 2e^4h + ... = h[1 + 2(e^2 + e^4 + e^6 + ...)]$

$$= \left[1 + 2\left(\frac{e^2}{1 - e^2}\right)\right]h = \left(\frac{1 + e^2}{1 - e^2}\right)h.$$

The total time taken by the ball till it stops bouncing $T = \sqrt{\frac{2h}{g}} + 2\sqrt{\frac{2h_1}{g}} + 2\sqrt{\frac{2h_2}{g}} + ...$

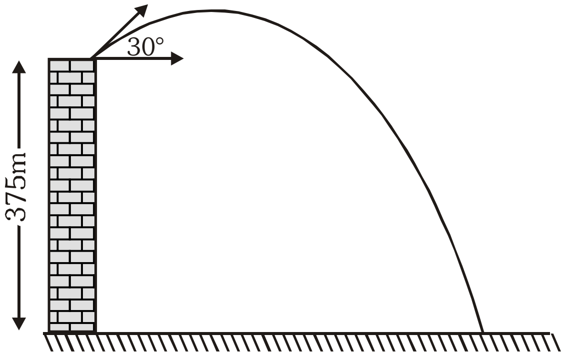
Putting $h_1 = e^2h$, $h_2 = e^4h$, $T = \sqrt{\frac{2h}{g}} + 2\sqrt{\frac{2e^2h}{g}} + 2\sqrt{\frac{2e^4h}{g}} + ...$

$\Rightarrow T = \sqrt{\frac{2h}{g}}\,[1 + 2(e + e^2 + ...)] = \sqrt{\frac{2h}{g}}\left[1 + \frac{2e}{1-e}\right] = \frac{1+e}{1-e}\sqrt{\frac{2h}{g}}$

Therefore, average speed of the ball for its total time of motion, $\vec{v} = \frac{\text{total distance}}{\text{total time}} = \frac{d}{T}$

Putting the values of d and T, we obtain $\vec{v} = \frac{1+e^2}{(1+e)^2}\sqrt{\frac{gh}{2}}$

Ex. A particle of mass 1 kg is projected from a tower of height 375m with initial velocity 100 ms^{-1} at an angle 30° with the horizontal. Find out its kinetic energy in joule just after collision with ground if collision is inelastic with $e = \frac{1}{2}$ $(g = 10\ ms^{-2})$



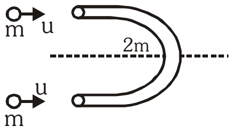
Sol. $v_y^2 = u_y^2 + 2gh \Rightarrow v_y = \sqrt{(50)^2 + 2 \times 10 \times 375} = 100ms^{-1}$

Horizontal velocity just after collision = $50\sqrt{3}$ ms^{-1}

Vertical velocity just after collision = $100 \times \frac{1}{2} = 50\ ms^{-1}$

Kinetic energy just after collision = $\frac{1}{2} \times 1 \times \left[(50\sqrt{3})^2 + (50)^2\right] = 5000J$

Ex A U shaped tube of mass 2m is placed on a horizontal surface. Two spheres each of diameter d (just less than the inner diameter of tube) and mass m enter into the tube with a velocity u as shown in figure. Taking all collisions to be elastic and all surfaces smooth. Match the following-



Column-I	Column-II
(A) The speed of the tube with respect to ground, when spheres are just about to collide inside the tube.	(p) u
(B) The speed of spheres when spheres are just about to collide.	(q) u/2
(C) The speed of the spheres when they comes out the tube.	(r) $\frac{\sqrt{3}}{2}u$
(D) The speed of the tube when spheres comes out the	(s) zero

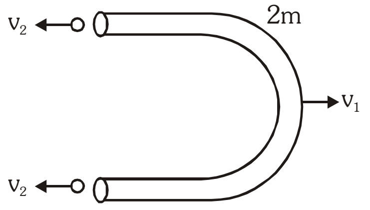
Sol. **For (A)** From conservation of linear momentum $2mu = (m + m)v \Rightarrow v = \frac{u}{2}$

For (B) Let v_1 be the velocity of spheres w.r.t. tube when they are just about to collide then by using conservation of kinetic energy $\frac{1}{2}(2m)u^2 = \frac{1}{2}(4m)\left(\frac{u}{2}\right)^2 + 2\frac{1}{2}mv_1^2$

$$\Rightarrow v_1 = \frac{u}{\sqrt{2}}$$

$$\Rightarrow \text{Required speed of spheres} = \sqrt{\left(\frac{u}{2}\right)^2 + \left(\frac{u}{\sqrt{2}}\right)^2} = \sqrt{\frac{u^2}{4} + \frac{u^2}{2}} = \frac{\sqrt{3}u}{2}$$

For (C) $2mu = 2mv_1 - 2mv_2$



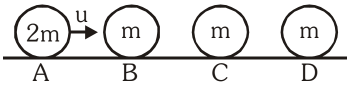
$$2 \times \frac{1}{2}mu^2 = 2 \times \frac{1}{2}mv_2^2 + \frac{1}{2}(2m)v_1^2$$

$$\Rightarrow u^2 = v_1^2 + v_2^2 - 2v_1v_2 \ \& \ u^2 = v_1^2 + v_2^2$$

$$\Rightarrow v_1v_2 = 0 \text{ but } v_1 \neq 0 \text{ so } v_2 = 0$$

For (D) Speed of tube $v_1 = u$

Ex. Four balls A,B,C and D are kept on a smooth horizontal surface as shown in figure. Ball A is given velocity u towards B-

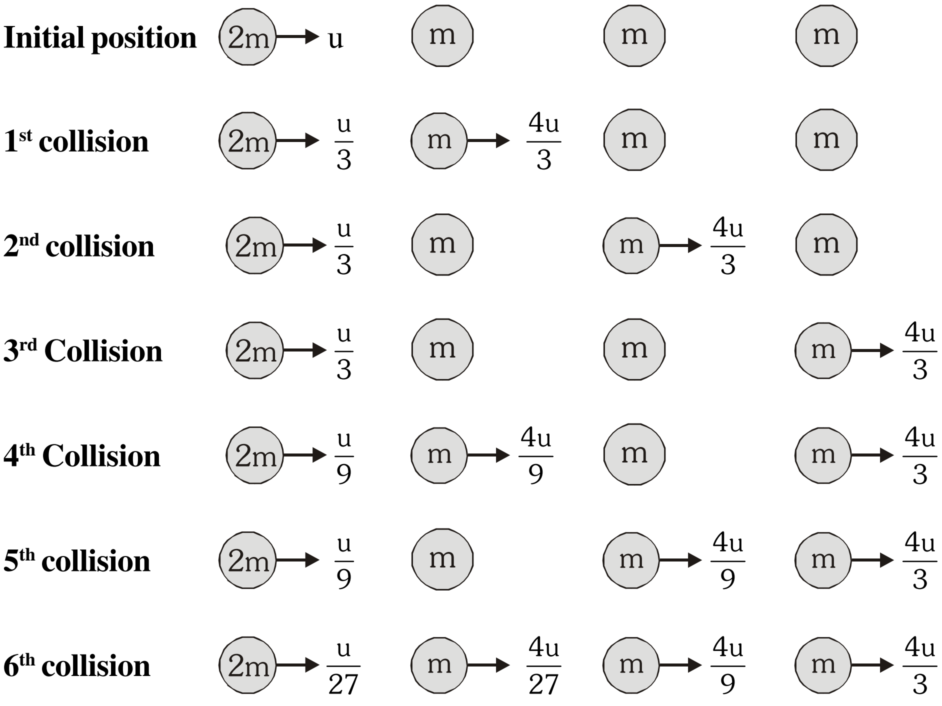


	Column-I		**Column-II**
(A)	Total impulse of all collisions on A	(p)	$\frac{4mu}{9}$
(B)	Total impulse of all collisions on B	(q)	$\frac{4mu}{27}$
(C)	Total impulse of all collision on C	(r)	$\frac{4mu}{3}$
(D)	Total impulse of all collisions on D	(s)	$\frac{52}{27}mu$

Sol. In 1st collision between A & B

$$2mu = 2mv_A + 2mv_B \ \& \ e = 1 = \frac{v_B - v_A}{u} \Rightarrow v_A = \frac{u}{3}, v_B = \frac{4u}{3}$$

Situation of all collisions is shown in figure.



For (A) Total impulse on A $= 2m\left(u - \frac{u}{27}\right) = \frac{52}{27}mu$

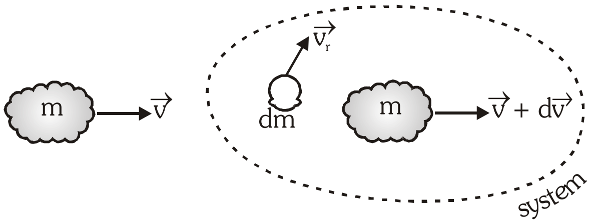
For (B) Total impulse on B $= m\left(\frac{4u}{27} - 0\right) = \frac{4}{27}mu$

For (C) Total impulse on C $= m\left(\frac{4u}{9} - 0\right) = \frac{4}{9}mu$

For (D) Total impulse on D $= m\left(\frac{4u}{3} - 0\right) = \frac{4}{3}mu$

Variable mass system:

In previous discussion of the conservation of linear momentum, we assume that system's mass remains constant. Now we are consider those system whose mass is variable i.e. those in which mass enters or leaves the system. Suppose at some moment $t = t$ mass of a body is m and its velocity is $\vec{v}$. After some time at $t=t + dt$ its mass becomes $(m–dm)$ and velocity becomes $\vec{v} + d\vec{v}$. The mass dm is ejected with relative velocity $\vec{v}_r$.



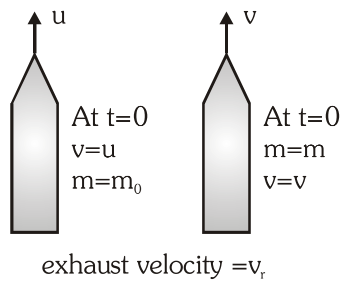
If no forces are acting on the system then the linear momentum of the system will remain conserved.

$$\Rightarrow \quad \vec{F}_{ex}\,dt = (m - dm)\left(\vec{v} + d\vec{v}\right) + dm\left(\vec{v}_r + \vec{v} + d\vec{v}\right) - m\vec{v}$$

$$\because \quad F_{ex} = 0 \Rightarrow m d\vec{v} = -\vec{v}_r dm \Rightarrow m\left(\frac{d\vec{v}}{dt}\right) = \vec{v}_r\left(-\frac{dm}{dt}\right)$$

Rocket propulsion :

Thrust force on the rocket $= v_r\left(-\frac{dm}{dt}\right)$



So for motion of rocket $m\frac{dv}{dt} = v_r\left(-\frac{dm}{dt}\right) - mg$

$$\Rightarrow dv = v_r\left(-\frac{dm}{m}\right) - gdt \Rightarrow \int_{u_0}^{v} dv = -v_r\int_{m_0}^{m}\frac{dm}{m} - g\int_0^t dt$$

$$\Rightarrow v - u = v_r \ell n\left(\frac{m_0}{m}\right) - gt \Rightarrow v = u - gt + v_r \ell n\left(\frac{m_0}{m}\right)$$

Ex An open topped rail road car of mass M has an initial velocity v_0 along a straight horizontal frictionless track. It suddenly starts raining at time t=0. The rain drops fall vertically with velocity u and add a mass m kg/sec of water. Find the velocity of car after t second (assuming that it is not completely filled with water).

Sol. According to law of conservation of momentum, $Mv_0 = (M + m \times t)\,v$. Where m is the mass of water added per second and v is the velocity of the car after t second. $\therefore\ v = \dfrac{Mv_0}{M+mt}$

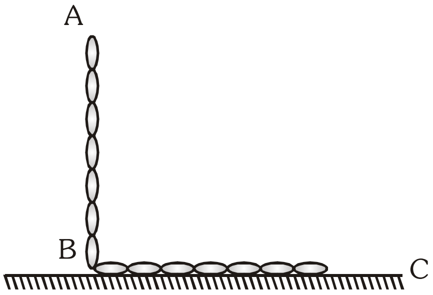
Ex. A uniform chain of mass m and length ℓ hangs on a thread and touches the surface of a table by its lower end. Find the force exerted by the chain on the table when half of its length has fallen on the table. The fallen part does not form heap.

Sol. At given condition force exerted by the chain on the table consists of two parts

(i) Weight of portion BC $= \dfrac{mg}{2}$

(ii) Thrust force $= v_r\left(-\dfrac{dm}{dt}\right) = v\left(\dfrac{m}{\ell}v\right) = \dfrac{m}{\ell}v^2$

but $v = \sqrt{2g\left(\dfrac{\ell}{2}\right)} = \sqrt{g\ell}$



$\Rightarrow$ Thrust force $= \dfrac{m}{\ell}(g\ell) = mg$

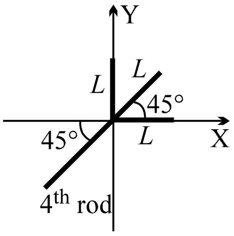
$\therefore$ Net force exerted by falling chain $= \dfrac{mg}{2} + mg = \dfrac{3mg}{2}$

EXERCISE (S-1)

1. Four particles of mass 5, 3, 2, 4 kg are at the points (1, 6), (–1, 5), (2, –3), (–1, –4). Find the coordinates of their centre of mass. **CM0001**

2. A rigid body consists of a 3 kg mass connected to a 2 kg mass by a massless rod. The 3kg mass is located at $\vec{r}_1 = (2\hat{i} + 5\hat{j})$ m and the 2 kg mass at $\vec{r}_2 = (4\hat{i} + 2\hat{j})$ m. Find the length of rod and the coordinates of the centre of mass. **CM0002**

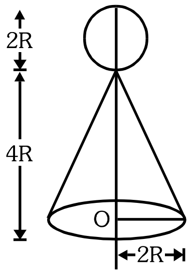
3. Three identical uniform rods of the same mass *M* and length *L* are arranged in xy plane as shown in the figure. A fourth uniform rod of mass *3M* has been placed as shown in the xy plane. What should be the value of the length of the fourth rod such that the center of mass of all the four rods lie at the origin?



CM0003

4. From a circle of radius a, an isosceles right angled triangle with the hypotenuse as the diameter of the circle is removed. The distance of the centre of gravity of the remaining position from the centre of the circle is **CM0004**

5. A man has constructed a toy as shown in figure. If density of the material of the sphere is 12 times of the cone compute the position of the centre of mass. [Centre of mass of a cone of height h is at height of h/4 from its base.]



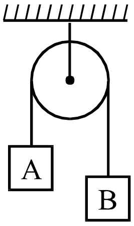
CM0005

6. The figure shows the positions and velocities of two particles. If the particles move under the mutual attraction of each other, then find the position of centre of mass at t = 1 s.

CM0006

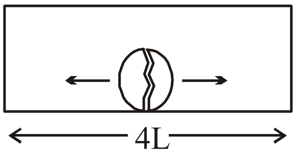
7. Mass centers of a system of three particles of masses 1, 2, 3 kg is at the point (1 m, 2 m, 3 m) and mass center of another group of two particles of masses 2 kg and 3 kg is at point (−1 m, 3 m, −2 m). Where a 5 kg particle should be placed, so that mass center of the system of all these six particles shifts to mass center of the first system? **CM0007**

8. In the arrangement shown in the figure, $m_A = 2$ kg and $m_B = 1$ kg. String is light and inextensible. Find the acceleration of centre of mass of both the blocks. Neglect friction everywhere.



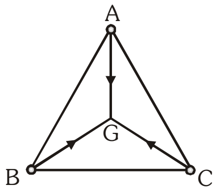
CM0008

9. A bomb of mass 3m is kept inside a closed box of mass 3m and length 4L at it's centre. It explodes in two parts of mass m & 2m. The two parts move in opposite direction and stick to the opposite side of the walls of box. Box is kept on a smooth horizontal surface. What is the distance moved by the box during this time interval.



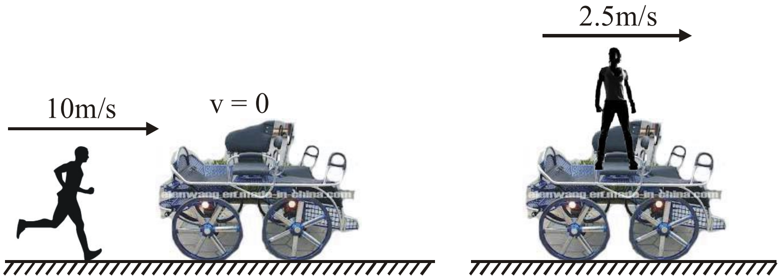
CM0009

10. Three particles A, B and C of equal mass move with equal speed v along the medians of an equilateral triangle as shown in fig. They collide at the centroid G of the triangle. After the collision, A comes to rest, B retraces its path with the speed v. What is the velocity of C ?



CM0010

11. A 50 kg boy runs at a speed of 10 m/s and jumps onto a cart as shown in the figure. The cart is initially at rest. If the speed of the cart with the boy on it is 2.50 m/s, what is the mass of the cart ? (Assuming friction is absent between cart and ground)



CM0011

12. Two cars initially at rest are free to move in the x direction. Car A has mass 4 kg and car B has mass 2 kg. They are tied together, compressing a spring in between them. When the spring holding them together is burned, car A moves off with a speed of 2 m/s.

(i) With what speed does car B leave.

(ii) How much energy was stored in the spring before it was burned. **CM0012**

13. A 24 kg projectile is fired at an angle of 53° above the horizontal with an initial speed of 50 m/s. At the highest point in its trajectory, the projectile explodes into two fragments of equal mass, the first of which falls vertically with zero initial speed.

(i) How far from the point of firing does the second fragment strike the ground? (Assume the ground is level.)

(ii) How much energy was released during the explosion? **CM0013**

14. A particle of mass m, moving in a circular path of radius R with a constant speed v_2 is located at point (2R, 0) at time t = 0 and a man starts moving with a velocity v_1 along the +ve y-axis from origin at time t = 0. Calculate the linear momentum of the particle w.r.t. the man as a function of time.

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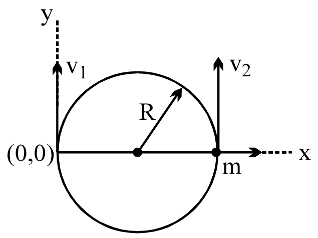
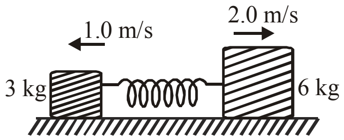


CM0014

15. A spaceship is moving with constant speed v_0 in gravity free space along +Y-axis suddenly shoots out one third of its part with speed $2v_0$ along + X-axis. Find the speed of the remaining part.

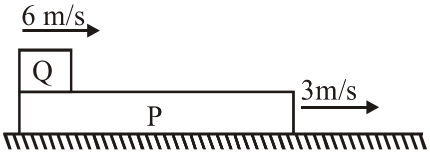
CM0015

16. Two blocks of mass 3 kg and 6 kg respectively are placed on a smooth horizontal surface. They are connected by a light spring of force constant k = 200 N/m. Initially the spring is unstretched. The indicated velocities are imparted to the blocks. The maximum extension of the spring will be :-



CM0016

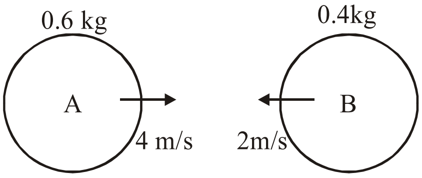
17. A plank P and block Q are arranged as shown on a smooth table top. They are given velocities 3 m/s and 6 m/s respectively. The length of plank is 1m and block is of negligible size. After some time when the block has reached the other end of plank it stops slipping on plank. Find the coefficient of friction between plank P and block Q if mass of plank is double of block).



CM0017

18. A bullet of mass m strikes an obstruction and deviates off at 60° to its original direction. If its speed is also changed from u to v, find the magnitude of the impulse acting on the bullet. **CM0018**

19. The velocities of two steel balls before impact are shown. If after head on impact the velocity of ball B is observed to be 3 m/s to the right, the coefficient of restitution is

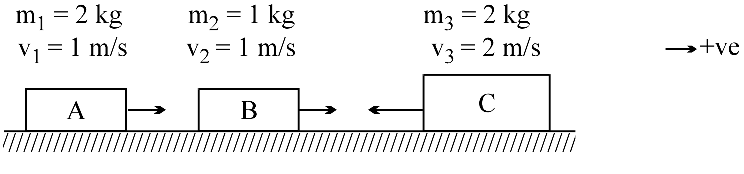


CM0019

20. Three carts move on a frictionless track with inertias and velocities as shown. The carts collide and stick together after successive collisions.

(i) Find loss of mechanical energy when B & C stick together.

(ii) Find magnitude of impulse experienced by A when it sticks to combined mass (B & C).

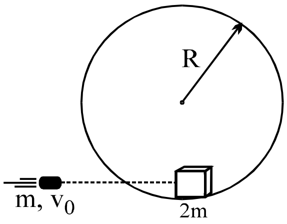


CM0020

21. A small block of mass 2m initially rests at the bottom of a fixed circular, vertical track, which has a radius of R. The contact surface between the mass and the loop is frictionless. A bullet of mass m strikes the block horizontally with initial speed v_0 and remain embedded in the block as the block and the bullet circle the loop. Determine each of the following in terms of m, v_0, R and g.

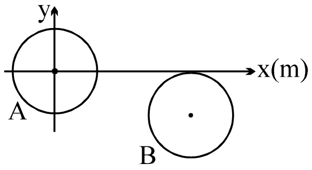
(i) The speed of the masses immediately after the impact.

(ii) The minimum initial speed of the bullet if the block and the bullet are to successfully execute a complete ride on the loop



CM0021

22. Two smooth balls A and B, each of mass m and radius R, have their centres at (0,0,R) and at (5R,–R,R) respectively, in a coordinate system as shown. Ball A, moving along positive x axis, collides with ball B. Just before the collision, speed of ball A is 4 m/s and ball B is stationary. The collision between the balls is elastic. Find Velocity of the ball A just after the collision and impulse of the force exerted by A on B during the collision.

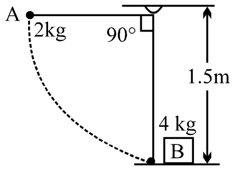


CM0022

23. A sphere A is released from rest in the position shown and strikes the block B which is at rest. If e = 0.75 between A and B and $\mu_k = 0.5$ between B and the support, determine

(i) the velocity of A just after the impact

(ii) the maximum displacement of B after the impact.



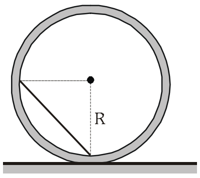
CM0023

24. Bullets of mass 10 g each are fired from a machine gun at rate of 60 bullets/minute. The muzzle velocity of bullets is 100 m/s. The thrust force due to firing bullets experienced by the person holding the gun stationary is _________. **CM0024**

EXERCISE (S-2)

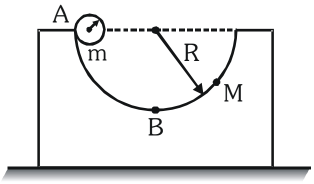
1. The linear mass density of a ladder of length ℓ increases uniformly from one end A to the other end B, (i) Form an expression for linear mass density as a function of distance x from end A where linear mass density λ_0. The density at one end being twice that of the other end. (ii) Find the position of the centre of mass from end A. **CM0025**

2. Inside a hollow uniform sphere of inner radius R a uniform rod of length $R\sqrt{2}$ is released from the state of rest as shown. The mass of the rod is same as that of the sphere. Assume friction to be absent everywhere. Horizontal displacement of sphere with respect to earth in the time in which the rod becomes horizontal, is



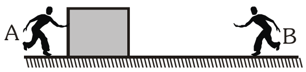
CM0026

3. A block of mass M with a semicircular track of radius R, rests on a horizontal frictionless surface. A uniform cylinder of radius r and mass m is released from rest at the top point A (see Fig). The cylinder slips on the semicircular frictionless track. How far has the block moved when the cylinder reaches the bottom (point B) of the track ? How fast is the block moving when the cylinder reaches the bottom of the track?



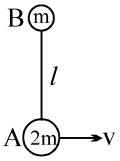
CM0027

4. Two persons A and B each of mass 100 kg are on a frictionless horizontal surface. Person A at rest is holding a block of mass 50 kg, suddenly pushes the block with some velocity (v) towards the person B approaching at a velocity of 5 m/s. B catches the block & slows down. Now the separation between A and B becomes constant. Find the speed v (in m/s).



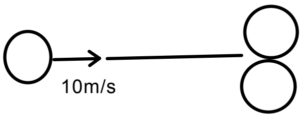
CM0028

5. Two masses A and B connected with an inextensible string of length ℓ lie on a smooth horizontal plane. A is given a velocity of v m/s along the ground perpendicular to line AB as shown in figure. Find the tension in string during their subsequent motion.



CM0029

6. A ball with initial speed of 10 m/s collides elastically with two other identical ball whose centres are on a line perpendicular to the initial velocity and which are initially in contact with each other. All the three ball are lying on a smooth horizontal table. The first ball is aimed directly at the contact point of the other two balls All the balls are smooth. Find the velocities of the three balls after the collision.



CM0030

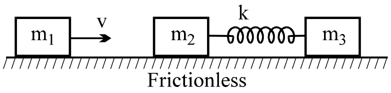
7. Mass m_1 hits & sticks with m_2 while sliding horizontally with velocity v along the common line of centres of the three equal masses ($m_1 = m_2 = m_3 = m$). Initially masses m_2 and m_3 are stationary and the spring is unstretched. Find the

(i) velocities of m_1, m_2 and m_3 immediately after impact.

(ii) maximum kinetic energy of m_3.

(iii) minimum kinetic energy of m_2.

(iv) maximum compression of the spring.



CM0031

8. A sphere of mass m is moving with a velocity $4\hat{i}-\hat{j}$ when it hits a smooth wall and rebounds with velocity $\hat{i}+3\hat{j}$. Find the impulse it receives. Find also the coefficient of restitution between the sphere and the wall. **CM0032**

9. Two particles A and B of mass 2m and m respectively are attached to the ends of a light inextensible string of length 4a which passes over a small smooth peg at a height 3a from an inelastic table. The system is released from rest with each particle at a height a from the table. Find

(i) The speed of B when A strikes the table.

(ii) The time that elapses before A first hits the table.

(iii) The time for which A is resting on the table after the first collision & before it is first jerked off.

CM0033

10. Two particles, each of mass m, are connected by a light inextensible string of length 2ℓ. Initially they lie on a smooth horizontal table at points A and B distant ℓ apart. The particle at A is projected across the table with velocity u. Find the speed with which the second particle begins to move if the direction of u is :-

(i) along BA.

(ii) at an angle of 120° with AB.

(iii) perpendicular to AB.

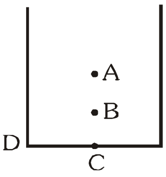
In each case calculate (in terms of m & u) the impulsive tension in the string.

CM0034

EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

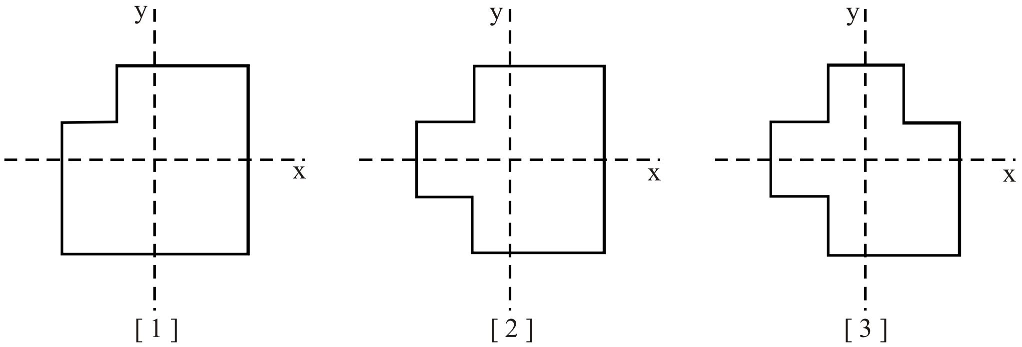
1. A thick uniform wire is bent into the shape of the letter "U" as shown. Which point indicates the location of the center of mass of this wire? A is the midpoint of the line joining mid points of two parallel sides of 'U' shaped wire.



(A) D (B) A (C) B (D) C

CM0035

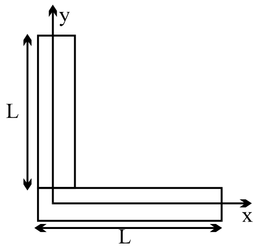
2. A machinist starts with three identical square plates but cuts one corner from one of them, two corners from the second, and three corners from the third. Rank the three plates according to the x-coordinate of their centers of mass, from smallest to largest.



(A) 3, 1, 2 (B) 1, 3, 2 (C) 3, 2, 1 (D) 1 and 3 tie, then 2

CM0036

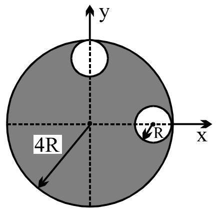
3. Centre of mass of two thin uniform rods of same length but made up of different materials & kept as shown, can be, if the meeting point is the origin of co-ordinates



(A) (L/2, L/2) (B) (2L/3, L/2) (C) (L/3, L/3) (D) (L/3, L/6)

CM0037

4. From the circular disc of radius 4R two small disc of radius R are cut off. The centre of mass of the new structure will be :



(A) $i\frac{R}{5}+j\frac{R}{5}$ (B) $-i\frac{R}{5}+j\frac{R}{5}$ (C) $\frac{-3R}{14}(\hat{i}+\hat{j})$ (D) None of these

CM0038

5. Seven identical birds are flying south together at constant velocity. A hunter shoots one of them, which immediately dies and falls to the ground. The other six continue flying south at the original velocity. After the one bird has hit the ground, the centre of mass of all seven birds

(A) continues south at the original speed, but is now located some distance behind the flying birds

(B) continues south, but at 6/7 the original velocity

(C) continues south, but at 1/7 the original velocity

(D) stops with the dead bird

CM0039

6. A man weighing 80 kg is standing at the centre of a flat boat and he is 20 m from the shore. He walks 8 m on the boat towards the shore and then halts. The boat weight 200 kg. How far is he from the shore at the end of this time ?

(A) 11.2 m (B) 13.8 m (C) 14.3 m (D) 15.4 m

CM0040

7. There are some passengers inside a stationary railway compartment. The track is frictionless. The centre of mass of the compartment itself (without the passengers) is C_1, while the centre of mass of the 'compartment plus passengers' system is C_2. If the passengers move about inside the compartment along the track.

(A) both C_1 and C_2 will move with respect to the ground.

(B) neither C_1 nor C_2 will move with respect to the ground.

(C) C_1 will move but C_2 will be stationary with respect to the ground.

(D) C_2 will move but C_1 will be stationary with respect to the ground.

CM0041

8. A non–zero external force acts on a system of particles. The velocity and acceleration of the centre of mass are found to be v_0 and a_C respectively at any instant t. It is possible that

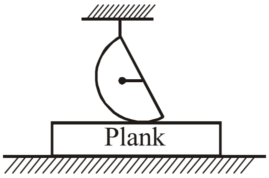
(i) $v_0 = 0, a_C = 0$ (ii) $v_0 \neq 0, a_C = 0$ (iii) $v_0 = 0, a_C \neq 0$ (iv) $v_0 \neq 0, a_C \neq 0$

Then

(A) (iii) and (iv) are true.
(B) (i) and (ii) are true.
(C) (i) and (iii) are true.
(D) (ii), (iii) and (iv) are true.

CM0042

9. Lower surface of a plank is rough and lying at rest on a rough horizontal surface. Upper surface of the plank is smooth and has a smooth hemisphere placed over it through a light string as shown in the figure. After the string is burnt, trajectory of centre of mass of the sphere is :-



(A) a circle
(B) an ellipse
(C) a straight line
(D) a parabola

CM0043

10. Three interacting particles of masses 100 g, 200 g and 400 g each have a velocity of 20 m/s magnitude along the positive direction of x-axis, y-axis and z-axis. Due to force of interaction the third particle stops moving. The velocity of the second particle is $\left(10\hat{j}+5\hat{k}\right)$. What is the velocity of the first particle?

(A) $20\hat{i}+20\hat{j}+70\hat{k}$
(B) $10\hat{i}+20\hat{j}+8\hat{k}$
(C) $30\hat{i}+10\hat{j}+7\hat{k}$
(D) $15\hat{i}+5\hat{j}+60\hat{k}$

CM0044

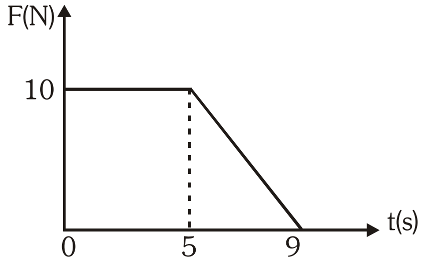
11. A system of N particles is free from any external forces.

(i) Which of the following is true for the magnitude of the total momentum of the system?
(A) It must be zero
(B) It could be non–zero, but it must be constant
(C) It could be non–zero, and it might not be constant
(D) The answer depends on the nature of the internal forces in the system

(ii) Which of the following must be true for the sum of the magnitudes of the momenta of the individual particles in the system?
(A) It must be zero
(B) It could be non–zero, but it must be constant
(C) It could be non–zero, and it might not be constant
(D) It could be zero, even if the magnitude of the total momentum is not zero

CM0045

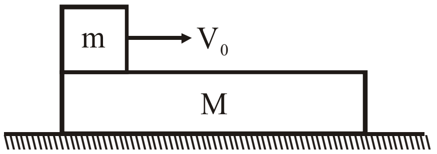
12. A body of mass 4 kg is acted on by a force which varies as shown in the graph below. The momentum acquired is



(A) 280 N–s (B) 140 N–s (C) 70 N–s (D) 210 N–s

CM0046

13. The coefficient of friction between the block and plank is μ and ground is smooth. The value of μ is such that block becomes stationary with respect to plank before it reaches the other end. Then which of the following statement is **incorrect.**



(A) The work done by friction on the block is negative.

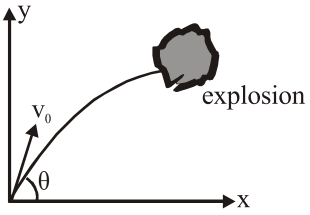
(B) The work done by friction on the plank is positive .

(C) The net work done by friction is negative.

(D) Net work done by the friction is zero.

CM0047

14. A projectile is projected in x-y plane with velocity v_0. At top most point of its trajectory projectile explodes into two identical fragments. Both the fragments land simultaneously on ground and stick there. Taking point of projection as origin and R as range of projectile if explosion had not taken place. Which of the following can not be position vectors of two pieces, when they land on ground.



(A) $\frac{R}{2}\hat{i},\frac{3R}{2}\hat{i}$ (B) $0\hat{i},2R\hat{i}$ (C) $R\hat{i}-R\hat{k},R\hat{i}+R\hat{k}$ (D) $2R\hat{i}+\frac{R}{2}\hat{k},R\hat{i}-\frac{R}{2}\hat{k}$

CM0048

15. A boy hits a baseball with a bat and imparts an impulse J to the ball. The boy hits the ball again with the same force, except that the ball and the bat are in contact for twice the amount of time as in the first hit. The new impulse equals:

(A) half the original impulse (B) the original impulse

(C) twice the original impulse (D) four times the original impulse

CM0049

16. Two balls of same mass are dropped from the same height h, on to the floor. The first ball bounces to a height h/4 ,after the collision & the second ball to a height h/16. The impulse applied by the first & second ball on the floor are I_1 and I_2 respectively. Then

(A) $5I_1 = 6I_2$ (B) $6I_1 = 5I_2$ (C) $I_1 = 2I_2$ (D) $2I_1 = I_2$

CM0050

17. Ball A of mass 5.0 kilograms moving at 20 m/s collides with ball B of unknown mass moving at 10m/s in the same direction. After the collision, ball A moves at 10 m/s and ball B at 15 m/s, both still in the same direction. What is the mass of ball B?

(A) 6.0 kg (B) 10. kg (C) 2.0 kg (D) 12 kg

CM0051

18. A smooth small spherical ball of mass m, moving with velocity u collides head on with another small spherical ball of mass 3m, which was initially at rest. Two-third of the initial kinetic energy of the system is lost. The coefficient of restitution between the spheres is

(A) $\frac{1}{3}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\frac{1}{2}$ (D) zero

CM0052

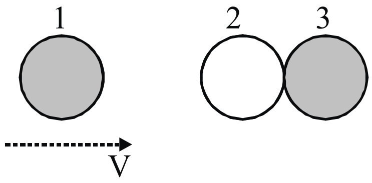
19. A ball strikes a smooth horizontal ground at an angle of 45° with the vertical. What cannot be the possible angle of its velocity with the vertical after the collision. (Assume $e \le 1$).

(A) 45° (B) 30° (C) 53° (D) 60°

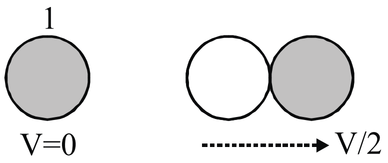
CM0053

20. Two identical ball bearings in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed *V* as shown in figure (i). If the collision is elastic, which of the following is a possible result after collision?

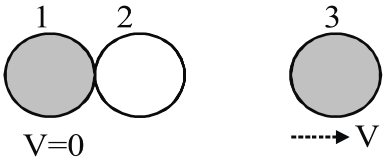
fig. (i)



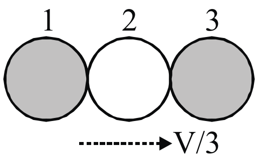
(A)



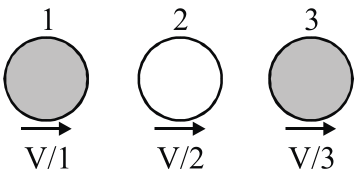
(B)



(C)



(D)



CM0054

21. A ball is projected from ground with a velocity V at an angle θ to the vertical. On its path it makes an elastic collison with a vertical wall and returns to ground. The total time of flight of the ball is

(A) $\frac{2v\sin\theta}{g}$ (B) $\frac{2v\cos\theta}{g}$ (C) $\frac{v\sin 2\theta}{g}$ (D) $\frac{v\cos\theta}{g}$

CM0055

22. A ball is thrown downwards with initial speed = 6 m/s, from a point at height = 3.2 m above a horizontalfloor. If the ball rebounds back to the same height then coefficient of restitution equals to

(A) 1/2 (B) 0.75 (C) 0.8 (D) None

CM0056

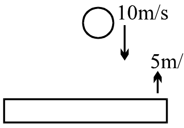
23. A particle is projected from a smooth horizontal surface with velocity v at an angle θ from horizontal. Coefficient of restitution between the surface and ball is e. The distance of the point where ball strikes the surface second time from the point of projection is

(A) $\frac{v^2\sin 2\theta(1+e^2)}{g}$ (B) $\frac{v^2\sin 2\theta(1+e^4)}{g}$

(C) $\frac{v^2\sin 2\theta(1+e^3)}{g}$ (D) $\frac{v^2\sin 2\theta(1+e)}{g}$

CM0057

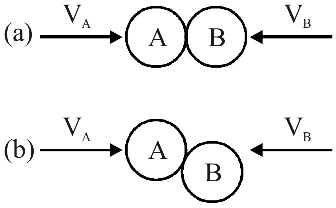
24. A ball of mass 1kg strikes a heavy platform, elastically, moving upwards with a velocity of 5m/s. The speed of the ball just before the collision is 10m/s downwards. Then the impulse imparted by the platform on the ball is :-



(A) 15 N – s (B) 10 N – s (C) 20 N – s (D) 30 N – s

CM0058

25. Two bodies, A and B, collide as shown in figures a and b below. Circle the true statement :



(A) They exert equal and opposite forces on each other in (a) but not in (b)

(B) They exert equal and opposite force on each other in (b) but not in (a)

(C) They exert equal and opposite force on each other in both (a) and (b)

(D) The forces are equal and opposite to each other in (a), but only the components of the forces parallel to the velocities are equal in (b).

CM0059

26. A mass ‘m’ moves with a velocity ‘v’ and collides inelastically with another identical mass at rest. After collision the 1st mass moves with velocity $\frac{v}{\sqrt{3}}$ in a direction perpendicular to the initial direction of motion. Find the speed of the 2nd mass after collision :-

(A) $\frac{2v}{\sqrt{3}}$

(B) $\frac{v}{\sqrt{3}}$

(C) $v\sqrt{\frac{2}{3}}$

(D) the situation of the problem is not possible without external impulse

CM0060

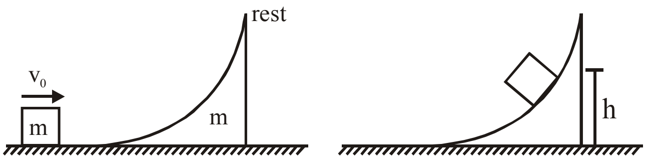
MULTIPLE CORRECT TYPE QUESTIONS

27. Two charges moving under their only own mutual attraction separated by large distance initially. Then choose the correct statement(s)

(A) If both are free, mechanical energy is conserved.
(B) If one is fixed and other is free, mechanical energy is conserved.
(C) If one is fixed and other is free, momentum is conserved.
(D) If both are free momentum is conserved.

CM0061

28. In the arrangement shown, horizontal surface is smooth, but friction is present between the block and the surface of the wedge. Block is given velocity v_0 at $t = 0$. After achieving height ‘h’ on the wedge, block comes to rest with respect to wedge at $t = t_0$. Then from $t = 0$ to $t = t_0$:-

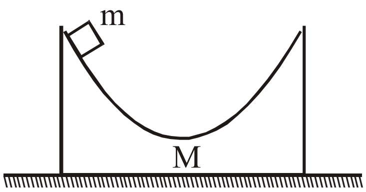


Initial Final

(A) Work done by friction on the block is negative
(B) Work done by friction on the wedge is negative
(C) Work done by block on the wedge is positive
(D) Work done by wedge on the block is positive

CM0062

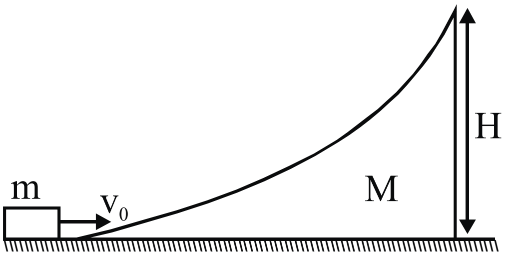
29. Figure shows a wedge on which a small block is released from rest. All the surfaces are smooth system comprises of wedge and blocks. Mark the correct statement(s) regarding motion of block on wedge till block attains maximum height on wedge.



(A) Acceleration of centre of mass of system is initially vertically down then vertically up.
(B) Initially centre of mass moves down and then up.
(C) At the maximum height block and wedge move with common velocity.
(D) Centre of mass of wedge moves towards left then right

CM0063

30. Figure shows a block of mass m projected with velocity v_0 towards a wedge. Consider all the surfaces to be smooth. Block does not have sufficient energy to negotiate (over come) wedge. Mark the correct option(s)



(A) when block is at the maximum height on wedge, block and wedge have velocity equal to velocity of centre of mass of block wedge system
(B) wedge acquires maximum speed with respect to ground when block returns to lowest point on wedge.
(C) momentum of wedge and block is conserved at all times
(D) centre of mass of wedge and block remains stationary

CM0064

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 31 and 32

A projectile of mass "m" is projected from ground with a speed of 50 m/s at an angle of 53° with the horizontal. It breaks up into two equal parts at the highest point of the trajectory. One particle coming to rest immediately after the explosion.

31. The ratio of the radii of curvatures of the moving particle just before and just after the explosion are:

(A) 1 : 4 (B) 1 : 3 (C) 2 : 3 (D) 4 : 9

CM0065

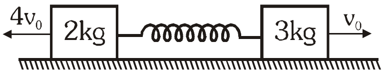
32. The distance between the pieces of the projectile when they reach the ground are:

(A) 240 (B) 360 (C) 120 (D) none

CM0065

Paragraph for Question 33 to 35

2 kg and 3 kg blocks are placed on a smooth horizontal surface and connected by spring which is unstretched initially. The blocks are imparted velocities as shown in the figure.



33. The maximum energy stored in the spring in the subsequent motion will be

(A) $5v_0^2$ (B) $15v_0^2$ (C) zero (D) $10v_0^2$

CM0066

34. Maximum speed of 3 kg block in the subsequent motion will be

(A) v_0 (B) $2v_0$ (C) $3v_0$ (D) $4v_0$

CM0066

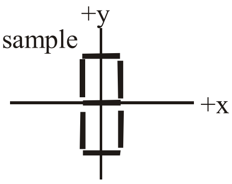
35. Maximum speed of 2 kg block in the subsequent motion will be

(A) v_0 (B) $2v_0$ (C) $3v_0$ (D) $4v_0$

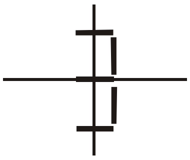
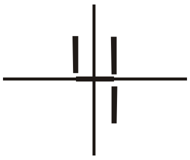
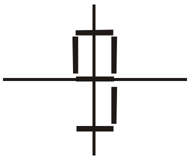
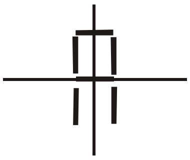
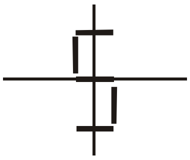
CM0066

MATRIX MATCH TYPE QUESTIONS

36. On the left are statements about the location of the center of mass of the objects depicted on the right. The objects on the right are symbols constructed out of sticks of equal length and mass. The location of the center of mass is described using the coordinate system depicted in the sample.

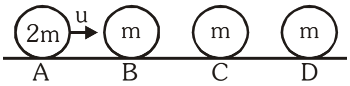


The centre of mass lies at x = 0, y = 0

Column I		**Column II**	
(A)	The center of mass is at $x > 0$ and $y = 0$	(P)	
(B)	The center of mass is at $x = 0$ and $y > 0$	(Q)	
(C)	The center of mass is at $x > 0$ and $y > 0$	(R)	
(D)	The center of mass is at $x = 0$ and $y = 0$	(S)	
		(T)	

CM0067

37. Four balls A,B,C and D are kept on a smooth horizontal surface as shown in figure. Ball A is given velocity u towards B- (Assume each collision to be elastic)



	Column-I		**Column-II**
(A)	Total impulse of all collisions on A	(p)	$\frac{4mu}{9}$
(B)	Total impulse of all collisions on B	(q)	$\frac{4mu}{27}$
(C)	Total impulse of all collision on C	(r)	$\frac{4mu}{3}$
(D)	Total impulse of all collisions on D	(s)	$\frac{52}{27}mu$

CM0068

38. In Column-I, 4 situations are depicted and in column-II, 4 possible kinds of collision are listed. Match the situation with type of collision.

	Column-I **Before**		**Column-II** **After**
(A)		(P)	Elastic
(B)		(Q)	Perfectly Inelastic
(C)		(R)	Partially elastic
		(S)	Collision is not possible

CM0069

39. A particle of mass m, kinetic energy K and momentum p collides head on elastically with another particle of mass 2 m at rest. After collision :

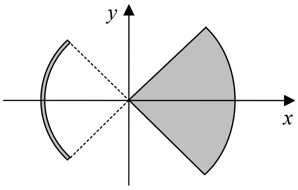
	Column I		**Column II**
(A)	Momentum of first particle	(P)	3/4 p
(B)	Momentum of second particle	(Q)	– K/9
(C)	Kinetic energy of first particle	(R)	– p/3
(D)	Kinetic energy of second particle	(S)	$\frac{8K}{9}$
		(T)	None

CM0070

EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

1. A sector cut from a uniform disk of radius 12 cm and a uniform rod of the same mass bent into shape of an arc are arranged facing each other as shown in the figure. If center of mass of the combination is at the origin, what is the radius of the arc?



(A) 8 cm (B) 9 cm (C) 12 cm (D) 18 cm

CM0071

2. A piece of paper (shown in figure-1) is in form of a square. Two corners of this square are folded to make it appear like figure-2. Both corners are put together at centre of square 'O'. If O is taken to be (0, 0), the centre of mass of new system will be at

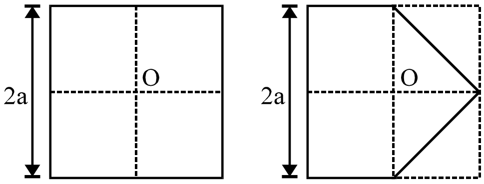
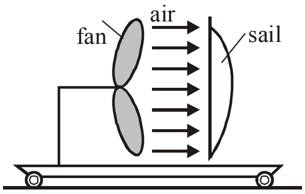


Figure-1 Figure-2

(A) $\left(\frac{-a}{8}, 0\right)$ (B) $\left(\frac{-a}{6}, 0\right)$ (C) $\left(\frac{a}{12}, 0\right)$ (D) $\left(\frac{-a}{12}, 0\right)$

CM0072

3. A fan and a sail are mounted vertically on a cart that is initially at rest on a horizontal table as shown in the diagram. When the fan is turned on, an air stream is blown towards the right and is incident on the sail. The cart is free to move with negligible resistance forces. After the fan has been turned on the cart will



(A) move to the right and then to the left (B) remain at rest
(C) move towards the right (D) move towards the left

CM0073

4. Two identical carts constrained to move on a straight line, on which sit two twins of same mass, are moving with same velocity. At some time snow begins to drop uniformly vertically downward. Ram, sitting on one of the trolleys, throws off the falling snow sideways with respect to himself and in the second cart shyam is asleep. (Assume that friction is absent)

(A) Cart carrying Ram will speed up while cart carrying shyam will slow down

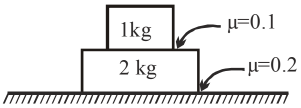
(B) Cart carrying Ram will remain at the same speed while cart carrying shyam will slow down

(C) Cart carrying Ram will speed up while cart carrying shyam will remain at the same speed

(D) Cart carrying Ram as well as shyam will slow down

CM0074

5. If both the blocks as shown in the given arrangement are given together a horizontal velocity towards right. If a_{cm} be the subsequent acceleration of the centre of mass of the system of blocks then a_{cm} equals



(A) 0 m/s^2

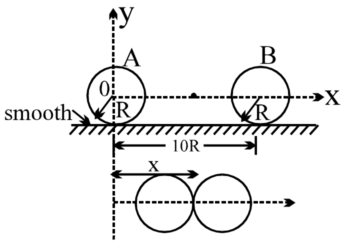
(B) $\frac{5}{3}$ m/s^2

(C) $\frac{7}{3}$ m/s^2

(D) 2 m/s^2

CM0075

6. Two uniform non conducting balls A & B have identical size having radius R but made of different density material (density of A = 2 density of B). The ball A is +vely charged & ball B is –vely charged. The balls are released on the horizontal smooth surface at the separation 10R as shown in figure. Because of mutual attraction the balls start moving towards each other. They will collide at a point.



(A) x = $\frac{10R}{3}$

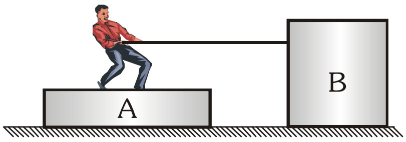
(B) x = $\frac{11R}{3}$

(C) x = 5R

(D) x = $\frac{7R}{5}$

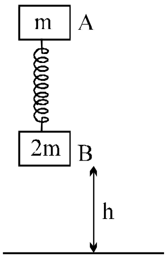
CM0076

7. In adjacent figure a boy, on a horizontal platform A, kept on a smooth horizontal surface, holds a rope attached to a box B. Boy pulls the rope with a constant force of 50N. The coefficient of friction between boy and platform is 0.5. (Mass of boy = 80 kg, mass of platform = 120kg and mass of box = 100 kg)



(A) Velocity of platform relative to box after 4 sec. is 2m/s
(B) Velocity of boy relative to platform after 4sec is 2m/s
(C) Friction force between boy and platform is 30N
(D) Friction force between boy and platform is 50N **CM0077**

8. From what minimum height h must the system be released when spring is unstretched so that after perfectly inelastic collision (e = 0) with ground, B may be lifted off the ground (Spring constant = k).



(A) mg/(4k) (B) 4mg/k (C) mg/(2k) (D) none

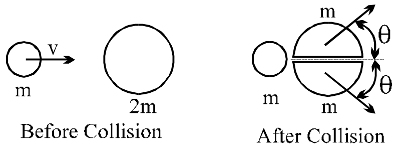
CM0078

9. An isolated particle of mass m is moving in horizontal plane (x–y), along the x–axis, at a certain height above the ground. It suddenly explodes into two fragment of masses $\frac{m}{4}$ and $\frac{3m}{4}$. An instant later, the smaller fragment is at y = +15 cm. The larger fragment at this instant is at :–

(A) y = –5 cm (B) y = +20 cm (C) y = +5 cm (D) y = –20 cm

CM0079

10. A particle of mass m is moving along the x-axis with speed v when it collides with a particle of mass 2m initially at rest. After the collisions, the first particle has come to rest, and the second particle has split into two equal-mass pieces that move at equal angles θ > 0 with the x-axis, as shown in the figure. Which of the following statements correctly describes the speeds of the two pieces?



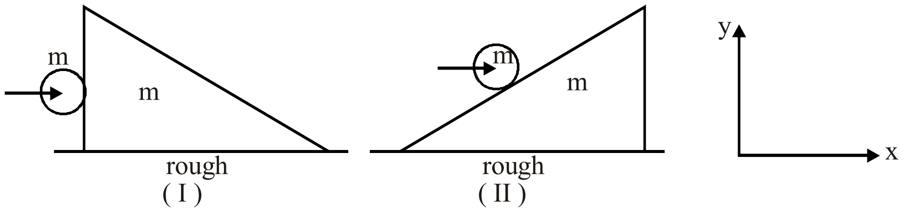
(A) Each piece moves with speed v
(B) One of the pieces moves with speed v, the other moves with speed less than v
(C) Each piece moves with speed $v/2$
(D) Each piece moves with speed greater than $v/2$

CM0080

11. A ball of mass m collides horizontally with a stationary wedge on a rough horizontal surface, in the two orientations as shown. Neglect friction between ball and wedge. Two student comment on system of ball and wedge in these situations

Saurav : Momentum of system in x-direction will change by significant amount in both cases.

Rahul : There are no impulsive external forces in y-direction in both cases hence the total momentum of system in y-direction can be treated as conserved in both cases.

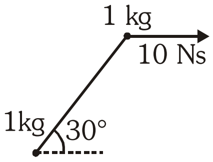


(A) **Saurav** is wrong and **Rahul** is correct (B) **Saurav** is correct and **Rahul** is wrong

(C) Both are correct (D) Both are wrong

CM0081

12. Two balls of masses 1 kg each are connected by an inextensible massless string. The system is resting on a smooth horizontal surface. An impulse of 10 Ns is applied to one of the balls at an angle 30° with the line joining two balls in horizontal direction as shown in the figure. Assuming that the string remains taut after the impulse, the magnitude of impulse of tension is :-



(A) 6Ns (B) $\frac{5}{2}\sqrt{3}$Ns (C) 5 Ns (D) $\frac{5}{\sqrt{3}}$Ns

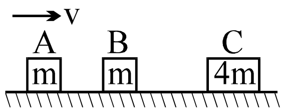
CM0082

13. A force exerts an impulse I on a particle changing its speed from u to 2u. The applied force and the initial velocity are oppositely directed along the same line. The work done by the force is

(A) $\frac{3}{2}Iu$ (B) $\frac{1}{2}Iu$ (C) I u (D) 2 I u

CM0083

14. Three blocks are initially placed as shown in the figure. Block A has mass m and initial velocity v to the right. Block B with mass m and block C with mass 4m are both initially at rest. Neglect friction. All collisions are elastic. The final velocity of block A is



(A) 0.6v to the left (B) 1.4v to the left (C) v to the left (D) 0.4v to the right

CM0084

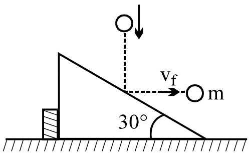
15. Two billiard balls undergo a head-on collision. Ball 1 is twice as heavy as ball 2. Initially, ball 1 moves with a speed v towards ball 2 which is at rest. Immediately after the collision, ball 1 travels at a speed of v/3 in the same direction. What type of collision has occured?

(A) inelastic (B) elastic

(C) completely inelastic (D) cannot be determined from the information given

CM0085

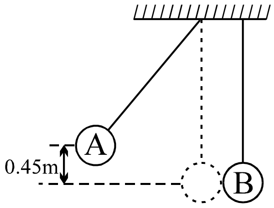
16. As shown in the figure a body of mass m moving vertically with speed 3 m/s hits a smooth fixed inclined plane and rebounds with a velocity v_f in the horizontal direction. If ∠ of inclined is 30°, the velocity v_f will be



(A) 3 m/s (B) $\sqrt{3}$ m/s (C) $1/\sqrt{3}$ m/s (D) this is not possible

CM0086

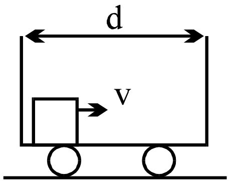
17. Two massless string of length 5 m hang from the ceiling very near to each other as shown in the figure. Two balls A and B of masses 0.25 kg and 0.5 kg are attached to the string. The ball A is released from rest at a height 0.45 m as shown in the figure. The collision between two balls is completely elastic. Immediately after the collision, the kinetic energy of ball B is 1 J. The velocity of ball A just after the collision is



(A) 5 ms^{-1} to the right (B) 5 ms^{-1} to the left (C) 1 ms^{-1} to the right (D) 1 ms^{-1} to the left

CM0087

18. In a smooth stationary cart of length d, a small block is projected along it's length with velocity v towards front. Coefficient of restitution for each collision is e. The cart rests on a smooth ground and can move freely. The time taken by block to come to rest w.r.t. cart is



(A) $\frac{ed}{(1-e)v}$ (B) $\frac{ed}{(1+e)v}$ (C) $\frac{d}{e}$ (D) infinite

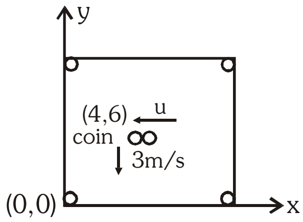
CM0088

19. A smooth sphere is moving on a horizontal surface with a velocity vector $(2\hat{i}+2\hat{j})$ m/s immediately before it hit a vertical wall. The wall is parallel to vector $\hat{j}$ and coefficient of restitution between the sphere and the wall is $e = 1/2$. The velocity of the sphere after it hits the wall is

(A) $\hat{i}-\hat{j}$ (B) $-\hat{i}+2\hat{j}$ (C) $-\hat{i}-\hat{j}$ (D) $2\hat{i}-\hat{j}$

CM0089

20. On a smooth carom board, a coin moving in negative y–direction with a speed of 3 m/s is being hit at the point (4, 6) by a striker moving along negative x–axis. The line joining centres of the coin and the striker just before the collision is parallel to x–axis. After collision the coin goes into the hole located at the origin. Masses of the striker and the coin are equal. Considering the collision to be elastic, the initial and final speeds of the striker in m/s will be



(A) (1.2, 0) (B) (2, 0) (C) (3, 0) (D) None of these

CM0090

21. Figure shows a block A of mass 5 kg kept at rest on a horizontal smooth surface. A spring (K = 200 N/m) which is compressed by 10 cm and tied with the help of a string to maintain the compression is attached to block A as shown in figure. Block B also of mass 5 kg moving with 2 m/s collides with A, as shown. During the collision the string breaks and after the collision the spring is in its natural state. Assume the bodies to be elastic and let the velocities of A and B be v_1 and v_2 respectively assuming positive direction towards right, after collision. Then

(A) $v_1 + v_2 > 2$

(B) Initial kinetic energy of system = final kinetic energy of system

(C) $v_1^2 + v_2^2 = 4.4\ (m/s)^2$

(D) $v_1 - v_2 = 2$

CM0091

22. An open water tight railway wagon of mass 5×10^3 kg coasts at an initial velocity 1.2 m/s without friction on a railway track. Rain drops fall vertically downwards into the wagon. The velocity of the wagon after it has collected 10^3 kg of water will be

(A) 0.5 m/s (B) 2 m/s (C) 1 m/s (D) 1.5 m/s

CM0092

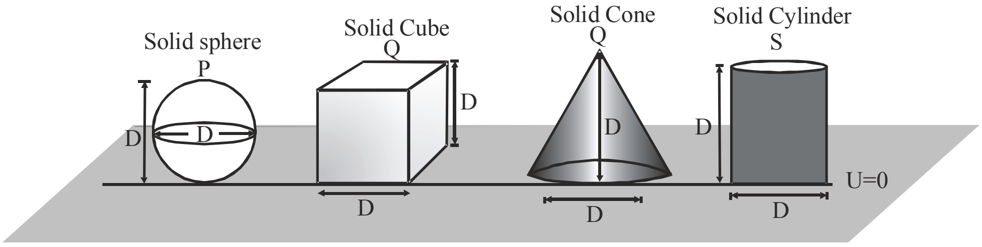
23. A rocket of mass 4000 kg is set for vertical firing. How much gas must be ejected per second so that the rocket may have initial upwards acceleration of magnitude 19.6 m/s^2. [Exhaust speed of fuel = 980 m/s.]

(A) 240 kg s^{-1} (B) 60 kg s^{-1} (C) 120 kg s^{-1} (D) None

CM0093

MULTIPLE CORRECT TYPE QUESTIONS

24. Assuming potential energy 'U' at ground level to be zero.



All objects are made up of same material.

U_P = Potential energy of solid sphere

U_Q = Potential energy of solid cube

U_R = Potential energy of solid cone

U_S = Potential energy of solid cylinder

(A) $U_S > U_P$ (B) $U_Q > U_S$ (C) $U_P > U_Q$ (D) $U_S > U_R$

CM0094

25. A blast breaks a body initially at rest of mass 0.5 kg into three pieces, two smaller pieces of equal mass and the third double the mass of either of small piece. After the blast the two smaller masses move at right angles to one another with equal speed. Find the statements that is/are true for this case assuming that the energy of blast is totally transferred to masses.

(A) All the three pieces share the energy of blast equally

(B) The speed of bigger mass is $\sqrt{2}$ times the speed of either of the smaller mass

(C) The direction of motion of bigger mass makes an angle of 135° with the direction of smaller pieces

(D) The bigger piece carries double the energy of either piece.

CM0095

26. A particle moving with kinetic energy = 3 joule makes an elastic head on collision with a stationary particle which has twice its mass during the impact.

(A) The minimum kinetic energy of the system is 1 joule.

(B) The maximum elastic potential energy of the system is 2 joule.

(C) Momentum and total kinetic energy of the system are conserved at every instant.

(D) The ratio of kinetic energy to potential energy of the system first decreases and then increases.

CM0096

27. In a one dimensional collision between two identical particles A and B, B is stationary and A has momentum p before impact. During impact, B gives impulse J to A.

(A) The total momentum of the 'A plus B' system is p before and after the impact, and (p-J) during the impact.

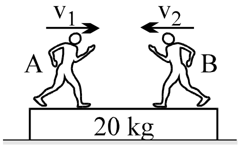
(B) During the impact A gives impulse of magnitude J to B

(C) The coefficient of restitution is $\frac{2J}{p}-1$

(D) The coefficient of restitution is $\frac{J}{p}+1$

CM0097

28. In the figure shown the system is at rest initially. Two persons 'A' and 'B' of masses 40 kg each move with speeds v_1 and v_2 respectively towards each other on a plank lying on a smooth horizontal surface as shown in figure. Plank travels a distance of 20 m towards right direction in 5 sec. (Here v_1 and v_2 are given with respect to the plank). Then the possible condition(s) can be



(A) $v_1 = 0$ m/s, $v_2 = 10$ m/s

(B) $v_1 = 5$ m/s, $v_2 = 15$ m/s

(C) $v_1 = 10$ m/s, $v_2 = 20$ m/s

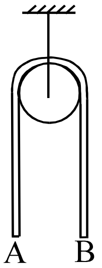
(D) $v_1 = 2$ m/s, $v_2 = 12$ m/s

CM0098

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 29 and 30

A uniform chain of length 2L is hanging in equilibrium position, if end B is given a slightly downward displacement the imbalance causes an acceleration. Here pulley is small and smooth & string is inextensible



29. The acceleration of end B when it has been displaced by distance x, is

(A) $\frac{x}{L}g$ (B) $\frac{2x}{L}g$ (C) $\frac{x}{2}g$ (D) g

CM0099

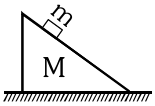
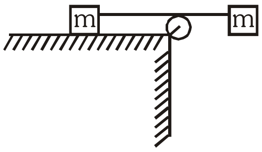
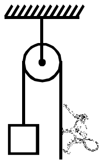
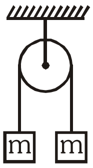
30. The velocity v of the string when it slips out of the pulley (height of pulley from floor > 2L)

(A) $\sqrt{\frac{gL}{2}}$ (B) $\sqrt{2gL}$ (C) $\sqrt{gL}$ (D) none of these

CM0099

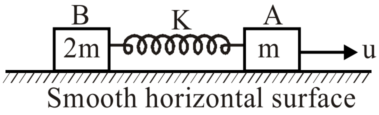
MATRIX MATCH TYPE QUESTION

31. In each situation of column–I, a system involving two bodies is given. All strings and pulleys are light and friction is absent everywhere. Initially each body of every system is at rest. Consider the system in all situation of column I from rest till any collision occurs. Then match the statements in column–I with the corresponding results in column–II

Column I	**Column II**
(A) The block plus wedge system is placed over smooth horizontal surface. After the system is released from rest, the centre of mass of system 	(P) Shifts towards right
(B) The string connecting both the blocks of mass m is horizontal. Left block is placed over smooth horizontal table as shown. After the two block system is released from rest, the centre of mass of system 	(Q) Shifts downwards
(C) The block and monkey have same mass. The monkey starts climbing up the rope. After the monkey starts climbing up, the centre of mass of monkey+block system 	(R) Shifts upwards
(D) Both block of mass m are initially at rest. The left block is given initial velocity u downwards. Then, the centre of mass of two block system afterwards 	(S) Does not shift

CM0100

32. Two blocks A and B of mass m and 2m respectively are connected by a massless spring of spring constant K. This system lies over a smooth horizontal surface. At t = 0 the block A has velocity u towards right as shown while the speed of block B is zero, and the length of spring is equal to its natural length at that instant.

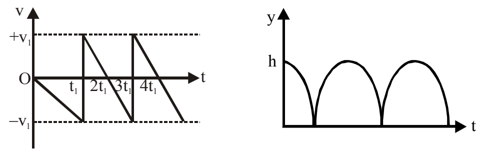


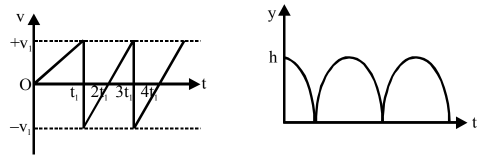
Column-I	**Column-II**
(A) The velocity of block A	(P) can never be zero
(B) The velocity of block B	(Q) may be zero at certain instants of time
(C) The kinetic energy of system of two block	(R) is minimum at maximum compression of spring
(D) The potential energy of spring	(S) is maximum at maximum extension of spring

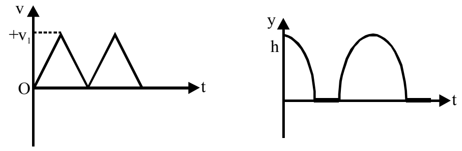
CM0101

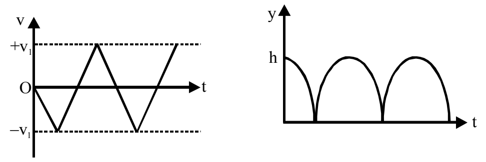
EXERCISE (J-M)

1. Consider a rubber ball freely falling from a height h = 4.9 m onto a horizontal elastic plate. Assume that the duration of collision is negligible and the collision with the plate is totally elastic.
Then the velocity as a function of time and the height as a function of time will be :- **[AIEEE - 2009]**

(1)


(2)


(3)


(4)


CM0102

Directions : Question number 4 contain Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best discribes the two statements.

2. **Statement-1 :** Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.

Statement-2 : Principle of conservation of momentum holds true for all kinds of collisions.

[AIEEE - 2010]

(1) Statement–1 is true, Statement–2 is false

(2) Statement–1 is true, Statement–2 is true; Statement–2 is the correct explanation of Statement–1

(3) Statement–1 is true, Statement–2 is true; Statement–2 is not the correct explanation of Statement–1

(4) Statement–1 is false, Statement–2 is true

CM0103

3. This question has Statement I and Statement II. Of the four choices given after the Statements, choose the one that best describes the two Statements. **[JEE Main-2013]**

Statement - I : A point particle of mass m moving with speed v collides with stationary point particle of mass M. If the maximum energy loss possible is given as $f\left(\frac{1}{2}mv^2\right)$ then $f=\left(\frac{m}{M+m}\right)$.

Statement - II : Maximum energy loss occurs when the particles get stuck together as a result of the collision.

(1) Statement–I is true, Statement–II is true, Statement–II is a correct explanation of Statement–I.

(2) Statement–I is true, Statement–II is true, Statement–II is a not correct explanation of Statement–I.

(3) Statement–I is true, Statement–II is false.

(4) Statement–I is false, Statement–II is true

CM0104

4. A particle of mass m moving in the x direction with speed 2υ is hit by another particle of mass 2m moving in the y direction with speed υ. If the collisions perfectly inelastic , the percentage loss in the energy during the collision is close to : **[JEE Main-2015]**

(1) 56 % (2) 62% (3) 44% (4) 50%

CM0105

5. Distance of the centre of mass of a solid uniform cone from its vertex is z_0. If the radius of its base is R and its height is h then z_0 is equal to :- **[JEE Main-2015]**

(1) $\frac{5h}{8}$ (2) $\frac{3h^2}{8R}$ (3) $\frac{h^2}{4R}$ (4) $\frac{3h}{4}$

CM0106

6. It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is p_d ; while for its similar collision with carbon nucleus at rest, fractional loss of energy is p_c. The values of p_d and p_c are respectively : **[JEE Main-2018]**

(1) (·28, ·89) (2) (0, 0) (3) (0, 1) (4) (·89, ·28)

CM0107

7. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50 % greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is : **[JEE Main-2018]**

(1) $\sqrt{2}v_0$ (2) $\frac{v_0}{2}$ (3) $\frac{v_0}{\sqrt{2}}$ (4) $\frac{v_0}{4}$

CM0108

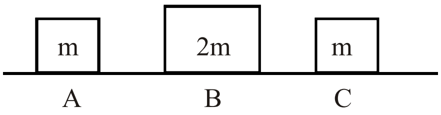
8. The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike, per second, a fixed wall of area 2 cm^2 at an angle of 45° to the normal, and rebound elastically with a speed of 10^3 m/s, then the pressure on the wall is nearly : **[JEE Main-2018]**

(1) 4.70×10^3 N/m^2 (2) 2.35×10^2 N/m^2 (3) 4.70×10^2 N/m^2 (4) 2.35×10^3 N/m^2

CM0109

EXERCISE (J-A)

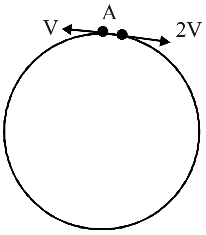
1. Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. These have masses m, 2m and m, respectively. The object A moves towards B with a speed 9 m/s and makes an elastic collision with it. Thereafter, B makes completely inelastic collision with C. All motions occur on the same straight line. Find the final speed (in m/s) of the object C. **[IIT-JEE-2009]**



CM0110

2. Two small particles of equal masses start moving in opposite directions from a point A in a horizontal circular orbit. Their tangential velocities are v and 2v, respectively, as shown in the figure. Between collisions, the particles move with constant speeds. After making how many elastic collisions, other than that at A, these two particles will again reach the point A?

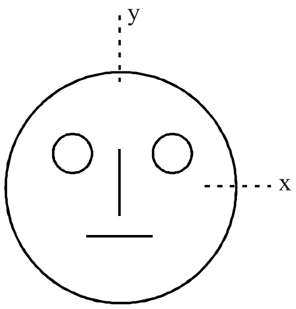
[IIT-JEE-2009]



(A) 4 (B) 3 (C) 2 (D) 1

CM0111

3. Look at the drawing given in the figure which has been drawn with ink of uniform line-thickness. The mass of ink used to draw each of the two inner circles, and each of the two line segments is m. The mass of the ink used to draw the outer circle is 6m. The coordinates of the centres of the different parts are: outer circle (0, 0), left inner circle (–a, a), right inner circle (a, a), vertical line (0, 0) and horizontal line (0, –a). The y-coordinate of the centre of mass of the ink in this drawing is **[IIT-JEE-2009]**



(A) $\frac{a}{10}$ (B) $\frac{a}{8}$ (C) $\frac{a}{12}$ (D) $\frac{a}{3}$

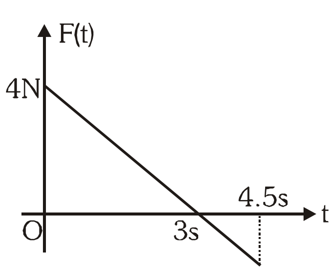
CM0112

4. A point mass of 1 kg collides elastically with a stationary point mass of 5 kg. Aftr their collision, the 1 kg mass reverses its direction and moves with a speed of 2 m/s. Which of the following statemet(s) is (are) correct for the system of these two masses ? **[IIT-JEE 2010]**

(A) Total momentum of the system is 3 kg m/s.
(B) Momentum of 5 kg mass after collision is 4 kg m/s.
(C) Kinetic energy of the centre of mass is 0.75 J.
(D) Total kinetic energy of the system is 4 J.

CM0113

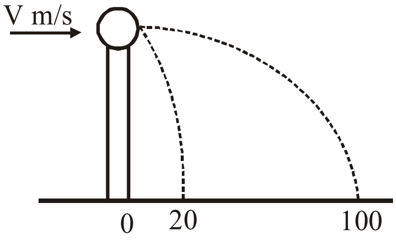
5. A block of mass 2 kg is free to move along the x–axis. It is at rest and from t=0 onwards it is subjected to a time–dependent force F(t) in the x–direction. The force F(t) varies with t as shown in the figure. The kinetic energy of the block after 4.5 second is **[IIT-JEE-2010]**



(A) 4.50 J
(B) 7.50 J
(C) 5.06 J
(D) 14.06 J

CM0114

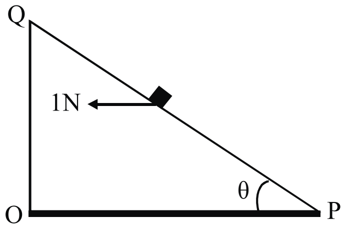
6. A ball of mass 0.2 kg rests on a vertical post of height 5m. A bullet of mass 0.01 kg, traveling with a velocity V m/s in a horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post. The initial velocity V of the bullet is + **[IIT-JEE 2011]**



(A) 250 m/s
(B) $250\sqrt{2}$ m/s
(C) 400 m/s
(D) 500 m/s

CM0115

7. A small block of mass of 0.1 kg lies on a fixed inclined plane PQ which makes an angle θ with the horizontal. A horizontal force of 1 N acts on the block through its center of mass as shown in the figure. The block remains stationary if (take $g = 10\ m/s^2$) **[IIT-JEE 2012]**



(A) $\theta = 45°$

(B) $\theta > 45°$ and a frictional force acts on the block towards P

(C) $\theta > 45°$ and a frictional force acts on the block towards Q

(D) $\theta < 45°$ and a frictional force acts on the block towards Q

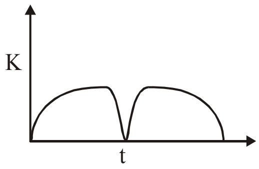
CM0116

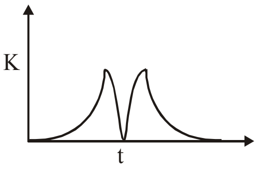
8. A bob of mass m, suspended by a string of length ℓ_1 is given a minimum velocity required to complete a full circle in the vertical plane. At the highest point, it collides elastically with another bob of mass m suspended by a string of length ℓ_2, which is initially at rest. Both the strings are mass-less and inextensible. If the second bob, after collision acquires the minimum sped required to complete a full circle in the vertical plane, the ratio $\frac{\ell_1}{\ell_2}$ is. **[JEE Advanced-2013]**

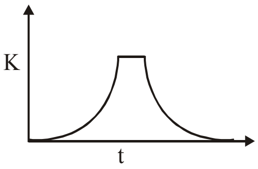
CM0117

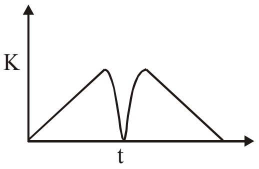
9. A tennis ball is dropped on a horizontal smooth surface. It bounces back to its original position after hitting the surface. The force on the ball during the collision is proportional to the length of compression of the ball. Which one of the following sketches describes the variation of its kinetic energy K with time t most appropriately ? The figures are only illustrative and not to the scale.

[JEE Advanced-2014]

(A) 

(B) 

(C) 

(D) 

CM0118

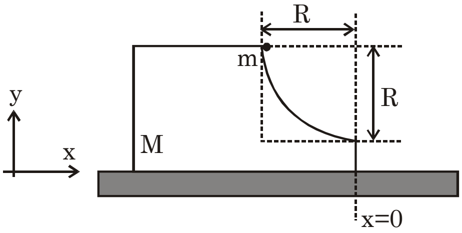
10. A block of mass M has a circular cut with a frictionless surface as shown. The block rests on the horizontal frictionless surface of a fixed table. Initially the right edge of the block is at x = 0, in a *co-ordinate system fixed to the table*. A point mass m is released from rest at the topmost point of the path as shown and it slides down. When the mass loses contact with the block, its position is x and the velocity is v. At that instant, which of the following options is/are **correct**? **[JEE Advanced-2017]**

(A) The x component of displacement of the centre of mass of the block M is : $-\dfrac{mR}{M+m}$

(B) The position of the point mass is : $x=-\sqrt{2}\dfrac{mR}{M+m}$

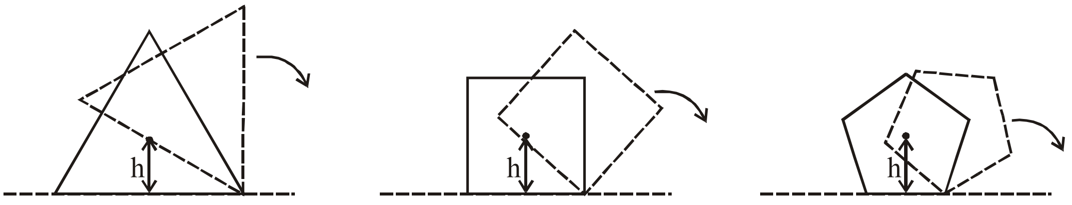
(C) The velocity of the point mass m is : $v=\sqrt{\dfrac{2gR}{1+\dfrac{m}{M}}}$

(D) The velocity of the block M is : $V=-\dfrac{m}{M}\sqrt{2gR}$



CM0119

11. Consider regular polygons with number of sides n = 3, 4, 5 ..... as shown in the figure. The center of mass of all the polygons is at height h from the ground. They roll on a horizontal surface about the leading vertex without slipping and sliding as depicted. The maximum increase in height of the locus of the center of mass for each polygon is Δ. Then Δ depends on n and h as :**[JEE Advanced-2017]**



(A) $\Delta = h\sin^2\left(\dfrac{\pi}{n}\right)$

(B) $\Delta = h\sin\left(\dfrac{2\pi}{n}\right)$

(C) $\Delta = h\left(\dfrac{1}{\cos\left(\dfrac{\pi}{n}\right)}-1\right)$

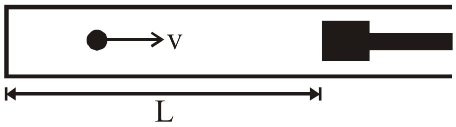
(D) $\Delta = h\tan^2\left(\dfrac{\pi}{2n}\right)$

CM0120

12. A solid horizontal surface is covered with a thin layer of oil. A rectangular block of mass $m = 0.4kg$ is at rest on this surface. An impulse of 1.0 N s is applied to the block at time to $t = 0$ so that it starts moving along the x-axis with a velocity $v(t) = v_0 e^{-t/\tau}$, where v_0 is a constant and $\tau = 4$ s. The displacement of the block, in metres, at $t = \tau$ is........... Take $e^{-1} = 0.37$? **[JEE Advanced-2018]**

CM0121

13. A small particle of mass m moving inside a heavy, hollow and straight tube along the tube axis undergoes elastic collision at two ends. The tube has no friction and it is closed at one end by a flat surface while the other end is fitted with a heavy movable flat piston as shown in figure. When the distance of the piston from closed end is $L = L_0$ the particle speed is $v = v_0$. The piston is moved inward at a very low speed V such that $V << \frac{dL}{L} v_0$, where dL is the infinitesimal displacement of the piston. Which of the following statement(s) is/are correct ? **[JEE Advanced-2019]**



(1) The rate at which the particle strikes the piston is v/L

(2) After each collision with the piston, the particle speed increases by 2V

(3) The particle's kinetic energy increases by a factor of 4 when the piston is moved inward from L_0 to $\frac{1}{2}L_0$

(4) If the piston moves inward by dL, the particle speed increases by $2v\frac{dL}{L}$

CM0122

ANSWER KEY

EXERCISE (S-1)

1. Ans. (1/7, 23/14)

2. Ans. √13 m, $\left(\frac{14}{5}, \frac{19}{5}\right)$

3. Ans. L($\sqrt{2}$ +1)/3

4. Ans. $\frac{a}{3(\pi - 1)}$

5. Ans. 4R from O

6. Ans. x = 6m

7. Ans. (3 m, 1 m, 8 m)

$$\frac{6\left(\hat{i}+2\hat{j}+3\hat{k}\right)+5\left(-\hat{i}+3\hat{j}-2\hat{k}\right)+5\vec{r}}{16}=\left(\hat{i}+2\hat{j}+3\hat{k}\right)$$

$$\vec{r}=\left(3\hat{i}+\hat{j}+8\hat{k}\right)$$

8. Ans. g/9 downwards

9. Ans. $\frac{L}{3}$

10. Ans. $\vec{v}_C = -\vec{v}_B$

11. Ans. 150 kg

12. Ans. (i) 4 m/s, (ii) 24 J

13. Ans. (i) 360 m, (ii) 10800 J

14. Ans. $\vec{P}_{PM} = m\,\overline{v}_{PM}$

$$= -mv_2 \sin\omega t\,\hat{i} + m(v_2\cos\omega t - v_1)\,\hat{j}$$

15. Ans. $\frac{\sqrt{13}}{2} v_0$

16. Ans. 30 cm

17. Ans. 0.3

18. Ans. $m \times \sqrt{u^2 - uv + v^2}$

19. Ans. $\frac{7}{18}$

20. Ans. (i) 3 J, (ii) $\frac{12}{5}$ N-s

21. Ans. (i) $v_0/3$, (ii) $3\sqrt{5gR}$.

22. Ans. $(\hat{i}+\sqrt{3}\hat{j})$ m/s, $(3m\hat{i}-\sqrt{3}m\hat{j})$ kg-m/s

23. Ans. (i) $v_A = \sqrt{g/12}$ m/s, (ii) $S_{max} = 49/48$ m

24. Ans. 1 N

EXERCISE (S-2)

1. Ans. (i) $\lambda(x) = \lambda_0 + \frac{\lambda_0 x}{\ell}$, (ii) $\frac{5}{9}\ell$

2. Ans. $\frac{R}{4}$

3. Ans. $\frac{m(R-r)}{M+m}$, $m\sqrt{\frac{2g(R-r)}{M(M+m)}}$

4. Ans. 4

5. Ans. $\frac{2mv^2}{3\ell}$

6. Ans. –2m/s, 6.93 m/s $\angle 30°$

7. Ans. (i) v/2, v/2, 0; (ii) $2mv^2/9$; (iii) $mv^2/72$; (d) $x = \sqrt{m/6k}\, v$

8. Ans. $m\left(-3\hat{i}+4\hat{j}\right)$, $e = \frac{9}{16}$

9. Ans. (i) $\sqrt{\frac{2ag}{3}}$ (ii) $\frac{3v}{g}$ (iii) $\frac{2v}{g}$

10. Ans. (i) $\frac{u}{2}$, $\frac{mu}{2}$ (ii) $\frac{u\sqrt{13}}{8}$, $\frac{mu\sqrt{13}}{8}$ (iii) $\frac{u\sqrt{3}}{4}$, $\frac{mu\sqrt{3}}{4}$

EXERCISE (O-1)

1. Ans. (C) 2. Ans. (B) 3. Ans. (D) 4. Ans. (C) 5. Ans. (B) 6. Ans. (C)
7. Ans. (C) 8. Ans. (A) 9. Ans. (C) 10. Ans. (A) 11. Ans. (i) (B) (ii) (C)
12. Ans. (C) 13. Ans. (D) 14. Ans. (D) 15. Ans. (C) 16. Ans. (A) 17. Ans. (B)
18. Ans. (A) 19. Ans. (B) 20. Ans. (B) 21. Ans. (B) 22. Ans. (C) 23. Ans. (D)
24. Ans. (D) 25. Ans. (C) 26. Ans. (D) 27. Ans. (A, B,D) 28. Ans. (A, C)
29. Ans. (B,C) 30. Ans. (AB) 31. Ans. (A) 32. Ans. (A) 33. Ans. (B)
34. Ans. (C) 35. Ans. (D) 36. Ans. (A)-P; (B)-S; (C)-Q, R; (D)-T
37. Ans. A-(s), B-(q), C-(p), D-(r) 38. Ans. (A)-Q (B)-S (C)-P
39. Ans. (A) R, (B) T, (C) T, (D)S

EXERCISE (O-2)

1. Ans. (A) 2. Ans. (D) 3. Ans. (B) 4. Ans. (D) 5. Ans. (D) 6. Ans. (B)
7. Ans. (C) 8. Ans. (B) 9. Ans. (A) 10. Ans. (D) 11. Ans. (D) 12. Ans. (B)
13. Ans. (B) 14. Ans. (A) 15. Ans. (B) 16. Ans. (B) 17. Ans. (D) 18. Ans. (D)
19. Ans. (B) 20. Ans. (B) 21. Ans. (C) 22. Ans. (C) 23. Ans. (C) 24. Ans. (A,B,D)
25. Ans. (A,C) 26. Ans. (A,B,D) 27. Ans. (B,C)
28. Ans. (A,B,C,D) 29. Ans. (A) 30. Ans. (C)
31. Ans. (A)-Q; (B)-P, Q; (C)-R; (D) S 32. Ans. (A)-Q; (B)-Q; (C)-P,R;, (D)-Q,S

EXERCISE (J-M)

1. Ans. (1) 2. Ans. (2) 3. Ans. (4) 4. Ans. (1) 5. Ans. (4) 6. Ans. (4)
7. Ans. (1) 8. Ans. (4)

EXERCISE (J-A)

1. Ans. 4m/s 2. Ans. (C) 3. Ans. (A) 4. Ans. (A,C) 5. Ans. (C) 6. Ans. (D)
7. Ans. (A,C) 8. Ans. 5 9. Ans. (B) 10. Ans. (A,C) 11. Ans. (C)
12. Ans. 6.3 [6.29, 6.31] 13. Ans. (2, 3)

Important Notes

Chapter Contents

04 ROTATIONAL DYNAMICS

Important Notes

KINEMATICS OF ROTATION MOTION

Rigid Body

A rigid body is an assemblage of a large number of material particles, which do not change their mutual distances under any circumstance or in other words, they are not deformed under any circumstance.

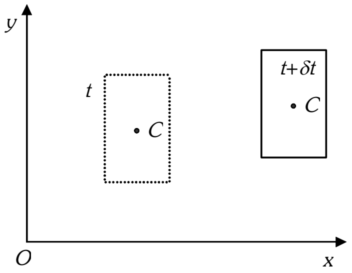
Actual material bodies are never perfectly rigid and are deformed under action of external forces. When these deformations are small enough to be considered during their course of motion, the body is assumed a rigid body. Hence, all solid objects such as stone, ball, vehicles etc are considered as rigid bodies while analyzing their translation as well as rotation motion.

To analyze rotation of a body relative motion between its particles cannot be neglected and size of the body becomes a considerable factor. This is why study of rotation motion is also known as *mechanics of rigid bodies*.

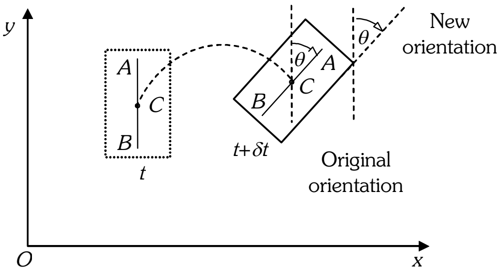
Rotation Motion of a Rigid Body

Any kind of motion of a body is identified by change in position or change in orientation or change in both. If a body changes its orientation during its motion it said to be in rotation motion.

In the following figures, a rectangular plate is shown moving in the *x-y* plane. The point *C* is its mass center. In the first case it does not changes orientation, therefore is in pure translation motion. In the second case it changes its orientation by during its motion. It is a combination of translation and rotation motion.



Pure Translation

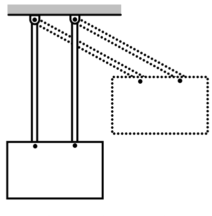


Combination of translation and rotation

Rotation i.e. change in orientation is identified by the angle through which a linear dimension or a straight line drawn on the body turns. In the figure this angle is shown by θ.

Ex. Identify Translation and rotation motion

A rectangular plate is suspended from the ceiling by two parallel rods each pivoted at one end on the plate and at the other end on the ceiling. The plate is given a side-push to oscillate in the vertical plane containing the plate. Identify motion of the plate and the rods.



Sol.

Neither of the linear dimensions of the plate turns during the motion. Therefore, the plate does not change its orientation. Here edges of the body easily fulfill our purpose to measure orientation; therefore, no line is drawn on it.

The plate is in *curvilinear translation motion* and the rods are in rotation motion.

Types of Motions involving Rotation

Motion of body involving rotation can be classified into following three categories.

I Rotation about a fixed axis.

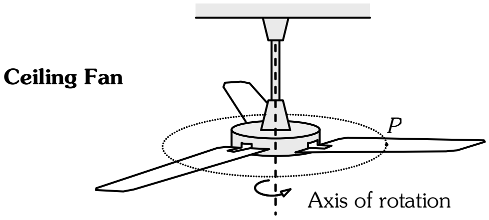
II Rotation about an axis in translation.

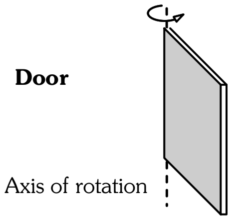
III Rotation about an axis in rotation

Rotation about a fixed axis

Rotation of ceiling fan, potter's wheel, opening and closing of doors and needles of a wall clock etc. come into this category.

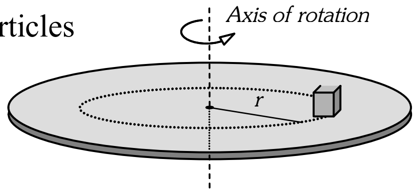
When a ceiling fan it rotates, the vertical rod supporting it remains stationary and all the particles on the fan move on circular paths. Circular path of a particle P on one of its blades is shown by dotted circle. Centers of circular paths followed by every particle are on the central line through the rod. This central line is known as *axis of rotation* and is shown by a dashed line. All the particles on the axis of rotation are at rest, therefore the axis is stationary and the fan is in rotation about this fixed axis.





A door rotates about a vertical line that passes through its hinges. This vertical line is the axis of rotation. In the figure, the axis of rotation is shown by dashed line.

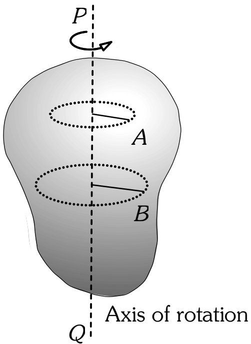
Axis of rotation



An imaginary line perpendicular to plane of circular paths of particles of a rigid body in rotation and containing the centers of all these circular paths is known as *axis of rotation*.

It is not necessary that the axis of rotation pass through the body. Consider system shown in the figure, where a block is fixed on a rotating disk. The axis of rotation passes through the center of the disk but not through the block.

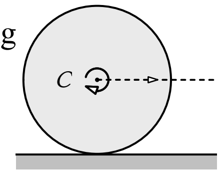
Important observations



Let us consider a rigid body of arbitrary shape rotating about a fixed axis PQ passing through the body. Two of its particles A and B are shown moving on their circular paths.

- ⇨ All of its particles, not on the axis of rotation, move on circular paths with centers on the axis or rotation. All these circular paths are in parallel planes that are perpendicular to the axis of rotation.
- ⇨ All the particles of the body cover same angular displacement in the same time interval, therefore all of them move with the same angular velocity and angular acceleration.
- ⇨ Particles moving on circular paths of different radii move with different speeds and different magnitudes of linear acceleration. Furthermore, no two particles in the same plane perpendicular to the axis of rotation have same velocity and acceleration vectors.
- ⇨ All the particles on a line parallel to the axis of rotation move circular paths of the same radius therefore have same velocity and acceleration vectors.
- ⇨ Consider two particles in a plane perpendicular to the rotational axis. Every such particle on a rigid body in rotation motion moves on circular path relative to another one. Radius of the circular path equals to the distance between the particles. In addition, angular velocity and angular acceleration equals to that of rotation motion of the body.

Rotation about an axis in translation



Rotation about an axis in translation includes a broad category of motions. Rolling is an example of this kind of motion. A rod lying on table when pushed from its one in its perpendicular direction also executes this kind of motion. To understand more let us discuss few examples.

Consider rolling of wheels of a vehicle, moving on straight level road. Relative to a reference frame, moving with the vehicle wheel appears rotating about its stationary axel. The rotation of the wheel from this frame is rotation about fixed axis. Relative to a reference frame fixed with the ground, the wheel appears rotating about the moving axel, therefore, rolling of a wheel is superposition two simultaneous but distinct motions – rotation about the axel fixed with the vehicle and translation of the axel together with the vehicle.

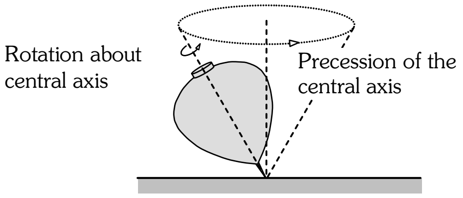
Important observations

- ⇨ Every particle of the body always remains in a plane perpendicular to the rotational axis. Therefore, this kind of motion is also known as *general plane motion*.
- ⇨ Relative to every particle another particle in a plane perpendicular to axis of rotation moves on circular path. Radius of the circular path equals to the distance between the particles and angular velocity and angular acceleration equals to that of rotation motion of the body.
- ⇨ Rotation about axis in translation is superposition of pure rotation about the axis and simultaneous translation motion of the axis.

Rotation about an axis in rotation.

In this kind of motion, the body rotates about an axis that also rotates about some other axis. Analysis of rotation about rotating axes is not in the scope of JEE, therefore we will discus it to have an elementary idea only.

As an example consider a rotating top. The top rotates about its central axis of symmetry and this axis sweeps a cone about a vertical axis. The central axis continuously changes its orientation, therefore is in rotation motion. This type of rotation in which the axis of rotation also rotates and sweeps out a cone is known as *precession*.

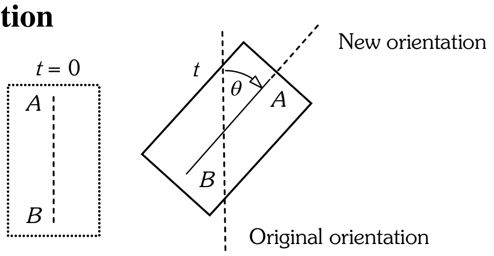


Another example of rotation about axis in rotation is a table-fan swinging while rotating. Table-fan rotates about its horizontal shaft along which axis of rotation passes. When the rotating table-fan swings, its shaft rotates about a vertical axis.

Angular displacement, angular velocity and angular acceleration

Rotation motion is the change in orientation of a rigid body with time.It is measured by turning of a linear dimension or a straight line drawn on the body.

In the figure is shown at two different instants $t = 0$ and t a rectangular plate moving in its own plane. Change in orientation during time t equals to the angle θ through which all the linear dimensions of the plate or a line AB turns.

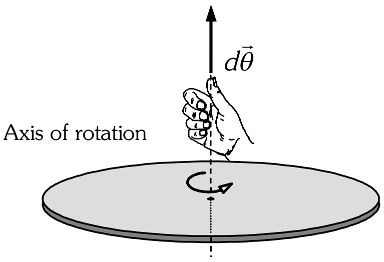


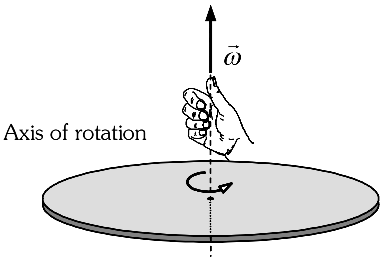
If the angle θ continuously changes with time t, instantaneous angular velocity ω and angular acceleration α for rotation of the body are defined by the following equations.

$$\omega = \frac{d\theta}{dt} \quad [1] \qquad\qquad \alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = \omega\frac{d\omega}{d\theta} \quad [2]$$

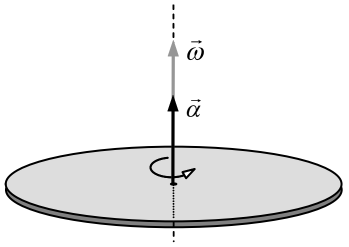
Direction of angular motion quantities

Angular displacement, angular velocity and angular acceleration are known as angular motion quantities. Infinitesimally small angular displacement, instantaneous angular velocity and angular acceleration are vector quantities. Direction of infinitesimally small angular displacement and instantaneous angular velocity is given by the right hand rule. For a disk rotating as shown in the figure, the angular velocity points upwards along the axis of rotation.

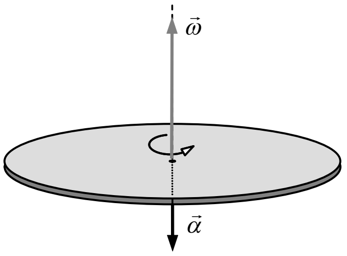




The direction of angular acceleration depends, whether angular velocity increases or decreases with time. For increasing angular velocity, the angular acceleration vector points in the direction of angular velocity vector and for decreasing angular velocity, the angular acceleration vector points opposite to the angular velocity vector.



Angular acceleration: Increasing angular speed



Angular acceleration: Decreasing angular speed

In rotation about fixed axis and rotation about axis in translation, the axis of rotation does not rotate and angular velocity and acceleration always point along the axis of rotation. Therefore, in dealing these kinds of motions, the angular motion quantities can used in scalar notations by assigning them positive sign for one direction and negative sign for the opposite direction.

These quantities have similar mathematical relations as position coordinate, velocity, acceleration and time have in rectilinear motion.

⇨ A body rotating with constant angular velocity ω and hence zero angular acceleration is said to be uniform rotation. Angular position θ is given by equation

$$\theta = \theta_o + \omega t \qquad [3]$$

⇨ Thus for a body rotating with uniform angular acceleration α, the angular position θ and angular velocity ω can be expressed by the following equation.

$$\omega = \omega_o + \alpha t \qquad [4]$$

$$\theta = \theta_o + \omega_o t + \tfrac{1}{2}\alpha t^2 = \theta_o + \tfrac{1}{2}\left(\omega_o + \omega\right)t \qquad [5]$$

$$\omega^2 = \omega_o^2 + 2\alpha\left(\theta - \theta_o\right) \qquad [6]$$

Ex. A disk rotates about a fixed axis. Its angular velocity ω varies with time according to equation $\omega = at + b$. At the instant $t = 0$ its angular velocity is 1.0 rad/s at angular position is 2 rad and at the instant $t = 2$ s, angular velocity is 5.0 rad/s. Determine angular position θ and angular acceleration α when $t = 4$ s.

Sol. The given equation $\omega = at + b$ has form similar to eq.[4], therefore motion is rotation with uniform angular acceleration. Initial angular velocity = $\omega_o = b = 1.0$ rad/s, Angular acceleration $\alpha = a$,

$$\theta = \tfrac{1}{2}at^2 + t + c$$

Since at $t = 0$, ω = 1.0 rad/s, we obtain the constant c.

Initial angular position = $\theta_o = c = 2.0$ rad

Since at $t = 2.0$ s angular velocity is 5.0 rad/s, from given expression of angular velocity, we have

$\omega = at + b \rightarrow$ Substituting $b = 1.0$ rad/s, $t = 2.0$ s and $\omega = 5.0$ rad/s, we have $a = 2.0$ rad/s

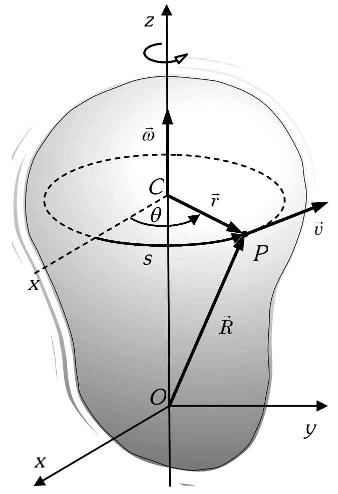
Now we can write expressions for angular position, angular velocity and angular acceleration.

$$\theta = t^2 + t + 2.0 \quad (1) \qquad \omega = 2.0t + 1.0 \quad (2)$$

From the above equations, we can calculate angular position, angular velocity and angular acceleration at t =4.0 s

$$\theta_4 = 22 \text{ rad}, \ \omega_4 = 9.0 \text{ rad/s}, \ \alpha = 2.0 \text{ rad/s}^2$$ **Ans.**

Kinematics of rotation about fixed axis



In figure is shown a rigid body of arbitrary shape rotating about the z-axis. In the selected frame (here the coordinate system) all the three axes are at rest, therefore the z-axis that is the axis of rotation is at rest and the body is in fixed axis rotation. All of its particles other than those on the z-axis move on circular paths with their centers on the z-axis. All these circular paths are parallel to the x-y plane. In the figure, one of its particles P is shown moving with velocity $\vec{v}$ on a circular path of radius r and center C. Its position vector is $\vec{R}$. It were at the line Cx at $t = 0$ and at the position shown at the instant t. During time interval t, it covers the circular arc of length s and its radius vector turns through angle θ.

In an infinitesimally small time interval dt let, the particle covers infinitesimally small distance ds along its circular path.

$$d\vec{s} = d\vec{\theta} \times \vec{r} = d\vec{\theta} \times \vec{R} \qquad [7]$$

$$\vec{v} = \frac{d\vec{s}}{dt} = \frac{d\vec{r}}{dt} = \frac{d\vec{R}}{dt} \qquad [8]$$

From eq. [7] and [8] we have

$$\vec{v} = \vec{\omega} \times \vec{r} = \vec{\omega} \times \vec{R} \qquad [9]$$

The above equation tells us the relation between the liner and angular velocity. Now we explore relation between the linear and angular accelerations. For the purpose, differentiate the above equation with respect to time.

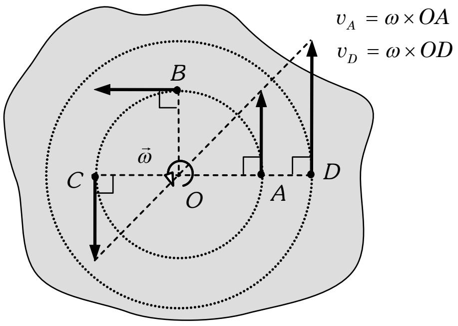
$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v} \qquad [10]$$

The first term on the RHS points along the tangent in the direction of the velocity vector and it is known as tangential acceleration $\vec{a}_T$ same as we have in circular motion. In addition, the second term point towards the center C. It is known as centripetal acceleration of normal component $\vec{a}_n$ of acceleration same as in circular motion. Now we have

Tangential acceleration $\qquad \vec{a}_T = \vec{\alpha} \times \vec{r} \qquad$ [11]

Normal acceleration $\qquad \vec{a}_n = \vec{\omega} \times \vec{v} = -\omega^2 \vec{r} \qquad$ [12]

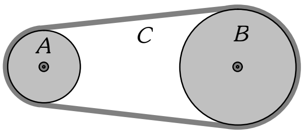
How to Locate Axis of Rotation



Every particle in a plane perpendicular to the axis of rotation move with different velocities and accelerations, moreover, they all have the same angular velocity and angular acceleration. Such a section of a body in rotation is shown here. The particles *A*, *B* and *C* at equal distance from the axis of rotation move with equal speeds v_A and the particle *D* moves with speed v_D on concentric circular paths. The location of rotational axis can be determined by any of the two graphical techniques.

- ⇨ Lines perpendicular to velocity vectors and passing through the particles, whose velocity vectors are neither parallel nor antiparallel intersect at the axis of rotation. See pairs of particles *A* and *B*, *B* and *C* and *B* and *D*.
- ⇨ Lines perpendicular to velocity vectors and passing through the particles, whose velocity vectors are either parallel or antiparallel, coincide and intersect the line joining tips of their velocity vectors at the axis of rotation. Refer pairs of particles *A* and *C*, *A* and *D* and *C* and *D*.

Ex. A belt moves over two pulleys *A* and *B* as shown in the figure. The pulleys are mounted on two fixed horizontal axels. Radii of the pulleys *A* and *B* are 50 cm and 80 cm respectively. Pulley *A* is driven at constant angular acceleration 0.8 rad/s^2 until the pulley *B* acquires an angular velocity of 10 rad/s. The belt does not slide on either of the pulleys.



(a) Find acceleration of a point *C* on the belt and angular acceleration of the pulley *B*.

(b) How long after the pulley *B* achieve angular velocity of 10 rad/s.

Sol. Since the belt does not slide on the pulleys, magnitude of velocity and acceleration of any point on the belt are same as velocity tangential acceleration of any point on periphery of either of the pulleys. Using the above fact with eq.[11], we have

$$\vec{a}_T = \vec{\alpha} \times \vec{r} \rightarrow \qquad a_C = \alpha_A r_A = \alpha_B r_B$$

Substituting $r_A = 0.5$ m, $r_B = 0.8$ m and $\alpha = 0.8$ rad/s^2, we have

$$a_C = 5\ \text{m/s}^2 \text{ and } \alpha_B = \frac{a_C}{r_B} = \frac{\alpha_A r_A}{r_B} = 0.5\ \text{rad/s}^2$$ **Ans.**

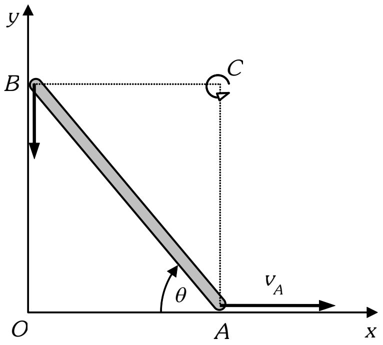
From eq. [4], we have

$$\omega = \omega_o + \alpha t \rightarrow \qquad t = \frac{\omega_B - \omega_{Bo}}{\alpha_B}$$

Substituting $\omega_{Bo} = 0$, $\omega_B = 10$ rad/s and $\alpha_B = 0.5$ rad/s^2,

we have $t = 20$ s **Ans.**

Kinematics of rotation about axis in translation



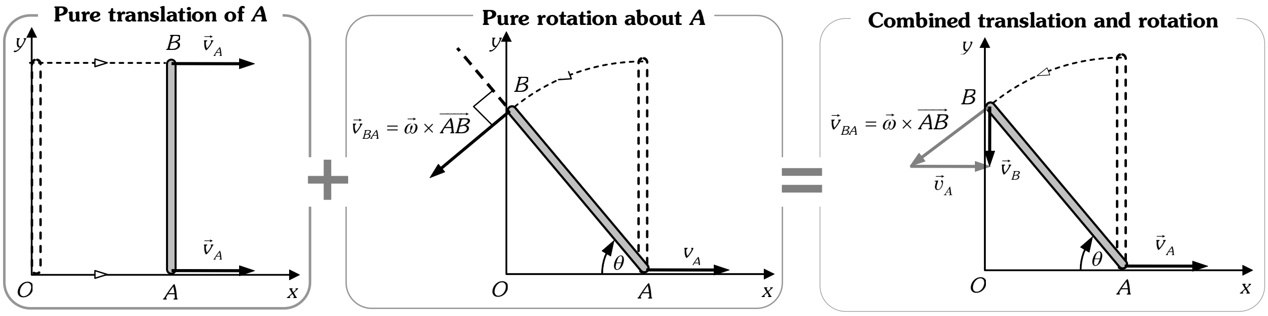
In this kind of motion, the body rotates about an axis and the axis moves without rotation. Rolling is a very common example of this kind of motion.

As an example consider a rod whose ends *A* and *B* are sliding on the *x* and *y*-axis as shown in the figure. Change in its orientation measured by change in angle θ indicates that the rod is in rotation. Perpendiculars drawn to velocity vector of its end points intersect at the axis of rotation, which is continuously changing it position.

Instantaneous Axis of Rotation (IAR)

It is a mathematical line about that a body in combined translation and rotation can be conceived in pure rotation at an instant. It continuously changes its location.

Now we explore how the combined translation and rotational motion of the rod is supper position of translation motion of any of its particle and pure rotation about an axis through that particle.
Consider motion of the rod from beginning when it was parallel to the y-axis. In the following figure translation motion of point A is superimposed with pure rotation about A.



The motion of the rod can be conceived as superposition of translation of point A and simultaneous rotation about an axis through A.
The same experiment can be repeated to demonstrate that motion of the rod can be conceived as superposition of translation of any of its particle and simultaneous rotation about an axis through that particle.
Considering translation of A and rotation about A this fact can be expressed by the following equation.

Combined Motion = Translation of point A + Pure rotation about point A

$$\vec v_B = \vec v_A + \vec v_{B/A} \qquad [13]$$

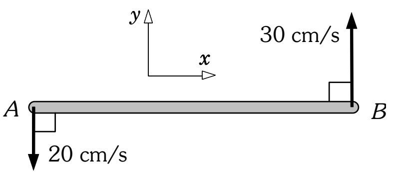
Since point B moves relative A moving on circular path its velocity relative to A is given by the equation

$$\vec v_{BA} = \vec\omega \times \overrightarrow{AB}\,.$$

Now we have $$\vec v_B = \vec v_A + \vec\omega \times \overrightarrow{AB} \qquad [14]$$

The above fact is true for any rigid body in combined translation and rotation motion.
Rotation about an axis in translation of a rigid body can be conceived as well as analyzed as superposition of translation motion of any of its particle and simultaneous rotation about an axis passing through that particle provided that the axis is parallel to the actual one.
Similar to eq.[13], we can write equation for acceleration.

$$\left.\begin{aligned} \vec a_A &= \vec a_B + \vec a_{BA} \\ \vec a_A &= \vec a_B + \overbrace{\vec a_{BAT} + \vec a_{BAn}} \\ \vec a_A &= \vec a_B + \vec\alpha \times \overrightarrow{AB} + -\omega^2 \overrightarrow{AB} \end{aligned}\right\} \qquad [15]$$



Ex. A 100 cm rod is moving on a horizontal surface. At an instant, when it is parallel to the *x*-axis its ends *A* and *B* have velocities 30 cm/s and 20 cm/s as shown in the figure.
(a) Find its angular velocity and velocity of its center.
(b) Locate its instantaneous axis of rotation.

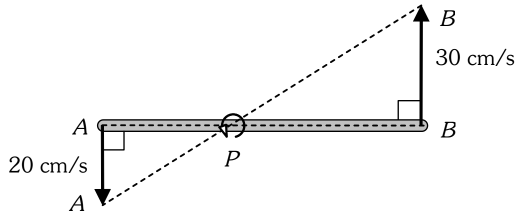
Sol.

Let the rod is rotating anticlockwise, therefore its angular velocity is given by $\vec{\omega} = \omega\hat{k}$. Velocity vectors of all the points on the rod and its angular velocity must satisfy the relative motion eq.[14].

(a) Substituting velocities $\vec{v}_A = -20\hat{j}$ cm/s and $\vec{v}_A = 30\hat{j}$ cm/s and angular velocity $\vec{\omega}$ in eq.[14], we have $\vec{v}_B = \vec{v}_A + \vec{\omega} \times \overrightarrow{AB} \rightarrow \quad \omega = 0.5$ rad/s **Ans.**

Velocity vector of the center *C* of the rod also satisfy the following equation.

$\vec{v}_C = \vec{v}_A + \vec{\omega} \times \overrightarrow{AC} \rightarrow \qquad \vec{v}_C = -20\hat{j} + 0.5\hat{k} \times 50\hat{i} = 5.0\hat{j}$ cm/s **Ans.**



(b) Here velocity vectors of the particles *A* and *B* are antiparallel, therefore the instantaneous axis of rotation passes through intersection of the common perpendicular to their velocity vectors and a line joining tips of the velocity vectors. The required geometrical construction is shown in the following figure.

Since triangles *AA'P* and *BB'P* are similar and $AB = 100$ cm, we have $AP = 40$ cm.

The instantaneous axis of rotation passes through the point *P*, which is 40 cm from *A*. **Ans.**

Analytical Approach.

The instantaneous center of rotation is at instantaneous rest. Using this fact in eq.[14], we have

$\vec{v}_P = \vec{v}_A + \vec{\omega} \times \overrightarrow{AP} \rightarrow \qquad \vec{0} = -20\hat{j} + 0.5\hat{k} \times (AP)\hat{j} \Rightarrow AP = 40$ cm **Ans.**

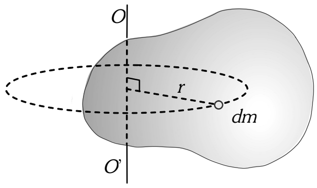
Concept of Rotational Inertia (Moment of inertia)

Total mass of a body in translation motion is the measure of its inertia to translation motion. Similarly if a point mass m is rotating about an axis at a distance r from the axis, then term mr^2 provides suitable measure of its inertia to rotation motion. *The inertia to rotation motion is known as rotational inertia or more commonly moment of inertia.*

Moment of inertia of a rigid body

A rigid body is continuous distribution of mass and can be assumed consisting of infinitely large number of point particles. If one of the point particle of infinitely small mass *dm* is at a distance *r* from the axis of rotation *OO*', the moment of inertia of this point particle is given by

$$dI_o = r^2 dm$$



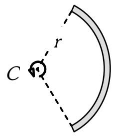
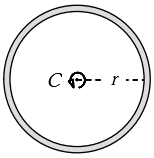
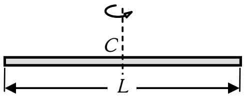
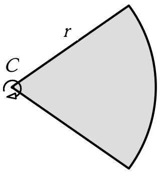
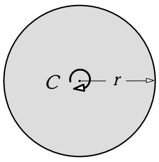
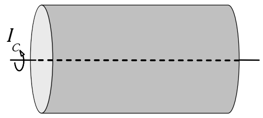
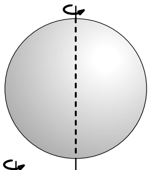
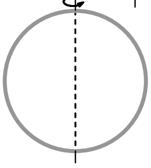
The moment of inertia of the whole body about the axis *OO*' can now be obtained by integrating term of the above equation over the limits to cover whole of the body.

$$I_o = \int dI_o = \int r^2 dm$$

Expression for moment of inertia contains product of two terms. One of them is the mass of the body and the other is a characteristic dimension, which depends on the manner how mass of the body is distributed relative to the axis of rotation. Therefore moment of inertia of a rigid body depends on the mass of the body and distribution of the mass relative to the axis of rotation. Obviously for uniform bodies expression of moment of inertia depends on their shape and location and orientation of the axis of rotation. Based on these facts we can conclude

1. If mass distribution is similar for two bodies about an axis, expressions of their moment of inertia must be of the same form about that axis.
2. If the whole body or any of its portions is shifted parallel to the axis of rotation, moment of inertia remains unchanged.

Moment of Inertia for some commonly used bodies

Body	Axis	Moment of Inertia	
Uniform thin rod bent into shape of an arc of mass *m*		Passing through center and perpendicular to the plane containing the arc	$I_C = mr^2$
Uniform ring of mass *m*		Passing through center and perpendicular to the plane containing the arc or the centroidal axis.	$I_C = mr^2$
Straight uniform rod		Passing through center and perpendicular to the rod or the centroidal axis.	$I_C = \frac{mL^2}{12}$
Sector of a uniform disk of mass *m*		Passing through center and perpendicular to the plane containing the sector.	$I_C = \frac{mr^2}{2}$
Uniform disk of mass *m*		Passing through center and perpendicular to the plane containing the disk or the centroidal axis.	$I_C = \frac{mr^2}{2}$
Homogeneous cylinder of mass *m*		Axis of the cylinder or the centroidal axis.	$I_C = \frac{mr^2}{2}$
Homogeneous sphere of mass *m*		Diameter or the centroidal axis	$I_C = \frac{2}{5}mR^2$
Spherical shell of mass *m*		Diameter or the centroidal axis	$I = \frac{2}{3}mR^2$

Theorems on Moment of Inertia

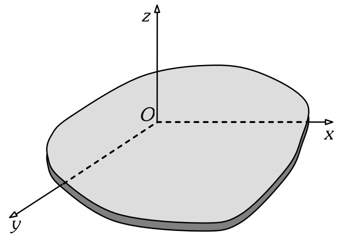
Moment of inertias of a rigid body about different axes may be different. There are two theorems known as *theorem of perpendicular axes* and *theorem of parallel axes*, which greatly simplify calculation of moment of inertia about an axis if moment of inertia of a body about another suitable axis is known.

Theorem of Perpendicular Axes

This theorem is applicable for a rigid body that lies entirely within a plane i.e. a laminar body or a rod bent into shape of a plane curve. The moment of inertia I_x, I_y and I_z of the body about the *x*, *y* and *z*-axis can be expressed by the following equations.

$$I_z = I_y + I_x$$

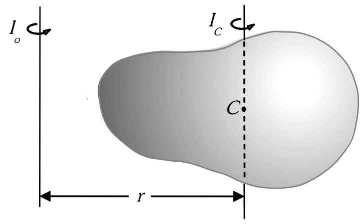
For a planar body, the moment of inertia about an axis perpendicular to the plane of the body is the sum of the moment of inertias about two perpendicular axes in the plane of the object provided that all the three axes are concurrent.



Theorem of Parallel Axes

This theorem also known as Steiner's theorem can be used to determine the moment of inertia of a rigid body about any axis, if the moment of inertia of the body about a parallel axis passing through mass center of the body and perpendicular distance between both the axes is known.

Consider a body of arbitrary shape and mass *m* shown in the figure. Its moment of inerta I_o and I_C are defined about two parallel axes. The axis about which moment of ienertia I_C is defined passes through the mass center *C*. Seperation between the axes is *r*. These two moment of inertias are related by the following equation.



$$I_O = I_C + Mx_C^2$$

The above equation is known as the theorem of parallel axes or Steiner's theorem.

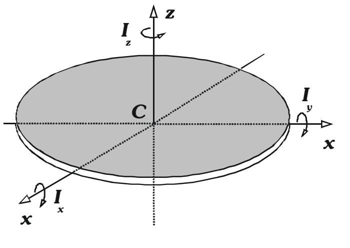
⇨ *The moment of inertia about any axis parallel to an axis through the mass center is given by sum of moment of inertia about the axis through the mass center and product term of mass of the body and square of the distance between the axes.*

⇨ *Among all the parallel axes the moment of inertia of a rigid body about the axis through the mass center is the minimum moment of inertia.*

The second term added to the moment of inertia I_C about the centroidal axis in the above equation can be recognized as the moment of inertia of a particle of mass equal to that of the body and located at its mass center. It again reveals that *the plane motion of a rigid body is superposition of pure rotation about the mass center or centroidal rotation and translation of its mass center*.

Ex. Find moment of inertia of a uniform disk of mass *m* and radius *r* about one of its diameter.

Sol. In the adjoining figure a disk is shown with two of its diameter perpendicular to each other. These diameters are along the *x* and the *y*-axis of a coordinate system. The *x*-axis is perpendicular to the plane of the disk and passes through its center is also shown. Since the disk is symmetric about both the diameters, moment of inertias about both the diameters must be equal. Thus substituting this in the theorem of perpendicular axes, we have



$$I_z = I_y + I_x \rightarrow \qquad I_z = 2I_x = 2I_y$$

Moment of inertia of the disk about the *z*-axis is $I_z = \frac{1}{2}mr^2$. Substituting it in the above equation, we have

$$I_x = I_y = \tfrac{1}{2}I_z = \tfrac{1}{4}mr^2 \qquad \textbf{Ans.}$$

Radius of Gyration

It is the radial distance from a rotation axis at which the mass of an object could be concentrated without altering the moment of inertia of the body about that axis.

If the mass *m* of the body were actually concentrated at a distance *k* from the axis, the moment of inertia about that axis would be mk^2.

$$k=\sqrt{\frac{I}{m}}$$

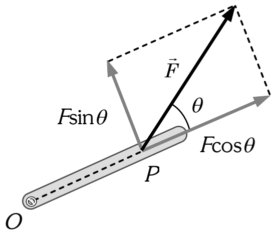
The radius of gyration has dimensions of length and is measured in appropriate units of length such as meters.

DYNAMICS OF RIGID BODY

Torque: Moment of a force

Torque is rotational analogue of force and expresses tendency of a force applied to an object to cause the object to rotate about a given point.

To investigate further let us discuss an experiment. Consider a rod pivoted at the point *O*. A force $\vec{F}$ is applied on it at the point *P*. The component $F\cos\theta$ of the force along the rod is counterbalanced by the reaction force of the pivot and cannot contribute in rotating the rod. It is the component $F\sin\theta$ of the force perpendicular to the rod, which is responsible for rotation of the rod. Moreover, farther is the point *P* from *O*, where the force is applied easier is to rotate the rod. This is why handle on a door is attached as far away as possible from the hinges.



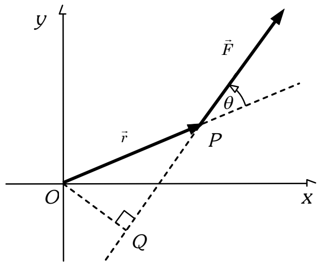
Magnitude of torque of a force is proportional to the product of distance of point of application of the force from the pivot and magnitude of the perpendicular component $F\sin\theta$ of the force. Denoting torque by symbol τ, the distance of point of application of force from the pivot by *r*, we can write

$$\tau_o \propto rF\sin\theta$$

Since rotation has sense of direction, torque should also be a vector. Its direction is given by right hand rule. Now we can express torque by the cross product of $\vec{r}$ and $\vec{F}$.

$$\vec{\tau}_o=\vec{r}\times\vec{F}$$

Here constant of proportionality has been assumed a dimensionless number unity because a unit of torque has been chosen as product of unit of force and unit of length.



The geometrical construction shown in figure suggests a simple way to calculate torque. The line *OQ* $(=r\sin\theta)$ known as *moment arm*, is the length of perpendicular drawn from *O* on the line of action of the force. The magnitude of the torque equals to the product of *OQ* and magnitude of the force $\vec{F}$..

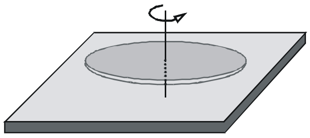
Torque about a Point and Torque about an Axes

We have defined *torque of a force about a point* as the moment of the force about that point. In dealing with rotation about a fixed axis we need to know torque about the axis rotation.

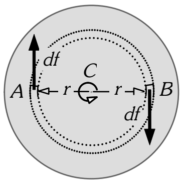
When a body is in plane motion the net torque of all the forces including the forces necessary to restrain rotation of the axis is along the axis of rotation. It is known as torque about the axis. Torque of a force about an axis of rotation equals to the moment of force about the point where plane of motion of the point of application of the force intersects the axis.

In analyzing plane motion we always consider torque about an axis under consideration and in rest of the book by the term torque of force we mean torque about an axis.

Ex. A uniform disk of mass M and radius R rotating about a vertical axis passing through its center and perpendicular to its plane is placed gently on a rough horizontal ground, where coefficient of friction is μ. Calculate torque of the frictional forces.



Sol. When the disk rotates on the ground, kinetic friction acts at every contact point. Since the gravity acts uniformly everywhere and the disk is also uniform, the normal reaction form the ground is uniformly distributed over the entire contact area. Consider two diametrically opposite identical portions A and B of the disk each of mass dm at distance r from the center as shown in the adjacent figure. The normal reaction form the ground on each of these portions equals to their weights and hence frictional forces are $df = \mu dmg$.



Consider a ring of radius r and width dr shown by dashed lines. Net torque $d\tau_C$ of friction force on this ring can easily be expressed by the following equation.

$$d\tau_C = r\mu(\text{mass of the ring})g = r\mu\left(\frac{\text{Mass of the disk}}{\text{Area of the disk}} \times \text{Area of the ring}\right)g = r\mu\left(\frac{2Mrdr}{R^2}\right)g$$

Integrating both sides of the above equation, we have

$$\tau_C = \frac{2\mu Mg}{R^2}\int_{r=0}^{R} r^2 dr = \tfrac{2}{3}\mu MgR \qquad \textbf{Ans.}$$

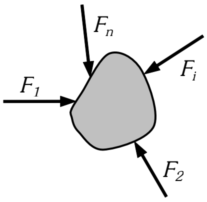
Rotational equilibrium

A rigid body is said to be state of rotational equilibrium if its angular acceleration is zero. Therefore a body in rotational equilibrium must either be in rest or rotation with constant angular velocity.

Since scope of JEE syllabus is confined only to rotation about a fixed axis or rotation about an axis in translation motion, the discussion regarding rotational equilibrium is limited here to situations involving only coplanar forces. Under these circumstances the necessary and sufficient condition for rotational equilibrium is

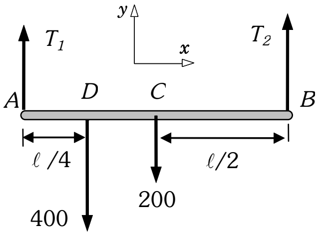
If a rigid body is in rotational equilibrium under the action of several coplanar forces, the resultant torque of all the forces about any axis perpendicular to the plane containing the forces must be zero.

In the figure a body is shown under the action of several external coplanar forces F_1, F_2, …… F_i, and F_n.



$$\sum \vec{\tau}_P = 0$$

Here P is a point in the plane of the forces about which we calculate torque of all the external forces acting on the body. The flexibility available in selection of the point P provides us with advantages that we can select such a point about which torques of several unknown forces will become zero or we can make as many number of equations as desired by selecting several different points. The first situation yields to a simpler equation to be solved and second situation though does not give independent equation, which can be used to determine additional unknowns yet may be used to check the solution.



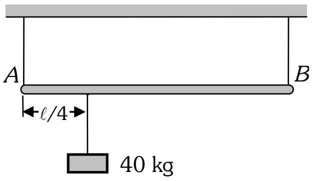
The above condition reveals that a body cannot be in rotational equilibrium under the action of a single force unless the line of action passes through the mass center of the body.

A case of particular interest arises where only three coplanar forces are involved and the body is in rotational equilibrium. It can be shown that *if a body is in rotational equilibrium under the action of three forces, the lines of action of the three forces must be either concurrent or parallel.* This condition provides us with a graphical technique to analyze rotational equilibrium.

Equilibrium of Rigid Bodies

A rigid body is said to be in equilibrium, if it is in translational as well as rotational equilibrium both. To analyze such problems conditions for both the equilibriums must be applied.

Ex. A uniform rod of 20 kg is hanging in horizontal position with the help of two threads. It also supports a 40 kg mass as shown in the figure. Find the tension developed in each thread.



Sol. Free body diagram of the rod is shown in the figure.

Translational equilibrium

$\Sigma F_y = 0 \rightarrow \qquad T_1 + T_2 = 400 + 200 = 600\ N \qquad (1)$

Rotational equilibrium: Applying the condition about *A*, we get T_2.

$\Sigma \vec{\tau}_A = \vec{0} \rightarrow \qquad 400(l/4) + 200(l/2) - T_2 l = 0$

$T_2 = 200$ N **Ans.**

Similarly writing torque equation about *B*, we have

$\Sigma \vec{\tau}_B = \vec{0} \rightarrow \qquad T_1 = 400$ N. **Ans.**

Force and Torque equations in General Plane Motion

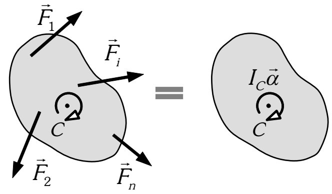
Motion of a rigid body either pure rotation or rotation about axis in translation can be thought and analyzed as superposition of translation of any of its particle and simultaneous rotation about an axis passing through that particle provided that the axis remain parallel to the original one. As far as kinematics in concerned this particle may or may not be the mass center. Whereas in dealing with kinetics, general plane motion is conceived as superposition of translation motion of the mass center and simultaneous centroidal rotation.

To make use the above idea and equations developed in the previous section we classify pure rotation i.e. rotation about fixed axis into two categories and deal with general plane motion as the third category.

Pure centroidal rotation: Rotation about fixed axis through mass centre

In this kind of rotation motion the axis of rotation passes through the mass center and remain fixed in space. Rotation of ceiling fan is a common example of this category. It is a subcategory of pure rotation. The axis of rotation passes through the mass center and remains fixed. In this kind of rotation the mass center of the body does not move.

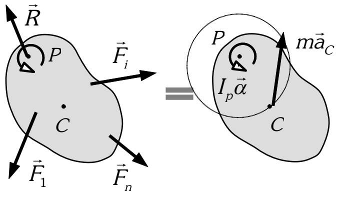
In the figure, free body diagram and kinetic diagram of a body rotating about a fixed axis passing through its mass center *C* is shown. The mass center of the body does not accelerate; therefore we only need to write the torque equation.



$$\Sigma \vec{\tau}_C = I_C \vec{\alpha}$$

Rotation about fixed axis not passing through mass center

In this kind of rotation the axis of rotation remains fixed and does not passes through the mass center. Rotation of door is a common example of this category. Doors are hinged about their edges; therefore their axis of rotation does not pass through the mass center. In this kind of rotation motion the mass center executes circular motion about the axis of rotation.



In the figure, free body diagram and kinetic diagram of a body rotating about a fixed axis through point *P* is shown. It is easy to conceive that as the body rotates its mass center moves on a circular path of radius $\vec{r}_{P/C}$. The mass center of the body is in translation motion with acceleration $\vec{a}_C$ on circular path of radius $r_{P/C}$. To deal with this kind of motion, we have to make use of both the force and the torque equations.

Translation of mass center $\quad \Sigma\vec{F}_i = M\vec{a}_C = M\vec{\alpha}\times\vec{r}_{C/P} - M\omega^2\vec{r}_{C/P}$

Centroidal Rotation $\quad \Sigma\vec{\tau}_C = I_C\vec{\alpha}$

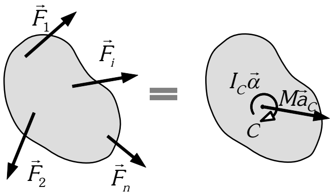
Making use of parallel axis theorem $\quad \left(I_P = Mr_{P/C}^2 + I_C\right)$ and $\vec{a}_{C/P} = \vec{\alpha}\times\vec{r}_{C/P} - \omega^2\vec{r}_{C/P}$

we can write the following equation also.

Pure Rotation about *P* $\quad \Sigma\vec{\tau}_P = I_P\vec{\alpha}$

General Plane Motion: Rotation about axis in translation motion

Rotation of bodies about an axis in translation motion can be dealt with either as superposition of translation of mass center and centroidal rotation or assuming pure rotation about the instantaneous axis of rotation. In the figure is shown the free body diagram and kinetic diagram of a body in general plane motion.



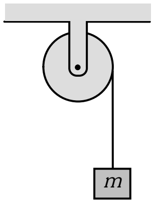
Translation of mass center $\quad \sum_{i=0}^{n}\vec{F}_i = M\vec{a}_C$

Centroidal Rotation $\quad \sum_{i-1}^{n}\vec{\tau}_C = I_C\vec{\alpha}$

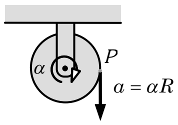
This kind of situation can also be dealt with considering it rotation about IAR. It gives sometimes quick solutions, especially when IAR is known and forces if acting at the IAR are not required to be found.

Example

A block of mass *m* is suspended with the help of a light cord wrapped over a cylindrical pulley of mass *M* and radius *R* as shown in the figure. The system is released from rest. Find the angular acceleration of the pulley and the acceleration of the block.



Solution.



After the system is released, the block is in translation motion and the pulley in rotation about an axis passing through its mass center i.e. in pure rotation.

Let the block moves vertically down with acceleration *a* pulling the cord down and causing the pulley to rotate clockwise. Since the cord is inextensible every point on its vertical portion and point of contact *P* of the pulley move down with acceleration *a* as shown in the adjacent figure. It is the tangential acceleration of point *P* so the angular acceleration α of the pulley rotating in clockwise sense is given by

$$a = \alpha R \qquad (1)$$

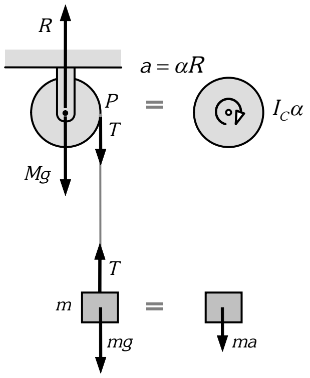
The forces acting on the pulley and on the block are shown in their free-body diagrams along with the effective torque $I_C\alpha$ of the pulley and effective force *ma* of the block. Here *T* is the tension in the string, *R* is the reaction by the axel of the pulley, *Mg* is weight of the pulley and *mg* is weight of the block.

The pulley is in rotation about fixed axis through its mass center so we use eq. .

$$\sum \vec{\tau}_C = I_C\vec{\alpha} \rightarrow \qquad TR = I_C\alpha$$

After substituting $I_C = \frac{1}{2}MR^2$ and α from eq. (1), we have

$$T = \frac{1}{2}Ma \qquad (2)$$



The block is in translation motion, so we use Newton's second law

$$\sum \vec{F} = m\vec{a} \rightarrow \qquad mg - T = ma \qquad (3)$$

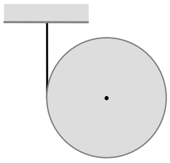
From equation (2) and (3), we have

Acceleration of the block $\quad a = \dfrac{2mg}{M+2m}$ **Ans.**

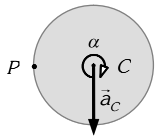
From eq. (1) and the above, we have

$$\alpha = \frac{2mg}{R(M+2m)} \qquad \textbf{Ans.}$$

Ex. A thread is wrapped around a uniform disk of radius *r* and mass *m*. One end of the thread is attached to a fixed support on the ceiling and the disk is held stationary in vertical plane below the fixed support as shown in the figure. When the disk is set free, it rolls down due to gravity. Find the acceleration of the center of the disk and tension in the thread.



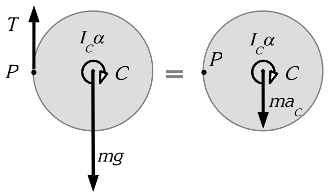
Sol. The point *P*, where the thread leaves the disk is always at instantaneous rest; therefore the disk can be assumed rolling without slipping with ICR at point *P*. Acceleration of the mass center $\vec{a}_C$ and angular acceleration of the disk are shown in the adjacent figure. Applying condition for rolling on stationary surface, we have



$$\vec{a}_C = \vec{\alpha} \times \vec{r}_{C/P} \rightarrow \qquad a_C = \alpha r \qquad (1)$$

The disk rolls down on the vertical stationary thread. Its motion can either be analyzed as superposition of translation of the mass center and simultaneous centroidal rotation or a pure rotation about ICR. Since tension, which acts at the ICR is asked; we prefer superposition of translation of the mass center and simultaneous centroidal rotation.

Forces acting on the disk are tension *T* applied by the thread at point *P* and weight of the disk. These forces and the effective force ma_C and effective torque $I_C\alpha$ are shown in the adjacent figure.



Applying Newton's second law for translation of mass center, we have

$$\sum \vec{F}_i = M\vec{a}_C \rightarrow \qquad mg - T = ma_C \qquad (2)$$

Applying torque equation for centroidal rotation, we have

$$\sum \vec{\tau}_C = I_C\vec{\alpha} \rightarrow \qquad Tr = I_C\alpha$$

Substituting $\frac{1}{2}mr^2$ for I_C and α from eq. (1), we have

$$T = \frac{1}{2}ma_C \qquad (3)$$

From eq. (2) and (3), we have

Acceleration of the mass center $a_C = \frac{2}{3}g$ **Ans.**

Tension in the string $T = \frac{1}{3}mg$ **Ans.**

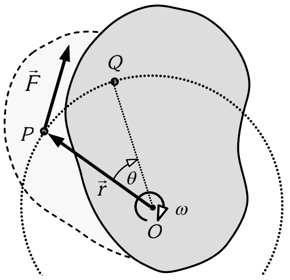
Energy Methods

Newton's laws of motion tell us what is happening at an instant, while method of work and energy equips us to analyze what happens when a body moves from one place to other or a system changes its configuration. In this section, we introduce how to use methods of work and energy to analyze motion of rigid bodies.

Concept of Work in rotation motion

Work of a force is defined as the scalar product of the force vector and displacement vector of the point of application of the force. If during the action of a force $\vec{F}$ its point of application moves from position $\vec{r}_1$ to $\vec{r}_2$, the work $W_{1\to 2}$ done by the force is expressed by the following equation.

$$W_{1\to 2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r}$$

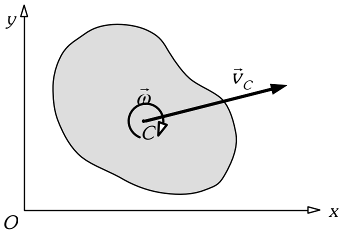


Either we can use of this idea to calculate work of a force or its modified version in terms of torque and angular displacement.

The work done by a torque during a finite rotation of the rigid body from initial value θ_i of the angle θ to final value θ_f, can be obtained by integrating both the sides of the equation given

$$W_{i\to f} = \int_{\theta_i}^{\theta_f} \vec{\tau}_o \cdot d\vec{\theta}$$

Kinetic Energy of a rigid body in rotation motion



A rigid body can be represented as a system of large number of particles, which keep their mutual distances unchanged in all circumstances. Kinetic energy of the whole body must be sum of kinetic energies of all of its particles. In this section we develop expressions for kinetic energy of a rigid body.

Kinetic Energy of a rigid body in plane motion

In the figure is shown a body in plane motion. Its mass center at an instant is moving with velocity $\vec{v}_C$ and rotating with angular velocity $\vec{\omega}$. Both these motions are shown superimposed in the given figure.

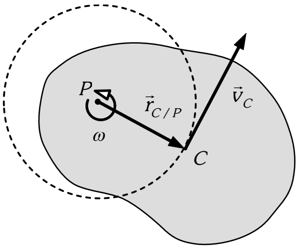
Kinetic energy too can be written as sum of kinetic energy $\left(\frac{1}{2}Mv_C^2\right)$ due to translation motion of the mass center and kinetic energy $\left(\frac{1}{2}I_C\omega^2\right)$ due to centroidal rotation.

$$K = \frac{1}{2}Mv_C^2 + \frac{1}{2}I_C\omega^2$$

If location of the instantaneous axis of rotation (IAR) is known, making use of the parallel axis theorem we can write kinetic energy by the following equation also.

$$K = \frac{1}{2}I_{IAR}\omega^2$$

Kinetic Energy of a rigid body in rotation about fixed axis not passing through the mass centre



In this kind of motion the mass center is in circular motion about the axis of rotation. In the figure is shown a body rotation with angular velocity ω about a fixed axis through pint P and perpendicular to plane of the paper. Mass center moves with speed $v_C = \omega r$. Kinetic energy of the body can now be expressed by the following equation.

$$K = \tfrac{1}{2} M v_C^2 + \tfrac{1}{2} I_C \omega^2$$

Making use of the parallel axis theorem $\left(I_P = M r_{P/C}^2 + I_C\right)$ we can write kinetic energy by the following equation also.

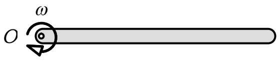
$$K = \tfrac{1}{2} I_P \omega^2$$

Kinetic Energy of a rigid body in pure centroidal rotation

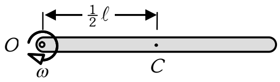
In pure centroidal rotation the mass center remain at rest; therefore kinetic energy due to translation of mass center vanishes.

$$K = \tfrac{1}{2} I_C \omega^2$$

Ex. A rod of mass m and length ℓ is pivoted to a fixed support at one of its ends O. It is rotating with constant angular velocity ω. Write expression for its kinetic energy.



Sol. If the point C is the mass center of the rod, from theorem of parallel axes, the moment of inertia I_O of the rod about the fixed axis is



$$I_O = I_C + m(OC)^2 \rightarrow \quad I_O = I_C + \tfrac{1}{4} m\ell^2$$

Substituting $\frac{1}{12} m\ell^2$ for I_C, we have

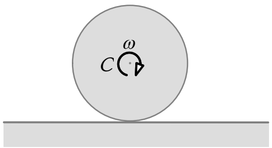
$$I_O = \tfrac{1}{3} m\ell^2$$

Kinetic energy of the rod equals to kinetic energy due to rotation about the fixed axis.

$K = \frac{1}{2} I_o \omega^2 \rightarrow$ Using above expression for I_O, we have

$$K = \tfrac{1}{6} m\ell^2\omega^2$$ **Ans.**

Ex. A uniform rigid body of mass m and round section of radius r is rolling on horizontal ground with angular velocity ω. Its radius of gyration about the centroidal axis is k.

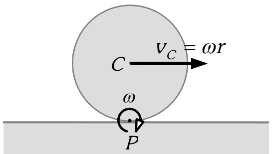


(a) Write expression of its kinetic energy.

(b) Also express the kinetic energy as sum of kinetic energy due to translation of mass center and kinetic energy due to simultaneous centroidal rotation.

Sol. *(a)* The point of contact with ground of a body rolling on the ground is its ICR. Let the point P is the ICR as shown in the adjacent figure. The geometrical center C of a uniform body and the mass center coincide. Therefore moment of inertia I_P of the body about the ICR can be written by using the theorem of parallel axes.

$$I_P = I_C + m(PC)^2 \rightarrow \qquad I_P = I_C + mr^2$$



Substituting $I_C = mk^2$, we have

$$I_P = m\left(k^2 + r^2\right) \qquad (1)$$

Kinetic energy of a rigid body equals to kinetic energy due to rotation about the ICR.

$K=\frac{1}{2}I_P\omega^2 \rightarrow$ Substituting I_P from eq. (1), we have

$$K=\tfrac{1}{2}m\left(k^2+r^2\right)\omega^2 \qquad \textbf{Ans.}$$

(b) Kinetic energy of the body also equals to sum of kinetic energy due to translation of its mass center and kinetic energy due to simultaneous centroidal rotation.

$K=\frac{1}{2}mv_C^2+\frac{1}{2}I_C\omega^2 \rightarrow$Substituting condition for rolling $v_C=\omega r$ and $I_C=mk^2$, we have

$$K=\tfrac{1}{2}m(\omega r)^2+\tfrac{1}{2}mk^2\omega^2=\tfrac{1}{2}m\left(r^2+k^2\right)\omega^2 \qquad \textbf{Ans.}$$

Rolling as rotation about an axis in translation

If the point of contact of the of the rolling body does not slide it is known as *rolling without slipping* or *pure rolling* or simply *rolling* and if the point of contact slides it is known as *rolling with slipping*. All kind of rolling motion is examples of rotation abut an axis in translation.

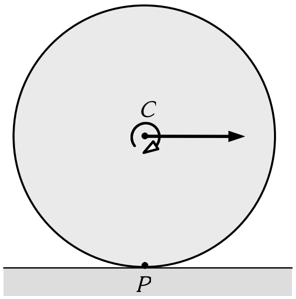
Rolling without slipping on stationary surface.

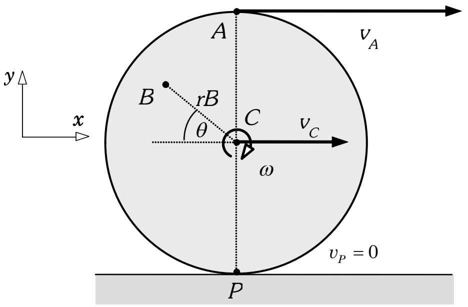
We first discuss velocity relations and thereafter accelerations relations of two points of a body of round section rolling on a stationary surface. For the purpose, we can use any of the following methods.

I	Analytical Method:	By using relative motion equations.
II	Superposition Method:	By superimposing translation of a point and pure rotation about that point.

Velocity relations by Analytical Method

Its point of contact *P* does not slide on the surface, therefore velocity of the point of contact relative to the surface is zero. In the next figure, velocity vectors of its center *C* and top point *A* are shown.





Velocity of the center *C* can be obtained with the help of relative motion equation.

$$\vec{v}_C=\vec{v}_P+\vec{\omega}\times\overrightarrow{PC}\rightarrow \quad \vec{v}_C=\vec{0}+\left(-\omega\hat{k}\right)\times R\hat{j}$$

$$\vec{v}_C=\omega R\hat{i} \qquad [16]$$

The above equation is used as condition of rolling without slipping on stationary surface.

Velocity of the top point *A* can be obtained by relative motion equation.

$$\vec{v}_A=\vec{v}_P+\vec{\omega}\times\overrightarrow{PA}\rightarrow \quad \vec{v}_C=\vec{0}+\left(-\omega\hat{k}\right)\times\left(2R\hat{j}\right)$$

$$\vec{v}_A=2\omega R\hat{i}=2\vec{v}_C \qquad [17]$$

Once velocity of the center is obtained, we can use relative motion between *A* and *C* as well.

$$\vec{v}_A=\vec{v}_C+\vec{\omega}\times\overrightarrow{CA}\rightarrow \quad \vec{v}_C=\omega R\hat{i}+\left(-\omega\hat{k}\right)\times\left(2R\hat{j}\right)$$

$$\vec{v}_A=2\omega R\hat{i}=2\vec{v}_C \qquad [18]$$

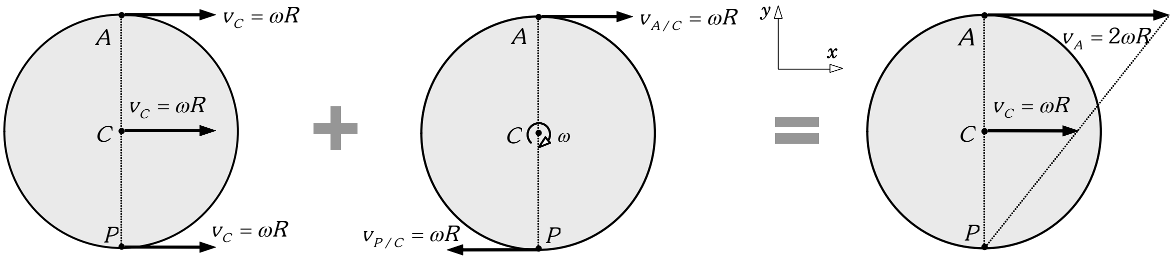
In similar fashion, velocity vector of an arbitrarily chosen point *B*.

$$\vec{v}_B = \vec{v}_C + \vec{\omega}\times\overrightarrow{CB} \rightarrow \quad \vec{v}_B = v_C\hat{i} + \left(-\omega\hat{k}\right)\times\left(-r\cos\theta\hat{i} + r\sin\theta\hat{j}\right)$$

$$\vec{v}_B = \left(v_C + \omega r\sin\theta\right)\hat{i} + \omega r\cos\theta\hat{j} \qquad [19]$$

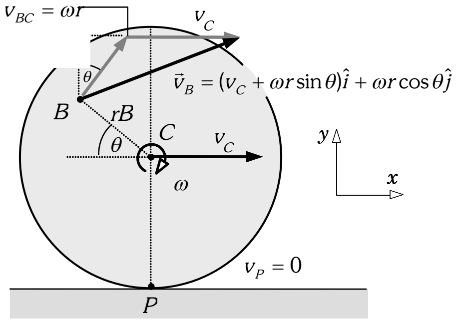
Velocity relations by Superposition Method

Now we will see that the above velocity relation can also be obtained by assuming rolling of the wheel as superposition of translation of its center and simultaneous rotation about the center.

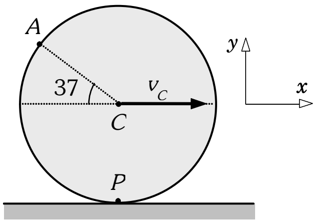


Translation of the center **Pure rotation about the center** **Rolling**

Velocity of an arbitrary point *B* as superposition of translation the center and rotation about the center.



Ex. A cylinder of radius 5 m rolls on a horizontal surface. Velocity of its center is 25 m/s. Find its angular velocity and velocity of the point *A*.



Sol. In rolling the angular velocity $\vec{\omega}$ and velocity of the center of a round section body satisfy condition described in the relative motion eq.[14]. So we have

$$\vec{v}_C = \vec{\omega}\times\vec{r}_{C/P} \rightarrow \qquad 25\hat{i} = \omega\hat{k}\times 5\hat{j} \Rightarrow \vec{\omega} = -5\hat{k}\text{ rad/s}$$

Angular velocity vector points in the negative *z*-axis so the cylinder rotates in clockwise sense.

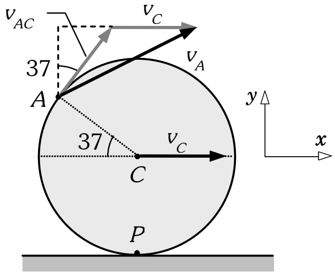
Velocity of the point *A* can be calculated by either analytical method, superposition method.

Analytical Method

$$\vec{v}_A = \vec{v}_C + \vec{\omega}\times\overrightarrow{AC} \rightarrow \quad \vec{v}_A = 25\hat{i} + \left(-5\hat{k}\right)\times\left(-5\cos 37^\circ\hat{i} + 5\sin 37^\circ\hat{j}\right)$$

$$\vec{v}_A = \left(40\hat{i} + 20\hat{j}\right)\text{ m/s}$$

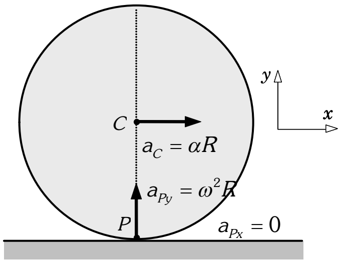
Superposition Method



In rolling $v_C = v_{AC} = \omega R = 25\text{m/s}$. The superposition i.e. vector addition of the terms of equation $\vec{v}_A = \vec{v}_C + \vec{v}_{AC}$ are shown in the following figure. Resolving $v_{AC} = \omega R = 25\text{m/s}$ into its Cartesian components and adding to $\vec{v}_C$, we obtain

$$\vec{v}_A = \vec{v}_C + \vec{v}_{A/C} \rightarrow \qquad \vec{v}_A = 25\hat{i} + 15\hat{i} + 20\hat{j} = \left(40\hat{i} + 20\hat{j}\right)\text{ m/s}$$

Acceleration relations by Analytical Method



The point of contact P does not slide on the surface, therefore component of its acceleration parallel to the surface must be zero. However, it has an acceleration component towards the center. The center always moves parallel to the horizontal surface and does not changes direction of its velocity; therefore, its acceleration can only be parallel to the surface.

Relation between acceleration of acceleration vector of the center C and point of contact P can be obtained with the help of relative motion [15] equation together with the above fact.

$$\vec{a}_C = \vec{a}_P + \vec{\alpha} \times \overrightarrow{PC} - \omega^2 \overrightarrow{PC} \rightarrow \qquad a_C\hat{i} = a_{Py}\hat{j} + \left(-\alpha\hat{k}\right) \times R\hat{j} - \omega^2 R\hat{j} = a_{Py}\hat{j} + \alpha R\hat{i} - \omega^2 R\hat{j}$$

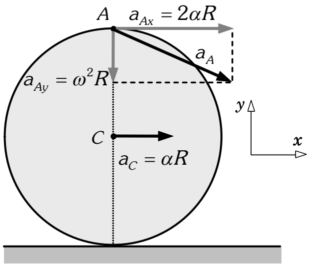
Equating coefficients of x and y-components on both the sides of the above equation, we have

$$\vec{a}_C = \alpha R\hat{i} \qquad [20]$$

$$\vec{a}_P = \omega^2 R\hat{j} \qquad [21]$$

The eq. [20] *is used as condition for rolling without slipping together with eq.* [16]

In the given figure, acceleration vectors the point of contact; center and the top point are shown. Now we will see how these accelerations can be calculated by using relative motion equation.



Once velocity of the center is obtained, we can use relative motion between A and C as well. Now we calculate acceleration of the top point A.

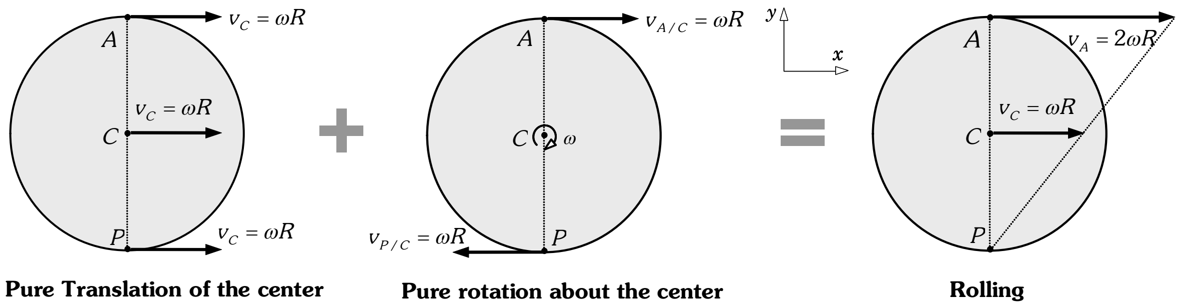
$$\vec{a}_A = \vec{a}_C + \vec{\alpha} \times \overrightarrow{CA} - \omega^2 \overrightarrow{CA} \rightarrow \qquad \vec{a}_A = \alpha R\hat{i} + \left(-\alpha\hat{k}\right) \times R\hat{j} - \omega^2 R\hat{j}$$

$$\vec{a}_A = 2\alpha R\hat{i} - \omega^2 R\hat{j} \qquad [22]$$

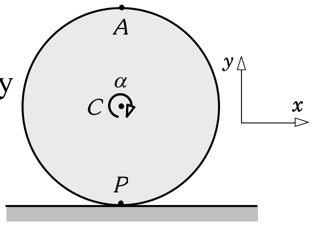
Acceleration vector of point A and its components are shown in the given figure.

Acceleration relations by Superposition Method

Now we see how acceleration relations are expressed for a rolling wheel by assuming its rolling as superposition of its translation with the velocity of center and simultaneous rotation about the centre.



Pure Translation of the center **Pure rotation about the center** **Rolling**



Ex. A body of round section of radius 10 cm starts rolling on a horizontal stationary surface with uniform angular acceleration 2 rad/s^2.

(a) Find initial acceleration of the center C and top point A.

(b) Find expression for acceleration of the top point A as function of time.

Sol. Initially when the body starts, it has no angular velocity; therefore, the last term in relative motion equation [15] for acceleration vanishes and for a pair of two points A and B the equation reduces to

$$\vec{a}_A = \vec{a}_B + \vec{\alpha} \times \overrightarrow{BA}$$

The angular acceleration vector is $\vec{\alpha} = -2\hat{k}$ rad/s^2.

(a) Acceleration of the center C is obtained by using condition for rolling without slipping.

$\vec{a}_C = \vec{\alpha} \times \overrightarrow{PC} \rightarrow$ $\qquad$ $\vec{a}_C = -2\hat{k} \times 10\hat{j} = 20\hat{i}$ cm/s^2 **Ans.**

Acceleration of the point A can be obtained either by analytical method, superposition method or by use of ICR. These methods for calculation of acceleration of the top point are already described; therefore, we use the result.

$\vec{a}_A = 2\alpha R\hat{i} \rightarrow$ $\qquad$ $\vec{a}_A = 40\hat{i}$ cm/s^2 **Ans.**

(b) Initially at the instant $t = 0$, when the body starts, its angular velocity is zero. At latter time it acquires angular velocity $\vec{\omega}$, therefore acceleration of any point on the body, other than its center, has an additional component of acceleration. This additional component is accounted for by the last term in the relative motion equation [15].

Angular velocity acquired by the body at time t is obtained by eq.[4] used for a body rotating with constant angular acceleration.

$\vec{\omega} = \vec{\omega}_o + \vec{\alpha}t \rightarrow$ $\qquad$ Substituting $\omega_o = 0$, we have

$\vec{\omega} = -2t\hat{k}$

Analytical Method

Using the relative motion equation for the pair of points C and A, we have

$\vec{a}_A = \vec{a}_C + \vec{\alpha} \times \overrightarrow{CA} - \omega^2\overrightarrow{CA} \rightarrow$ $\qquad$ $\vec{a}_A = \alpha R\hat{i} + \left(-\alpha\hat{k}\right) \times R\hat{j} - \omega^2 R\hat{j} = 2\alpha R\hat{i} - \omega^2 R\hat{j}$

Substituting the known values

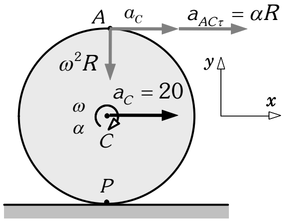
$\vec{\alpha} = -2\hat{k}$ rad/s^2, $\vec{\omega} = -2t\hat{k}$ rad/s and $R = 10$ cm

we have $\vec{a}_A = 40\hat{i} - 40t^2\hat{j}$ cm/s^2 **Ans.**

Superposition Method

We superimpose translation motion of the center and rotation motion about the center. In fact it is vector addition of terms of above equation used in analytical method.

From the above figure, we have



$\vec{a}_A = \left(a_C + \alpha R\right)\hat{i} - \omega^2 R\hat{j}$

Substituting known values $\qquad$ $\vec{\alpha} = -2\hat{k}$ rad/s^2, $\vec{\omega} = -2t\hat{k}$ rad/s and R=10 cm,

we have $\vec{a}_A = 40\hat{i} - 40t^2\hat{j}$ cm/s^2 **Ans.**

Methods of Impulse and Momentum

Methods of impulse and momentum describe what happens over a time interval. When motion of a body involves rotation we have to consider angular impulse as well as angular momentum. In this section we discuss concept of angular impulse, angular momentum of rigid body, angular impulse momentum principle and conservation of angular momentum.

Angular Impulse

Like impulse of a force angular impulse of a constant torque equals to product of the torque and concerned time interval and if the torque is not constant it must be integrated with time over the concerned time interval.

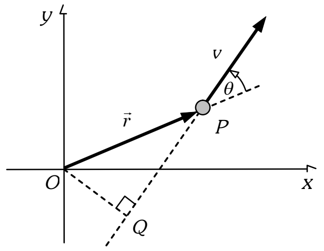
If torque $\vec{\tau}_o$ about an axis passing through O is constant, its angular impulse during a time interval from t_1 to t_2 denoted by $\vec{J}_{o,1\rightarrow 2}$ is given by the following equation.

$$\vec{J}_{o,1\rightarrow 2} = \vec{\tau}_o\left(t_2 - t_1\right)$$

If torque $\vec{\tau}_o$ about an axis passing through *O* is time varying, its angular impulse during a time interval from t_1 to t_2 denoted by $\vec{J}_{o,1\to2}$ is given by the following equation.

$$\vec{J}_{o,1\to2} = \int_{t_1}^{t_2} \vec{\tau}_o dt$$

Angular momentum of a particle



Angular momentum $\vec{L}_o$ about the origin *O* of a particle of mass *m* moving with velocity $\vec{v}$ is defined as the moment of its linear momentum $\vec{p} = m\vec{v}$ about the point *O*.

$$\vec{L}_o = \vec{r} \times (m\vec{v})$$

Angular Momentum of a Rigid Body

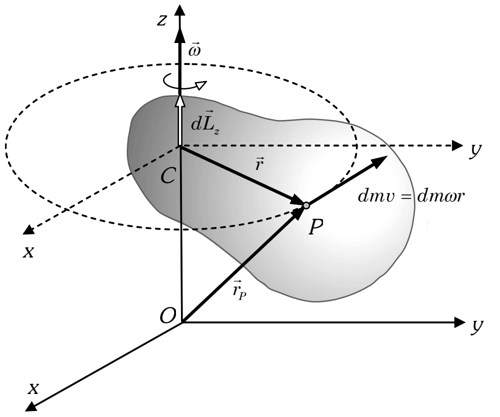
Angular momentum is quantity of rotation motion in a body. The angular momentum of a system of particles is the sum of angular momentum all the particles within the system. A rigid body is an assemblage of large number of particles maintaining their mutual distances intact under all circumstances, therefore angular momentum of a rigid body must be sum of angular momenta of all of its particles.

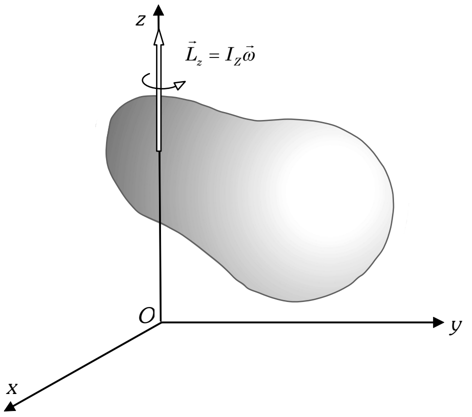
Angular Momentum about a point and about an axis

Angular momentum of a particle is not defined about an axis instead it is defined about a point. Therefore above idea of summing up angular momenta of all the particles about a point gives angular momentum of the rigid body about a point. But while dealing with fixed axis rotation or rotation about axis in translation we need angular momentum about an axis.

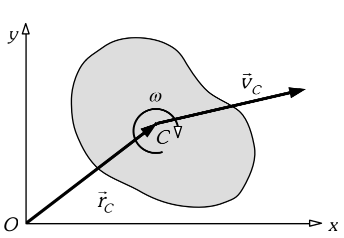
Angular momentum about an axis is calculated similar to torque abut an axis. To calculate angular momentum of a particle of rigid body about an axis we take moment of momentum of the particle about the point where plane of motion of the point of application of the force intersects the axis.

In the following figure is shown angular momentum $d\vec{L}_z = \vec{r} \times (dm\vec{v}) = r^2 dm\omega$ of a particle *P* of a rigid body rotating about the *z*-axis. It is along the *z*-axis i.e. axis of rotation. In the next figure total angular momentum $\vec{L}_z = \int d\vec{L}_z = I_z\vec{\omega}$ about the axis of rotation is shown. It is also along the axis of rotation.





Angular Momentum in general plane motion



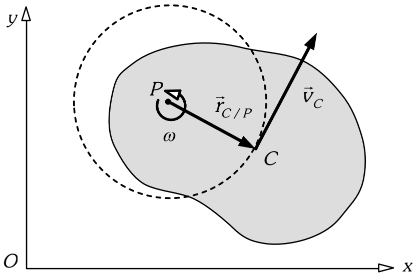
Angular momentum of a body in plane motion can also be written similar to torque equation or kinetic energy as sum of angular momentum about the axis due to translation of mass center and angular momentum of centroidal rotation about centroidal axis parallel to the original axis.

Consider a rigid body of mass M in plane motion. At the instant shown its mass center has velocity $\vec{v}$ and it is rotating with angular velocity $\vec{\omega}$ about an axis perpendicular to the plane of the figure. It angular momentum $\vec{L}_o$ about an axis passing though the origin and parallel to the original is expressed by the following equation.

$$\vec{L}_o = \vec{r}_C \times \left(M\vec{v}_C\right) + I_C\vec{\omega}$$

The first term of the above equation represent angular momentum due to translation of the mass center and the second term represents angular momentum in centroidal rotation.

Angular momentum in rotation about fixed axis

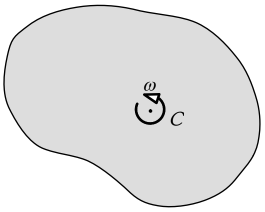


Consider a body of mass M rotating with angular velocity ω about a fixed axis perpendicular to plane of the figure passing through point P. Making use of the parallel axis theorem $I_P = Mr_{C/P}^2 + I_C$ and equation $\vec{v}_C = \vec{\omega} \times \vec{r}_{C/P}$ we can express the angular momentum $\vec{L}_P$ of the body about the fixed rotational axis.

$$\vec{L}_P = I_P\vec{\omega}$$

The above equation reveals that the angular momentum of a rigid body in plane motion can also be expressed in a single term due to rotation about the instantaneous axis of rotation.

Angular momentum in pure centroidal rotation



In pure centroidal rotation, mass center remains at rest, therefore angular momentum due to translation of the mass center vanishes.

$$\vec{L}_C = I_C\vec{\omega}$$

Rotational Equivalent of the Newton's Laws of Motion

Differentiating terms on both the sides of equation $\vec{L}_o = \vec{r}_C \times \left(M\vec{v}_C\right) + I_C\vec{\omega}$ with respect to time, and making substitution of $\vec{v}_C = d\vec{r}_C/dt$, $\vec{a}_C = d\vec{v}_C/dt$ and $\vec{\alpha} = d\vec{\omega}/dt$ we have

$$\frac{d\vec{L}_o}{dt} = \vec{v}_C \times \left(M\vec{v}_C\right) + \vec{r}_C \times M\vec{a}_C + I_C\vec{\alpha}$$

The first on the right hand side vanishes, so we can write

$$\frac{d\vec{L}_o}{dt} = \vec{r}_C \times M\vec{a}_C + I_C\vec{\alpha}$$

Now comparing the above equation with torque equation $\Sigma\vec{\tau}_o = \vec{r}_C \times M\vec{a}_C + I_C\vec{\alpha}$, we have

$$\sum\vec{\tau}_o = \frac{d\vec{L}_o}{dt}$$

The above equation though developed for plane motion only yet is valid for rotation about an axis in rotation also. It states that the net torque about the origin of an inertial frame equals to the time rate of change in angular momentum about the origin and can be treated as a parallel to Newton's second law which states that net external force on a body equals to time rate of change in its linear momentum.

Angular Impulse Momentum Principle

Rearranging the terms and integrating both the sides obtained form previous equation, we can write

$$\Sigma\int_{t_1}^{t_2}\vec{\tau}_o dt = \vec{L}_{o2} - \vec{L}_{o1}$$

The left hand side of the above equation is the angular impulse of torque of all the external forces in the time interval in the time interval t_1 to t_2.

$$\Sigma\vec{J}_{o,1\to 2} = \vec{L}_{o2} - \vec{L}_{o1}$$

The idea expressed by the above equation is known as angular impulse momentum principle and states that increment in the angular momentum of a body about a point in a time interval equals to the net angular impulse of all the external forces acting on it during the concerned time interval.

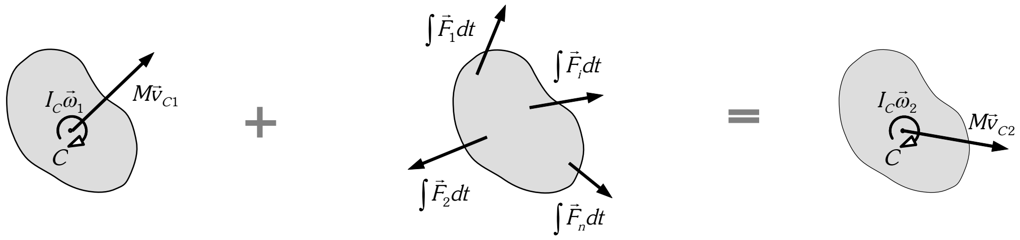
For the ease of application the above equation is rearranged as

$$\vec{L}_{o1} + \sum\vec{J}_{o,1\to 2} = \vec{L}_{o2}$$

Like linear impulse momentum principle, the angular impulse momentum principle provides us solution of problems concerned with change in angular velocity in a time interval or change in angular velocity during very short interval interactions.

Method of Impulse Momentum Principle for Plane motion of a Rigid Body

Linear momentum and angular momentum serve as measures of amount of translation and rotation motion respectively. The external forces acting on a rigid body can change its state of translation as well as rotation motion which is reflected by change in linear as well as angular momentum according to the principles of linear impulse and momentum and angular impulse and momentum.



Linear and angular momenta at the instant t_1

Impulse of all the forces during time interval t_1 to t_2

Linear and angular momenta at the instant t_2

In the above figure is shown strategy to apply method of impulse and momentum. Consider a rigid body of mass M in plane motion. Its moment of inertial about the centroidal axis perpendicular to plane of motion is I_C. Let $\vec{v}_{C1}$ and $\vec{\omega}_1$ represent velocity of its mass center and its angular velocity at the beginning of a time interval t_1 to t_2. Under the action of several forces $\vec{F}_1$, $\vec{F}_2$ ……. $\vec{F}_i$ …..$\vec{F}_n$ during the time interval its mass center velocity and angular velocity become $\vec{v}_{C2}$ and $\vec{\omega}_2$ respectively. The adjacent figure shows strategy representing how to write equations for linear and angular impulse momentum principles.

While applying the principle it becomes simpler to consider translation of the mass center and centroidal rotation separately. Thus in an alternative way we apply linear impulse momentum principle for translation of the mass center and angular impulse momentum principle for centroidal rotation.

Translation of mass center: Linear impulse momentum principle.

$$\vec{p}_1 + \sum\vec{I}_{mp1\to 2} = \vec{p}_2$$

Here $\vec{p}_1 = M\vec{v}_{C1}$ and $\vec{p}_2 = M\vec{v}_{C2}$ represent linear momentums at the beginning and end of the time interval and $\sum\vec{I}_{mp1\to 2}$ stands for impulse of all the external forces during the time interval.

Centroidal rotation: Angular impulse momentum principle.

$$\vec{L}_{C1} + \sum\vec{J}_{C,1\to 2} = \vec{L}_{C2}$$

Here $\vec{L}_{C1} = I_C\vec{\omega}_1$ and $\vec{L}_{C2} = I_C\vec{\omega}_2$ represent angular momentums about the centroidal axis at the beginning and end of the time interval and $\sum \vec{J}_{C,1\to 2}$ stands for angular impulse of all the external forces about the centroidal axis during the time interval.

Conservation of Angular Momentum

If angular impulse of all the external forces about an axis in time interval vanishes, the angular momentum of the system about the same axis in that time interval remain unchanged.

$$\text{If } \Sigma\int_{t_1}^{t_2} \vec{\tau}_o dt = 0 \text{, we have } \vec{L}_{o1} = \vec{L}_{o2}$$

The condition of zero net angular impulse required for conservation of angular momentum can be fulfilled in the following cases.

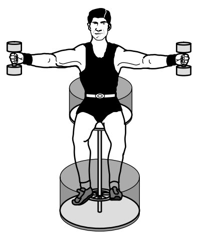
- If no external force acts, the angular impulse about all axes will be zero and hence angular momentum remains conserved about all axes.
- If net torque of all the external forces or torques of each individual force about an axis vanishes the angular momentum about that axes will be conserved.
- If all the external forces are finite in magnitude and the concerned time interval is infinitely small, the angular momentum remain conserved.
- If a system of rigid bodies changes its moment of inertia by changing its configuration due to internal forces only its angular momentum about any axes remains conserved. If we denote the moment of inertias in two configurations by I_1 and I_2 and angular velocities by ω_1 and ω_2, we can write

$$I_1\vec{\omega}_1 = I_2\vec{\omega}_2$$

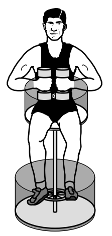
The principle of conservation of angular momentum governs a wide range of physical processes from subatomic to celestial world. The following examples explicate some of these applications.

Student on rotating turntable

The student, the turntable and dumbbells make an isolated system on which no external torque acts, if we ignore friction in the bearing of the turntable and air friction. Initially the student has his arm stretched on rotating turntable. When he pulls dumbbells close to his body, angular velocity increases due to conservation of angular momentum.

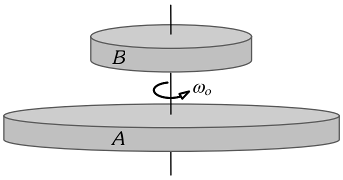


Larger moment of inertia and smaller angular velocity



Smaller moment of inertia and larger angular velocity

Ex. Consider the disk *A* of moment of inertia I_1 rotating freely in horizontal plane about its axis of symmetry with angular velocity ω_o. Another disk *B* of moment of inertia I_2 held at rest above the disk *A*. The axis of symmetry of the disk *B* coincides with that of the disk *A* as shown in the figure. The disk *B* is released to land on the disk *A*. When sliding stops, what will be the angular velocity of both the disks?



Sol. Both the disks are symmetric about the axis of rotation therefore does not require any external torque to keep the axis stationary. When the disk *B* lands on *A* slipping starts. The force of friction provides an internal torque to system of both the disk. It slows down rotation rate of *A* and increases that of *B* till both acquire same angular velocity ω.

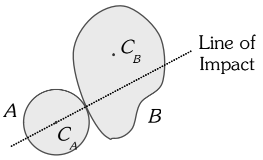
Since there is no external torques on the system of both the disks about the axis of rotation, the total angular momentum of the system remains conserved. The total angular momentum of the system is the sum of angular momentum of both disks. Denoting the angular momentum of the disk *A* before *B* lands on it and long after slipping between them stops by symbols $\vec{L}_{A1}$, $\vec{L}_{B1}$, $\vec{L}_{A2}$ and $\vec{L}_{B2}$ respectively, we can express conservation of angular momentum by the following equation.

$$\vec{L}_{A1}+\vec{L}_{B1}=\vec{L}_{A2}+\vec{L}_{B2} \rightarrow I_1\omega_o+0=I_1\omega+I_2\omega \Rightarrow \omega=\frac{I_1\omega_o}{I_1+I_2} \qquad \textbf{Ans.}$$

Eccentric Impact

In eccentric impact the line of impact which is the common normal drawn at the point of impact does not passes through mass center of at least one of the colliding bodies. It involves change in state of rotation motion of either or both the bodies.

Consider impact of two *A* and *B* such that the mass center C_B of *B* does not lie on the line of impact as shown in figure. If we assume bodies to be frictionless their mutual forces must act along the line of impact. The reaction force of *A* on *B* does not passes through the mass center of *B* as a result state of rotation motion of *B* changes during the impact.



Problems of Eccentric Impact

Problems of eccentric impact can be divided into two categories. In one category both the bodies under going eccentric impact are free to move. No external force act on either of them. There mutual forces are responsible for change in their momentum and angular momentum. In another category either or both of the bodies are hinged.

Eccentric Impact of bodies free to move

Since no external force acts on the two body system, we can use principle of conservation of linear momentum, principle of conservation of angular momentum about any point and concept of coefficient of restitution.

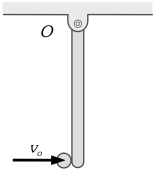
The coefficient of restitution is defined for components of velocities of points of contacts of the bodies along the line of impact.

While applying principle of conservation of angular momentum care must be taken in selecting the point about which we write the equation. The point about which we write angular momentum must be at rest relative to the selected inertial reference frame and as far as possible its location should be selected on line of velocity of the mass center in order to make zero the first term involving moment of momentum of mass center.

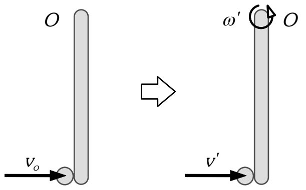
Eccentric Impact of hinged bodies

When either or both of the bodies are hinged the reaction of the hinge during the impact act as external force on the two body system, therefore linear momentum no longer remain conserved and we cannot apply principle of conservation of linear momentum. When both the bodies are hinged we cannot also apply conservation of angular momentum, and we have to use impulse momentum principle on both the bodies separately in addition to making use of coefficient of restitution. But when one of the bodies is hinged and other one is free to move, we can apply conservation of angular momentum about the hinge.

Ex. A uniform rod of mass m and length ℓ is suspended from a fixed support and can rotate freely in the vertical plane. A small ball of mass m moving horizontally with velocity v_o strikes elastically the lower end of the rod as shown in the figure. Find the angular velocity of the rod and velocity of the ball immediately after the impact.



Sol. The rod is hinged and the ball is free to move. External forces acting on the rod ball system are their weights and reaction from the hinge. Weight of the ball as well as the rod are finite and contribute negligible impulse during the impact, but impulse of reaction of the hinge during impact is considerable and cannot be neglected. Obviously linear momentum of the system is not conserved. The angular impulse of the reaction of hinge about the hinge is zero. Therefore angular momentum of the system about the hinge is conserved. Let velocity of the ball after the impact becomes v'_B and angular velocity of the rod becomes ω'.



Before the impact

Immediately after the impact

We denote angular momentum of the ball and the rod about the hinge before the impact by L_{B1} and L_{R1} and after the impact by L_{B2} and L_{R2}.

Applying conservation of angular momentum about the hinge, we have

$$\vec{L}_{B1}+\vec{L}_{R1}=\vec{L}_{B2}+\vec{L}_{R2} \rightarrow \quad mv_o\ell+0=mv'_B\ell+I_o\omega'$$

Substituting $\frac{1}{3}M\ell^2$ for I_o, we have

$$3mv'_B+M\ell\omega'=3mv_o \qquad (1)$$

The velocity of the lower end of the rod before the impact was zero and immediately after the impact it becomes $\ell\omega'$ towards right. Employing these facts we can express the coefficient of restitution according to eq.

$$e=\frac{v'_{Qn}-v'_{Pn}}{v_{pn}-v_{Qn}} \rightarrow \qquad \ell\omega'-v'_B=ev_o \qquad (2)$$

From eq. (1) and (2), we have

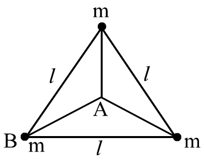
Velocity of the ball immediately after the impact $v'_B=\dfrac{(3m-eM)v_o}{3m+M}$ **Ans.**

Angular velocity of the rod immediately after the impact $\omega'=\dfrac{3(1+e)mv_o}{(3m+M)\ell}$ **Ans.**

EXERCISE (S-1)

Moment of inertia

1. Three equal masses m are rigidly connected to each other by massless rods of length l forming an equilateral triangle, as shown in the figure. What is the ratio of the moment of inertia of the assembly for an axis through B compared with that for an axis through A (centroid). Both the axis are perpendicular to the plane of triangle.



RD0001

2. Two rods of equal mass m and length l lie along the x axis and y axis with their centres at origin. What is the moment of inertia of the system about the line x = y :

RD0002

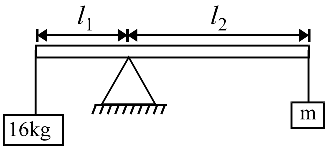
Torque and its calculation

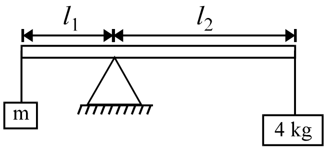
3. A force $\vec{F} = 2\hat{i} + 3\hat{j}$ N is applied to an object that is pivoted about a fixed axle aligned along the z coordinate axis. If the force is applied at the point $\vec{r} = 4\hat{i} + 5\hat{j}$ m, find (a) the magnitude of the net torque about the z axis and (b) the direction of the torque vector τ.

RD0003

Equilibrium

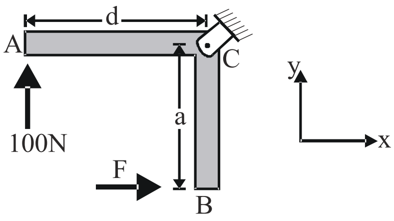
4. In an experiment with a massless beam balance an unknown mass m is balanced by two known masses of 16kg and 4 kg as shown in figure. Find the value of the unknown mass m.





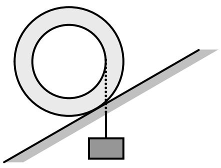
RD0004

5. Find the force F required to keep the system in equilibrium. The dimensions of the system are d = 0.3 m and a = 0.2 m. Assume the rods to be massless.



RD0005

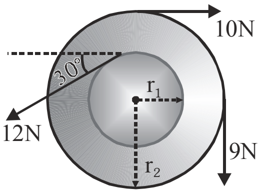
6. A 3.0 kg bobbin consists of a central cylinder of radius 5.0 cm and two end plates each of radius 6.0 cm. It is placed on a slotted incline, where friction is sufficient to prevent sliding. A block of mass 4.5 kg is suspended from a cord wound around the bobbin and passing through the slot under the incline. If the bobbin is in static equilibrium, what is the angle of tilt of the incline?



RD0006

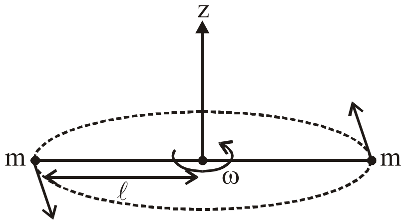
$\tau = I\alpha$ and its calculation

7. In the following figure r_1 and r_2 are 5 cm and 30 cm respectively. If the moment of inertia of the wheel is 5100 kg-m^2 about the axis passing through its centre and perpendicular to the plane of wheel, then what will be its angular acceleration?



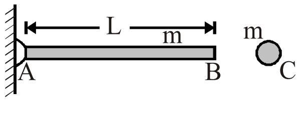
RD0007

8. A dumbbell consists of two identical particles of mass m connected by a rigid light rod of length 2ℓ. The dumbbell is set spinning with angular speed ω_0 on a surface with a small friction coefficient μ_k. If dumbbell stops in time $t = \dfrac{K\omega_0\ell}{2\mu g}$ where K is a constant, then find the value of K.



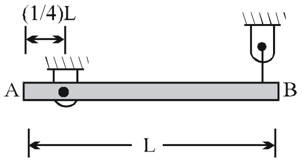
RD0008

9. A uniform bar AB of mass m and a ball of the same mass are released from rest from the same horizontal position. The bar is hinged at end A. There is gravity downwards. What is the distance of the point from point B that has the same acceleration as that of ball, immediately after release?



RD0009

10. A uniform beam of length L and mass m is supported as shown. If the cable at B suddenly breaks, determine; (a) the acceleration of end B. (b) the reaction at the pin support.



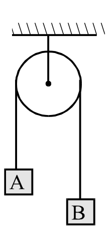
RD0010

11. In the figure, A & B are two blocks of mass 4 kg & 2 kg respectively attached to the two ends of a light string passing over a disc C of mass 40 kg and radius 0.1 m. The disc is free to rotate about a fixed horizontal axes, coinciding with its own axis. The system is released from rest and the string does not slip over the disc. Find:

(i) the linear acceleration of mass B .

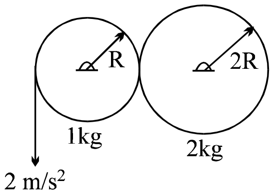
(ii) the number of revolutions made by the disc at the end of 10sec. from the start.

(iii) the tension in the string segment supporting the block A.



RD0011

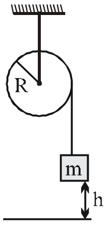
12. Two discs A and B touch each other as shown in figure. A rope tightly wound on A is pulled down at 2 m/s^2. Find the friction force between A and B if slipping is absent



RD0012

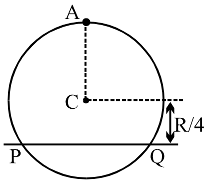
Kinetic energy in pure rotation

13. A mass m is attached to a pulley through a cord as shown in the figure. The pulley is a solid disk with radius R. The cord does not slip on the disk. The mass is released from rest at a height h from the ground and at the instant the mass reaches the ground, the disk is rotating with angular velocity ω. Find the mass of the disk.



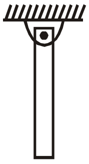
RD0013

14. A uniform circular disc has radius R and mass m. A particle also of mass m is fixed at a point A on the edge of the disc as in figure. The disc can rotate freely about a fixed horizontal chord PQ that is at a distance R/4 from the centre C of the disc. The line AC is perpendicular to PQ. Initially the disc is held vertical with the point A at its highest position. It is then allowed to fall so that it starts rotating about PQ. Find the linear speed of the particle at it reaches its lowest position.



RD0014

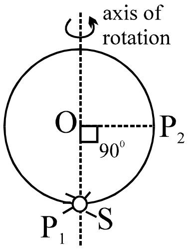
15. A rod of mass m & length ℓ is hinged at it's upper end. It can rotate in vertical plane. It is given angular velocity ω so that it can complete vertical circle. Find (a) ω (b) Tension at centre of rod at initial moment.



RD0015

Angular momentum and its conservation

16. A uniform ring is rotating about vertical axis with angular velocity ω initially. A point insect (S) having the same mass as that of the ring starts walking from the lowest point P_1 and finally reaches the point P_2 (as shown in figure). What is the final angular velocity of the ring?

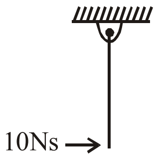


RD0016

17. Two men, each of mass 75 kg , stand on the rim of a horizontal large disc, diametrically opposite to each other. The disc has a mass 450 kg and is free to rotate about its axis. Each man simultaneously start along the rim clockwise with the same speed and reaches their original starting points on the disc. Find the angle turned by the disc with respect to the ground in this duration.

RD0017

18. A thin uniform straight rod of mass 2 kg and length 1 m is free to rotate about its upper end when at rest. It receives an impulsive blow of 10 Ns at its lowest point, normal to its length as shown in figure. Find the kinetic energy of rod just after impact.



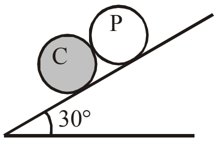
RD0018

Combined rotation and translation

19. A solid uniform disk of mass m rolls down a fixed inclined plane without slipping with an acceleration a. Find the frictional force on the disk due to surface of the plane :

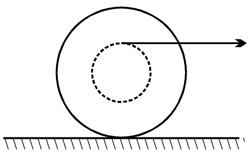
RD0019

20. A solid cylinder C and a hollow pipe P of same diameter are in contact when they are released from rest as shown in the figure on a long incline plane. Cylinder C and pipe P roll without slipping. Determine the clear gap (in m) between them after 6 seconds.



RD0020

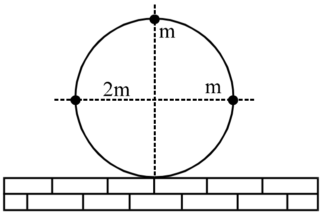
21. A spool of inner radius R and outer radius 3R has a moment of inertia = MR^2 about an axis passing through its geometric centre, where M is the mass of the spool. A thread wound on the inner surface of the spool is pulled horizontally with a constant force = Mg. Find the acceleration of the point on the thread which is being pulled assuming that the spool rolls purely on the floor.



RD0021

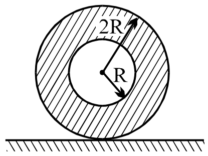
Kinetic energy in rolling

22. A ring of mass m and radius R has three particles attached to the ring as shown in the figure. The centre of the ring has a speed v_0. Find the kinetic energy of the system. (Slipping is absent)



RD0022

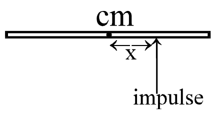
23. A hollow cylinder with inner radius R, outer radius 2R and mass M is rolling with speed of its axis v. Its kinetic energy is :-



RD0023

Angular impulse

24. A uniform rod of length *l* is given an impulse at right angle to its length as shown. Find the distance of instantaneous centre of rotation from the centre of the rod.



RD0024

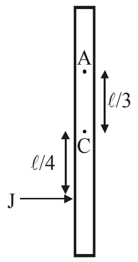
25. A uniform rod of mass m and length ℓ is placed in gravity free space and linear impulse J is given to the rod at a distance $x = \ell/4$ from centre 'C' and perpendicular to the rod. Point A is at a distance $\ell/3$ from centre as shown in the figure. Then find

(i) Speed of centre of rod

(ii) Speed of point A

(iii) Speed of upper end of rod

(iv) Speed of lower end of rod



RD0025

26. A solid sphere of mass m and radius R is placed on a smooth horizontal surface. A sudden blow is given horizontally to the sphere at a height h = 4R/5 above the centre line. If I is the impulse of the blow then find

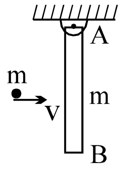
(a) the minimum time after which the highest point B will touch the ground

(b) the displacement of the centre of mass during this internal.

RD0026

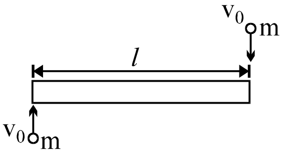
Eccentric collision

27. A uniform rod AB of length L and mass m is suspended freely at A and hangs vertically at rest when a particle of same mass m is fired horizontally with speed v to strike the rod at its mid point. If the particle comes to rest after the impact, then find the impulsive reaction at A.



RD0027

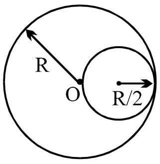
28. On a smooth table two particles of mass m each, travelling with a velocity v_0 in opposite directions, strike the ends of a rigid massless rod of length *l*, kept perpendicular to their velocity. The particles stick to the rod after the collision.Find the tension in rod during subsequent motion.



RD0028

EXERCISE (S-2)

1. A hole of radius R/2 is cut from a solid sphere of radius R. If the mass of the remaining part of sphere is M , then find moment of inertia of the body about an axis through O.

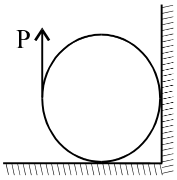


RD0029

2. Find the MI of a rod about an axis through its centre of mass and perpendicular to the length whose linear density varies as $\lambda = ax$ where a is a constant and x is the position of an element of the rod relative to its left end. The length of the rod is ℓ.

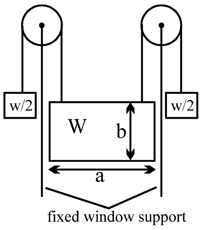
RD0030

3. Figure shows a vertical force applied tangentially to a uniform cylinder of weight F_g. The coefficient of static friction between the cylinder and both surfaces is 0.500. In terms of F_g, find the maximum force **P** that can be applied that does not cause the cylinder to rotate.



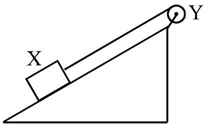
RD0031

4. A slight loosely fit window is balanced by two strings which are connected to weights w/2 each. The strings pass over the frictionless pulleys as shown in the figure. The strings are tied almost at the corner of the window. The string on the right is cut and then the window accelerates downwards. If the coefficients of friction between the window and the side supports is μ then calculate the acceleration of the window in terms of μ, a, b and g, where a is width and b is the length of the window.



RD0032

5. A block X of mass 0.5kg is held by a long massless string on a frictionless fixed inclined plane of inclination 30° to the horizontal. The string is wound on a uniform solid cylindrical drum Y of mass 2kg and of radius 0.2 m as shown in the fig. The drum is given an initial angular velocity such that the block X starts moving up the plane. ($g = 9.8\ m/s^2$)
(i) Find the tension in the string during the motion
(ii) At a certain instant of time the magnitude of the angular velocity of Y is 10 rad/sec.
Calculate the distance travelled by X from that instant of time until it comes to rest.

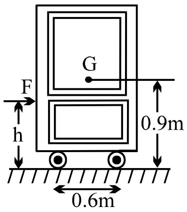


RD0033

6. A 20 kg cabinet is mounted on small casters that allow it to move freely ($\mu = 0$) on the floor. If a 100 N force is applied as shown, determine

(a) the acceleration of the cabinet,

(b) the range of values of h for which the cabinet will not tip.

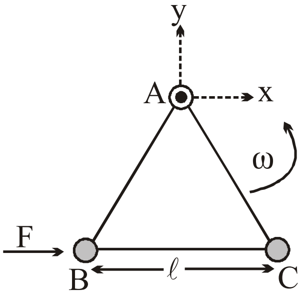


RD0034

7. Three particles A,B, and C each of mass m, are connected to each other by three massless rigid rods to form a rigid, equilateral triangular body of side ℓ. This body is placed on a horizontal frictionless table (x-y plane) and is hinged to it at the point A, so that it can move without friction about the vertical axis through A (see figure). The body is set into rotational motion on the table about A with a constant angular velocity ω.

(a) Find the magnitude of the horizontal force exerted by the hinge on the body.

(b) At time T, when the side BC is parallel to the x-axis, a force F is applied on B along BC (as shown).Obtain the x–component and the y–component of the force exerted by the hinge on the body, immediately after time T. **[IIT-JEE' 2001]**



RD0035

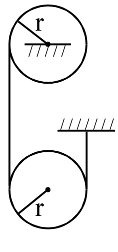
8. The door of an automobile is open and perpendicular to the body. The automobile starts with an acceleration of 2 ft/sec^2, and the width of the door is 30 inches. Treat the door as a uniform rectangle, and neglect friction to find the speed of its outside edge as seen by the driver when the door closes.

RD0036

9. Two uniform cylinders, each of mass m = 10 kg and radius r = 150 mm, are connected by a rough belt as shown. If the system is released from rest, determine

(a) the velocity of the centre of cylinder A after it has moved through 1.2 m &

(b) the tension in the portion of the belt connecting the two cylinders.



RD0037

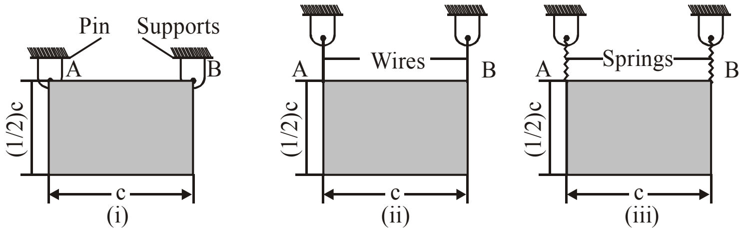
10. A circular disc of mass 300 gm and radius 20 cm can rotate freely about a vertical axis passing through its centre O. A small insect of mass 100 gm is initially at a point A on the disc (which is initially stationary) the insect starts walking from rest along the rim of the disc with such a time varying relative velocity that the disc rotates in the opposite direction with a constant angular acceleration = 2π rad/s^2. After some time T, the insect is back at the point A. By what angle θ has the disc rotated till now ; as seen by a stationary earth observer? Also find the time T.

RD0038

11. A uniform plate of mass m is suspended in each of the ways shown. For each case determine immediately after the connection at B has been released ;

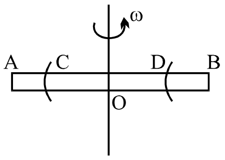
(a) the angular acceleration of the plate .

(b) the acceleration of its mass center .



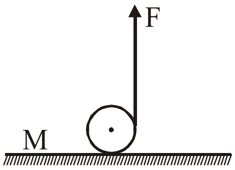
RD0039

12. A rigid horizontal smooth rod AB of mass 0.75 kg and length 40 cm can rotate freely about a fixed vertical axis through its mid point O. Two rings each of mass 1 kg are initially at rest at a distance of 10 cm from O on either side of the rod. The rod is set in rotation with an angular velocity of 30 radians per second. Find the velocity of each ring along the length of the rod in m/s when they reach the ends of the rod



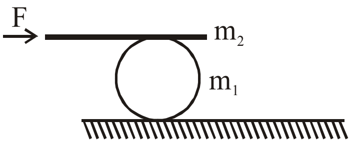
RD0040

13. The free end of the string wound on the surface of a solid cylinder of mass M & radius r is pulled up by a force F as shown. If there is sufficient friction between cylinder & floor so that the cylinder is able to roll without slipping, find the maximum angular acceleration that the cylinder can have (without cylinder leaving contact with surface).



RD0041

14. A man pushes a cylinder of mass m_1 with help of a plank of mass m_2 as shown. There is no slipping at any contact. the horizontal component of the force applied by the man is F. Find:



(a) the accelerations of the plank and the centre of mass of the cylinder, and

(b) the magnitudes and directions of frictional forces at contact points. **[JEE '99 , 6 + 4]**

RD0042

15. A 160 mm diameter pipe of mass 6 kg rests on a 1.5 kg plate. The pipe and plate are initially at rest when a force P of magnitude 25 N is applied for 0.75s. Knowing that $\mu_s = 0.25$ & $\mu_k = 0.20$ between the plate and both the pipe and the floor, determine;

(a) whether the pipe slides with respect to the plate .

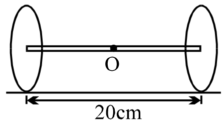
(b) the resulting velocities of the pipe and of the plate .

RD0043

16. Two thin circular disks of mass 2kg and radius 10cm each are joined by a rigid massless rod of length 20cm. The axis of the rod is along the perpendicular to the planes of the disks through their centre. The object is kept on a truck in such a way that the axis of the object is horizontal and perpendicular to the direction of motion of the truck. Its friction with the floor of the truck is large enough so that the object can roll on the truck without slipping. Take x-axis as the direction of motion of the truck and z -axis as the vertically upwards direction. If the truck has an acceleration of $9 m/s^2$ calculate :

(a) the force of friction on each disk

(b) The magnitude and the direction of the frictional torque acting on each disk about the centre of mass O of the object. Express the torque in the vector form of unit vectors in the x-y and z directions.



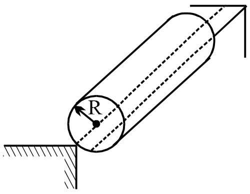
RD0044

17. A rectangular rigid fixed block has a long horizontal edge. A solid homogeneous cylinder of radius R is placed horizontally at rest with its length parallel to the edge such that the axis of the cylinder and the edge of the block are in the same vertical plane as shown in figure. there is sufficient friction present at the edge so that a very small displacement cause the cylinder to roll of the edge without slipping. Determine

(a) the angle θ_C through which the cylinder rotates before it leaves contact with the edge,

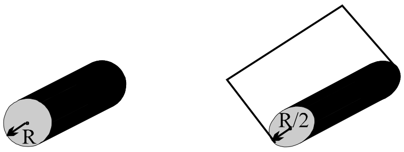
(b) the speed of the centre of mass of the cylinder before leaving contact with the edge, and

(c) the ratio of the translational to rotational kinetic energies of the cylinder when its centre of mass is in horizontal line with the edge.



RD0045

18. A carpet of mass 'M' made of inextensible material is rolled along its length in the form of a cylinder of radius 'R' and is kept on a rough floor. The carpet starts unrolling without sliding on the floor when a negligibly small push is given to it. The horizontal velocity of the axis of the cylindrical part of the carpet when its radius reduces to R/2 will be:

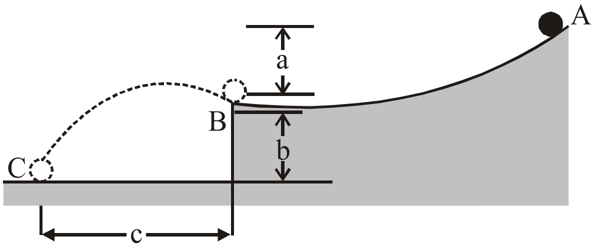


RD0046

19. A small sphere of mass m and radius r is released from rest at A and rolls without sliding on the curved surface to point B where it leaves the surface with a horizontal velocity. Knowing that a = 1.5 m & b = 1.2 m, determine;

(a) the speed of the sphere as it strikes the ground at C .

(b) the corresponding distance c .

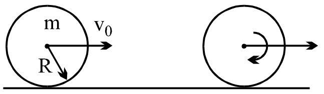


RD0047

20. A uniform disk of mass m and radius R is projected horizontally with velocity v_0 on a rough horizontal floor so that it starts off with a purely sliding motion at t = 0. After t_0 seconds it acquires a purely rolling motion as shown in figure.

(i) Calculate the velocity of the centre of mass of the disk at t_0.

(ii) Assuming the coefficient of friction to be μ calculate t_0. Also calculate the work done by the frictional force as a function of time and the total work done by it over a time t much longer than t_0.



RD0048

21. A uniform disc of mass m and radius R rotates about a fixed vertical axis passing through its centre with angular velocity ω. A particle of same mass m and having velocity 2ωR towards centre of the disc collides with the disc moving horizontally and sticks to its rim. Find

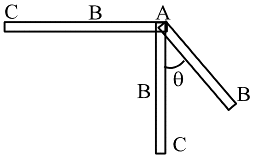
(a) the angular velocity of the disc.

(b) the impulse on the particle due to disc.

(c) the impulse on the disc due to hinge.

RD0049

22. A rod hinged at one end is released from the horizontal position as shown in the figure. When it becomes vertical its lower half separates without exerting any reaction at the breaking point. Then find the maximum angle 'θ' made by the hinged upper half with the vertical.



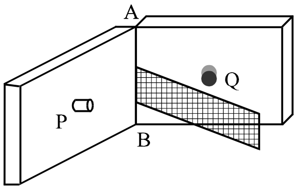
RD0050

23. Two heavy metallic plates are joined together at 90^0 to each other. A laminar sheet of mass 30 Kg is hinged at the line AB joining the two heavy metallic plates. The hinges are frictionless. The moment of inertia of the laminar sheet about an axis parallel to AB and passing through its centre of mass is 1.2 Kg-m^2. Two rubber obstacles P and Q are fixed, one on each metallic plate at a distance 0.5 m from the line AB. This distance is chosen so that the reaction due to the hinges on the laminar sheet is zero during the impact. Initially the laminar sheet hits one of the obstacles with an angular velocity 1 rad/s and turns back. If the impulse on the sheet due to each obstacle is 6 N–s. **[IIT-JEE' 2001]**

(a) Find the location of the centre of mass of the laminar sheet from AB.

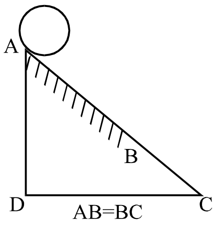
(b) At what angular velocity does the laminar sheet come back after the first impact ?

(c) After how many impacts, does the laminar sheet come to rest ?



RD0051

24. Portion AB of the wedge shown in figure is rough and BC is smooth. A solid cylinder rolls without slipping from A to B. Find the ratio of translational kinetic energy to rotational kinetic energy, when the cylinder reaches point C.



RD0052

25. A rod AB of mass M and length L is lying on a horizontal frictionless surface. A particle of mass m travelling along the surface hits the end 'A' of the rod with a velocity v_0 in the direction perpendicular to AB. The collision is completely elastic. After the collision the particle comes to rest.

(a) Find the ratio m/M . **[IIT-JEE' 2000]**

(b) A point P on the rod is at rest immediately after the collision. Find the distance AP.

(c) Find the linear speed of the point P at a time $\pi L/(3v_0)$ after the collision.

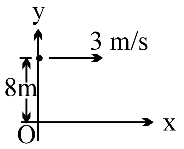
RD0053

EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

Conecpt of ω, α, v, a

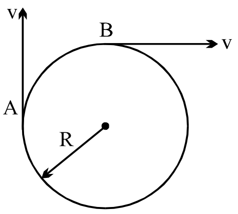
1. A particle starts from the point (0m, 8m) and moves with uniform velocity of $3\,\hat{i}$ m/s. After 5 seconds, the angular velocity of the particle about the origin will be :



(A) $\frac{8}{289}$ rad/s (B) $\frac{3}{8}$ rad/s (C) $\frac{24}{289}$ rad/s (D) $\frac{8}{17}$ rad/s

RD0054

2. Two points of a rigid body are moving as shown. The angular velocity of the body is:



(A) $\frac{\upsilon}{2R}$ (B) $\frac{\upsilon}{R}$ (C) $\frac{2\upsilon}{R}$ (D) $\frac{2\upsilon}{3R}$

RD0055

Moment of inertia

3. Three bodies have equal mass m. Body A is solid cylinder of radius R, body B is a square lamina of side R, and body C is a solid sphere of radius R. Which body has the smallest moment of inertia about an axis passing through their centre of mass and perpendicular to the plane (in case of lamina)

(A) A (B) B (C) C (D) A and C both

RD0056

4. For the same total mass which of the following will have the largest moment of inertia about an axis passing through its centre of mass and perpendicular to the plane of the body

(A) a disc of radius a (B) a ring of radius a

(C) a square lamina of side 2a (D) four rods forming a square of side 2a

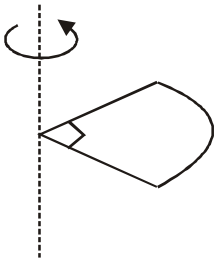
RD0057

5. Let I be the moment of inertia of a uniform square plate about an axis AB that passes through its centre and is parallel to two of its sides. CD is a line in the plane of the plate that passes through the centre of the plate and makes an angle θ with AB. The moment of inertia of the plate about the axis CD is then equal to

(A) I (B) $I\sin^2\theta$ (C) $I\cos^2\theta$ (D) $I\cos^2(\theta/2)$

RD0058

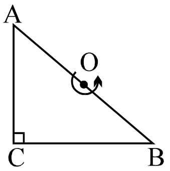
6. One quarter sector is cut from a uniform circular disc of radius R. This sector has mass M. It is made to rotate about a line perpendicular to its plane and passing through the center of the original disc. Its moment of inertia about the axis of rotation is **[IIT-JEE 2000]**



(A) $\frac{1}{2}MR^2$ (B) $\frac{1}{4}MR^2$ (C) $\frac{1}{8}MR^2$ (D) $\sqrt{2}\,MR^2$

RD0059

7. Find the moment of inertia of a plate cut in shape of a right angled triangle of mass M, about an axis perpendicular to the plane of the plate and passing through the mid point of side AB. (Side AC = BC = a)



(A) $\frac{Ma^2}{12}$ (B) $\frac{Ma^2}{6}$ (C) $\frac{Ma^2}{3}$ (D) $\frac{2Ma^2}{3}$

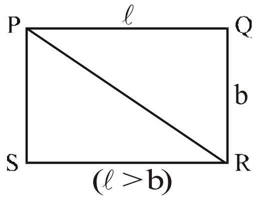
RD0060

8. A circular disc X of radius R is made from an iron plate of thickness t and another disc Y of radius 4R is made from an iron plate of thickness t/4. Then the relation between the moment of inertia I_X and I_Y is- **[AIEEE - 2003]**

(A) $I_Y = 32\,I_X$ (B) $I_Y = 16I_X$ (C) $I_Y = I_X$ (D) $I_Y = 64\,I_X$

RD0061

9. Moment of inertia of a rectangular plate about an axis passing through P and perpendicular to the plate is I. Then moment of PQR about an axis perpendicular to the plane of the plate:



(A) about P = I/2 (B) about R = I/2 (C) about P > I/2 (D) about R > I/2

RD0062

10. A thin uniform rod of mass M and length L has its moment of inertia I_1 about its perpendicular bisector. The rod is bend in the form of a semicircular arc. Now its moment of inertia through the centre of the semi circular arc and perpendicular to its plane is I_2. The ratio of $I_1 : I_2$ will be _______

(A) < 1 (B) > 1 (C) = 1 (D) can't be said

RD0063

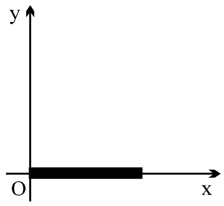
11. One solid sphere A and another hollow sphere B are of same mass and same outer radii. Their moment of inertia about their diameters are respectively I_A and I_B such that- **[AIEEE - 2004]**

(A) $I_A = I_B$ (B) $I_A > I_B$ (C) $I_A < I_B$ (D) $\frac{I_A}{I_B} = \frac{d_A}{d_B}$

where d_A and d_B are their densities.

RD0064

12. The figure shows a uniform rod lying along the x-axis. The locus of all the points lying on the xy-plane, about which the moment of inertia of the rod is same as that about O is :-



(A) an ellipse (B) a circle (C) a parabola (D) a straight line

RD0065

13. A rigid body can be hinged about any point on the x-axis. When it is hinged such that the hinge is at x, the moment of inertia is given by $I = 2x^2 - 12x + 27$. The x-coordinate of centre of mass is

(A) x = 2 (B) x = 0 (C) x = 1 (D) x = 3

RD0066

Equilibrium

14. A weightless rod is acted on by upward parallel forces of 2N and 4N at ends A and B respectively. The total length of the rod is AB = 3m. To keep the rod in equilibrium a force of 6N should act in the following manner:

(A) Downwards at any point between A and B.

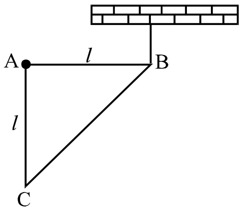
(B) Downwards at mid point of AB.

(C) Downwards at a point C such that AC = 1m.

(D) Downwards at a point D such that BD = 1m.

RD0067

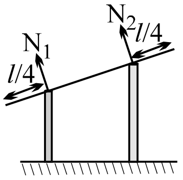
15. A right triangular plate ABC of mass m is free to rotate in the vertical plane about a fixed horizontal axis through A. It is supported by a string such that the side AB is horizontal. The reaction at the support A in equilibrium is:



(A) $\frac{mg}{3}$ (B) $\frac{2mg}{3}$ (C) $\frac{mg}{2}$ (D) mg

RD0068

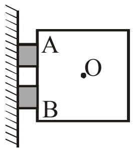
16. A uniform rod of length l is placed symmetrically on two walls as shown in figure. The rod is in equilibrium. If N_1 and N_2 are the normal forces exerted by the walls on the rod then :-

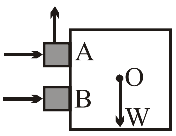


(A) $N_1 > N_2$
(B) $N_1 < N_2$
(C) $N_1 = N_2$
(D) N_1 and N_2 would be in the vertical directions

RD0069

17. A vertical rectangular door with its centre of gravity at O (see figure) is fixed on two hinges A and B along one vertical length side of the door. The entire weight of the door is supported by the hinge A. Then the free body force diagram for the door (the arrows indicate the direction of the forces) is :-



(A)


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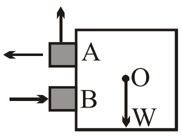
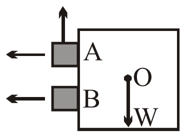
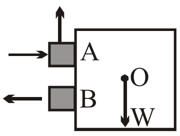
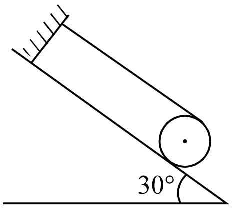


(C)


(D)


RD0070

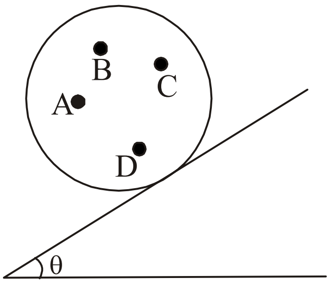
18. A thin hoop of weight 500 N and radius 1 m rests on a rough inclined plane as shown in the figure. The minimum coefficient of friction needed for this configuration to be in equilibrium is:



(A) $\frac{1}{3\sqrt{3}}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\frac{1}{2}$ (D) $\frac{1}{2\sqrt{3}}$

RD0071

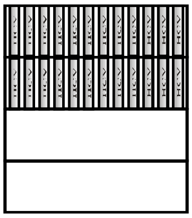
19. A non uniform sphere can be kept on a rough inclined plane so that it is in equilibrium. In the figure below the dots represents location of centre of mass. In which one of the positions can sphere be in equilibrium?

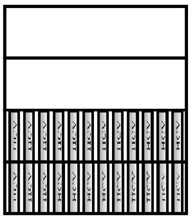


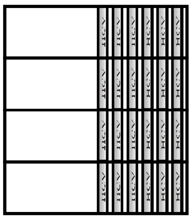
RD0072

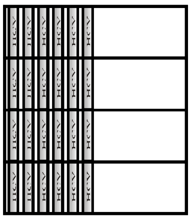
Toppling

20. Same number of books are placed in four book cases as shown. Which bookcase is most likely to topple forward if pulled a little at the top towards right :

(A)


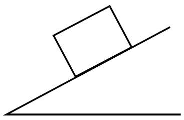
(B)


(C)


(D)


RD0073

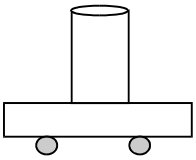
21. A homogeneous cubical brick lies motionless on a rough inclined surface. The half of the brick which applies greater pressure on the plane is :



(A) left half (B) right half
(C) both applies equal pressure (D) the answer depend upon coefficient of friction

RD0074

22. A uniform 2 kg cylinder rests on a laboratory cart as shown. The coefficient of static friction between the cylinder and the cart is 0.5. If the cylinder is 4 cm in diameter and 10 cm in height, which of the following is closest to the maximum acceleration of the cart such that cylinder neither slips nor tips over?

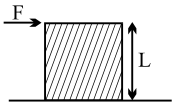


(A) 2 m/s^2 (B) 4 m/s^2 (C) 5 m/s^2 (D) 6 m/s^2

RD0075

23. A cubical block of side L rests on a rough horizontal surface with coefficient of friction μ. A horizontal force F is applied on the block as shown. If the coefficient of friction is sufficiently high so that the block does not slide before toppling, the minimum force required to topple the block is:

[IIT-JEE'(Scr)'2000]

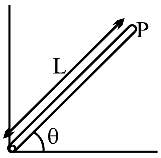


(A) infinitesimal (B) mg/4 (C) mg/2 (D) mg(1-μ)

RD0076

τ = Iα

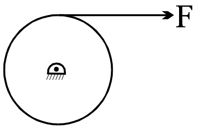
24. A uniform flag pole of length L and mass M is pivoted on the ground with a frictionless hinge. The flag pole makes an angle θ with the horizontal. The moment of inertia of the flag pole about one end is (1/3)ML^2. If it starts falling from the position shown in the accompanying figure, the linear acceleration of the free end of the flag pole (labeled P) immediately after it starts falling off would be:



(A) (2/3) gcosθ (B) (2/3) g (C) g (D) (3/2) gcosθ

RD0077

25. A pulley is hinged at the centre and a massless thread is wrapped around it. The thread is pulled with a constant force F starting from rest. As time increases,



(A) its angular velocity increases, but force on hinge remains constant

(B) its angular velocity remains same, but force on hinge increases

(C) its angular velocity increases and force on hinge increases

(D) its angular velocity remains same and force on hinge is constant

RD0078

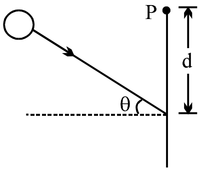
Angular momentum and its conservation

26. A particle of mass 2 kg located at the position $(\hat{i}+\hat{j})$ m has a velocity $2(+\hat{i}-\hat{j}+\hat{k})$ m/s. Its angular momentum about z-axis in kg-m^2/s is:

(A) zero (B) +8 (C) 12 (D) –8

RD0079

27. A ball of mass m moving with velocity v, collide with the wall elastically as shown in the figure. After impact the change in angular momentum about P is:



(A) 2 *mvd* (B) 2 *mvd* cosθ (C) 2 *mvd* sin θ (D) zero

RD0080

28. Two uniform spheres of mass M have radii R and 2R. Each sphere is rotating about a fixed axis through a diameter. The rotational kinetic energies of the spheres are identical. What is the ratio of the magnitude of the angular momenta of these spheres? That is, $\frac{L_{2R}}{L_R} =$

(A) 4 (B) $2\sqrt{2}$ (C) 2 (D) $\sqrt{2}$

RD0081

29. A spinning ice skater can increase his rate of rotation by bringing his arms and free leg closer to his body. How does this procedure affect the skater's angular momentum and kinetic energy and what is the work done by the skater?

(A) angular momentum remains the same while kinetic energy increases and work done is positive.

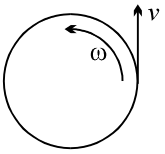
(B) angular momentum remains the same while kinetic energy decreases and work done is negative.

(C) both angular momentum and kinetic energy remain the same and work done is zero.

(D) angular momentum increases while kinetic energy remains the same and work done may be positive or negative.

RD0082

30. A child with mass m is standing at the edge of a disc with moment of inertia I, radius R, and initial angular velocity ω. See figure given below. The child jumps off the edge of the disc with tangential velocity v with respect to the ground. The new angular velocity of the disc is

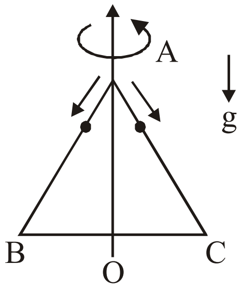


(A) $\sqrt{\dfrac{I\omega^2 - mv^2}{I}}$ (B) $\sqrt{\dfrac{(I + mR^2)\omega^2 - mv^2}{I}}$

(C) $\dfrac{I\omega - mvR}{I}$ (D) $\dfrac{(I + mR^2)\omega - mvR}{I}$

RD0083

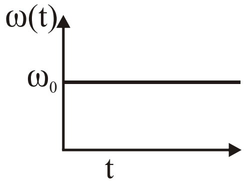
31. An equilateral triangle ABC formed from a uniform wire has two small identical beads initially located at A. The triangle is set rotating about the vertical axis AO. Then the beads are released from rest simultaneously and allowed to slide down, one along AB and the other along AC as shown. Neglecting frictional effects, the quantities that are conserved as the beads slide down, are

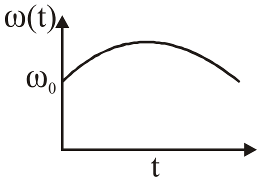


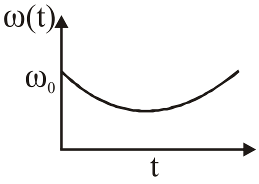
(A) Angular velocity and total energy (kinetic and potential)
(B) Total angular momentum and total energy
(C) Angular velocity and moment of inertia about the axis of rotation. **[IIT-JEE 2000]**
(D) Total angular momentum and moment of inertia about the axis of rotation.

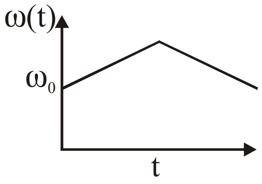
RD0084

32. A circular platform is free to rotate in a horizontal plane about a vertical axis passing through its centre. A tortoise is sitting at the edge of the platform. Now, the platform is given an angular velocity ω_0. When the tortoise move along a chord of the platform with a constant velocity (with respect to the platform), the angular velocity of the platform ω(t). will vary with time t as **[IIT-JEE 2002]**

(A)


(B)


(C)


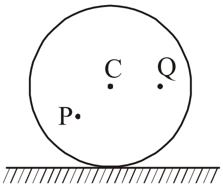
(D)


RD0085

Combined rotation and translation

Kinematics

33. A disc is rolling without slipping with angular velocity ω. P and Q are two points equidistant from the centre C. The order of magnitude of velocity is :- **[IIT-JEE 2004]**

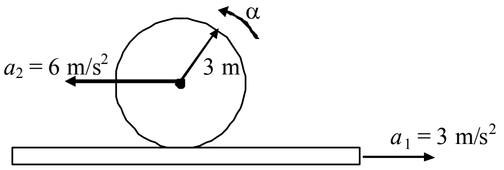


(A) $v_Q > v_C > v_P$
(B) $v_P > v_C > v_Q$
(C) $v_P = v_C, v_Q = v_C/2$
(D) $v_P < v_C > v_Q$

RD0086

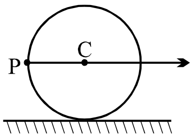
34. In the following figure, a sphere of radius 3 m rolls on a plank. The accelerations of the sphere and the plank are indicated. The value of α is

(A) 3 rad/s^2
(B) 6 rad/s^2
(C) 3 rad/s^2 (opposite to the direction shown in figure)
(D) 1 rad/s^2



RD0087

35. A disc of radius R is rolling purely on a flat horizontal surface, with a constant angular velocity. The angle between the velocity and acceleration vectors of point P is

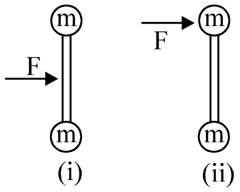


(A) zero
(B) 45°
(C) 135°
(D) $\tan^{-1}(1/2)$

RD0088

Dynamics

36. A force F is applied to a dumbbell for a time interval, t, first as in (i) and then as in (ii). In which case does the dumbbell acquire the greater centre-of-mass speed?



(A) (i)
(B) (ii)
(C) there is no difference
(D) the answer depends on the rotational inertia of the dumbbell

RD0089

37. A cylinder rolls up an inclined plane, reaches some height, and then rolls down (without slipping throughout these motions). The directions of the frictional force acting on the cylinder are

(A) Up the incline while ascending and down the incline while descending **[IIT-JEE 2002]**

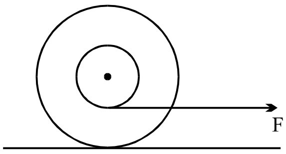
(B) Up the incline while ascending as well as descending

(C) down the incline while ascending and up the incline while descending

(D) down the incline while ascending as well as descending

RD0090

38. Inner and outer radii of a spool are r and R respectively. A thread is wound over its inner surface and placed over a rough horizontal surface. Thread is pulled by a force F as shown in fig. then in case of pure rolling



(A) Thread unwinds, spool rotates anticlockwise and friction act leftwards

(B*)* Thread winds, spool rotates clockwise and friction acts leftwards

(C) Thread winds, spool moves to the right and friction act rightwards

(D) Thread winds, spool moves to the right and friction does not come into existence.

RD0091

39. A body kept on a smooth horizontal surface is pulled by a constant horizontal force applied at the top point of the body. If the body rolls purely on the surface, its shape can be :

(A) thin pipe (B) uniform cylinder

(C) uniform sphere (D) thin spherical shell

RD0092

40. A solid sphere with a velocity (of centre of mass) v and angular velocity ω is gently placed on a rough horizontal surface. The frictional force on the sphere:

(A) must be forward (in direction of v) (B) must be backward (opposite to v)

(C) cannot be zero (D) none of the above

RD0093

Kinetic energy

41. A hoop and a solid cylinder have the same mass and radius. They both roll, without slipping, on a horizontal surface. If their kinetic energies are equal

(A) the hoop has a greater translational speed than the cylinder

(B) the cylinder has a greater translational speed than the hoop

(C) the hoop and the cylinder have the same translational speed

(D) the hoop has a greater rotational speed than the cylinder

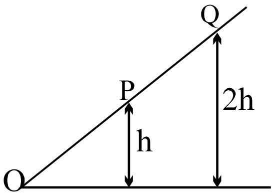
RD0094

42. A ball rolls down an inclined plane as shown in figure. The ball is first released from rest from P and then later from Q. Which of the following statement is/ are correct?

(i) The ball takes twice as much time to roll from Q to O as it does to roll from P to O.

(ii) The acceleration of the ball at Q is twice as large as the acceleration at P.

(iii) The ball has twice as much K.E. at O when rolling from Q as it does when rolling from P.



(A) i, ii only (B) ii, iii only (C) i only (D) iii only

RD0095

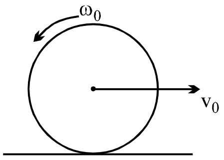
43. The moment of inertia of a solid cylinder about its axis is given by (1/2)MR^2. If this cylinder rolls without slipping, the ratio of its rotational kinetic energy to its translational kinetic energy is

(A) 1 : 1 (B) 2 : 2 (C) 1 : 2 (D) 1 : 3

RD0096

Angular momentum

44. A uniform circular disc placed on a rough horizontal surface has initially a velocity v_0 and an angular velocity ω_0 as shown in the figure. The disc comes to rest after moving some distance in the direction of motion. Then $\frac{v_0}{r\omega_0}$ is



(A) $\frac{1}{2}$ (B) 1 (C) $\frac{3}{2}$ (D) 2

RD0097

45. A hollow sphere of radius R and mass m is fully filled with non viscous liquid of mass m. It is rolled down a horizontal plane such that its centre of mass moves with a velocity v. If it purely rolls

(A) Kinetic energy of the sphere is $\frac{5}{6}mv^2$

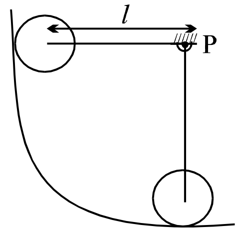
(B) Kinetic energy of the sphere is $\frac{4}{5}mv^2$

(C) Angular momentum of the sphere about a fixed point on ground is $\frac{8}{3}mvR$

(D) Angular momentum of the sphere about a fixed point on ground is $\frac{14}{5}mvR$

RD0098

46. A sphere of mass M and radius R is attached by a light rod of length ℓ to a point P. The sphere rolls without slipping on a circular track as shown. It is released from the horizontal position. The angular momentum of the system about P when the rod becomes vertical is :



(A) $M\sqrt{\frac{10}{7}gl}\,[l+R]$

(B) $M\sqrt{\frac{10}{7}gl}\left[l+\frac{2}{5}R\right]$

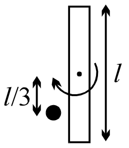
(C) $M\sqrt{\frac{10}{7}gl}\left[l+\frac{7}{5}R\right]$

(D) $M\sqrt{\frac{10}{7}gl}\left[l-\frac{2}{5}R\right]$

RD0099

Eccentric collision

47. A uniform rod of length l and mass M rotating about a fixed vertical axis on a smooth horizontal table. It elastically strikes a particle placed at a distance $l/3$ from its axis and stops. Mass of the particle is



(A) 3M (B) $\frac{3M}{4}$ (C) $\frac{3M}{2}$ (D) $\frac{4M}{3}$

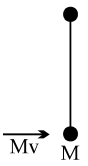
RD0100

48. A mass m is moving at speed v perpendicular to a rod of length d and mass $M = 6m$ which pivots around a frictionless axle running through its centre. It strikes and sticks to the end of the rod. The moment of inertia of the rod about its centre is $Md^2/12$. Then the angular speed of the system right after the collision is

(A) $2v/d$ (B) $2v/(3d)$ (C) v/d (D) $3v/(2d)$

RD0101

49. Two particles each of mass M are connected by a massless rod of length l. The rod is lying on the smooth sufrace. If one of the particle is given an impulse MV as shown in the figure then angular velocity of the rod would be **[IIT-JEE'(Scr)2003]**

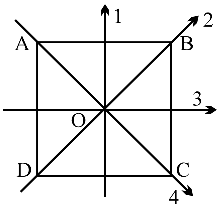


(A) v/l (B) $2v/l$ (C) $v/2l$ (D) None

RD0102

MULTIPLE CORRECT TYPE QUESTIONS

50. ABCD is a square plate with centre O. The moments of inertia of the plate about the perpendicular axis through O is I and about the axes 1, 2, 3 & 4 are I_1, I_2, I_3 & I_4 respectively. It follows that :



(A) $I_2 = I_3$ (B) $I = I_1 + I_4$ (C) $I = I_2 + I_4$ (D) $I_1 = I_3$

RD0103

51. A rod of weight w is supported by two parallel knifes at end points A and B and is in equilibrium in a horizontal position. The knives are at a distance 'd' from each other. The centre of mass of the rod is at a distance 'x' from A.

(A) the normal reaction at A is $\frac{wx}{d}$

(B) the normal reaction at A is $\frac{w(d-x)}{d}$

(C) the normal reaction at B is $\frac{wx}{d}$

(D) the normal reaction at B is $\frac{w(d-x)}{d}$

RD0104

52. A block with a square base measuring 'a' × 'a' and height h, is placed on an inclined plane. The coefficient of friction is μ. The angle of inclination (θ) of the plane is gradually increased. The block will

(A) topple before sliding if $\mu > \frac{a}{h}$

(B) topple before sliding if $\mu < \frac{a}{h}$

(C) slide before toppling if $\mu > \frac{a}{h}$

(D) slide before toppling if $\mu < \frac{a}{h}$

RD0105

53. A particle falls freely near the surface of the earth. Consider a fixed point O (not vertically below the particle) on the ground.

(A) Angular momentum of the particle about O is increasing .

(B) Torque of the gravitational force on the particle about O is decreasing.

(C) The moment of inertia of the particle about O is decreasing .

(D) The angular velocity of the particle about O is increasing.

RD0106

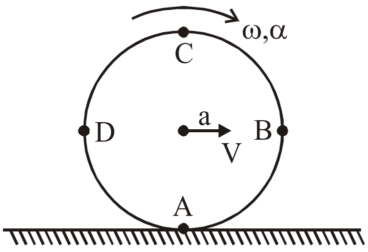
54. A man spinning in free space changes the shape of his body, eg. by spreading his arms or curling up. By doing this, he can change his

(A) moment of inertia (B) angular momentum

(C) angular velocity (D) rotational kinetic energy

RD0107

55. A circular disc of radius R rolls without slipping on a rough horizontal surface. At the instant shown its linear velocity is V, linear acceleration a, angular velocity ω and angular acceleration α. Four points A, B, C and D lie on its circumference such that the diameter AC is vertical & BD horizontal then choose the **CORRECT** option(s).



(A) $V_B = \sqrt{V^2 + (R\omega)^2}$

(B) $V_C = V + R\omega$

(C) $a_A = \sqrt{(a - R\alpha)^2 + (\omega^2 R)^2}$

(D) $a_D = \sqrt{(a + \omega^2 R)^2 + (R\alpha)^2}$

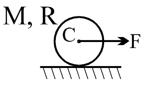
RD0108

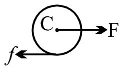
COMPREHENSION TYPE QUESTIONS

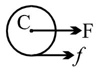
Paragraph for Question No. 56 to 58

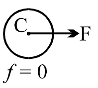
In the following problems, indicate the correct direction of friction force acting on the cylinder, which is pulled on a rough surface by a constant force F.

56. A cylinder of mass M and radius R is pulled horizontally by a force F. The friction force can be given by which of the following diagrams :-



(A) 

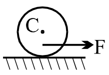
(B) 

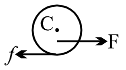
(C) 

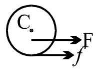
(D) cannot be interpreted

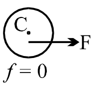
RD0109

57. A cylinder is pulled horizontally by a force F acting at a point below the centre of mass of the cylinder, as shown in figure. The friction force can be given by which of the following diagrams :-



(A) 

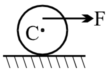
(B) 

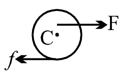
(C) 

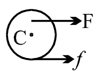
(D) cannot be interpreted

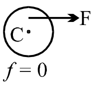
RD0109

58. A cylinder is pulled horizontally by a force F acting at a point above the centre of mass of the cylinder, as shown in figure. The friction force can be given by which of the following diagrams



(A) 

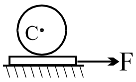
(B) 

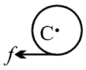
(C) 

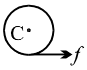
(D) cannot be interpreted

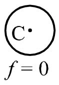
RD0109

59. A cylinder is placed on a rough plank which in turn is placed on a smooth surface. The plank is pulled with a constant force F. The friction force can be given by which of the following diagrams



(A) 

(B) 

(C) 

(D) canot be interpreted

RD0109

Paragraph for Question No. 60 to 62

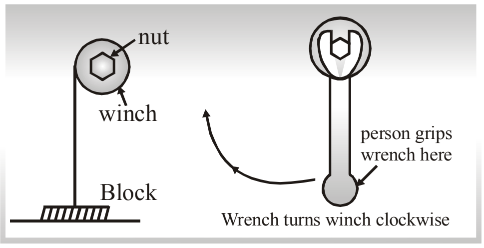
In figure, the winch is mounted on an axle, and the 6-sided nut is welded to the winch. By turning the nut with a wrench, a person can rotate the winch. For instance, turning the nut clockwise lifts the block off the ground, because more and more rope gets wrapped around the winch.

Three students agree that using a longer wrench makes it easier to turn the winch. But they disagree about why. All three students are talking about the case where the winch is used, over a 10 s time interval, to lift the block one metre off the ground.

Student 1 : By using a longer wrench, the person decreases the average force he must exert on the wrench, in order to lift the block one metre in 10 s.

Student 2 : Using a longer wrench reduces the work done by the person as he uses the winch to lift the block 1m in 10s.

Student 3 : Using a longer wrench reduces the power that the person must exert to lift the block 1m in 10s.



60. Student 1 is :-

(A) correct, because the torque that the wrench must exert to lift the block doesn't depend on the wrench's length

(B) correct, because using a longer wrench decreases the torque it must exert on the winch

(C) incorrect, because the torque that the wrench must exert to lift the block doesn't depend on the wrench's length

(D) Incorrect, because using a longer wrench decreases the torque it must exert on the winch.

RD0110

61. Which of the following is true about student 2 and 3 :-

(A) Student 2 and 3 are both correct

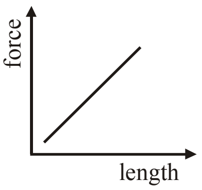
(B) Student 2 is correct, but student 3 is incorrect

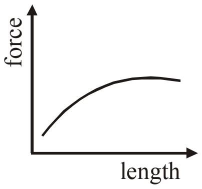
(C) Student 3 is correct, but student 2 is incorrect

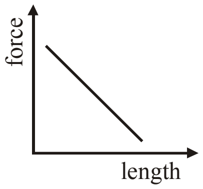
(D) Student 2 and 3 are both incorrect

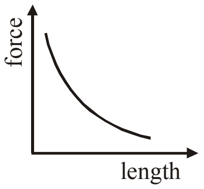
RD0110

62. If several wrenches all apply the same torque to a nut, which graph best expresses the relationship between the force the person must apply to the wrench, and the length of the wrench :-

(1) 

(2) 

(3) 

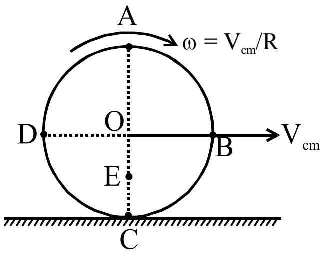
(4) 

(A) 1 (B) 2 (C) 3 (D) 4

RD0110

MATRIX MATCH TYPE QUESTION

63. Consider a body rolling on a horizontal surface as shown in figure (Symbols have their usual meaning)



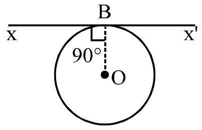
	Column-I		**Column-II**
(A)	Velocity is zero	(P)	Point A
(B)	Speed in maximum	(Q)	Point B
(C)	$0 <$ speed $< v_{cm}$	(R)	Point C
(D)	$1.3v_{cm} <$ speed $< 2\ v_{cm}$	(S)	Point D
		(T)	Point E

RD0111

EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

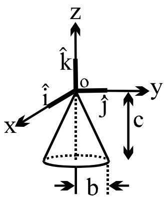
1. A thin wire of length L and uniform linear mass density ρ is bent into a circular loop with centre at O as shown. The moment of inertia of the loop about the axis XX' is : **[IIT-JEE'(Scr)'2000]**



(A) $\rho L^3/8\pi^2$ (B) $\rho L^3/16\pi^2$ (C) $5\rho L^3/16\pi^2$ (D) $3\rho L^3/8\pi^2$

RD0112

2. A solid cone hangs from a frictionless pivot at the origin O, as shown. If $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors, and a, b, and c are positive constants, which of the following forces F applied to the rim of the cone at a point P results in a torque τ on the cone with a negative component τ_Z?



(A) $F = a\hat{k}$, P is (0,b,–c) (B) $F = -a\hat{k}$, P is (0,–b,–c)

(C) $F = a\hat{j}$, P is (–b,0,–c) (D) None

RD0113

3. A heavy seesaw (i.e., not massless) is out of balance. A light girl sits on the end that is tilted downward, and a heavy body sits on the other side so that the seesaw now balances. If they both move forward so that they are one-half their original distance from the pivot point (the fulcrum) what will happen to the seesaw ?

(A) The side the body is sitting on will tilt downward

(B) The side the girl is sitting on will once again tilt downward

(C) Nothing ; the seesaw will still be balanced

(D) It is impossible to say without knowing the masses and the distances

RD0114

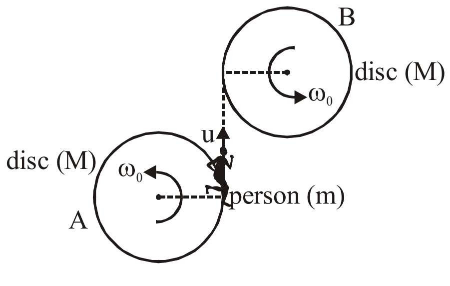
4. Two identical discs each of mass M(= 4m) and radius R are rotating in opposite sense with equal angular speed ω_0 about vertical axes passing through their centres, (as shown in figure). A person of mass m sitting on circumference of disc A jumps with a tangential relative velocity u (after jumping) w.r.t one rotating disc (A) and lands on other disc (B) also tangential. Now the second disc (B) comes to a stop. Find the relative velocity u.

(A) $u = \dfrac{\omega_0 R}{2}$

(B) $u = \omega_0 R$

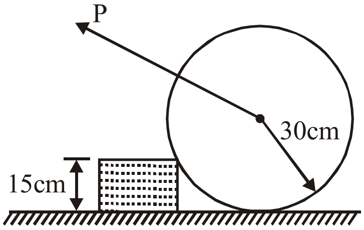
(C) $u = \dfrac{3\omega_0 R}{2}$

(D) $u = 2\omega_0 R$



RD0115

5. In the given figure a uniform wheel of radius 30cm rests against a rigid rectangular block 15cm high. The wheel weighs 1000 N. The minimum pull *P* through the center which will turn the wheel over the block is :-



(A) $500\sqrt{3}$ N (B) $1000\sqrt{3}$ N

(C) 1000 N (D) $400\sqrt{3}$ N

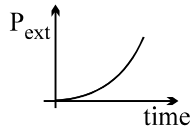
RD0116

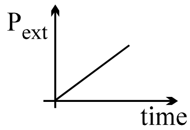
6. A uniform rod AB of mass m and length l is at rest on a smooth horizontal surface. An impulse J is applied to the end B, perpendicular to the rod in the horizontal direction. Speed of particle P at a distance $\frac{l}{6}$ from the centre towards A of the rod after time $t = \frac{\pi m l}{12J}$ is

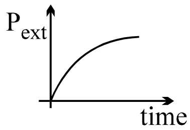
(A) $2\frac{J}{m}$ (B) $\frac{J}{\sqrt{2}m}$ (C) $\frac{J}{m}$ (D) $\sqrt{2}\frac{J}{m}$

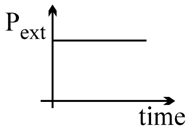
RD0117

7. A rod is hinged at its centre and rotated by applying a constant torque starting from rest. The power developed by the external torque as a function of time is :

(A) 

(B) 

(C) 

(D) 

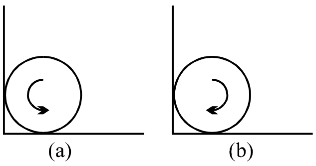
RD0118

8. A straight rod of length L is released on a frictionless horizontal floor in a vertical position. As it falls + slips, the distance of a point on the rod from the lower end, which follows a quarter circular locus is :-

(A) L/2 (B) L/4 (C) L/8 (D) None

RD0119

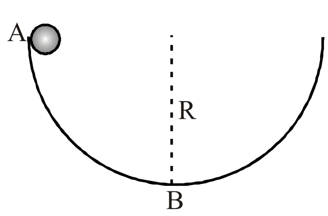
9. A sphere is placed rotating with its centre initially at rest in a corner as shown in figure (a) & (b). Coefficient of friction between all surfaces and the sphere is 1/3. Find the ratio of the frictional force $\frac{f_a}{f_b}$ by ground in situations (a) & (b).



(A) 1 (B) $\frac{9}{10}$ (C) $\frac{10}{9}$ (D) none

RD0120

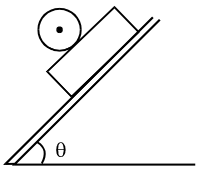
10. A small sphere A of mass m and radius r rolls without slipping inside a large fixed hemispherical bowl of radius R (>> r) as shown in figure. If the sphere starts from rest at the top point of the hemisphere find the normal force exerted by the small sphere on the hemisphere when it is at the bottom B of the hemisphere.



(A) $\frac{10}{7}$ mg (B) $\frac{17}{7}$ mg (C) $\frac{5}{7}$ mg (D) $\frac{7}{5}$ mg

RD0121

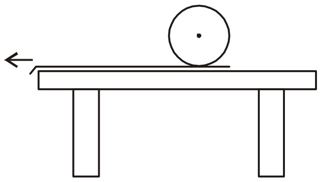
11. A plank of mass M is placed over smooth inclined plane and a sphere is also placed over the plank. Friction is sufficient between sphere and plank. If plank and sphere are released from rest, the frictional force on sphere is:



(A) up the plane (B) down the plane (C) horizontal (D) zero

RD0122

12. A thin table cloth covers a horizontal table and a uniform body of round shape lies on top of it. The table cloth is pulled from under the body, and friction causes the body to slide and rotate. What is the body's final motion on the table? (Assume that the table is so large that the body does not fall off it.)



(A) Body will finally roll towards left

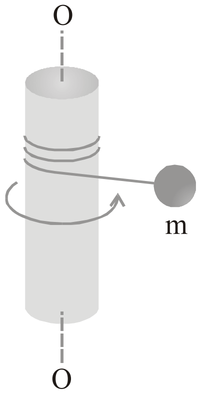
(B) Body will finally roll towards right

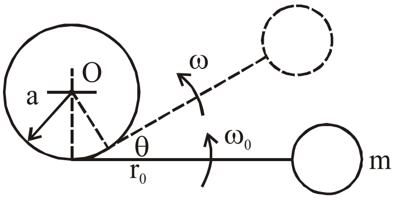
(C) Body will finally come to rest

(D) Any of the above is possible depending on shape of body

RD0123

13. The small particle of mass m is given an initial high velocity in the horizontal plane and winds its cord around the fixed vertical shaft of radius 1m. All motion occurs essentially in the horizontal plane. If the angular velocity of the cord is 0.8 rad/s when the distance from the particle to the tangential point is 5m, determine the angular velocity ω (in rad/s) of the cord after it has turned through an angle 1 rad.

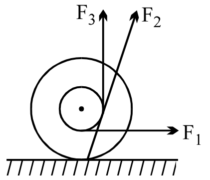




(A) 1 (B) 2 (C) 4 (D) 6

RD0124

14. A yo-yo is resting on a perfectly rough horizontal table. Forces F_1, F_2 and F_3 are applied separately as shown. The correct statement is :-



(A) when F_3 is applied the centre of mass will move to the right.

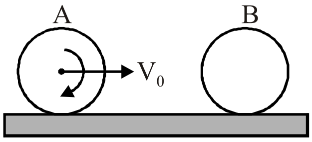
(B) when F_2 is applied the centre of mass will move to the left.

(C) when F_1 is applied the centre of mass will move to the right.

(D) when F_2 is applied the centre of mass will move to the right.

RD0125

15. A hollow smooth uniform sphere A of mass 'm' rolls without sliding on a smooth horizontal surface. It collides head on elastically with another stationary smooth solid sphere B of the same mass m and same radius. The ratio of kinetic energy of 'B' to that of 'A' just after the collision is :



(A) 1 : 1 (B) 2 : 3 (C) 3 : 2 (D) None

RD0126

16. A wheel of radius r rolling on a straight line, the velocity of its centre being v. At a certain instant the point of contact of the wheel with the grounds is M and N is the highest point on the wheel (diametrically opposite to M). The incorrect statement is:

(A) The velocity of any point P of the wheel is proportional to MP.

(B) Points of the wheel moving with velocity greater than v form a larger area of the wheel than points moving with velocity less than v.

(C) The point of contact M is instantaneously at rest.

(D) The velocities of any two parts of the wheel which are equidistant from centre are equal.

RD0127

17. A ladder of length L is slipping with its ends against a vertical wall and a horizontal floor. At a certain moment, the speed of the end in contact with the horizontal floor is v and the ladder makes an angle $\alpha = 30°$ with the horizontal. Then the speed of the ladder's center must be :-

(A) $2v/\sqrt{3}$ (B) v/2 (C) v (D) None

RD0128

18. In the previous question, if dv/dt = 0, then the angular acceleration of the ladder when $\alpha = 45°$ is :-

(A) $2v^2/L^2$ (B) $v^2/2L^2$ (C) $\sqrt{2}[v^2/L^2]$ (D) None

RD0129

19. One ice skater of mass m moves with speed 2v to the right, while another of the same mass m moves with speed v toward the left, as shown in figure I. Their paths are separated by a distance b. At t = 0, when they are both at x = 0, they grasp a pole of length b and negligible mass. For t > 0, consider the system as a rigid body of two masses m separated by distance b, as shown in figure II. Which of the following is the correct formula for the motion after t = 0 of the skater initially at y = b/2?

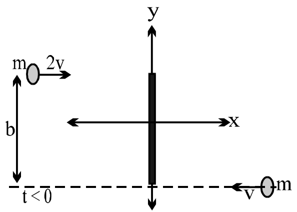


Figure I

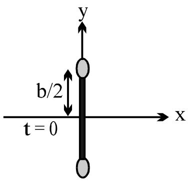


Figure II

(A) $x = 2vt, y = b/2$

(B) $x = vt + 0.5b\sin(3vt/b), y = 0.5b\cos(3vt/b)$

(C) $x = 0.5vt + 0.5b\sin(3vt/b), y = 0.5b\cos(3vt/b)$

(D) $x = 0.5vt + 0.5b\sin(6vt/b), y = 0.5b\cos(6vt/b)$

RD0130

MULTIPLE CORRECT TYPE QUESTIONS

20. A rigid object is rotating in a counterclockwise sense around a fixed axis. If the rigid object rotates through more than 180° but less than 360°, which of the following pairs of quantities can represent an initial angular position and a final angular position of the rigid object. Which of the sets can only occur.

(A) 3 rad, 6 rad (B) – 1 rad, 1 rad (C) 1 rad, 5 rad (D) –1 rad, 2.5 rad

RD0131

21. A body is in equilibrium under the influence of a number of forces. Each force has a different line of action. The minimum number of forces required is

(A) 2, if their lines of action pass through the centre of mass of the body.

(B) 3, if their lines of action are not parallel.

(C) 3, if their lines of action are parallel.

(D) 4, if their lines of action are parallel and all the forces have the same magnitude.

RD0132

22. The torque $\vec{\tau}$ on a body about a given point is found to be equal to $\vec{A} \times \vec{L}$ where $\vec{A}$ is a constant vector and $\vec{L}$ is the angular momentum of the body about that point. From this it follows that

(A) $d\vec{L}/dt$ is perpendicular to $\vec{L}$ at all instants of time

(B) the components of $\vec{L}$ in the direction of $\vec{A}$ does not change with time

(C) the magnitude of $\vec{L}$ does not change with time

(D) $\vec{L}$ does not change with time

RD0133

23. A block of mass m moves on a horizontal rough surface with initial velocity v. The height of the centre of mass of the block is h from the surface. Consider a point A on the surface.

(A) angular momentum about A is mvh initially

(B) the velocity of the block decreases at time passes.

(C) torque of the forces acting on block is zero about A

(D) angular mometum is not conserved about A.

RD0134

24. If a cylinder is rolling down the incline with sliding.

(A) after some time it may start pure rolling

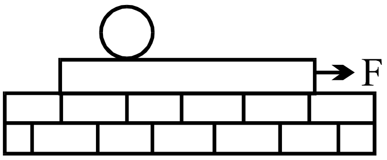
(B) after sometime it will start pure rolling

(C) it may be possible that it will never start pure rolling

(D) none of these

RD0135

25. A plank with a uniform sphere placed on it rests on a smooth horizontal plane. Plank is pulled to right by a constant force F. If sphere does not slip over the plank. Which of the following is correct.



(A) Acceleration of the centre of sphere is less than that of the plank.

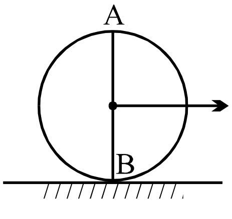
(B) Work done by friction acting on the sphere is equal to its total kinetic energy.

(C) Total kinetic energy of the system is equal to work done by the force F

(D) None of the above

RD0136

26. A uniform disc is rolling on a horizontal surface. At a certain instant B is the point of contact and A is at height 2R from ground, where R is radius of disc.



(A) The magnitude of the angular momentum of the disc about B is thrice that about A.

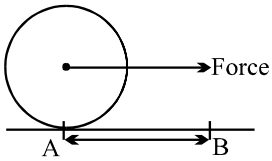
(B) The angular momentum of the disc about A is anticlockwise.

(C) The angular momentum of the disc about B is clockwise

(D) The angular momentum of the disc about A is equal to that about B.

RD0137

27. A disc of circumference s is at rest at a point A on a horizontal surface when a constant horizontal force begins to act on its centre. Between A and B there is sufficient friction to prevent slipping, and the surface is smooth to the right of B. AB = s. The disc moves from A to B in time T. To the right of B,



(A) the angular acceleration of the disc will disappear, linear acceleration will remain unchanged

(B) linear acceleration of the disc will increase

(C) the disc will make one rotation in time T/2

(D) the disc will cover a distance greater than s in further time T.

RD0138

28. A ring rolls without slipping on the ground. Its centre C moves with a constant speed u. P is any point on the ring. The speed of P with respect to the ground is ν.

(A) $0 \le \nu \le 2u$

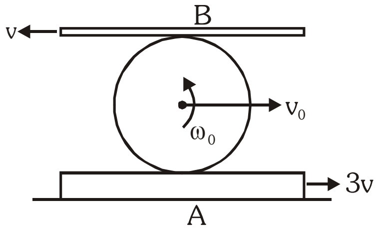
(B) ν = u, if CP is horizontal

(C) ν = u, if CP makes an angle of 30° with the horizontal and P is below the horizontal level of C.

(D) $\nu = \sqrt{2}u$, if CP is horizontal

RD0139

29. The disc of radius r is confined to roll without slipping at A and B. If the plates have the velocities shown, then



(A) linear velocity $v_0 = v$

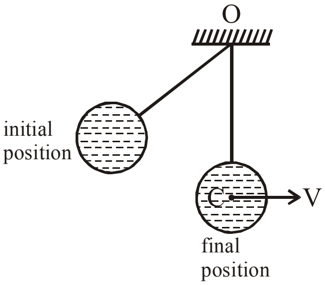
(B) angular velocity of disc is $\frac{3v}{2r}$

(C) angular velocity of disc is $\frac{2v}{r}$

(D) None of these

RD0140

30. A massles rod has a massless hollow sphere attached to it. This sphere can be fully filled either with a liquid (non viscous) or with a solid (rigidly fitted into sphere) of same mass. System is released from rest from initial position (as shown). When it reaches final position which of the following is/are true for the system.



(A) Kinetic energy in case of liquid will be more than in case of solid.

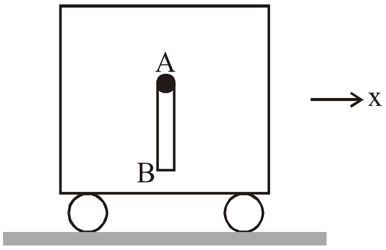
(B) Velocity of centre (V) in case of liquid will be more than in case of solid.

(C) About C angular momentum in case of liquid will be more than in case of solid.

(D) About C angular momentum in case of liquid will be less than in case of solid.

RD0141

31. A uniform rod AB of length ℓ and mass M hangs from point A at which it is freely hinged in a car moving with velocity v_0. The rod can rotate in vertical plane about the axis at A. If the car suddenly stops,



(A) The angular speed ω with which the rod starts rotating is $\frac{3v_0}{2\ell}$

(B) The minimum value of v_0 so that the rod completes the rotation $\sqrt{\frac{8}{3}g\ell}$

(C) Loss of energy during the process $\frac{1}{8}Mv_0^2$

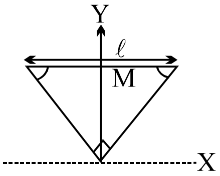
(D) There is no loss of energy

RD0142

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 32 to35

The figure shows an isosceles triangular plate of mass M and base L. The angle at the apex is 90°. The apex lies at the origin and the base is parallel to X–axis



32. The moment of inertia of the plate about the z-axis is :-

(A) $\frac{ML^2}{12}$ (B) $\frac{ML^2}{24}$ (C) $\frac{ML^2}{6}$ (D) none of these

RD0143

33. The moment of inertia of the plate about the x-axis is :-

(A) $\frac{ML^2}{8}$ (B) $\frac{ML^2}{32}$ (C) $\frac{ML^2}{24}$ (D) $\frac{ML^2}{6}$

RD0144

34. The moment of inertia of the plate about its base parallel to the x-axis is :-

(A) $\frac{ML^2}{18}$ (B) $\frac{ML^2}{36}$ (C) $\frac{ML^2}{24}$ (D) none of these

RD0143

35. The moment of inertia of the plate about the y-axis is :-

(A) $\frac{ML^2}{6}$ (B) $\frac{ML^2}{8}$ (C) $\frac{ML^2}{24}$ (D) none of these

RD0143

Paragraph for Question on 36 and 37

In the treatment of moments of inertia, introductory textbooks often present two theorems, generally called the parallel axis theorem and the perpendicular axis theorem. There is another theorem of this same genre, which is not usually included, but which is interesting and useful. It is

$$I_x + I_y + I_z = 2\sum_i m_i r_i^2$$

Here, I_x, I_y and I_z are the moments of inertia about three mutually perpendicular intersecting axes, m_1 is the mass of the ith particle and r_i is the distance from the intersection. The proof is simple: Taking the three axes as coordinate axes, we have :

$$I_x + I_y + I_z$$

$$= \sum_i m_i\left(y_i^2 + z_i^2\right) + \sum_i m_i\left(z_i^2 + x_i^2\right) + \sum_i m_i\left(x_i^2 + y_i^2\right)$$

$$= 2\sum_i m_i\left(x_i^2 + y_i^2 + z_i^2\right) = 2\sum_i m_i r_i^2$$

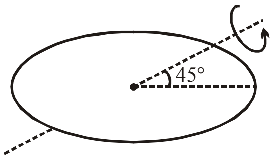
One important application is the calculation of the moment of inertia I_d of a uniform thin-walled spherical shell, of mass M and radius R, about a diameter. Taking the centre as the origin of coordinates, we have $I_x = I_y = I_z = I_d$, and $r_i = R$. The theorem gives $3I_d = 2\Sigma_i m_i R^2 = 2(\Sigma_i m_i)R^2 = 2MR^2$, where $I_d = 2MR^2/3$.

36. Consider a solid cube of mass m and side L. What will be the value of $\Sigma m_i r_i^2$ for this body when the point of intersection of axes is the centre of the cube :

(A) $\frac{ML^2}{2}$ (B) $\frac{ML^2}{4}$ (C) $\frac{ML^2}{3}$ (D) $\frac{ML^2}{6}$

RD0145

37. Find the moment of inertia of ring of mass m and radius R about an axis passing through its centre and making an angle 45° with its plane :

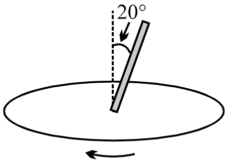


(A) $\frac{MR^2}{4}$ (B) $\frac{MR^2}{2}$ (C) $\frac{3}{4}MR^2$ (D) MR^2

RD0145

Paragraph for Question No. 38 and 39

A uniform rod is fixed to a rotating turntable so that its lower end is on the axis of the turntable and it makes an angle of 20° to the vertical. (The rod is thus rotating with uniform angular velocity about a vertical axis passing through one end.) If the turntable is rotating clockwise as seen from above.



38. What is the direction of the rod's angular momentum vector (calculated about its lower end)?

(A) vertically downwards
(B) down at 20° to the horizontal
(C) up at 20° to the horizontal
(D) vertically upwards

RD0146

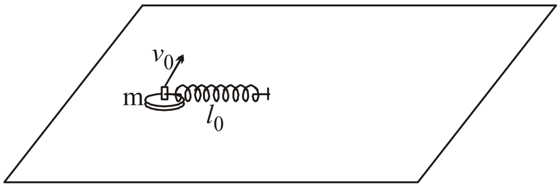
39. Is there a torque acting on it, and if so in what direction?

(A) yes, vertically (B) yes, horizontally
(C) yes at 20° to the horizontal (D) no

RD0146

Paragraph for Question No. 40 to 43

A spring having initial unstretched length ℓ_0 is lying on a smooth table. It's one end is fixed and the other one is fastened to a small particle of mass m. The particle is imparted an initial speed v_0 horizontally in a direction perpendicular to the spring. In the course of the motion in horizontal plane, the maximum elongation of the spring is $\Delta\ell = \ell_0/10$. (Given : m = 0.1 kg, ℓ_0 = 1m, v_0 = 11 m/s).



40. In the course of motion, which of the quantities relating to spring block system are conserved ?
(A) kinetic energy (B) momentum (C) angular momentum (D) potential energy

RD0147

41. Which of the following is correct about initial situation and situation at maximum elogation ?
(A) the orientiation of spring in both positions should be perpendicular to each other.
(B) the velocity at maximum extension should be zero
(C) the velocity at maximum extension as well as at initial position should be perpendicular to spring.
(D) Acceeration should be zero at the maximum extension as well as at initial position.

RD0147

42. **Student-A :** at maximum extension $mv'(\ell_0 + \Delta\ell) = mv_0\ell_0$.

Student-B : at maximum extension $k\Delta\ell = \dfrac{mv'^2}{(\ell_0 + \Delta\ell)}$. **Student-C :** $\frac{1}{2}mv_0^2 = \frac{1}{2}mv'^2 + \frac{1}{2}k\Delta\ell^2$.

where v' is velocity at instant of maximum extension :
(A) Only Student-A and B are correct (B) Only Student-A and C are correct
(C) Only Student-B and C are correct (D) All are correct

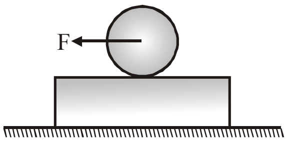
RD0147

43. The value of spring constant (in N/m) is :
(A) 100 (B) 210 (C) 420 (D) 105

RD0147

Paragraph for Question No. 44 and 45

A disc of mass m and radius R is placed over a plank of same mass m. There is sufficient friction between disc and plank to prevent slipping. A force F is applied at the centre of the disc.



44. Acceleration of the plank is :-

(A) $\dfrac{F}{2m}$ (B) $\dfrac{3F}{4m}$ (C) $\dfrac{F}{4m}$ (D) $\dfrac{3F}{2m}$

RD0148

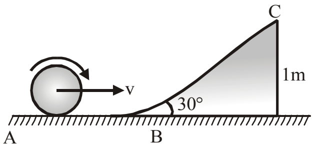
45. Force of friction between the disc and the plank is :-

(A) $\frac{F}{2}$ (B) $\frac{F}{4}$ (C) $\frac{F}{3}$ (D) $\frac{2F}{3}$

RD0148

Paragraph for Question No. 46 and 47

A small sphere of mass 1 kg is rolling without slipping on a rough stationary base with linear speed $v = \sqrt{\frac{200}{7}}$ m/s. It leaves the inclined plane at point C.



46. Find its linear speed at point C :-

(A) $\sqrt{\frac{100}{7}}$ m/s (B) $\sqrt{\frac{50}{7}}$ m/s (C) $\sqrt{\frac{100}{35}}$ m/s (D) $\sqrt{\frac{200}{35}}$ m/s

RD0149

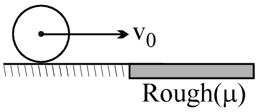
47. Find ratio of rotational and translational kinetic energy of the sphere when it strikes the ground after leaving from point C :-

(A) $\frac{2}{5}$ (B) $\frac{2}{3}$ (C) $\frac{1}{6}$ (D) $\frac{1}{2}$

RD0149

Paragraph for Question No. 48 to 51

A ring of mass M and radius R sliding with a velocity v_0 suddenly enters into rough surface where the coefficient of friction is μ, as shown in figure.



48. Choose the correct statement(s)

(A) As the ring enters on the rough surface, the limiting friction force acts on it

(B) The direction of friction is opposite to the direction of motion

(C) The friction force accelerates the ring in the clockwise sense about its centre of mass

(D) As the ring enters on the rough surface it starts rolling

RD0150

49. Choose the correct statement(s)

(A) The momentum of the ring is conserved

(B) The angular momentum of the ring is conserved about its centre of mass

(C) The angular momentum of the ring conserved about any point on the horizontal surface in line of friction.

(D) The mechanical energy of the ring is conserved

RD0150

50. Choose the correct statement(s)

(A) The ring starts its rolling motion when the centre of mas stationary

(B) The ring starts rolling motion when the point of contact becomes stationary

(C) The time after which the ring starts rolling is $\frac{v_0}{2\mu g}$

(D) The rolling velocity is $\frac{v_0}{2}$

RD0150

51. Choose the correct alternative(s) :-

(A) The linear distance moved by the centre of mass before the ring starts rolling is $\frac{3v_0^2}{8\mu g}$

(B) The net work done by friction force is $-\frac{3}{8}mv_0^2$

(C) The loss is kinetic energy of the ring is $\frac{mv_0^2}{4}$

(D) The gain in rotational kinetic energy is $+\frac{mv_0^2}{8}$

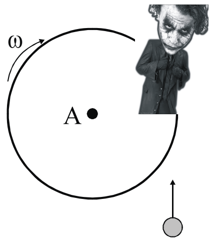
RD0150

MATRIX MATCH TYPE QUESTION

52. Column-I depicts various situations where some sudden events are taking place. Column-II describes changes in various parameters of systems immediately after the events taking place in column-I.

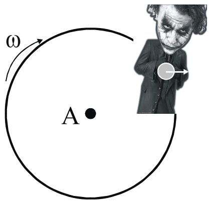
Column-I

(A) Joker is standing on revolving platform and batman throws the ball and joker catches the ball while it was moving horizontally.



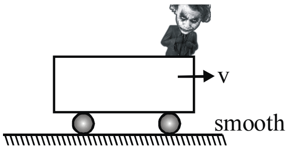
Joker, ball and platform is system.

(B) Joker throws the ball horizontally and perpendicular to his motion while standing on the revolving platform.



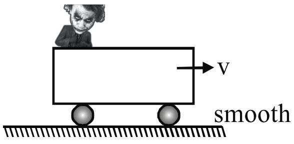
Joker, ball and platform is system.

(C) Joker jumps horizontally towards right from the cart which is moving at speed v on smooth horizontal floor.



Joker and cart is the system

(D) Joker drops himself vertically from the moving cart with no horizontal velocity relative to cart.



Joker and cart is the system

Column-II

(P) Linear momentum remains conserved.

(Q) Mechanical energy is conserved

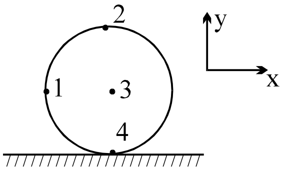
(R) Mechanical energy increases.

(S) Mechanical energy decreases.

(T) v or ω changes

RD0151

53. A rigid cylinder is kept on a smooth horizontal surface as shown. If **Column-I** indicates velocities of various points (3-centre of cylinder, 2- top point, 4-bottom point, 1- on the level of 3 at the rim) on it shown, choose correct state of motion from **Column-II**.



Column-I		**Column-II**	
(A)	$\vec{v}_1 = \hat{i} + \hat{j}, \vec{v}_2 = 2\hat{i}$	(P)	Pure rotation about centre
(B)	$\vec{v}_1 = \hat{i} + \hat{j}, \vec{v}_3 = -\hat{i}$	(Q)	Rolling without slipping to left
(C)	$\vec{v}_2 = \hat{i}, \vec{v}_3 = 0$	(R)	Rolling without slipping to right
(D)	$\vec{v}_4 = 0, \vec{v}_1 = -\hat{i} - \hat{j}$	(S)	Not possible

RD0152

EXERCISE (JM)

1. A thin uniform rod of length l and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω. Its centre of mass rises to a maximum height of:-

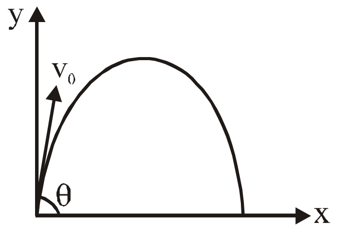
[AIEEE - 2009]

(1) $\frac{1}{2}\frac{l^2\omega^2}{g}$ (2) $\frac{1}{6}\frac{l^2\omega^2}{g}$ (3) $\frac{1}{3}\frac{l^2\omega^2}{g}$ (4) $\frac{1}{6}\frac{l\omega}{g}$

RD0153

2. A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity v_0 in the x-y plane as shown in the figure. At a time $t < \frac{v_0 \sin\theta}{g}$, the angular momentum of the particle is:

Where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axis respectively. **[AIEEE-2010]**



(1) $\frac{1}{2} mg\, v_0 t^2 \cos\theta\, \hat{i}$ (2) $- mg\, v_0 t^2 \cos\theta\, \hat{j}$

(3) $mg\, v_0 t \cos\theta\, \hat{k}$ (4) $-\frac{1}{2} mg\, v_0 t^2 \cos\theta\, \hat{k}$

RD0154

3. A pulley of radius 2 m is rotated about its axis by a force $F = (20t - 5t^2)$ newton (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10kg m^2, the number of rotations made by the pulley before its direction of motion it reversed, is :-

(1) more than 6 but less than 9 (2) more than 9 **[AIEEE-2011]**

(3) less than 3 (4) more than 3 but less than 6

RD0155

4. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. Euring the fjourney of the insect, then angular speed of the disc :-

[AIEEE-2011]

(1) continuously increases (2) first increases and then decreases

(3) remains unchanged (4) continuously decreses

RD0156

5. A particle of mass 'm' is projected with a velocity v making an angle of 30° with the horizontal. The magnitude of angular momentum of the projectile about the point of projection when the particle is at its maximum height 'h' is :- **[AIEEE-2011]**

(1) $\frac{\sqrt{3}}{2}\frac{mv^2}{g}$ (2) zero (3) $\frac{mv^3}{\sqrt{2}g}$ (4) $\frac{\sqrt{3}}{16}\frac{mv^3}{g}$

RD0157

6. A hoop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip? **[JEE Mains-2013]**

(1) $\frac{r\omega_0}{4}$ (2) $\frac{r\omega_0}{3}$ (3) $\frac{r\omega_0}{2}$ (4) $r\omega_0$

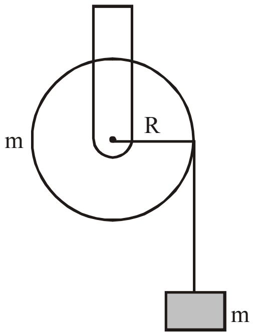
RD0158

7. A bob of mass m attached to an inextensible string of length ℓ is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed ω rad/s about the vertical. About the point of suspension : **[JEE Mains-2014]**

(1) Angular momentum changes in direction but not in magnitude
(2) Angular momentum changes both in direction and magnitude
(3) Angular momentum is conserved
(4) Angular momentum changes in magnitude but not in direction.

RD0159

8. A mass 'm' is supported by a massless string wound around a uniform hollow cylinder of mass m and radius R. If the string does not slip on the cylinder, with what acceleration will the mass fall on release? **[JEE Mains-2014]**



(1) $\frac{5g}{6}$ (2) g (3) $\frac{2g}{3}$ (4) $\frac{g}{2}$

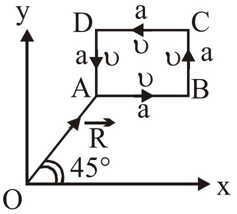
RD0160

9. From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its centre and perpendicular to one of its faces is:- **[JEE Mains-2015]**

(1) $\frac{4MR^2}{9\sqrt{3}\pi}$ (2) $\frac{4MR^2}{3\sqrt{3}\pi}$ (3) $\frac{MR^2}{32\sqrt{2}\pi}$ (4) $\frac{MR^2}{16\sqrt{2}\pi}$

RD0161

10. A particle of mass m is moving along the side of a square of side 'a', with a uniform speed υ in the x-y plane as shown in the figure : **[JEE Mains-2016]**



Which of the following statement is false for the angular momentum $\vec{L}$ about the origin ?

(1) $\vec{L}=\frac{m\upsilon}{\sqrt{2}}R\hat{k}$ when the particle is moving from D to A

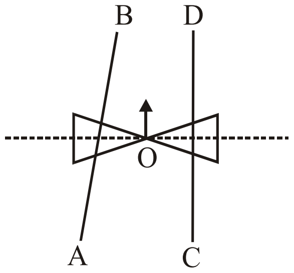
(2) $\vec{L}=-\frac{m\upsilon}{\sqrt{2}}R\hat{k}$ when the particle is moving from A to B

(3) $\vec{L}=m\upsilon\left[\frac{R}{\sqrt{2}}-a\right]\hat{k}$ when the particle is moving from C to D

(4) $\vec{L}=m\upsilon\left[\frac{R}{\sqrt{2}}+a\right]\hat{k}$ when the particle is moving from B to C

RD0162

11. A roller is made by joining together two cones at their vertices O. It is kept on two rails AB and CD which are placed asymmetrically (see figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and CD (see figure). It is given a light push so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to :- **[JEE Mains-2016]**



(1) turn left and right alternately.
(2) turen left.
(3) turn right.
(4) go straight.

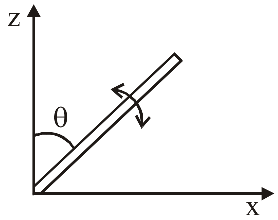
RD0163

12. The moment of inertia of a uniform cylinder of length ℓ and radius R about its perpendicular bisector is I. What is the ratio ℓ/R such that the moment of inertia is minimum ? **[JEE Main-2017]**

(1) 1
(2) $\frac{3}{\sqrt{2}}$
(3) $\sqrt{\frac{3}{2}}$
(4) $\frac{\sqrt{3}}{2}$

RD0164

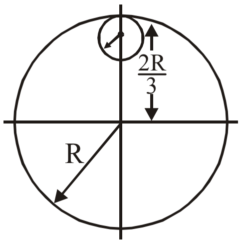
13. A slender uniform rod of mass M and length ℓ is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is : **[JEE Main-2017]**



(1) $\frac{3g}{2\ell}\cos\theta$ (2) $\frac{2g}{3\ell}\cos\theta$ (3) $\frac{3g}{2\ell}\sin\theta$ (4) $\frac{2g}{3\ell}\sin\theta$

RD0165

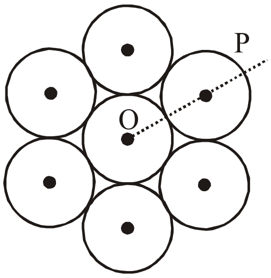
14. From a uniform circular disc of radius R and mass 9M, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is : **[JEE Main-2018]**



(1) $\frac{40}{9}MR^2$ (2) 10 MR^2 (3) $\frac{37}{9}MR^2$ (4) 4 MR^2

RD0166

15. Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is : **[JEE Main-2018]**



(1) $\frac{55}{2}$ MR^2 (2) $\frac{73}{2}$ MR^2 (3) $\frac{181}{2}$ MR^2 (4) $\frac{19}{2}$ MR^2

RD0167

EXERCISE (JA)

1. A block of base 10 cm × 10 cm and height 15 cm is kept on an inclined plane. The coefficient of friction between them is $\sqrt{3}$. The inclination θ of this inclined plane from the horizontal plane is gradually increased from 0°. Then :- **[IIT-JEE 2009]**

(A) at θ =30°, the block will start sliding down the plane

(B) the block will remain at rest on the plane up to certain θ and then it will topple

(C) at θ = 60°, the block will start sliding down the plane and continue to do so at higher angles

(D) at θ = 60°, the block will start sliding down the plane and on further increasing θ, it will topple at certain θ

RD0168

2. If the resultant of the external forces acting on a system of particles is zero, then from an inertial frame, one can surely say that **[IIT-JEE 2009]**

(A) linear momentum of the system does not change in time

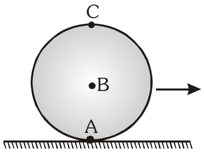
(B) kinetic energy of the system does not change in time

(C) angular momentum of the system does not change in time

(D) potential energy of the system does not change in time

RD0169

3. A sphere is rolling without slipping on a fixed horizontal plane surface. In the figure, A is the point of contact, B is the centre of the sphere and C is its topmost point. Then **[IIT-JEE 2009]**



(A) $\vec{v}_C - \vec{v}_A = 2(\vec{v}_B - \vec{v}_C)$

(B) $\vec{v}_C - \vec{v}_B = \vec{v}_B - \vec{v}_A$

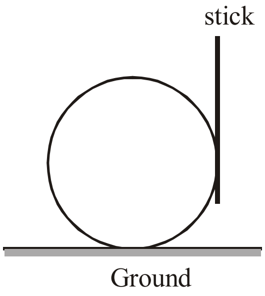
(C) $|\vec{v}_C - \vec{v}_A| = 2|\vec{v}_B - \vec{v}_C|$

(D) $|\vec{v}_C - \vec{v}_A| = 4|\vec{v}_B|$

RD0170

4. A boy is pushing a ring of mass 2 kg and radius 0.5 m with a stick as shown in the figure. The stick applies a force of 2N on the ring and rolls it without slipping with an acceleration of 0.3 m/s^2. The coefficient of friction between the ground and the ring is large enough that rolling always occurs and the coefficient of friction between the stick and the ring is (P/10). The value of P is

[IIT-JEE 2011]

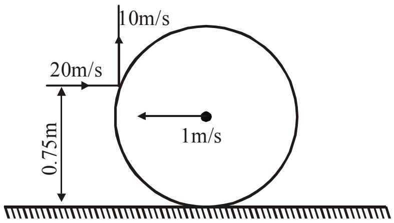


RD0171

5. Four solid spheres each of diameter $\sqrt{5}$ cm and mass 0.5 kg are placed with their centers at the corners of a square of side 4 cm. The moment of inertia of the system about the diagonal of the square is N × 10^{-4} kg-m^2, then N is **[IIT-JEE 2011]**

RD0172

6. A thin ring of mass 2 kg and radius 0.5 m is rolling without slipping on a horizontal plane with velocity 1 m/s. A small ball of mass 0.1 kg, moving with velocity 20 m/s in the opposite direction, hits the ring at a height of 0.75 m and goes vertically up with velocity 10 m/s. Immediately after the collision **[IIT-JEE 2011]**



(A) the ring has pure rotation about its stationary CM

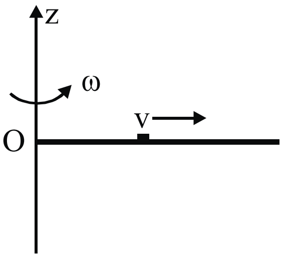
(B) the ring comes to a complete stop

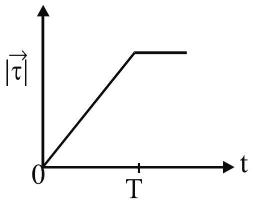
(C) friction between the ring and the ground is to the left

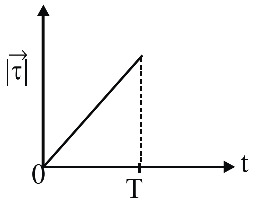
(D) there is no friction between the ring and the ground

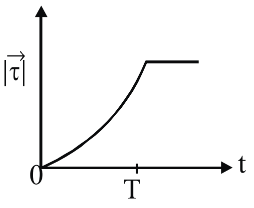
RD0173

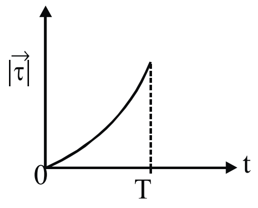
7. A thin uniform rod, pivoted at OP, is rotating in the horizontal plane with constant angular speed ω. as shown in the figure. At time t = 0, a small insect starts from O and moves with constant speed v with respect to the rod towards the other end. If reaches the end of the rod at t = T and stops. The angular speed of the system remains ω throughout. The magnitude of the torque $(|\tau|)$ on the system about O, as a function of time is best represented by which plot? **[IIT-JEE 2012]**



(A)


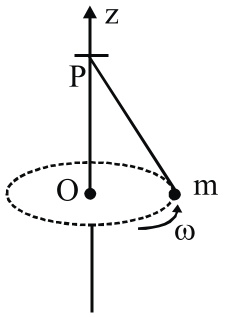
(B)


(C)


(D)


RD0174

8. A small mass m is attached to a massless string whose other end is fixed at P as shown in the figure. The mass is undergoing circular motion in the x-y plane with centre at O and constant angular speed ω. If the angular momentum of the system, calculated about O and P are denoted by $\vec{L}_O$ and $\vec{L}_P$ respectively, then **[IIT-JEE 2012]**



(A) $\vec{L}_O$ and $\vec{L}_P$ do not vary with time

(B) $\vec{L}_O$ varies with time while $\vec{L}_P$ remains constant

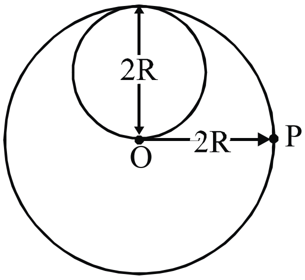
(C) $\vec{L}_O$ remains constant while $\vec{L}_P$ varies with time

(D) $|\vec{L}_O|$ and $|\vec{L}_P|$ both do not vary with time

RD0175

9. A lamina is made by removing a small disc of diameter 2R from a bigger disc of uniform mass density and radius 2R, as shown in the figure. The moment of inertia of this lamina about axes assing through O and P is I_0 and I_P respectively. Both these axes are perpendicular to the plane of the lamina.

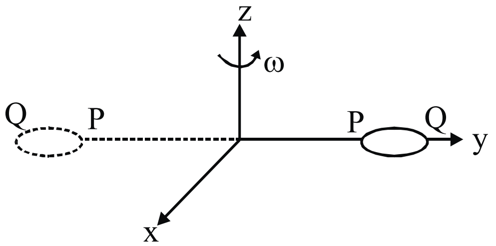
The ratio $\frac{I_P}{I_O}$ to the nearest integer is **[IIT-JEE 2012]**



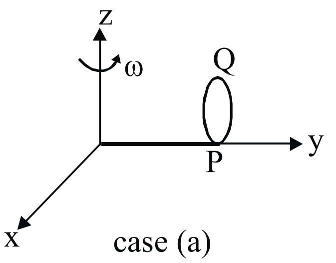
RD0176

Paragraph for Questions 10 and 11

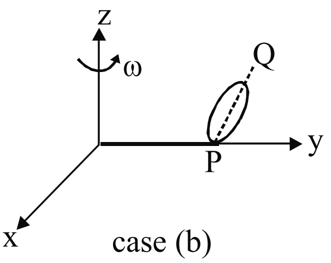
The general motion of a rigid body can be considered to be a combination of (i) a motion of its centre of mass about an axis, and (ii) its motion about an instantaneous axis passing through the centre of mass. These axes need not be stationary. Consider, for example, a thin uniform disc welded (rigidly fixed) horizontally at its rim to a massless stick, as shown in the figure. When the disc-stick system is rotated about the origin on a horizontal frictionless plane with angular speed ω, the motion at any instant can be taken as a combination of (i) a rotation of the centre of mass of the disc about the z-axis, and (ii) a rotation of the disc through an instantaneous vertical axis passing through its centre of mass (as is seen from the changed orientation of points P and Q). Both these motions have the same angular speed ω in this case.



Now consider two similar systems as shown in the figure : case (A) the disc with its face vertical and parallel to x-z plane; Case (B) the disc with its face making an angle of 45° with x-y plane and its horizontal diameter parallel to x-axis. In both the cases, the disc is welded at point P, and the systems are rotated with constant angular speed ω about the z-axis. **[IIT-JEE 2012]**



case (a)



case (b)

10. Which of the following statements regarding the angular speed about the instantaneous axis (passing through the centre of mass) is correct?

(A) It is $\sqrt{2\omega}$ for both the cases.
(B) It is ω for case (a); and $\frac{\omega}{\sqrt{2}}$ for case (b).
(C) It is ω for case (a); and $\sqrt{2\omega}$ for case (b).
(D) It is ω for both the cases.

RD0177

11. Which of the following statements about the instantaneous axis (passing through the centre of mass) is correct?
(A) It is vertical for both the cases (a) and (b).
(B) It is vertical for case (a); and is at 45° to the x-z plane and lies in the plane of the disc for case (b).
(C) It is horizontal for case (a); and is at 45° to the x-z plane and is normal to the plane of the disc for case (b).
(D) It is vertical for case (a); and is at 45° to the x-z plane and is normal to the plane of the disc for case (b).

RD0177

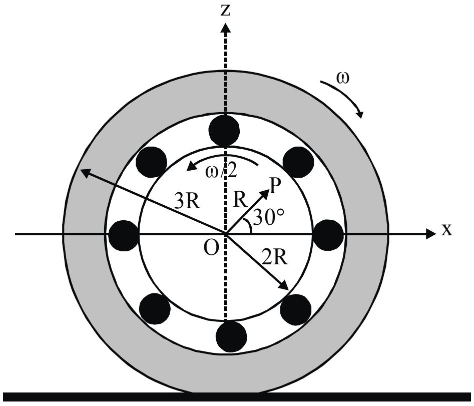
12. The figure shows a system consisting of (i) a ring of outer radius 3R rolling clockwise without slipping on a horizontal surface with angular speed ω and (ii) an inner disc of radius 2R rotating anti-clockwise with angular speed ω/2. The ring and disc are separated by frictionless ball bearing. The system is in the x-z plane. The point P on the inner disc is at a distance R from the origin, where OP makes an angle of 30° with the horizontal. Then with respect to the horizontal surface, **[IIT-JEE 2012]**

(A) the point O has a linear velocity $3R\omega\hat{i}$

(B) the point P has a linear velocity $\frac{11}{4}R\omega\hat{i}+\frac{\sqrt{3}}{4}R\omega\hat{k}$

(C) the point P has a linear velocity $\frac{13}{4}R\omega\hat{i}-\frac{\sqrt{3}}{4}R\omega\hat{k}$

(D) the point P has a linear velocity $\left(3-\frac{\sqrt{3}}{4}\right)R\omega\hat{i}+\frac{1}{4}R\omega\hat{k}$



RD0178

13. Two solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement(s) is(are) correct?
(A) Both cylinders P and Q reach the ground at the same time. **[IIT-JEE 2012]**
(B) Cylinder P has larger acceleration than cylinder Q.
(C) Both cylinders reach the ground with same translational kinetic energy.
(D) Cylinder Q reaches the ground with larger angular speed.

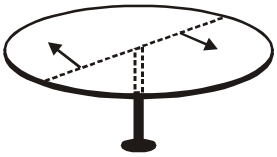
RD0179

14. A uniform circular disc of mass 50 kg and radius 0.4 m is rotating with an angular velocity of 10 rad s^{-1} about its own axis, which is vertical. Two uniform circular rings, each of mass 6.25 kg and radius 0.2 m, are gently placed symmetrically on the disc in such a manner that they are touching each other along the axis of the disc and are horizontal. Assume that the friction is large enough such that the rings are at rest relative to the disc and the system rotates about the original axis. The new angular velocity (in rad s^{-1}) of the system is **[IIT-JEE 2013]**

RD0180

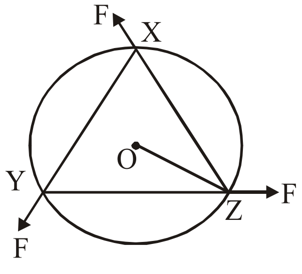
15. A horizontal circular platform of radius 0.5 m and mass 0.45 kg is free to rotate about its axis. Two massless spring toy-guns, each carrying a steel ball of mass 0.05 kg are attached to the platform at a distance 0.25 m from the centre on its either sides along its diameter (see figure). Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platform, the balls have horizontal speed of 9 ms^{-1} with respect to the ground. The rotational speed of the platform in rad s^{-1} after the balls leave the platform is

[JEE Advanced-2014]



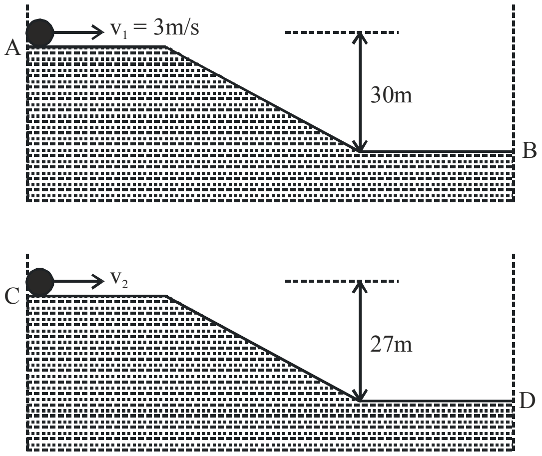
RD0181

16. A uniform circular disc of mass 1.5 kg and radius 0.5 m is initially at rest on a horizontal frictionless surface. Three forces of equal magnitude F = 0.5 N are applied simultaneously along the three sides of an equilateral triangle XYZ with its vertices on the perimeter of the disc (see figure). One second after applying the forces, the angular speed of the disc in rad s^{-1} is **[JEE Advanced-2014]**



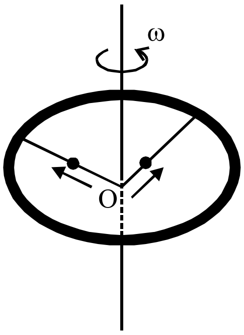
RD0182

17. Two identical uniform discs roll without slipping on two different, surfaces AB and CD (see figure) starting at A and C with linear speeds v_1 and v_2 respectively, and always remain in contact with the surfaces. If they reach B and D with the same linear speed and $v_1 = 3$m/s, then v_2 in m/s is ($g = 10$ m/s^2) **[JEE Advanced-2015]**



RD0183

18. A ring of mass M and radius R is rotating with angular speed ω about a fixed vertical axis passing through its centre O with two point masses each of mass $\frac{M}{8}$ at rest at O. These masses can move radially outwards along two massless rods fixed on the ring as shown in the figure. At some instant the angular speed of the system is $\frac{8}{9}\omega$ and one of the masses is at a distance of $\frac{3}{5}$ R from O. At this instant the distance of the other mass from O is : **[JEE Advanced-2015]**



(A) $\frac{2}{3}$R (B) $\frac{1}{3}$R (C) $\frac{3}{5}$R (D) $\frac{4}{5}$R

RD0184

19. The densities of two solid spheres A and B of the same radii R vary with radial distance r as $\rho_A(r) = k\left(\frac{r}{R}\right)$ and $\rho_B(r) = k\left(\frac{r}{R}\right)^5$, respectively, where k is a constant. The moments of inertia of the indivisual spheres about axes passing through their centres are I_A and I_B, respectively. If $\frac{I_B}{I_A} = \frac{n}{10}$, the value of n is. **[JEE Advanced-2015]**

RD0185

20. A uniform wooden stick of mass 1.6 kg and length ℓ rests in an inclined manner on a smooth, vertical wall of height h ($<\ell$) such that a small portion of the stick extends beyond the wall. The reaction force of the wall on the stick is perpendicular to the stick. The stick makes an angle of 30° with the wall and the bottom of the stick is on a rough floor. The reaction of the wall on the stick is equal in magnitude to the reaction of the floor on the stick. The ratio h/ℓ and the frictional force f at the bottom of the stick are: ($g = 10\ \text{ms}^{-2}$) **[JEE Advanced-2016]**

(A) $\frac{h}{\ell} = \frac{\sqrt{3}}{16}, f = \frac{16\sqrt{3}}{3}\text{N}$

(B) $\frac{h}{\ell} = \frac{3}{16}, f = \frac{16\sqrt{3}}{3}\text{N}$

(C) $\frac{h}{\ell} = \frac{3\sqrt{3}}{16}, f = \frac{8\sqrt{3}}{3}\text{N}$

(D) $\frac{h}{\ell} = \frac{3\sqrt{3}}{16}, f = \frac{16\sqrt{3}}{3}\text{N}$

RD0186

21. The position vector $\vec{r}$ of a particle of mass m is given by the following equation $\vec{r}(t) = \alpha t^3\hat{i} + \beta t^2\hat{j}$, where $\alpha = \frac{10}{3}\text{ms}^{-3}$, $\beta = 5\ \text{ms}^{-2}$ and m = 0.1 kg. At t = 1 s, which of the following statement(s) is(are) true about the particle? **[JEE Advanced-2016]**

(A) The velocity $\vec{v}$ is given by $\vec{v} = (10\hat{i} + 10\hat{j})\text{ms}^{-1}$

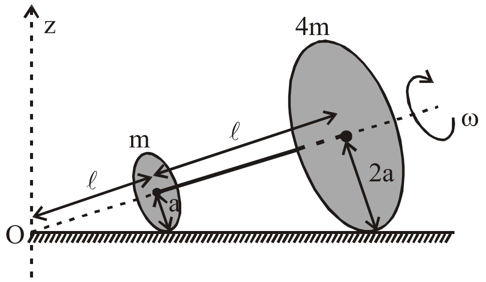
(B) The angular momentum $\vec{L}$ with respect to the origin is given by $\vec{L} = -\left(\frac{5}{3}\right)\hat{k}\ \text{Nms}$

(C) The force $\vec{F}$ is given by $\vec{F} = (\hat{i} + 2\hat{j})\text{N}$

(D) The torque $\vec{\tau}$ with respect to the origin is given by $\vec{\tau} = -\left(\frac{20}{3}\right)\hat{k}\ \text{Nm}$

RD0187

22. Two thin circular discs of mass m and 4m, having radii of a and 2a, respectively, are rigidly fixed by a massless, right rod of length $\ell = \sqrt{24}a$ throught their center. This assembly is laid on a firm and flat surface, and set rolling without slipping on the surface so that the angular speed about the axis of the rod is ω. The angular momentum of the entire assembly about the point 'O' is $\vec{L}$ (see the figure). Which of the following statement(s) is(are) true ? **[JEE Advanced-2016]**



(A) The magnitude of angular momentum of the assembly about its center of mass is $17ma^2\omega/2$

(B) The magnitude of the z-component of $\vec{L}$ is $55\ ma^2\omega$

(C) The magnitude of angular momentum of center of mass of the assembly about the point O is $81ma^2\omega$

(D) The center of mass of the assembly rotates about the z-axis with an angular speed of ω/5

RD0188

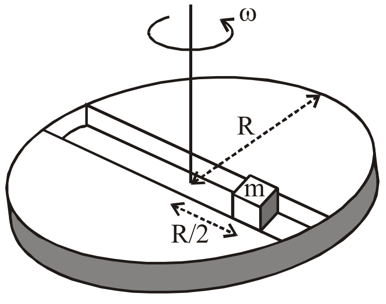
Paragraph for Question No. 23 and 24

A frame of reference that is accelerated with respect to an inertial frame of reference is called a non-inertial frame of reference. A coordinate system fixed on a circular disc rotating about a fixed axis with a constant angular velocity ω is an example of a non-inertial frame of reference. The relationship between the force $\vec{F}_{rot}$ experienced by a particle of mass m moving on the rotating disc and the force $\vec{F}_{in}$ experienced by the particle in an inertial frame of reference is

$$\vec{F}_{rot} = \vec{F}_{in} + 2m\left(\vec{\upsilon}_{rot} \times \vec{\omega}\right) + m\left(\vec{\omega} \times \vec{r}\right) \times \vec{\omega},$$

where $\vec{\upsilon}_{rot}$ is the velocity of the particle in the rotating frame of reference and $\vec{r}$ is the position vector of the particle with respect to the centre of the disc.

Now consider a smooth slot along a diameter of a disc of radius R rotating counter-clockwise with a constant angular speed ω about its vertical axis through its center. We assign a coordinate system with the origin at the centre of the disc, the x-axis along the slot, the y-axis perpendicular to the slot and the z-axis along the rotation axis $\left(\vec{\omega} = \omega\hat{k}\right)$. A small block of mass m is gently placed in the slot at $\vec{r} = (R/2)\hat{i}$ at t = 0 and is constrained to move only along the slot. **[JEE Advanced-2016]**



23. The distance r of the block at time t is :

(A) $\frac{R}{4}\left(e^{2\omega t}+e^{-2\omega t}\right)$

(B) $\frac{R}{2}\cos 2\omega t$

(C) $\frac{R}{2}\cos \omega t$

(D) $\frac{R}{4}\left(e^{\omega t}+e^{-\omega t}\right)$

RD0189

24. The net reaction of the disc on the block is :

(A) $-m\omega^2 R\cos\omega t\hat{j}-mg\hat{k}$

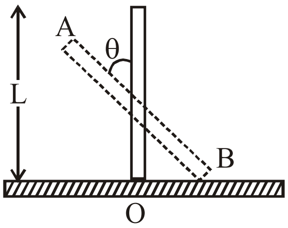
(B) $m\omega^2 R\sin\omega t\hat{j}-mg\hat{k}$

(C) $\frac{1}{2}m\omega^2 R\left(e^{\omega t}-e^{-\omega t}\right)\hat{j}+mg\hat{k}$

(D) $\frac{1}{2}m\omega^2 R\left(e^{2\omega t}-e^{-2\omega t}\right)\hat{j}+mg\hat{k}$

RD0189

25. A rigid uniform bar AB of length L is slipping from its vertical position on a frictionless floor (as shown in the figure). At some instant of time, the angle made by the bar with the vertical is θ. Which of the following statements about its motion is/are correct ? **[JEE Advanced-2017]**



(A) When the bar makes an angle θ with the vertical, the displacement of its midpoint from the initial position is proportional to $(1-\cos\theta)$

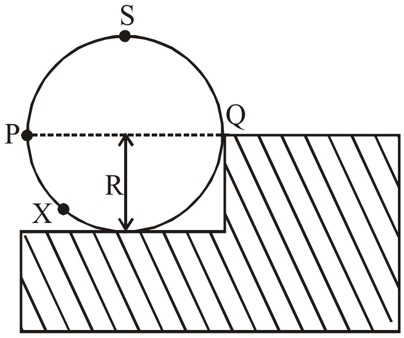
(B) The midpoint of the bar will fall vertically downward

(C) Instantaneous torque about the point in contact with the floor is proportional to $\sin\theta$

(D) The trajectory of the point A is a parabola

RD0190

26. A wheel of radius R and mass M is placed at the bottom of a fixed step of height R as shown in the figure. A constant force is continuously applied on the surface of the wheel so that it just climbs the step without slipping. Consider the torque τ about an axis normal to the plane of the paper passing through the point Q. Which of the following options is/are correct ? **[JEE Advanced-2017]**



(A) If the force is applied normal to the circumference at point X then τ is constant

(B) If the force is applied tangentially at point S then $\tau \neq 0$ but the wheel never climbs the step

(C) If the force is applied normal to the circumference at point P then τ is zero

(D) If the force is applied at point P tangentially then τ decreases continuously as the wheel climbs

RD0191

Paragraph for Question no. 27 & 28

One twirls a circular ring (of mass M and radius R) near the tip of one's finger as shown in Figure 1. In the process the finger never loses contact with the inner rim of the ring. The finger traces out the surface of a cone, shown by the dotted line. The radius of the path traced out by the point where the ring and the finger is in contact is r. The finger rotates with an angular velocity ω_0. The rotating ring rolls without slipping on the outside of a smaller circle described by the point where the ring and the finger is in contact (Figure 2). The coefficient of friction between the ring and the finger is μ and the acceleration due to gravity is g. **[JEE Advanced-2017]**

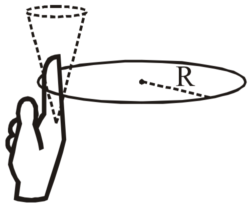


Figure 1

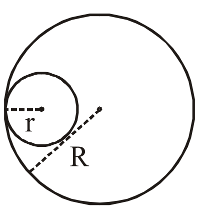


Figure 2

27. The total kinetic energy of the ring is :-

(A) $M\omega_0^2R^2$ (B) $M\omega_0^2(R-r)^2$ (C) $\frac{1}{2}M\omega_0^2(R-r)^2$ (D) $\frac{3}{2}M\omega_0^2(R-r)^2$

RD0192

28. The minimum value of ω_0 below which the ring will drop down is :-

(A) $\sqrt{\frac{3g}{2\mu(R-r)}}$ (B) $\sqrt{\frac{g}{\mu(R-r)}}$ (C) $\sqrt{\frac{2g}{\mu(R-r)}}$ (D) $\sqrt{\frac{2g}{2\mu(R-r)}}$

RD0192

29. The potential energy of a particle of mass m at a distance r from a fixed point O is given by V (r) = $kr^2/2$, where k is a positive constant of appropriate dimensions. This particle is moving in a circular orbit of radius R about the point O. If v is the speed of the particle and L is the magnitude of its angular momentum about O, which of the following statements is (are) true ?

[JEE Advanced-2018]

(A) $v=\sqrt{\frac{k}{2m}}R$ (B) $v=\sqrt{\frac{k}{m}}R$ (C) $L=\sqrt{mk}R^2$ (D) $L=\sqrt{\frac{mk}{2}}R^2$

RD0193

30. Consider a body of mass 1.0 kg at rest at the origin at time t = 0. A force $\vec{F}=(\alpha t\hat{i}+\beta\hat{j})$ is applied on the body, where $\alpha = 1.0\ Ns^{-1}$ and $\beta = 1.0$ N. The torque acting on the body about the origin at time t = 1.0 s is $\vec{\tau}$. Which of the following statements is (are) true? **[JEE Advanced-2018]**

(A) $|\vec{\tau}|=\frac{1}{3}Nm$

(B) The torque $\vec{\tau}$ is in the direction of the unit vector $+\hat{k}$

(C) The velocity of the body at t = 1 s is $\vec{v}=\frac{1}{2}(\hat{i}+2\hat{j})ms^{-1}$

(D) The magnitude of displacement of the body at t = 1 s is $\frac{1}{6}m$

RD0194

31. A ring and a disc are initially at rest, side by side, at the top of an inclined plane which makes an angle 60° with the horizontal. They start to roll without slipping at the same instant of time along the shortest path. If the time difference between their reaching the ground is $(2-\sqrt{3})/\sqrt{10}s$, then the height of the top of the inclined plane, in meters, is _____. Take g = 10 ms^{-2}.

[JEE Advanced-2018]

RD0195

32. A thin and uniform rod of mass M and length L is held vertical on a floor with large friction. The rod is released from rest so that it falls by rotating about its contact-point with the floor without slipping. Which of the following statement(s) is/are correct, when the rod makes an angle 60° with vertical ? [g is the acceleration due to gravity] **[JEE Advanced-2019]**

(1) The radial acceleration of the rod's center of mass will be $\frac{3g}{4}$

(2) The angular acceleration of the rod will be $\frac{2g}{L}$

(3) The angular speed of the rod will be $\sqrt{\frac{3g}{2L}}$

(4) The normal reaction force from the floor on the rod will be $\frac{Mg}{16}$

RD0196

ANSWER KEY

EXERCISE (S-1)

1. Ans. 2

2. Ans. $\frac{ml^2}{12}$

3. Ans. 2.00 N·m, (b) $\hat{k}$

4. Ans. 8 kg

5. Ans. $150(+\hat{i})$

6. Ans. 30°

7. Ans. 10^{-3} rad/s^2 clockwise

8. Ans. 2

9. Ans. $\frac{L}{3}$

10. Ans. (a) $\frac{9g}{7}\downarrow$ (b) $\frac{4mg}{7}\uparrow$

11. Ans. (i) 10/13m/s^2, (ii) 5000/26π, (iii) 480/13N

12. Ans. 2N

13. Ans. $M = 2m\left(\frac{2gh}{R^2\omega^2} - 1\right)$

14. Ans. $\sqrt{5gR}$

15. Ans. (a) $\omega = \sqrt{\frac{6g}{\ell}}$, (b) $T = \frac{11mg}{4}$

16. Ans. $\frac{\omega}{3}$

17. Ans. $\frac{4\pi}{5}$

18. Ans. 75 J

19. Ans. 1/2 ma

20. Ans. 15

21. Ans. 16 m/s^2

22. Ans. $6\,mv_0^2$

23. Ans. $\frac{13}{16}$ Mv^2

24. Ans. $\frac{l^2}{12x}$

25. Ans. (i) $\frac{J}{m}$ (ii) zero (iii) $\frac{J}{2m}$ (iv) $\frac{5}{2}\frac{J}{m}$

26. Ans. (a) $t = \frac{\pi Rm}{2I}$; (b) $s = \frac{\pi R}{2}$

27. Ans. $\frac{mv}{4}$

28. Ans. $\frac{2m{v_0}^2}{l}$

EXERCISE (S-2)

1. Ans. 57/140MR^2

2. Ans. $\frac{a\ell^4}{36}$

3. Ans. (3/8) F_g

4. Ans. $a = \left[\frac{b-\mu a}{3b+\mu a}\right]g$

5. Ans. 1.63 N, 1.224 m

6. Ans. (a) 5 m/s^2 →, (b) $0.3 < h < 1.5$ m

7. Ans. (a) $\sqrt{3}$ $ml\omega^2$ (b) $(F_{net})_x = -\frac{F}{4}$, $(F_{net})_y = \sqrt{3}$ $ml\omega^2$

8. Ans. $\sqrt{15}$ ft/sec

9. Ans. (a) $4\sqrt{\frac{3}{7}}$ m/s, (b) $\frac{200}{7}$N

10. Ans. $t = 2/\sqrt{5}$ sec, $\theta = 4\pi/5$ rad

11. Ans. (i) (a) $\frac{1.2g}{c}$ (cw) (b) $-03.(\hat{i} + 2\hat{j})g$ (ii) (a) 24g/17c (cw) (b) 12g/17↓ (iii) (a) 2.4g/c (cw) (b) 0.5g↓

12. Ans. 3

13. Ans. $\alpha_{max} = \frac{2g}{3r}$

14. Ans. (a) $a_c = \frac{4F}{3m_1 + 8m_2}$, $a_p = 2a_c$

(b) friction at the top of the cylinder = $3m_1F(3m_1+8m_2)$ towards right; friction at the bottom = $m_1F/(3m_1+8m_2)$ towards right.

15. Ans. (a) pipe rolls without sliding

(b) pipe : 5/6 m/s →, 125/12 rad/s (anticlockwise); plate : 5/3 m/s →

16. Ans. 6N, $-0.6\hat{j} \pm 0.6\hat{k}$

17. Ans. (a) $\theta_C = \cos^{-1}(4/7)$, (b) $v = \sqrt{4/7\,gR}$, (c) $K_T/K_R = 6$

18. Ans. $v=\sqrt{\frac{14gR}{3}}$ **19. Ans. (a)** 6.68 m/s **(b)** 2.27 m

20. Ans. (i) $2v_0/3$, (ii) $t=v_0/3\mu g$, $W=\frac{1}{2}[3\mu^2 mg^2t^2-2\mu mgtv_0]$ $(t<t_0)$, $W=-\frac{1}{6}mv_0^2$ $(t>t_0)$

21. Ans. (a) $\omega/3$, (b) $\frac{\sqrt{37}}{3}m\omega R$, (c) $\frac{\sqrt{37}}{3}m\omega R$ **22. Ans.** 60°

23. Ans. (a) $\ell=0.1$m; (b) $\omega'=1$rad/s ; (c) laminar sheet will never come to rest

24. Ans. 5 **25. Ans.** (a) $\frac{m}{M}=\frac{1}{4}$; (b) $x=\frac{2L}{3}$; (c) $\frac{v_0}{2\sqrt{2}}$

EXERCISE (O-1)

1. Ans. (C) 2. Ans. (B) 3. Ans. (B) 4. Ans. (D) 5. Ans. (A) 6. Ans. (A)
7. Ans. (B) 8. Ans. (D) 9. Ans. (C) 10. Ans. (A) 11. Ans. (C) 12. Ans. (B)
13. Ans. (D) 14. Ans. (D) 15. Ans. (B) 16. Ans. (C) 17. Ans. (B) 18. Ans. (D)
19. Ans. (C) 20. Ans. (C) 21. Ans. (A) 22. Ans. (B) 23. Ans. (C) 24. Ans. (D)
25. Ans. (A) 26. Ans. (D) 27. Ans. (B) 28. Ans. (C) 29. Ans. (A) 30. Ans. (D)
31. Ans. (B) 32. Ans. (B) 33. Ans. (A) 34. Ans. (A) 35. Ans. (B) 36. Ans. (C)
37. Ans. (B) 38. Ans. (B) 39. Ans. (A) 40. Ans. (D) 41. Ans. (B) 42. Ans. (D)
43. Ans. (C) 44. Ans. (A) 45. Ans. (C) 46. Ans. (D) 47. Ans. (B) 48. Ans. (B)
49. Ans. (A) 50. Ans. (A,B,C,D) 51. Ans. (B,C) 52. Ans. (A,D) 53. Ans. (A,C,D)
54. Ans. (A,C,D) 55. Ans. (A,B,C,D) 56. Ans. (A) 57. Ans. (A)
58. Ans. (D) 59. Ans. (B) 60. Ans. (A) 61. Ans. (D) 62. Ans. (D)
63. Ans. (A)→(R); (B)→(P); (C)→(T); (D)→(Q,S)

EXERCISE (O-2)

1. Ans. (D) 2. Ans. (C) 3. Ans. (B) 4. Ans. (C) 5. Ans. (A) 6. Ans. (D)
7. Ans. (B) 8. Ans. (B) 9. Ans. (B) 10. Ans. (B) 11. Ans. (D) 12. Ans. (C)
13. Ans. (A) 14. Ans. (C) 15. Ans. (C) 16. Ans. (D) 17. Ans. (C) 18. Ans. (A)
19. Ans. (C) 20. Ans. (C,D) 21. Ans. (B,C,D) 22. Ans. (A,B,C) 23. Ans. (A,B,D)
24. Ans. (A,C) 25. Ans. (A,B,C) 26. Ans. (A,B,C) 27. Ans. (B,C,D)
28. Ans. (A,C,D) 29. Ans. (A,C) 30. Ans. (B,D) 31. Ans. (A, B, C)
32. Ans. (C) 33. Ans. (A) 34. Ans. (C) 35. Ans. (C) 36. Ans. (B) 37. Ans. (C)
38. Ans. (B) 39. Ans. (B) 40. Ans. (C) 41. Ans. (C) 42. Ans. (B) 43. Ans. (B)
44. Ans. (C) 45. Ans. (B) 46. Ans. (A) 47. Ans. (C) 48. Ans. (A,B,C)
49. Ans. (C) 50. Ans. (B,C,D) 51. Ans. (A,C,D)
52. Ans. (A) ST (B) R (C) PRT (D) PQ 53. Ans. (A)-R, (B)-S, (C)-P, (D)-Q

EXERCISE (JM)

1. Ans. (2) 2. Ans. (4) 3. Ans. (4) 4. Ans. (2) 5. Ans. (4) 6. Ans. (3)
7. Ans. (1) 8. Ans. (4) 9. Ans. (1) 10. Ans. (1 or 3) 11. Ans. (2) 12. Ans. (3)
13. Ans. (3) 14. Ans. (4) 15. Ans. (3)

EXERCISE (JA)

1. Ans. (B) 2. Ans. (A) 3. Ans. (B,C) 4. Ans. 4 5. Ans. 9 6. Ans. (AC or C)
7. Ans. (B) 8. Ans. (C, D) 9. Ans. 3 10. Ans. (D) 11. Ans. (A) 12. Ans. (A,B)
13. Ans. (D) 14. Ans. 8 15. Ans. 4 16. Ans. 2 17. Ans. 7 18. Ans. (C, D)
19. Ans. 6 20. Ans. (D) 21. Ans. (A, B, D) 22. Ans. (D) 23. Ans. (D)
24. Ans. (C) 25. Ans. (A), (B), (C) 26. Ans. (B,C or C) 27. Ans. (Bonus)
28. Ans. (B) 29. Ans. (B,C) 30. Ans. (A,C) 31. Ans. 0.75 [0.74, 0.76] 32. Ans. (1,3,4)

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E

PHYSICS

Chapter 01 Contents

Elasticity, Thermal Expansion, Calorimetry & Heat Transfer

Important Notes

ELASTICITY

A body is said to be rigid if the relative positions of its constituent particles remains unchanged when external deforming forces are applied to it. The nearest approach to a rigid body is diamond or carborundum.

Actually no body is perfectly rigid and every body can be deformed more or less by the application of suitable forces. All these deformed bodies however regain their original shape or size, when the deforming forces are removed.

The property of matter by virtue of which a body tends to regain its original shape and size after the removal of deforming forces is called elasticity.

SOME TERMS RELATED TO ELASTICITY :

- **Deforming Force**

  External force which try to change in the length, volume or shape of the body is called deforming force.
- **Perfectly Elastic Body**

  The body which perfectly regains its original form on removing the external deforming force, is defined as a perfectly elastic body. Ex. : quartz – Very nearly a perfect elastic body.
- **Plastic Body**

  (a) The body which does not have the property of opposing the deforming force, is known as a plastic body.

  (b) The bodies which remain in deformed state even after removed of the deforming force are defined as plastic bodies.
- **Internal restoring force**

  When a external force acts at any substance then due to the intermolecular force there is a internal resistance produced into the substance called internal restoring force.

  At equilibrium the numerical value of internal restoring force is equal to the external force.

STRESS

The internal restoring force acting per unit area of cross–section of the deformed body is called stress.

$$\text{Stress} = \frac{\text{Internal restoring force}}{\text{Area of cross section}} = \frac{F_{\text{int ernal}}}{A} = \frac{F_{\text{external}}}{A}$$

Stress depends on direction of force as well as direction of area of application so it is tensor.

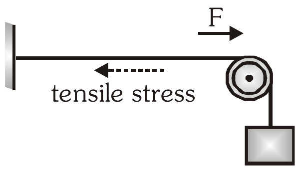
SI UNIT : N–m^{-2} **Dimensions :** $M^1 L^{-1} T^{-2}$

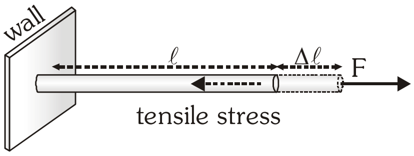
There are three types of stress :-

- **Longitudinal Stress**

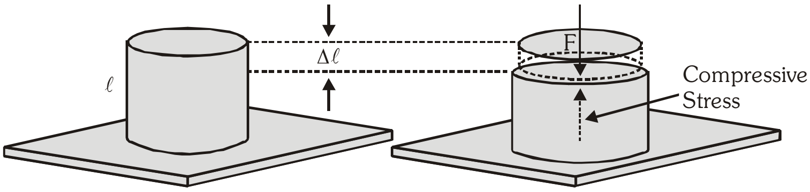
  When the stress is normal to the surface of body, then it is known as longitudinal stress. There are two types of longitudinal stress

  (a) Tensile Stress : The longitudinal stress, produced due to increase in length of a body, is defined as tensile stress.





(b) Compressive Stress : The longitudinal stress, produced due to decrease in length of a body, is defined as compressive stress.

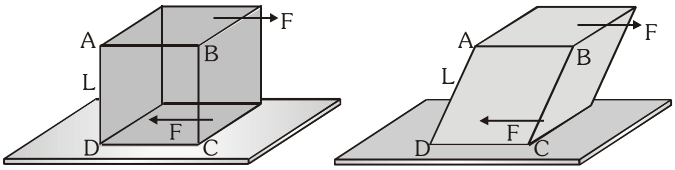


(c) Volume Stress

If equal normal forces are applied every one surface of a body, then it undergoes change in volume. The force opposing this change in volume per unit area is defined as volume stress.

(d) Tangential Stress or Shear Stress

When the stress is tangential or parallel to the surface of a body then it is known as shear stress. Due to this stress, the shape of the body changes or it gets twisted.



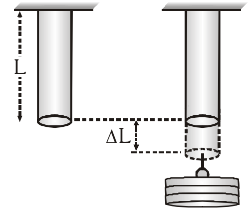
STRAIN

- The ratio of change of any dimension to its original dimension is called strain.

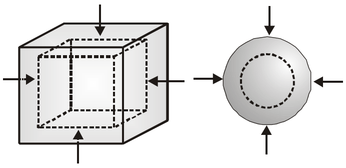
$$\text{Strain} = \frac{\text{change in size of the body}}{\text{original size of the body}}$$

- It is a unitless and dimensionless quantity.
- There are three types of strain : Type of strain depends upon the directions of applied force.

- **Longitudinal strain** $= \dfrac{\text{change in length of the body}}{\text{initial length of the body}} = \dfrac{\Delta L}{L}$

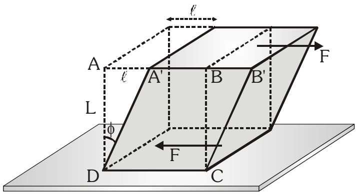


- **Volume strain**$= \dfrac{\text{change in volume of the body}}{\text{original volume of the body}} = \dfrac{\Delta V}{V}$



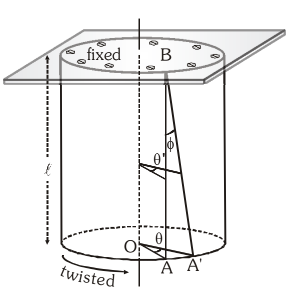
- **Shear strain**

When a deforming force is applied to a body parallel to its surface then its shape (not size) changes. The strain produced in this way is known as shear strain. The strain produced due to change of shape of the body is known as shear strain.



$$\tan\phi = \frac{\ell}{L} \text{ or } \phi = \frac{\ell}{L} = \frac{\text{displacement of upper face}}{\text{distance between two faces}}$$

Relation Between angle of twist and Angle of shear



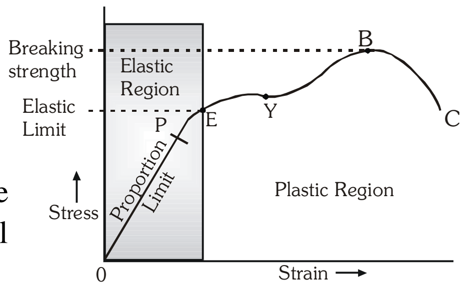
When a cylinder of length 'ℓ' and radius 'r' is fixed at one end and and tangential force is applied at the other end, then the cylinder gets twisted. Figure shows the angle of shear ABA' and angle of twist AOA'. Arc AA' = r θ and Arc AA' = ℓφ so rθ = ℓφ

$$\Rightarrow \phi = \frac{r\theta}{\ell}$$

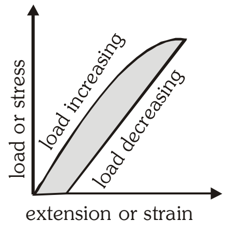
where θ = angle of twist, φ = angle of shear

- When a material is under tensile stress restoring force are caused by intermolecular attraction while under compressive stress, the restoring force are due to intermolecular repulsion.
- If the deforming force is inclined to the surface at an angle θ such that $\theta \neq 0$ and $\theta \neq 90°$ then both tangential and normal stress are developed.
- Linear strain in the direction of force is called **longitudinal** strain while in a direction perpendicular to force **lateral strain**.

STRESS – STRAIN GRAPH

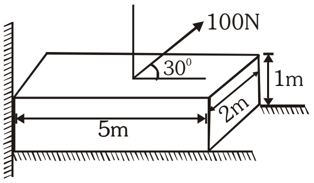


- **Proportion Limit** : The limit in which Hooke's law is valid and stress is directly proportional to strain is called proportion limit. Stress ∝ Strain
- **Elastic limit** : That maximum stress which on removing the deforming force makes the body to recover completely its original state.
- **Yield Point :** The point beyond elastic limit, at which the length of wire starts increasing without increasing stress, is defined as the yield point.
- **Breaking Point :** The position when the strain becomes so large that the wire breaks down at last, is called breaking point. At this position the stress acting in that wire is called breaking stress and strain is called breaking strain.



- **Elastic Hysteresis :** The strain persists even when the stress is removed. This lagging behind of strain is called elastic hysteresis. This is the reason why the values of strain for same stress are different while increasing the load and while decreasing the load.
- **Breaking Stress :** The stress required to cause actual facture of a material is called the breaking stress
  Breaking stress = F/A

- Breaking stress also measures the tensile strength.
- Metals with small plastic deformation are called brittle.
- Metals with large plastic deformation are called ductile.
- Elasticity restoring forces are strictly conservative only when the elastic hysteresis is zero. i.e. the loading and unloading stress – strain curves are identical.



Ex. Find out longitudinal stress and tangential stress on a fixed block.

Sol. Longitudinal or normal stress $\sigma_1 = \dfrac{100\sin 30^0}{5\times 2} = 5\ \text{N/m}^2$

Tangential stress $\sigma_2 = \dfrac{100\cos 30^0}{5\times 2} = 5\sqrt{3}\,\text{N/m}^2$

Ex. The breaking stress of aluminium is 7.5×10^8 dyne cm^{-2}. Find the greatest length of aluminium wire that can hang vertically without breaking. Density of aluminium is 2.7 g cm^{-3}.
Given : g = 980 cm s^{-2}.

Sol. Let ℓ be the greatest length of the wire that can hang vertically without breaking.
Mass of wire m = cross–sectional area (A) × length (ℓ) × density (ρ), Weight of wire = mg = $A\ell\rho g$
This is equal to the maximum force that the wire can withstand.

$\therefore$ Breaking stress $= \dfrac{\ell A\rho g}{A} = \ell\rho g \Rightarrow 7.5\times 10^8 = \ell\times 2.7\times 980$

$\Rightarrow \quad \ell = \dfrac{7.5\times 10^8}{2.7\times 980}\ \text{cm} = 2.834\times 10^5\text{cm} = 2.834\ \text{km}$

HOOKE'S LAW

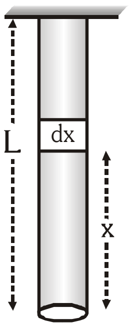
If the deformation is small, the stress in a body is proportional to the corresponding strain, this fact is known as Hooke's Law. Within elastic limit : stress $\propto$ strain $\Rightarrow \dfrac{\text{stress}}{\text{strain}} = \text{constant}$

This constant is known as modulus of elasticity or coefficient of elasticity.
The modulus of elasticity depends only on the type of material used. It does not depend upon the value of stress and strain.

YOUNG'S MODULUS OF ELASTICITY 'Y'

- Within elastic limit the ratio of longitudinal stress and longitudinal strain is called Young's modulus of elasticity.
- $Y = \dfrac{\text{longitudinal stress}}{\text{longitudinal strain}} = \dfrac{F/A}{\ell/L} = \dfrac{FL}{\ell A}$
- Within elastic limit the force acting upon a unit area of a wire by which the length of a wire becomes double, is equivalent to the Young's modulus of elasticity of material of a wire.
- If L is the length of wire, r is radius and ℓ is the increase in length of the wire by suspending a weight Mg at its one end then Young's modulus of elasticity of the material of wire $Y = \dfrac{(Mg/\pi r^2)}{(\ell/L)} = \dfrac{MgL}{\pi r^2\ell}$
- Unit of Y : N/m^2 ● Dimensions of Y : $M^1L^{-1}T^{-2}$

Increment of length due to own weight



Let a rope of mass M and length L is hanged vertically. As the tension of different point on the rope is different. Stress as well as strain will be different at different point.

(i) maximum stress at hanging point (ii) minimum stress at lower point

Consider a dx element of rope at x distance from lower end then tension $T = \left(\dfrac{M}{L}\right)x\ g$

So stress $= \dfrac{T}{A} = \left(\dfrac{M}{L}\right)\dfrac{xg}{A}$

Let increase in length of dx is dy then strain = $\frac{dy}{dx}$

So Young modulus of elasticity $Y = \frac{\text{stress}}{\text{strain}} = \frac{\frac{M}{L}\frac{xg}{A}}{dy/dx} \Rightarrow \left(\frac{M}{L}\right)\frac{xg}{A}dx = Y\,dy$

For full length of rope $\frac{Mg}{LA}\int_0^L x\,dx = Y\int_0^{\Delta\ell} dy \Rightarrow \frac{Mg}{LA}\frac{L^2}{2} = Y\Delta\ell \Rightarrow \Delta\ell = \frac{MgL}{2AY}$

[Since the stress is varying linearly we may apply average method to evaluate strain.]

BULK MODULUS OF ELASTICITY 'K' or 'B'

- Within elastic limit the ratio of the volume stress and the volume strain is called bulk modulus of elasticity.
- $K \text{ or } B = \frac{\text{volume stress}}{\text{volume strain}} = \frac{F/A}{\frac{-\Delta V}{V}} = \frac{\Delta P}{\frac{-\Delta V}{V}}$

  The minus sign indicates a decrease in volume with an increase in stress.
- Unit of K : N m^{-2} or pascal
- **Bulk modulus of an ideal gas is process dependence.**
  - For isothermal process PV = constant $\Rightarrow PdV + VdP = 0 \Rightarrow P = \frac{-dP}{dV/V}$ So bulk modulus = P
  - For adiabatic process PV^{γ} = constant $\Rightarrow \gamma PV^{\gamma-1}dV + V^{\gamma}dP = 0$

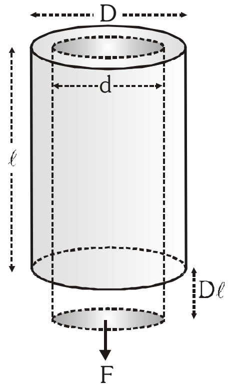
    $\Rightarrow \gamma PdV + VdP = 0 \Rightarrow \gamma P = \frac{-dP}{dV/V}$ So bulk modulus = γP
  - For any polytropic process PV^{n} = constant

    $\Rightarrow nPV^{n-1}dV + V^{n}dP = 0 \Rightarrow PdV + VdP = 0 \Rightarrow nP = \frac{-dP}{dV/V}$ So bulk modulus = nP
  - **Compressibility** : The reciprocal of bulk modulus of elasticity is defined as compressibility. $C = \frac{1}{K}$

MODULUS OF RIGIDITY 'η'

- Within elastic limit the ratio of shearing stress and shearing strain is called modulus of rigidity of a material.
- $\eta = \frac{\text{shearing stress}}{\text{Shearing strain}} = \left(\frac{\frac{F_{\text{tangential}}}{A}}{\phi}\right) = \frac{F_{\text{tangential}}}{A\phi}$

  Note : Angle of shear 'ϕ' always taking in radian



Poisson's Ratio (σ)

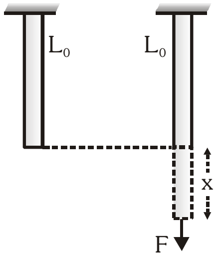
In elastic limit, the ratio of lateral strain and longitudinal strain is called Poisson's ratio.

$\sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{\beta}{\alpha}$; $\beta = \frac{-\Delta D}{D} = \frac{d-D}{D}$ & $\alpha = \frac{\Delta L}{L}$

WORK DONE IN STRETCHING A WIRE

(Potential energy of a stretched wire)

When a wire is stretched, work is done against the interatomic forces, which is stored in the form of elastic potential energy.



For a wire of length L_o stretched by a distance x, the restoring elastic force is :

$$F = \text{stress} \times \text{area} = Y\left[\frac{x}{L_o}\right]A$$

The work has to be done against the elastic restoring forces in stretching dx

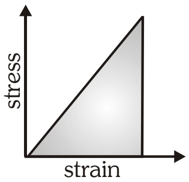
$$dW = Fdx = \frac{YA}{L_o}xdx$$

The total work done in stretching the wire from x = 0 to x = $\Delta\ell$ is, then

$$W = \int_0^{\Delta\ell}\frac{YA}{L_o}xdx = \frac{YA}{L_o}\left[\frac{x^2}{2}\right]_0^{\Delta\ell} = \frac{YA(\Delta\ell)^2}{2L_o}$$

$$W = \frac{1}{2} \times Y \times (\text{strain})^2 \times \text{original volume} = \frac{1}{2}(\text{stress})(\text{strain})(\text{volume})$$

- The value of K is maximum for solids and minimum for gases.



- For any ideal rigid body all three elastic modulus are infinite .
- η is the characteristic of solid material only as the fluids do not have fixed shape.
- Potential energy density = area under the stress–strain curve.
- Young's modulus = Slope of the stress–strain curve

Ex. A steel wire of 4.0 m in length is stretched through 2.00 mm. The cross–sectional area of the wire is 2.0 mm^2. If Young's modulus of steel is 2.0×10^{11} N/m^2 find (i) the energy density of wire (ii) the elastic potential energy stored in the wire.

Sol. (i) The energy density of stretched wire $= \frac{1}{2} \times \text{stress} \times \text{strain} = \frac{1}{2} \times Y \times (\text{strain})^2$

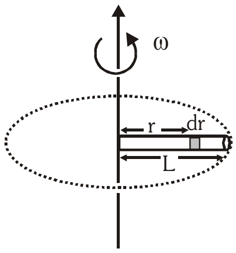
$$= \frac{1}{2} \times 2.0 \times 10^{11} \times \left[\frac{2 \times 10^{-3}}{4}\right]^2 = 2.5 \times 10^4 \text{ J/m}^3$$

(ii) Elastic potential energy = energy density × volume = $2.5 \times 10^4 \times (2.0 \times 10^{-6}) \times 4.0$ J

$= 20 \times 10^{-2} = 0.20$J

Ex. A thin uniform metallic rod of length 0.5 m and radius 0.1 m rotates with an angular velocity 400 rad/s in a horizontal plane about a vertical axis passing through one of its ends. Calculate (a) tension in the rod and (b) the elongation of the rod. The density of material of the rod is 10^4 kg/m^3 and the Young's modulus is 2×10^{11} N/m^2.

Sol. (a) Consider an element of length dr at a distance r from the axis of rotation as shown in figure. The centripetal force acting on this element will be $dT = dmr\omega^2 = (\rho A dr)r\omega^2$. As this force is provided by tension in the rod (due to elasticity), so the tension in the rod at a distance r from the axis of rotation will be due to the centripetal force due to all elements between x = r to x = L



i.e., $T = \int_r^L \rho A\omega^2 r dr = \frac{1}{2}\rho A\omega^2\left[L^2 - r^2\right]$...(i)

So here $T = \frac{1}{2} \times 10^4 \times \pi \times 10^{-2} \times (400)^2 \left[\left(\frac{1}{2}\right)^2 - r^2\right] = 8\pi \times 10^6\left[\frac{1}{4} - r^2\right]$N

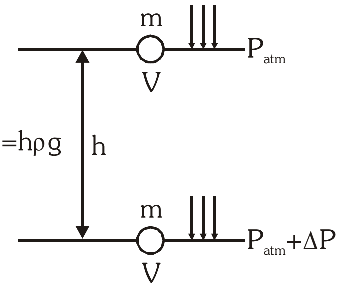
(b) Now if dy is the elongation in the element of length dr at position r then strain

$$\frac{dy}{dr} = \frac{\text{stress}}{Y} = \frac{T}{AY} = \frac{1}{2}\frac{\rho\omega^2}{Y}[L^2 - r^2]\ dr$$

So the elongation of the whole rod

$$\Delta L = \frac{\rho\omega^2}{2Y}\int_0^L \left(L^2 - r^2\right) dr = \frac{1}{3}\frac{\rho\omega^2 L^3}{Y} = \frac{1}{3} \times \frac{10^4 \times (400)^2 (0.5)^3}{2 \times 10^{11}} = \frac{1}{3} \times 10^{-3} \text{m}$$

Ex. Find the depth of lake at which density of water is 1% greater than at the surface.
Given compressibility $K = 50 \times 10^{-6}$ /atm.



Sol. $B = \frac{\Delta P}{-\Delta V/V} \Rightarrow \frac{\Delta V}{V} = -\frac{\Delta P}{B}$

We know $P = P_{atm} + h\rho g$ and $m = \rho V$ = constant

$d\rho V + dV\rho = 0 \Rightarrow \frac{d\rho}{\rho} = -\frac{dV}{V}$ i.e. $\frac{\Delta\rho}{\rho} = \frac{\Delta P}{B}$

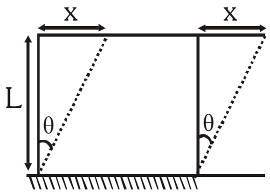
$\Rightarrow \frac{\Delta\rho}{\rho} = \frac{1}{100} = \frac{h\rho g}{B}$

[assuming ρ = constant]; $h\rho g = \frac{B}{100} = \frac{1}{100K}$

$\Rightarrow h\rho g = \frac{1 \times 1 \times 10^5}{100 \times 50 \times 10^{-6}}$

$h = \frac{10^5}{5000 \times 10^{-6} \times 1000 \times 10} = \frac{100 \times 10^3}{50}$ m = 2km

Ex. A rubber cube of side 5 cm has one side fixed while a tangential force equal to 1800 N is applied to opposite face. Find the shearing strain and the lateral displacement of the strained face. Modulus of rigidity for rubber is 2.4×10^6 N/m^2.



Sol. Stress = $\frac{F}{A} = \eta \frac{x}{L}$

Strain = $\theta = \frac{F}{A\eta} = \frac{1800}{25 \times 10^{-4} \times 2.4 \times 10^{6}} = \frac{180}{25 \times 24} = \frac{3}{10} = 0.3$ radian

as $\frac{x}{L} = 0.3 \Rightarrow x = 0.3 \times 4 \times 10^{-2} = 1.5 \times 10^{-2}$ m = 1.5 mm

EFFECT OF TEMPERATURE ON ELASTICITY

When temperature is increased then due to weakness of inter molecular force the elastic properties in general decreases i.e. elastic constant decreases. Plasticity increases with temperature. For example, at ordinary room temperature, carbon is elastic but at high temperature, carbon becomes plastic. Lead is not much elastic at room temperature but when cooled in liquid nitrogen exhibit highly elastic behaviour. For a special kind of steel, elastic constants do not vary appreciably temperature. This steel is called 'INVAR steel'.

EFFECT OF IMPURITY ON ELASTICITY

Y is slightly increase by impurity. The inter molecular attraction force inside wire effectively increase by impurity due to this external force can be easily opposed.

TEMPERATURE & DIFFERENT TYPE OF TEMPERATURE SCALES

TEMPERATURE

- Temperature is a macroscopic physical quantity related to our sense of hot and cold.
- The natural flow of heat is from higher temperature to lower temperature, i.e. temperature determines the thermal state of a body whether it can give or receive heat.

DIFFERENT TYPES OF TEMPERATURE SCALES

- The Kelvin temperature scale is also known as thermodynamic scale. The SI unit of temperature is the kelvin and is defined as (1/273.16) of the temperature of the triple point of water. The triple point of water is that point on a P–T diagram where the three phase of water, the solid, the liquid and the gas, can coexist in equilibrium.
- In addition to Kelvin temperature scale, there are other temperature scales also like Celsius, Fahrenheit, Reaumur, Rankine, etc. Temperature on one scale can be converted into other scale by using the following identity

$$\frac{\text{Reading on any scale} - \text{lower fixed point (LFP)}}{\text{Upper fixed point (UFP)} - \text{lower fixed point (LFP)}} = \text{constant for all scales}$$

Hence $$\frac{C-0^\circ}{100^\circ-0^\circ} = \frac{F-32^\circ}{212^\circ-32^\circ} = \frac{K-273.15}{373.15-273.15}$$

- Different temperature scales :

Name of the scale	Symbol for each degree	Lower fixed point (LFP)	Upper fixed point (UFP)	Number of divisions on the scale
Celsius	℃	0℃	100°C	100
Fahrenheit	℉	32℉	212℉	180
Kelvin	K	273.15 K	373.15 K	100

Ex. Express a temperature of 60°F in degree celsius and in kelvin.

Sol. By using $\frac{C-0^\circ}{100^\circ-0^\circ} = \frac{F-32^\circ}{212^\circ-32^\circ} = \frac{K-273.15}{373.15-273.15}$

$$\Rightarrow \frac{C-0^\circ}{100^\circ-0^\circ} = \frac{60^\circ-32^\circ}{212^\circ-32^\circ} = \frac{K-273.15}{373.15-273.15} \Rightarrow C = 15.15^\circ\ C \text{ and } K = 288.7\ K$$

Ex. The temperature of an iron piece is heated from 30° to 90°C. What is the change in its temperature on the fahrenheit scale and on the kelvin scale?

Sol. $\Delta C = 90^\circ - 30^\circ = 60^\circ C$

Temperature difference on Fahrenheit Scale $\Delta F = \frac{9}{5}\Delta C = \frac{9}{5}(60^\circ C) = 108^\circ F$

Temperature difference on Kelvin Scale $\Delta K = \Delta C = 60K$

THERMAL EXPANSION

THERMAL EXPANSION

When matter is heated without any change in it's state, it usually expands. According to atomic theory of matter, asymmetry in potential energy curve is responsible for thermal expansion. As with rise in temperature the amplitude of vibration increases and hence energy of atoms increases, hence the average distance between the atom increases. So the matter as a whole expands.

- Thermal expansion is minimum in case of solids but maximum in case of gases because intermolecular force is maximum in solids but minimum in gases.
- Solids can expand in one dimension (Linear expansion), two dimension (Superficial expansion) and three dimension (Volume expansion) while liquids and gases usually suffers change in volume only.

Linear expansion :

$$\ell = \ell_0 (1 + \alpha\Delta\theta) \Rightarrow \Delta\ell = \ell_0\alpha\Delta\theta$$

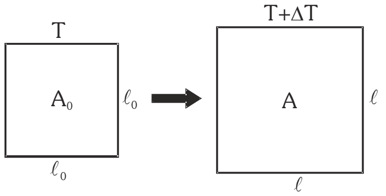
Before heating After heating

Superficial (areal) expansion :

$$A = A_0 (1 + \beta\Delta\theta)$$

Also $A_0 = \ell_0^2$ and $A = \ell^2$

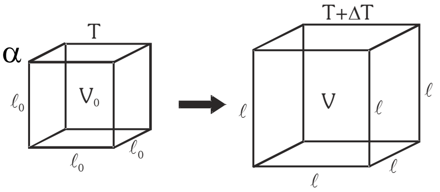
So $\ell^2 = \ell_0^2(1 + \beta\Delta\theta) = [\ell_0(1 + \alpha\Delta\theta)]^2 \Rightarrow \beta = 2\alpha$



Volume expansion :

$V = V_0 (1 + \gamma\Delta\theta)$ Also $V = \ell^3$ and $V_0=\ell_0^3$ so $\gamma = 3\alpha$

$\Rightarrow 6\alpha = 3\beta = 2\gamma$ or $\alpha : \beta : \gamma = 1 : 2 : 3$

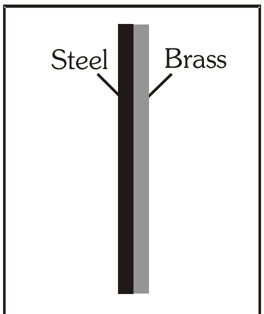


Contraction on heating :

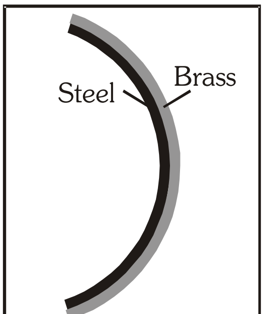
Some rubber like substances contract on heating because transverse vibration of atoms of substance dominate over longitudinal vibration which is responsible for expansion.

Application of thermal Expansion in Solids

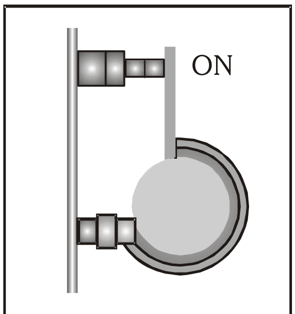
(a) **Bi–metallic strip :** Two strips of equal length but of different materials (different coefficient of linear expansion) when join together, it is called "Bi–metallic strip" and can be used in thermostat to break or make electrical contact. This strip has the characteristic property of bending on heating due to unequal linear expansion of the two metals. The strip will bend with metal of greater α on outer side.



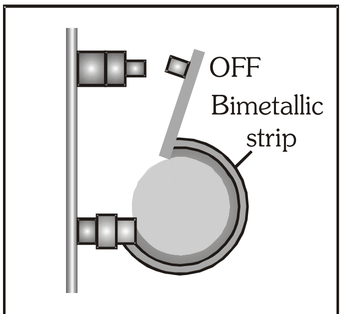
Room temperature



Higher temperature



At room temperature



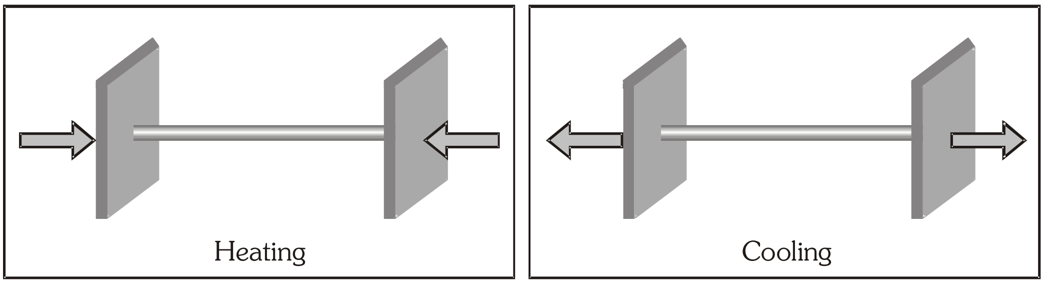
At high temperature

(b) **Effect of temperature on the time period of a simple pendulum :** A pendulum clock keeps proper time at temperature θ. If temperature is increased to θ' (> θ) then due to linear expansion, length of pendulum increases and hence its time period will increase.

Fractional change in time period $\frac{\Delta T}{T} = \frac{1}{2}\alpha\Delta\theta$ $(\because T \propto \sqrt{\ell} \;\therefore\; \frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta\ell}{\ell})$

- Due to increment in its time period, a pendulum clock becomes slow in summer and will lose time. Loss of time in a time period $\Delta T = \frac{1}{2}\alpha\,\Delta\theta T$
- The clock will lose time i.e. will become slow if θ' > θ (in summer) and will gain time i.e will become fast if θ' < θ (in winter).
- Since coefficient of linear expansion (α) is very small for invar, hence pendulums are made of invar to show the correct time in all seasons.

(c) When a rod whose ends are rigidly fixed such as to prevent expansion or contraction, undergoes a change in temperature due to thermal expansion or contraction, a compressive or tensile stress is developed in it. Due to this thermal stress the rod will exert a large force on the supports. If the change in temperature of a rod of length L is Δθ then :–

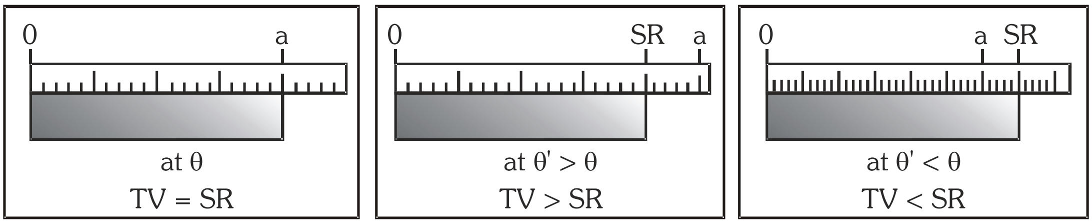


Thermal strain $= \frac{\Delta L}{L} = \alpha\Delta\theta$ $\because \alpha = \frac{\Delta L}{L} \times \frac{1}{\Delta\theta}$ So thermal stress = $Y\alpha\Delta\theta$

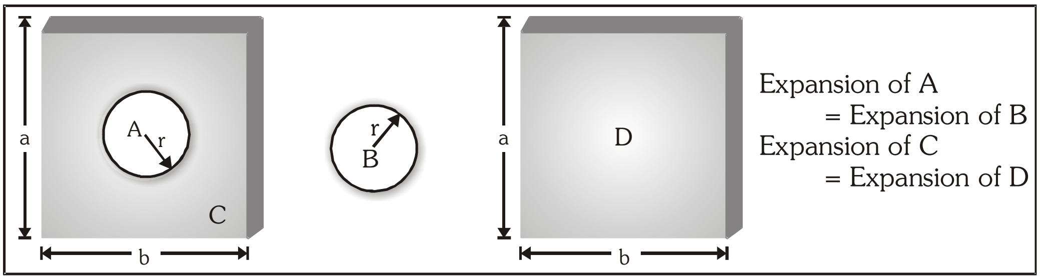
$\because Y = \frac{\text{stress}}{\text{strain}}$

So force on the supports $F = YA\alpha\Delta\theta$

(d) **Error in scale reading due to expansion or contraction:** If a scale gives correct reading at temperature θ. At temperature θ'(>θ) due to linear expansion of scale, the scale will expand and scale reading will be lesser than true value so that,



(e) Expansion of cavity: Thermal expansion of an isotropic object may be imagined as a photographic enlargement.



(f) Some other application

- When rails are laid down on the ground, space is left between the ends of two rails
- The transmission cable are not tightly fixed to the poles
- Test tubes, beakers and cubicles are made of pyrex–glass or silica because they have very low value of coefficient of linear expansion
- The iron rim to be put on a cart wheel is always of slightly smaller diameter than that of wheel
- A glass stopper jammed in the neck of a glass bottle can be taken out by warming the neck of the bottle.

Ex. A steel ruler exactly 20 cm long is graduated to give correct measurements at 20^0C. ($\alpha_{steel} = 1.2 \times 10^{-5}\ °C^{-1}$)

(a) Will it give readings that are too long or too short at lower temperatures?

(b) What will be the actual length of the ruler be when it is used in the desert at a temperature of 40^0C?

Sol. (a) If the temperature decreases, the length of the ruler also decreases through thermal contraction. Below 20^0C, each centimeter division is actually somewhat shorter than 1.0 cm, so the steel ruler gives readings that are too long.

(b) At 40^0C, the increases in length of the ruler is

$$\Delta\ell = \ell\alpha\Delta T = (20)\,(1.2 \times 10^{-5})\,(40^0 - 20^0) = 0.48 \times 10^{-2} \text{ cm}$$

$\therefore$ The actual length of the ruler is, $\ell' = \ell + \Delta\ell = 20.0048$ cm

Ex. A second's pendulum clock has a steel wire. The clock is calibrated at 20^0C. How much time does the clock lose or gain in one week when the temperature is increased to 30^0C? ($\alpha_{steel} = 1.2 \times 10^{-5}\ °C^{-1}$)

Sol. The time period of second's pendulum is 2 second. As the temperature increases length time period increases. Clock becomes slow and it loses the time. The change in time period is

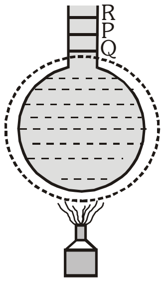
$$\Delta T = \frac{1}{2}T\alpha\Delta\theta = \left(\frac{1}{2}\right)(2)(1.2 \times 10^{-5})\,(30^0 - 20^0) = 1.2 \times 10^{-4} \text{ s}$$

$\therefore$ New Time period is $\quad T' = T + \Delta T = (2 + 1.2 \times 10^{-4}) = 2.00012$ s

$\therefore$ Time lost in one week $\quad \Delta t = \left(\frac{\Delta T}{T'}\right)t = \frac{(1.2 \times 10^{-4})}{(2.00012)}\,(7 \times 24 \times 3600) = 36.28$ s

Thermal Expansion in Liquids

- Liquids do not have linear and superficial expansion but these only have volume expansion.
- Since liquids are always to be heated along with a vessel which contains them so initially on heating the system (liquid + vessel), the level of liquid in vessel falls (as vessel expands more since it absorbs heat and liquid expands less) but later on, it starts rising due to faster expansion of the liquid.



PQ → represents expansion of vessel

QR → represents the real expansion of liquid.

- The actual increase in the volume of the liquid
  = The apparent increase in the volume of liquid + the increase in the volume of the vessel.
- Liquids have two coefficients of volume expansion.

(i) Co–efficient of apparent expansion (γ_a)

- It is due to apparent (that appears to be, but in not) increase in the volume of liquid if expansion of vessel containing the liquid is not taken into account.

$$\gamma_a = \frac{\text{Apparent expansion in volume}}{\text{Initial volume} \times \Delta\theta} = \frac{(\Delta V)}{V \times \Delta\theta}$$

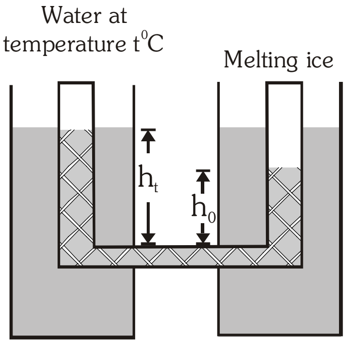
(ii) Co–efficient of real expansion (γ_r)

- It is due to the actual increase in volume of liquid due to heating.

$$\gamma_r = \frac{\text{Real increase in volume}}{\text{Initial volume} \times \Delta\theta} = \frac{(\Delta V)}{V \times \Delta\theta}$$

- Also coefficient of expansion of flask $\gamma_{\text{Vessel}} = \frac{(\Delta V)_{\text{Vessel}}}{V \times \Delta\theta}$
- $\gamma_{\text{Real}} = \gamma_{\text{Apparent}} + \gamma_{\text{Vessel}}$
- Change (apparent change) in volume in liquid relative to vessel is
  $\Delta V_{\text{app}} = V(\gamma_{\text{Real}} - \gamma_{\text{Vessel}})\,\Delta\theta = V(\gamma_r - 3\alpha)\Delta\theta$
  α = Coefficient of linear expansion of the vessel.

Ex. In figure shown, left arm of a U–tube is immersed in a hot water bath at temperature t°C, and right arm is immersed in a bath of melting ice; the height of manometric liquid in respective columns is h_t and h_0. Determine the coefficient of expansion of the liquid.



Sol. The liquid is in hydrostatic equilibrium ⇒ $\rho_t g h_t = \rho_0 g h_0$

Where, ρ_t is density of liquid in hot bath, ρ_0 is density of liquid in cold bath.

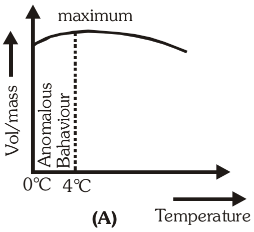
Volumes of a given mass M of liquid at temperatures t and 0^0C

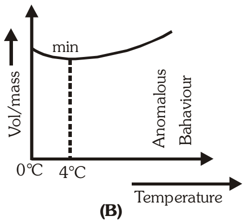
are related by $V_t = V_0(1+\gamma t)$ Since $\rho_t V_t = \rho_0 V_0 \Rightarrow \rho_t = \frac{\rho_0 V_0}{V_t} = \frac{\rho_0}{(1+\gamma t)}$

Since $h_t = \frac{\rho_0 h_0}{\rho_t} = h_0(1+\gamma t)$ which on solving for γ, yields $\gamma = \frac{(h_t - h_0)}{h_0 t}$

Anomalous expansion of water

Generally matter expands on heating and contracts on cooling. In case of water, it expands on heating if its temperature is greater than 4°C. In the range 0°C to 4°C, water contracts on heating and expands on cooling, i.e. γ is negative. This behaviour of water in the range from 0°C to 4°C is called anomalous expansion.





This anomalous behaviour of water causes ice to form first at the surface of a lake in cold weather. As winter approaches, the water temperature increases initially at the surface. The water there sinks because of its increased density. Consequently, the surface reaches 0°C first and the lake becomes covered with ice. Aquatic life is able to survive the cold winter as the lake bottom remains unfrozen at a temperature of about 4°C. At 4°C, density of water is maximum while its specific volume is minimum.

Ex. The difference between lengths of a certain brass rod and of a steel rod is claimed to be constant at all temperatures. Is this possible ?

Sol. If L_B and L_S are the lengths of brass and steel rods respectively at a given temperature, then the lengths of the rods when temperature is changed by θ °C.

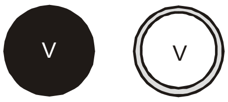
$L'_B = L_B(1 + \alpha_B \Delta\theta)$ and $L'_S = L_S(1 + \alpha_B \Delta\theta)$ So that $L'_B = L'_S(L_B - L_S) + (L_B \alpha_B - L_S\alpha_S) \Delta\theta$

So $(L'_B - L'_S)$ will be equal to $(L_B - L_S)$ at all temperatures if , $L_B\alpha_B - L_S\alpha_S = 0$ [as $\Delta\theta \neq 0$] or $\frac{L_B}{L_S} = \frac{\alpha_S}{\alpha_B}$

i.e., the difference in the lengths of the two rods will be independent of temperature if the lengths are in the inverse ratio of their coefficients of linear expansion.

Ex. There are two spheres of same radius and material at same temperature but one being solid while the other hollow. Which sphere will expand more if

(a) they are heated to the same temperature, (b) same heat is given to them ?



Sol. (a) As thermal expansion of isotropic solids is similar to true photographic enlargement,

expansion of a cavity is same as if it had been a solid body of the same material

i.e. $\Delta V = V\gamma \Delta\theta$

As here V, γ and $\Delta\theta$ are same for both solid and hollow spheres treated (cavity) ;

so the expansion of both will be equal.

(b) If same heat is given to the two spheres due to lesser mass, rise in temperature of hollow sphere will be more [as $\Delta\theta = \frac{Q}{mc}$] and hence its expansion will be more [as $\Delta V = V\gamma\Delta\theta$].

CALORIMETRY

HEAT

When a hot body is put in contact with a cold one, the former gets colder and the latter warmer. From this observation it is natural to conclude that a certain quantity of heat has passed from the hot body to the cold one. Heat is a form of energy.

Heat is felt by its effects. Some of the effects of heat are :

(a) Change in the degree of hotness (b) Expansion in length, surface area and volume

(c) Change in state of a substance (d) Change in the resistance of a conductor

(e) Thermo e.m.f. effect

SI UNIT : J (joule) Also measured in the unit calorie.

- **Calorie**

  It is defined as the amount of heat required to raise the temperature of 1 g water by 1°C.

- **International calorie**

  International calorie is the amount of heat required to raise the temperature of 1g water from 14.5 °C to 15.5 °C rise of temperature.

MECHANICAL EQUIVALENT OF HEAT

According to Joule, work may be converted into heat and vice–versa. The ratio of work done to heat produced is always constant. $\frac{W}{H}$ = constant (J) $\Rightarrow W = J\,H$

W must be in joule, irrespective of nature of energy or work and H must be in calorie.

J is called mechanical equivalent of heat. It is not a physical quantity but simply a conversion factor. It converts unit of work into that of heat and vice–versa.

J = 4.18 joule/cal or 4.18 × 10^3 joule per kilo–cal. For rough calculations we take J = 4.2 joule/cal

SPECIFIC HEAT (s or c)

It is the amount of energy required to raise the temperature of unit mass of that substance by 1°C (or 1K) is called specific heat. It is represented by s or c.

If the temperature of a substance of mass m changes from T to T + dT when it exchanges an amount of heat dQ with its surroundings then its specific heat is $c = \frac{1}{m}\frac{dQ}{dT}$

The specific heat depends on the pressure, volume and temperature of the substance.

For liquids and solids, specific heat measurements are most often made at a constant pressure as functions of temperature, because constant pressure is quite easy to produce experimentally.

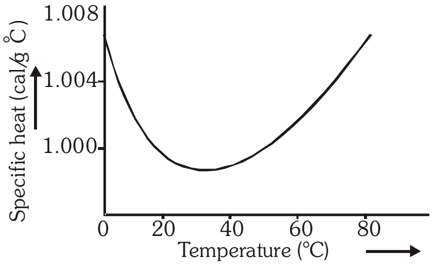
SI UNIT : joule/kg–K **CGS UNIT :** cal/g –°C

Specific heat of water : c_{water} = 1 cal/g–°C = 1 cal/g–K = 1 kcal/kg–K = 4200 joule/kg–K

When a substance does not undergo a change of state (i.e., liquid remains liquid or solid remains solid), then the amount of heat required to raise the temperature of mass m of thesubstance by an amount $\Delta\theta$ is $Q = ms\Delta\theta$.

The temperature dependence of the specific heat of water at 1 atmospheric pressure is shown in figure. Its variation is less than 1% over the interval from 0 to 100°C. Such a small variation is typical for most solids and liquids, so their specific heats can generally be taken to be constant over fairly large temperature ranges.

The temperature dependence of the specific heat of water at 1 atm



- There are many processes possible to give heat to a gas.

  A specific heat can be associated to each such process which depends on the nature of process.
- Value of specific heats can vary from zero (0) to infinity.
- Generally two types of specific heat are mentioned for a gas –

  (a) Specific heat at constant volume (C_V) (b) Specific heat at constant pressure (C_P)
- These specific heats can be molar or gram.

MOLAR HEAT CAPACITY

The amount of energy needed to raise the temperature of one mole of a substance by 1°C (or 1K) is called molar heat capacity. The molar heat capacity is the product of molecular weight and specific heat i.e.,

Molar heat capacity C = Molecular weight (M) × Specific heat(c) $\Rightarrow C = \frac{1}{\mu}\left(\frac{dQ}{dT}\right)$

If the molecular mass of the substance is M and the mass of the substance is m then number of moles of the substance $\mu = \frac{m}{M} \Rightarrow C = \frac{M}{m}\left(\frac{dQ}{dT}\right)$ **SI UNIT :** J/mol–K

THERMAL CAPACITY

The quantity of heat required to raise the temperature of the whole of that substance through 1°C is called thermal capacity. The thermal capacity of mass m of the whole of substance of specific heat s is = ms

Thermal capacity = mass × specific heat

Thermal capacity depends on property of material of the body and mass of the body.

SI UNIT : cal/°C or cal/K, **Dimensions :** $ML^2T^{-2}K^{-1}$

WATER EQUIVALENT OF A BODY

As the specific heat of water is unity so the thermal capacity of a body (ms) represents its water equivalent also.

- Mass of water having the same thermal capacity as the body is called the water equivalent of the body
- The water equivalent of a body is the amount of water that absorbs or gives out the same amount of heat as is done by the body when heated or cooled through 1°C.

  Water equivalent= mass of body × specific heat of the material $\Rightarrow$ (w = ms).

LATENT HEAT OR HIDDEN HEAT

When state of a body changes, change of state takes place at constant temperature [melting point or boiling point] and heat released or absorbed is $Q = mL$ where L is latent heat. Heat is absorbed if solid converts into liquid (at melting point) or liquid converts into vapours (at boiling point) and heat is released if liquid converts into solid or vapours converts into liquid.

- **Latent heat of fusion**

  It is the quantity of heat (in kilocalories) required to change its 1 kg mass from solid to liquid state at its melting point. Latent heat of fusion for ice : 80 kcal/kg = 80 cal /g.

- **Latent heat of vaporization**

  The quantity of heat required to change its 1 kg mass from liquid to vapour state at its boiling point.

  Latent heat of vaporisation for water : 536 kcal/kg = 536 cal/g

CHANGE OF STATE

- **Melting**

  Conversion of solid into liquid state at constant temperature is known as melting.

- **Boiling**

  Evaporation within the whole mass of the liquid is called boiling. Boiling takes place at a constant temperature known as boiling point. A liquid boils when the saturated vapour pressure on its surface is equal to atmospheric pressure. Boiling point reduces on decreasing pressure.

- **Evaporation**

  Conversion of liquid into vapours at all temperatures is called evaporation. It is a surface phenomenon. Greater the temperature, faster is the evaporation. Smaller the boiling point of liquid, more rapid is the evaporation. Smaller the humidity, more is the evaporation. Evaporation increases on decreasing pressure that is why evaporation is faster in vacuum.

  - **Heat of evaporation**

    Heat required to change unit mass of liquid into vapour at a given temperature is called heat of evaporation at that temperature.

- **Sublimation**

  Direct conversion of solid in to vapour state is called sublimation.

  - **Heat of sublimation**

    Heat required to change unit mass of solid directly into vapours at a given temperature is called heat of sublimation at that temperature.

  - Camphor and ammonium chloride sublimates on heating in normal conditions.
  - A block of ice sublimates into vapours on the surface of moon because of very–very low pressure on its surface

- **Condensation**

  The process of conversion from gaseous or vapour state to liquid state is known as condensation .

  These materials again get converted to vapour or gaseous state on heating.

PHASE OF A SUBSTANCE

The phase of a substance is defined as its form which is homogeneous, physically distinct and mechanically separable from the other forms of that substance.

Phase diagram

- A phase diagram is a graph in which pressure (P) is represented along the y–axis and temperature (T) is represented along the x–axis.
- **Characteristics of Phase diagram**

(i) Different phases of a substances can be shown on a phase diagram.

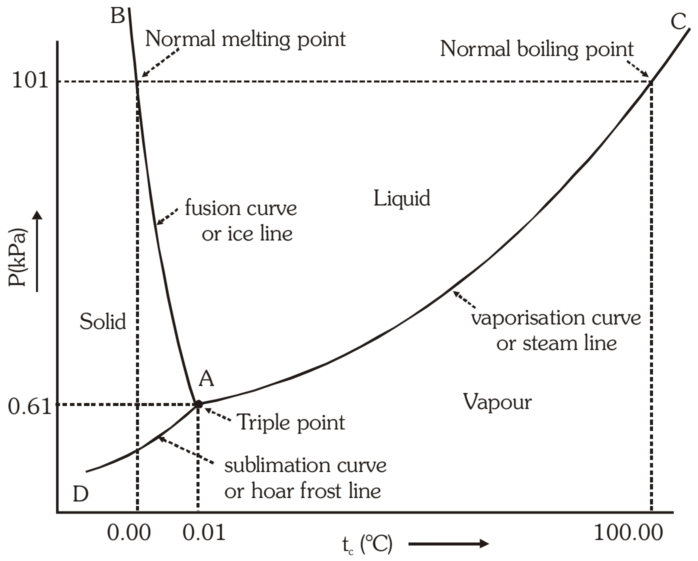
(ii) A region on the phase diagram represents a single phase of the substance, a curve represents equilibrium between two phases and a common point represents equilibrium between three phases.

(iii) A phase diagram helps to determine the condition under which the different phases are in equilibrium.

(iv) A phase diagram is useful for finding a convenient way in which a desired change of phase can be produced.

PHASE DIAGRAM FOR WATER

The phase diagram for water consists of three curves AB, AC and AD meeting each other at the point A, these curves divide the phase diagram into three regions.



Phase diagram for water
(Scale are not linear)

At the triple point solid, liquid and vapour phases are all in equilibrium

Region to the left of the curve AB and above the curve AD represents the solid phase of water (ice). The region to the right of the curve AB and above the curve AC represents the liquid phase of water. The region below the curves AC and AD represents the gaseous phase of water (i.e. water vapour). A curve on the phase diagram represents the boundary between two phases of the substance.

Along any curve the two phases can coexist in equilibrium

- Along curve AB, ice and water can remain in equilibrium.This curve is called fusion curve or ice line. This curve shows that the melting point of ice decreases with increase in pressure.
- Along the curve AC, water and water vapour can remain in equilibrium. The curve is called vaporisation curve or steam line. *The curve shows that the boiling point of water increases with increase in pressure.*
- Along the curve AD, ice and water vapour can remain in equilibrium.
  This curve is called sublimation curve or hoar frost line.

TRIPLE POINT OF WATER

The three curves in the phase diagram of water meet at a single point A, which is called the triple point of water. The triple point of water represents the co–existance of all the three phases of water ice water and water vapour in equilibrium. The pressure corresponding to triple point of water is 6.03×10^{-3} atmosphere or 4.58 mm of Hg and temperature corresponding to it is 273.16K.

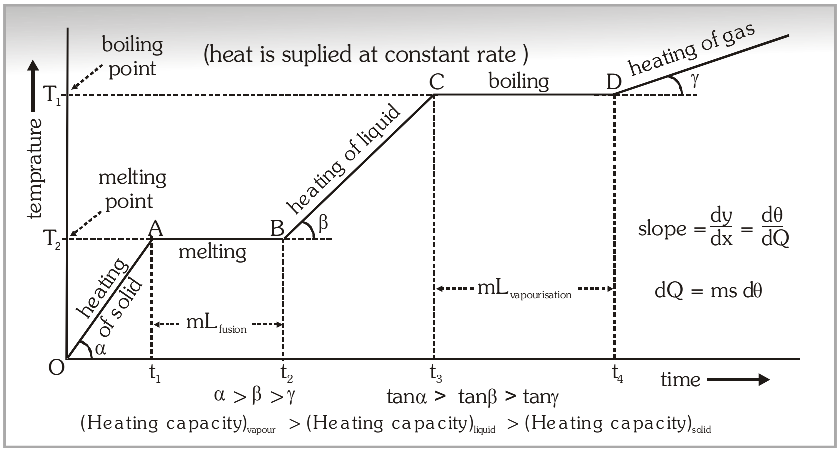
- **Significance of triple point of water**

  Triple point of water represents a unique condition and it is used to define the absolute temperature. While making Kelvin's absolute scale, upper fixed point is 273.16 K and lower fixed point is 0 K.

  One kelvin of temperature is fraction $\frac{1}{273.16}$ of the temperature of triple point of water.

HEATING CURVE

If to a given mass (m) of a solid, heat is supplied at constant rate and a graph is plotted between temperature and time as shown in figure is called heating curve.



- **In the region OA**

  Rate of heat supply P is constant and temperature of solid is changing with time

  So, $Q = mc_S\Delta T \Rightarrow P\,\Delta t = mc_S\,\Delta T$ $[\because Q = P\,\Delta t]$ $\because \frac{\Delta T}{\Delta t} =$ The slope of temperature–time curve so

  specific heat of solid $c_S \propto \frac{1}{\text{slope of line OA}}$ specific heat (or thermal capacity) is inversely proportional

  to the slope of temperature–time curve.

- **In the region AB**

  Temperature is constant, so it represents change of state, i.e., melting of solid with melting point T_1. At point A melting starts and at point B all solid is converted into liquid. So between A and B substance is partly solid and partly liquid. If L_F is the latent heat of fusion then

  $$Q = mL_F \Rightarrow L_F = \frac{P(t_2 - t_1)}{m} \quad [\text{as } Q = P(t_2 - t_1)] \Rightarrow L_F \propto \text{length of line AB}$$

  i.e., Latent heat of fusion is proportional to the length of line of zero slope.

  [In this region specific heat $\propto \frac{1}{\tan 0^\circ} = \infty$]

- **In the region BC**

  Temperature of liquid increases so specific heat (or thermal capacity) of liquid will be inversely proportional to the slope of line BC, $c_L \propto \frac{1}{\text{slope of line BC}}$

- **In the region CD**

  Temperature in constant, so it represents change of state, i.e., liquid is boiling with boiling point T_2. At C all substance is in liquid state while at D is vapour state and between C and D partly liquid and partly gas. The length of line CD is proportional to latent heat of vaporisation, i.e., $L_V \propto$ Length of line CD.

  [In this region specific heat $\propto \frac{1}{\tan 0^\circ} = \infty$]

  The line DE represents gaseous state of substance with its temperature increasing linearly with time. The reciprocal of slope of line will be proportional to specific heat or thermal capacity of substance in vapour state.

LAW OF MIXTURES

- When two bodies (one being solid and other liquid or both being liquid) at different temperatures are mixed, heat will be transferred from body at higher temperature to a body at lower temperature till both acquire same temperature. The body at higher temperature released heat while body at lower temperature absorbs it, so that Heat lost = Heat gained. Principle of calorimetry represents the law of conservation of heat energy.
- Temperature of mixture (T) is always ≥ lower temperature (T_L) and ≤ higher temperature (T_H), $T_L \le T \le T_H$

  The temperature of mixture can never be lesser than lower temperature (as a body cannot be cooled below the temperature of cooling body) and greater than higher temperature (as a body cannot be heated above the temperature of heating body). Further more usually rise in temperature of one body is not equal to the fall temperature of the other body though heat gained by one body is equal to the heat lost by the other.

Ex. 5g ice at 0°C is mixed with 5g of steam at 100°C . What is the final temperature?

Sol. Heat required by ice to raise its temperature to 100°C,

$Q_1 = m_1L_1 + m_1c_1\Delta\theta_1 = 5 \times 80 + 5 \times 1 \times 100 = 400 + 500 + 900 = 1800$ cal

Heat given by steam when condensed $Q_2 = m_2L_2 = 5 \times 536 = 2680$ cal

As $Q_2 > Q_1$. This means that whole steam is not even condensed.

Hence temperature of mixture will remain at 100°C.

Ex. A calorimeter of heat capacity 100 J/K is at room temperature of 30°C. 100 g of water at 40°C of specific heat 4200 J/kg–K is poured into the calorimeter. What is the temperature of water in calorimeter?

Sol. Let the temperature of water in calorimeter is t. Then heat lost by water = heat gained by calorimeter

$(0.1) \times 4200 \times (40 - t) = 100\,(t - 30) \Rightarrow 42 \times 40 - 42t = 10t - 300 \Rightarrow t = 38.07°C$

Ex. Find the quantity of heat required to convert 40 g of ice at –20°C into water at 20°C.

Given $L_{ice} = 0.336 \times 10^6$ J/kg.

Specific heat of ice = 2100 J/kg–K, specific heat of water = 4200 J/kg–K

Sol. Heat required to raise the temperature of ice from –20°C to 0°C = $0.04 \times 2100 \times 20 = 1680$ J

Heat required to convert the ice into water at 0°C = $mL = 0.04 \times 0.336 \times 10^6 = 13440$ J

Heat required to heat water from 0°C to 20°C = $0.04 \times 4200 \times 20 = 3360$ J

Total heat required = $1680 + 13440 + 3360 = 18480$ J

Ex. Steam at 100°C is passed into 1.1 kg of water contained in a calorimeter of water equivalent 0.02 kg at 15°C till the temperature of the calorimeter and its contents rises to 80°C. What is the mass of steam condensed? Latent heat of steam = 536 cal/g.

Sol. Heat required by (calorimeter + water)

$$Q = (m_1c_1 + m_2c_2)\,\Delta\theta = (0.02 + 1.1 \times 1)\,(80 - 15) = 72.8 \text{ kcal}$$

If m is mass of steam condensed, then heat given by steam

$$Q = mL + mc\,\Delta\theta = m \times 536 + m \times 1 \times (100 - 80) = 556\,m \quad \therefore\ 556\,m = 72.8$$

$\therefore$ Mass of steam condensed $m = \dfrac{72.8}{556} = 0.130$ kg

MODE OF HEAT TRANSFER

Heat is a form of energy which transfers from a body at higher temperature to a body at lower temperature. The transfer of heat from one body to another may take place by any one of the following modes :

- **Conduction**

  The process in which the material takes an active part by molecular action and energy is passed from one particle to another is called conduction. It is predominant in solids.

- **Convection**

  The transfer of energy by actual motion of particle of medium from one place to another is called convection. It is predominant is fluids (liquids and gases).

- **Radiation**

  Quickest way of transmission of heat is known as radiation. In this mode of energy transmission, heat is transferred from one place to another without effecting the inter–venning medium.

Conduction	Convection	Radiation
Heat Transfer due to Temperature difference	Heat transfer due to density difference	Heat transfer with out any medium
Due to free electron or vibration motion of molecules	Actual motion of particles	Electromagnetic radiation
Heat transfer in solid body (in mercury also)	Heat transfer in fluids (Liquid + gas)	All
Slow process	Slow process	Fast process (3 × 10^8 m/sec)
Irregular path	Irregular path	Straight line (like light)

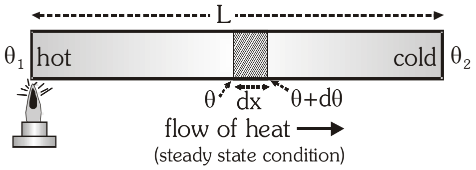
THERMAL CONDUCTION

The process by which heat is transferred from hot part to cold part of a body through the transfer of energy from one particle to another particle of the body without the actual movement of the particles from their equilibrium positions is called conduction. The process of conduction only in solid body (except Hg) Heat transfer by conduction from one part of body to another continues till their temperatures become equal.

Steady state : When temperature of the each cross–section of the bar becomes constant through different for different cross–sections is called steady state.

- **Equation of thermal conduction**

Rate of heat flow $\frac{dQ}{dt} = -KA\frac{d\theta}{dx}$ $\frac{dQ}{dt} = \frac{KA}{L}(\theta_1 - \theta_2)$



Cross section area = A ; Length = L

Thermal conductivity of material = K

- **Thermal (temperature) gradient**

The decrease in temperature with distance from hot end of the rod is known as temperature gradient or in the direction of heat energy flow, the rate of fall in temperature w.r.t. distance is called as temperature gradient. It is denoted by –dT/dx

- **Thermal conductivity (K) :** It's depends on nature of material.
  - **SI UNIT :** J s^{-1} m^{-1} K^{-1} **Dimensions :** $M^1L^1T^{-3}\theta^{-1}$ K [For Ag maximum is (410 W/mK) / For Freon minimum is 12 (0.008 W/mK)
  - For an ideal or perfect conductor of heat the value of $K = \infty$
  - For an ideal or perfect bad conductor or insulator the value of K = 0
  - For cooking the food, low specific heat and high conductivity utensils are most suitable.

APPLICATION OF THERMAL CONDUCTION

- In winter, the iron chairs appear to be colder than the wooden chairs.
- Cooking utensils are made of aluminium and brass whereas their handles are made of wood.
- Ice is covered in gunny bags to prevent melting of ice.
- We feel warm in woollen clothes and fur coat.
- Two thin blankets are warmer than a single blanket of double the thickness.
- Birds often swell their feathers in winter.
- A new quilt is warmer than old one.

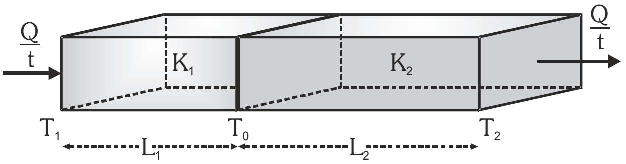
THERMAL RESISTANCE (R)

The thermal resistance of a body is a measure of its opposition of the flow of heat through it. $R = \frac{L}{KA}$

- **Heat flow through slabs in series**

$$R_{eq} = R_1 + R_2$$

$$\frac{L_1+L_2}{K_{eq}A} = \frac{L_1}{K_1A} + \frac{L_2}{K_2A}$$



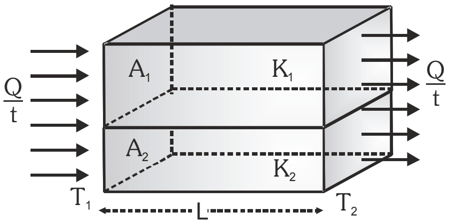
Equivalent thermal conductivity of the system is

$$K_{eq} = \frac{L_1+L_2}{\frac{L_1}{K_1}+\frac{L_2}{K_2}} = \frac{\Sigma L_i}{\Sigma\frac{L_i}{K_i}}$$

equivalent to

- **Heat flow through slabs in parallel**

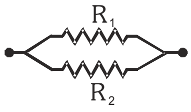
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2},\ R = \frac{L}{KA};\ \frac{K_{eq}}{L}(A_1 + A_2) = \frac{K_1A_1}{L} + \frac{K_2A_2}{L}$$



Equivalent thermal conductivity

$$K_{eq} = \frac{K_1A_1+K_2A_2}{A_1+A_2} = \frac{\Sigma K_iA_i}{\Sigma A_i}$$

equivaleent to



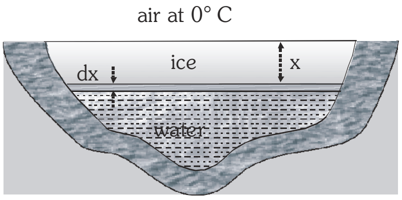
GROWTH OF ICE ON LAKES

In winter atmospheric temperature falls below 0°C and water in the lake start freezing.

Let at time t thickness of ice on the surface of the lake = x and air temperature = $-\theta°$ C

The temperature of water in contact with the lower surface of ice = 0°C

Let area of the lake = A



Heat escaping through ice in time dt is $dQ = KA\frac{[0-(-\theta)]}{x}dt$

Due to escape of this heat increasing extra thickness of ice = dx

Mass of this extra thickness of ice is m = ρV = ρ A.dx

dQ = mL = (ρ A.dx) L

$\therefore \quad KA\frac{\theta}{x}dt = (\rho\ A.dx)\ L \Rightarrow \quad dt = \frac{\rho L}{K\theta}x\,dx$

So time taken by ice to grow a thickness x is $t = \frac{\rho L}{K\theta}\int_0^x x\,dx = \frac{1}{2}\frac{\rho L}{K\theta}x^2$

So time taken by ice to grow from thickness x_1 to thickness x_2 is

$$t = t_2 - t_1 = \frac{1}{2}\frac{\rho L}{KT}(x_2^{\,2} - x_1^{\,2}) \quad \text{and} \quad t \propto (x_2^{\,2} - x_1^{\,2})$$

Time taken to double and triple the thickness ratio $t_1 : t_2 : t_3 :: 1^2 : 2^2 : 3^2$

So $t_1 : t_2 : t_3 :: 1 : 4 : 9$

Ex. One end of a brass rod 2m long and having 1 cm radius is maintained at 250°C. When a steady state is reached, the rate of heat flow across any cross–section is 0.5 cal s^{-1}. What is the temperature of the other end K = 0.26 cal s^{-1} cm^{-1} °C^{-1}.

Sol. $\frac{Q}{t} = 0.5$ cal s^{-1}; r = 1 cm $\quad \therefore$ Area $A = \pi r^2 = 3.142 \times 1\ cm^2 = 3.142\ cm^2$

L = Length of rod = 2m = 200 cm, $T_1 = 250°C$, $T_2 = ?$

We know $\frac{Q}{t} = \frac{KA(T_1 - T_2)}{L}$ or $(T_1 - T_2) = \frac{Q}{t} \times \frac{\Delta x}{kA} = \frac{0.5 \times 200}{0.26C^{-1} \times 3.142} = 122.4°C$

$\therefore \quad T_2 = 250°C - 122.4°C = 127.6°C$

Ex. Steam at 373 K is passed through a tube of radius 10 cm and length 2 m. The thickness of the tube is 5 mm and thermal conductivity of the material is 390 W m^{-1} K^{-1}, calculate the heat lost per second. The outside temp. is 0°C.

Sol. Using the relation $Q = \frac{KA(T_1 - T_2)t}{L}$

Here, heat is lost through the cylindrical surface of the tube.

A = 2πr (radius of the tube) (length of the tube) = 2π × 0.1 × 2 = 0.4 πm^2

K = 390 W m^{-1} K^{-1}

$T_1 = 373$ K, $\quad T_2 = 0°C = 273$ K, $\quad L = 5$ mm $= 0.005$ m $\quad$ and $\quad t = 1$ s

$$\therefore Q = \frac{390 \times 0.4\pi \times (373-273) \times 1}{0.005} = \frac{390 \times 0.4\pi \times 100}{0.005} = 98 \times 10^5\ J.$$

Ex. The thermal conductivity of brick is 1.7 W m^{-1} K^{-1}, and that of cement is 2.9 W m^{-1} K^{-1}. What thickness of cement will have same insulation as the brick of thickness 20 cm.

Sol. Since $Q = \frac{KA(T_1 - T_2)t}{L}$. For same insulation by the brick and cement Q, A $(T_1 - T_2)$ and t do not change. Hence, $\frac{K}{L}$ remain constant. If K_1 and K_2 be the thermal conductivities of brick and cement respectively and L_1 and L_2 be the required thickness then $\frac{K_1}{L_1} = \frac{K_2}{L_2}$ or $\frac{1.7}{20} = \frac{2.9}{L_2}$

$\therefore L_2 = \frac{2.9}{1.7} \times 20 = 34.12$ cm

Ex. Two vessels of different material are identical in size and wall–thickness. They are filled with equal quantities of ice at 0°C. If the ice melts completely, in 10 and 25 minutes respectively then compare the coefficients of thermal conductivity of the materials of the vessels.

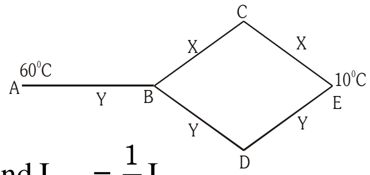
Sol. Let K_1 and K_2 be the coefficients of thermal conductivity of the materials, and t_1 and t_2 be the time in which ice melts in the two vessels. Since both the vessels are identical, so A and x in both the cases is same.

Now, $Q = \frac{K_1A(\theta_1 - \theta_2)t_1}{L} = \frac{K_2A(\theta_1 - \theta_2)t_2}{L} \Rightarrow \frac{K_1}{K_2} = \frac{t_2}{t_1} = \frac{25\text{ min}}{10\text{ min}} = \frac{5}{2}$

Ex. Three rods of material X and three rods of material Y are connected as shown in figure. All the rods are identical in length and cross–sectional area. If the end A is maintained at 60^0C and the junction E at at 10^0C, calculate the temp. of the junctions B,C,D. The thermal conductivity of X is 0.92 CGS units and that of Y is 0.46 CGS units.

Sol. $R_X \propto \frac{1}{K_X}$, $R_Y \propto \frac{1}{K_Y} \Rightarrow \frac{R_X}{R_Y} = \frac{K_Y}{K_X} = \frac{0.46}{0.92} = \frac{1}{2}$ Let $R_X = R \therefore R_Y = 2R$

The total resistance $\Sigma R = R_Y +$ effective resistance in the bridge



$\Sigma R = 2R + \frac{2R \times 4R}{2R + 4R} = 2R + \frac{4}{3}R = \frac{10}{3}R$ & $\because \Delta\theta = \ell \times R$

Further $I_{BCE}(2R) = I_{BDE}(4R)$ and $I_{BCE} + I_{BDE} = I \Rightarrow I_{BCE} = \frac{2}{3}I$ and $I_{BDE} = \frac{1}{3}I$

For A and B $\theta_A - \theta_B = 60^0 - \theta_B \Rightarrow 60 - \theta_B = 2R \times I$ ...(i)

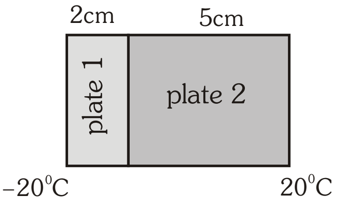
For B and C $\theta_B - \theta_C = \frac{2}{3}(I \times R)$ ....(ii) $\theta_C - \theta_E = \frac{2}{3} \times R \times I$

For A and E $\theta_A - \theta_E = 60 - 10 = 50 \Rightarrow \frac{10}{3}(R \times I) = 50$ ....(iii) $\therefore R \times I = 15$

$\therefore \theta_A - \theta_B - 2 \times 15 = 30$, $\theta_B = 60 - 30 = 30^0$C, $\theta_B - \theta_C = \left(\frac{2}{3}\right) \times 15 = 10$

$\therefore \theta_C = 30 - 10 = 20^0$C Obviously, $\theta_C = \theta_D = 20^0$C

Ex. Two plates of equal areas are placed in contact with each other. Their thickness are 2.0 cm and 5.0 cm respectively. The temperature of the external surface of the first plate is –20°C and that of the external surface of the second plate is 20°C. What will be the temperature of the contact surface if the plate (i) are of the same material, (ii) have thermal conductivities in the ratio 2 : 5.



Sol. Rate of flow of heat in the plates is $\frac{Q}{t} = \frac{K_1A(\theta_1-\theta)}{L_1} = \frac{K_2A(\theta-\theta_2)}{L_2}$ ...(i)

(i) Here $\theta_1 = -20°C$, $\theta_2 = 20°C$,
$L_1 = 2$ cm $= 0.02$ m, $L_2 = 5$ cm $= 0.05$ m and $K_1 = K_2 = K$

$\therefore$ equation (i) becomes $\frac{KA(-20-\theta)}{0.02} = \frac{KA(\theta-20)}{0.05}$

$\therefore 5(-20-\theta) = 2(\theta-20) \Rightarrow -100-5\theta = 2\theta-40 \Rightarrow 7\theta = -60 \Rightarrow \theta = -8.6°C$

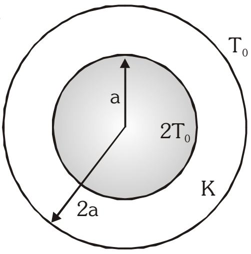
(ii) $\frac{K_1}{K_2} = \frac{2}{5}$ or $K_1 = \frac{2}{5}K_2$

$\therefore$ from equation (i) $\frac{2/5K_2A(-20-\theta)}{0.02} = \frac{K_2A(\theta-20)}{0.05}$ $-20-\theta = \theta-20$ or $-2\theta = 0 \therefore \theta = 0°C$

Ex. An ice box used for keeping eatables cold has a total wall area of 1 metre^2 and a wall thickness of 5.0 cm. The thermal conductivity of the ice box is K = 0.01 joule/metre–°C. It is filled with ice at 0°C along with eatables on a day when the temperature is 30°C. The latent heat of fusion of ice is 334 × 10^3 jule/kg. Calculate the amount of ice melted in one day.

Sol. $\frac{dQ}{dt} = \frac{KA}{L}d\theta = \frac{0.01\times1}{0.05}\times30 = 6$ joule/s So $\frac{dQ}{dt}\times86400 = 6\times86400$

Q = mL (L – latent heat), $m = \frac{Q}{L} = \frac{6\times86400}{334\times10^3} = 1.552$ kg



Ex. A hollow spherical ball of inner radius a and outer radius 2a is made of a uniform material of constant thermal conductivity K. The temperature within the ball is maintained at $2T_0$ and outside the ball it is T_0. Find, (a) the rate at which heat flows out of the ball in the steady state, (b) the temperature at r = 3a/2, where r is radial distance from the centre of shell. Assume steady state condition.

Sol. In the steady state, the net outward thermal current is constant, and does not depend on the radial position.

Thermal current, $C_1 = \left(\frac{dQ}{dt}\right) = -K.(4\pi r^2)\frac{dT}{dr} \Rightarrow \frac{dT}{dr} = -\frac{C_1}{4\pi K}\frac{1}{r^2} + C_2$

At r=a, T = $2T_0$ and r =2a, T=T_0 $\Rightarrow T = \frac{2a}{r}T_0$ (a) $\frac{dQ}{dt} = 8\pi aKT_0$ (b) $T(r = 3a/2) = 4T_0/3$

Thermal Radiation

The process of the transfer of heat from one place to another place without heating the intervening medium is called radiation. When a body is heated and placed in vacuum, it loses heat even when there is no medium surrounding it. The heat can not go out from the body by the process of conduction or convection since both of these process require the presence of a material medium between source and surrounding objects. The process by which heat is lost in this case is called radiation. This does not require the presence of any material medium.

It is by radiation that the heat from the Sun reaches the Earth. Radiation has the following properties:

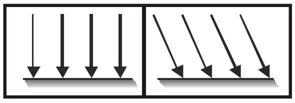
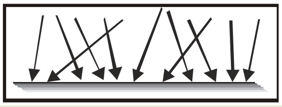
(a) Radiant energy travels in straight lines and when some object is placed in the path, it's shadow is formed at the direction.

(b) It can travel through vacuum.

(c) Intensity of radiation follows the law of inverse square.

All these and many other properties establish that heat radiation has nearly all the properties possessed by light and these are also electromagnetic waves with the only difference of wavelength or frequency. The wavelength of heat radiation is larger than that of visible light.

Types of thermal Radiation :– Two types of thermal radiation.

Plane Radiation	**Diffuse Radiation**
	
Radiations which are incident on a surface at certain angle	Incident on the surface at all angles

- When radiation passes through any medium then radiations slightly absorbed by medium according to its absorptive power so temperature of medium slightly increases.
- Heat radiation are always obtained in infra–red region of electromagnetic wave spectrum so they are called Infra red rays.

BASIC FUNDAMENTAL DEFINITIONS

- **Energy Density (u)**

  The radiation energy of whole wavelength (0 to ∞) present in unit volume at any point in space is defined as energy density. **S I UNIT :** J/m^3

- **Spectral energy density (u_λ) :** Energy density per unit spectral region. $u = \int_0^\infty u_\lambda d\lambda$

  SI UNIT : J/m^3 Å

- **Absorptive power or absorptive coefficient 'a' :** The ratio of amount of radiation absorbed by a surface (Q_a) to the amount of radiation incident (Q) upon it is defined as the coefficient of absorption

  $a = \frac{Q_a}{Q}$ .It is unitless

- **Spectral absorptive power (a_λ)** $a_\lambda = \frac{Qa_\lambda}{Q_\lambda}$: Also called monochromatic absorptive coefficient

  At a given wavelength $a = \int_0^\infty a_\lambda d\lambda$. For ideal black body a_λ and $a = 1$, a and a_λ are unitless

- **Emissive power (e) :** The amount of heat radiation emitted by unit area of the surface in one second at a particular temperature. **SI UNIT :** J/m^{2}s

- **Spectral Emmisive power (e_λ) :** The amount of heat radiation emitted by unit area of the body in one second in unit spectral region at a given wavelength. Emissive power or total emissive power $e = \int_0^\infty e_\lambda \, d\lambda$

  SI UNIT : W/m^2 Å

EMISSIVITY (e)

- **Absolute emissivity or emissivity :** Radiation energy given out by a unit surface area of a body in unit time corresponding to unit temperature difference w.r.t. the surroundings is called Emissivity.

  S I UNIT : W/m^2 °K

- **Relative emissivity (e_r) :** $e_r = \dfrac{Q_{GB}}{Q_{IBB}} = \dfrac{e_{GB}}{E_{IBB}} = \dfrac{\text{emitted radiation by gray body}}{\text{emitted radiation by ideal black body}}$

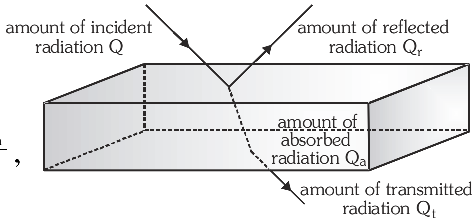
  GB = gray or general body, IBB = Ideal black body

  (i) No unit (ii) For ideal black body $e_r = 1$ (iii) range $0 < e_r < 1$

SPECTRAL, EMISSIVE, ABSORPTIVE AND TRANSMITTIVE POWER OF A GIVEN BODY SURFACE

Due to incident radiations on the surface of a body following phenomena occur by which the radiation is divided into three parts. (a) Reflection (b) Absorption (c) Transmission

- **From energy conservation**



$Q = Q_r + Q_a + Q_t \quad \Rightarrow \dfrac{Q_r}{Q} + \dfrac{Q_a}{Q} + \dfrac{Q_t}{Q} = 1 \Rightarrow \; r + a + t = 1$

Reflective Coefficient $r = \dfrac{Q_r}{Q}$, Absorptive Coefficient $a = \dfrac{Q_a}{Q}$,

Transmittive Coefficient $t = \dfrac{Q_t}{Q}$

$r = 1$ and $a = 0$, $t = 0$ ⇒ Perfect reflector

$a = 1$ and $r = 0$, $t = 0$ ⇒ Ideal absorber (ideal black body)

$t = 1$ and $a = 0$, $r = 0$ ⇒ Perfect transmitter

Reflection power (r) = $\left[\dfrac{Q_r}{Q} \times 100\right]\%$, Absorption power (a) = $\left[\dfrac{Q_a}{Q} \times 100\right]\%$

Transmission power (t) = $\left[\dfrac{Q_t}{Q} \times 100\right]\%$

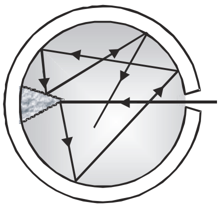
Ex. Total radiations incident on body = 400 J, 20% radiation reflected and 120 J absorbs. Then find out % of transmittive power

Sol. $Q = Q_t + Q_r + Q_a \Rightarrow 400 = 80 + 120 + Q_t \;\Rightarrow\; Q_t = 200$. So % of transmittive power is 50%

IDEAL BLACK BODY

- For a body surface which absorbs all incident thermal radiations at low temperature irrespective of their wave length and emitted out all these absorbed radiations at high temperature assumed to be an ideal black body surface.
- The identical parameters of an ideal black body is given by

  $a = a_\lambda = 1$ and $r = 0 = t$, $e_r = 1$
- The nature of emitted radiations from surface of ideal black body only depends on its temperature
- The radiations emitted from surface of ideal black body called as either full or white radiations.
- At low temperature surface of ideal black body is a perfect absorber and at a high temperature it proves to be a good emitter.
- An ideal black body need not be black colour (eg. Sun)



Ferry's ideal black body

PREVOST'S THEORY OF HEAT ENERGY EXCHANGE

According to Prevost at every possible temperature (Not absolute temperature) there is a continuous heat energy exchange between a body and its surrounding and this exchange carry on for infinite time.

The relation between temperature difference of body with its surrounding decides whether the body experience cooling effect or heating effect.

When a cold body is placed in the hot surrounding : The body radiates less energy and absorbs more energy from the surrounding, therefore the temperature of body increases.

When a hot body placed in cooler surrounding : The body radiates more energy and absorb less energy from the surroundings. Therefore temperature of body decreases.

When the temperature of a body is equal to the temperature of the surrounding

The energy radiated per unit time by the body is equal to the energy absorbed per unit time by the body, therefore its temperature remains constant.

- At absolute zero temperature (0 kelvin) all atoms of a given substance remains in ground state, so, at this temperature emission of radiation from any substance is impossible, so Prevost's heat energy exchange theory does not applied at this temperature, so it is called **limited temperature** of prevosts theory.
- With the help of Prevost's theory rate of cooling of any body w.r.t. its surroundings can be worked out (applied to Stefen Boltzman law, Newton's law of cooling.)

KIRCHHOFF'S LAW

At a given temperature for all bodies the ratio of their spectral emissive power (e_λ) to spectral absorptive power (a_λ) is constant and this constant is equal to spectral emissive power (E_λ) of the ideal black body at same temperature

$$\frac{e_\lambda}{a_\lambda} = E_\lambda = \text{constant} \qquad \left[\frac{e_\lambda}{a_\lambda}\right]_1 = \left[\frac{e_\lambda}{a_\lambda}\right]_2 = \text{constant} \quad e_\lambda \propto a_\lambda$$

Good absorbers are good emitters and bad absorbers are bad emitters

- For a constant temperature the spectral emmisive power of an ideal black body is a constant parameter
- The practical confirmation of Kirchhoff's law carried out by Rishi apparatus and the main base of this apparatus is a Lessilie container.
- The main conclusion predicted from Kirchhof's law can be expressed as

Good absorber	$\rightleftharpoons$	Good emitter
Bad absorber	$\rightleftharpoons$	Bad emitter
(at Low temperature)		(at high temperature)

APPLICATIONS OF KIRCHOFF LAW

- **In deserts days are hot and nights cold**

Sand is rough and black, so it is a good absorber and hence in deserts, days (When radiation from Sun is incident on sand) will be very hot. Now in accordance with Kirchhoff's Law, good absorber is a good emitter.

So nights (when send emits radiation) will be cold.

STEFAN'S LAW

The amount of radiation emitted per second per unit area by a black body is directly proportional to the fourth power of its absolute temperature.

Amount of radiation emitted $E \propto T^4$ where T = temperature of ideal black body (in K)

$E = \sigma T^4$ This law is true for only ideal black body

SI Unit : $E = \text{watt/m}^2$ σ = Stefen's constant = 5.67×10^{-8} watt /m^2 K^4

Dimensions of σ : $M^1 L^0 T^{-3} \theta^{-4}$

Total radiation energy emitted out by surface of area A in time t : If T is constant.

Ideal black body $Q_{IBB} = \sigma A T^4 t$ and for any other body $Q_{GB} = e_r \sigma A T^4 t$

Rate of emission of radiation

When Temperature of surrounding T_0 (Let $T_0 < T$)

Rate of emission of radiation from ideal black body surface $E_1 = \sigma T^4$

Rate of absorption of radiation from surrounding $E_2 = \sigma T_0^4$

Net rate of loss of radiation from ideal black body surface is $E = E_1 - E_2 = \sigma T^4 - \sigma T_0^4 = \sigma(T^4 - T_0^4)$

Net loss of radiation energy from entire surface area in time t is $Q_{IBB} = \sigma A(T^4 - T_0^4)t$

For any other body $Q_{GB} = e_r A\sigma(T^4 - T_0^4)t$

If in time dt the net heat energy loss for ideal black body is dQ and because of this its temperature falls by $d\theta$

Rate of loss of heat $R_H = \dfrac{dQ}{dt} = \sigma A(T^4 - T_0^4)$

It is also equal to emitted power or radiation emitted per second

Rate of fall in temperature (Rate of cooling) $R_F = \dfrac{d\theta}{dt} = \dfrac{\sigma A}{msJ}(T^4 - T_0^4)\left[\because \dfrac{dQ}{dt} = msJ\dfrac{d\theta}{dt}\right]$

Note :

(i) If all of T, T_0, m, s, V, ρ, are same for different shape body then R_F and R_H will be maximum in the flat surface.

(ii) If a solid and hollow sphere are taken with all the parameters same then hollow will cool down at fast rate.

(iii) Rate of temperature fall , $R_F \propto \dfrac{1}{s} \propto \dfrac{d\theta}{dt}$ so $dt \propto s$. If condition in specific heat is $\Rightarrow s_1 > s_2 > s_3$

If all cooled same temperature i.e. temperature fall is also identical for all then required time $t \propto s \therefore t_1 > t_2 > t_3$

- **When a body cools by radiation the cooling depends on :**

(i) Nature of radiating surface : greater the emissivity (e_r), faster will be the cooling.

(ii) Area of radiating surface : greater the area of radiating surface, faster will be the cooling.

(iii) Mass of radiating body : greater the mass of radiating body slower will be the cooling.

(iv) Specific heat of radiating body : greater the specific heat of radiating body slower will be the cooling.

(v) Temperature of radiating body : greater the temperature of radiating body faster will be the cooling.

Ex. The operating temperature of a tungesten filament in an incandescent lamp is 2000 K and its emissivity is 0.3. Find the surface area of the filament of a 25 watt lamp. Stefan's constant $\sigma = 5.67 \times 10^{-8}\ \mathrm{Wm^{-2}K^{-4}}$

Sol. $\because$ Rate of emission = wattage of the lamp

$$\therefore\ W = Ae\sigma T^4 \Rightarrow A = \frac{W}{e\sigma T^4} = \frac{25}{0.3 \times 5.67 \times 10^{-8} \times (200)^4} = 0.918\ \mathrm{m^2}$$

NEWTON'S LAW OF COOLING

Rate of loss of heat $\left(\frac{dQ}{dt}\right)$ is directly proportional to excess of temperature of the body over that of surrounding. [(when $(\theta - \theta_0) \not> 35°C$] $\frac{dQ}{dt} \propto (\theta - \theta_0)$ $\Rightarrow \frac{dQ}{dt} = ms\frac{d\theta}{dt}$

θ = temperature of body [in °C], θ_o = temperature of surrounding, $\theta - \theta_0$ = excess of temperature ($\theta > \theta_0$)

If the temperature of body decrease $d\theta$ in time dt then rate of fall of temperature $-\frac{d\theta}{dt} \propto (\theta - \theta_0)$

Where negative sign indictates that the rate of cooling is decreasing with time.

Excess of temperature

If the temperature of body decreases from θ_1 to θ_2 and temperature of surroundings is θ_0 then average excess of temperature = $\left[\frac{\theta_1 + \theta_2}{2} - \theta_0\right]$ $\Rightarrow \left[\frac{\theta_1 - \theta_2}{t}\right] = -K\left[\frac{\theta_1 + \theta_2}{2} - \theta_0\right]$

Ex. If a liquid takes 30 seconds in cooling of 80°C to 70°C and 70 seconds in cooling 60°C to 50°C, then find the room temperature.

Sol. $\frac{\theta_1 - \theta_2}{t} = K\left(\frac{\theta_1 + \theta_2}{2} - \theta_0\right)$

In first case, $\frac{80 - 70}{30} = K\left(\frac{80 + 70}{2} - \theta_0\right)$ $\frac{1}{3} = K(75 - \theta_0)$ ...(i)

In second case , $\frac{60 - 50}{70} = K\left(\frac{60 + 50}{2} - \theta_0\right)$ $\frac{1}{7} = K(55 - \theta_0)$ ... (ii)

Equation (i) divide by equation (ii) $\frac{7}{3} = \frac{(75 - \theta_0)}{(55 - \theta_0)} \Rightarrow 385 - 7\theta_0 = 225 - 3\theta_0 \Rightarrow \theta_0 = \frac{160}{4} = 40°C$

Limitations of Newton's Law

- Temperature difference should not exceed 35° C, $(\theta - \theta_0) \not> 35°$ C
- Loss of heat should only be by radiation.
- This law is an extended form of Stefan–Boltzman's law.

For Heating, Newton's law of heating $\frac{\theta_1 - \theta_2}{t} = +H\left[\theta_0 - \frac{\theta_1 + \theta_2}{2}\right]$ H heating constant.

Derivation of Newton's law from Steafen's Boltzman law

$\frac{d\theta}{dt} = \frac{\sigma A}{msJ}(T^4 - T_0^4)$ $\left\{\begin{matrix} T - T_0 = \Delta T \\ T = T_0 + \Delta T \end{matrix}\right\}$

$\frac{d\theta}{dt} = \frac{\sigma A}{msJ}(T^4 - T_0^4)$ $T - T_0 = \Delta T$, $\Delta T <<< T_0$

$\frac{d\theta}{dt} = \frac{\sigma A}{msJ}\left[(T_0 + \Delta T)^4 - T_0^4\right]$ If $x <<< 1$ then $(1 + x)^n = 1 + nx$

$$\frac{d\theta}{dt}=\frac{\sigma A}{msJ}\left[T_0^4(1+\frac{\Delta T}{T_0})^4-T_0^4\right]=\frac{\sigma A}{msJ}T_0^4\left[(1+\frac{\Delta T}{T_0})^4-1\right]=\frac{\sigma A}{msJ}T_0^4\left[1+4\frac{\Delta T}{T_0}-1\right]$$

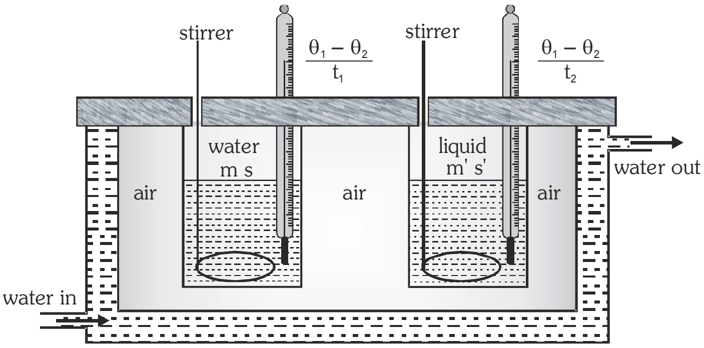
$$\frac{d\theta}{dt}=\left[4\frac{\sigma A}{msJ}T_0^3\right]\Delta T \Rightarrow \frac{d\theta}{dt}=K\Delta T \qquad \text{constant } K=\frac{4\sigma A T_0^3}{msJ}$$

Newton's law of cooling $\frac{d\theta}{dt}\propto \Delta T$ (for small temperature difference)

APPLICATION OF NEWTON'S LAW OF COOLING

- **To find out specific heat of a given liquid**

  If for the two given liquids their volume, radiating surface area, nature of surface, initial temperature are allowed to cool down in a common environments then rate of loss of heat of these liquids are equal .

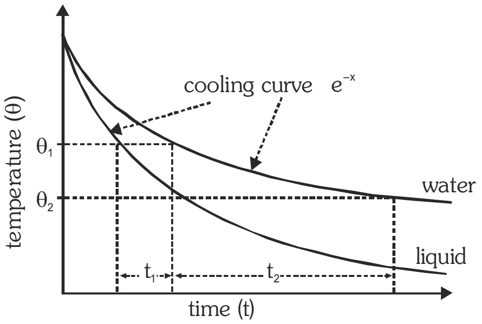


fall of temperature from $\theta_1 \rightarrow \theta_2$ in time t_1 for water and t_2 for liquid

$$\because \left[\frac{dQ}{dt}\right]_{Water}=\left[\frac{dQ}{dt}\right]_{Liquid} \quad \therefore (ms+w)\left[\frac{\theta_1-\theta_2}{t_1}\right]=(m's'+w)\left[\frac{\theta_1-\theta_2}{t_2}\right] \Rightarrow \frac{ms+w}{t_1}=\frac{m's'+w}{t_2}$$

where w = water equivalent of calorimeter.

Cooling curve :



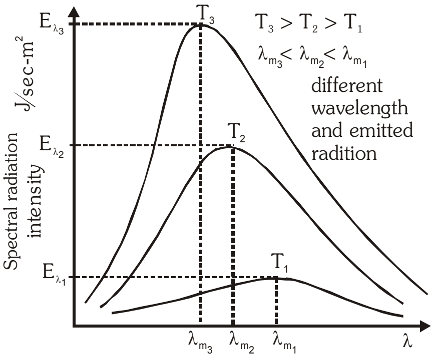
Ex. When a calorimeter contains 40g of water at 50°C, then the temperature falls to 45°C in 10 minutes. The same calorimeter contains 100g of water at 50°C, it takes 20 minutes for the temperature to become 45°C. Find the water equivalent of the calorimeter.

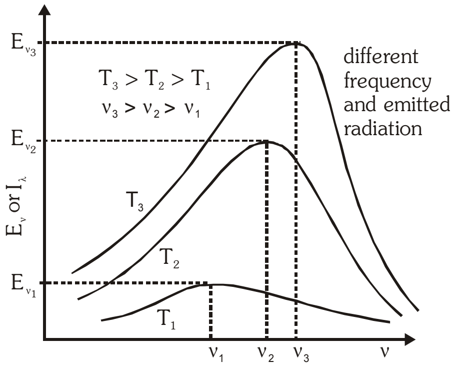
Sol. $\frac{m_1s_1+W}{t_1}=\frac{m_2s_2+W}{t_2}$ where W is the water equivalent

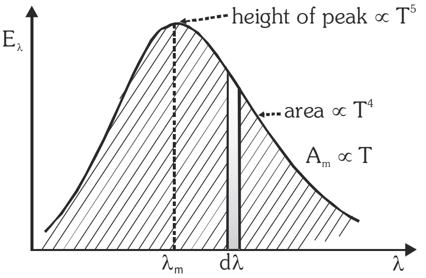
$$\Rightarrow \frac{40\times 1+W}{10}=\frac{100\times 1+W}{20} \Rightarrow 80+2W=100+W \Rightarrow W=20\text{ g}$$

SPECTRAL ENERGY DISTRIBUTION CURVE OF BLACK BODY RADIATIONS

Practically given by : Lumers and Pringshem Mathematically given by : Plank







spectral energy distribution curve (E_λ–λ)

(i) $\lambda_m \propto \frac{1}{T}$

(ii) $E_{\lambda_m} \propto T^5$

(iii) Area $\int_0^\infty E_\lambda\, d\lambda = E = \sigma T^4$ $\frac{A_1}{A_2}=\left[\frac{T_1}{T_2}\right]^4$

- Spectral energy distribution curves are continuous. At any temperature in between possible wavelength $(0-\infty)$ radiation emitted but for different wavelength quantity of radiations are different.
- As the wave length increases, the amount of radiation emitted first increase, becomes maximum and then decreases.
- At a particular temperature the area enclosed between the spectral energy curve shows the spectral emissive power of the body. Area $= \int_0^\infty E_\lambda d\lambda = E = \sigma T^4$

WEIN'S DISPLACEMENT LAW

The wavelength corresponding to maximum emission of radiation decrease with increasing temperature $\left[\lambda_m \propto \frac{1}{T}\right]$. This is known as Wein's displacement law. $\lambda_m T = b$ where b Wein's constant $= 2.89 \times 10^{-3}$ mK.

Dimensions of b : $= M^0 L^1 T^0 \theta^1$

Relation between frequency and temperature $\nu_m = \frac{c}{b}T$

Ex. The temperature of furnace is 2000°C, in its spectrum the maximum intensity is obtained at about 4000Å, If the maximum intensity is at 2000Å calculate the temperature of the furnace in °C.

Sol. by using $\lambda_m T = b$, $4000\,(2000+273) = 2000(T) \Rightarrow T = 4546\text{K}$

The temperature of furnace = 4546 – 273 = 4273 °C

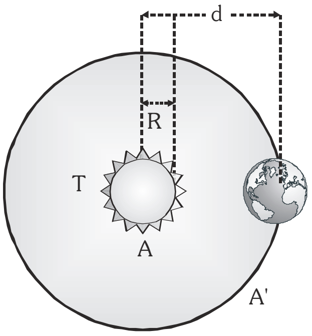
SOLAR CONSTANT 'S'

The Sun emits radiant energy continuously in space of which an in significant part reaches the Earth. The solar radiant energy received per unit area per unit time by a black surface held at right angles to the Sun's rays and placed at the mean distance of the Earth (in the absence of atmosphere) is called solar constant.

The solar constant S is taken to be 1340 watts/m^2 or 1.937 Cal/cm^2–minute

- **Temperature of the Sun**

Let R be the radius of the Sun and 'd' be the radius of Earth's orbit around the Sun. Let E be the energy emitted by the Sun per second per unit area. The total energy emitted by the Sun in one second = E.A = E × $4\pi R^2$. (This energy is falling on a sphere of radius equal to the radius of the Earth's orbit around the Sun i.e., on a sphere of surface area $4\pi d^2$)



So, The energy falling per unit area of Earth = $\dfrac{4\pi R^2 \times E}{4\pi d^2} = \dfrac{E\,R^2}{d^2}$

$R = 7\times 10^8\text{m}$, $d = 1.5\times 10^{11}\text{ m}$, $s = 5.7\times 10^{-8}\text{ W m}^{-2}\text{ K}^{-4}$

Solar constant $S = \dfrac{E\,R^2}{d^2}$

By Stefan's Law $E = \sigma T^4$

$$S = \frac{\sigma T^4\,R^2}{d^2} \Rightarrow T = \left[\frac{S\times d^2}{\sigma\times R^2}\right]^{\frac{1}{4}} = \left[\frac{1340\times(1.5\times10^{11})^2}{5.7\times10^{-8}\times(7\times10^{8})^2}\right]^{\frac{1}{4}} = 5732\text{ K}$$

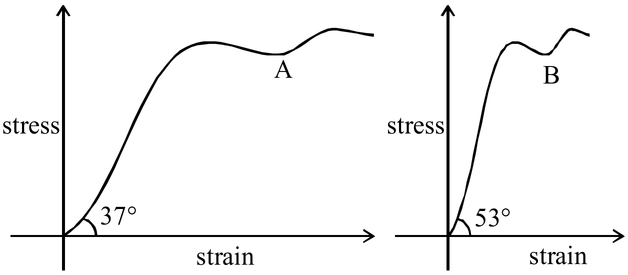
EXERCISE (S-1)

Elasticity

1. A steel wire of length 4.5 m and a copper wire of length 3.5 m are stretched same amount under a given load. If ratio of youngs modulli of steel to that of copper is $\frac{12}{7}$, then what is the ratio of cross sectional area of steel wire to copper wire ?

EH0001

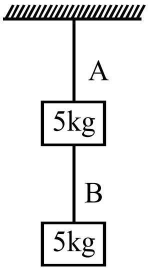
2. Diagram shows stress-strain graph for two material A & B. The graphs are drawn to scale.



The ratio of young modulli of material A to material B is.

EH0002

3. Two identical wires A & B of same material are loaded as shown in figure. If the elongation in wire B is 1.5 mm, what is the elongation in A. (Mass of A & B can be neglected)



EH0003

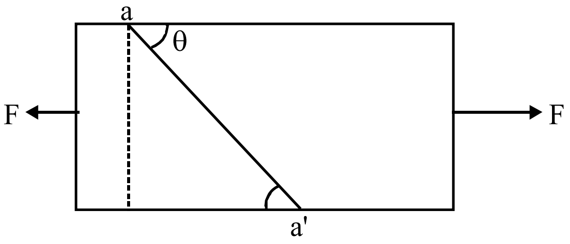
4. A wire of length L and radius r is clamped rigidly at one end. When the other end of the wire is pulled by a force f, its length increases by l. Another wire of the same material of length $2L$ and radius $2r$, is pulled by a force $2f$. Find the increase in length of this wire.

EH0004

5. Consider a long steel bar under a tensile stress due to forces $\vec{F}$ acting at the edges along the length of the bar (Fig.). Consider a plane making an angle θ with the length. What are the tensile and shearing stresses on this plane?

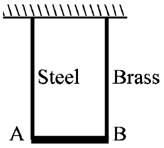
(a) For what angle is the tensile stress a maximum?

(b) For what angle is the shearing stress a maximum?



EH0005

6. A light rigid bar AB is suspended horizontally from two vertical wires, one of steel and one of brass, as shown in figure. Each wire is 2.00 m long. The diameter of the steel wire is 0.60 mm and the length of the bar AB is 0.20 m. When a mass of 10 kg is suspended from the centre of AB bar remains horizontal.



(i) What is the tension in each wire?

(ii) Calculate the extension of the steel wire and the energy stored in it.

(iii) Calculate the diameter of the brass wire.

(iv) If the brass wire were replaced by another brass wire of diameter 1 mm, where should the mass be suspended so that AB would remain horizontal? The Young modulus for steel = 2.0×10^{11} Pa, the Young modulus for brass = 1.0×10^{11} Pa.

EH0006

Calorimetry

7. One day in the morning, Ramesh filled up 1/3 bucket of hot water from geyser, to take bath. Remaining 2/3 was to be filled by cold water (at room temperature) to bring mixture to a comfortable temperature. Suddenly Ramesh had to attend to something which would take some times, say 5-10 minutes before he could take bath. Now he had two options: (i) fill the remaining bucket completely by cold water and then attend to the work, (ii) first attend to the work and fill the remaining bucket just before taking bath. Which option do you think would have kept water warmer ? Explain.

EH0007

8. An aluminium container of mass 100 gm contains 200 gm of ice at – 20°C. Heat is added to the system at the rate of 100 cal/s. Find the temperature of the system after 4 minutes (specific heat of ice = 0.5 and L = 80 cal/gm, specific heat of A*l* = 0.2 cal/gm/°C)

EH0008

9. A hot liquid contained in a container of negligible heat capacity loses temperature at rate 3 K/min, just before it begins to solidify. The temperature remains constant for 30 min. Find the ratio of specific heat capacity of liquid to specific latent heat of fusion is in K^{-1} (given that rate of losing heat is constant).

EH0009

10. Two 50 gm ice cubes are dropped into 250 gm of water into a glass. If the water was initially at a temperature of 25°C and the temperature of ice –15°C. Find the final temperature of water. (specific heat of ice = 0.5 cal/gm/°C and L = 80 cal/gm). Find final amount of water and ice.

EH0010

11. A flow calorimeter is used to measure the specific heat of a liquid. Heat is added at a known rate to a stream of the liquid as it passes through the calorimeter at a known rate. Then a measurement of the resulting temperature difference between the inflow and the outflow points of the liquid stream enables us to compute the specific heat of the liquid. A liquid of density 0.2 g/cm^3 flows through a calorimeter at the rate of 10 cm^3/s. Heat is added by means of a 250-W electric heating coil, and a temperature difference of 25°C is established in steady-state conditions between the inflow and the outflow points. Find the specific heat of the liquid.

EH0011

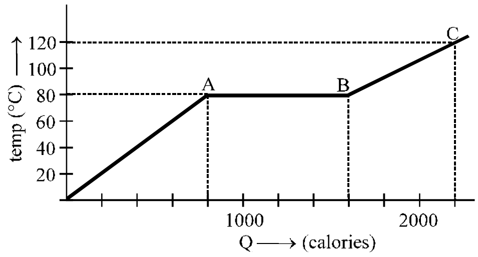
12. Two identical calorimeter A and B contain equal quantity of water at 20°C. A 5 gm piece of metal X of specific heat 0.2 cal g^{-1} (C°)$^{-1}$ is dropped into A and a 5 gm piece of metal Y into B. The equilibrium temperature in A is 22°C and in B 23°C. The initial temperature of both the metals is 40°C. Find the specific heat of metal Y in cal g^{-1} (C°)$^{-1}$.

EH0012

13. The temperature of 100 gm of water is to be raised from 24°C to 90°C by adding steam to it. Calculate the mass of the steam required for this purpose.

EH0013

14. A substance is in the solid form at 0°C. The amount of heat added to this substance and its temperature are plotted in the following graph. If the relative specific heat capacity of the solid substance is 0.5, find from the graph



(i) the mass of the substance ;
(ii) the specific latent heat of the melting process, and
(iii) the specific heat of the substance in the liquid state.

EH0014

Thermal expansion

15. If two rods of length L and 2 L having coefficients of linear expansion α and 2α respectively are connected so that total length becomes 3 L, determine the average coefficient of linear expansion of the composite rod.

EH0015

16. A clock pendulum made of invar has a period of 0.5 sec at 20°C. If the clock is used in a climate where average temperature is 30°C, approximately. How much fast or slow will the clock run in 10^6 sec. ($\alpha_{\text{invar}}=1\times10^{-6}$/°C)

EH0016

17. An iron bar (Young's modulus = 10^{11} N/m^2, $\alpha = 10^{-6}$ /°C) 1 m long and 10^{-3} m^2 in area is heated from 0°C to 100°C without being allowed to bend or expand. Find the compressive force developed inside the bar.

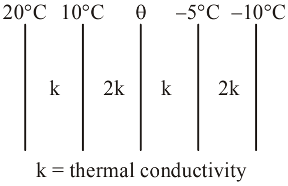
EH0017

Conduction

18. A thin walled metal tank of surface area $5m^2$ is filled with water and contains an immersion heater dissipating 1 kW. The tank is covered with 4 cm thick layer of insulation whose thermal conductivity is 0.2 W/m/K. The outer face of the insulation is 25°C. Find the temperature of the tank in the steady state

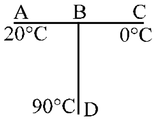
EH0018

19. The figure shows the face and interface temperature of a composite slab containing of four layers of two materials having identical thickness. Under steady state condition, find the value of temperature θ.



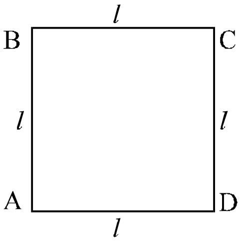
EH0019

20. Three conducting rods of same material and cross-section are shown in figure. Temperature of A, D and C are maintained at 20°C, 90°C and 0°C. Find the ratio of length BD and BC if there is no heat flow in AB.



EH0020

21. In the square frame of side l of metallic rods, the corners A and C are maintained at T_1 and T_2 respectively. The rate of heat flow from A to C is ω. If A and D are instead maintained T_1 & T_2 respectively. Find the total rate of heat flow.



EH0021

22. One end of copper rod of uniform cross-section and of length 1.5 meters is in contact with melting ice and the other end with boiling water. At what point along its length should a temperature of 200°C be maintained, so that in steady state, the mass of ice melting is equal to that of steam produced in the same interval of time? Assume that the whole system is insulated from the surroundings.

EH0022

Radiation

23. Two spheres of same radius R have their densities in the ratio 8 : 1 and the ratio of their specific heats are 1 : 4. If by radiation their rates of fall of temperature are same, then find the ratio of their rates of losing heat.

EH0023

24. A solid receives heat by radiation over its surface at the rate of 4 kW. The heat convection rate from the surface of solid to the surrounding is 5.2 kW, and heat is generated at a rate of 1.7 kW over the volume of the solid. The rate of change of the average temperature of the solid is $0.5°Cs^{-1}$. Find the heat capacity of the solid.

EH0024

25. A solid copper cube and sphere, both of same mass & emissivity are heated to same initial temperature and kept under identical conditions. What is the ratio of their initial rate of fall of temperature?

EH0025

26. A vessel containing 100 gm water at 0°C is suspended in the middle of a room. In 15 minutes the temperature of the water rises by 2°C. When an equal amount of ice is placed in the vessel, it melts in 10 hours. Calculate the specific heat of fusion of ice.

EH0026

27. The maximum in the energy distribution spectrum of the sun is at 4753 Å and its temperature is 6050K. What will be the temperature of the star whose energy distribution shows a maximum at 9506 Å.

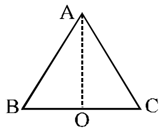
EH0027

28. A pan filled with hot food cools from 50.1 °C to 49.9 °C in 5 sec. How long will it take to cool from 40.1 °C to 39.9°C if room temperature is 30°C ?

EH0028

EXERCISE (S-2)

1. Three aluminium rods of equal length form an equilateral triangle ABC. Taking O (mid point of rod BC) as the origin. Find the increase in Y-coordinate of center of mass per unit change in temperature of the system. Assume the length of the each rod is 2m, and $\alpha_{al} = 4\sqrt{3} \times 10^{-6} / ^\circ C$



EH0029

2. A metal rod A of 25cm lengths expands by 0.050 cm, when its temperature is raised from 0°C to 100°C. Another rod B of a different metal of length 40cm expands by 0.040 cm for the same rise in temperature. A third rod C of 50cm length is made up of pieces of rods A and B placed end to end expands by 0.03 cm on heating from 0°C to 50°C. Find the lengths of each portion of the composite rod.

EH0030

3. A wire of cross-sectional area 4×10^{-4} m^2, modulus of elasticity 2×10^{11} N/m^2 and length 1 m is stretched between two vertical rigid poles. A mass of 1 kg is suspended at its middle. Calculate the angle it makes with the horizontal.

EH0031

4. A copper calorimeter of mass 100 gm contains 200 gm of a mixture of ice and water. Steam at 100°C under normal pressure is passed into the calorimeter and the temperature of the mixture is allowed to rise to 50°C. If the mass of the calorimeter and its contents is now 330 gm, what was the ratio of ice and water in the beginning? Neglect heat losses.

Given : Specific heat capacity of copper = 0.42×10^3 J kg^{-1}K^{-1},
Specific heat capacity of water = 4.2×10^3 J kg^{-1}K^{-1},
Specific heat of fusion of ice = 3.36×10^5 J kg^{-1}
Latent heat of condensation of steam = 22.5×10^5 Jkg^{-1}

EH0032

5. An isosceles triangle is formed with a rod of length l_1 and coefficient of linear expansion α_1 for the base and two thin rods each of length l_2 and coefficient of linear expansion α_2 for the two pieces, if the distance between the apex and the midpoint of the base remain unchanged as the temperatures varied show that $\frac{l_1}{l_2} = 2\sqrt{\frac{\alpha_2}{\alpha_1}}$.

EH0033

6. A steel drill making 180 rpm is used to drill a hole in a block of steel. The mass of the steel block and the drill is 180 gm. If the entire mechanical work is used up in producing heat and the rate of raise in temperature of the block and the drill is 0.5 °C/s. Find (a) the rate of working of the drill in watts, and (b) the torque required to drive the drill. Specific heat of steel = 0.1 and J = 4.2 J/cal. Use : P = τ ω

EH0034

7. Ice at -20°C is filled upto height h = 10 cm in a uniform cylindrical vessel. Water at temperature θ°C is filled in another identical vessel upto the same height h = 10 cm. Now, water from second vessel is poured into first vessel and it is found that level of upper surface falls through Δh = 0. 5 cm when thermal equilibrium is reached. Neglecting thermal capacity of vessels, change in density of water due to change in temperature and loss of heat due to radiation, calculate initial temperature θ of water. Given,

Density of water :	$\rho_w = 1$ gm cm^{-3}	Density of ice :	$\rho_i = 0.9$ gm/cm^3
Specific heat of water :	$s_w = 1$ cal/gm ^{0}C	Specific heat of ice :	$s_i = 0.5$ cal/gm^{0}C

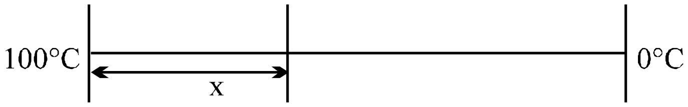
Specific latent heat of ice : L = 80 cal/gm

EH0035

8. A highly conducting solid cylinder of radius a and length *l* is surrounded by a co-axial layer of a material having thermal conductivity K and negligible heat capacity. Temperature of surrounding space (out side the layer) is T_0, which is higher than temperature of the cylinder. If heat capacity per unit volume of cylinder material is s and outer radius of the layer is b, calculate time required to increase temperature of the cylinder from T_1 to T_2. Assume end faces to be thermally insulated.

EH0036

9. A lagged stick of cross section area 1 cm^2 and length 1 m is initially at a temperature of 0°C. It is then kept between 2 reservoirs of temperature 100°C and 0°C. Specific heat capacity is 10 J/kg°C and linear mass density is 2 kg/m. Find



(a) temperature gradient along the rod in steady state.

(b) total heat absorbed by the rod to reach steady state.

EH0037

10. A cylindrical block of length 0.4 m an area of cross-section 0.04m^2 is placed coaxially on a thin metal disc of mass 0.4 kg and of the same cross-section. The upper face of the cylinder is maintained at a constant temperature of 400K and the initial temperature of the disc is 300K. If the thermal conductivity of the material of the cylinder is 10 watt/m-K and the specific heat of the material of the disc in 600 J/kg-K, how long will it take for the temperature of the disc to increase to 350K? Assume, for purposes of calculation, the thermal conductivity of the disc to be very high and the system to be thermally insulated except for the upper face of the cylinder.

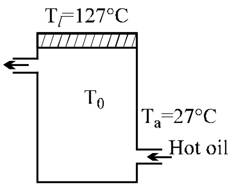
EH0038

11. A liquid takes 5 minutes to cool from 80°C to 50°C. How much time will it take to cool from 60°C to 30°C ? The temperature of surrounding is 20°C. Use exact method.

EH0039

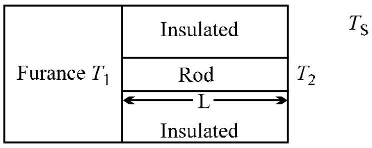
12. Hot oil is circulated through an insulated container with a wooden lid at the top whose conductivity K = 0.149 J/(m-°C-sec), thickness t = 5 mm, emissivity = 0.6. Temperature of the top of the lid in steady state is at $T_l = 127°$. If the ambient temperature $T_a = 27°C$. Calculate **[JEE' 2003]**
(a) rate of heat loss per unit area due to radiation from the lid.

(b) temperature of the oil. (Given $\sigma = \frac{17}{3} \times 10^{-8}$)



EH0040

13. One end of a rod of length L and cross-sectional area A is kept in a furnace of temperature T_1. The other end of the rod is kept at a temperature T_2. The thermal conductivity of the material of the rod is K and emissivity of the rod is e. It is given that $T_2 = T_S + \Delta T$ where $\Delta T << T_S$, T_S being the temperature of the surroundings. If $\Delta T \propto (T_1 - T_S)$, find the proportionality constant. Consider that heat is lost only by radiation at the end where the temperature of the rod is T_2. **[JEE 2004]**



EH0041

EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

Elasticity

1. The maximum load a wire can withstand without breaking, when its length is reduced to half of its original length, will

(A) be double. (B) be half. (C) be four times. (D) remain same.

EH0042

2. The temperature of a wire is doubled. The Young's modulus of elasticity

(A) will also double. (B) will become four times.

(C) will remain same. (D) will decrease.

EH0043

3. A spring is stretched by applying a load to its free end. The strain produced in the spring is

(A) volumetric. (B) shear.

(C) longitudinal and shear. (D) longitudinal.

EH0044

4. Overall changes in volume and radii of a uniform cylindrical steel wire are 0.2% and 0.002% respectively when subjected to some suitable force. Longitudinal tensile stress acting on the wire is :-

($Y = 2.0 \times 10^{11}\ Nm^{-2}$)

(A) $3.2 \times 10^9\ Nm^{-2}$ (B) $3.2 \times 10^7\ Nm^{-2}$ (C) $3.6 \times 10^9\ Nm^{-2}$ (D) $3.9 \times 10^8\ Nm^{-2}$

EH0045

5. A solid sphere of radius R made of material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area A floats on the surface of the liquid. When a mass m is placed on the piston to compress the liquid, the fractional change in the radius of the sphere $\delta R/R$ is

(A) mg/AK (B) mg/3AK (C) mg/A (D) mg/3AR

EH0046

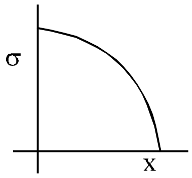
6. A wire suspended vertically from one of its ends is stretched by attaching a weight of 200 N to the lower ends. The weight stretches the wire by 1mm. Then the elastic energy stored in the wire is-

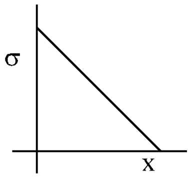
[AIEEE - 2003]

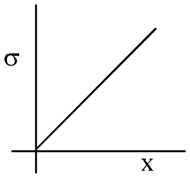
(A) 0.2 J (B) 10 J (C) 20 J (D) 0.1 J

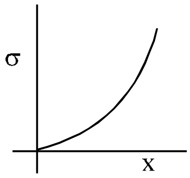
EH0047

7. A uniform rod rotating in gravity free region with certain constant angular velocity. The variation of tensile stress with distance x from axis of rotation is best represented by which of the following graphs.

(A) 

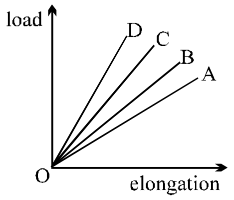
(B) 

(C) 

(D) 

EH0048

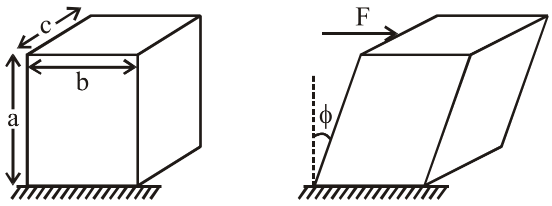
8. The load versus strain graph for four wires of the same material is shown in the figure. The thickest wire is represented by the line :-



(A) OB
(B) OA
(C) OD
(D) OC

EH0049

9. A cuboidal block of sides a, b & c is fixed on ground. The top is pushed by a horizontal force F as shown. The angle ϕ by which the block deforms is : (η is modulus of rigidity)



(A) $\frac{F}{ab\eta}$
(B) $\frac{F}{ac\eta}$
(C) $\frac{F}{bc\eta}$
(D) $\frac{F}{\sqrt{b^2+c^2}\,\eta}$

EH0050

Calorimetry

10. Heat is associated with
(A) kinetic energy of random motion of molecules.
(B) kinetic energy of orderly motion of molecules.
(C) total kinetic energy of random and orderly motion of molecules.
(D) kinetic energy of random motion in some cases and kinetic energy of orderly motion in other.

EH0051

11. Equal amount of heat energy are transferred into equal mass of ethyl alcohol and water sample. The rise in temperature of water sample is 25°C. The temperature rise of ethyl alcohol will be. (Specific heat of ethyl alcohol is one half of the specific heat of water).

(A) 12.5°C (B) 25°C

(C) 50°C (D) It depends on the rate of energy transfer.

EH0052

12. A block of mass 2.5 kg is heated to temperature of 500°C and placed on a large ice block. What is the maximum amount of ice that can melt (approx.). Specific heat for the body = 0.1 Cal/gm°C.

(A) 1 kg (B) 1.5 kg (C) 2 kg (D) 2.5 kg

EH0053

13. 10 gm of ice at 0°C is kept in a calorimeter of water equivalent 10 gm. How much heat should be supplied to the apparatus to evaporate the water thus formed? (Neglect loss of heat)

(A) 6200 cal (B) 7200 cal (C) 13600 cal (D) 8200 cal

EH0054

14. A continuous flow water heater (geyser) has an electrical power rating = 2 kW and efficienty of conversion of electrical power into heat = 80%. If water is flowing through the device at the rate of 100 cc/sec, and the inlet temperature is 10°C, the outlet temperature will be

(A) 12.2°C (B) 13.8°C (C) 20°C (D) 16.5°C

EH0055

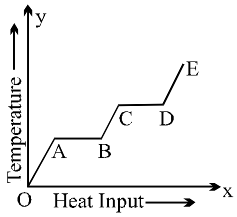
15. A solid material is supplied with heat at a constant rate. The temperature of material is changing with heat input as shown in the figure. What does slope DE represents?

(A) latent heat of liquid

(B) latent heat of vapour

(C) heat capacity of vapour

(D) inverse of heat capacity of vapour



EH0056

16. A block of ice with mass *m* falls into a lake. After impact, a mass of ice *m*/5 melts. Both the block of ice and the lake have a temperature of 0°C. If L represents the heat of fusion, the minimum distance the ice fell before striking the surface is

(A) $\frac{L}{5g}$ (B) $\frac{5L}{g}$ (C) $\frac{gL}{5m}$ (D) $\frac{mL}{5g}$

EH0057

17. The specific heat of a metal at low temperatures varies according to $S = aT^3$ where a is a constant and T is the absolute temperature. The heat energy needed to raise unit mass of the metal from T = 1 K to T = 2 K is :-

(A) 3 a (B) $\frac{15a}{4}$ (C) $\frac{2a}{3}$ (D) $\frac{12a}{5}$

EH0058

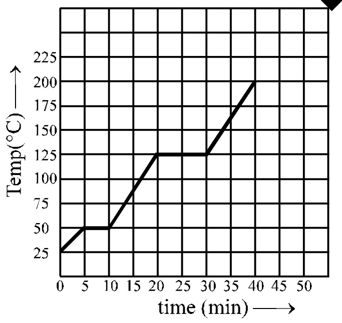
18. The graph shown in the figure represent change in the temperature of 5 kg of a substance as it abosrbs heat at a constant rate of 42 kJ min^{-1}. The latent heat of vapourazation of the substance is :

(A) 630 kJ kg^{-1}

(B) 126 kJ kg^{-1}

(C) 84 kJ kg^{-1}

(D) 12.6 kJ kg^{-1}



EH0059

19. The density of a material A is 1500 kg/m^3 and that of another material B is 2000 kg/m^3. It is found that the heat capacity of 8 volumes of A is equal to heat capacity of 12 volumes of B. The ratio of specific heats of A and B will be

(A) 1 : 2 (B) 3 : 1 (C) 3 : 2 (D) 2 : 1

EH0060

20. 2 kg ice at – 20°C is mixed with 5 kg water at 20°C. Then final amount of water in the mixture would be; Given specific heat of ice = 0.5cal/g°C, specific heat of water = 1 cal/g°C, Latent heat of fusion of ice = 80 cal/g. **[JEE' (Scr) 2003]**

(A) 6 kg (B) 5 kg (C) 4 kg (D) 2 kg

EH0061

21. Some steam at 100°C is passed into 1.1 kg of water contained in a calorimeter of water equivalent 0.02 kg at 15°C so that the temperature of the calorimeter and its contents rises to 80°C. What is the mass of steam condensing. (in kg)

(A) 0.130 (B) 0.065 (C) 0.260 (D) 0.135

EH0062

22. Find the amount of heat supplied to decrease the volume of an ice water mixture by 1 cm^3 without any change in temperature. ($\rho_{ice} = 0.9\ \rho_{water}$, $L_{ice} = 80$ cal/gm).

(A) 360 cal (B) 500 cal (C) 720 cal (D) none of these

EH0063

Thermal expansion

23. The radius of a metal sphere at room temperature T is R, and the coefficient of linear expansion of the metal is α. The sphere is heated a little by a temperature ΔT so that its new temperature is $T + \Delta T$. The increase in the volume of the sphere is approximately

(A) $2\pi R\ \alpha\ \Delta T$ (B) $\pi R^2\ \alpha\ \Delta T$ (C) $4\pi R^3\ \alpha\ \Delta T/3$ (D) $4\pi R^3\ \alpha\ \Delta T$

EH0064

24. A hole is made in a metal plate, when the temperature of metal is raised then the diameter of the hole will :-

(A) Decrease

(B) Increase

(C) Remain same

(D) Answer depends upon the initial temperature of the metal

EH0065

25. A rod of length 2m rests on smooth horizontal floor. If the rod is heated from 0°C to 20°C. Find the longitudinal strain developed? ($\alpha = 5 \times 10^{-5}$/°C)

(A) 10^{-3} (B) 2×10^{-3} (C) Zero (D) None

EH0066

26. A steel tape gives correct measurement at 20°C. A piece of wood is being measured with the steel tape at 0°C. The reading is 25 cm on the tape, the real length of the given piece of wood must be:

(A) 25 cm (B) <25 cm (C) >25 cm (D) can not say

EH0067

27. The bulk modulus of copper is 1.4×10^{11} Pa and the coefficient of linear expansion is $1.7 \times 10^{-5}\ (C^\circ)^{-1}$. What hydrostatic pressure is necessary to prevent a copper block from expanding when its temperature is increased from 20°C to 30°C?

(A) 6.0×10^5 Pa (B) 7.1×10^7 Pa (C) 5.2×10^6 Pa (D) 40 atm

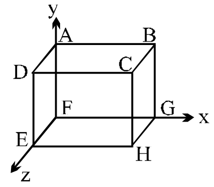
EH0068

28. A thin copper wire of length L increase in length by 1% when heated from temperature T_1 to T_2. What is the percentage change in area when a thin copper plate having dimensions 2L × L is heated from T_1 to T_2?

(A) 1% (B) 2% (C) 3% (D) 4%

EH0069

29. A cuboid ABCDEFGH is anisotropic with $\alpha_x = 1 \times 10^{-5}$ /°C, $\alpha_y = 2 \times 10^{-5}$ /°C, $\alpha_z = 3 \times 10^{-5}$ /°C. Coefficient of superficial expansion of faces can be



(A) $\beta_{ABCD} = 5 \times 10^{-5}$ /°C (B) $\beta_{BCGH} = 4 \times 10^{-5}$ /°C

(C) $\beta_{CDEH} = 3 \times 10^{-5}$ /°C (D) $\beta_{EFGH} = 2 \times 10^{-5}$ /°C

EH0070

30. The coefficient of apparent expansion of a liquid in a copper vessel is C and in a silver vessel is S. The coefficient of volume expansion of copper is γ_c. What is the coefficient of linear expansion of silver?

(A) $\frac{(C+\gamma_c+S)}{3}$ (B) $\frac{(C-\gamma_c+S)}{3}$ (C) $\frac{(C+\gamma_c-S)}{3}$ (D) $\frac{(C-\gamma_c-S)}{3}$

EH0071

31. Two rods one of aluminium of length l_1 having coefficient of linear expansion α_a, and other steel of length l_2 having coefficient of linear expansion α_s are joined end to end. The expansion in both the rods is same on variation of temperature. Then the value of $\frac{l_1}{l_1+l_2}$ is **[JEE' (Scr) 2003]**

(A) $\frac{\alpha_s}{\alpha_a+\alpha_s}$ (B) $\frac{\alpha_s}{\alpha_a-\alpha_s}$ (C) $\frac{\alpha_a+\alpha_s}{\alpha_s}$ (D) None of these

EH0072

32. An open vessel is filled completely with oil which has same coefficient of volume expansion as that of the vessel. On heating both oil and vessel,

(A) the vessel can contain more volume and more mass of oil

(B) the vessel can contain same volume and same mass of oil

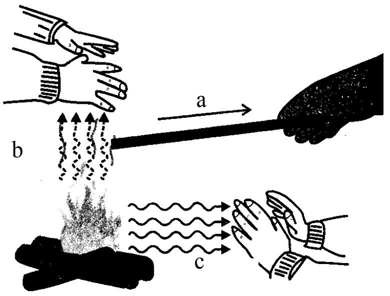
(C) the vessel can contain same volume but more mass of oil

(D) the vessel can contain more volume but same mass of oil

EH0073

Conduction

33. Diagram shows a heat source 'S' and three position of heat recover (hand). The main made of heat transfer is given as 'a', 'b' & 'c'. Choose the correct matching :-



(A) a – conduction ; b – convection ; c – radiation

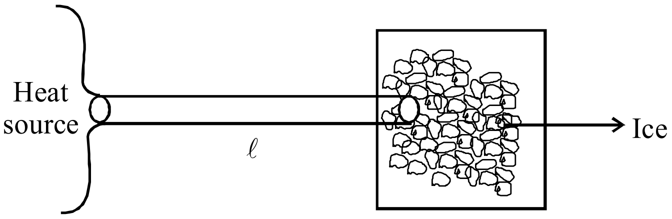
(B) b – conduction ; a – convection ; c – radiation

(C) a – conduction ; c – convection ; b – radiation

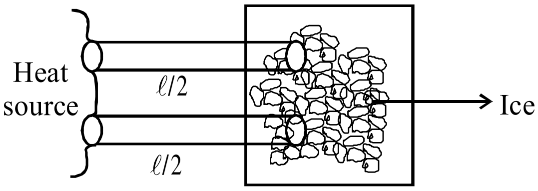
(D) c – conduction ; b – radiation ; a – convection

EH0074

34. A rod of length 'ℓ' and cross-section 'A' is used to melt a piece of ice as shown.



Now if the rod broken into two equal parts and is arranged as shown.



Time taken to melt ice in second use becomes.

(A) Half (B) One-forth (C) Twice (D) Four times

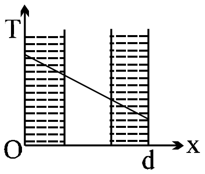
EH0075

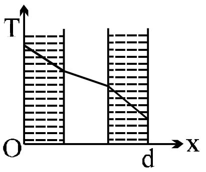
35. One end of a 2.35m long and 2.0cm radius aluminium rod ($K = 235\ W.m^{-1}K^{-1}$) is held at 20^0C. The other end of the rod is in contact with a block of ice at its melting point. The rate in $kg.s^{-1}$ at which ice melts is [Take latent heat of fusion for ice as $\frac{10}{3}\times10^5 J.kg^{-1}$]

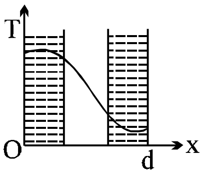
(A) $48\pi \times 10^{-6}$ (B) $24\pi \times 10^{-6}$ (C) $2.4\pi \times 10^{-6}$ (D) $4.8\pi \times 10^{-6}$

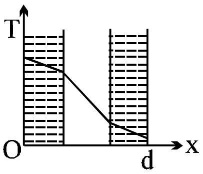
EH0076

36. The wall with a cavity consists of two layers of brick separated by a layer of air. All three layers have the same thickness and the thermal conductivity of the brick is much greater than that of air. The left layer is at a higher temperature than the right layer and steady state condition exists. Which of the following graphs predicts correctly the variation of temperature T with distance d inside the cavity?

(A)


(B)


(C)


(D)


EH0077

37. A wall has two layer A and B each made of different material, both the layers have the same thickness. The thermal conductivity of the material A is twice that of B. Under thermal equilibrium the temperature difference across the wall B is 36°C. The temperature difference across the wall A is

(A) 6°C (B) 12°C (C) 18°C (D) 72°C

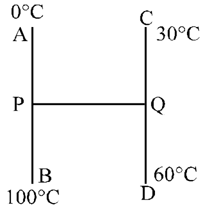
EH0078

38. Two identical conducting rods are first connected independently to two vessels, one containing water at 100°C and the other containing ice at 0° C. In the second case, the rods are joined end to end and connected to the same vessels. Let q_1 and q_2 g/s be the rate of melting of ice in the two cases respectively. The ratio q_2/q_1 is **[JEE' 2004 (Scr.)]**

(A) 1/2 (B) 2/1 (C) 4/1 (D) 1/4

EH0079

39. Three identical rods AB, CD and PQ are joined as shown. P and Q are mid points of AB and CD respectively. Ends A, B, C and D are maintained at 0°C, 100°C, 30°C and 60°C respectively. The direction of heat flow in PQ is

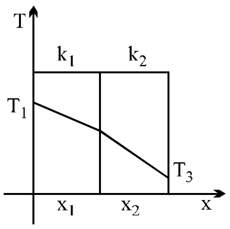


(A) from P to Q (B) from Q to P

(C) heat does not flow in PQ (D) data not sufficient

EH0080

40. The temperature drop through each layer of a two layer furnace wall is shown in figure. Assume that the external temperature T_1 and T_3 are maintained constant and $T_1 > T_3$. If the thickness of the layers x_1 and x_2 are the same, which of the following statements are correct.



(A) $k_1 > k_2$

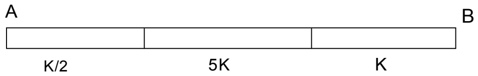
(B) $k_1 < k_2$

(C) $k_1 = k_2$ but heat flow through material (1) is larger then through (2)

(D) $k_1 = k_2$ but heat flow through material (1) is less than that through (2)

EH0081

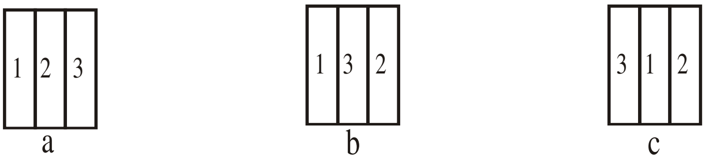
41. A composite rod made of three rods of equal length and cross-section as shown in the fig. The thermal conductivities of the materials of the rods are K/2, 5K and K respectively. The end A and end B are at constant temperatures. All heat entering the end A goes out of the end B, there being no loss of heat from the sides of the bar. The effective thermal conductivity of the bar is



(A) 15K/16 (B) 6K/13 (C) 5K/16 (D) 2K/13.

EH0082

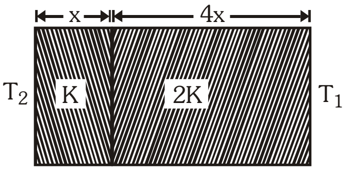
42. Figure shows three different arrangements of materials 1, 2 and 3 to form a wall. Thermal conductivities are $k_1 > k_2 > k_3$. The left side of the wall is 20°C higher than the right side. Temperature difference ΔT across the material 1 has following relation in three cases :



(A) $\Delta T_a > \Delta T_b > \Delta T_c$ (B) $\Delta T_a = \Delta T_b = \Delta T_c$ (C) $\Delta T_a = \Delta T_b > \Delta T_c$ (D) $\Delta T_a = \Delta T_b < \Delta T_c$

EH0083

43. The temperature of the two outer surfaces of a composite slab, consisting of two materials having coefficients of thermal conductivity K and 2K and thickness x and 4x, respectively are T_2 and T_1 ($T_2 > T_1$). The rate of heat transfer through the slab, in a steady state is $\left(\frac{A(T_2 - T_1)K}{x}\right)$f, with f equals to- **[AIEEE - 2004]**



(A) 1 (B) 1/2 (C) 2/3 (D) 1/3

EH0084

Radiation

44. A black metal foil is warmed by radiation from a small sphere at temperature 'T' and at a distance 'd'. It is found that the power received by the foil is P. If both the temperature and distance are doubled, the power received by the foil will be :

(A) 16 P (B) 4 P (C) 2 P (D) P

EH0085

45. The rate of emission of radiation of a black body at 273°C is E, then the rate of emission of radiation of this body at 0°C will be :-

(A) $\frac{E}{16}$ (B) $\frac{E}{4}$ (C) $\frac{E}{8}$ (D) 0

EH0086

46. The power radiated by a black body is P and it radiates maximum energy around the wavelength λ_0. If the temperature of the black body is now changed so that it radiates maximum energy around wavelength $3/4\lambda_0$, the power radiated by it will increase by a factor of :-

(A) 4/3 (B) 16/9 (C) 64/27 (D) 256/81

EH0087

47. Spheres P and Q are uniformly constructed from the same material which is a good conductor of heat and the radius of Q is thrice the radius of P. The rate of fall of temperature of P is x times that of Q when both are at the same surface temperature. The value of x is :

(A) 1/4 (B) 1/3 (C) 3 (D) 4

EH0088

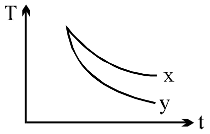
48. Two spheres of the same material have radii 1 m and 4 m and temperatures 4000 K and 2000 K respectively. The ratio of the energy radiated per second by the first sphere to that by the second is-

(A) 1 : 1 (B) 16 : 1 (C) 4 : 1 (D) 1 : 9

EH0089

49. If emissivity of bodies X and Y are e_x and e_y and absorptive power are A_x and A_y then

[JEE' (Scr) 2003]



(A) $e_y > e_x$; $A_y > A_x$ (B) $e_y < e_x$; $A_y < A_x$ (C) $e_y > e_x$; $A_y < A_x$ (D) $e_y = e_x$; $A_y = A_x$

EH0090

50. Three discs A, B, and C having radii 2 m, 4 m and 6 m respectively are coated with carbon black on their outer surfaces. The wavelengths corresponding to maximum intensity are 300 nm, 400 nm and 500 nm respectively. The power radiated by them are Q_A, Q_B and Q_C respectively.

[JEE' 2004 (Scr.)]

(A) Q_A is maximum (B) Q_B is maximum (C) Q_C is maximum (D) $Q_A = Q_B = Q_C$

EH0091

51. A black body calorimeter filled with hot water cools from 60°C to 50°C in 4 min and 40°C to 30°C in 8 min. The approximate temperature of surrounding is :

(A) 10°C (B) 15°C (C) 20°C (D) 25°C

EH0092

52. A system S receives heat continuously from an electrical heater of power 10W. The temperature of S becomes constant at 50°C when the surrounding temperature is 20°C. After the heater is switched off, S cools from 35.1°C to 34.9°C in 1 minute. The heat capacity of S is

(A) 100J/°C (B) 300J/°C (C) 750J/°C (D) 1500J/°C

EH0093

53. If the temperature of the sun were to increase from T to 2T and its radius from R to 2R, then the ratio of the radiant energy received on earth to what it was previously, will be- **[AIEEE - 2004]**

(A) 4 (B) 16 (C) 32 (D) 64

EH0094

MULTIPLE CORRECT TYPE QUESTIONS

Elasticity

54. A wire is suspended from the ceiling and stretched under the action of a weight *F* suspended from its other end. The force exerted by the ceiling on it is equal and opposite to the weight.

(A) Tensile stress at any cross section *A* of the wire is *F/A*.

(B) Tensile stress at any cross section is zero.

(C) Tensile stress at any cross section *A* of the wire is *2F/A*.

(D) Tension at any cross section A of the wire is *F*.

EH0095

55. A copper and a steel wire of the same diameter are connected end to end. A deforming force *F* is applied to this composite wire which causes a total elongation of 1cm. The two wires will have

(A) the same stress. (B) different stress. (C) the same strain. (D) different strain.

EH0096

56. A body of mass M is attached to the lower end of a metal wire, whose upper end is fixed. The elongation of the wire is *l*.

(A) Loss in gravitational potential energy of M is Mg*l*

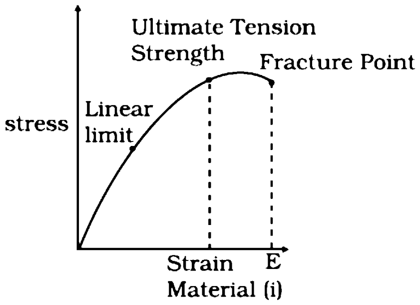
(B) The elastic potential energy stored in the wire is Mg*l*

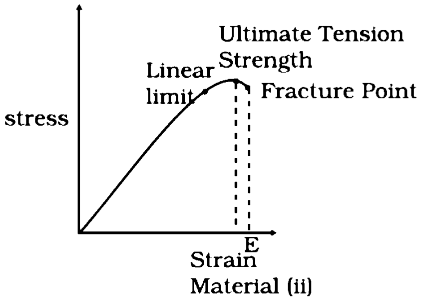
(C) The elastic potential energy stored in the wire is 1/2 Mg*l*

(D) Heat produced is 1/2 Mg*l*.

EH0097

57. The stress-strain graphs for two materials are shown in figure (assume same scale).





(A) Material (ii) is more elastic than material (i) and hence material (ii) is more brittle.

(B) Material (i) and (ii) have the same elasticity and the same brittleness.

(C) Material (ii) is elastic over a larger region of strain as compared to (i).

(D) Material (ii) is more brittle than material (i).

EH0098

58. A composite rod consists of a steel rod of length 25 cm and area 2A and a copper rod of length 50cm and area A. The composite rod is subjected to an axial load F. If the Young's modulus of steel and copper are in the ratio 2 : 1.

(A) the extension produced in copper rod will be more .

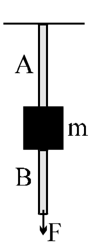
(B) the extension in copper and steel parts will be in the ratio 2 : 1.

(C) the stress applied to the copper rod will be more.

(D) no extension will be produced in the steel rod.

EH0099

59. The wires A and B shown in the figure are made of the same material and have radii r_A and r_B respectively. The block between them has a mass m. When the force F is mg/3, one of the wires breaks.



(A) A breaks if $r_A = r_B$
(B) A breaks if $r_A < 2r_B$
(C) Either A or B may break if $r_A = 2r_B$
(D) The lengths of A and B must be known to predict which wire will break

EH0100

Calorimetry

60. Mark the **CORRECT** options:

(A) A system *X* is in thermal equilibrium with *Y* but not with *Z*. System *Y* and *Z* may be in thermal equilibrium with each other.
(B) A system *X* is in thermal equilibrium with *Y* but not with *Z*. Systems *Y* and *Z* are not in thermal equilibrium with each other.
(C) A system *X* is neither in thermal equilibrium with *Y* nor with *Z*. The systems *Y* and *Z* must be in thermal equilibrium with each other.
(D) A system *X* is neither in thermal equilibrium with *Y* nor with *Z*. The system *Y* and *Z* may be in thermal equilibrium with each other.

EH0101

61. 50 gm ice at – 10°C is mixed with 20 gm steam at 100°C. When the mixture finally reaches its steady state inside a calorimeter of water equivalent 1.5 gm then : [Assume calorimeter was initially at 0°C, Take latent heat of vaporization of water = 540 cal/gm, Latent heat of fusion of water = 80 cal/gm, specific heat capacity of water = 1 cal/gm-°C, specific heat capacity of ice = 0.5 cal/gm°C]

(A) Mass of water remaining is : 67.4 gm
(B) Mass of steam remaining is : 2.6 gm
(C) Mass of water remaining is : 67.87 gm
(D) Mass of steam remaining is : 2.13 gm

EH0102

Thermal expansion

62. When the temperature of a copper coin is raised by 80°C, its diameter increases by 0.2%.

(A) Percentage rise in the area of a face is 0.4 %
(B) Percentage rise in the thickness is 0.4 %
(C) Percentage rise in the volume is 0.6 %
(D) Coefficient of linear expansion of copper is 0.25×10^{-4} $C^{\circ\,-1}$.

EH0103

Radiation

63. Two metallic sphere A and B are made of same material and have got identical surface finish. The mass of sphere A is four times that of B. Both the spheres are heated to the same temperature and placed in a room having lower temperature but thermally insulated from each other.

(A) The ratio of heat loss of A to that of B is $2^{4/3}$.
(B) The ratio of heat loss of A to that of B is $2^{2/3}$.
(C) The ratio of the initial rate of cooling of A to that of B is $2^{-2/3}$.
(D) The ratio of the initial rate of cooling of A to that of B is $2^{-4/3}$.

EH0104

64. Two bodies A and B have thermal emissivities of 0.01 and 0.81 respectively. The outer surface areas of the two bodies are the same. The two bodies radiate energy at the same rate. The wavelength λ_B, corresponding to the maximum spectral radiancy in the radiation from B, is shifted from the wavelength corresponding to the maximum spectral radiancy in the radiation from A by 1.00 μm. If the temperature of A is 5802 K,

(A) the temperature of B is 1934 K
(B) λ_B =1.5 μm
(C) the temperature of B is 11604 K
(D) the temperature of B is 2901 K

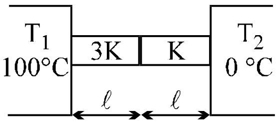
EH0105

COMPREHENSION TYPE QUESTIONS

Conduction

Paragraph for Question No. 65 to 67

Two rods A and B of same cross-sectional are A and length *l* connected in series between a source (T_1 = 100°C) and a sink (T_2 = 0°C) as shown in figure. The rod is laterally insulated



65. The ratio of the thermal resistance of the rods is

(A) $\frac{R_A}{R_B}=\frac{1}{3}$ (B) $\frac{R_A}{R_B}=3$ (C) $\frac{R_A}{R_B}=\frac{3}{4}$ (D) $\frac{4}{3}$

EH0106

66. If T_A and T_B are the temperature drops across the rod A and B, then

(A) $\frac{T_A}{T_B}=\frac{3}{1}$ (B) $\frac{T_A}{T_B}=\frac{1}{3}$ (C) $\frac{T_A}{T_B}=\frac{3}{4}$ (D) $\frac{T_A}{T_B}=\frac{4}{3}$

EH0106

67. If G_A and G_B are the temperature gradients across the rod A and B, then

(A) $\frac{G_A}{G_B}=\frac{3}{1}$ (B) $\frac{G_A}{G_B}=\frac{1}{3}$ (C) $\frac{G_A}{G_B}=\frac{3}{4}$ (D) $\frac{G_A}{G_B}=\frac{4}{3}$

EH0106

EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

1. A cylindrical wire of radius 1 mm, length 1 m, Young's modulus = 2×10^{11} N/m^2, poisson's ratio $\mu = \pi/10$ is stretched by a force of 100 N. Its radius will become

(A) 0.99998 mm (B) 0.99999 mm (C) 0.99997 mm (D) 0.99995 mm

EH0107

2. A thermally insulated vessel contains some water at 0^0C. The vessel is connected to a vacuum pump to pump out water vapour. This results in some water getting frozen. It is given Latent heat of vaporization of water at 0°C = 21×10^5 J/kg and latent heat of freezing of water = 3.36×10^5 J/kg. The maximum percentage amount of water that will be solidified in this manner will be :-

(A) 86.2% (B) 33.6% (C) 21% (D) 24.36%

EH0108

3. Ice at 0°C is added to 200 g of water initially at 70°C in a vacuum flask. When 50 g of ice has been added and has all melted the temperature of the flask and contents is 40°C. When a further 80g of ice has been added and has all metled, the temperature of the whole is 10°C. Calculate the specific latent heat of fusion of ice. [Take S_w =1 cal /gm °C.]

(A) 3.8×10^5 J/ kg (B) 1.2×10^5 J/ kg (C) 2.4×10^5 J/ kg (D) 3.0×10^5 J/ kg

EH0109

4. The coefficient of linear expansion of copper is 17×10^{-6} $(°C)^{-1}$. A copper statue is 93 m tall on the summer morning of temperature 25°C. What is maximum order of increase in magnitude of the height in statue (maximum temperature of day is 45°C)

(A) 0.1 mm (B) 1 mm (C) 10 mm (D) 100 mm

EH0110

5. The coefficients of thermal expansion of steel and a metal X are respectively 12×10^{-6} and 2×10^{-6} per°C. At 40°C, the side of a cube of metal X was measured using a steel vernier callipers. The reading was 100 mm. Assuming that the calibration of the vernier was done at 0°C, then the actual length of the side of the cube at 0°C will be

(A) > 100 mm (B) < 100 mm

(C) = 100 mm (D) data insufficient to conclude

EH0111

6. The volume of the bulb of a mercury thermometer at 0°C is V_0 and cross section of the capillary is A_0. The coefficient of linear expansion of glass is α_g per °C and the cubical expansion of mercury γ_m per °C. If the mercury just fills the bulb at 0°C, what is the length of mercury column in capillary at T°C.

(A) $\dfrac{V_0T(\gamma_m+3\alpha_g)}{A_0(1+2\alpha_gT)}$ (B) $\dfrac{V_0T(\gamma_m-3\alpha_g)}{A_0(1+2\alpha_gT)}$ (C) $\dfrac{V_0T(\gamma_m+2\alpha_g)}{A_0(1+3\alpha_gT)}$ (D) $\dfrac{V_0T(\gamma_m-2\alpha_g)}{A_0(1+3\alpha_gT)}$

EH0112

7. A rod of length 2m at 0°C and having expansion coefficient $\alpha = (3x + 2) \times 10^{-6}\,°C^{-1}$ where x is the distance (in cm) from one end of rod. The length of rod at 20°C is :

(A) 2.124 m (B) 3.24 m (C) 2.0120 m (D) 3.124 m

EH0113

8. A liquid is given some heat.

Statement A : Some liquid evaporates. **Statement B** : The liquid starts boiling.

(A) A implies B and B implies A
(B) B implies A but, A does not imply B
(C) A implies B but B does not imply A
(D) Neither A implies B nor B implies A

EH0114

9. A long solid cylinder is radiating power. It is remoulded into a number of smaller cylinders, each of which has the same length as original cylinder. Each small cylinder has the same temperature as the original cylinder. The total radiant power emitted by the pieces is twice that emitted by the original cylinder. How many smaller cylinders are there ? Neglect the energy emitted by the flat faces of cylinder.

(A) 3 (B) 4 (C) 5 (D) 6

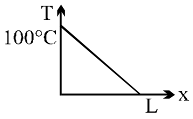
EH0115

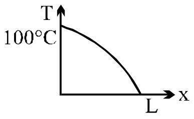
10. Four rods of same material with different radii r and length ℓ are used to connect two reservoirs of heat at different temperatures. Which one will conduct most heat ?

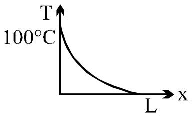
(A) r =2cm, ℓ=0.5m
(B) r = 2cm, ℓ = 2m
(C) r =0.5cm, ℓ =0.5m
(D) r = 1 cm, ℓ = 1 m

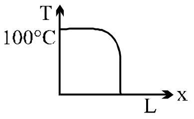
EH0116

11. A rod of length L and uniform cross-sectional area has varying thermal conductivity which changes linearly from 2K at end A to K at the other end B. The ends A and B of the rod are maintained at constant temperature 100°C and 0°C, respectively. At steady state, the graph of temperature : T = T(x) where x = distance from end A will be

(A)


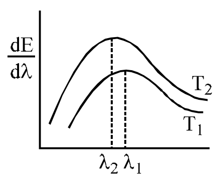
(B)


(C)


(D)


EH0117

12. The spectral emissive power E_λ for a body at temperature T_1 is plotted against the wavelength and area under the curve is found to be A. At a different temperature T_2 the area is found to be 9A. Then λ_1/λ_2 =



(A) 3 (B) 1/3 (C) $1/\sqrt{3}$ (D) $\sqrt{3}$

EH0118

13. 'Gulab Jamuns' (assumed to be spherical) are to be heated in an oven. They are available in two sizes, one twice bigger (in radius) than the other. Pizzas (assumed to be discs) are also to be heated in oven. They are also in two sizes, one twice big (in radius) than the other. All four are put together to be heated to oven temperature. Choose the correct option from the following:

(A) Both size gulab jamuns will get heated in the same time.
(B) Smaller gulab jamuns are heated before bigger ones.
(C) Smaller pizzas are heated before bigger ones.
(D) Bigger pizzas are heated before smaller ones.

EH0119

14. An experiment is perfomed to measure the specific heat of copper. A lump of copper is heated in an oven, then dropped into a beaker of water. To calculate the specific heat of copper, the experimenter must know or measure the value of all of the quantities below EXCEPT the

(A) heat capacity of water and beaker
(B) original temperature of the copper and the water
(C) final (equilibrium) temperature of the copper and the water
(D) time taken to achieve equilibrium, after the copper is dropped into the water

EH0120

15. One end of a conducting rod is maintained at temperature 50°C and at the other end, ice is melting at 0°C. The rate of melting of ice is doubled if:

(A) the temperature is made 200°C and the area of cross-section of the rod is doubled
(B) the temperature is made 100°C and length of rod is made four times
(C) area of cross-section of rod is halved and length is doubled
(D) the temperature is made 100°C and the area of cross-section of rod and length both are doubled.

EH0121

16. A black body is at a temperature of 2880 K. The energy of radiation emitted by this object with wavelength between 499 nm and 500 nm is U_1, between 999 nm and 1000 nm is U_2 and between 1499 nm and 1500 nm is U_3. The Wien constant $b = 2.88 \times 10^6$ nm K. Then

(A) $U_1 = 0$ (b) $U_3 = 0$ (C) $U_1 > U_2$ (D) $U_2 > U_1$

EH0122

MULTIPLE CORRECT TYPE QUESTIONS

17. A bimetallic strip is formed out of two identical strips one of copper and the other of brass. The coefficient of linear expansion of the two metals are α_C and α_B. On heating, the temperature of the strip goes up by ΔT and the strip bends to form an arc of radius of curvature R. Then R is

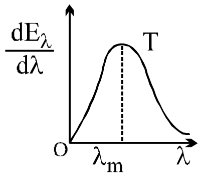
(A) proportional at ΔT (B) inversely proportional to ΔT
(C) proportional to $|\alpha_B - \alpha_C|$ (D) inversely proportional to $|\alpha_B - \alpha_C|$

EH0123

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 18 and 19

The figure shows a radiant energy spectrum graph for a black body at a temperature T.



18. Choose the **CORRECT** statement(s)

(A) The radiant energy is not equally distributed among all the possible wavelengths

(B) For a particular wavelength the spectral intensity is maximum

(C) The area under the curve is equal to the total rate at which heat is radiated by the body at that temperature

(D) None of these

EH0124

19. If the temperature of the body is raised to a higher temperature T', then choose the correct statement(s)

(A) The intensity of radiation for every wavelength increases

(B) The maximum intensity occurs at a shorter wavelength

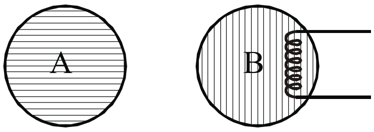
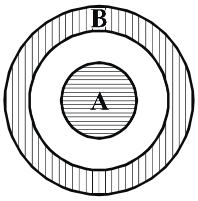
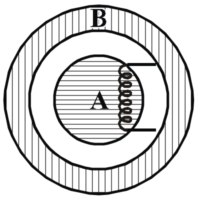
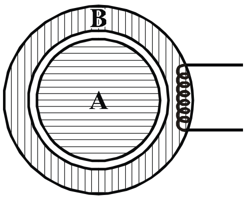
(C) The area under the graph increases

(D) The area under the graph is proportional to the fourth power of temperature

EH0124

MATRIX MATCH TYPE QUESTION

20. A & B are two black bodies of radii r_A and r_B respectively, placed in surrounding of temperature T_0. At steady state the temperature of A & B is T_A & T_B respectively.

Column I	Column II
(A)  • A & B are solid sphere • $r_A = r_B$ • Body 'B' is being heated by a heater of constant power 'P'	(P) $T_A = T_B$
(B)  • B is thin spherical shell • A is a solid sphere • $r_A < r_B$	(Q) $T_A < T_B$
(C)  • B is thin spherical shell • A is a solid sphere • $r_A < r_B$ • Body A is being heated by a heater of constant power 'P'	(R) Heat received by A is more than heat radiated by it at steady state.
(D)  • B is thin spherical shell • A is a solid sphere • $r_A \approx r_B$ • Body B is being heated by a heater of constant power 'P'	(S) Radiation spectrum of A & B is distinguishable (T) Steady state can't be achieved

EH0125

21. A sample 'A' of liquid water and a sample B of ice of equal mass are kept in 2 nearby containers so that they can exchange heat with each other but are thermally insulated from the surroundings. The graphs in column-II show the sketch of temperature T of samples versus time t. Match with appropriate description in column-I.

Column I

(A) Equilibrium temperature is above melting point of ice.

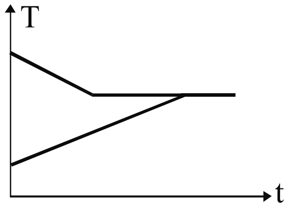
(B) At least some of water freezes.

(C) At least some of ice melts.

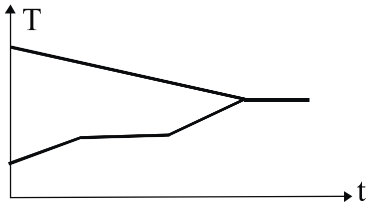
(D) Equilibrium temperature is below freezing point of water

Column II

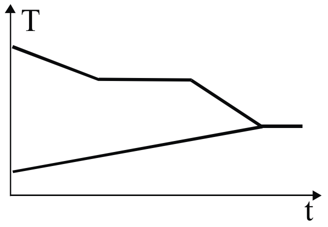
(P)



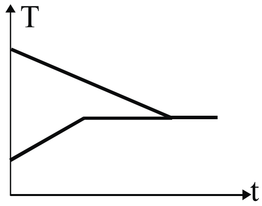
(Q)



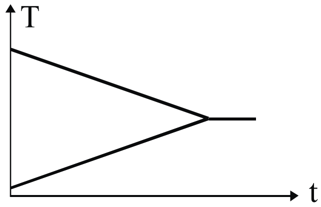
(R)



(S)



(T)



EH0126

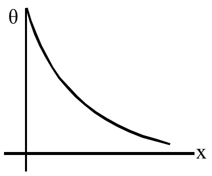
EXERCISE (JM)

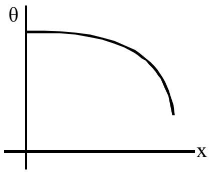
1. Two wires are made of the same material and have the same volume. However wire 1 has cross-sectional area A and wire 2 has cross-sectional area 3A. If the length of wire 1 increases by Δx on applying force F, how much force is needed to stretch wire 2 by the same amount ? **[AIEEE-2009]**

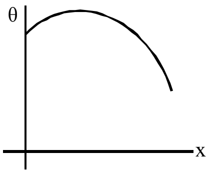
(1) 6F (2) 9F (3) F (4) 4F

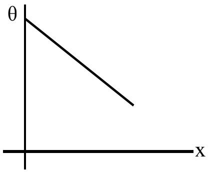
EH0127

2. A long metallic bar is carrying heat from one of its ends to the other end under steady-state. The variation of temperature θ along the length x of the bar from its hot end is best described by which of the following figures ? **[AIEEE - 2009]**

(1)


(2)


(3)


(4)


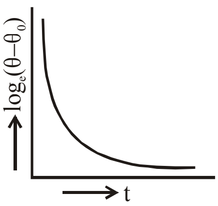
EH0128

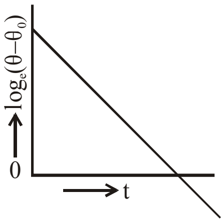
3. 100 g of water is heated from 30°C to 50°C Ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is 4184 J/kg/K) :- **[AIEEE - 2011]**

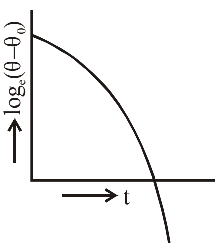
(1) 84 kJ (2) 2.1 kJ (3) 4.2 kJ (4) 8.4 kJ

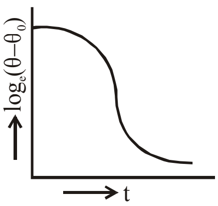
EH0129

4. A liquid in a beaker has temperature $\theta(t)$ at time t and θ_0 is temperature of surroundings, then according to Newton's law of cooling the correct graph between $\log_e(\theta - \theta_0)$ and t is :- **[AIEEE 2012]**

(1)


(2)


(3)


(4)


EH0130

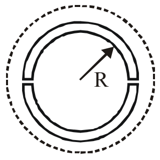
5. A wooden wheel of radius R is made of two semicircular parts (see figure). The two parts are held together by a ring made of a metal strip of cross sectional area S and Length L. L is slightly less than $2\pi R$. To fit the ring on the wheel, it is heated so that its temperature rises by ΔT and it just steps over the wheel. As it cools down to surrounding temperature, it presses the semicircular parts together. If the coefficient of linear expansion of the metal is α, and its Young's modulus is Y, the force that one part of the wheel applies on the other part is : **[AIEEE 2012]**

(1) $2SY\alpha\Delta T$

(2) $2\pi SY\alpha\Delta T$

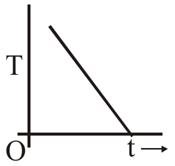
(3) $SY\alpha\Delta T$

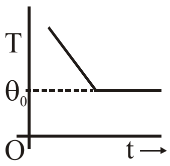
(4) $\pi SY\alpha\Delta T$

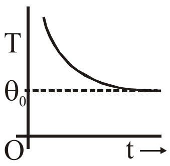


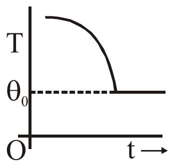
EH0131

6. If a piece of metal is heated to temperature θ and then allowed to cool in a room which is at temperature θ_0 the graph between the temperature T of the metal and time t will be closed to: **[JEE-Main- 2013]**

(1)


(2)


(3)


(4)


EH0132

7. The pressure that has to be applied to the ends of a steel wire of length 10 cm to keep its length constant when its temperature is raised by 100°C is :

(For steel Young's modulus is 2×10^{11} N m^{-2} and coefficient of thermal expansion is 1.1×10^{-5} K^{-1})

[JEE-Main-2014]

(1) 2.2×10^7 Pa (2) 2.2×10^6 Pa (3) 2.2×10^8 Pa (4) 2.2×10^9 Pa

EH0133

8. Three rods of Copper, Brass and Steel are welded together to form a Y-shaped structure. Area of cross-section of each rod = 4 cm^2. End of copper rod is maintained at 100°C where as ends of brass and steel are kept at 0°C. Lengths of the copper, brass and steel rods are 46, 13 and 12 cms respectively. The rods are thermally insulated from surroundings except at ends. Thermal conductivities of copper, brass and steel are 0.92, 0.26 and 0.12 CGS units respectively. Rate of heat flow through copper rod is : **[JEE-Main-2014]**

(1) 4.8 cal/s (2) 6.0 cal/s (3) 1.2 cal/s (4) 2.4 cal/s

EH0134

9. A pendulum made of a uniform wire of cross sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to T_M. If the Young's modulus of the material of the wire is Y then $\frac{1}{Y}$ is equal to :- (g = gravitational acceleration) **[JEE-Main-2015]**

(1) $\left[1-\left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$ (2) $\left[1-\left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$ (3) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{A}{Mg}$ (4) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{Mg}{A}$

EH0135

10. A pendulume clock loses 12s a day if the temperature is 40°C and gains 4s a day if the temperature is 20°C. The temperature at which the clock will show correct time, and the coeffecient of linear expansion (α) of the metal of the pendulum shaft are respectively :- **[JEE-Main-2016]**

(1) 55°C ; $\alpha = 1.85 \times 10^{-2}$ / °C (2) 25°C ; $\alpha = 1.85 \times 10^{-5}$ / °C

(3) 60°C ; $\alpha = 1.85 \times 10^{-4}$ / °C (4) 30°C ; $\alpha = 1.85 \times 10^{-3}$ / °C

EH0136

11. A copper ball of mass 100 gm is at a temperature T. It is dropped in a copper calorimeter of mass 100gm, filled with 170 gm of water at room temperature. Subsequently, the temperature of the system is found to be 75°C. T is given by : (Given : room temperature = 30° C, specific heat of copper = 0.1 cal/gm°C) **[JEE-Main-2017]**

(1) 1250°C (2) 825°C (3) 800°C (4) 885° C

EH0137

12. A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains same, the stress in the leg will change by a factor of : **[JEE-Main-2017]**

(1) 81 (2) $\frac{1}{81}$ (3) 9 (4) $\frac{1}{9}$

EH0138

13. An external pressure P is applied on a cube at 0°C so that it is equally compressed from all sides. K is the bulk modulus of the material of the cube and α is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by :

[JEE-Main-2017]

(1) $\frac{3\alpha}{PK}$ (2) $3PK\alpha$ (3) $\frac{P}{3\alpha K}$ (4) $\frac{P}{\alpha K}$

EH0139

14. A solid sphere of radius r made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid, covering entire cross section of cylindrical container. When a mass m is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere, $\left(\frac{dr}{r}\right)$, is : **[JEE-Main-2018]**

(1) $\frac{Ka}{3mg}$ (2) $\frac{mg}{3Ka}$ (3) $\frac{mg}{Ka}$ (4) $\frac{Ka}{mg}$

EH0140

EXERCISE (JA)

1. A metal rod AB of length 10x has its one end A in ice at 0°C and the other end B in water at 100°C. If a point P on the rod is maintained at 400°C, then it is found that equal amounts of water and ice evaporate and melt per unit time. The latent heat of evaporation of water is 540 cal/g and latent heat of melting of ice is 80 cal/g. If the point P is at a distance of λx from the ice end A, find the value of λ.[Neglect any heat loss to the surrounding] **[JEE 2009]**

EH0141

2. Two spherical bodies A (radius 6 cm) and B (radius 18 cm) are at temperatures T_1 and T_2, respectively. The maximum intensity in the emission spectrum of A is at 500 nm and in that of B is at 1500 nm. Considering then to be black bodies, what will be the ratio of the rate of total energy radiated by A to that of B? **[JEE 2010]**

EH0142

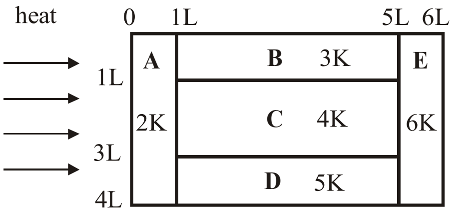
3. A piece of ice (heat capacity = 2100 J kg^{-1} $°C^{-1}$ and latent heat = 3.36×10^5 J kg^{-1}) of mass m grams is at –5°C at atmospheric pressure. It is given 420 J of heat so that the ice starts melting. Finally when the ice–water mixture is in equilibrium, it is found that 1 gm of ice has melted. Assuming there is no other heat exchange in the process, the value of m is **[JEE 2010]**

EH0143

4. A 0.1 kg mass is suspended from a wire of negligible mass. The length of the wire is 1m and its cross–sectional area is 4.9×10^{-7} m^2. If the mass is pulled a little in the vertically downward direction and released, it performs simple harmonic motion of angular frequency 140 rad s^{-1}. If the Young's modulus of the material of the wire is $n \times 10^9$ Nm^{-2}, the value of n is **[IIT-JEE 2010]**

EH0144

5. A composite block is made of slabs A, B, C, D and E of different thermal conductivities (given in terms of a constant K) and sizes (given in terms of length, L) as shown in the figure. All slabs are of same width. Heat 'Q' flows only from left to right through the blocks. Then in steady state **[JEE 2011]**



(A) heat flow through A and E slabs are same

(B) heat flow through slab E is maximum

(C) temperature difference across slab E is smallest

(D) heat flow through C= heat flow through B + Heat flow through D

EH0145

6. Steel wire of length 'L' at 40°C is suspended from the ceiling and then a mass 'm' is hung from its free end. The wire is cooled down from 40° to 30° C to regain its original length 'L'. The coefficient of linear thermal expansion of the steel is $10^{-5}/°C$, Young's modulus of steel is 10^{11} N/m^2 and radius of the wire is 1 mm. Assume that L>> diameter of the wire. Then the value of 'm' in kg is nearly

[JEE 2011]

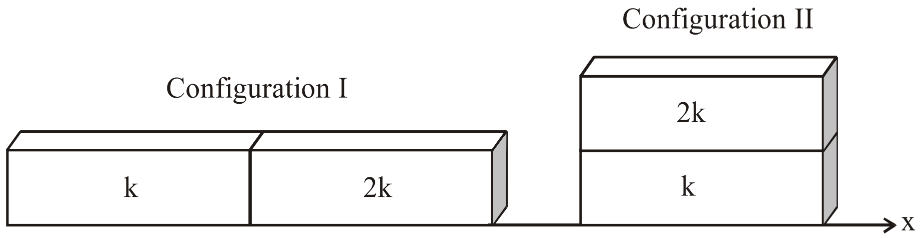
EH0146

7. Three very large plates of same area are kept parallel and close to each other. They are considered as ideal black surfaces and have very high thermal conductivity. The first and third plates are maintained at temperatures 2T and 3T respectively. The temperature of the middle (i.e. second) plate under steady state condition is **[JEE 2012]**

(A) $\left(\frac{65}{2}\right)^{1/4} T$ (B) $\left(\frac{97}{4}\right)^{1/4} T$ (C) $\left(\frac{97}{2}\right)^{1/4} T$ (D) $(97)^{1/4} T$

EH0147

8. Two rectangular blocks, having identical dimensions, can be arranged either in configuration I or in configuration II as shown in the figure. One of the blocks has thermal conductivity k and the other 2k. The temperature difference between the ends along the x-axis is the same in both the configurations. It takes 9s to transport a certain amount of heat from the hot end to the cold end in the configuration I. The time to transport the same amount of heat in the configuration II is :- **[JEE-Advance-2013]**



(A) 2.0 s (B) 3.0 s (C) 4.5 s (D) 6.0 s

EH0148

9. One end of a horizontal thick copper wire of length 2L and radius 2R is welded to an end of another horizontal thin copper wire of length L and radius R. When the arrangement is stretched by applying forces at two ends, the ratio of the elongation in the thin wire to that in the thick wire is :-

[JEE-Advance-2013]

(A) 0.25 (B) 0.50 (C) 2.00 (D) 4.00

EH0149

10. Parallel rays of light of intensity $I = 912\ Wm^{-2}$ are incident on a spherical black body kept in surroundings of temperature 300 K. Take Stefan-Boltzmann constant $\sigma = 5.7 \times 10^{-8}\ Wm^{-2}\ K^{-4}$ and assume that the energy exchange with the surroundings is only through radiation. The final steady state temperature of the black body is close to :- **[JEE-Advance-2014]**

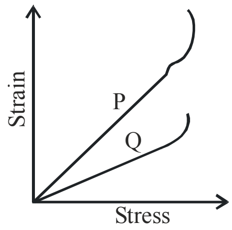
(A) 330 K (B) 660 K (C) 990 K (D) 1550 K

EH0150

11. Two spherical stars A and B emit blackbody radiation. The radius of A is 400 times that of B and A emits 10^4 times the power emitted from B. The ratio $\left(\frac{\lambda_A}{\lambda_B}\right)$ of their wavelengths λ_A and λ_B at which the peaks occur in their respective radiation curves is. **[JEE-Advance-2015]**

EH0151

12. In plotting stress versus strain curves for two materials P and Q, a student by mistake puts strain on the y-axis and stress on the x-axis as shown in the figure. Then the correct statement(s) is (are):- **[JEE-Advance-2015]**

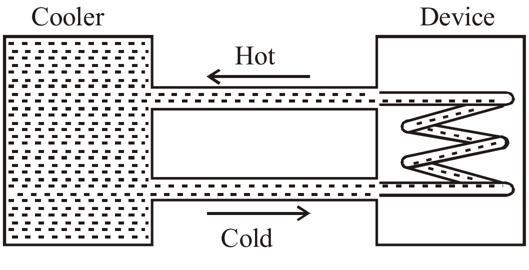


(A) P has more tensile strength than Q (B) P is more ductile than Q

(C) P is more brittle than Q (D) The Young's modulus of P is more than that of Q

EH0152

13. A water cooler of storage capacity 120 litres can cool water at constant rate of P watts. In a closed circulation system (as shown schematically in the figure), the water from the cooler is used to cool an external device that generates constantly 3 kW of heat (thermal load). The temperature of water fed into the device cannot exceed 30°C and the entire stored 120 litres of water is initially cooled to 10°C. The entire system is thermally insulated. The minimum value of P (in watts) for which the device can be operated for 3 hours is : **[JEE-Advance-2016]**



(Specific heat of water is $4.2\ kJ\ kg^{-1}\ K^{-1}$ and the density of water is $1000\ kg\ m^{-3}$)

(A) 1600 (B) 2067 (C) 2533 (D) 3933

EH0153

14. A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated (P) by the metal. The sensor has a scale that displays $\log_2(P/P_0)$, where P_0 is a constant. When the metal surface is at a temperature of 487 °C, the sensor shows a value 1. Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to 2767 °C ? **[JEE-Advance-2016]**

EH0154

15. The ends Q and R of two thin wires, PQ and RS, are soldered (joined) together. Initially each of the wires has a length of 1m at 10°C. Now the end P is maintained at 10°C, while the end S is heated and maintained at 400°C. The system is thermally insulated from its surroundings. If the thermal conductivity of wire PQ is twice that of the wire RS and the coefficient of linear thermal expansion of PQ is $1.2 \times 10^{-5}\,K^{-1}$, the change in length of the wire PQ is **[JEE-Advance-2016]**

(A) 0.78 mm (B) 0.90 mm (C) 1.56 mm (D) 2.34 mm

EH0155

16. A human body has a surface area of approximately 1 m^2. The normal body temperature is 10 K above the surrounding room temperature T_0. Take the room temperature to be T_0 = 300 K. For T_0 = 300 K, the value of $\sigma T_0^4 = 460\ Wm^{-2}$ (where σ is the Stefan-Boltzmann constant). Which of the following options is/are **correct** ? **[JEE-Advance-2017]**

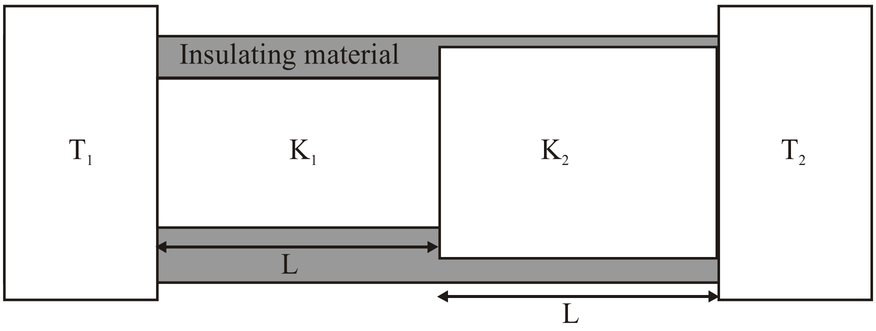
(A) The amount of energy radiated by the body in 1 second is close to 60 Joules

(B) If the surrounding temperature reduces by a small amount $\Delta T_0 << T_0$, then to maintain the same body temperature the same (living) human being needs to radiate $\Delta W = 4\sigma T_0^3 \Delta T_0$ more energy per unit time

(C) Reducing the exposed surface area of the body (e.g. by curling up) allows humans to maintain the same body temperature while reducing the energy lost by radiation

(D) If the body temperature rises significantly then the peak in the spectrum of electromagnetic radiation emitted by the body would shift to longer wavelengths **EH0156**

17. Two conducting cylinders of equal length but different radii are connected in series between two heat baths kept at temperatures $T_1 = 300$ K and $T_2 = 100$ K, as shown in the figure. The radius of the bigger cylinder is twice that of the smaller one and the thermal conductivities of the materials of the smaller and the larger cylinders are K_1 and K_2 respectively. If the temperature at the junction of the two cylinders in the steady state is 200 K, then K_1/K_2 = _________. **[JEE-Advance-2018]**

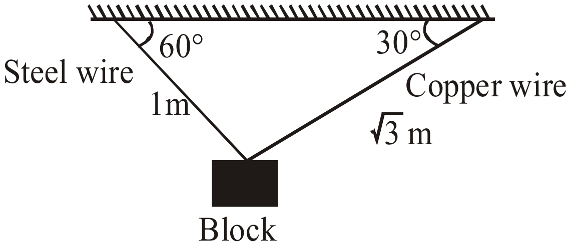


EH0157

18. A block of weight 100 N is suspended by copper and steel wires of same cross sectional area 0.5 cm^2 and, length $\sqrt{3}$ m and 1 m, respectively. Their other ends are fixed on a ceiling as shown in figure. The angles subtended by copper and steel wires with ceiling are 30° and 60°, respectively. If elongation in copper wire is $(\Delta\ell_C)$ and elongation in steel wire is $(\Delta\ell_S)$, then the ratio $\frac{\Delta\ell_C}{\Delta\ell_S}$ is _____.

[Young's modulus for copper and steel are 1×10^{11} N/m^2 and 2×10^{11} N/m^2 respectively]

[JEE-Advance-2019]



EH0158

19. A liquid at 30°C is poured very slowly into a Calorimeter that is at temperature of 110°C. The boiling temperature of the liquid is 80°C. It is found that the first 5 gm of the liquid completely evaporates. After pouring another 80 gm of the liquid the equilibrium temperature is found to be 50°C. The ratio of the Latent heat of the liquid to its specific heat will be ______ °C.

[Neglect the heat exchange with surrounding] **[JEE-Advance-2019]**

EH0159

ANSWER KEY

EXERCISE (S-1)

1. Ans. 0.75 **2. Ans.** $\frac{9}{16}$ **3. Ans.** 3mm **4. Ans.** ℓ'

5. Ans. (a) $\theta = \frac{\pi}{2}$ (b) $\theta = \pi/4$.

6. Ans. (i) 50 N, (ii) 0.045 J, 1.8×10^{-3} m (iii) 8.4×10^{-4} m, (iv) x = 0.12 m **7. Ans.** The first one
8. Ans. 25.5°C **9. Ans.** 1/90 **10. Ans.** 0 °C, 125/4 g ice, 1275/4 g water
11. Ans. 5000 J/°C kg **12. Ans.** 27/85 **13. Ans.** 12 gm
14. Ans. (i) 0.02kg, (ii) 40,000calkg^{-1}, (iii) 750calkg^{-1}K^{-1} **15. Ans.** $5\alpha/3$
16. Ans. 5 sec slow **17. Ans.** 10, 000 N **18. Ans.** 65°C **19. Ans.** 5°C
20. Ans. 7/2 **21. Ans.** $(4/3)\omega$ **22. Ans.** 10.34 cm **23. Ans.** 2:1

24. Ans. 1000 J $(C°)^{-1}$ **25. Ans.** $\left(\frac{6}{\pi}\right)^{1/3}$ **26. Ans.** 80 kcal/kg **27. Ans.** 3025 K

28. Ans. 10 sec

EXERCISE (S-2)

1. Ans. 4×10^{-6} m/°C **2. Ans.** 10cm , 40cm **3. Ans.** 1/200 rad **4. Ans.** 1 : 1.26
6. Ans. (a) 37.8 J/s (Watts), (b) 2.005 N-m **7. Ans.** 45°C

8. Ans. $\frac{a^2 s}{2K}\log_e\left(\frac{b}{a}\right)\log_e\left(\frac{T_0 - T_1}{T_0 - T_2}\right)$ **9. Ans.** (a) – 100°C (b) 1000 J

10. Ans. 166.3 sec **11. Ans.** 10 min. **12. Ans.** (a) 595 watt/m^2; (b) $T_0 \approx 420$ K

13. Ans. $\frac{K}{4e\sigma L T_S^3 + K}$

EXERCISE (O-1)

1. Ans. (D) 2. Ans. (D) 3. Ans. (C) 4. Ans. (D) 5. Ans. (B) 6. Ans. (D)
7. Ans. (A) 8. Ans. (C) 9. Ans. (C) 10. Ans. (A) 11. Ans. (C) 12. Ans. (B)
13. Ans. (D) 14. Ans. (B) 15. Ans. (D) 16. Ans. (A) 17. Ans. (B) 18. Ans. (C)
19. Ans. (D) 20. Ans. (A) 21. Ans. (A) 22. Ans. (C) 23. Ans. (D) 24. Ans. (B)
25. Ans. (C) 26. Ans. (B) 27. Ans. (B) 28. Ans. (B) 29. Ans. (C) 30. Ans. (C)
31. Ans. (A) 32. Ans. (D) 33. Ans. (A) 34. Ans. (B) 35. Ans. (C) 36. Ans. (D)
37. Ans. (C) 38. Ans. (D) 39. Ans. (A) 40. Ans. (A) 41. Ans. (A) 42. Ans. (B)
43. Ans. (D) 44. Ans. (B) 45. Ans. (A) 46. Ans. (D) 47. Ans. (C) 48. Ans. (A)
49. Ans. (A) 50. Ans. (B) 51. Ans. (B) 52. Ans. (D) 53. Ans. (D) 54. Ans. (A,D)
55. Ans. (A,D) 56. Ans. (A,C,D) 57. Ans. (C,D) 58. Ans. (A,C)
59. Ans. (A,B,C) 60. Ans. (B,D) 61. Ans. (A,B) 62. Ans. (A,C,D)
63. Ans. (A,C) 64. Ans. (A,B) 65. Ans. (A) 66. Ans. (B)
67. Ans. (B)

EXERCISE (O-2)

1. Ans. (D) **2. Ans. (A)** **3. Ans. (A)** **4. Ans. (C)** **5. Ans. (A)** **6. Ans. (B)**

7. Ans. (C) **8. Ans. (B)** **9. Ans. (B)** **10. Ans. (A)** **11. Ans. (B)** **12. Ans. (D)**

13. Ans. (B) **14. Ans. (D)** **15. Ans. (D)** **16. Ans. (D)** **17. Ans. (B,D)** **18. Ans. (A,B)**

19. Ans. (A,B,C,D) **20. Ans. A - Q,S; B - P; C - S; D - P**

21. Ans. A - Q; B - P,R; C - Q,S; D - R

EXERCISE (JM)

1. Ans. (2) **2. Ans. (4)** **3. Ans. (4)** **4. Ans. (2)** **5. Ans. (1)** **6. Ans. (3)**

7. Ans. (3) **8. Ans. (1)** **9. Ans. (3)** **10. Ans. (2)** **11. Ans. (4)** **12. Ans. (3)**

13. Ans. (3) **14. Ans. (2)**

EXERCISE (JA)

1. Ans. 9 **2. Ans. 9** **3. Ans. 8** **4. Ans. 4** **5. Ans. (ACD or ABCD)**

6. Ans. 3 **7. Ans. (C)** **8. Ans. (A)** **9. Ans. (C)** **10. Ans. (A)** **11. Ans. 2**

12. Ans. (A,B) **13. Ans. (B)** **14. Ans. 9** **15. Ans. (A)** **16. Ans. (C)**

17. Ans. 4 [3.99, 4.01] **18. Ans. (2.00)** **19. Ans. (270.00)**

Important Notes

Chapter Contents 02

Kinetic Theory of Gases & Thermodynamics

Important Notes

KINETIC THEORY OF GASES

The properties of the gases are entirely different from those of solid and liquid. In case of gases, thermal expansion is very large as compared to solids and liquids .To state the conditions of a gas, its volume, pressure and temperature must be specified.

Intermolecular force Solid > liquid > real gas > ideal gas (zero)

Potential energy Solid < liquid < real gas < ideal gas (zero)

Internal energy, internal kinetic energy, internal potential energy

At a given temperature for solid, liquid and gas:

(i) Internal kinetic energy : Same for all

(ii) Internal potential Energy : Maximum for ideal gas (PE = 0) and Minimum for solids (PE = –ve)

(iii) Internal Energy : Maximum for Ideal gas and Minimum for solid

At a given temperature for rared and compressed gas :

(i) Internal kinetic energy → Same

(ii) Internal potential energy → $(PE)_{Rared} > (PE)_{compressed}$

(iii) Internal Energy → $(U)_{Rared} > (U)_{compressed}$

	N.T.P.	**S.T.P.**
Temperature	0° C = 273.15 K	0.01° C = 273.16K
Pressure	1 atm = 1.01325×10^5 N/m^2 = 1.01325×10^5 pascal	1 atm
Volume	22.4 litre	22.4 litre

IDEAL GAS CONCEPT

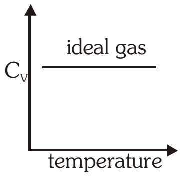
- A gas which follows all gas laws and gas equation at every possible temperature and pressure is known as ideal or perfect gas.
- Volume of gas molecules is negligible as compared to volume of container so volume of gas = volume of container (Except 0 K)
- No intermoleculer force act between gas molecules.
- Potential energy of ideal gas is zero so internal energy of ideal gas is perfectly translational K.E. of gas. It is directly proportional to absolute temperature.

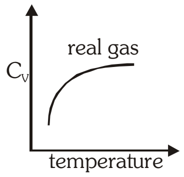
  So, internal energy depends only and only on its temperature.

  $E_{trans} \propto T$

  For a substance $U = U_{KE} + U_{PE}$

  U_{KE} : depends only on T, U_{PE} : depends upon intermolecular forces (Always negative)





- Specific heat of ideal gas is constant quantity and it does not change with temperature
- All real gases behaves as ideal gas at high temperature and low pressure.
- Volume expansion coefficient (α) and pressure expansion coefficient (β) is same for a ideal gas and value of each is $\frac{1}{273}$ per °C $\alpha = \beta = \frac{1}{273}$ per °C
- Gas molecule have point mass and negligible volume and velocity is very high (10^7 cm/s). That's why there is no effect of gravity on them.

EQUATION OF STATE FOR IDEAL GAS

$PV = \mu RT$ where μ = number of moles of gas $\Rightarrow PV = \frac{M}{M_w}RT = \left[\frac{mN}{mN_0}\right]RT = \left[\frac{R}{N_0}\right]N\,T = NkT$

Ex. By increasing temperature of gas by 5° C its pressure increases by 0.5% from its initial value at constant volume then what is initial temperature of gas ?

Sol. $\because$ At constant volume $T \propto P$ $\therefore \frac{\Delta T}{T}\times 100 = \frac{\Delta P}{P}\times 100 = 0.5 \Rightarrow T = \frac{5\times 100}{0.5} = 1000\text{K}$

Ex. Calculate the value of universal gas constant at STP.

Sol. Universal gas constant is given by $R = \frac{PV}{T}$

One mole of all gases at S.T.P. occupy volume $V = 22{\cdot}4$ litre $= 22{\cdot}4 \times 10^{-3}\ \text{m}^3$

$P = 760$ mm of Hg $= 760 \times 10^{-3} \times 13{\cdot}6 \times 10^3 \times 9.80\ \text{N m}^{-2}$ $\quad T = 273$ K

$$\therefore R = \frac{760\times 10^{-3}\times 13.6\times 10^3\times 9.80\times 22.4\times 10^{-3}}{273} = 8{\cdot}31\ \text{J mol}^{-1}\ \text{K}^{-1}$$

Ex. A closed container of volume 0.02 m^3 contains a mixture of neon and argon gases at a temperature of 27°C and pressure of $1 \times 10^5\ \text{Nm}^2$. The total mass of the mixture is 28 g. If the gram molecular weights of neon and argon are 20 and 40 respectively, find the masses of the individual gases in the container, assuming them to be ideal. Given : R = 8.314 J/mol/K.

Sol. Let m gram be the mass of neon. Then, the mass of argon is (28 – m)g.

Total number of moles of the mixture, $\mu = \frac{m}{20} + \frac{28-m}{40} = \frac{28+m}{40}$ ...(i)

Now, $\mu = \frac{PV}{RT} = \frac{1\times 10^5\times 0.02}{8.314\times 300} = 0.8$ ...(ii)

By (i) and (ii), $\frac{28+m}{40} = 0.8 \Rightarrow 28 + m = 32 \Rightarrow m = 4$ gram or mass of argon = (28 – 4)g = 24 g

Ex. Calculate the temperature of the Sun if density is 1.4 g cm^{-3}, pressure is 1.4×10^9 atmosphere and average molecular weight of gases in the Sun in 2 g/mole. **[Given $R = 8.4\ \text{J mol}^{-1}\text{K}^{-1}$]**

Sol. $PV = \mu RT \Rightarrow T = \frac{PV}{\mu R}$ ...(i) But $\mu = \frac{M}{M_w}$ and $\rho = \frac{M}{V}$ $\therefore \mu = \frac{\rho V}{M_w}$

From equation (i) $T = \frac{PVM_w}{\rho VR} = \frac{PM_w}{\rho R} = \frac{1.4\times 10^9\times 1.01\times 10^5\times 2\times 10^{-3}}{1.4\times 1000\times 8.4} = 2.4 \times 10^7$ K

Ex. At the top of a mountain a thermometer reads 7°C and barometer reads 70 cm of Hg. At the bottom of the mountain they read 27°C and 76 cm of Hg respectively. Compare the density of the air at the top with that at the bottom.

Sol. By gas equation $PV = \frac{M}{M_w}RT \Rightarrow \frac{P}{\rho T} = \frac{R}{M_w}\left[\because \mu = \frac{M}{M_w} \text{ and } \frac{M}{V} = \rho\right]$

Now as M_W and R are same for top and bottom $\left[\frac{P}{\rho T}\right]_T = \left[\frac{P}{\rho T}\right]_B$ So $\frac{\rho_T}{\rho_B} = \frac{P_T}{P_B} \times \frac{T_B}{T_T} = \frac{70}{76} \times \frac{300}{280} = \frac{75}{76} = 0.9868$

Ex. During an experiment an ideal gas is found to obey an additional law VP^2 = constant. The gas is initially at temperature T and volume V. What will be the temperature of the gas when it expands to a volume 2V.

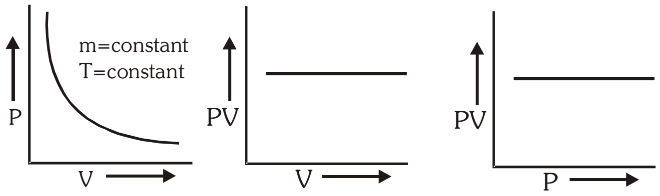
Sol. By gas equation $PV = \mu RT$ and VP^2 = constant on eliminating P

$$\left[\frac{A}{\sqrt{V}}\right]V = \mu RT \Rightarrow \sqrt{V} = \frac{\mu R}{A}T \quad \therefore \frac{\sqrt{V_1}}{\sqrt{V_2}} = \left[\frac{T_1}{T_2}\right] \Rightarrow \frac{\sqrt{V}}{\sqrt{2V}} = \frac{T}{T'} \quad \Rightarrow T' = \left(\sqrt{2}\right)T$$

GAS LAWS

- **Boyle's Law**

According to it for a given mass of an ideal gas at constant temperature, the volume of a gas is inversely proportional to its pressure, i.e., $V \propto \frac{1}{P}$ if m and T = Constant



Ex. A sample of oxygen with volume of 500 cc at a pressure of 2 atm is compressed to a volume of 400 cc. What pressure is needed to do this if the temperature is kept constant ?

Sol. Temperature is constant, so $P_1V_1 = P_2V_2$ $\therefore P_2 = P_1\frac{V_1}{V_2} = 2\left[\frac{500}{400}\right] = 2.5$ atm

Ex. An air bubble doubles in radius on rising from bottom of a lake to its surface. If the atmosphere pressuer is equal to that due to a column of 10 m of water, then what will be the depth of the lake. (Assuming that surface tension is negligible) ?

Sol. Given that constant temperature, we use $P_1V_1 = P_2V_2$

$P_2 = (10)$ dg (for water column) $\qquad P_1 = (10+h)$ dg (where h=depth of lake)

$$V_1 = \frac{4\pi}{3}r^3,\ V_2 = \frac{4\pi}{3}(2r)^3 = 8\left(\frac{4\pi}{3}r^3\right) = 8V_1 \text{ Thus for } P_2V_2 = P_1V_1,$$

We have $10\,dg\,(8V_1) = (10 + h)\,dg\,V_1 \Rightarrow 80 = 10 + h \Rightarrow h = 70$ m

Ex. A vessel of volume 8.0×10^{-3} m^3 contains an ideal gas at 300 K and 200 k Pa. The gas is allowed to leak till the Pressure falls to 125 kPa. Calculate the amount of the gas leaked assuming that the temperature remains constant.

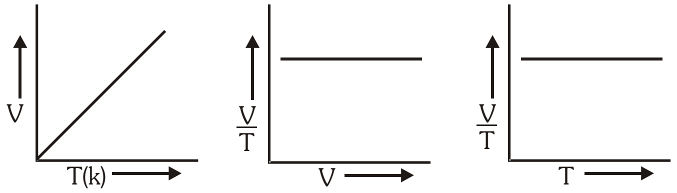
Sol. As the gas leaks out, the volume and the temperature of the remaining gas do not change. The number of moles of the gas in the vessel in given by $n = \frac{PV}{RT}$.

The number of moles in the vessel before the leakage is $n_1 = \frac{P_1V}{RT}$ and that after the leakage is $n_2 = \frac{P_2V}{RT}$.

The amount leaked is $n_1 - n_2 = \frac{(P_1 - P_2)V}{RT} = \frac{(200-125)\times 10^3 \times 8.0 \times 10^{-3}}{8.3 \times 300} = 0.24$ mole

- **Charle's Law**

According to it for a given mass of an ideal gas at constant pressure, volume of a gas is directly proportional to its absolute temperature, i.e. $V \propto T$ if m and P = Constant



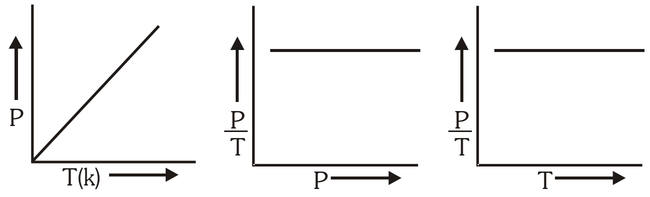
Ex. 1500 ml of a gas at a room temperature of 23°C is inhaled by a person whose body temperature is 37°C, if the pressure and mass stay constant, what will be the volume of the gas in the lungs of the person ?

Sol. $T_1 = 273 + 37 = 310$ K; $T_2 = 273 + 23 = 296$ K. Pressure and amount of the gas are kept constant,

So $\frac{V_1}{T_1} = \frac{V_2}{T_2} \therefore V_2 = V_1 \times \frac{T_2}{T_1} = 1500 \times \frac{293}{310} = 1417.74$ ml

Gay–Lussac's Law

According to it, for a given mass of an ideal gas at constant volume, pressure of a gas is directly proportional to its absolute temperature, i.e., $P \propto T$ if m and V = constant



Ex. A sample of O_2 is at a pressure of 1 atm when the volume is 100 ml and its temperature is 27°C. What will be the temperature of the gas if the pressure becomes 2 atm and volume remains 100 ml.

Sol. $T_1 = 273 + 27 = 300$ K

For constant volume $\frac{P_1}{T_1} = \frac{P_2}{T_2} \Rightarrow T_2 = T_1 \times \frac{P_2}{P_1} = 300 \times \frac{2}{1} = 600$ K $= 600 - 273 = 327$°C

Avogadro's Law

According to it, at same temperature and pressure of equal volumes of all gases contain equal number of molecules, i.e., $N_1 = N_2$ if P,V and T are same.

The kinetic theory of gases

- Rudolph Claussius (1822–88) and James Clark Maxwell (1831–75) developed the kinetic theory of gases in order to explain gas laws in terms of the motion of the gas molecules. The theory is based on following assumptions as regards to the motion of molecules and the nature of the gases.

Basic postulates of Kinetic theory of gases

- Every gas consists of extremely small particles known as molecules. The molecules of a given as are all identical but are different than those another gas.
- The molecules of a gas are identical, spherical, rigid and perfectly elastic point masses.
- The size is negligible in comparision to inter molecular distance (10^{-9} m)

Assumptions regarding motion :

- Molecules of a gas keep on moving randomly in all possible direction with all possible velocities.
- The speed of gas molecules lie between zero and infinity (very high speed).
- The number of molecules moving with most probable speeds is maximum.

Assumptions regarding collision:

- The gas molecules keep on colliding among themselves as well as with the walls of containing vessel. These collision are perfectly elastic. (ie., the total energy before collision = total energy after the collisions.)

Assumptions regarding force:

- No attractive or repulsive force acts between gas molecules.
- Gravitational attraction among the molecules is ineffective due to extremely small masses and very high speed of molecules.

Assumptions regarding pressure:

- Molecules constantly collide with the walls of container due to which their momentum changes. This change in momentum is transferred to the walls of the container. Consequently pressure is exerted by gas molecules on the walls of container.

Assumptions regarding density:

- The density of gas is constant at all points of the container.

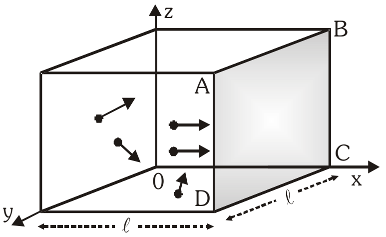
PROPERTIES/ASSUMPTIONS OF IDEAL GAS

- The molecules of a gas are in a state of continuous random motion. They move with all possible velocities in all possible directions. They obey Newton's law of motion.
- Mean momentum = 0; Mean velocity = 0.$< \vec{v} > = 0$; $< v^2 > \neq 0$ (Non zero); $< v^3 > = < v^5 > = 0$
- The average distance travelled by a molecule between two successive collisions is called as mean free path (λ_m) of the molecule.
- The time during which a collision takes place is negligible as compared to time taken by the molecule to cover the mean free path so NTP ratio of time of collision to free time of motion 10^{-8} : 1.
- When a gas taken into a vessel it is uniformly distributed in entire volume of vessel such that its density, moleculer density, motion of molecules etc. all are identical for all direction, therefore root mean velocity

  $\bar{v}_x^2 = \bar{v}_y^2 = \bar{v}_z^2 \rightarrow$ equal Pressure exerted by the gas in all direction $P_x = P_y = P_z = P \rightarrow$ equal
- All those assumptions can be justified, if number of gas molecules are taken very large

EXPRESSION FOR PRESSURE OF AN IDEAL GAS

Consider an ideal gas enclosed in a cubical vessel of length ℓ. Suppose there are 'N' molecules in a gas which are moving with velocities $\vec{v}_1, \vec{v}_2 \ldots\ldots \vec{v}_N$.



If we consider any single molecule than its instantaneous velocity $\vec{v}$ can be expressed as $\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$

Due to random motion of the molecule $v_x = v_y = v_z$

$$|v| = v_x\sqrt{3} = v_y\sqrt{3} = v_z\sqrt{3} = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

Suppose a molecule of mass m is moving with a velocity v_x towards the face ABCD. It strikes the face of the cubical vessel and returns back to strike the opposite face.

Change in momentum of the molecule per collision $\Delta p = - mv_x - mv_x = - 2\ mv_x$

Momentum transferred to the wall of the vessel per molecule per collision $\Delta p = 2\ mv_x$

The distance travelled by the molecule in going to face ABCD and coming back is 2ℓ.

So, the time between two successive collision is $\Delta t = \dfrac{2\ell}{v_x}$

Number of collision per sec per molecule is $f_c = \dfrac{v_x}{2\ell} = \dfrac{\text{molecule velocity}}{\text{mean free path}}$, $f_c = \dfrac{v_{rms}}{\lambda_m}$ or $f_c = \dfrac{v_m}{\lambda_m}$

Hence momentum transferred in the wall per second by the molecule is = force on the wall

$$\text{force } F = (2\ mv_x)\ \frac{v_x}{2\ell} = \frac{mv_x^2}{\ell} = \frac{mv^2}{3\ell}$$

Pressure exerted by gas molecule $P = \dfrac{F}{A} = \dfrac{1}{3}\dfrac{mv^2}{\ell \times A} \quad \Rightarrow P = \dfrac{1}{3}\dfrac{mv^2}{V} \quad [\because A \times \ell = V]$

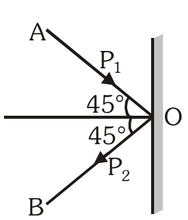
Pressure exerted by gas $P = \sum \dfrac{1}{3}\dfrac{mv^2}{V} = \sum \dfrac{1}{3}\dfrac{mv^2}{V} \times \dfrac{N}{N} = \dfrac{1}{3}\dfrac{mN}{V}\dfrac{\sum v^2}{N} = \dfrac{1}{3}\dfrac{mN}{V}v_{rms}^2$

$$v_{rms}^2 = \frac{3PV}{M} = \frac{3\mu RT}{\mu M_w} \Rightarrow v_{rms} = \sqrt{\frac{3RT}{M_w}},\ P = \frac{1}{3}\frac{M}{V}v_{rms}^2 = \frac{1}{3}\rho v_{rms}^2$$

- Average number of molecules for each wall $= \dfrac{N}{6}$. No. of molecules along each axis $= \dfrac{N}{3}$ $(N_x = N_y = N_z)$
- $\overline{v}_x^2 = \overline{v}_y^2 = \overline{v}_z^2 = \dfrac{\overline{v}_{rms}^2}{3}$ Root mean square velocity along any axis for gas molecule is $(v_{rms})_\chi = (v_{rms})_y$ $= (v_{rms})_z = \dfrac{v_{rms}}{\sqrt{3}}$

All gas laws and gas equation can be obtained by expression of pressure of gas (except Joule's law)

Ex. The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} molecules are colliding per second on a stationary wall of area 2 cm^2 at an angle of 45° to the normal to the wall and reflected elastically with a speed 10^3 m/s. Find the pressure exerted on the wall will be (in N/m^2)



Sol. As the impact is elastic $\therefore |\vec{p}_1| = |\vec{p}_2| = p = mv = 3.32 \times 10^{-24}$ kg m/s

The change in momentum along the normal $\Delta p = |\vec{p}_2 - \vec{p}_1| = 2p\cos 45° = \sqrt{2}p$

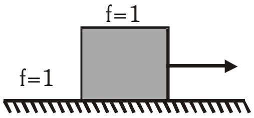
If f is the collision frequency then force applied on the wall $F = \dfrac{\Delta p}{\Delta t} = \Delta p \times f = \sqrt{2}pf$

$$\therefore \quad \text{Pressure } P = \frac{F}{A} = \frac{\sqrt{2}pf}{A} = \frac{\sqrt{2} \times 3.32 \times 10^{-24} \times 10^{23}}{2 \times 10^{-4}} = 2.347 \times 10^{3}\ \text{N/m}^2$$

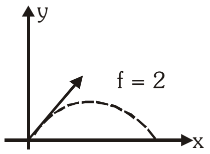
DEGREE OF FREEDOM (f)

- The number of independent ways in which a molecule or an atom can exhibit motion or have energy is called it's degrees of freedom.

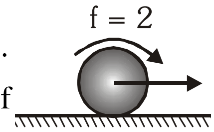
For example



(a) Block has one degree of freedom, because it is confined to move in a straight line and has only one transistional degree of freedom.



(b) The projectile has two degrees of freedom becomes it is confined to move in a plane and so it has two translational degrees of freedom.

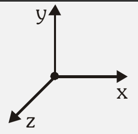
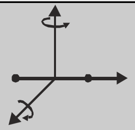


(c) The sphere has two degrees of freedom one rotational and another translational. Similarly a particle free to move in space will have three translational degrees of freedom.

Note : In pure rolling sphere has one degree of freedom as KE = $\dfrac{1}{2}mv^2\left(1+\dfrac{K^2}{R^2}\right) = \dfrac{7}{10}mv^2$

- The degrees of freedom are of three types :
  - **(a) Translational Degree of freedom :** Maximum three degree of freedom are there corresponding to translational motion.
  - **(b) Rotational Degree of freedom** : The number of degrees of freedom in this case depends on the structure of the molecule.
  - **(c) Vibrational Degree of freedom :** It is exhibited at high temperatures.

Degree of freedom for different gases according to atomicity of gas at low temperature

Atomicity of gas	Translational	Rotational	Total	
Monoatomic **Ex.** Ar, Ne, Ideal gas etc	3	0	3	
Diatomic **Ex.** O_2, Cl_2, N_2 etc.	3	2	5	

At high temperatures a diatomic molecule has 7 degrees of freedom. (3 translational, 2 rotational and 2 vibrational)

Ex. Calculate the total number of degrees of freedom possessed by the molecules in one cm^3 of H_2 gas at NTP.

Sol. 22400 cm^3 of every gas constains 6.02×10^{23} molecules.

$\therefore$ Number of molecules in 1 cm^3 of H_2 gas $= \dfrac{6.02 \times 10^{23}}{22400} = 0.26875 \times 10^{20}$

Number of degrees of freedom of a H_2 gas molecule = 5

$\therefore$ Total number of degrees of freedom of $0.26875 \times 10^{20} \times 5 = 1.34375 \times 10^{20}$.

MAXWELL'S LAW OF EQUIPARTITION OF ENERGY

The total kinetic energy of a gas molecules is equally distributed among its all degree of freedom and the energy associated with each degree of freedom at absolute temperature T is $\frac{1}{2}kT$

For one molecule of gas

Energy related with each degree of freedom = $\frac{1}{2}kT$

Energy related with all degree of freedom = $\frac{f}{2}kT$ $\because \overline{v}_x^2 = \overline{v}_y^2 = \overline{v}_z^2 = \frac{\overline{v}_{rms}^2}{3} \Rightarrow \frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT$

So energy related with one degree of freedom = $\frac{1}{2}m\frac{v_{rms}^2}{3} = \frac{3}{2}\frac{kT}{3} = \frac{1}{2}kT$

Ex. A cubical box of side 1 meter contains helium gas (atomic weight 4) at a pressure of 100 N/m^2. During an observation time of 1 second, an atom travelling with the root–mean–square speed parallel to one of the edges of the cube, was found to make 500 hits with a particular wall, without any collision with other atoms. Take $R = \frac{25}{3}$ J/mol–K and $k = 1.38 \times 10^{-23}$ J/K.

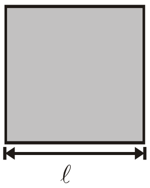
(a) Evaluate the temperature of the gas.

(b) Evaluate the average kinetic energy per atom.

(c) Evaluate the total mass of helium gas in the box.

Sol. Volume of the box = 1m^3, Pressure of the gas = 100 N/m^2. Let T be the temperature of the gas

(a) Time between two consecutive collisions with one wall = $\frac{1}{500}$ sec



This time should be equal to $\frac{2I}{v_{rms}}$, where ℓ is the side of the cube.

$$2\ell v_{rms} = \frac{1}{500} \Rightarrow v_{rms} = 1000 \text{ m/s} \therefore \sqrt{\frac{3RT}{M}} = 1000 \Rightarrow T = \frac{(1000)^2 M}{3R} = \frac{(10)^6(3\times10^{-3})}{3\left(\frac{25}{3}\right)} = 160\text{K}$$

(b) Average kinetic energy per atom = $\frac{3}{2}kT = \frac{3}{2}$ $[(1.38 \times 10^{-23}) = 160]$ J $= 3.312 \times 10^{-21}$ J

(c) From $PV = nRT = \frac{m}{M}RT$, Mass of helium gas in the box $m = \frac{PVM}{RT}$

Substituting the values, $m = \dfrac{(100)(1)(4\times10^{-3})}{\left(\frac{25}{3}\right)(160)} = 3.0 \times 10^{-4}$ kg

DIFFERENT K.E. OF GAS (INTERNAL ENERGY)

- **Translatory kinetic energy (E_T)** $E_T = \frac{1}{2}Mv_{rms}^2 = \frac{3}{2}PV$

  Kinetic energy of volume V is $= \frac{1}{2}Mv_{rms}^2$ Note : Total internal energy of ideal gas is kinetic

- **Energy per unit volume or energy density (E_V)**

  $E_V = \frac{\text{Total energy}}{\text{Volume}} = \frac{E}{V}$; $E_V = \frac{1}{2}\left[\frac{M}{V}\right]v_{rms}^2 = \frac{1}{2}\rho v_{rms}^2 \because P = \frac{2}{3}\left[\frac{1}{2}\rho v_{rms}^2\right] \therefore E_V = \frac{3}{2}P$

- **Molar K.E. or Mean Molar K.E. (E)**

  $E = \frac{1}{2}M_w v_{rms}^2$ for N_0 molecules or M_w (gram) $\quad E = \frac{3}{2}RT = \frac{3}{2}N_0 kT$

- **Molecular kinetic energy or mean molecular K.E. ($\bar{E}$)**

  $E = \frac{1}{2}M_w v_{rms}^2, \bar{E} = \frac{E}{N_0} = \frac{3}{2}\frac{RT}{N_0} \Rightarrow \bar{E} = \frac{3}{2}\ kT$

GOLDEN KEY POINT

- Except 0 K, at any temperature T , $E > E_m > \bar{E}$
- At a common temperature, for all ideal gas

  E and $\bar{E}$ are same while E_m is different and depends upon nature of gas (M_w or m)
- For thermal equilibrium of gases, temperature of each gas is same and this temperature called as temperature of mixture (T_m) which can be find out on basis of conservation of energy (All gases are of same atomicity). $T_m = \frac{\sum NT}{\sum N} = \frac{N_1T_1 + N_2T_2 + \ldots\ldots\ldots + N_nT_n}{N_1 + N_2 \ldots\ldots N_n}$
- **1 mole gas :** Mean kinetic energy $= \frac{3}{2}RT$; Total kinetic energy $= \frac{f}{2}RT$

  1 molecule of gases : Mean kinetic energy $= \frac{3}{2}kT$; Total kinetic energy $= \frac{f}{2}kT$

  f → Degree of freedom

Ex. Two ideal gases at temperature T_1 and T_2 are mixed. There is no loss of energy. If the masses of molecules of the two gases are m_1 and m_2 and number of their molecules are n_1 and n_2 respectively. Find the temperature of the mixture.

Sol. Total energy of molecules of first gas $= \frac{3}{2}n_1kT_1$, Total energy of molecules of second gas $= \frac{3}{2}n_2kT_2$

Let temperature of mixture be T then total energy of molecules of mixture $= \frac{3}{2}k(n_1 + n_2)T$

$$\therefore \frac{3}{2}(n_1 + n_2)kT = \frac{3}{2}k(n_1T_1 + n_2T_2) \Rightarrow T = \frac{n_1T_1 + n_2T_2}{(n_1 + n_2)}$$

Ex. The first excited state of hydrogen atom is 10.2 eV above its ground state. What temperature is needed to excite hydrogen atoms to first excited level.

Sol. K.E. of the hydrogen atom $\frac{3}{2}kT = 10.2$ eV $= 10.2 \times \left(1.6 \times 10^{-19}\right)$ J

$$\Rightarrow T = \frac{2}{3} \times \frac{10.2 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23}} = 7.88 \times 10^4 \text{ K}$$

DIFFERENT SPEEDS OF GAS MOLECULES

- **Average velocity**

Because molecules are in random motion in all possible direction in all possible velocity. Therefore, the average velocity of the gas in molecules in container is zero. $<\vec{v}> = \dfrac{\vec{v}_1+\vec{v}_2+............\vec{v}_N}{N} = 0$

rms speed of molecules $v_{rms} = \sqrt{\dfrac{3P}{\rho}} = \sqrt{\dfrac{3RT}{M_w}} = \sqrt{\dfrac{3kT}{m}} = 1.73\sqrt{\dfrac{kT}{m}}$

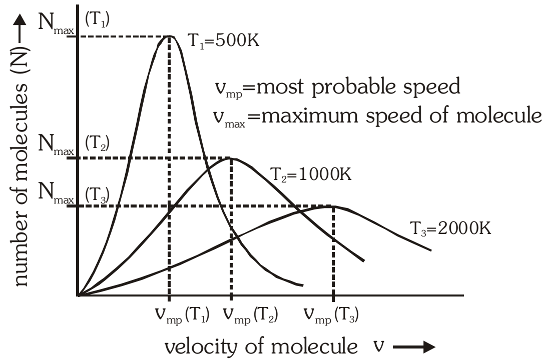
Mean speed of molecules :By maxwell's velocity distribution law v_M or $<|\vec{v}|> = v_{mean}$

$$<|\vec{v}|> = v_{mean} = \frac{|\vec{v}_1|+|\vec{v}_2|+..........|\vec{v}_n|}{N} = \sqrt{\frac{8}{\pi}\frac{P}{\rho}} = \sqrt{\frac{8RT}{\pi M_w}} = \sqrt{\frac{8kT}{\pi m}} = 1.59\sqrt{\frac{kT}{m}}$$

Most probable speed of molecules (v_{mp})

At a given temperature, the speed to which maximum number of molecules belongs is called as most probable speed (v_{mp}) $v_{mp} = \sqrt{\dfrac{2P}{\rho}} = \sqrt{\dfrac{2RT}{M_w}} = \sqrt{\dfrac{2kT}{m}} = 1.41\sqrt{\dfrac{kT}{m}}$

MAXWELL'S LAW OF DISTRIBUTION OF VELOCITIES



GOLDEN KEY POINT

- At any given temperature graph drawn in between molecular velocity and number of molecules is known as velocity distribution curve.
- The velocities of molecules of a gas are in between zero and infinity ($0-\infty$)
- With the increase in the temperature, the most probable velocity and maximum molecule velocity both increases.
- The number of molecules within certain velocity range is constant although the velocity of molecule changes continuously at particular temperature.
- The area enclosed between the (N – v) curve and the velocity axis presents the total number of molecules.

On the basis of velocity distribution Maxwell established gives the law of equipartition of energy for gases of any temperature.

Ex. The velocities of ten particles in ms^{-1} are 0, 2, 3, 4, 4, 4, 5, 5, 6, 9. Calculate
(i) average speed and (ii) rms speed (iii) most probable speed.

Sol. (i) average speed, $v_{av} = \dfrac{0+2+3+4+4+4+5+5+6+9}{10} = \dfrac{42}{10} = 4{\cdot}2\ ms^{-1}$

(ii) rms speed, $v_{rms} = \left[\dfrac{(0)^2+(2)^2+(3)^2+(4)^2+(4)^2+(4)^2+(5)^2+(5)^2+(6)^2+(9)^2}{10}\right]^{1/2} = \left[\dfrac{228}{10}\right]^{1/2} = 4.77\ ms^{-1}$

(iii) most probable speed $v_{mp} = 4$ m/s

Ex. At what temperature, will the root mean square velocity of hydrogen be double of its value at S.T.P., pressure remaining constant ?

Sol. Let v_1 be the r.m.s. velocity at S.T.P. and v_2 be the r.m.s. velocity at unknown temperature T_2.

$$\therefore \quad \frac{v_1^2}{v_2^2} = \frac{T_1}{T_2} \quad \text{or} \quad T_2 = T_1\left[\frac{v_2}{v_1}\right]^2 = 273 \times (2)^2 = 273 \times 4 = 1092\ \text{K} = (1092 - 273) = 819°\text{C}$$

Ex. Calculate rms velocity of oxygen molecule at 27°C

Sol. Temperature, T = 27° C $\Rightarrow$ 273 + 27 = 300 K,

Molecular weight of oxygen = 32×10^{-3} kg and $R = 8{\cdot}31\ \text{J mol}^{-1}\ \text{K}^{-1}$

rms velocity is $v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8\cdot 31 \times 300}{32 \times 10^{-3}}} = 483{\cdot}5\ \text{ms}^{-1}$

Ex. Calculate the kinetic energy of a gram moelcule of argon at 127°C.

Sol. Temperature, T = 127°C = 273 + 127 = 400 K, R = 8.31 J/mol K

K.E. per gram molecule of argon = $\frac{3}{2}\,RT = \frac{3}{2} \times 8.31 \times 400 = 4986$ J

THERMODYNAMICS

Branch of physics which deals with the inter–conversion between heat energy and any other form of energy is known as thermodynamics. In this branch of physics we deals with the processes involving heat, work and internal energy. In this branch of science the conversion of heat into mechanical work and vice versa is studied.

- **Thermodynamical System**
  The system which can be represented in of pressure (P), volume (V) and temperature (T), is known thermodynamic system. A specified portion of matter consisting of one or more substances on which the effects of variables such as temperature, volume and pressure are to be studied, is called a system. e.g. A gas enclosed in a cylinder fitted with a piston is a system.

- **Surroundings**
  Anything outside the system, which exchanges energy with the system and which tends to change the properties of the system is called its surroundings.

- **Heterogeneous System**
  A system which is not uniform throughout is said to be heterogeneous. e.g. A system consisting of two or more immiscible liquids.

- **Homogeneous System**
  A system is said to be homogeneous if it is completely uniform throughout. e.g. Pure solid or liquid.

- **Isolated System**
  A system in which there can be no exchange of matter and energy with the surroundings is said to be an isolated system.

- **Universe**
  The system and its surroundings are together known as the universe.

- **Thermodynamic variables of the system**
  (i) Composition (μ) (ii) Temperature (T) (iii) Volume (V) (iv) Pressure (P)

- **Thermodynamic state**
  The state of a system can be described completely by composition, temperature, volume and pressure. If a system is homogeneous and has definite mass and composition, then the state of the system can be described by the remaining three variables namely temperature, pressure and volume. These variables are interrelated by equation $PV = \mu RT$ The thermodynamic state of the system is its condition as identified by two independent thermodynamic variables (P, V or P, T or V, T).

- **Zeroth law of thermodynamics**
  If objects A and B are separately in thermal equilibrium with a third object C (say thermometer), then objects A and B are in thermal equilibrium with each other. Zeroth law of thermodynamics introduce the concept of temperature. Two objects (or systems) are said to be in thermal equilibrium if their temperatures are the same.
  In measuring the temperature of a body, it is important that the thermometer be in the thermal equilibrium with the body whose temperature is to be measured.

- **Thermal equilibrium**
  Thermal equilibrium is a situation in which two objects in thermal contact cease to exchange energy by the process of heat. Heat is the transfer of energy from one object to another object as a result of a difference in temperature between them.

- **Internal Energy**

Internal energy of a system is the energy possessed by the system due to molecular motion and molecular configuration. The energy due to molecular motion is called internal kinetic energy (U_k) and that due to molecular configuration is called internal potential energy (U_p). $dU = dU_k + dU_p$

If there no intermolecular forces, then $dU_p = 0$ and $dU = dU_k = m\, c_v\, dT$

c_v = Specific heat at constant volume and dT = Infinitesimal change in temperature

m = Mass of system M = Molecular weight

Molar heat capacity $C_v = Mc_v$ For μ–moles of ideal gas $dU = \mu C_v dT = \frac{m}{M} C_v dT$

Internal energy in the absence of inter–molecular forces is simply the function of temperature and state only, it is independent of path followed. $\Delta U = U_f - U_i$

U_i = Internal energies in initial state and U_f = Internal energies in final state

- **Thermodynamic Processes**

In the thermodynamic process pressure, volume, temperature and entropy of the system change with time.

Thermodynamic process is said to take place if change occurs in the state of a thermodynamic system.

- **Sign convention used for the study of thermodynamic processes**

Heat gained by a system	Positive
Heat lost by a system	Negative
The work done by a system	Positive
Work done on the system	Negative
Increase in the internal energy of system	Positive
Decrease in the internal energy of system	Negative

- **Indicator Diagram or P–V Diagram**

In the equation of state of a gas PV = μRT

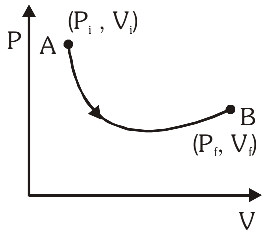
Two thermodynamic variables are sufficient to describe the behavior of a thermodynamic system.

If any two of the three variables P, V and T are known then the third can be calculated.

P–V diagram is a graph between the volume V and the pressure P of the system.

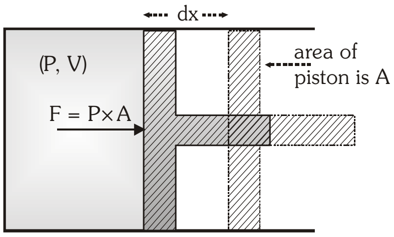
The volume is plotted against X–axis while the pressure is plotted against Y–axis.

The point A represents the initial stage of the system. Initial pressure of the system is P_i and initial volume of the system V_i .



The point B represents the final state of the system. P_f and V_f are the final pressure and final volume respectively of the system. The points between A and B represent the intermediate states of the system. With the help of the indicator diagram we calculate the amount of work done by the gas or on the gas during expansion or compression.

WORK DONE BY THERMODYNAMIC SYSTEM



One of the simple example of a thermodynamic system is a gas in a cylinder with a movable piston.

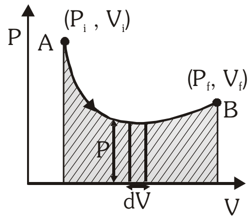
- **If the gas expands against the piston**

  Gas exerts a force on the piston and displace it through a distance and does work on the piston.

- **If the piston compresses the gas**

  When piston moved inward, work is done on the gas.

- **The work associated with volume changes**



If pressure of gas on the piston = P.

Then the force on the piston due to gas is F = PA

When the piston is pushed outward an infinitesimal distance dx, the work done by the gas is dW = F × dx = PA dx

The change in volume of the gas is dV = Adx, ∴ dW = PdV

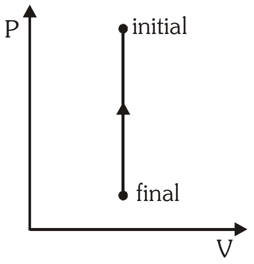
For a finite change in volume from V_i to V_f, this equation is then integrated between V_i to V_f to find the

$$\text{net work done } W = \int_{V_i}^{V_f} dW = \int_{V_i}^{V_f} PdV$$

Hence the work done by a gas is equal to the area under P–V graph.

Following different cases are possible.

(i) Volume is constant



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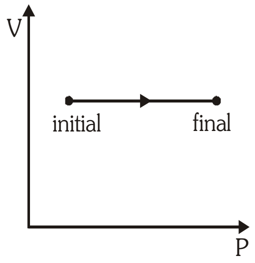
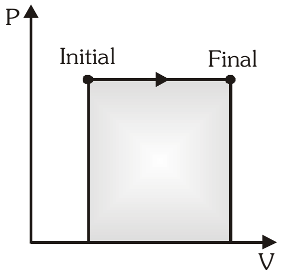


V = constant and $W_{AB} = 0$

(ii) Volume is increasing



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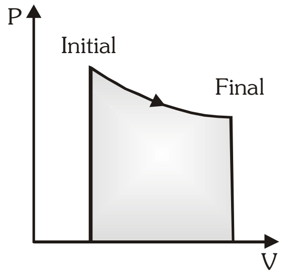
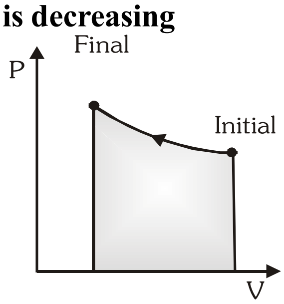


V is increasing $W_{AB} > 0$ W_{AB} = Shaded area

(iii) Volume is decreasing



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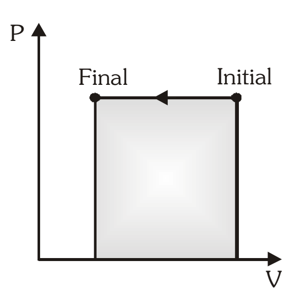
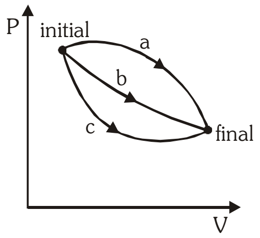


V is decreasing $W_{AB} < 0$ W_{AB} = – Shaded area

FIRST LAW OF THERMODYNAMICS



Let a gas in a cylinder with a moveable piston changes from an initial equilibrium state to a final equilibrium state.

System change its state through path **'a'** :

The heat absorbed by the system in this process = δQ_a

The work done by the system = δW_a

- **Again for path 'b' :**

Heat absorbed by the system = δQ_b , Work done by the system = δW_b.

It is experimental fact that the $\delta Q_a - \delta W_a = \delta Q_b - \delta W_b$

Both δQ and δW depend on the thermodynamic path taken between two equilibrium states, but difference (δQ – δW) does not depends on path in between two definite states of the system.

So, there is a function (internal energy) of the thermodynamic coordinates (P, V and T) whose final value (U_f) minus its initial value (U_i) equals the change δQ – δW in the process.

- **$dU = \delta Q - \delta W$.** This is the first law of thermodynamics.

Heat supplied to the system and work done by the system are path dependent so they are denoted by δQ and δW respectively. Change in internal energy $\Delta U = U_f - U_i$ does not depends on path it depends only on initial and final positions of the system. So, it is denoted by dU (or ΔU)

- **First Law of Thermodynamics**

If some quantity of heat is supplied to a system capable of doing external work, then the quantity of heat absorbed by the system is equal to the sum of the increase in the internal energy of the system and the external work done by the system. $\delta Q = dU + \delta W$ or $Q = W + \Delta U$

* This law is applicable to every process in nature

* The first law of thermodynamics introduces the concept of internal energy.

* The first law of thermodynamics is based on the law of conservation of energy.

* δQ, dU and δW must be expressed in the same units (either in units of work or in units of heat).

* This law is applicable to all the three phases of matter, i.e., solid, liquid and gas.

* dU is a characteristic of the state of a system, it may be any type of internal energy–translational kinetic energy, rotational kinetic energy, binding energy etc.

- **Limitations of first law of thermodynamics** :

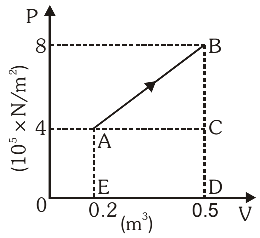
It does not explain the direction of heat flow and it does not explain how much amount of heat given will be converted into work.

- **Significance of the first law of thermodynamics** :

The first law of thermodynamics tells us that it is impossible to get work from any machine without giving it an equivalent amount of energy.

Ex. The pressure in monoatomic gas increases linearly from 4×10^5 Nm^{-2} to $8 \times 10^{+5}$ Nm^{-2} when its volume increases from $0.2m^3$ to 0.5 m^3. Calculate.

(i) Work done by the gas, (ii) Increase in the internal energy,
(iii) Amount of heat supplied, (iv) Molar heat capacity of the gas $R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$



Sol. $P_1 = 4 \times 10^5 \text{ Nm}^{-2}$ $P_2 = 8 \times 10^{+5} \text{ Nm}^{-2}$, $V_1 = 0.2 \text{ m}^3$, $V_2 = 0.5 \text{ m}^3$

(i) Work done by the gas = Area under P–V graph (Area ABCDEA)

$$= \frac{1}{2}(AE + BD) \times AC = \frac{1}{2}(4 \times 10^5 + 8 \times 10^5) \times (0.5 - 0.2)$$

$$= \frac{1}{2} \times 12 \times 10^5 \times 0.3 = 1.8 \times 10^5 \text{ J}$$

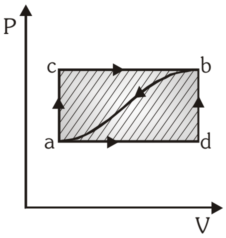
(ii) Increase in internal energy $\Delta U = C_V(T_2 - T_1) = \frac{C_V}{R} R(T_2 - T_1) = \frac{C_V}{R}(P_2V_2 - P_1V_1)$

For monoatomic gas

$$C_V = \frac{3}{2}R \quad \therefore \Delta U = \frac{3}{2}[(8 \times 10^5 \times 0.5) - (4 \times 10^5 \times 0.2)] = \frac{3}{2}[4 \times 10^5 - 0.8 \times 10^5] = 4.8 \times 10^5 \text{ J}$$

(iii) $Q = \Delta U + W = 4.8 \times 10^5 + 1.8 \times 10^5 = 6.6 \times 10^5 \text{ J}$

(iv) $$C = \frac{Q}{\eta \Delta T} = \frac{QR}{\eta R \Delta T} = \frac{QR}{\eta(P_2V_2 - P_1V_1)} = \frac{6.6 \times 10^5 \times 8.31}{1 \times 3.2 \times 10^5} = 17.14 \text{ J/mole K}$$



Ex. When a system is taken from state a to state b, in figure along the path a → c → b, 60 J of heat flow into the system, and 30 J of work is done :

(i) How much heat flows into the system along the path a → d → b if the work is 10 J.

(ii) When the system is returned from b to a along the curved path, the work done by the system is –20 J. Does the system absorb or liberate heat, and how much?

(iii) If, $U_a = 0$ and $U_d = 22$ J, find the heat absorbed in the process a → d and d → b.

Sol. For the path a, c, b, $\Delta U = Q - W = 60 - 30 = 30$ J or $U_b - U_a = 30$ J

(i) Along the path a, d, b, $Q = \Delta U + W = 30 + 10 = 40$ J

(ii) Along the curved path b, a, $Q = (U_a - U_b) + W = (-30) + (-20) = -50$ J, heat flows out the system

(iii) $Q_{ad} = 32$ J; $Q_{db} = 8$ J

ISOMETRIC OR ISOCHORIC PROCESS

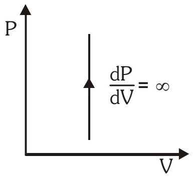
- Isochoric process is a thermodynamic process that takes place at constant volume of the system, but pressure and temperature varies for change in state of the system.

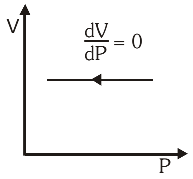
  Equation of state P = constant × T (P and T are variable, V is constant)

  Work done In this process volume remains constant $\Delta V = 0$ or $dV = 0 \Rightarrow W = \int_{V_i}^{V_f} P dV = 0$

  Form of first Law $Q = \Delta U$

  It means whole of the heat supplied is utilized for change in internal energy of the system. $Q = \Delta U = \mu C_v \Delta T$





  Slope of the P–V curve $\frac{dP}{dV} = \infty$

Specific heat at constant volume (C_V)

The quantity of heat required to raise the temperature of 1 gram mole gas through 1 °C at constant volume is equal to the specific heat at constant volume.

- A gas enclosed in a cylinder having rigid walls and a fixed piston. When heat is added to the gas, there would be no change in the volume of the gas.

- When a substance melts, the change in volume is negligibly small. So, this may be regarded as a nearly isochoric process.
- Heating process in pressure cooker is an example of isometric process.

Ex. An ideal gas has a specific heat at constant pressure $C_P = \frac{5R}{2}$. The gas is kept in a closed vessel of volume $0.0083\ m^3$ at a temperature of 300K and a pressure of $1.6 \times 10^6\ Nm^{-2}$. An amount of 2.49×10^4 J of heat energy is supplied to the gas. Calculate the final temperature and pressure of the gas.

Sol. $C_V = C_P - R = \frac{5R}{2} - R = \frac{3R}{2}$, $\Delta V = 0$, $T_1 = 300$ K, $V = 0.0083\ m^3$, $P_1 = 1.6 \times 10^6 Nm^{-2}$

From first law of thermodynamics $Q = \Delta U + P\Delta V \Rightarrow \Delta U = Q = 2.49 \times 10^4$ J

From gas equation $n = \frac{PV}{RT} = \frac{1.6 \times 10^6 \times 0.0083}{8.3 \times 300} = \frac{16}{3}$

$\therefore \Delta U = nC_V\Delta T \Rightarrow \Delta T = \frac{\Delta U}{nC_v} = \frac{2.49 \times 10^4 \times 6}{3 \times 8.3 \times 16} = 375$ K

Final temperature = 300 + 375 = 675K

According to pressure law $P \propto T \Rightarrow \frac{P_2}{P_1} = \frac{T_2}{T_1} \Rightarrow P_2 = \frac{T_2}{T_1} \times P_1 = \frac{1.6 \times 10^6 \times 675}{300} = 3.6 \times 10^6\ Nm^{-2}$

Ex. 5 moles of oxygen is heated at constant volume from 10°C to 20°C. What will be change in the internal energy of the gas? The gram molecular specific heat of oxygen at constant pressure.
$C_P = 8$ cal/mole and R = 8.36 joule/mole °C.

Sol. $\because C_V = C_P - R = 8 - 2 = 6$ cal/mole °C

$\therefore$ Heat absorbed by 5 moles of oxygen at constant volume

$Q = nC_V \Delta T = 5 \times 6\ (20 - 10) = 30 \times 10 = 300$ cal

At constant volume $\Delta V = 0$. $\therefore \Delta W = 0$

$\therefore$ From first law of thermodynamics $Q = \Delta U + W \Rightarrow 300 = \Delta U + 0 \Rightarrow \Delta U = 300$ cal

ISOBARIC PROCESS

Isobaric process is a thermodynamic process that takes place at constant pressure, but volume and temperature varies for change in state of the system.

- **Equation of state** $V = \text{constant} \times T$ or $V \propto T$
- **Work done** In this process pressure remains constant $\Delta P = 0$

  Work done $W = \int_{V_i}^{V_f} PdV = P(V_f - V_i)$
- **Form of first Law** $Q = \Delta U + P\ (V_f - V_i)$

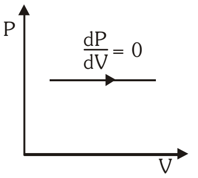
  $\mu\ C_p\ dT = \mu\ C_v\ dT + P(V_f - V_i)$

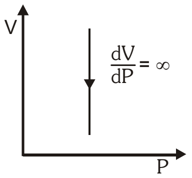
It is clear that heat supplied to the system is utilized for :

(i) Increasing internal energy and

(ii) Work done against the surrounding atmosphere.

- **Slope of the PV curve :** $\left(\frac{dP}{dV}\right)_{\text{isobaric}} = 0$





- **Specific heat at constant pressure (C_P)**

  The quantity of heat required to raise the temperature of 1 gram mole gas through 1°C at constant pressure is equal to the specific heat. Heating of water at atmospheric pressure. • Melting of solids and boiling of liquids at atmospheric pressure.

Ex. At normal pressure and 0°C temperature the volume of 1 kg of ice is reduced by 91 cm^3 on melting. Latent heat of melting of ice is 3.4×10^5 J/kg. Calculate the change in the internal energy when 2kg of ice melts at normal pressure and 0°C. ($P=1.01 \times 10^5\ Nm^{-2}$)

Sol. Heat energy absorbed by 2 kg of ice for melting $Q = mL = 2 \times 3.4 \times 10^5 = 6.8 \times 10^5 J$

Change in volume of 2 kg of ice $= 2 \times 91 = 182\ cm^3 = 182 \times 10^{-6}\ m^3$

$\therefore\ W = P\Delta V = 1.01 \times 10^5 \times (-182 \times 10^{-6}) = -18.4\ J$

Since, work is done on ice so work W is taken –ve. Now from first law of thermodynamics

$$Q = \Delta U + W \Rightarrow \Delta U = Q - W \quad = 6.8 \times 10^5 - (-18.4) = (6.8 \times 10^5 + 18.4)J$$

Ex. What amount of heat must be supplied to 2.0×10^{-2} kg of nitrogen (at room temperature) to raise the temperature by 45°C at constant pressure. Molecular mass of $N_2 = 28$, $R = 8.3\ J\ mol^{-1}\ K^{-1}$.

Sol. Here $m = 2 \times 10^{-2}$ kg, $\Rightarrow n = \frac{m}{M} = \frac{2\times10^{-2}}{28\times10^{-3}} = \frac{5}{7}$ & $C_P = \frac{7}{2}R$

$\therefore\ Q = nC_P\Delta T = \frac{5}{7}\times\frac{7}{2}\times 8.3\times 45 = 933.75$ J

ISOTHERMAL PROCESS

In this process pressure and volume of system change but temperature remains constant.

In an isothermal process, the exchange of heat between the system and the surroundings is allowed.

Isothermal process is carried out by either supplying heat to the substance or by extracting heat from it.

A process has to be extremely slow to be isothermal.

- **Equation of state**

$$P V = \text{constant } (\mu RT) \quad (T \text{ is constant})$$

- **Work Done**

  Consider μ moles of an ideal gas, enclosed in a cylinder, at absolute temperature T, fitted with a frictionless piston. Suppose that gas undergoes an isothermal expansion from the initial state (P_1, V_1) to the final state (P_2, V_2).

$$\therefore \text{ Work done : } W = \int_{V_1}^{V_2}\frac{\mu RT}{V}dV = \mu RT\int_{V_1}^{V_2}\frac{dV}{V} = \mu RT[\log_e V]_{V_1}^{V_2}$$

$$= \mu RT[\log_e V_2 - \log_e V_1] = \mu RT\log_e\left[\frac{V_2}{V_1}\right]$$

$$\Rightarrow W = 2.303\mu RT\log_{10}\left[\frac{P_1}{P_2}\right] \quad [\because P_1V_1 = P_2V_2]$$

Form of First Law

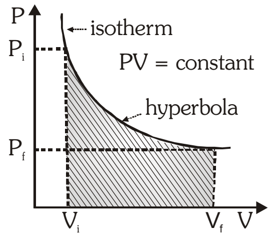
There is no change in temperature and internal energy of the system depends on temperature only

So $\Delta U = 0,\ Q = 2.303\ \mu RT\log_{10}\left[\frac{V_2}{V_1}\right]$

It is clear that Whole of the heat energy supplied to the system is utilized by the system in doing external work. There is no change in the internal energy of the system.

Slope of the isothermal curve

For an isothermal process, PV = constant



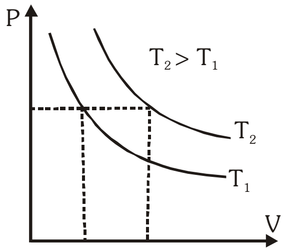
Differentiating, $PdV + VdP = 0 \quad \Rightarrow VdP = -PdV \Rightarrow \frac{dP}{dV} = -\frac{P}{V}$

Slope of isothermal curve, $\left[\frac{dP}{dV}\right]_{\text{isothermal}} = -\frac{P}{V}$

For a given system :

- The product of the pressure and volume of a given mass of a perfect gas remains constant in an isothermal process.
- Boyle's law is obeyed in an isothermal process.
- A graph between pressure and volume of a given mass of a gas at constant temperature is known as isotherm or isothermal of the gas.
- Two isotherms for a given gas at two different temperatures T_1 and T_2 are shown in figure.
- The curves drawn for the same gas at different temperatures are mutually parallel and do not cut each other.
- If two isotherms intersect each other at a single point we get same value of P and V at intersection point.
- $PV = \mu RT_1$ for temperature T_1 and $PV = \mu RT_2$ for temperature T_2.

  It means $T_1 = T_2$ which is not possible.
- An ideal gas enclosed in a conducting cylinder fitted with a conducting piston. Let the gas be allowed to expand very–very slowly.



at const P $V \propto T$ so $T_2 > T_1$

- This shall cause a very slow cooling of the gas, but heat will be conducted into the cylinder from the surrounding. Hence the temperature of the gas remains constant. If the gas is compressed very–very slowly, heat will be produced, but this heat will be conducted to the surroundings and the temperature of the gas shall remain constant.
- The temperature of a substance remains constant during melting. So, the melting process is an isothermal process.
- Boiling is an isothermal process, when a liquid boils, its temperature remains constant.
- If sudden changes are executed in a vessel of infinite conductivity then they will be isothermal.

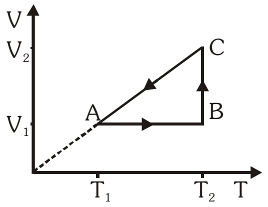
Ex. Two moles of a gas at 127°C expand isothermally until its volume is doubled. Calculate the amount of work done.

Sol. $n = 2, \quad T = 127 + 273 = 400K, \ \frac{V_2}{V_1} = 2$

From formula $W = 2.3026\, nRT \log_{10} \dfrac{V_2}{V_1} = 2.3026 \times 2 \times 8.3 \times 400 \times \log_{10} 2$

$$= 2.3026 \times 2 \times 8.3 \times 400 \times 0.3010 \approx 4.6 \times 10^3 J$$

Ex. Figure shows a process ABCA performed on an ideal gas.



Find the net heat given to the system during the process.

Sol. Since the process is cyclic, hence the change in internal energy is zero.

The heat given to the system is then equal to the work done by it.

The work done in part AB is $W_1 = 0$ (the volume remains constant). The part BC represents an isothermal process so that the work done by the gas during this part is $W_2 = nRT_2 \, \ell n \dfrac{V_2}{V_1}$

During the part CA $V \propto T$ So, V/T is constant and hence, $P = \dfrac{nRT}{V}$ is constant

The work done by the gas during the part CA is $W_3 = P(V_1 - V_2) = nRT_1 - nRT_2 = - nR\,(T_2 - T_1)$.

The net work done by the gas in the process ABCA is $W = W_1 + W_2 + W_3 = nR\left[T_2 \ell n \dfrac{V_2}{V_1} - (T_2 - T_1)\right]$

The same amount of heat is given to the gas.

ADIABATIC PROCESS

It is that thermodynamic process in which pressure, volume and temperature of the system change but there is no exchange of heat between the system and the surroundings.

A sudden and quick process will be adiabatic since there is no sufficient time available for exchange of heat so process adiabatic.

Equation of state : $PV = \mu RT$

Equation for adiabatic process PV^{γ} = constant

Work done

Let initial state of system is (P_1, V_1, T_1) and after adiabatic change final state of system is (P_2, V_2, T_2) then we can write $P_1 V_1^{\gamma} = P_2 V_2^{\gamma} = K$ (here K is const.)

So $$W = \int_{V_1}^{V_2} PdV = K\int_{V_1}^{V_2} V^{-\gamma} dV = K\left(\frac{V^{-\gamma+1}}{-\gamma+1}\right)_{V_1}^{V_2} = \frac{K}{(-\gamma+1)}\left[V_2^{-\gamma+1} - V_1^{-\gamma+1}\right] \quad \left(\because\ K = P_1V_1^{\gamma} = P_2V_2^{\gamma}\right)$$

$$\Rightarrow W = \frac{1}{(\gamma-1)}\left[P_1V_1^{\gamma}V_1^{-\gamma}.V_1 - P_2V_2^{\gamma}V_2^{-\gamma}.V_2\right] = \frac{1}{(\gamma-1)}\,[P_1V_1 - P_2V_2]$$

$$\Rightarrow W = \frac{\mu R}{(\gamma-1)}(T_1 - T_2) \quad (\because\ PV = \mu RT)$$

Form of first law : dU = – δW

It means the work done by an ideal gas during adiabatic expansion (or compression) is on the cost of change in internal energy proportional to the fall (or rise) in the temperature of the gas.

If the gas expands adiabatically, work is done by the gas. So, W_{adia} is positive.

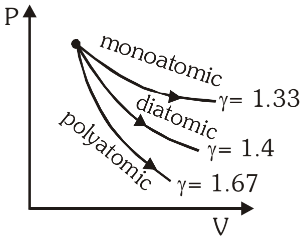
The gas cools during adiabatic expansion and $T_1 > T_2$.

If the gas is compressed adiabatically, work is done on the gas. So, W_{adia} is negative.

The gas heats up during adiabatic compression and $T_1 < T_2$.

Slope of the adiabatic curve

For an adiabatic process, PV^γ = constant

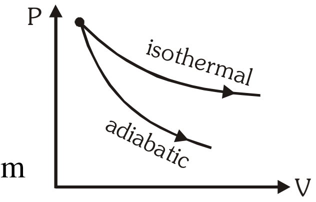

(Adiabatic expansion of gas)

Differentiating, $P\gamma V^{\gamma-1} dV + V^\gamma dP = 0$

$$\Rightarrow V^\gamma dP = -\gamma PV^{\gamma-1} dV \Rightarrow \frac{dP}{dV} = -\frac{\gamma PV^{\gamma-1}}{V^\gamma} = -\gamma\frac{P}{V} = \gamma\left(-\frac{P}{V}\right)$$

Slope of adiabatic curve, $\left[\frac{dP}{dV}\right]_{adiabatic} = -\frac{\gamma P}{V}$

Slope of adiabatic is greater than the slope of isotherm


(expansion of gas)

$$\left[\frac{dP}{dV}\right]_{adia} = \gamma\left[-\frac{P}{V}\right] = \gamma\left[\frac{dP}{dV}\right]_{iso} \Rightarrow \frac{\text{slope of adiabatic changes}}{\text{slope of isothermal changes}} = \gamma$$

Since γ is always greater than one so an adiabatic is steeper than an isotherm

Examples of adiabatic process

- A gas enclosed in a thermally insulated cylinder fitted with a non–conducting piston. If the gas is compressed suddenly by moving the piston downwards, some heat is produced. This heat cannot escape the cylinder. Consequently, there will be an increase in the temperature of the gas.
- If a gas is suddenly expanded by moving the piston outwards, there will be a decrease in the temperature of the gas.
- Bursting of a cycle tube.
- Propagation of sound waves in a gas.
- In diesel engines burning of diesel without spark plug is done due to adiabatic compression of diesel vapour and air mixture

Ex. Why it is cooler at the top of a mountain than at sea level?

Sol. Pressure decreases with height. Therefore if hot air rises, it suffers adiabatic expansion.

From first law of thermodynamics $\Delta Q = \Delta U + \Delta W \Rightarrow \Delta U = -\Delta W\ [\because \Delta Q = 0]$

This causes a decrease in internal energy and hence a fall of temperature.

Ex. 2m^3 volume of a gas at a pressure of 4×10^5 Nm^{-2} is compressed adiabatically so that its volume becomes 0.5m^3. Find the new pressure. Compare this with the pressure that would result if the compression was isothermal. Calculate work done in each process. $\gamma = 1.4$

Sol. $V_1 = 2m^3$, $P_1 = 4 \times 10^5$ Nm^{-2}, $V_2 = 0.5m^3$

In adiabatic process $P_1V_1^{\gamma} = P_2V_2^{\gamma} \Rightarrow P_2 = 4 \times 10^5\left[\frac{2}{0.5}\right]^{1.4} = 4 \times 10^5 (4)^{1.4} = 2.8 \times 10^6$ Nm^{-2}

In isothermal process $P_1V_1 = P_2V_2 \Rightarrow P_2 = \frac{P_1V_1}{V_2} = \frac{4 \times 10^5 \times 2}{0.5} = 1.6 \times 10^6$ Nm^{-2}.

Now work done in adiabatic process $W = \frac{P_2V_2 - P_1V_1}{\gamma - 1} = \frac{(2.8 \times 10^6 \times 0.5) - (4 \times 10^5 \times 2)}{1.4 - 1} = 1.48 \times 10^6$ J.

Work done in isothermal process $W = 2.3026RT \log\frac{V_2}{V_1} = 2.3026P_1V_1 \log\frac{V_2}{V_1}$

$$= 2.3026 \times 4 \times 10^5 \times 2 \times \log\left[\frac{0.5}{2.0}\right] = 2.3026 \times 4 \times 10^5 \times 2\log\left(\frac{1}{4}\right) = -1.1 \times 10^6 \text{ J}$$

Ex. Two samples of a gas initially at same temperature and pressure are compressed from a volume V to $\frac{V}{2}$. One sample is compressed isothermally and the other adiabatically. In which sample is the pressure greater?

Sol. Let initial volume, $V_1 = V$ and pressure, $P_1 = P$, final volume, $V_2 = \frac{V}{2}$ and final pressure, $P_2 = ?$

For isothermal compression $P_2V_2 = P_1V_1$ or $P_2 = \frac{P_1V_1}{V_2} = \frac{PV}{\frac{V}{2}} = 2P$

For adiabatic compression $P_2' = P_1\left[\frac{V_1}{V_2}\right]^{\gamma} \Rightarrow P_2' = P\left[\frac{V}{V/2}\right]^{\gamma} = 2^{\gamma}P$

$\Rightarrow P_2' = 2^{\gamma}P \quad \gamma > 1 \therefore 2^{\gamma} > 2$ and $P_2' > P_2$

Pressure during adiabatic compression is greater than the pressure during isothermal compression.

Ex. Calculate the work done when 1 mole of a perfect gas is compressed adiabatically. The initial pressure and volume of the gas are 10^5 N/m^2 and 6 litre respectively. The final volume of the gas is 2 liters. Molar specific heat of the gas at constant volume is $\frac{3R}{2}$ $[(3)^{5/3} = 6.19]$

Sol. For an adiabatic change PV^{γ} = constant $P_1V_1^{\gamma} = P_2V_2^{\gamma}$

As molar specific heat of gas at constant volume $C_V = \frac{3}{2}R$

$$C_P = C_V + R = \frac{3}{2}R + R = \frac{5}{2}R \Rightarrow \gamma = \frac{C_P}{C_V} = \frac{\frac{5}{2}R}{\frac{3}{2}R} = \frac{5}{3}$$

$$\therefore\ P_2 = \left[\frac{V_1}{V_2}\right]^{\gamma} P_1 = \left[\frac{6}{2}\right]^{5/3} \times 10^5 = (3)^{5/3} \times 10^5 = 6.19 \times 10^5\ \text{N/m}^2$$

Work done $W = \dfrac{1}{1-\gamma}[P_2V_2 - P_1V_1] = \dfrac{1}{1-(5/3)}[6.19 \times 10^5 \times 2 \times 10^{-3} - 10^{-5} \times 6 \times 10^{-3}]$

$$= -\left[\frac{2 \times 10^2 \times 3}{2}(6.19 - 3)\right] = -3 \times 10^2 \times 3.19 = -957\ \text{joules}$$

– ive sign shows external work done on the gas

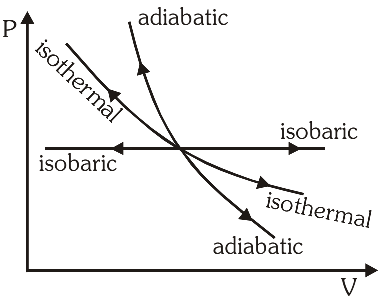
Ex. A motor tyre pumped to a pressure of 3 atm. It suddenly bursts. Calculate the fall in temperature due to adiabatic expansion. The temperature of air before expansion is 27°C. Given $\gamma=1.4$.

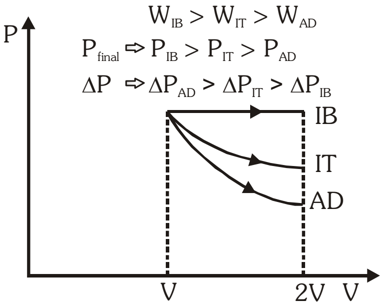
Sol. We know that $T_2^{\gamma}P_2^{1-\gamma} = T_1^{\gamma}P_1^{1-\gamma} \Rightarrow \left[\dfrac{T_2}{T_1}\right]^{\gamma} = \left[\dfrac{P_1}{P_2}\right]^{1-\gamma} \Rightarrow \left[\dfrac{T_2}{300}\right]^{1.4} = \left[\dfrac{3}{1}\right]^{1-1.4}$

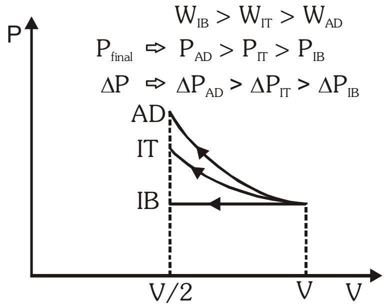
$$\Rightarrow \left[\frac{T_2}{300}\right]^{1.4} = \left[\frac{1}{3}\right]^{0.4} \Rightarrow T_2 = 219.2\ \text{K} \Rightarrow T_1 - T_2 = (300 - 219.2)\ \text{K} = 80.8\ \text{K}$$

GOLDEN KEY POINT

- When a gas expands its volume increases, then final pressure is less for adiabatic expansion. But, when a gas compresses its volume decreases, then the final pressure is more in case of adiabatic compression.



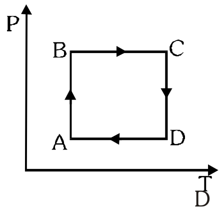




First Law of Thermodynamics Applied to Different Processes

Process	Q	ΔU	W
Cyclic	W	0	Area of the closed curve
Isochoric	ΔU	$\mu C_V\Delta T$ (μ mole of gas)	0
Isothermal	W	0	$\mu RT\log_e\left[\dfrac{V_f}{V_i}\right] = \mu RT\log_e\left[\dfrac{P_i}{P_f}\right]$
Adiabatic	0	–W	$\dfrac{\mu R(T_f - T_i)}{1-\gamma}$
Isobaric	$\mu C_P\Delta T$	$\mu C_V\Delta T$	$P(V_f - V_i) = \mu R(T_f - T_i)$

Ex. Plot P – V , V – T graph corresponding to the P–T graph for an ideal gas shown in figure. Explain your answers.

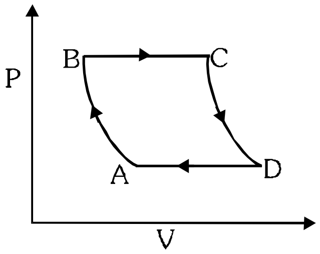


Sol. For process AB T = constant so $P \propto \frac{1}{V}$

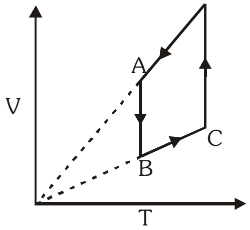
For process BC P = constant so $V \propto T$

For process CD T = constant so $V \propto \frac{1}{P}$

For process DA P = constant so $V \propto T$



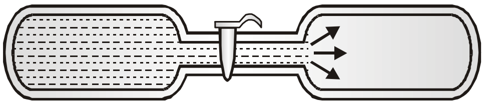
(P - V curve)



(V - T curve)

FREE EXPANSION

Take a thermally insulated bottle with ideal gas at some temperature T_1 and, by means of a pipe with a stopcock, connect this to another insulated bottle which is evacuated. If we suddenly open the stopcock, the gas will rush from the first bottle into the second until the pressures are equalized.



free expansion of a gas

Experimentally, we find that this process of free expansion does not change the temperature of the gas – when the gas attains equilibrium and stops flowing, the final temperature of both botles are equal to the initial temperature T_1.

This process is called a free expansion.

The change in the internal energy of the gas can be calculated by applying the first law of thermodynamics to the free–expansion process.

The process is adiabatic because of the insulation, So Q = 0.

No part of the surroundings moves so the system does no work on its surroundings.

- **For ideal gas** $(\delta W)_{ext.}$ = Work done against external atmosphere

  $= P\,dV = 0$ (because P = 0)

  $(\delta W)_{int.}$ = Work done against internal molucular forces = 0

  $\delta Q = dU + \delta W \quad \Rightarrow \quad 0 = dU + 0$

The internal energy does not change dU = 0 So U and T are constant.

The initial and final states of this gas have the same internal energy.

Which implies that the internal energy of an ideal gas does not depend on the volume at all.

The free–expansion process has led us to the following conclusion :

The internal energy U(T) of an ideal gas depends only on the temperature.

- **For real gas**

In free expansion of real gases, measurements show that the temperature changes slightly in a free expansion. Which implies that the internal energy of a real gas depends on the volume also.

$\delta Q = 0 \quad (\delta W)_{ext.} = 0 \quad (\because P = 0)$

$(\delta W)_{int.} \neq 0$ (Intermolucular forces are present in real gases)

$\delta Q = dU + \delta W \Rightarrow 0 = dU + (\delta W)_{int.} \Rightarrow dU = -(\delta W)_{int.}$

$\Rightarrow$ U decreases. So T decreases.

RELATION BETWEEN DEGREE OF FREEDOM AND SPECIFIC HEAT OF GAS

Energy related with each degree of freedom $=\frac{1}{2}kT$, Energy related with all degree of freedom $=\frac{f}{2}kT$

Internal energy of one mole of ideal gas (total K.E.) $U=\frac{f}{2}RT$ for Isometric process (volume constant)

$\delta W = 0$

By first law of thermodynamics $\quad \delta Q = \delta W + dU \;\Rightarrow\; C_V\, dT = dU \;\Rightarrow\; C_V = \frac{dU}{dT}$

$$C_V = \frac{dU}{dT} = \frac{f}{2}R = \frac{R}{\gamma-1} \quad . C_P = C_V + R = \left[\frac{f}{2}+1\right]R = \frac{\gamma R}{\gamma-1} \quad \text{and} \quad \gamma = \frac{C_P}{C_V} = 1+\frac{2}{f}$$

$$C_V = \frac{R}{\gamma-1} \quad , \quad C_P = \frac{\gamma R}{\gamma-1} \quad \text{and} \quad \gamma = 1+\frac{2}{f}$$

General expression for C (C_P or C_V) in the process PV^x = constant $C=\frac{R}{\gamma-1}+\frac{R}{1-x}$

For isobaric process P = constant so $x = 0$ $\quad \therefore\; C = C_P = \frac{R}{\gamma-1}+R = C_V + R$

For isothermal process, PV = constant so $x = 1$ $\quad \therefore\; C = \infty$

For adiabatic process PV^γ = constant so $x = \gamma$ $\quad \therefore\; C = 0$

Values of f, U, C_V, C_P and γ for different gases are shown in table below.

Atomicity of gas	f	C_V	C_P	γ
Monoatomic	3	$\frac{3}{2}R$	$\frac{5}{2}R$	$\frac{5}{3}=1.67$
Diatomic	5	$\frac{5}{2}R$	$\frac{7}{2}R$	$\frac{7}{5}=1.4$
Triatomic and Triatomic linear	7	$\frac{7}{2}R$	$\frac{9}{2}R$	$\frac{9}{7}=1.28$
Poly atomic Triangular Non-linear	6	$\frac{6}{2}R=3R$	$\frac{8}{2}R=4R$	$\frac{4}{3}=1.33$

- $1<\gamma<2$
- If atomicity of gases is same U, C_P, C_V and γ is same for gas mixture.
- If in a gas mixture gases are of different atomicity, then for gas mixture γ changes according to following condition. Diatomic $\gamma_1 \le$ mixture $\le \gamma_2$ mono atomic where $\gamma_1<\gamma_2$
- If 'f' is the degree of freedom per molecule for a gas, then

Total energy of each molecule $=\frac{fkT}{2}$

Total energy per mole of gas $= N_0\frac{f}{2}kT = \frac{f}{2}RT$

- According to kinetic theory of gases, the molecule are not interacting with each other. So potential energy is zero and internal energy of gas molecules is only their kinetic energy.
- For 'μ' mole of a gas : Internal energy at temperature T is $U = \frac{\mu f R T}{2} = \mu C_V T$
- Change in internal energy is given by $dU = \frac{\mu f R}{2}(dT) = \mu C_V dT$

  This change is process independent.

C_P is greater than C_V

If a gas is heated at constant volume, the gas does no work against external pressure. In this case, the whole of the heat energy supplied to the gas is spend in raising the temperature of the gas.

If a gas is heated at constant pressure, its volume increases. In this case, heat energy is required for the following two purpose :

(i) To increase the volume of the gas against external pressure.

(ii) To increase the temperature of 1 mole of gas through 1 K.

Thus, more heat energy is required to raise the temperature of 1 mole of gas through 1 K when it is heated at constant pressure than when it is heated at constant volume. $\therefore C_P > C_V$

The difference between C_P and C_V is equal to thermal equivalent of the work done by the gas in expanding against external pressure.

Mayer's formula : $C_P - C_V = R$

$\because$ At constant pressure $dQ = \mu C_P dT$, $dU = \mu C_V dT$ & $dW = PdV = \mu R dT$

Now from first law of thermodynamics $dQ = dW + dU$

$\Rightarrow \mu C_P dT = \mu R dT + \mu C_V dT \quad \Rightarrow C_P = R + C_V \quad \Rightarrow C_P - C_V = R$

Ex. Calculate the difference between two specific heats of 1 g of helium gas at NTP. Molecular weight of helium = 4 and J = 4.186 × 10^7 erg cal^{-1}.

Sol. Gas constant for 1 g of helium, $r = \frac{R}{M_w} = \frac{PV}{T \times M_w} = \frac{76 \times 13.6 \times 981 \times 22400}{273 \times 4} = 2.08 \times 10^7$ erg g^{-1} K^{-1}

$$C_P - C_V = \frac{r}{J} = \frac{2.08 \times 10^7}{4.186 \times 10^7} = 0.5 \text{ cal g}^{-1} \text{ K}^{-1}$$

Ex. Calculate the molar specific heat at constant volume. Given : specific heat of hydrogen at constant pressure is 6.85 cal mol^{-1} K^{-1} and density of hydrogen = 0.0899 g cm^{-3}. One mole of gas = 2.016 g, J = 4.2 × 10^7 erg cal^{-1} and 1 atmosphere = 10^6 dyne cm^{-2}.

Sol. Since the density of hydrogen is 0.0899 g cm^{-3} therefore volume occupied by 0.0899 g of hydrogen at NTP is 1000 cm^3. So, volume of 1 mole (2.016 g) of gas, $V = \frac{1000}{0.0899} \times 2.016 \text{ cm}^3$

$$C_P - C_V = \frac{R}{J} = \frac{PV}{T \times J} = \frac{10^6 \times 1000 \times 2.016}{0.0899 \times 273 \times 4.2 \times 10^7} = 1.96 \text{ cal mol}^{-1} \text{ K}^{-1}$$

$\therefore C_V = C_P - 1.96 = (6.85 - 1.96) = 4.89$ cal mol^{-1} K^{-1}

Ex. The specific heat of argon at constant volume is 0.075 kcal/kg K. Calculate its atomic weight, [R = 2cal/mol K]

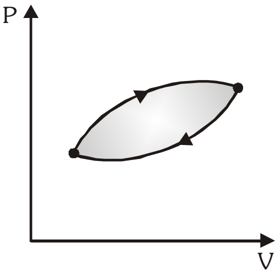
Sol. As argon is monoatomic, its molar specific heat at constant volume will be

$C_V = \frac{3}{2}R = \frac{3}{2} \times 2 = 3$ cal/mol K, $C_V = M_w c_V$ and $c_V = 0.075$ cal/g K

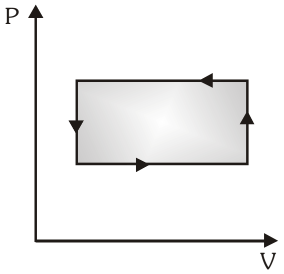
So $3 = M_w \times 0.075 \Rightarrow M_w = \frac{3}{0.075} = 40$ gram/mole

Cyclic process

Cyclic process is that thermodynamic process in which the system returns to its initial stage after undergoing a series of changes. A typical cyclic process is represented on PV diagram as shown in figure.

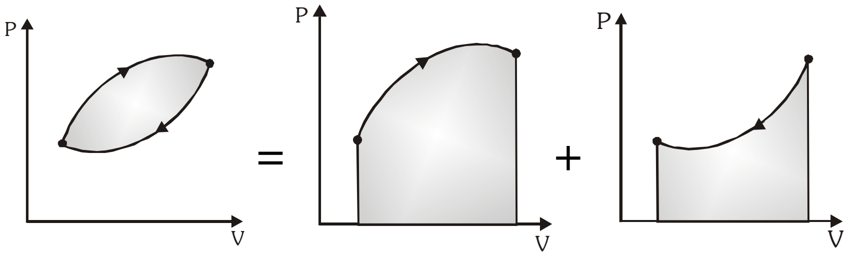


$W_{\text{clockwise cycle}} = +$ Shaded area



$W_{\text{anticlockwise cycle}} = -$ Shaded area

WORK DONE IN CLOCKWISE CYCLE

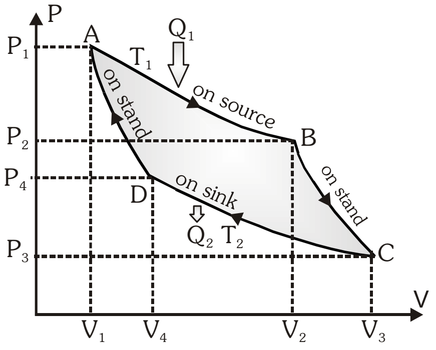


$W_{AB\ I}$ Positive $W_{AB\ II}$ Negative

$W_{\text{cyclic}} = W_{AB\ I}(\text{positive}) + W_{AB\ II}$ (negative) =

Similarly work done in anticlociwise cyclic process is negative.

8. CARNOT CYCLE



Carnot devised an ideal engine which is based on a reversible cycle of four operations in succession :

(i) isothermal expansion, A → B

(ii) adiabatic expansion, B → C

(iii) isothermal compression C → D

(iv) adiabatic compression. D → A

Main parts of Carnot's engine are

Source of heat

It is a hot body of very large heat capacity kept at a constant high temperature T_1.

Mechanical arrangements and working substance

It is a cylinder whose walls are perfectly non-conducting and its base is perfectly conducting fitted with non-conducting piston. This piston move without any friction. Ideal gas enclosed in cylinder as a working substance.

Heat sink

It is a cold body at low temperature T_2. It is a body of large heat capacity.

Stand

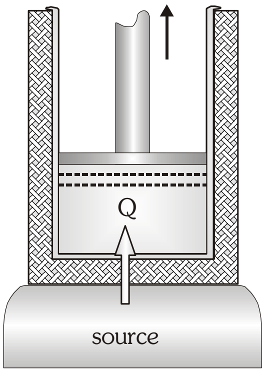
It is of two types : conducting and nonconducting.

Working

Through a set of reversible processes the working substance is taken back to initial condition to get maximum work from this type of ideal engine.

Processes of Carnot's cycle can be denoted by an indicator diagram.

Isothermal expansion A → B



Initially the cyclinder is taken to be in thermal equilibrium with the high temperature T_1, this is initial state of working substance denoted by point A (P_1, V_1, T_1).

After that the piston is allowed to move outward slowly. With the movement of the piston the process is very slow so that it is isothermal.

Heat from reservoir flows through the base of cylinder into the gas so temperature of the gas remains T_1.

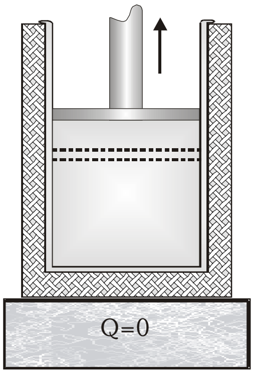
Gas expand and receive heat Q_1 from source and gets state B(P_2, V_2, T_1)

This heat input Q_1 to the gas from path A to B is utilized for doing work W_1.

By path A to B the heat input to the gas = the work done against the external pressure.

$$W_1 = Q_1 = \int_{V_1}^{V_2} PdV = \int_{V_1}^{V_2} \frac{\mu RT_1}{V} dV = \mu RT_1 \ell n \frac{V_2}{V_1}$$

Adiabatic expansion B → C

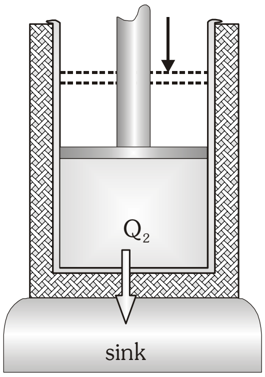


Now cylinder is put in contact with a non-conducting stand and piston is allowed to move outward,because no heat can enter in or leave out so the expansion of gas is adiabatic. The tempeature falls to T_2K and gas describes the adiabatic process from B to **point C** (P_3, V_3, T_2)

during this expansion more work is done (W_2) at the expense of the internal energy.

Work done in adiabatic path BC is $W_2 = \frac{\mu R}{\gamma - 1}(T_1 - T_2)$

Isothermal compression C → D



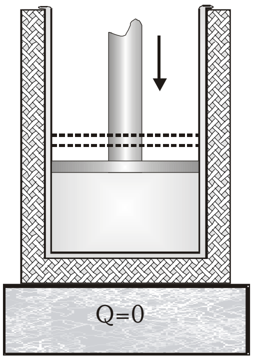
Now the gas cylinder is placed in contact with sink at temperature T_2. The piston is moved slowly inward so that heat produced during compression passes to the sink. The gas is isothermally compressed from C to **point D**. (P_4, V_4, T_2)

The heat rejected Q_2 to the cold reservoir (sink) at T_2 occurs over this path.

Amount of work done on gas W_3 = amount of heat rejected to the sink

$$Q_2 = W_3 = \mu RT_2 \ell n\left(\frac{V_4}{V_3}\right) \Rightarrow Q_2 = \mu RT_2 \ell n\left(\frac{V_4}{V_3}\right)$$

Adiabatic compression D → A



The cylinder is removed from the sink and is put in contact with insulating stand now piston moves inward. Heat is not allowed to go out and it increases the internal energy of the system. Now work is done on the gas during adiabatic compression from state D to initial point A (P_1, V_1, T_1).

No heat exchanges occur over the adiabatic path.

Work done on the system $W_4 = \frac{\mu R}{\gamma - 1}(T_2 - T_1)$

This cycle of operations is called a Carnot cycle.

In first two steps work is done by engine W_1 and W_2 are positive

In last two steps work is done on gas W_3 and W_4 are negative

The work done in complete cylce W = the area of the closed part of the P–V cycle.

$$W = W_1 + W_2 + W_3 + W_4$$

$$\therefore \quad W = \mu RT_1 \ell n\frac{V_2}{V_1} + \frac{\mu R}{\gamma-1}(T_1 - T_2) + \mu RT_2 \ell n\frac{V_4}{V_3} + \frac{\mu R}{\gamma-1}(T_2 - T_1) = \mu RT_1 \ell n\frac{V_2}{V_1} + \mu RT_2 \ell n\frac{V_4}{V_3}$$

Efficiency of Carnot Engine, $\eta = \frac{W}{Q_1} = \frac{\mu RT_1 \ell n\frac{V_2}{V_1} + \mu RT_2 \ell n\frac{V_4}{V_3}}{\mu RT_1 \ell n\frac{V_2}{V_1}}$

B to C and D to A are adiabatic paths

so $T_1V_2^{(\gamma-1)} = T_2V_3^{(\gamma-1)}$ and $T_1V_1^{(\gamma-1)} = T_2V_4^{(\gamma-1)} \Rightarrow \frac{V_2}{V_1} = \frac{V_3}{V_4}$

$$\eta = \frac{T_1 - T_2}{T_1} = \frac{Q_1 - Q_2}{Q_1} \Rightarrow \eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1} \qquad \frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

$$\eta = \frac{T_1 - T_2}{T_1} \times 100\% \Rightarrow \eta = \frac{Q_1 - Q_2}{Q_1} \times 100\%$$

The efficiency for the Carnot engine is the best that can be obtained for any heat engine.

The efficiency of a Carnot engine is never 100% because it is 100% only if temperature of sink $T_2 = 0\text{k}$ and $T_1 = \infty$ which is impossible.

CARNOT THEOREM

No irreversible engine (I) can have efficiency greater than Carnot reversible engine (R) working between same hot and cold reservoirs. $\eta_R > \eta_I \Rightarrow 1 - \frac{T_2}{T_1} > 1 - \frac{Q_2}{Q_1}$

9. SECOND LAW OF THERMODYNAMICS

The first law of thermodynamics is a generalization of the law of conservation of energy to include heat energy. It tells us that heat and mechanical work are mutually interconvertible.

Second law of thermodynamics tells us in what conditions heat can be converted into useful work.

The following three conditions must be fulfilled to utilize heat for useful work :

(i) A device called engine with a working substance is essential.

(ii) The engine must work in a reversible cyclic process.

(iii) The engine must operate between two temperatures. It will absorb heat from a hot body (called source), and converts a part of it into useful work and reject the rest to a cold body (called sink).

There are two conventional statements of second law :

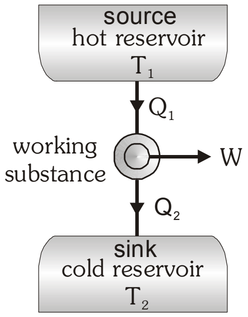
Kelvin-Planck Statement

It is impossible for an engine working between a cyclic process to extract heat from a reservoir and convert completely into work. In other words, 100% conversion of heat into work is impossible.

Clausius Statement

It is impossible for a self-acting machine, unaided by any external agency to transfer heat from a cold to hot reservoir. In other words heat can not in itself flow from a colder to a hotter body.

10. HEAT ENGINE



Heat engine is a device which converts heat into work.

Three parts of a heat engine:

(i) Source of high temperature reservoir at temperature T_1

(ii) Sink or low temperature reservoir at temperature T_2

(iii) Working substance.

In a cycle of heat engine the working substance extracts heat Q_1 from source, does some work W and rejects remaining heat Q_2 to the sink.

Efficiency of heat engine $\eta = \frac{\text{work done (W)}}{\text{heat taken from source }(Q_1)}$

$$\eta = \frac{T_1 - T_2}{T_1} = \frac{Q_1 - Q_2}{Q_1}$$

Ex. A carnot engine working between 400 K and 800 K has a work output of 1200 J per cycle. What is the amount of heat energy supplied to the engine from source per cycle?

Sol. W = 1200 J, T_1 = 800 K, T_2 = 400 K

$$\therefore \eta = 1 - \frac{T_2}{T_1} = \frac{W}{Q_1} \Rightarrow 1 - \frac{400}{800} = \frac{1200}{Q_1} \Rightarrow 0.5 = \frac{1200}{Q_1}$$

Heat energy supplied by source $Q_1 = \frac{1200}{0.5}$ = 2400 joule per cycle

Ex. The temperatures T_1 and T_2 of the two heat reservoirs in an ideal carnot engine are 1500°C and 500°C respectively. Which of the following : increasing T_1 by 100°C or decreasing T_2 by 100°C would result in a greater improvement in the efficiency of the engine?

Sol. T_1 = 1500°C = 1500 + 273 = 1773 K and T_2 = 500°C = 500 + 273 = 773 K.

The efficiency of a carnot's engine $\eta = 1 - \frac{T_2}{T_1}$

When the temperature of the source is increased by 100°C, keeping T_2 unchanged, the new temperature of the source is T'_1 = 1500 + 100 = 1600°C = 1873 K. The efficiency becomes

$$\eta' = 1 - \frac{T_2}{T_1'} = 1 - \frac{773}{1873} = 0.59$$

On the other hand, if the temperature of the sink is decreased by 100°C, keeping T_1 unchanged, the new temperature of the sink is T'_2 = 500 – 100 = 400°C = 673 K. The efficiency now becomes

$$\eta'' = 1 - \frac{T'_2}{T_1} = 1 - \frac{673}{1773} = 0.62$$

Since η′′ is greater than η′, decreasing the temperature of the sink by 100°C results in a greater efficiency than increasing the temperatue of the source by 100°C.

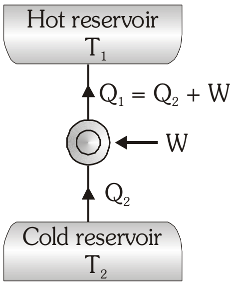
Ex. A heat engine operates between a cold reservoir at temperature T_2 = 300 K and a hot reservoir at temperature T_1. It takes 200 J of heat from the hot reservoir and delivers 120 J of heat to the cold reservoir in a cycle. What could be the minimum temperature of hot reservoir?

Sol. Work done by the engine in a cycle is $W = 200 - 120 = 80$ J. $\eta = \frac{W}{Q} = \frac{80}{200} = 0.4$

From carnot's Theorem $0.4 \le 1 - \frac{T_2}{T_1} = 1 - \frac{300}{T_1}$ or $\frac{300}{T_1} \le 0.6$ or $T_1 \ge \frac{300}{0.6}$ or $T_1 \ge 500$

11. REFRIGERATOR

It is inverse of heat engine. It extracts heat (Q_2) from a cold reservoir, External work W is done on it and rejects heat (Q_1) to hot reservoir. The coefficient of performance of a refrigerator.



$$\beta = \frac{\text{heat extracted from cold reservoir}}{\text{work done on refrigerator}} = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{1}{\frac{Q_1}{Q_2} - 1}$$

For Carnot reversible refrigerator $\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$

$$\therefore\ \beta = \frac{Q_2}{W} = \frac{1}{\left[\frac{Q_1}{Q_2} - 1\right]} = \frac{1}{\left[\frac{T_1}{T_2} - 1\right]} \Rightarrow \beta = \frac{T_2}{T_1 - T_2}$$

Ex. A carnot engine works as a refrigrator between 250 K and 300 K. If it receives 750 cal of heat from the reservoir at the lower temperature. Calculate the amount of heat rejected at the higher temperature.

Sol. T_1 = 300 K $\quad T_2$ = 250 K $\quad Q_2$ = 750 $\quad Q_1$ = ?

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \qquad Q_1 = \frac{300}{250} \times 750 = 900 \text{ cal}$$

Ex. The temperature insides & outside of refrigerator are 260 K and 315 K respectively. Assuming that the refrigerator cycle is reversible, calculate the heat delivered to surroundings for every joule of work done.

Sol. T_2 = 260 K, T_1 = 315 K, W = 1 joule

Coefficient of performance of Carnot refrigerator $\beta = \frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$

$$\therefore\ \frac{Q_2}{1} = \frac{260}{315 - 260} = \frac{260}{55} \Rightarrow Q_2 = \frac{260}{55} = 4.7 \text{ J}$$

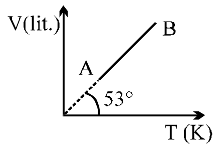
Ex. A refrigerator takes heat from water at 0°C and transfer it to room at 27°C. If 100 kg of water is converted in ice at 0°C then calculate the workdone. (Latent heat of ice is 3.4×10^5 J/kg)

Sol. Coefficient of performance (COP) $= \frac{T_2}{T_1 - T_2} = \frac{273}{300 - 273} = \frac{273}{27}$

$$W = \frac{Q_2}{COP} = \frac{mL}{COP} = \frac{100 \times 3.4 \times 10^5}{273/27} = \frac{100 \times 3.4 \times 10^5 \times 27}{273} = 3.36 \times 10^6 \text{ J}$$

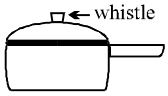
EXERCISE (S-1)

1. V-T curve for 2 moles of a gas is straight line as shown in the graph here. Find the pressure of gas at A.



KT0001

2. An empty pressure cooker of volume 10 litres contains air at atmospheric pressure 10^5 Pa and temperature of 27°C. It contains a whistle which has area of 0.1 cm^2 and weight of 100 g. What should be the temperature of air inside so that the whistle is just lifted up?

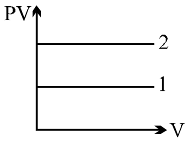


KT0002

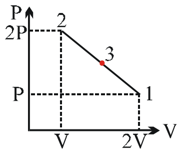
3. Examine the following plots and predict whether in

(i) $P_1 < P_2$ and $T_1 > T_2$, in (ii) $T_1 = T_2 < T_3$, in

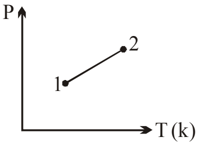
(iii) $V_1 > V_2$, in (iv) $P_1 > P_2$ or otherwise.



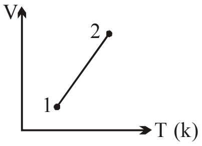
(i)



(ii)



(iii)



(iv)

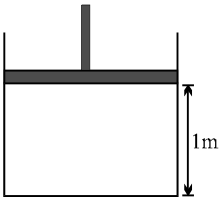
KT0003

4. Find the molecular mass of a gas if the specific heats of the gas are C_P=0.2 cal/gm°C and C_V = 0.15 cal/gm°C. [Take R = 2 cal/mole°C]

KT0004

5. The piston cylinder arrangement shown contains a diatomic gas at temperature 300 K. The cross-sectional area of the cylinder is1 m^2. Initially the height of the piston above the base of the cylinder is 1 m. The temperature is now raised to 400 K at constant pressure. Find the new height of the piston above the base of the cylinder. If the piston is now brought back to its original height without any heat loss, find the new equilibrium temperature of the gas. You can leave the answer in fraction.

[JEE' 2004]



KT0005

6. A mixture of 4 gm helium and 28 gm of nitrogen is enclosed in a vessel of constant volume 300 K. Find the quantity of heat absorbed by the mixture to doubled the root mean velocity of its molecules. (R = Universal gas constant)

KT0006

7. An insulated container containing monoatomic gas of molar mass m is moving with a velocity v_0. If the container is suddenly stopped, find the change in temperature. **[JEE 2003]**

KT0007

8. If heat is added at constant volume, 6300 J of heat are required to raise the temperature of an ideal gas by 150 K. If instead, heat is added at constant pressure, 8800 joules are required for the same temperature change. When the temperature of the gas changes by 300 K, determine the change is the internal energy of the gas.

KT0008

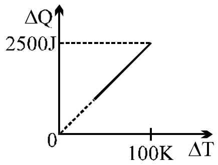
9. Ideal diatomic gas is taken through a process $\Delta Q = 2\Delta U$. Find the molar heat capacity for the process (where ΔQ is the heat supplied and ΔU is change in internal energy)

KT0009

10. A cylinder with a movable piston contains 3 moles of hydrogen at standard temperature and pressure. The walls of the cylinder are made of a heat insulator, and the piston is insulated by having a pile of sand on it. By what factor does the pressure of the gas increase if the gas is compressed to half its original volume ?

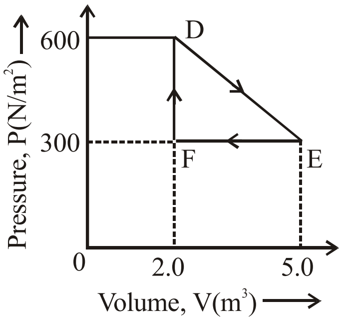
KT0010

11. One mole of a gas mixture is heated under constant pressure, and heat required ΔQ is plotted against temperature difference acquired. Find the value of γ for mixture.



KT0011

12. A thermodynamic system is taken from an original state to an intermediate state by the linear process shown in figure. Its volume is then reduced to the original value from E to F by an isobaric process. Calculate the total work done by the gas from D to E to F

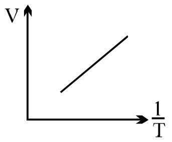


KT0012

13. In changing the state of a gas adiabatically from an equilibrium state A to another equilibrium state B, an amount of work equal to 22.3 J is done on the system. If the gas is taken from state A to B via a process in which the net heat absorbed by the system is 9.35 cal, how much is the net work done by the system in the latter case ? (Take 1 cal = 4.19 J)

KT0013

14. One mole of an ideal monoatomic gas undergoes a process as shown in the figure. Find the molar specific heat of the gas in the process.



KT0014

15. Two cylinders A and B of equal capacity are connected to each other via a stopcock. A contains a gas at standard temperature and pressure. B is completely evacuated. The entire system is thermally insulated. The stopcock is suddenly opened. Answer the following :

(a) What is the final pressure of the gas in A and B ?

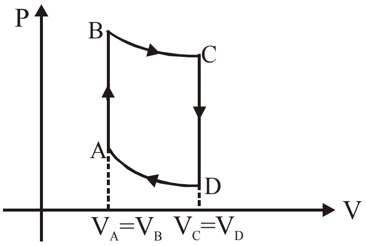
(b) What is the change in internal energy of the gas ?

(c) What is the change in the temperature of the gas ?

(d) Do the intermediate states of the system (before settling to the final equilibrium state) lie on its P-V-T surface ?

KT0015

16. A cycle followed by an engine (made of one mole of perfect gas in a cylinder with a piston) is shown in figure.



A to B : volume constant

B to C : adiabatic

C to D : volume constant

D to A : adiabatic

$V_C = V_D = 2V_A = 2V_B$

(a) In which part of the cycle heat is supplied to the engine from outside?

(b) In which part of the cycle heat is being given to the surrounding by the engine?

(c) What is the work done by the engine in one cycle? Write your answer in term of P_A, P_B, V_A

(d) What is the efficiency of the engine?

$[\gamma = \frac{5}{3}$ for the gas], $(C_V = \frac{3}{2}R$ for one mole)

KT0016

EXERCISE (S-2)

1. A barometer is faulty. When the true barometer reading are 73 and 75 cm of Hg, the faulty barometer reads 69 cm and 70 cm respectively.

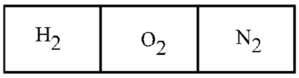
(i) What is the total length of the barometer tube?

(ii) What is the true reading when the faulty barometer reads 69.5 cm ?

(iii) What is the faulty barometer reading when the true barometer reads 74 cm?

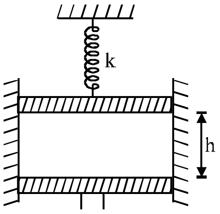
KT0017

2. A vessel of volume V = 30*l* is separated into three equal parts by stationary semipermeable thin membranes as shown in the Figure. The left , middle and right parts are filled with m_{H_2} = 30g of hydrogen, m_{O_2} =160g of oxygen, and m_{N_2} = 70g of nitrogen respectively. The left partition lets through only hydrogen, while the right partition lets through hydrogen and nitrogen. What will be the pressure in each part of the vessel after the equilibrium has been set in if the vessel is kept at a constant temperature T = 300K?



KT0018

3. An ideal gas at NTP is enclosed in a adiabatic vertical cylinder having area of cross section A = 27cm^2, between two light movable pistons as shown in the figure. Spring with force constant k = 3700 N/m is in a relaxed state initially. Now the lower piston is moved upwards a height h/2, h being the initial length of gas column. It is observed that the upper piston moves up by a distance h/16. Find h taking γ for the gas to be 1.5. Also find the final temperature of the gas.



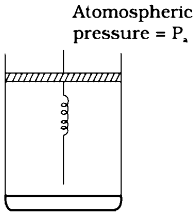
KT0019

4. Consider one mole of perfect gas in a cylinder of unit cross section with a piston attached (Fig.). A spring (spring constant *k*) is attached (unstretched length *L*) to the piston and to the bottom of the cylinder. Initially the spring is unstretched and the gas is in equilibrium. A certain amount of heat Q is supplied to the gas causing an increase of volume from V_0 to V_1.

(a) What is the initial pressure of the system?

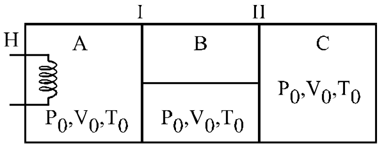
(b) What is the final pressure of the system?

(c) Using the first law of thermodynamics, write down a relation between Q, P_a, V, V_0 and k.



KT0020

5. The figure shows an insulated cylinder divided into three parts A, B and C. Pistons I and II are connected by a rigid rod and can move without friction inside the cylinder. Piston I is perfectly conducting while piston II is perfectly insulating. The initial state of the gas ($\gamma = 1.5$) present in each compartment A, B and C is as shown. Now, compartment A is slowly given heat through a heater H such that the final volume of C becomes $\frac{4V_0}{9}$. Assume the gas to be ideal and find.



(a) Final pressures in each compartment A, B and C

(b) Final temperatures in each compartment A, B and C

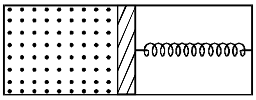
(c) Heat supplied by the heater

(d) Work done by gas in A and B.

(e) Heat flowing across piston I.

KT0021

6. A thermally insulated vessel is divided into two equal parts by a heat-insulating piston which can move in the vessel without the friction. The left part of the vessel contains one mole of an ideal monatomic gas, & the right part is empty. The piston is connected to the right wall of the vessel through a spring whose length in free state is equal to the length of the vessel as shown in the figure. Determine the heat capacity C of the system, neglecting the heat capacities of the vessel, piston and spring.



KT0022

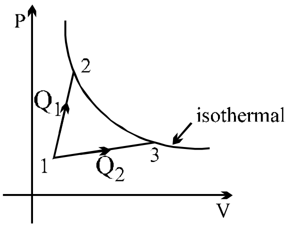
7. A weightless piston divides a thermally insulated cylinder into two parts of volumes V and 3V. 2 moles of an ideal gas at pressure P = 2 atmosphere are confined to the part with volume V = 1 litre. The remainder of the cylinder is evacuated. The piston is now released and the gas expands to fill the entire space of the cylinder. The piston is then pressed back to the initial position. Find the increase of internal energy in the process and final temperature of the gas. The ratio of the specific heat of the gas $\gamma = 1.5$.

KT0023

8. Two moles of an ideal monoatomic gas are confined within a cylinder by a massless & frictionless spring loaded piston of cross-sectional area 4×10^{-3} m^2. The spring is, initially in its relaxed state. Now the gas is heated by an electric heater, placed inside the cylinder, for some time. During this time, the gas expands and does 50 J of work in moving the piston through a distance 0.10 m. The temperature of the gas increases by 50 K. Calculate the spring constant & the heat supplied by the heater. [$P_0 = 10^5$ Pa]

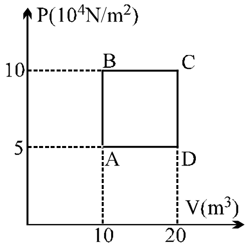
KT0024

9. A gas takes part in two processes in which it is heated from the same initial state 1 to the same final temperature. The processes are shown on the P-V diagram by the straight line 1-2 and 1-3. 2 and 3 are the points on the same isothermal curve. Q_1 and Q_2 are the heat transfer along the two processes. Then in which case will the heat transfer be more.



KT0025

10. A sample of 2 kg of monoatomic Helium (assumed ideal) is taken through the process ABC and another sample of 2 kg of the same gas is taken through the process ADC as in figure. Given, molecular mass of Helium = 4



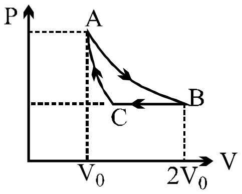
(i) What is the temperature of Helium in each of the states A, B, C & D ?

(ii) Is there any way of telling afterwards which sample of Helium went through the process ABC and which went through the process ADC? Write Yes or No.

(iii) How much is the heat involved in each of the processes ABC & ADC.

KT0026

11. In a cyclic process ABCA consisting of isothermal expansion AB, isobaric compression BC and adiabatic compression CA, find the efficiency of cycle (Given : $T_A = T_B = 400$ K, $\gamma = 1.5$)



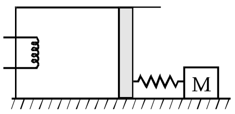
KT0027

12. An adiabatic cylinder has 8 gram of helium. A light smooth adiabatic piston is connected to a light spring of force constant 300 N/m. The other end of the spring is connected with a block of mass 1 kg kept on a rough horizontal surface of coefficient of friction $\mu = 0.3$. Area of cross-section of cylinder is A = 25 cm^2. Initially the spring is in a relaxed position and the temperature of the gas is 400 K. The gas is heated slowly for some time by means of an electric heater so as to bring the block M on the verge of motion. Take $P_{atm} = 10^5$ N/m^2. Find

(a) the work done by the gas

(b) the final temperature

(c) heat supplied by the heater



KT0028

EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

KTG

1. Find the approx. number of molecules contained in a vessel of volume 7 litres at 0°C at 1.3×10^5 pascal

(A) 2.4×10^{23} (B) 3×10^{23} (C) 6×10^{23} (D) 4.8×10^{23}

KT0029

2. An ideal gas mixture filled inside a balloon expands according to the relation $PV^{2/3}$ = constant. The temperature inside the balloon is

(A) increasing (B) decreasing (C) constant (D) can't be said

KT0030

3. At a temperature T K, the pressure of 4.0g argon in a bulb is p. The bulb is put in a bath having temperature higher by 50K than the first one. 0.8g of argon gas had to be removed to maintained original pressure. The temperature T is equal to

(A) 510 K (B) 200 K (C) 100 K (D) 73 K

KT0031

4. 28 gm of N_2 gas is contained in a flask at a pressure of 10 atm and at a temperature of 57°. It is found that due to leakage in the flask, the pressure is reduced to half and the temperature reduced to 27°C. The quantity of N_2 gas that leaked out is :-

(A) 11/20 gm (B) 20/11 gm (C) 5/63 gm (D) 63/5 gm

KT0032

5. A container X has volume double that of container Y and both are connected by a thin tube. Both contains same ideal gas. The temperature of X is 200K and that of Y is 400K. If mass of gas in X is m then in Y it will be:

(A) m/8 (B) m/6 (C) m/4 (D) m/2

KT0033

6. When 2 gms of a gas are introduced into an evacuated flask kept at 25^0C the pressure is found to be one atmosphere. If 3 gms of another gas added to the same flask the pressure becomes 1.5 atmospheres. The ratio of the molecular weights of these gases will be :-

(A) 1 : 3 (B) 3 : 1 (C) 2 : 3 (D) 3 : 2

KT0034

7. A rigid tank contains 35 kg of nitrogen at 6 atm. Sufficient quantity of oxygen is supplied to increase the pressure to 9 atm, while the temperature remains constant. Amount of oxygen supplied to the tank is :

(A) 5 kg (B) 10 kg (C) 20 kg (D) 40 kg

KT0035

8. During an experiment an ideal gas obeys an addition equation of state P^2V = constant. The initial temperature and pressure of gas are T and V respectively. When it expands to volume 2V, then its temperature will be:

(A) T (B) $\sqrt{2}$ T (C) 2 T (D) $2\sqrt{2}$ T

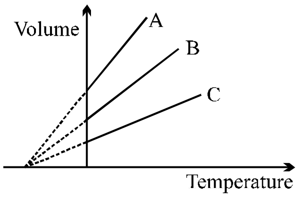
KT0036

9. A vessel contains 1 mole of O_2 gas (molar mass 32) at a temperature T. The pressure of the gas is P. An identical vessel containing one mole of He gas (molar mass 4) at a temperature 2T has a pressure of

(A) P/8 (B) P (C) 2P (D) 8P

KT0037

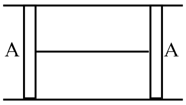
10. The expansion of an ideal gas of mass m at a constant pressure P is given by the straight line B. Then the expansion of the same ideal gas of mass 2 m at a pressure 2P is given by the straight line



(A) C (B) A (C) B (D) none

KT0038

11. A cylindrical tube of cross-sectional area A has two air tight frictionless pistons at its two ends. The pistons are tied with a straight piece of metallic wire. The tube contains a gas at atmospheric pressure P_0 and temperature T_0. If temperature of the gas is doubled then the tension in the wire is



(A) $4 P_0 A$ (B) $P_0A/2$ (C) $P_0 A$ (D) $2 P_0 A$

KT0039

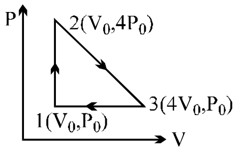
12. One mole of an ideal diatomic gas is taken through the cycle as shown in the figure.

$1 \to 2$: isochoric process

$2 \to 3$: straight line on P-V diagram

$3 \to 1$: isobaric process

The average molecular speed of the gas in the states 1, 2 and 3 are in the ratio



(A) 1 : 2 : 2 (B) $1 : \sqrt{2} : \sqrt{2}$ (C) 1 : 1 : 1 (D) 1 : 2 : 4

KT0040

13. One mole of an ideal gas at STP is heated in an insulated closed container until the average speed of its molecules is doubled. Its pressure would therefore increase by factor.

(A) 1.5 (B) $\sqrt{2}$ (C) 2 (D) 4

KT0041

14. Three particles have speeds of 2u, 10u and 11u. Which of the following statements is correct?

(A) The r.m.s. speed exceeds the mean speed by about u.

(B) The mean speed exceeds the r.m.s. speed by about u.

(C) The r.m.s. speed equals the mean speed.

(D) The r.m.s. speed exceeds the mean speed by more than 2u.

KT0042

15. One mole of an ideal gas is contained with in a cylinder by a frictionless piston and is initially at temperature T. The pressure of the gas is kept constant while it is heated and its volume doubles. If R is molar gas constant, the work done by the gas in increasing its volume is :-

(A) RT ln2 (B) 1/2 RT (C) RT (D) 3/2 RT

KT0043

16. A gas mixture consists of 2 moles of oxygen and 4 moles of argon at temperature T. Neglecting all vibrational modes, the total internal energy of the system is :-

(A) 4 RT (B) 15 RT (C) 9 RT (D) 11 RT

KT0044

First Law of Thermodynamics :

17. In thermodynamic process pressure of a fixed mass of gas is changed in such a manner that the gas releases 30 joule of heat and 18 joule of work was done on the gas. If the initial internal energy of the gas was 60 joule, then, the final internal energy will be :

(A) 32 joule (B) 48 joule (C) 72 joule (D) 96 joule

KT0045

18. Two monoatomic ideal gas at temperature T_1 and T_2 are mixed. There is no loss of energy. If the masses of molecules of the two gases are m_1 and m_2 and number of their molecules are n_1 and n_2 respectively. The temperature of the mixture will be :

(A) $\frac{T_1+T_2}{n_1+n_2}$ (B) $\frac{T_1}{n_1}+\frac{T_2}{n_2}$ (C) $\frac{n_2T_1+n_1T_2}{n_1+n_2}$ (D) $\frac{n_1T_1+n_2T_2}{n_1+n_2}$

KT0046

19. An ideal gas expands isothermally from a volume V_1 to V_2 and then compressed to original volume V_1 adiabatically. Initial pressure is P_1 and final pressure is P_3. The total work done is W. Then

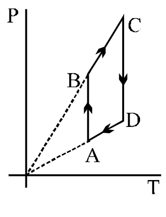
[JEE' 2004 (Scr)]

(A) $P_3 > P_1$, $W > 0$ (B) $P_3 < P_1$, $W < 0$

(C) $P_3 > P_1$, $W < 0$ (D) $P_3 = P_1$, $W = 0$

KT0047

20. Pressure versus temperature graph of an ideal gas is shown in figure



(A) During the process AB work done by the gas is positive

(B) During the process CD work done by the gas is negative

(C) During the process BC internal energy of the gas is increasing

(D) None

KT0048

21. A polyatomic gas with six degrees of freedom does 25J of work when it is expanded at constant pressure. The heat given to the gas is :-

(A) 100J (B) 150J (C) 200J (D) 250J

KT0049

22. A reversible adiabatic path on a P-V diagram for an ideal gas passes through state A where $P = 0.7\times10^5\ N/m^{-2}$ and $V = 0.0049\ m^3$. The ratio of specific heat of the gas is 1.4. The slope of path at A is :

(A) $2.0 \times 10^7\ Nm^{-5}$ (B) $1.0 \times 10^7\ Nm^{-5}$ (C) $-2.0 \times 10^7\ Nm^{-5}$ (D) $-1.0 \times 10^7\ Nm^{-5}$

KT0050

23. The adiabatic Bulk modulus of a diatomic gas at atmospheric pressure is

(A) $0\ Nm^{-2}$ (B) $1\ Nm^{-2}$ (C) $1.4 \times 10^4\ Nm^{-2}$ (D) $1.4 \times 10^5\ Nm^{-2}$

KT0051

24. A given quantity of an ideal gas is at pressure P and absolute temperature T. The isothermal bulk modulus of the gas is :

(A) 2P/3 (B) P (C) 3P/2 (D) 2P

KT0052

25. One mole of an ideal gas at temperature T_1 expends according to the law $\frac{P}{V^2} = a$ (constant). The work done by the gas till temperature of gas becomes T_2 is :

(A) $\frac{1}{2}R(T_2 - T_1)$ (B) $\frac{1}{3}R(T_2 - T_1)$ (C) $\frac{1}{4}R(T_2 - T_1)$ (D) $\frac{1}{5}R(T_2 - T_1)$

KT0053

26. The first law of thermodynamics can be written as $\Delta U = \Delta Q + \Delta W$ for an ideal gas. Which of the following statements is correct?

(A) ΔU is always zero when no heat enters or leaves the gas

(B) ΔW is the work done by the gas in this written law.

(C) ΔU is zero when heat is supplied and the temperature stays constant

(D) $\Delta Q = -\Delta W$ when the temperature increases very slowly.

KT0054

27. 2 moles of a monoatomic gas are expanded to double its initial volume, through a process P/V = constant. If its initial temperature is 300 K, then which of the following is not true.

(A) $\Delta T = 900$ K (B) $\Delta Q = 3200$ R (C) $\Delta Q = 3600$ R (D) W = 900 R

KT0055

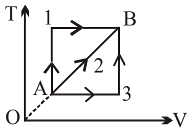
28. A student records ΔQ, ΔU & ΔW for a thermodynamic cycle A → B → C → A. Certain entries are missing. Find correct entry in following options.

	AB	BC	CA
ΔW	40J		30J
ΔU		50J	
ΔQ	150J	10J	

(A) $W_{BC} = -70$ J (B) $\Delta Q_{CA} = 130$ J (C) $\Delta U_{AB} = 190$ J (D) $\Delta U_{CA} = -160$ J

KT0056

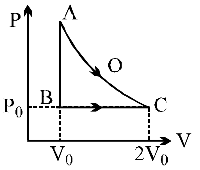
29. A given mass of a gas expands from a state A to the state B by three paths 1, 2 and 3 as shown in T-V indicator diagram. If W_1, W_2 and W_3 respectively be the work done by the gas along the three paths, then



(A) $W_1 > W_2 > W_3$ (B) $W_1 < W_2 < W_3$ (C) $W_1 = W_2 = W_3$ (D) $W_1 < W_2$, $W_1 > W_3$

KT0057

30. An ideal gas is taken from point A to point C on P-V diagram through two process AOC and ABC as shown in the figure. Process AOC is isothermal



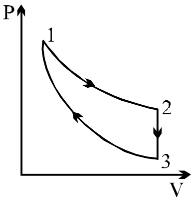
(A) Process AOC requires more heat than process ABC.
(B) Process ABC requires more heat than process AOC.
(C) Both process AOC & ABC require same amount of heat.
(D) Data is insufficient for comparison of heat requirement for the two processes.

KT0058

31. Monoatomic, diatomic and triatomic gases whose initial volume and pressure are same, are compressed till their volume becomes half the initial volume.
(A) If the compression is adiabatic then monoatomic gas will have maximum final pressure.
(B) If the compression is adiabatic then triatomic gas will have maximum final pressure.
(C) If the compression is adiabatic then their final pressure will be same.
(D) If the compression is isothermal then their final pressure will be different.

KT0059

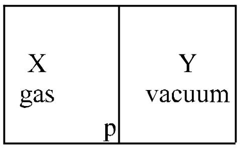
32. Three processes form a thermodynamic cycle as shown on P-V diagram for an ideal gas. Process $1 \rightarrow 2$ takes place at constant temperature (300K). Process $2 \rightarrow 3$ takes place at constant volume. During this process 40J of heat leaves the system. Process $3 \rightarrow 1$ is adiabatic and temperature T_3 is 275K. Work done by the gas during the process $3 \rightarrow 1$ is :-



(A) – 40J (B) – 20J (C) +40J (D) +20J

KT0060

33. A closed container is fully insulated from outside. One half of it is filled with an ideal gas X separated by a plate P from the other half Y which contains a vacuum as shown in figure. When P is removed, X moves into Y. Which of the following statements is correct?



(A) No work is done by X (B) X decreases in temperature
(C) X increases in internal energy (D) X doubles in pressure

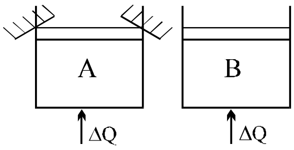
KT0061

34. Water is heated in an open pan where the air pressure is 10^{+5} Pa. The water remains a liquid, which expands by a small amount as it is heated. Determine the ratio of the heat absorbed by the water to the work done by water. (γ for water = 10^{-3}/C, S = 1 cal/gm°C)

(A) 4.2×10^3 (B) 4.2×10^5 (C) 4.2×10^2 (D) 4.2×10^4

KT0062

35. Two identical vessels A & B contain equal amount of ideal monoatomic gas. The piston of A is fixed but that of B is free. Same amount of heat is absorbed by A & B. If B's internal energy increases by 100 J the change in internal energy of A is :-



(A) 100 J (B) $\frac{500}{3}$ J (C) 250 J (D) none of these

KT0063

36. An ideal gas expands from volume V_1 to V_2. This may be achieved by either of the three processes: isobaric, isothermal and adiabatic. Let ΔU be the change in internal energy of the gas, Q be the quantity of heat added to the system and W be the work done by the gas. Identify which of the following statements is false for ΔU?

(A) ΔU is least under adiabatic process.

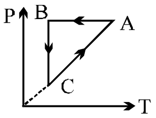
(B) ΔU is greatest under adiabatic process.

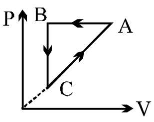
(C) ΔU is greatest under the isobaric process.

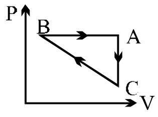
(D) ΔU in isothermal process lies in-between the values obtained under isobaric and adiabatic processes.

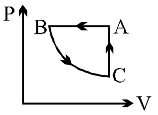
KT0064

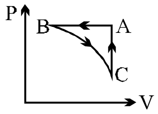
37. A cyclic process ABCA is shown in PT diagram. When presented on PV, it would



(A) 

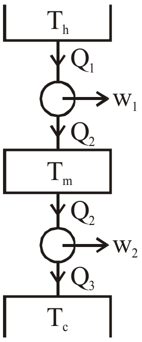
(B) 

(C) 

(D) 

KT0065

38. Suppose that two heat engines are connected in series, such that the heat exhaust of the first engine is used as the heat input of the second engine as shown in figure. The efficiencies of the engines are η_1 and η_2, respectively. The net efficiency of the combination is given by



(A) $\eta_{net} = \eta_2 + (1 - \eta_1)\eta_2$ (B) $\eta_{net} = \frac{\eta_1}{(1-\eta_1)\eta_2}$

(C) $\eta_{net} = \eta_1 + (1 - \eta_1)\eta_2$ (D) $\eta_{net} = \frac{1-\eta_1}{(1-\eta_2)\eta_2}$

KT0066

39. During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio C_P/C_V for the gas is- **[AIEEE-2003]**

(A) 4/3 (B) 2 (C) 5/3 (D) 3/2

KT0067

40. "Heat cannot be itself flow from a body at lower temperature to a body at higher temperature" is a statement or consequence of- **[AIEEE - 2003]**

(A) second law of thermodynamics (B) conservation of momentum

(C) conservation of mass (D) first law of thermodynamics

KT0068

41. A carnot engine takes 3×10^6 cal of heat from a reservoir at 627 °C and gives it to a sink at 27 °C. The work done by the engine is- **[AIEEE - 2003]**

(A) 4.2×10^6 J (B) 8.4×10^6 J (C) 16.8×10^6 J (D) zero

KT0069

MULTIPLE CORRECT TYPE QUESTIONS

42. Two gases have the same initial pressure, volume and temperature. They expand to the same final volume, one adiabatically and the other isothermally

(A) The final temperature is greater for the isothermal process

(B) The final pressure is greater for the isothermal process

(C) The work done by the gas is greater for the isothermal process

(D) All the above options are incorrect

KT0070

43. On the P–T graph of an ideal gas,

(A) adiabatic process will be a straight line

(B) isochoric process will be a straight line passing through the origin

(C) adiabatic curve will have a positive slope

(D) the slope of adiabatic curve will decrease with increase in T

KT0071

44. During the melting of a slab of ice at 273 K at atmospheric pressure

(A) Positive work is done by the ice-water system on the atmosphere

(B) Positive work is done on the ice-water system by the atmosphere

(C) The internal energy of the ice-water increases

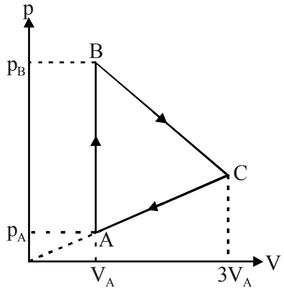
(D) The internal energy of ice-water system decreases

KT0072

COMPREHENSION TYPE QUESTIONS

Paragraph for question nos. 45 and 46

A sample of ideal gas is taken through the cyclic process shown in the figure. The temperature of the gas at state A is $T_A = 200$ K. At states B and C the temperature of the gas is the same.

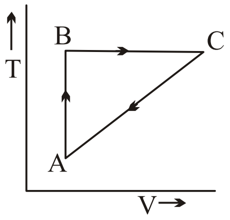


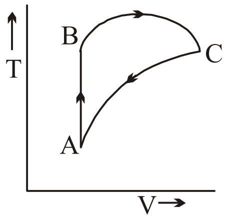
45. Net work done by the gas in the process is

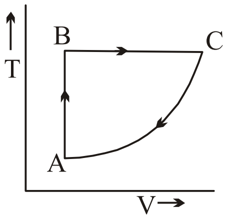
(A) $2P_AV_A$ (B) $4P_AV_A$ (C) $6P_AV_A$ (D) $8P_AV_A$

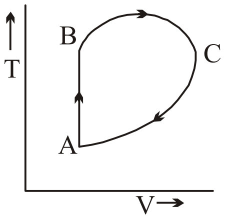
KT0073

46. Which of the following graphs best represent the cyclic process in T-V diagram.

(A)


(B)


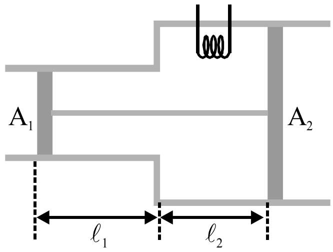
(C)


(D)


KT0073

Paragraph for Question Nos. 47 to 49

The walls of the two connecting cylinders shown in the figure are adiabatic (thermally insulating). The cross-sectional areas of the parts are A_1 and A_2. There is a well-fitting but freely moving, thermally insulating piston in each cylinder, at a distance $\ell_1 = \ell_2 = \ell$ from the point where the cross-sectional area changes. The pistons are connected to each other by a thin and rigid rod. The enclosed volume contains air. The temperature and air pressure are T_0 and p_0 both inside and outside. The heater filament inside is operated for time t at a power of P.



47. Mark the **CORRECT** statement :-

(A) Process performed is adiabatic
(B) Piston shifts towards left
(C) Process performed is polytropic
(D) Piston shifts towards right

KT0074

48. What is change in temperature (Q is heat energy supplied, v_0 is initial volume) :-

(A) $\Delta T = \frac{2}{5}\frac{QT_0}{p_0V_0}$
(B) $\Delta T = \frac{2}{7}\frac{QT_0}{p_0V_0}$
(C) $\Delta T = \frac{5}{7}\frac{QT_0}{p_0V_0}$
(D) $\Delta T = \frac{QT_0}{p_0V_0}$

KT0074

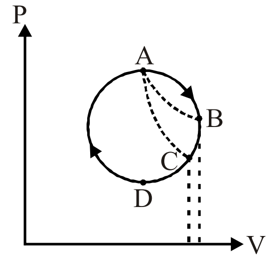
49. What is displacement (x) of piston till equilibrium is reached :-

(A) $x = \frac{2Pt}{(A_2 - A_1)7p_0}$
(B) $x = \frac{2Pt}{(A_2 - A_1)5p_0}$
(C) $x = \frac{2Pt}{(A_2 - A_1)3p_0}$
(D) $x = \frac{2Pt}{(A_2 - A_1)9p_0}$

KT0074

MATRIX MATCH TYPE QUESTIONS

50. For an ideal gas a process PV diagram is a circle. An adiabatic from A passes through C. An isotherm from A passes through B. We take a part of the circular cyclic process. Comment on the sign of the quantity of column-I.



	Column-I		**Column-II**
(A)	Heat given to the gas in going from A to C along circle	(P)	Positive
(B)	Heat given to the gas in going from B to C along circle	(Q)	Negative
(C)	Heat given to the gas in going from C to D along circle	(R)	Zero
		(S)	can't be said

KT0075

EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

1. An open and wide glass tube is immersed vertically in mercury in such a way that length 0.05 m extends above mercury level. The open end of the tube is closed and the tube is raised further by 0.43m. The length of air column above mercury level in the tube will be : Take P_{atm} = 76 cm of mercury

(A) 0.215 m (B) 0.2 m (C) 0.1 m (D) 0.4 m

KT0076

2. A barometer tube, containing mercury, is lowered in a vessel containing mercury until only 50 cm of the tube is above the level of mercury in the vessel. If the atmospheric pressure is 75 cm of mercury, what is the pressure at the top of the tube ?

(A) 33.3 kPa (B) 66.7 kPa (C) 3.33 MPa (D) 6.67 MPa

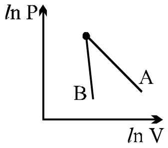
KT0077

3. At temperature T, N molecules of gas A each having mass m and at the same temperature 2N molecules of gas B each having mass 2m are filled in a container. The mean square velocity of molecules of gas B is v^2 and mean square of x component of velocity of molecules of gas A is w^2. The ratio of w^2/v^2 is :

(A) 1 (B) 2 (C) 1/3 (D) 2/3

KT0078

4. The figure, shows the graph of logarithmic reading of pressure and volume for two ideal gases A and B undergoing adiabatic process. From figure it can be concluded that



(A) gas B is diatomic (B) gas A and B both are diatomic

(C) gas A is monoatomic (D) gas B is monoatomic & gas A is diatomic

KT0079

5. When unit mass of water boils to become steam at 100^0C, it absorbs Q amount of heat. The densities of water and steam at 100^0C are ρ_1 and ρ_2 respectively and the atmospheric pressure is p_0. The increase in internal energy of the water is

(A) Q (B) $Q + p_0\left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right)$ (C) $Q + p_0\left(\frac{1}{\rho_2} - \frac{1}{\rho_1}\right)$ (D) $Q - p_0\left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right)$

KT0080

6. A cylinder made of perfectly non conducting material closed at both ends is divided into two equal parts by a heat proof piston. Both parts of the cylinder contain the same masses of a gas at a temperature t_0 = 27° C and pressure P_0 = 1 atm. Now if the gas in one of the parts is slowly heated to t = 57°C while the temperature of first part is maintained at t_0 the distance moved by the piston from the middle of the cylinder will be (length of the cylinder = 84 cm)

(A) 3 cm (B) 5 cm (C) 2 cm (D) 1 cm

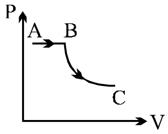
KT0081

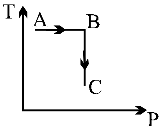
7. 1 kg of a gas does 20 kJ of work and receives 16 kJ of heat when it is expanded between two states. A second kind of expansion can be found between the initial and final state which requires a heat input of 9 kJ. The work done by the gas in the second expansion is :

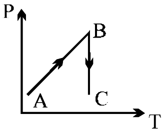
(A) 32 kJ (B) 5 kJ (C) – 4 kJ (D) 13 kJ

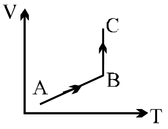
KT0082

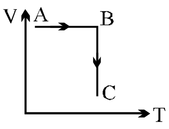
8. A process is shown in the diagram. Which of the following curves may represent the same process?



(A)


(B)


(C)


(D)


KT0083

9. An ideal gas expands in such a way that PV^2 = constant throughout the process.

(A) The graph of the process of T-V diagram is a parabola.

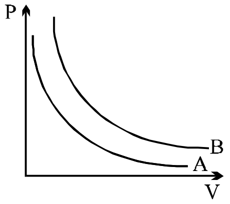
(B) The graph of the process of T-V diagram is a straight line.

(C) Such an expansion is possible only with heating.

(D) Such an expansion is possible only with cooling.

KT0084

10. Figure shows the pressure P versus volume V graphs for two different gas sample at a given temperature. M_A and M_B are masses of two samples, n_A and n_B are numbers of moles. Which of the following ***must be incorrect.***



(A) $M_A > M_B$ (B) $M_A < M_B$ (C) $n_A > n_B$ (D) $n_A < n_B$

KT0085

11. A vertical cylinder with heat-conducting walls is closed at the bottom and is fitted with a smooth light piston. It contains one mole of an ideal gas. The temperature of the gas is always equal to the surrounding's temperature, T_0. The piston is moved up slowly to increase the volume of the gas to η times. Which of the following is incorrect?

(A) Work done by the gas is $RT_0 \ln \eta$.

(B) Work done against the atmosphere is $RT_0(\eta - 1)$.

(C) There is no change in the internal energy of the gas.

(D) The final pressure of the gas is $\frac{1}{(\eta-1)}$ times its initial pressure.

KT0086

12. A gas is enclosed in a vessel at a constant temperature at a pressure of 5 atmosphere and volume 4 litre. Due to a leakage in the vessel, after some time, the pressure is reduced to 4 atmosphere. As a result, the

(A) volume of the gas decreased by 20%

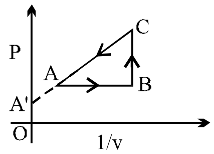
(B) average K.E. of gas molecule decreases by 20%

(C) 20% of the gas escaped due to the leakage

(D) 25% of the gas escaped due to the leakage

KT0087

13. An enclosed ideal gas is taken through a cycle as shown in the figure. Then



(A) Along AB, temperature decreases while along BC temperature increases

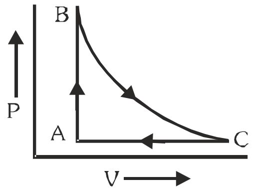
(B) Along AB, temperature increases while along BC the temperature decreases.

(C) Along CA work is done by the gas and the internal energy remains constant.

(D) Along CA work is done on the gas and internal energy of the gas increases.

KT0088

14. One mole of monoatomic ideal gas undergoes a cyclic process ABCA as shown in figure. Process BC is adiabatic. The temperatures at A, B and C are 300, 600 and 450K respectively. Choose the correct statement(s).



(A) In process CA change in internal energy is 225R.

(B) In process AB change in internal energy is –150R.

(C) In process BC change in internal energy is –225R.

(D) Change in internal energy during the whole cyclic process is +150R.

KT0089

MULTIPLE CORRECT TYPE QUESTIONS

15. Let v_{av}, v_{rms} and v_p respectively denote mean speed, root mean square speed and the most probable speed of the molecule in an ideal monoatomic gas at absolute temperature T. The mass of a molecule is m then :

(A) no molecule can have speed greater than $\sqrt{2}\,v_{rms}$

(B) no molecule can have speed less than $v_p/\sqrt{2}$

(C) $v_p<v_{av}<v_{rms}$

(D) the average kinetic energy of a molecule is $3/4\ mv_p^2$

KT0090

16. A gas expands such that its initial and final temperature are equal. Also, the process followed by the gas traces a straight line on the P-V diagram :

(A) The temperature of the gas remains constant throughout.

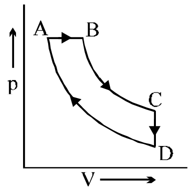
(B) The temperature of the gas first increases and then decreases

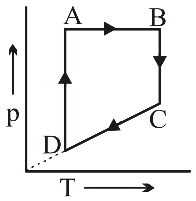
(C) The temperature of the gas first decreases and then increases

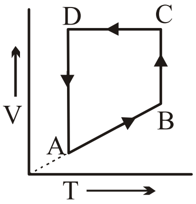
(D) The straight line has a negative slope.

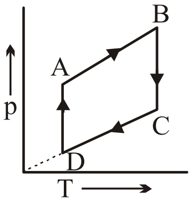
KT0091

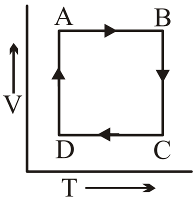
17. A cyclic process ABCD is shown in the p-V diagram. Which of the following curves represents the same process if BC & DA are isothermal processes



(A) 

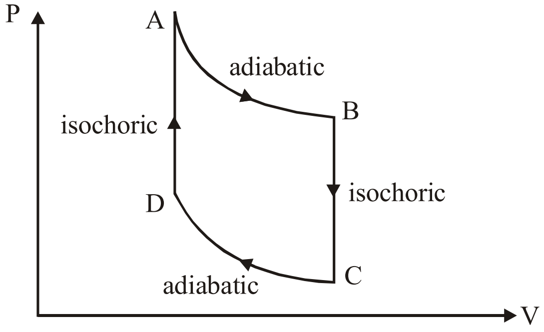
(B) 

(C) 

(D) 

KT0092

18. Two moles of helium gas is taken through the cycle ABCDA as shown in the figure. If $T_A = 1000$ K, $2P_A = 3P_B = 6P_C$.



(A) work done by the gas in the process A to B is 3741 J.

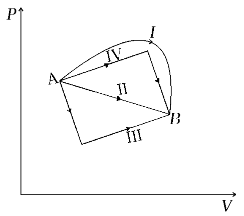
(B) heat lost by the gas in the process B to C is 10600 J.

(C) temperature T_D is 2000 K.

(D) none of these

KT0093

19. Figure shows the *P-V* diagram of an ideal gas undergoing a change of state from A to B. Four different parts I, II, III and IV as shown in the figure may lead to the same change of state.



(A) Change in internal energy is same in IV and III cases, but not in I and II.

(B) Change in internal energy is same in all the four cases.

(C) Work done is maximum in case I

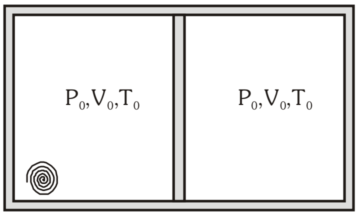
(D) Work done is minimum in case II.

KT0094

COMPREHENSION TYPE QUESTIONS

Paragraph for Question Nos. 20 to 22

One mole of a monoatomic ideal gas occupies two chambers of a cylinder partitioned by means of a movable piston. The walls of the cylinder as well as the piston are thermal insulators. Initially equal amounts of gas fill both the chambers at (P_0, V_0, T_0). A coil is burnt in the left chamber which absorbs heat and expands, pushing the piston to the right. The gas on the right chamber is compressed until to pressure becomes 32 P_0.



20. The final volume of left chamber is :-

(A) $\frac{V_0}{8}$ (B) $\frac{15}{8}V_0$

(C) $\frac{7}{8}V_0$ (D) $\frac{9}{8}V_0$

KT0095

21. The work done on the gas in the right chamber is :-

(A) $\frac{9}{2}P_0V_0$ (B) $-\frac{9}{2}P_0V_0$

(C) $\frac{13}{2}P_0V_0$ (D) $\frac{17}{2}P_0V_0$

KT0095

22. The change in internal energy of the gas in the left chamber is :-

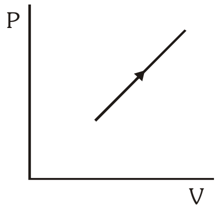
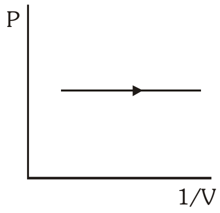
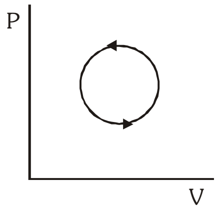
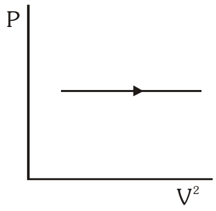
(A) $\frac{186}{4}RT_0$ (B) $\frac{177}{4}RT_0$

(C) $\frac{59}{2}RT_0$ (D) $\frac{131}{4}RT_0$

KT0095

MATRIX MATCH TYPE QUESTION

23.

Column I **(Pressure volume graph)**	**Column II** **(W is work done by gas, Q heat supply to gas)**
(A) 	(P) $W > 0$
(B) 	(Q) $W < 0$
(C) 	(R) $Q > 0$
(D) 	(S) $Q < 0$

KT0096

SUPPLEMENT FOR JEE-MAINS

1. Two engines A and B have their sources at 400 K and 350 K and sinks at 350 K and 300 K respectively. Which engine is more efficient and by how much ?

2. A carnot engine working between 400 K and 800 K has a work output of 1200 J per cycle. What is the amount of heat energy supplied to the engine from source per cycle?

3. A carnot engine works as a refrigrator between 250 K and 300 K. If it receives 750 cal of heat from the reservoir at the lower temperature. Calculate the amount of heat rejected at the higher temperature.

4. The temperature insides & outside of refrigerator are 260 K and 315 K respectively. Assuming that the refrigerator cycle is reversible, calculate the heat delivered to surroundings for every joule of work done.

5. A refrigerator takes heat from water at 0°C and transfer it to room at 27°C. If 100 kg of water is converted in ice at 0°C then calculate the workdone. (Latent heat of ice is 3.4×10^5 J/kg)

6. The coefficient of performance of a refrigerator working between 10° C and 20° C is :–

(A) 1 (B) 28.3 (C) 29.3 (D) 3.53

7. A refrigerator freezes 5 kg of water at 0°C into ice at 0°C in a time interval of 20 minutes. Assume that room temperature is 20°C. Calculate the minimum power needed to accomplish it ?

(A) 24.4 Watt (B) 0.1025 Watt (C) 0.0244 Watt (D) 102.5 Watt

8. A carnot engine whose source temperature is at 400 K takes 100 kcal of heat at this temperature in each cycle and gives 70 kcal to the sink. Calculate (i) the temperature of the sink and (ii) the efficiency of the engine.

(A) 280 K, 30 % (B) 270 K, 30% (C) 280 K, 10 % (D) None of these

9. An ionization gauge installed in the artificial satellite showed that 1 cm^3 of the atmosphere contained about a thousand million particles of gas at a height of 300 km from the Earth's surface. Find the mean free path of the gas particles at this height. Take the diameter of the particles equal to 2×10^{-10} m.

(A) $\ell = 5.6$ km (B) $\ell = 2.6$ km (C) $\ell = 8.4$ km (D) $\ell = 1.1$ km

10. What maximum number of molecules of a gas should be contained in 1 cm^3 of a spherical vessels with a diameter of 15 cm so that the molecules do not collide with each other? The diameter of a gas molecule is 3×10^{-8} cm.

(A) 1.7×10^{18}/CC (B) 1.7×10^{16}/CC (C) 2.5×10^{19}/CC (D) 1.1×10^{14}/CC

11. An ideal refrigerator operates according to the reverse Carnot cycle and transmits heat from a cold source with water at a temperature of 0°C to a boiler with water at a temperature of 100 °C. What amount of water must be frozen in the cooler to convert 1 kg of water into vapour in the boiler?

(A) 4.94 kg (B) 3.24 kg (C) 5.63 kg (D) 2.12 kg

12. Oxygen (10 grammes) is heated from $t_1 = 50°C$ to $t_2 = 150°C$. Find the change in entropy if the oxygen is heated: (1) isochorically, (2) isobarically.

(A) $\Delta S_V = 1.76$ J/K; $\Delta S_P = 2.46$ J/K (B) $\Delta S_V = \Delta S_P = 1.76$ J/K

(C) $\Delta S_V = \Delta S_P = 2.46$ J/K (D) $\Delta S_V = 2.46$ J/K; $\Delta S_P = 1.76$ J/K

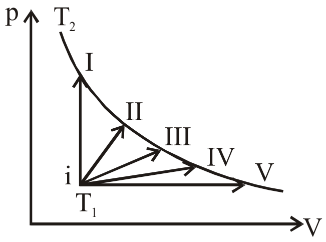
13. An ideal gas expands into a vacuum in a rigid vessel. As a result there is :

(A) a change in entropy (B) an increase of pressure

(C) a change in temperature (D) a decrease of internal energy

(E) a change in phase

14. An ideal gas is to taken reversibly from state i, at temperature T_1, to any of the other states labeled, I, II, III, IV and V on the p-V diagram below. All are at the same temperature T_2. Rank the five processes according to the change in entropy of the gas, least to greatest :



(A) I, II, III, IV, V (B) V, IV, III, II, I
(C) I, then II, III, IV, and V tied (D) I, II, III and IV tied, then V
(E) I and V tied, then II, III, IV

15. According to the second law of thermodynamics:
(A) Heat energy cannot be completely converted to work
(B) Work cannot be completely converted to heat energy
(C) For all cyclic processes we have $dQ/T < 0$
(D) The reason all heat engine efficiencies are less than 100% is friction, which is unavoidable
(E) All of the above are true

16. A reversible refrigerator operates between a low temperature reservoir at T_C and a high temperature reservoir at T_H. Its coefficient of performance is given by :
(A) $(T_H - T_C)/T_C$ (B) $T_C/(T_H - T_C)$ (C) $(T_H - T_C)/T_H$ (D) $T_H/(T_H - T_C)$
(E) $T_H(T_H + T_C)$

17. An inventor claims to have a heat engine that has an efficiency of 40% when it operates between a high temperature reservoir of 150°C and a low temperature reservoir of 30°C. This engine:
(A) Must violate the zeroth law of thermodynamics
(B) Must violate the first law of thermodynamics
(C) Must violate the second law of thermodynamics
(D) Must violate the third law of thermodynamics
(E) Does not necessarily violate any of the laws of thermodynamics

18. A carnot heat engine runs between a cold reservoir at temperature T_C and a hot reservoir at temperature T_H. You want to increase its efficiency. Of the following, which change results in the greatest increase in efficiency? The value of ΔT is the same for all changes.
(A) Raise the temperature of the hot reservoir by ΔT
(B) Raise the temperature of the cold reservoir by ΔT
(C) Lower the temperature of the hot reservoir by ΔT
(D) Lower the temperature of the cold reservoir by ΔT
(E) Lower the temperature of the hot reservoir by $1/2\Delta T$ and raise the temperature of the cold reservoir by $1/2\Delta T$.

19. A Carnot engine takes 3×10^6 cal of heat from reservoir at 627° and gives it to a sink at 27°C. Then work done by the engine is :-
(A) 4.2×10^6 J (B) 8.4×10^6 J (C) 16.8×10^6 J (D) zero

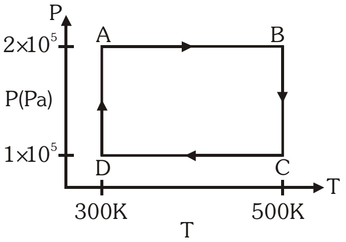
EXERCISE (J-M)

1. One kg of a diatomic gas is at a pressure of 8×10^4 N/m^2. The density of the gas is 4 kg/m^3. What is the energy of the gas due to its thermal motion ? **[AIEEE-2009]**

(1) 6×10^4 J (2) 7×10^4 J (3) 3×10^4 J (4) 5×10^4 J

KT0097

Directions : Question number 2, 3 and 4 are based on the following paragraph.
Two moles of helium gas are taken over the cycle ABCDA, as shown in the P–T diagram.



2. Assuming the gas to be ideal the work done on the gas in taking it from A to B is :- **[AIEEE-2009]**

(1) 400 R (2) 500 R (3) 200 R (4) 300

KT0098

3. The work done on the gas in taking it from D to A is :- **[AIEEE-2009]**

(1) –690 R (2) +690 R (3) –414 R (4) +414 R

KT0099

4. The net work done on the gas in the cycle ABCDA is :- **[AIEEE-2009]**

(1) 1076 R (2) 1904 R (3) Zero (4) 276 R

KT0100

5. A diatomic ideal gas is used in a carnot engine as the working substance. If during the adiabatic expansion part of the cycle the volume of the gas increases from V to 32 V, the efficiency of the engine is :- **[AIEEE-2010]**

(1) 0.25 (2) 0.5 (3) 0.75 (4) 0.99

KT0101

6. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ. It is moving with speed v and is suddenly broght to rest. Assuming no heat is lost to the surroundings, its temperature increases by :- **[AIEEE-2011]**

(1) $\frac{\gamma M v^2}{2R}$ (2) $\frac{(\gamma-1)}{2R}Mv^2$ (3) $\frac{(\gamma-1)}{2(\gamma+1)R}Mv^2$ (4) $\frac{(\gamma-1)}{2\gamma R}Mv^2$

KT0102

7. A Carnot engine operating between temperatures T_1 and T_2 has efficientcy $\frac{1}{6}$. When T_2 is lowered by 62 K, its efficiency increases to $\frac{1}{3}$. Then T_1 and T_2 are, respectively:- **[AIEEE-2011]**

(1) 330 K and 268 K (2) 310 K and 248 K (3) 372 K and 310 K (4) 372 K and 330 K

KT0103

8. Three perfect gases at absolute temperatures T_1, T_2 and T_3 are mixed. The masses of molecules are m_1, m_2, and m_3 and the number of molecules are n_1, n_2 and n_3 respectively. Assuming no loss of energy, then final temperature of the mixture is :- **[AIEEE-2011]**

(1) $\frac{n_1T_1^2+n_2T_2^2+n_3T_3^2}{n_1T_1+n_2T_2+n_3T_3}$ (2) $\frac{n_1^2T_1^2+n_2^2T_2^2+n_3^2T_3^2}{n_1T_1+n_2T_2+n_3T_3}$

(3) $\frac{T_1+T_2+T_3}{3}$ (4) $\frac{n_1T_1+n_2T_2+n_3T_3}{n_1+n_2+n_3}$

KT0104

9. The specific heat capacity of a metal at low temperautre (T) is given as $C_p(\text{kJk}^{-1}\text{kg}^{-1}) = 32\left(\frac{T}{400}\right)^3$

A 100 gram vessel of this metal is to be cooled from 20°K to 4°K by a special refrigerator operating at room temperature (27°C). The amount of work required to cool the vessel is:- **[AIEEE-2011]**

(1) equal to 0.002 kJ (2) greater than 0.148 kJ

(3) between 0.148 kJ and 0.028 kJ (4) less than 0.028 kJ

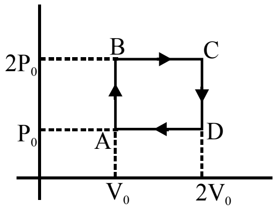
KT0105

10. A container with insulating walls is divided into two equal parts by a partition fitted with a valve. One part is filled with an ideal gas at a pressure P and temperature T, whereas the other part is completely evacuated. If the valve is suddenly opened, the pressure and temperature of the gas will be :- **[AIEEE-2011]**

(1) $\frac{P}{2}, T$ (2) $\frac{P}{2}, \frac{T}{2}$ (3) P, T (4) $P, \frac{T}{2}$

KT0106

11. Helium gas goes through a cycle ABCDA (consisting of two isochoric and two isobaric lines) as shown in figure. Efficiency of this cycle is nearly (Assume the gas to be close to ideal gas) :- **[AIEEE-2012]**



(1) 12.5% (2) 15.4% (3) 9.1% (4) 10.5%

KT0107

12. A Carnot engine, whose efficiency is 40% takes in heat from a source maintained at a temperature of 500 K. It is desired to have an engine of efficiency 60%. Then, the intake temperature for the same exhaust (sink) temperature must be :- **[AIEEE-2012]**

(1) 600 K

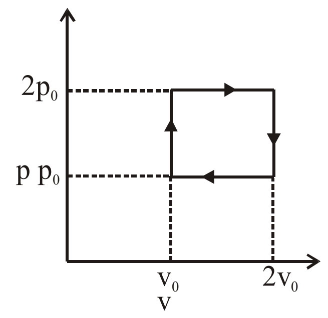
(2) efficiency of Carnot engine cannot be made larger than 50%

(3) 1200 K

(4) 750 K

KT0108

13. The above p-v diagram represents the thermodynamic cycle of an engine, operating with an ideal monoatomic gas. The amount of heat, extracted from the source in a single cycle is : **[JEE-Mains-2013]**



(1) p_0v_0 (2) $\left(\frac{13}{2}\right)p_0v_0$ (3) $\left(\frac{11}{2}\right)p_0v_0$ (4) $4p_0v_0$

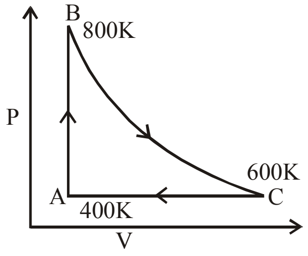
KT0109

14. An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open of the tube is then closed and sealed and the tube is raised vertically up by addition 46 cm. What will be length of the air column above mercury in the tube now ? (Atmospheric pressure = 76 cm of Hg) **[JEE-Mains-2014]**

(1) 38 cm (2) 6 cm (3) 16 cm (4) 22 cm

KT0110

15. One mole of diatomic ideal gas undergoes a cyclic process ABC as shown in figure. The process BC is adiabatic. The temperatures at A, B and C are 400 K, 800 K and 600 K respectively. Choose the correct statement : **[JEE-Mains-2014]**



(1) The change in internal energy in the process AB is –350 R.

(2) The change in internal energy in the process BC is –500R

(3) The change in internal energy in whole cyclic process is 250 R.

(4) The change in internal energy in the process CA is 700 R.

KT0111

16. A solid body of constant heat capacity 1 J°/C is being heated by keeping it in contact with reservoirs in two ways - **[JEE-Mains-2015]**

(i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.

(ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat.

In both the cases body is brought from initial temperature 100°C to final temperature 200°C. Entropy change of the body in the two cases respectively is -

(1) ln2, 2ln2 (2) 2ln2, 8ln2 (3) ln2, 4ln2 (4) ln2, ln2

KT0112

17. Consider a spherical shell of radius R at temperature T. The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u = \frac{U}{V} \propto T^4$ and pressure $p = \frac{1}{3}\left(\frac{U}{V}\right)$. If the shell now undergoes an adiabatic expansion the relation between T and R is -

[JEE-Mains-2015]

(1) $T \propto \frac{1}{R}$ (2) $T \propto \frac{1}{R^3}$ (3) $T \propto e^{-R}$ (4) $T \propto e^{-3R}$

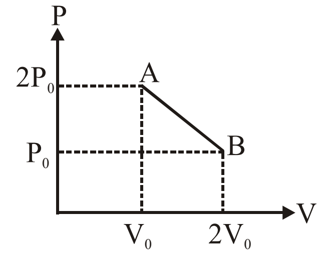
KT0113

18. Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increases as V^q, where V is the volume of the gas. The value of q is :- $\left(\gamma = \frac{C_p}{C_v}\right)$ **[JEE-Mains-2015]**

(1) $\frac{\gamma+1}{2}$ (2) $\frac{\gamma-1}{2}$ (3) $\frac{3\gamma+5}{6}$ (4) $\frac{3\gamma-5}{6}$

KT0114

19. 'n' moles of an ideal gas undergoes a process A → B as shown in the figure. The maximum temperature of the gas during the process will be : **[JEE-Mains-2016]**



(1) $\frac{9P_0V_0}{nR}$ (2) $\frac{9P_0V_0}{4nR}$ (3) $\frac{3P_0V_0}{2nR}$ (4) $\frac{9P_0V_0}{2nR}$

KT0115

20. An ideal gas undergoes a quasi static, reversible process in which its molar heat capacity C remains constant. If during this process the relation of pressure P and volume V is given by PV^n = constant, then n is given by (Here C_P and C_V are molar specific heat at constant pressure and constant volume, respectively) :- **[JEE-Mains-2016]**

(1) $n=\frac{C-C_V}{C-C_P}$ (2) $n=\frac{C_P}{C_V}$ (3) $n=\frac{C-C_P}{C-C_V}$ (4) $n=\frac{C_P-C}{C-C_V}$

KT0116

21. The temperature of an open room of volume 30 m^3 increases from 17°C to 27°C due to sunshine. The atmospheric pressure in the room remains 1×10^5 Pa. If n_i and n_f are the number of molecules in the room before and after heating, then $n_f - n_i$ will be :- **[JEE-Main 2017]**

(1) 2.5×10^{25} (2) -2.5×10^{25} (3) -1.61×10^{23} (4) 1.38×10^{23}

KT0117

22. C_p and C_v are specific heats at constant pressure and constant volume respectively. It is observed that

$C_p - C_v = a$ for hydrogen gas

$C_p - C_v = b$ for nitrogen gas

The correct relation between a and b is : **[JEE-Main 2017]**

(1) $a = 14\ b$ (2) $a = 28\ b$ (3) $a = \frac{1}{14} b$ (4) $a = b$

KT0118

23. Two moles of an ideal monoatomic gas occupies a volume V at 27°C. The gas expands adiabatically to a volume 2V. Calculate (a) the final temperature of the gas and (b) change in its internal energy. **[JEE-Main 2018]**

(1) (a) 195 K (b) –2.7 kJ (2) (a) 189 K (b) –2.7 kJ

(3) (a) 195 K (b) 2.7 kJ (4) (a) 189 K (b) 2.7 kJ

KT0119

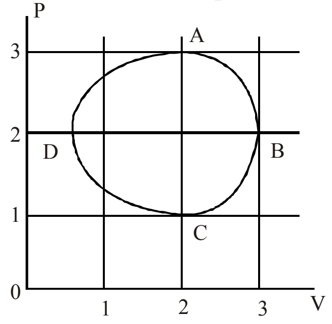
EXERCISE (JA)

1. C_v and C_p denote the molar specific heat capacities of a gas at constant volume and constant pressure, respectively. Then **[JEE-2009]**

(A) $C_p - C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas
(B) $C_p + C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas
(C) C_p / C_v is larger for a diatomic ideal gas than for a monoatomic ideal gas
(D) $C_p \cdot C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

KT0120

2. The figure shows the P-V plot of an ideal gas taken through a cycle ABCDA. The part ABC is a semicircle and CDA is half of an ellipse. Then, **[JEE-2009]**



(A) the process during the path A → B is isothermal

(B) heat flows out of the gas during the path B → C → D

(C) work done during the path A → B → C is zero

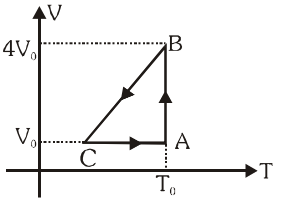
(D) positive work is done by the gas in the cycle ABCDA

KT0121

3. A real gas behaves like an ideal gas if its
(A) pressure and temperature are both high
(B) pressure and temperature are both low
(C) pressure is high and temperature is low
(D) pressure is low and temperature is high

KT0122

4. One mole of an ideal gas in initial state A undergoes 1 a cyclic process ABCA, as shown in the figure. its pressure at A is P_0. Choose the correct option(s) from the following. **[JEE-2010]**
(A) Internal energies at A and B are the same
(B) Work done by the gas in process AB is P_0V_0 ln4



(C) Pressure at C is $\frac{P_0}{4}$

(D) Temperature at C is $\frac{T_0}{4}$

KT0123

5. A diatomic ideal gas is compressed adiabatically to 1/32 of its initial volume. In the initial temperature of the gas is T_i (in Kelvin) and the final temperature is aT_i, the value of a is **[JEE-2010]**

KT0124

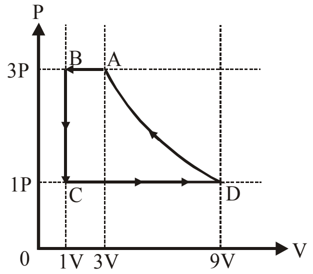
6. 5.6 liter of helium gas at STP is adiabatically compressed to 0.7 liter. Taking the initial temperature to be T_1, the work done in the process is **[JEE-2011]**

(A) $\frac{9}{8}RT_1$ (B) $\frac{3}{2}RT_1$ (C) $\frac{15}{8}RT_1$ (D) $\frac{9}{2}RT_1$

KT0125

7. One mole of a monatomic ideal gas is taken through a cycle ABCDA as shown in P-V diagram. **Column II** gives the characteristics involved in the cycle. Match them with each of the processes given in **Column I**. **[JEE-2011]**

Column I	**Column II**
(A) Process A→B	(P) Internal energy decreases.
(B) Process B→C	(Q) Internal energy increases.
(C) Process C→D	(R) Heat is lost.
(D) Process D→A	(S) Heat is gained.
	(T) Work is done on the gas.



KT0126

8. A mixture of 2 moles of helium gas (atomic mass = 4 amu) and 1 mole of argon gas (atomic mass = 40 amu) is kept at 300 K in a container. The ratio of the rms speeds $\left(\frac{v_{rms}(helium)}{v_{rms}(\text{arg}on)}\right)$ is

[JEE-2012]

(A) 0.32 (B) 0.45 (C) 2.24 (D) 3.16

KT0127

9. Two moles of ideal helium gas are in a rubber balloon at 30°C. The balloon is fully expandable and can be assumed to required no energy in its expansion. The temperature of the gas in the balloon is slowly changed to 35°C. The amount of heat required in raising the temperature is nearly

(Take : R = 8.31 J/mol.K) **[JEE-2012]**

(A) 62 J (B) 104 J (C) 124 J (D) 208 J

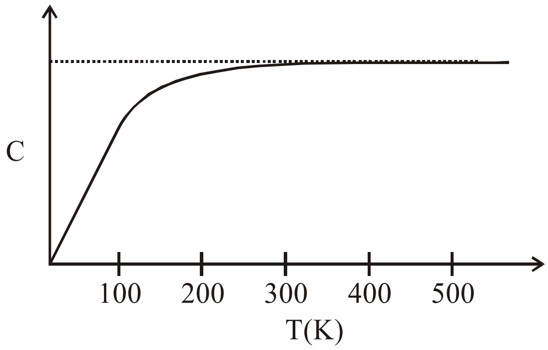
KT0128

10. Two non-reactive monoatomic ideal gases have their atomic masses in the ratio 2 : 3. The ratio of their partial pressures, when enclosed in a vessel kept at a constant temperature, is 4 : 3. The ratio of their densities is :- **[JEE-2013]**

(A) 1 : 4 (B) 1 : 2 (C) 6 : 9 (D) 8 : 9

KT0129

11. The figure below shows the variation of specific heat capacity (C) of a solid as a function of temperature (T). The temperature is increased continuously from 0 to 500 K at a constant rate. Ignoring any volume change, the following statement(s) is (are) correct to a reasonable approximation :



(A) the rate at which heat is absorbed in the range 0–100 K varies linearly with temperature T.

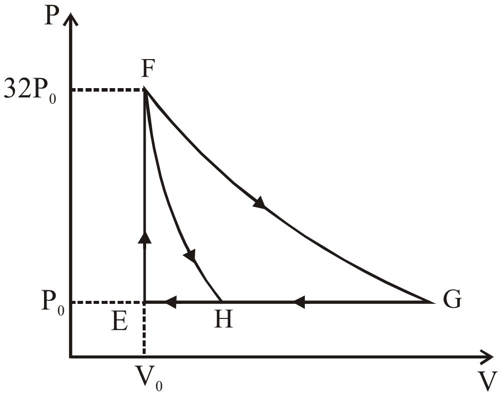
(B) heat absorbed in increasing the temperature from 0–100 K is less than the heat required for increasing the temperature from 400–500 K.

(C) there is no change in the rate of heat absorption in the range 400–500 K

(D) the rate of heat absorption increases in the range 200–300 K **[JEE-2013]**

KT0130

12. One mole of a monatomic ideal gas is taken along two cyclic processes E→F→G→E and E→F→H→E as shown in the PV diagram. The processes involved are purely isochoric, isobaric, isothermal or adiabatic. **[JEE-2013]**



Match the paths in List I with the magnitudes of the work done in the List II and select the correct answer using the codes given blow the lists.

	List I		**List II**
P.	$G \rightarrow E$	1.	$160\ P_0V_0 \ln 2$
Q.	$G \rightarrow H$	2.	$36\ P_0V_0$
R.	$F \rightarrow H$	3.	$24\ P_0V_0$
S.	$F \rightarrow G$	4.	$31\ P_0V_0$

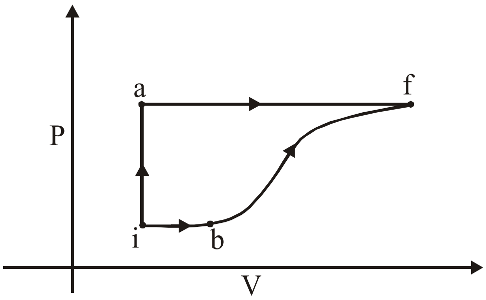
Codes :

	P	Q	R	S
(A)	4	3	2	1
(B)	4	3	1	2
(C)	3	1	2	4
(D)	1	3	2	4

KT0131

13. A thermodynamic system is taken from an initial state i with internal energy U_i = 100 J to the final state f along two different paths iaf and ibf, as schematically shown in the figure. The work done by the system along the paths af, ib and bf are W_{af} = 200 J, W_{ib} = 50 J and W_{bf} = 100 J respectively. The heat supplied to the system along the path iaf, ib and bf are Q_{iaf}, Q_{ib} and Q_{bf} respectively. If the internal energy of the system in the state b is U_b = 200 J and Q_{iaf} = 500 J, the ratio Q_{bf}/Q_{ib} is.

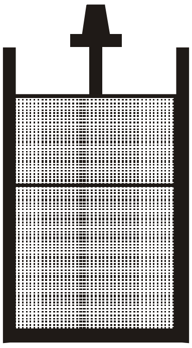
[JEE-Advance-2014]



Paragraph for Questions 14 and 15

In the figure a container is shown to have a movable (without friction) piston on top. The container and the piston are all made of perfectly insulating material allowing no heat transfer between outside and inside the container. The container is divided into two compartments by a rigid partition made of a thermally conducting material that allows slow transfer of heat. The lower compartment of the container is filled with 2 moles of an ideal monatomic gas at 700 K and the upper compartment is filled with 2 moles of an ideal diatomic gas at 400 K. The heat capacities per mole of an ideal monatomic gas are $C_V = \frac{3}{2}R$, $C_P = \frac{5}{2}R$, and those for an ideal diatomic gas are $C_V = \frac{5}{2}R, C_P = \frac{7}{2}R$.

[JEE-Advance-2014]



KT0132

14. Consider the partition to be rigidly fixed so that it does not move. When equilibrium is achieved, the final temperature of the gases will be

(A) 550 K (B) 525 K (C) 513 K (D) 490 K

KT0133

15. Now consider the partition to be free to move without friction so that the pressure of gases in both compartments is the same. Then total work done by the gases till the time they achieve equilibrium will be

(A) 250 R (B) 200 R (C) 100 R (D) –100 R

KT0133

16. A container of fixed volume has a mixutre of one mole of hydrogen and one mole of helium in equilibrium at temperature T. Assuming the gases are ideal, the correct statement(s) is (are) :-

[JEE-Advance-2015]

(A) The average energy per mole of the gas mixture is 2RT.

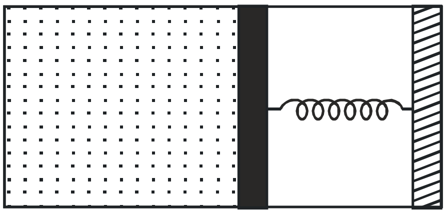
(B) The ratio of speed of sound in the gas mixture to that in helium gas is $\sqrt{6/5}$.

(C) The ratio of the rms speed of helium atoms to that of hydrogen molecules is 1/2.

(D) The ratio of the rms speed of helium atoms to that of hydrogen molecules is $1/\sqrt{2}$.

KT0134

17. An ideal monoatomic gas is confined in a horizontal cylinder by a spring loaded piston (as shown in the figure). Initially the gas is at temperature T_1, pressure P_1 and volume V_1 and the spring is in its relaxed state. The gas is then heated very slowly to temperature T_2, pressure P_2 and volume V_2. During this process the piston moves out by a distance x. Ignoring the friction between the piston and the cylinder, the correct statement(s) is(are) :- **[JEE-Advance-2015]**



(A) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the energy stored in the spring is $\frac{1}{4}P_1V_1$

(B) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the change in internal energy is $3P_1V_1$

(C) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the work done by the gas is $\frac{7}{3}P_1V_1$

(D) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the heat supplied to the gas is $\frac{17}{6}P_1V_1$

KT0135

18. A gas is enclosed in a cylinder with a movable frictionless piston. Its initial thermodynamic state at pressure $P_i = 10^5$ Pa and volume $V_i = 10^{-3}$ m^3 changes to a final state at $P_f = \left(\frac{1}{32}\right) \times 10^5 \text{Pa}$ and $V_f = 8 \times 10^{-3}$ m^3 in an adiabatic quasi-static process, such that P^3V^5 = constant. Consider another thermodynamic process that brings the system from the same initial state to the same final state in two steps: an isobaric expansion at P_i followed by an isochoric (isovolumetric) process at volumes V_f. The amount of heat supplied to the system in the two step process is approximately

[JEE-Advance 2016]

(A) 112 J (B) 294 J (C) 588 J (D) 813 J

KT0136

19. A flat plate is moving normal to its plane through a gas under the action of a constant force F. The gas is kept at a very low pressure. The speed of the plate v is much less than the average speed u of the gas molecules. Which of the following options is/are true ? **[JEE-Advance 2017]**

(A) The resistive force experienced by the plate is proportional to v

(B) The pressure difference between the leading and trailing faces of the plate is proportional to uv.

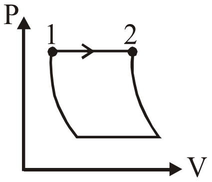
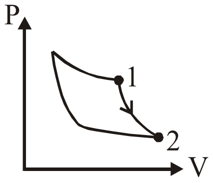
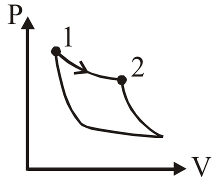
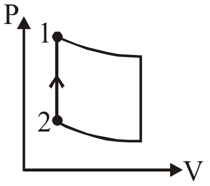
(C) The plate will continue to move with constant non-zero acceleration, at all times

(D) At a later time the external force F balances the resistive force.

KT0137

Answer Q.20, Q.21 and Q.22 by appropriately matching the information given in the three columns of the following table.

An ideal gas is undergoing a cyclic thermodynamics process in different ways as shown in the corresponding P–V diagrams in column 3 of the table. Consider only the path from state 1 to state 2. W denotes the corresponding work done on the system. The equations and plots in the table have standard notations as used in thermodynamics processes. Here γ is the ratio of heat capacities at constant pressure and constant volume. The number of moles in the gas is n. **[JEE-Advance 2017]**

Column-1	**Column-2**	**Column-3**
(I) $W_{1\to2} = \frac{1}{\gamma-1}(P_2V_2 - P_1V_1)$	(i) Isothermal	(P) 
(II) $W_{1\to2} = -PV_2 + PV_1$	(ii) Isochoric	(Q) 
(III) $W_{1\to2} = 0$	(iii) Isobaric	(R) 
(IV) $W_{1\to2} = -nRT \ln \frac{V_2}{V_1}$	(iv) Adiabatic	(S) 

20. Which of the following options is the only correct representation of a process in which $\Delta U = \Delta Q - P\Delta V$?

(A) (II) (iv) (R) (B) (II) (iii) (P) (C) (II) (iii) (S) (D) (III) (iii) (P)

KT0138

21. Which one of the following options is the correct combination ?

(A) (III) (ii) (S) (B) (II) (iv) (R) (C) (II) (iv) (P) (D) (IV) (ii) (S)

KT0138

22. Which one of the following options correctly represents a thermodynamics process that is used as a correction in the determination of the speed of sound in an ideal gas ?

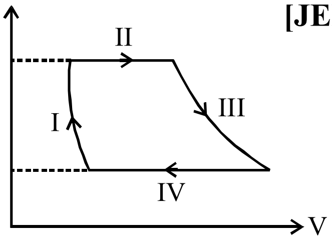
(A) (III) (iv) (R) (B) (I) (ii) (Q) (C) (IV) (ii) (R) (D) (I) (iv) (Q)

KT0138

23. One mole of a monatomic ideal gas undergoes a cyclic process as shown in the figure (whre V is the volume and T is the temperature). Which of the statements below is (are) true ?

[JEE-Advance 2018]

(A) Process I is an isochoric process
(B) In process II, gas absorbs heat
(C) In process IV, gas releases heat
(D) Processes I and II are not isobaric



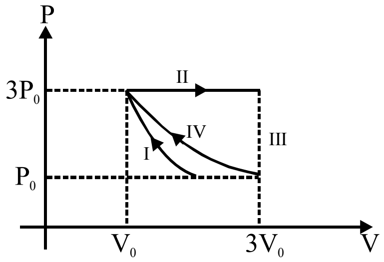
KT0139

24. One mole of a monatomic ideal gas undergoes an adiabatic expansion in which its volume becomes eight times its initial value. If the initial temperature of the gas is 100 K and the universal gas constant $R = 8.0 \text{ J mol}^{-1} \text{ K}^{-1}$, the decrease in its internal energy, in Joule, is............ .

[JEE-Advance 2018]

KT0140

25. One mole of a monatomic ideal gas undergoes four thermodynamic processes as shown schematically in the PV-diagram below. Among these four processes, one is isobaric, one is isochoric, one is isothermal and one is adiabatic. Match the processes mentioned in List-I with the corresponding statements in List–II. **[JEE-Advance 2018]**

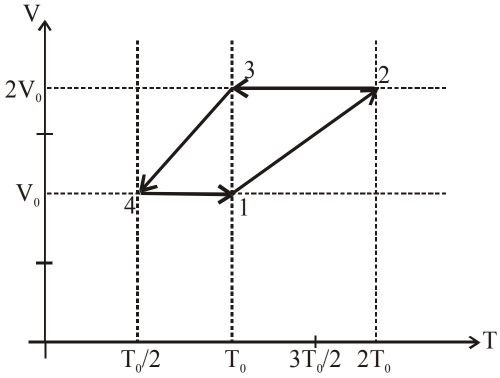


List–I		List–II	
P.	In process I	1.	Work done by the gas is zero
Q.	In process II	2.	Temperature of the gas remains unchanged
R.	In process III	3.	No heat is exchanged between the gas and its surroundings
S.	In process IV	4.	Work done by the gas is 6 P_0V_0

(A) $P \to 4$; $Q \to 3$; $R \to 1$; $S \to 2$
(B) $P \to 1$; $Q \to 3$; $R \to 2$; $S \to 4$
(C) $P \to 3$; $Q \to 4$; $R \to 1$; $S \to 2$
(D) $P \to 3$; $Q \to 4$; $R \to 2$; $S \to 1$

KT0141

26. One mole of a monoatomic ideal gas goes through a thermodynamic cycle, as shown in the volume versus temperature (V-T) diagram. The correct statement(s) is/are :
[R is the gas constant] **[JEE-Advance 2019]**



(1) Work done in this thermodynamic cycle (1→2→3→4→1) is $|W| = \frac{1}{2}RT_0$

(2) The ratio of heat transfer during processes 1→2 and 2→3 is $\left|\frac{Q_{1\to2}}{Q_{2\to3}}\right| = \frac{5}{3}$

(3) The above thermodynamic cycle exhibits only isochoric and adiabatic processes.

(4) The ratio of heat transfer during processes 1→2 and 3→4 is $\left|\frac{Q_{1\to2}}{Q_{3\to4}}\right| = \frac{1}{2}$

KT0142

27. A mixture of ideal gas containing 5 moles of monatomic gas and 1 mole of rigid diatomic gas is initially at pressure P_0, volume V_0 and temperature T_0. If the gas mixture is adiabatically compressed to a volume $V_0/4$, then the correct statement(s) is/are,

(Give $2^{1.2} = 2.3$; $2^{3.2} = 9.2$; R is gas constant) **[JEE-Advance 2019]**

(1) The final pressure of the gas mixture after compression is in between $9P_0$ and $10P_0$
(2) The average kinetic energy of the gas mixture after compression is in between $18RT_0$ and $19RT_0$

(3) The work |W| done during the process is $13RT_0$

(4) Adiabatic constant of the gas mixture is 1.6

KT0143

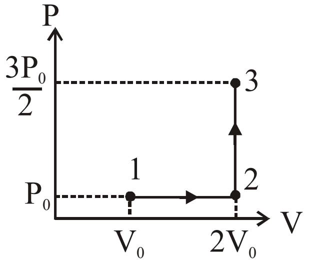
28. **Answer the following by appropriately matching the lists based on the information given in the paragraph.** **[JEE-Advance 2019]**

In a thermodynamics process on an ideal monatomic gas, the infinitesimal heat absorbed by the gas is given by $T\Delta X$, where T is temperature of the system and ΔX is the infinitesimal change in a thermodynamic quantity X of the system. For a mole of monatomic ideal gas $X = \frac{3}{2}R\ln\left(\frac{T}{T_A}\right) + R\ln\left(\frac{V}{V_A}\right)$. Here, R is gas constant, V is volume of gas, T_A and V_A are constants.

The List-I below gives some quantities involved in a process and List-II gives some possible values of these quantities.

List-I	**List-II**
(I) Work done by the system in process $1 \to 2 \to 3$	(P) $\frac{1}{3}RT_0 \ln 2$
(II) Change in internal energy in process $1 \to 2 \to 3$	(Q) $\frac{1}{3}RT_0$
(III) Heat absorbed by the system in process $1 \to 2 \to 3$	(R) RT_0
(IV) Heat absorbed by the system in process $1 \to 2$	(S) $\frac{4}{3}RT_0$
	(T) $\frac{1}{3}RT_0(3+\ln 2)$
	(U) $\frac{5}{6}RT_0$

If the process carried out on one mole of monatomic ideal gas is as shown in figure in the PV-diagram with $P_0V_0 = \frac{1}{3}RT_0$, the correct match is,



(1) I → Q, II → R, III → P, IV → U
(2) I → S, II → R, III → Q, IV → T
(3) I → Q, II → R, III → S, IV → U
(4) I → Q, II → S, III → R, IV → U

KT0144

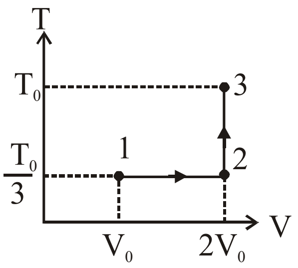
29. **Answer the following by appropriately matching the lists based on the information given in the paragraph.** **[JEE-Advance 2019]**

In a thermodynamic process on an ideal monatomic gas, the infinitesimal heat absorbed by the gas is given by $T\Delta X$, where T is temperature of the system and ΔX is the infinitesimal change in a thermodynamic quantity X of the system. For a mole of monatomic ideal gas $X = \frac{3}{2} R \ln\left(\frac{T}{T_A}\right) + R \ln\left(\frac{V}{V_A}\right)$. Here, R is gas constant, V is volume of gas, T_A and V_A are constants.

The List-I below gives some quantities involved in a process and List-II gives some possible values of these quantities.

List-I	**List-II**
(I) Work done by the system in process $1 \to 2 \to 3$	(P) $\frac{1}{3} RT_0 \ln 2$
(II) Change in internal energy in process $1 \to 2 \to 3$	(Q) $\frac{1}{3} RT_0$
(III) Heat absorbed by the system in process $1 \to 2 \to 3$	(R) RT_0
(IV) Heat absorbed by the system in process $1 \to 2$	(S) $\frac{4}{3} RT_0$
	(T) $\frac{1}{3} RT_0 (3 + \ln 2)$
	(U) $\frac{5}{6} RT_0$

If the process on one mole of monatomic ideal gas is an shown is as shown in the TV-diagram with $P_0V_0 = \frac{1}{3} RT_0$, the correct match is



(1) I → S, II → T, III → Q, IV → U (2) I → P, II → R, III → T, IV → S

(3) I → P, II → T, III → Q, IV → T (4) I → P, II → R, III → T, IV → P

KT0145

ANSWER KEY

EXERCISE (S-1)

1. Ans. 1.25×10^4 N/m^2 **2. Ans.** 327 °C

3. Ans. (i) $P_1 < P_2$, $T_1 < T_2$; (ii) $T_1 = T_2 < T_3$; (iii) $V_2 > V_1$; (iv) $P_1 > P_2$ **4. Ans.** 40

5. Ans. (a) 4/3 m, (b) $T_3 = 400\left(\frac{4}{3}\right)^{0.4}$ K **6. Ans.** 3600 R **7. Ans.** $\Delta T = \frac{mv_0^2}{3R}$

8. Ans. 12600 J **9. Ans.** 5R **10. Ans.** 2.64 **11. Ans.** 1.5

12. Ans. 450 J **13. Ans.** 16.9 J **14. Ans.** R/2

15. Ans. (a) 0.5 atm (b) zero (c) zero (d) no

16. Ans. (a) A to B (b) C to D

(c) $W_{AB} = \frac{1}{1-\gamma}(2^{1-\gamma}-1)(P_B - P_A)V_A = \frac{3}{2}\left(1-\left(\frac{1}{2}\right)^{2/3}\right)(P_B - P_A)V_A$

(d) $\left[1-\left(\frac{1}{2}\right)^{2/3}\right]$

EXERCISE (S-2)

1. Ans. (i) 74 cm, (ii) 73.94 cm, (iii) 69.52 cm

2. Ans. $p_1 = p_{H_2} \simeq 1.25 \times 10^6$ Pa ; $p_2 = p_{H_2} + p_{O_2} + p_{N_2} \simeq 2.8125 \times 10^6$ Pa ; $p_3 = p_{H_2} + p_{N_2} \simeq 1.5625 \times 10^6$ Pa

3. Ans. 1.6 m, 364 K

4. Ans. (a) $P_i = P_a$ (b) $P_f = P_a + k(V - V_0)/A^2$; $V_0 = \ell A$

(c) $\Delta Q = P_a(V - V_0) + \frac{1}{2}k(V - v_0)^2 + C_v(T - T_0)$; $A = 1$

5. Ans. (a) Final pressure in A = $\frac{27}{8}P_0$ = Final pressure in C, Final pressure in B = $\frac{21}{4}P_0$

(b) Final temperature in A (and B) = $\frac{21}{4}T_0$, Final temperature in C = $\frac{3}{2}T_0$,

(c) $18\,P_0V_0$

(d) work done by gas in A = $+\,P_0V_0$, work done by gas in B = 0,

(e) $\frac{17}{2}P_0V_0$

6. Ans. C = 2R **7. Ans.** 400 J, $2T_0$ **8. Ans.** 2000 N/m, 1295 J

9. Ans. $Q_1 < Q_2$

10. Ans. (i) $T_A = 120.33$ K, $T_B = 240.66$ K, $T_C = 481.32$ K, $T_D = 240.66$ K, (ii) No

(iii) $\Delta Q_{ABC} = 3.25 \times 10^6$ J; $\Delta Q_{ADC} = 2.75 \times 10^6$ J

11. Ans. $1 - \frac{3\left(1-\frac{1}{2^{1/3}}\right)}{\ln 2}$ **12. Ans.** 2.515, 404.8 K

EXERCISE (O-1)

1. Ans. (A) 2. Ans. (A) 3. Ans. (B) 4. Ans. (D) 5. Ans. (C) 6. Ans. (A)

7. Ans. (C) 8. Ans. (B) 9. Ans. (C) 10. Ans. (C) 11. Ans. (C) 12. Ans. (A)

13. Ans. (D) 14. Ans. (A) 15. Ans. (C) 16. Ans. (D) 17. Ans. (B) 18. Ans. (D)

19. Ans. (C) 20. Ans. (C) 21. Ans. (A) 22. Ans. (C) 23. Ans. (D) 24. Ans. (B)

25. Ans. (B) 26. Ans. (C) 27. Ans. (B) 28. Ans. (D) 29. Ans. (A) 30. Ans. (A)

31. Ans. (A) 32. Ans. (A) 33. Ans. (A) 34. Ans. (D) 35. Ans. (B) 36. Ans. (B)

37. Ans. (C) 38. Ans. (C) 39. Ans. (D) 40. Ans. (A) 41. Ans. (B) 42. Ans. (A,B,C)

43. Ans. (B,C) 44. Ans. (B,C) 45. Ans. (D) 46. Ans. (D) 47. Ans. (C, D)

48. Ans. (B) 49. Ans. (A) 50. Ans. (A) P (B) Q (C) Q

EXERCISE (O-2)

1. Ans. (C) 2. Ans. (A) 3. Ans. (D) 4. Ans. (D) 5. Ans. (B) 6. Ans. (C)

7. Ans. (D) 8. Ans. (C) 9. Ans. (D) 10. Ans. (C) 11. Ans. (D) 12. Ans. (C)

13. Ans. (A) 14. Ans. (C) 15. Ans. (C,D) 16. Ans. (B,D) 17. Ans. (A,B) 18. Ans. (A,B)

19. Ans. (B,C) 20. Ans. (B) 21. Ans. (A) 22. Ans. (B)

23. Ans. (A) → (P,R) ; (B) → (Q, S) ; (C) → (Q, S) ; (D) → (P, R)

SUPPLEMENT FOR JEE-MAINS

1. Ans. B & 1.8% more 2. Ans. 2400 joule per cycle 3. Ans. 900 cal 4. Ans. 5.7 J

5. Ans. 3.36×10^6 J 6. Ans. (B) 7. Ans. (D) 8. Ans. (A) 9. Ans. (A)

10. Ans. (A) 11. Ans. (A) 12. Ans. (A) 13. Ans. (A) 14. Ans. (A)

15. Ans. (A) 16. Ans. (B) 17. Ans. (C) 18. Ans. (D) 19. Ans. (B)

EXERCISE (J-M)

1. Ans. (4) 2. Ans. (1) 3. Ans. (4) 4. Ans. (4) 5. Ans. (3) 6. Ans. (2)

7. Ans. (3) 8. Ans. (4) 9. Ans. (2) 10. Ans. (1) 11. Ans. (2) 12. Ans. (4)

13. Ans. (2) 14. Ans. (3) 15. Ans. (2) 16. Ans. (Bonus) 17. Ans. (1)

18. Ans. (1) 19. Ans. (2) 20. Ans. (3) 21. Ans. (2) 22. Ans. (1) 23. Ans. (2)

EXERCISE (JA)

1. Ans. (BD) 2. Ans. (B,D) 3. Ans. (D) 4. Ans. (A,B,C,D) 5. Ans. 4

6. Ans. (A) 7. Ans. (A) → (P,R,T) ; (B) → (P,R) ; (C) → (Q,S) ; (D) → (R,T)

8. Ans. (D) 9. Ans. (D) 10. Ans. (D) 11. Ans. (A, B, C, D) or (B, C, D)

12. Ans. (A) 13. Ans. 2 14. Ans. (D) 15. Ans. (D) 16. Ans. (A,B,D)

17. Ans. (A,B,C) 18. Ans. (C) 19. Ans. (A,B,D) 20. Ans. (B)

21. Ans. (A) 22. Ans. (D) 23. Ans. (B,C,D) 24. Ans. 900 [899.95, 900.05]

25. Ans. (C) 26. Ans. (1,2) 27. Ans. (1,3,4) 28. Ans. (3) 29. Ans. (4)

Chapter 03

Contents

FLUID MECHANICS

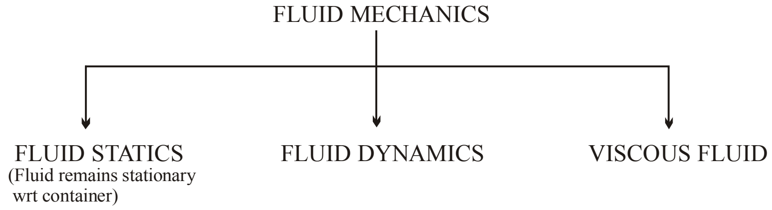
Important Notes

FLUID MECHANICS

KEY CONCEPTS

What Is a Fluid?

A **fluid,** in contrast to a solid, is a substance that can flow. Fluids conform to the boundaries of any container in which we put them. They do so because a fluid cannot sustain a force that is tangential to its surface. a fluid is a substance that flows because it cannot withstand a shearing stress. It can, however, exert a force in the direction perpendicular to its surface.



Fluid includes property → (A) Density (B) Viscosity (C) Bulk modulus of elasticity (D) Pressure (E) Specific gravity.

Assumptions used in fluid mechanics

1. Fluid is incompressible means density remains constant and volume also remains constant.
2. Fluid is non viscous. There is no tangential force between two layers.

DENSITY (ρ)

Mass per unit volume is defined as density. So density at a point of a fluid is represented as

$$\rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V} = \frac{dm}{dV}$$ Density is a positive scalar quantity.

SI UNIT : kg/m^3 **CGS UNIT :** g/cc **Dimensions :** $[ML^{-3}]$

RELATIVE DENSITY

It is defined as the ratio of the density of the given fluid to the density of pure water at 4°C.

$$\text{Relative density (R.D.)} = \frac{\text{density of given liquid}}{\text{density of pure water at } 4^\circ\text{C}}$$

Relative density or specific gravity is a unitless and dimensionless positive scalar physical quantity. Being a dimensionless/unitless quantity R.D. of a substance is same in SI and CGS system.

SPECIFIC GRAVITY

It is defined as the ratio of the specific weight of the given fluid to the specific weight of pure water at 4°C.

$$\text{Specific gravity} = \frac{\text{specific weight of given liquid}}{\text{specific weight of pure water at } 4^\circ\text{C } (9.81\ \text{kN/m}^3)} = \frac{\rho_\ell \times g}{\rho_w \times g} = \frac{\rho_\ell}{\rho_w} = \text{R.D. of the liquid}$$

Thus specific gravity of a liquid is numerically equal to the relative density of that liquid and for calculation purposes they are used interchangeably.

Density of a Mixture of substance in the proportion of mass

Let a number of substances of masses M_1, M_2, M_3 etc., and densities ρ_1, ρ_2, ρ_3 etc. respectively are mixed together. The total mass of the mixture = $M_1 + M_2 + M_3 + ...$

The total volume = $\frac{M_1}{\rho_1} + \frac{M_2}{\rho_2} + \frac{M_3}{\rho_3} + ...$ therefore, the density of the mixture is $\rho = \dfrac{M_1 + M_2 + M_3 ....}{\frac{M_1}{\rho_1} + \frac{M_2}{\rho_2} + \frac{M_3}{\rho_3} + ...}$

For two substances the density of the mixture $\rho = \dfrac{\rho_1 \rho_2 (M_1 + M_2)}{\rho_1 M_2 + \rho_2 M_1}$

Density of a mixture of substance in the proportion of volume

Suppose that a number of substances of volume V_1, V_2, V_3 etc. and densities ρ_1, ρ_2, ρ_3 etc. respectively are mixed. The total mass of the mixture is = $\rho_1 V_1 + \rho_2 V_2 + \rho_3 V_3 + ...$

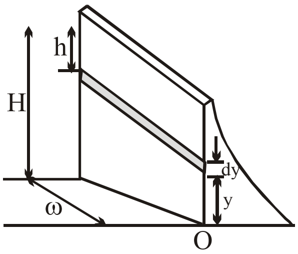
The total volume of the mixture is = $V_1 + V_2 + V_3 + ...$

Therefore, the density of the mixture is $\rho = \dfrac{\rho_1 V_1 + \rho_2 V_2 + \rho_3 V_3}{V_1 + V_2 + V_3 + ...}$

Therefore, for two substances we can write $\rho = \dfrac{\rho_1 V_1 + \rho_2 V_2}{V_1 + V_2}$

Example(a) :

Water is filled to a height H behind a dam of width w (fig.). Determine the resultant force exerted by the water on the dam.



Water exerts a force on a dam

Solution :

Let's consider a vertical y axis, starting from the bottom of the dam. Lets consider a thin horizontal strip at a height y above the bottom, such as shown in Fig. We need to consider force due to the pressure of the water only as atmospheric pressure acts on both sides of the dam.

The pressure due to the water at the depth h : $P = \rho gh = \rho g(H - y)$

The force exerted on the shaded strip of area $dA = wdy$:

$dF = P\,dA = \rho g(H - y)\,\omega\,dy$

Integrate to find the total force on the dam :

$$F = \int P\,dA = \int_0^H \rho g(H - y)\omega\,dy = 1/2\ \rho g \omega H^2$$

Example(b) :

In the previuous example find the total torque exerted by the water on dam about a horizontal axis through O. Also find the effective line of action of the total force exerted by the water is at a distance 1/3 H above O.

Solution : .

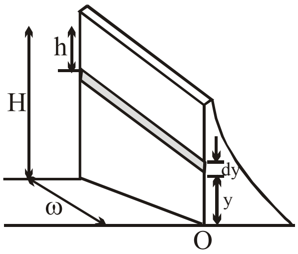
The torque is $\tau = \int d\tau = \int r dF$

From the figure $\tau = \int_0^H y[\rho g(H-y)w dy] = \frac{1}{6}\rho g w H^3$

The total force is given as $\frac{1}{2}\rho g w H^2$

If this were applied at a height y_{eff} such that the torque remains unchanged, we have

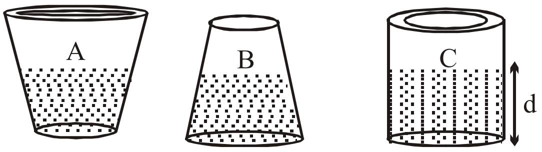
$$\frac{1}{6}\rho g w H^3 = y_{eff}\left[\frac{1}{2}\rho g w H^2\right] \text{ and } y_{eff} = \frac{1}{3}H$$



Water exerts a force on a dam

Example(a&b) :

Three vessels having different shapes are as shown in the figure below, they have same base area and the same weight when empty (Fig.). The vessels are filled with mercury to the same level. Neglect the effect of the atmosphere. (a) Which have the largest and which have the smallest pressures at the bottom of the vessel or are they same? (b)Which show the highest weight when weighed on a weighing scale or are they same?



Three differently shaped vessels filled with water to same level.

Solution (a) The mercury at the bottom of each vessel is at the same depth d below the surface. Neglecting the pressure at the surface, the pressures at the bottom must be equal hence:

$$P = \rho g d$$

(b) The weight of each filled vessel is equal to the weight of the vessel itself plus the weight of the mercury inside. The vessels themselves are of equal weights, but vessel A holds more mercury than C, while vessel B holds less mercury than C. Vessel A weighs the most and vessel B weighs the least.

Example(c) : As the mercury exerts the same downward force on the bottom of each vessel, then why does the vessels weigh differently ?

Soln. In vessel C ,forces due to fluid pressure on the sides of the container are horizontal. Forces on any two diametrically opposite points on the walls of the container are equal and opposite; thus, the net force on the container walls is zero. The force on the bottom is

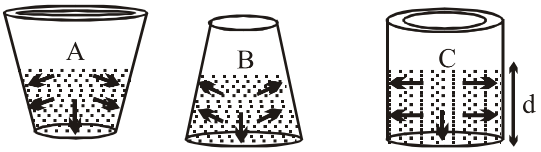
$$F = PA = (\rho g d)(\pi r^2)$$

The volume of water in the cylinder is $V = \pi r^2 d$, so

$$F = \rho g V = (\rho V) g = mg$$

The force on the bottom of vessel C is equal to the weight of the water, as expected. The forces due to fluid pressure on the sides of the containers A and B have vertical components also. Hence the force between the fluid and the base of container will not be equal to the weight of the fluid. These containers support the fluid by exerting an upward force equal in magnitude to the weight of the fluid but some force is being applied by the sidewalls and the remaining by the bottom. Figure shows the forces acting on each container due to the water.

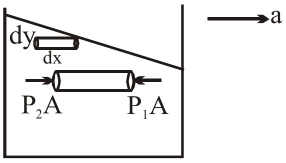
The force on the bottom of vessel A is less than the weight of the mercury in the container, while the force on the bottom of vessel B is greater than the weight of the mercury. In vessel A, the forces on the container walls have downward components as well as horizontal components. The sum of the downward components of the forces on the walls and the downward force on the bottom of the container is equal to the weight of the water. Similarly, the forces on the walls of vessel B have upward components. In each case, the total force on the bottom and sides of the container due to the water is equal to the weight of the water.



Forces exerted on the containers by the water.

Linear Accelerated Motion :

We consider an open container of a liquid that is moving along a straight line with a constant acceleration a as shown in Fig.



Lets consider a small horizontal cylinder of length dx and crossectional area A located y below the free surface of the fluid. This cylinder is accelerating in ground frame with accleration a hence the net horizontal force acting on it should be equal to the product of mass(dm) and acceleration.

$dm = Adx\rho$

$P_2A - P_1A = (Adx\rho)a$

If we say that the right face of the cylinder is y below the free surface of the fluid then the left surface is y + dy below the surface of liquid. Thus

$$P_2 - P_1 = \rho g dy$$

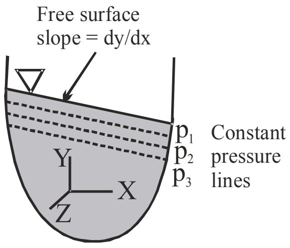
$$\therefore \quad \frac{dy}{dx} = \frac{a}{g}$$

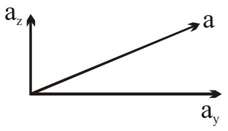
Since the slope of the free surface is coming out to be constant we can say that it must be straight line.

$$\tan\theta = \frac{a}{g}$$

If the container have acceleration along y also than the slope of this line is given by the relationship.

$$\frac{dy}{dx} = -\frac{a_x}{g + a_y}$$





Along a free surface the pressure is constant, so that for the accelerating mass shown in figure the free surface will be inclined if $a_x \neq 0$. In addition, all lines parallel to the free surface will have same presure. For the special circumstance in which $a_x = 0$, $a_y \neq 0$. which corresponds to the mass of fluid accelerating in the vertical direction, Equation indicates that the fluid surface will be horizontal. However, from below equation we see that the pressure variatio is not ρgdy, but is given by the equation.

$$dP = \rho(g + a_y)dy$$

Thus, the pressure on the bottom of a liquid-filled tank which is resting on the floor of an elevator that is accelerating upward will be more than, if the, tank would have been at rest (or moving with a constant velocity). It is to be noted that for a freely falling fluid mass ($a_y = -g$), the pressure variation in all three coordinate directions are zero, which means that the pressure throughout will be same. The pressure throughout a "blob" of a liquid floating in an orbiting space shuttle (a form of free fall) is zero.

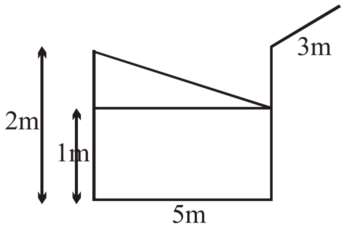
Ques : The cross section of a tank kept on a vehicle is shown in Fig.. The rectangular tank is open to the atmosphere. During motion of the vehicle, the tank is subjected to a constant linear acceleration, a = 2.5 m/s^2. How much fluid will be left inside the tank if initially the tank is half filled. The vessel is 5m wide and 2m high.

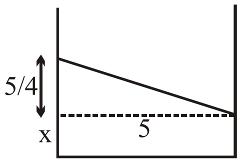
Ans.

If the height of the liquid on the left wall is greater than 2m the fluid will be spilled out.

Now we can find the angle that the fluid will make with the horizontal.

$$\tan\theta = \frac{2.5}{10} = \frac{1}{4}$$





Lets assume that the dimension of tank in the plane perpendicular to the page is d.

From the geometry its easy to see that free surface on RHS will go down and will rise on LHS. Thus if we assume that fluid on RHS has not touched the floor, we will have fluid taking ashape as described in the diagram. The cuboid part will have volume $x \times 5 \times d$, where x is the height above the bottom.

The wedge part will have the volume $\frac{1}{2} \times h \times 5 \times d$ where h can be found as following (h/5) = tan θ = (1/4)

Thus total volume will be $\frac{1}{2} \times (5/4) \times 5 \times d + x \times 5 \times d$ and if we assume there is no spilling than it must beequal to the final volume.

$$\frac{1}{2} \times (5/4) \times 5 \times d + x \times 5 \times d = 1 \times 5 \times d$$

solving we get $x = \frac{3}{8}$

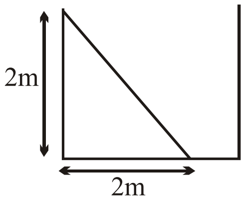
$\therefore$ Total length $\frac{5}{4} + \frac{3}{8} = \frac{10+3}{8} = \frac{13}{8} < 2$

Thus, height is less than 2.
Hence water will not spill.

Ex. How much fluid will be left inside the tank if the vehicle accelerates at accleration, a = 10 m/s^2 ?
If the height of the liquid on the left wall is greater than 2m the fluid will be spilled out.
Now we can find the angle that the fluid will make with the horizontal.

$$\tan\theta = \frac{10}{10} = 1, \text{ thus } \theta = \frac{\pi}{4}$$

In this case fluid can not remain inside. Fluid having an inclined free surface at 45^0 angle, and covering the bottom of length 5m, will also be 5 m high. This will require the wall to be of 5 m height, which is just 2m for the given vessel. Instead if we think it other way round to keep in contact with the LHS wall, bottom will have to be covered only 2m with the fluid as as shown in the diagram,



Fluid Inside = (1/2) $\times 2 \times 2 \times d$ m^3
Remain inside = 2d m^3
$\therefore$ Thus volume of fluid gone Outside = 3d m^3

Pascal's Principle

When you squeeze one end of a tube to get toothpaste out the other end, you are watching **Pascal's principle** in action. This principle is also the basis for the Heimlich maneuver, in which a sharp pressure increase properly applied to the abdomen is transmitted to the throat, forcefully ejecting food lodged there. The principle was first stated clearly in 1652 by Blaise Pascal (for whom the unit of pressure is named):

A change in the pressure applied to an enclosed incompressible fluid is transmitted undiminished to every portion of the fluid and to the walls of its container.

Demonstrating Pascal's Principle

Consider the case in which the incompressible fluid is a liquid contained in a tall cylinder, as in Fig. The cylinder is fitted with a piston on which a container of lead shot rests. The atmosphere, container, and shot exert pressure p_{ext} on the piston and thus on the liquid. The pressure p at any point P in the liquid is then

$$p = p_{ext} + \rho_{gh}$$

Let us add a little more lead shot to the container to increase pext by an amount p_{ext}. The quantities Δp_{ext}, g and h in Eq. are unchanged, so the pressure change at P is

$$\Delta p = \Delta p_{ext}$$

This pressure change is independent of h, so it must hold for all points within the liquid, as Pascal's principle states.

Pascal's Principle and the Hydraulic Lever

Figure shows how Pascal's principle can be made the basis of a hydraulic lever. In operation, let an external force of magnitude F_i be directed downwardon the left-hand (or input) piston, whose surface area is A_i. An incompressible liquid in the device then produces an upward force of magnitude F_0 on the right-hand (or output) piston, whose surface area is A_0. To keep the system in equilibrium, there must be a downward force of magnitude *Fo* on the output piston from an external load (not shown). The force $\vec{F}_1$ applied on the left and the downward force $\vec{F}_0$ from the load on the right produce a change Δp in the pressure of the liquid that is given by

$$\Delta p = \frac{F_i}{A_i} = \frac{F_0}{A_0}$$

$$F_0 = F_1 \frac{A_0}{A_i}$$

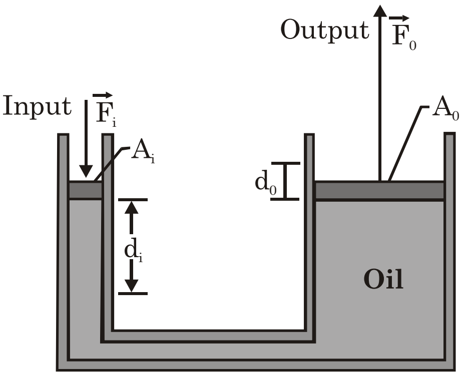


Figure (a)

Equation shows that the output force F_0 on the load must be greater than the input force F_i if $A_0 > A_p$ as is the case in figure.

If we move the input piston downward a distance di, the output piston moves upward a distance do, such that the same volume V of the incompressible liquid is displaced at both pistons.Then

$$V = A_i d_i = A_0 d_0$$

which we can write as

$$d_0 = d_i \frac{A_i}{A_0}$$

This shows that, if Ao > Ai (as in Figure), the output piston moves a smaller distance than the input piston moves.

From Eqs. we can write the output work as

$$W = F_0 d_0 = \left(F_i \frac{A_i}{A_0}\right)\left(d_i \frac{A_i}{A_0}\right) = F_i d_i$$

which shows that the work W done on the input piston by the applied force is equal to the work W done by the output piston in lifting the load placed on it.

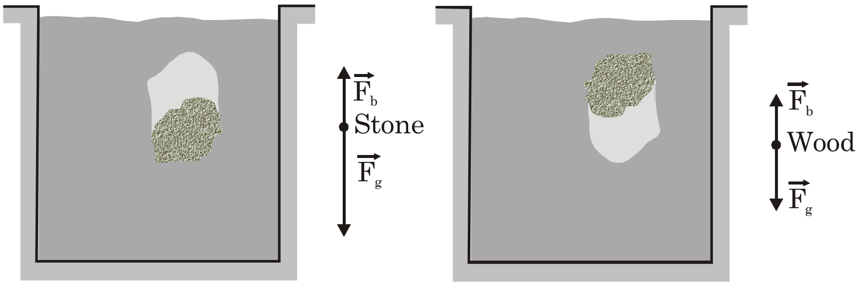
The advantage of a hydraulic lever is this:

With a hydraulic lever, a given force applied over a given distance can be transformed to a greater force applied over a smaller distance.

The product of force and distance remains unchanged so that the same work is done. However, there is often tremendous advantage in being able to exert the larger force. Most of us, for example, cannot lift an automobile directly but can with a hydraulic jack, even though we have to pump the handle farther than the automobile rises and in a series of small strokes.

Archimedes' Principle

Figure shows a student in a swimming pool, manipulating a very thin plastic sack (of negligible mass) that is filled with water. She finds that the sack and its contained water are in static equilibrium, tending neither to rise nor to sink.The downward gravitational force $\vec{F}_g$ on the contained water must be balanced by a net upward force from the water surrounding the sack.



(b) (c)

This net upward force is a **buoyant force** $\vec{F}_b$. It exists because the pressure in the surrounding water increases with depth below the surface.Thus, the pressure near the bottom of the sack is greater than the pressure near the top, which means the forces on the sack due to this pressure are greater in magnitude near the bottom of the sack than near the top. Some of the forces are represented in figure (a) where the space occupied by the sack has been left empty. Note that the force vectors drawn near the bottom of that space (with upward components) have longer lengths than those drawn near the top of the sack (with downward components). If we vectorially add all the forces on the sack from the water, the horizontal components cancel and the vertical components add to yield the upward buoyant force $\vec{F}_b$ on the sack. (Force $\vec{F}_b$ is shown to the right of the pool in Figure (a)

Because the sack of water is in static equilibrium, the magnitude $\vec{F}_b$ of is equal to the magnitude m_f g of the gravitational force $\vec{F}_g$ on the sack of water: $F_b = m_f g$. (Subscript *f* refers to fluid, here the water.) In words, the magnitude of the buoyant force is equal to the weight of the water in the sack.

In Fig. (b), we have replaced the sack of water with a stone that exactly fills the hole in Fig. (a). The stone is said to displace the water, meaning that the stone occupies space that would otherwise be occupied by water. We have changed nothing about the shape of the hole, so the forces at the hole's surface must be the same as when the water-filled sack was in place. Thus, the same upward buoyant force that acted on the water-filled sack now acts on the stone; that is, the magnitude $\vec{F}_b$ of the buoyant force is equal to $m_f g$, the weight of the water displaced by the stone.

Unlike the water-filled sack, the stone is not in static equilibrium. The downward gravitational force $\vec{F}_g$ on the stone is greater in magnitude than the upward buoyant force, as is shown in the free-body diagram in Fig. (*b*). The stone thus accelerates downward, sinking to the bottom of the pool. Let us next exactly fill the hole in Fig. (a) with a block of lightweight wood, as in Fig. (c*)*. Again, nothing has changed about the forces at the hole's surface, so the magnitude $\vec{F}_b$ of the buoyant force is still equal to $m_f g$, the weight of the displaced water. Like the stone, the block is not in static equilibrium. However, this time the gravitational force $\vec{F}_g$ is lesser in magnitude than the buoyant force (as shown to the right of the pool), and so the block accelerates upward, rising to the top surface of the water. Our results with the sack, stone, and block apply to all fluids and are summarized in **Archimedes' principle:**

When a body is fully or partially submerged in a fluid, a buoyant force $\vec{F}_b$ from the surrounding fluid acts on the body. The force is directed upward and has a magnitude equal to the weight $m_f g$ of the fluid that has been displaced by the body. The buoyant force on a body in a fluid has the magnitude

$$F_b = m_f g \text{ (buoyant force)}$$

where m_f is the mass of the fluid that is displaced by the body.

Floating

When we release a block of lightweight wood just above the water in a pool, the block moves into the water because the gravitational force on it pulls it downward.

As the block displaces more and more water, the magnitude F_b of the upward buoyant force acting on it increases. Eventually, F_b is large enough to equal the magnitude F_g of the downward gravitational force on the block, and the block comes to rest. The block is then in static equilibrium and is said to be floating in the water. In general, When a body floats in a fluid, the magnitude F_b of the buoyant force on the body is equal to the magnitude F_g of the gravitational force on the body.

We can write this statement as

$$F_b = F_g \text{ (Floating)}$$

From Eq. we know that $F_b = m_f g$. Thus,

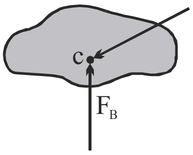
When a body floats in a fluid, the magnitude F_g of the gravitational force on the body is equal to the weight $m_f g$ of the fluid that has been displaced by the body.

We can write this statement as

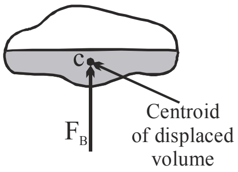
$$F_g = m_f g$$

In other words, a floating body displaces its own weight of fluid.

The location of the line of action of the buoyant force can be determined by adding torques of the forces due to pressure forces, with respect to some convenient axis. The buoyant force must pass through the center of mass of the desplaced volume, as shown in Fig. (c), as it was in translational and rotational equilibirium. The point through which the buoyant force acts is called the center of buoyancy.



(c)



(D)

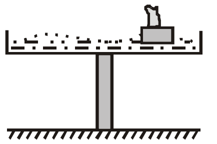
These same results apply to floating bodies which are only partially submerged, as shown in Fig.(d), if the density of the fluid above the liquid surface is very small compared with the liquid in which the body floats. Since the fluid above the surface is usually air, for practial purposes this condition is satisfied.

In the above discussion, the fluid is assumed to have a constant density. If a body is immersed in a fluid in which density varies with depth, such as having multiple layers of fluid, the magnitude of the buoyant force remains equal to the weight of the displaced fluid and the buoyant force passes through the center of mass of the displaced volume.

Q. A wooden block floats vertically in a glass filled with water. How will the level of the water in the glass change if the block is kept in a horizontal position ?

Solution : The level of the water will not change because the quantity of water displaced will remain the same.

Q. A vessel filled with water is placed exactly in middle of a thin wall(fig.). Will the system topple if a small wooden boat carrying some weight is floated in the vessel?



Solution : The system will not topple, since according to Pascal's law the pressure on the bottom of the vessel will be the same everywhere thus the body will still remain in rotational equilibirium

Ex. A homogeneous piece of ice floats in a glass filled with water. How will the level of the water in the glass change when the ice melts ?

Solution : Since the piece of ice floats, the weight of the water displaced by it is equal to the weight of the ice itself or the weight of the water it produces upon melting. For this reason the water formed by the piece of ice will occupy a volume equal to that of the submerged portion, and the level of the water will not change.

Q. A piece of ice is floating in a tub filled with water. How will the level of the water in the tub change when the ice melts ? Consider the following cases :

(1) a stone is frozen in the ice

(2) the ice contains an air bubble

Solution :

(1) The volume of the submerged portion of the piece with the stone is greater than the sum of the volumes of the stone and the water produced by the melting ice. Therefore, the level of the water in the glass will drop.

(2) The weight of the displaced water is equal to that of the ice (the weight of the air in the bubble may be neglected). For this reason, as in conceptual eg., the level of the water will not change.

Example :

A vessel with a body floating in it is kept in elevator accelerating downwards with acceleration a such that $a < g$. Will the body rise or sink further in the vessel?

Solution : The force of bouyancy on the body can be written as $F = \rho V_2(g - a)$, where V_2 is the volume of the submerged portion of the body in the lift. As pressure at a point h below the surface will become $\rho(g - a)h$ instead of ρgh. Applying the Newton's second Law, remembering that the body was accelerating upwards at a.

$$Mg - \rho V_2 (g - a) = Ma$$

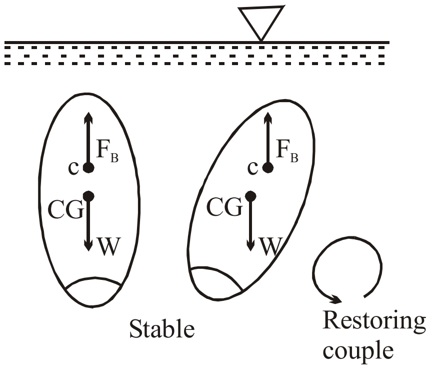
Hence, $V_2 = \dfrac{M}{\rho}$ thus $V_2 = V$, as in a stationary vessel, $V = \dfrac{M}{\rho}$. Thus the body does not rise to the surface.

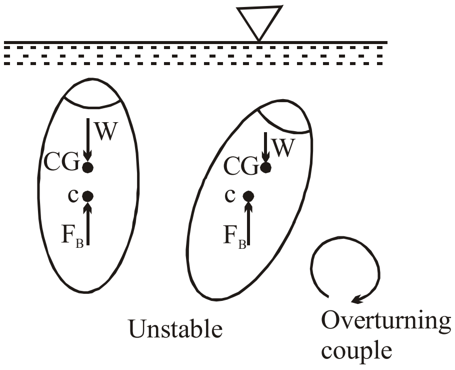
Stability :

The center of buoyancy and center of gravity do not necessarily coincide so the floating or submerged body may not be in stable equilibirium. A small rotation can cause the buoyant force to produce either a restoring or overturning torque. For example, for the completely submerged body shown in Fig., which has a center of gravity below the center of buoyancy, a rotation from its equilibrium position will create a restoring torque by the buoyant force, F_B, which causes the body to rotate back to its original position. Thus, if the center of gravity falls below the center of buoyancy, the body is stable.

However, as shown in Fig., if the center of gravity of the completely submerged body is above the center of buoyancy, the resulting torque formed by the weight and the buoyant force will cause the body to overturn and move to a new equilibrium position. Thus, a completely submerged body with its center of gravity above its center of buoyancy is in an unstable equilibrium position.

For floating bodies the stability problem is more complicated, since as the body rotates the location of the center of buoyancy may change.





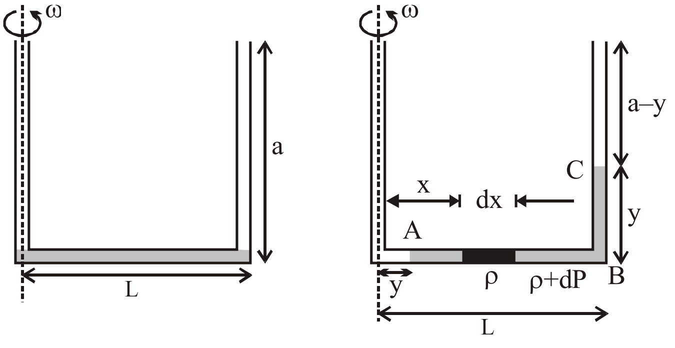
Concept : Length of a horizontal arm of a U-tube is L and ends of both the vertical arms are open to atmospheric pressure P_0. A liquid of density ρ is poured in the tube such that liquid just fills the horizontal part of the tube as shown in figure. Now one end of the opened ends is sealed and the tube is then rotated about a vertical axis passing through the other vertical arm with angular speed ω. If length of each vertical arm is a and in the sealed end liquid rises to a height y, find pressure in the sealed tube during rotation.

The pressure difference across an element of width dx, which is given as

$$dP = dx\rho\omega^2 x$$

Now integrating from A to B, we get

$$P_B - P_A = \int_y^L \rho\omega^2 x dx$$



(a) (b)

Thus pressure at point C an be given as

$$P_C = P_B - y\rho g$$

and at point A, pressure is atmospheric, thus we have

$$P_C = \frac{\rho\omega^2}{2}\left(L^2 - y^2\right) + \rho_A - y\rho g$$

Q. A hemisphere of radius R is just submerged is just sinking in water of density ρ. Find the

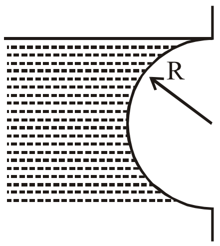
(a) horizontal thrust.

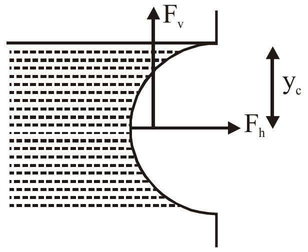
(b) vertical thrust.

(c) total hydrostatic force.

(d) angle of orientation of total hydrostatic force acting on the hemisphere.

Do not count atmospheric pressure





Sol. (a) Let the horizontal and vertical thrusts on the hemisphere be F_h and F_V respectively

We know that

$$F_h = \rho g y_c A_y$$

where $y_c = R$

and $A_v = \pi R^2$

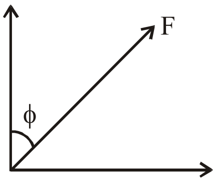
This gives $F_h = \rho g \pi R^3$ (right)

(b) Similarly using the formula $F_v = \rho V g$

Where V = volume of the hemisphere = $\frac{2}{3}\pi R^3$,

we have = $F_V = \frac{2}{3}\rho g \pi R^3$ (up)

(c) Hence the net dydrostatic force on the hemisphere is



$$F = \sqrt{F_h^2 + F_v^2}$$

$$F = \sqrt{\left(\rho g \pi R^3\right)^2 + \left(\frac{2}{3}\rho g \pi R^3\right)^2}$$

$$= \frac{\sqrt{13}}{3}\rho g \pi R^3$$

(d) The angle of orientation of the force F is

$$\phi = \tan^{-1}\frac{F_h}{F_v}$$

$$= \tan^{-1}\frac{\rho g \pi R^3}{\frac{2}{3}\rho g \pi R^3} = \tan^{-1}\frac{3}{2}$$

FLUID DYMANICS & VISCOSITY

Ideal Fluids in Motion

The motion of real fluids is very complicated and not yet fully understood. Instead, we shall discuss the motion of an **ideal fluid,** which is simpler to handle mathematically and yet provides useful results. Here are four assumptions that we make about our ideal fluid; they all are concerned with flow:

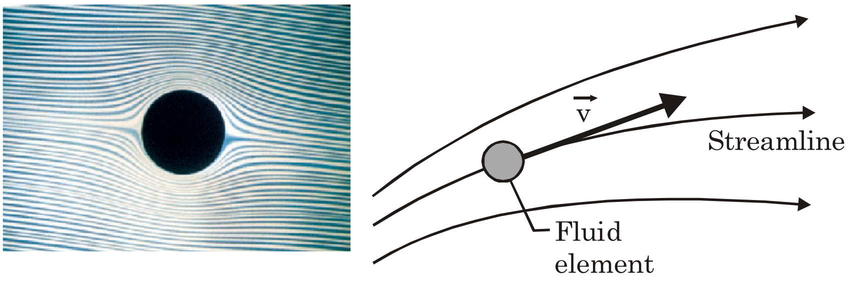
1. **Steady flow** In steady (or laminar) flow, the velocity of the moving fluid at any fixed point does not change with time, either in magnitude or in direction. The gentle flow of water near the center of a quiet stream is steady; the flow in a chain of rapids is not. Figure shows a transition from steady flow to nonsteady (or nonlaminar or turbulent) flow for a rising stream of smoke. The speed of the smoke particles increases as they rise and, at a certain critical speed, the flow changes from steady to nonsteady.
2. **Incompressible flow** We assume, as for fluids at rest, that our ideal fluid is incompressible; that is, its density has a constant, uniform value.

3. **Nonviscous flow** Roughly speaking, the viscosity of a fluid is a measure of how resistive the fluid is to flow. For example, thick honey is more resistive to flow than water, and so honey is said to be more viscous than water. Viscosity is the fluid analog of friction between solids; both are mechanisms by which the kinetic energy of moving objects can be transferred to thermal energy. In the absence of friction, a block could glide at constant speed along a horizontal surface. In the same way, an object moving through a nonviscous fluid would experience no viscous drag force-that is, no resistive force due to viscosity; it could move at constant speed through the fluid.

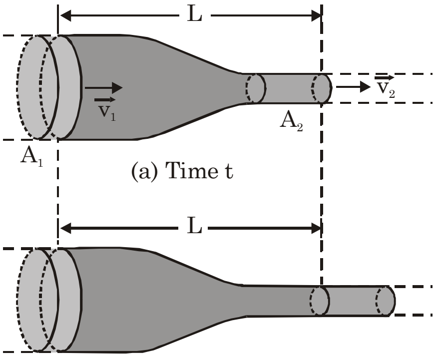
4. **Irrotational flow** : Although it need not concern us further, we also assume that the flow is irrotational. To test for this property, let a tiny grain of dust move with the fluid. Although this test body may (or may not) move in a circular path, in irrotational flow the test body will not rotate about an axis through its own center of mass. For a loose analogy, the motion of a Ferris wheel is rotational; that of its passengers is irrotational. T8hat the velocity of a particle is always tangent to the path taken by the particle. Here the particle is the fluid element, and its velocity is $\vec{v}$ always tangent to a streamline (Figure). For this reason, two streamlines can never intersect; if they did, then an element arriving at their intersection would have two different velocities simultaneously an impossibility.

The Equation of Continuity

You may have noticed that you can increase the speed of the water emerging from a garden hose by partially closing the hose opening with your thumb. Apparently the speed v of the water depends on the cross-sectional area A through which the water flows.

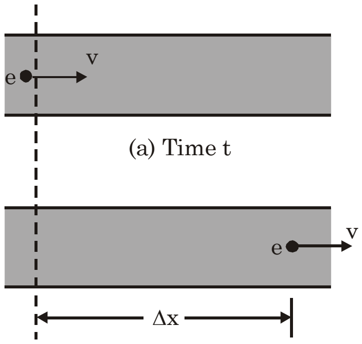


Here we wish to derive an expression that relates v and A for the steady flow of an ideal fluid through a tube with varying cross section, like that in Figure. The flow there is toward the right, and the tube segment shown (part of a longer tube) has length L. The fluid has speeds v_1 at the left end of the segment and v_2 at the right end. The tube has cross-sectional areas A_1 at the left end and A_2 at the right end. Suppose that in a time interval Δt a volume ΔV of fluid enters the tube segment at its left end (that volume is colored purple in Figure. Then, because the fluid is incompressible, an identical volume ΔV must emerge from the right end of the segment (it is colored green in Figure). We can use this common volume ΔV to relate the speeds and areas. To do so, we first consider Fig. , which shows a side view of a tube of uniform crosssectional area A. In Fig.(a), a fluid element e is about to pass through the dashed line drawn across the tube width.The element's speed is v, so during a time interval Δt, the element moves along the tube a distance Δx Δ v Δt.The volume ΔV of fluid that has passed through the dashed line in that time interval Δt is



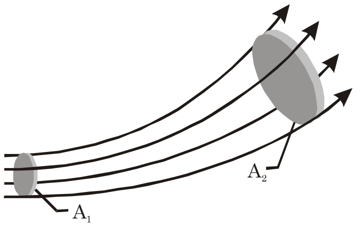
(a) Time t

(b) Time t + Δt



(a) Time t

(b) Time t + Δt



$$\Delta V = A\, \Delta x = Av\, \Delta t.$$

Applying Eq. to both the left and right ends of the tube segment in Fig., we have

$$\Delta V = A_1 v_1\, \Delta t = A_2 v_2\, \Delta t$$

Or $\quad A_1 v_1 = A_2 v_2$ (equation of continuity)

This relation between speed and cross-sectional area is called the **equation of continuity** for the flow of an ideal fluid. It tells us that the flow speed increases when we decrease the cross-sectional area through which the fluid flows (as when we partially close off a garden hose with a thumb). Equation applies not only to an actual tube but also to any so-called tube of flow, or imaginary tube whose boundary consists of streamlines. Such a tube acts like a real tube because no fluid element can cross a streamline; thus, all the fluid within a tube of flow must remain within its boundary. Figure shows a tube of flow in which the cross-sectional area increases from area A_1 to area A_2 along the flow direction. From Eq. we know that, with the increase in area, the speed must decrease, as is indicated by the greater spacing between streamlines at the right in Fig. . Similarly, you can see that in Fig. the speed of the flow is greatest just above and just below the cylinder. We can rewrite Eq. as

$R_V = A_v =$ a constant (volume flow rate, equation of continuity), in which R_V is the **volume flow rate** of the fluid (volume past a given point per unit time). Its SI unit is the cubic meter per second (m^3/s). If the density ρ of the fluid is uniform, we can multiply Eq. by that density to get the **mass flow rate** R_m (mass per unit time):

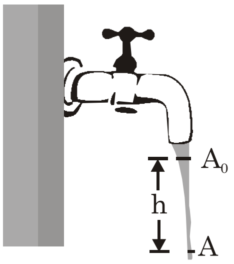
$$R_m = \rho R_V = A_v = \text{a constant (mass flow rate).}$$

The SI unit of mass flow rate is the kilogram per second (kg/s). Equation says that the mass that flows into the tube segment of Fig. each second must be equal to the mass that flows out of that segment each second.

Sample Problem

Figure shows how the stream of water emerging from a faucet "necks down" as it falls. The indicated cross-sectional areas are $A_0 = 1.2$ cm2 and A =0.35 cm^2. The two levels are separated by a vertical distance h = 45 mm.What is the volume flow rate from the tap?

The volume flow rate through the higher cross section must be the same as that through the lower cross section.



where v_0 and v are the water speeds at the levels corresponding to A_0 and A. From Eq. we can also write, because the water is falling freely with acceleration g,

$$v_2 = v^2_0 - 2gh.$$

Eliminating v between Eqs. and solving for v_0, we obtain

$$v_0 = \sqrt{\frac{2ghA^2}{A_0^2 - A^2}}$$

$$v_0 = \sqrt{\frac{(2)\times(9.8\text{m}/\text{s}^2)(0.045\text{m})(0.35\text{cm}^2)^2}{(1.2\text{cm}^2)-(0.35\text{cm}^2)^2}}$$

$$v_0 = 0.286 \text{ m/s} = 28.6 \text{ cm/s}.$$

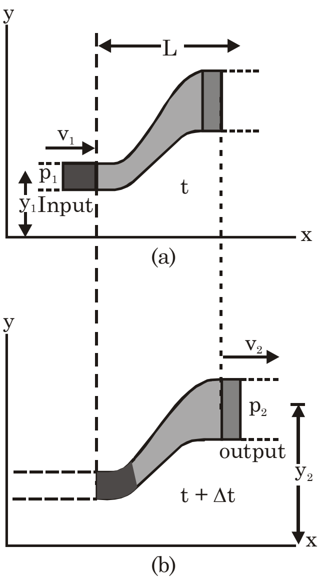
From Eq. , the volume flow rate R_V is then

$$R_V = A_0v_0 = (1.2 \text{ cm}^2)(28.6 \text{ cm/s})$$

$$= 34 \text{ cm}^3\text{/s. Ans.} \qquad]$$

Bernoulli's Equation

Figure represents a tube through which an ideal fluid is flowing at a steady rate. In a time interval Δt, suppose that a volume of fluid ΔV, colored purple in Fig. , enters the tube at the left (or input) end and an identical volume, colored green in Fig. , emerges at the right (or output) end. The emerging volume must be the same as the entering volume because the fluid is incompressible, with an assumed constant density ρ.



$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

Let y_1, v_1, and p_1 be the elevation, speed, and pressure of the fluid entering at the left, and y_2, v_2, and p_2 be the corresponding quantities for the fluid emerging at the right. By applying the principle of conservation of energy to the fluid, we shall show that these quantities are related by

In general, the term $\frac{1}{2}\rho v^2$ is called the fluid's **kinetic energy density** (kinetic energy per unit volume).We can also write Eq. as

$$p+\frac{1}{2}\rho v^2+\rho gy \text{ a constant (Bernoulli's equation).}$$

Equations are equivalent forms of **Bernoulli's equation,** after Daniel Bernoulli, who studied fluid flow in the 1700s.* Like the equation of continuity Eq., Bernoulli's equation is not a new principle but simply the reformulation of a familiar principle in a form more suitable to fluid mechanics. As a check, let us apply Bernoulli's equation to fluids at rest, by putting $v_1 = v_2 = 0$ in Eq. The result is

$$p_2 = p_1 + \rho g(y_1 - y_2)$$

Which is equation.

A major prediction of Bernoulli's equation emerges if we take y to be a constant (y = 0, say) so that the fluid does not change elevation as it flows. Equation then becomes

$$p_1+\frac{1}{2}\rho v_1^2 = p_2+\frac{1}{2}\rho v_2^2$$

Which tells us that :

If the speed of a fluid element increases as the element travels along a horizontal streamline, the pressure of the fluid must decrease, and conversely.

Put another way, where the streamlines are relatively close together (where the velocity is relatively great), the pressure is relatively low, and conversely. The link between a change in speed and a change in pressure makes sense if you consider a fluid element. When the element nears a narrow region, the higher pressure behind it accelerates it so that it then has a greater speed in the narrow region. When it nears a wide region, the higher pressure ahead of it decelerates it so that it then has a lesser speed in the wide region. Bernoulli's equation is strictly valid only to the extent that the fluid is ideal. If viscous forces are present, thermal energy will be involved. We take no account of this in the derivation that follows.

Proof of Bernoulli's Equation

Let us take as our system the entire volume of the (ideal) fluid shown in Fig. .We shall apply the principle of conservation of energy to this system as it moves from its initial state (Fig. (a)) to its final state (Fig. (b)). The fluid lying between the two vertical planes separated by a distance L in Fig. does not change its properties during this process; we need be concerned only with changes that take place at the input and output ends. First, we apply energy conservation in the form of the work–kinetic energy theorem,

$$W = \Delta K,$$

which tells us that the change in the kinetic energy of our system must equal the net work done on the system. The change in kinetic energy results from the change in speed between the ends of the tube and is

$$\Delta K = \frac{1}{2}\Delta m v_2^2 - \frac{1}{2}\Delta m v_1^2 = \frac{1}{2}\rho\Delta V(v_2^2 - v_1^2)$$

in which Δm (= ρ ΔV) is the mass of the fluid that enters at the input end and leaves at the output end during a small time interval Δt.

The work done on the system arises from two sources.The work W_g done by the gravitational force $(\Delta m\vec{g})$ on the fluid of mass Δm during the vertical lift of the mass from the input level to the output level is

$$W_g = \Delta m\, g(y_2 - y_1)$$
$$= -\rho g \Delta V(y_2 - y_1)$$

This work is negative because the upward displacement and the downward gravitational force have opposite directions.

Work must also be done on the system (at the input end) to push the entering fluid into the tube and by the system (at the output end) to push forward the fluid that is located ahead of the emerging fluid. In general, the work done by a force of magnitude F, acting on a fluid sample contained in a tube of area A to move the fluid through a distance Δx, is

$$F\Delta x = (pA)(\Delta x) = p(A\Delta x) = p\Delta v$$

The work done on the system is then $p_1\,\Delta V$, and the work done by the system is $-p_2\,\Delta V$. Their sum Wp is Wp

$$Wp = -p_2\,\Delta V - p_1\,\Delta V$$
$$= -(p_2 - p_1)\,\Delta V.$$

The work–kinetic energy theorem of Eq. now becomes

$$W = Wg + Wp = \Delta K.$$

Substituting from Eqs. yields

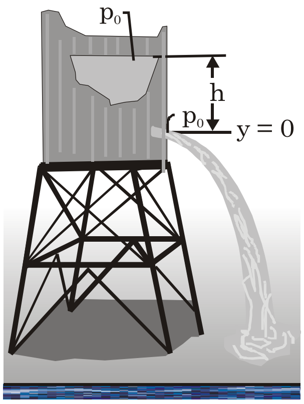
$$-\rho g \Delta V(y_2 - y_1) - \Delta V(p_2 - p_1) = \frac{1}{2}\rho \Delta v(v_2^2 - v_1^2)$$

This, after a slight rearrangement, matches Eq. , which we set out to prove.

Sample Problem : In the old West, a desperado fires a bullet into an open water tank (Fig.), creating a hole a distance h below the water surface.What is the speed v of the water exiting the tank?

Sol. From Eq. $R_V =$ av = Av_0 and thus

$$v_0 = \frac{a}{A}v$$



Because a <<A, we see that v_0 <<v.To apply Bernoulli's equation, we take the level of the hole as our reference level for measuring elevations (and thus gravitational potential energy). Noting that the pressure at the top of the tank and at the bullet hole is the atmospheric pressure p_0 (because both places are exposed to the atmosphere), we write Eq. as

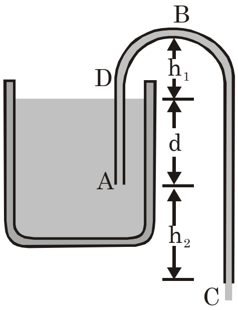
$$p_0 + \frac{1}{2}\rho v_0^2 + \rho g h = p_0 + \frac{1}{2}\rho v^2 + \rho g(0)$$

(Here the top of the tank is represented by the left side of the equation and the hole by the right side. The zero on the right indicates that the hole is at our reference level.) Before we solve Eq. for v, we can use our result that v_0 <<v to simplify it:We assume that v^2_0 and thus the term $\frac{1}{2}\rho v_0^2$ in Eq. , is negligible relative to the other terms, and we drop it. Solving the remaining equation for v then yields

$$v = \sqrt{2gh} \text{ Ans.}$$

This is the same speed that an object would have when falling a height h from rest.This is because the work done by atmospheric pressure is cancelling out at open surface and the hole.

Sample Problem : shows a siphon, which is a device for removing liquid from a container. Tube ABC must initially be filled, but once this has been done, liquid will flow through the tube until the liquid surface in the container is level with the tube opening at A. The liquid has density 1000 kg/m^3 and negligible viscosity. The dis-tances shown are $h_1 = 25$ cm, $d = 12$ cm, and $h_2 = 40$ cm. (a) With what speed does the liquid emerge from the tube at C? (b) If the atmospheric pressure is 1.0×10^5 Pa, what is the pressure in the liquid at the topmost point B? (c) Theoretically, what is the greatest possible height h_1 that a siphon can lift water?



Solution: You may have used siphon and you may recollect that lower the exit point of the fluid is, faster the fluid flows out.

We consider a point D on the surface of the liquid in the container, in the same tube of flow with points A, B and C. Applying Bernoulli's equation to points D and C, we obtain

$$p_D + \frac{1}{2}\rho v_D^2 + \rho g h_D = p_C + \frac{1}{2}\rho v_C^2 + \rho g h_C$$

$$v_C = \sqrt{\frac{2(p_D - p_C)}{\rho} + 2g(h_D - h_C) + v_D^2}$$

$$\approx \sqrt{2g(d + h_2)}$$

where in the last step we set $p_D = p_C = p_{air}$ and $v_D/v_C \approx 0$. Plugging in the values, we obtain

$$v_c = \sqrt{2(9.8 m/s^2)(0.40 m + 0.12 m)} = 3.2 \text{ m/s}$$

The result confirms our experience.

We now consider points B and C:

$$p_B + \frac{1}{2}\rho v_B^2 + \rho g h_B = p_C + \frac{1}{2}\rho v_C^2 + \rho g h_C$$

Since $v_B = v_C$ by equation of continuity, and $p_C = p_{air}$, Bernoulli's equation becomes

$$p_B = p_C + \rho g(h_C - h_g) = p_{air} - \rho g(h_1 + h_2 + d)$$

$$= 1.0 \times 10^5 \text{ Pa } (1.0 \times 10^3 \text{ kg/m })(9.8 \text{ m/s}^2)(0.25 \text{ m} + 0.40 \text{ m} + 0.12 \text{ m})$$

$$= 9.2 \times 10 \text{ Pa.}$$

Since $p_B \geq 0$, we must let $p_{air} - \rho g (h_1 + d + h_2) \geq 0$, which yields

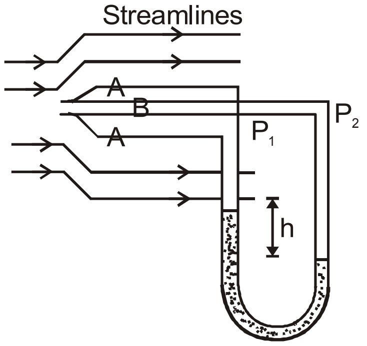
$$h_1 \leq h_{1\max} = \frac{p_{air}}{\rho} - d - h_2 \leq \frac{p_{air}}{\rho} = 10.3 m$$

Q. In the example just above consider a small hole in the hose at location (1) as indicated. When the siphon is used, will water leak out of the hose, or will air leak into the hose and thus causing the siphon to stop ?

Solution : Whether air will leak out of the hose depends on whether the pressure within the hose at (1) is less than or greater than atmospheric.

Since the hose diameter is constant, it follows from the continuity equation (AV = constant) that the water velocity in the hose is constant throughout. Also the pressure at the end of the hose is atmospheric so the pressure at all the points above it can be shown to be less than the atmospheric pressure. Thus air will leak into the hose pipe stoping it.

Example: Fig. shows a device called pitot's tube. It measures the velocity of moving fluids. Determine the velocity of the fluid in terms of the density ρ, the density of the fluid in manometer (U-tube) σ and the height 'h'.



figure

Solution.

The difference in the two tubes is that liquid will flow into the tube B with full Kinetic Energy while it will just pass over the tube A without directly entering into it.

This problem is based on the use of Bernoulli's principle, on two different situations.

The fluid inside the right tube must be at rest as the fluid exactly at the end is in contact with the fluid in pitot tube, which is at rest.

The velocity υ_1 is the fluid velocity υ_f. The velocity υ_2 of the fluid at point B is zero and the pressure in the right arm is P_2 (called stagnation pressure).

Thus using Bernoulli's principle $P_1 + \frac{1}{2} p\upsilon_1^2 = P_2 + \frac{1}{2} p\upsilon_2^2$

We get $P_2 = P_1 + \frac{1}{2} p\upsilon_f^2$

On the other hand the openings at point A is not along the flow lines, so we dont need to use Bernoulli's eqn. We can simply say that the pressure just outside the opening is same as that within the pitot tube.

Therefore the pressure at the left arm of the manometer is same as the fluid pressre P_f, i.e., $P_1 = P_f$.

Also $P_2 = P_1 + (\rho - \sigma) gh$ ....(3)

Generally σ << ρ, so it is ignored.

Thus $P_2 = P_f + \rho gh$ ....(4)

From eqns. (4) and (3), $\frac{1}{2} \sigma\upsilon_f^2 = \rho gh$

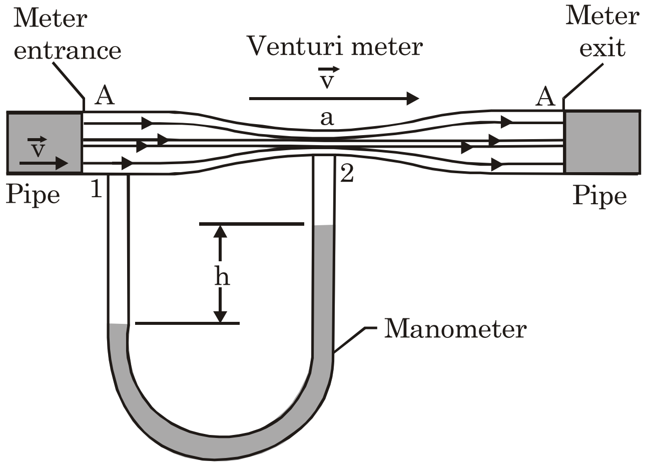
or $\upsilon_f = \sqrt{\frac{2\rho gh}{\sigma}}$

Thus we can see that we have measured the fluid velocity as this was the only difference between the two tubes leading to the pressure difference between the tubes.

Conceptual example:A venturi meter is used to measure the flow speed of a fluid in a pipe. The meter is connected between two sections A venturi meter is used to measure the flow speed of a fluid in a pipe. The meter is connected between two sections of the pipe (Fig.); the cross-sectional area A of the entrance and exit of the meter matches the pipe's crosssectional area. Between the entrance and exit, the fluid flows from the pipe with speed V and then through a narrow "throat" of cross-sectional area a with speed v A manometer connects the wider portion of the meter to the narrower portion. The change in the fluid's speed is accompanied by a change Dp in the fluid's pressure, which causes a height difference h of the liquid in the two arms of the manometer. (Here Dp means pressure in the throat minus pressure in the pipe.) (a) By applying Bernoulli's equation and the equation of continuity to points 1 and 2 in Fig. , show that

$$v=\sqrt{\frac{2a^2\Delta p}{\rho(a^2-A^2)}}$$

where r is the density of the fluid. (b) Suppose that the fluid is fresh water, that the cross-sectional areas are 64 cm^2 in the pipe and 32 cm^2 in the throat, and that the pressure is 55 kPa in the pipe and 41 kPa in the throat.What is the rate of water flow in cubic meters per second?



(a) The continuity equation yields Av = aV, and Bernoulli's equation yields $\Delta p+\frac{1}{2}\rho v^2=\frac{1}{2}\rho V^2$ where $\Delta p = p_1 - p_2$. The first equation gives V = (A/a)v. We use this to substitute for V in the second equation, and obtain $\Delta p+\frac{1}{2}\rho v^2=\frac{1}{2}\rho(A/a)^2v^2$ We solve for v. The result is

$$v=\sqrt{\frac{2\Delta p}{\rho((A/a)^2-1)}}=\sqrt{\frac{2a^2\Delta p}{\rho(A^2-a^2)}}$$

(b) We substitute values to obtain

$$v=\sqrt{\frac{2(32\times10^{-4}m^2)^2(55\times10^3Pa-41\times10^3Pa)}{(1000kg/m^3)((64\times10^{-4}m^2)^2-(32\times10^{-4}m^2)^2)}}=30.06\text{ m/s}$$

Consequently, the flow rate is

$$Av = (64\times10^{-4}m^2)(3.06\text{ m/s}) = 2.0\times10^{-2}m^3/s.$$

SURFACE TENSION & VISCOSITY

EXPLANATION OF SOME OBSERVED PHENOMENA

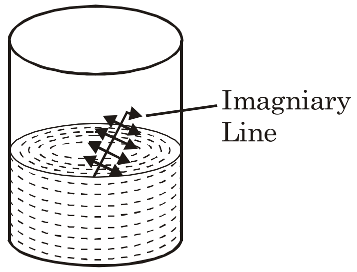
1. Lead balls are spherical in shape.
2. Rain drops and a globule of mercury placed on glass plate are spherical.
3. Hair of a shaving brush/painting brush, when dipped in water spread out, but as soon as it is taken out. Its hair stick together.
4. A greased needle placed gently on the free surface of water in a beaker does not sink.
5. Similarly, insects can walk on the free surface of water without drowning.
6. Bits of Camphor gum move inregularly when placed on water surface.

SURFACE TENSION

Surface Tension is a property of liquid at rest by virtue of which a liquid surface gets contracted to a minimum area and behaves like a stretched membrane.

Surface Tension of a liquid is measured by force per unit length on either side of any imaginary line drawn tangentially over the liquid surface, force being normal to the imaginary line as shown in fig. i.e. Surface tension.

$$(T) = \frac{\text{Total force on either of the imginary line (F)}}{\text{Length of the line } (l)}$$



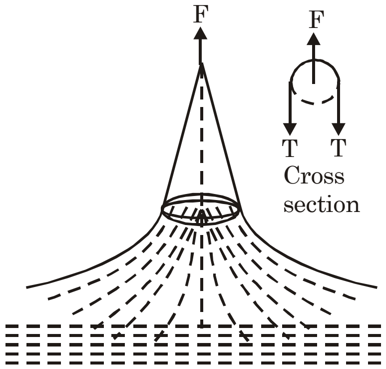
Unit of Surface Tension

In C. G. S. system the unit of surface tension is dyne/cm (dyne cm^{-1}) and SI system its units is Nm^{-1}

Solved Examples

Ex A ring is cut form a platinum tube of 8.5 cm internal and 8.7 cm external diameter. It is supported horizontally from a pan of a balance so that it comes in contact with the water in a glass vessel. What is the surface tension of water if an extra 3.97 g weight is required to pull it away from water? ($g = 980\ cm/s^2$).

Sol.



The ring is in contact with water along its inner and outer circumference ; so when pulled out the total force on it due to surface tension will be

$$F = T(2\pi\, r_1 + 2\pi t_2)$$

So, $T = \dfrac{mg}{2\pi(r_1 + r_2)}$ [$\because F = mg$]

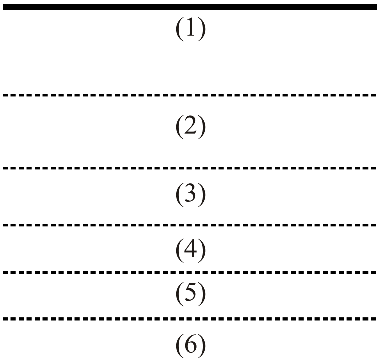
i.e., $$T = \frac{3.97 \times 980}{3.14 \times (8.5+8.7)} = 72.13 \text{ dyne/cm}$$

Surface energy

The course of reasoning given below is usually followed to prove that the molecules of the surface layer of a liquid have surplus potential energy. A molecule inside the liquid is acted upon by the forces of attraction from the other molecules which compensate each other on the average. If a molecule is singled out on the surface, the resulting force of attraction from the other molecule is directed into the liquid. For thi reason the molecule tends to move into the liquid, and definite work should be done to bring it to the surface. Therefore, each molecule of the surface layer has excess potential energy equal to this work. The average force that acts on any molecule from the side of all the others, however, is always equal to zero if the liquid is in equilibrium. This is why the work done to move the liquid from a depth to the surface should also be zero. What is the origin, in this case, of the surface energy ?

[Sol. The forces of attraction acting on a molecule in the surface layer from all the other molecules produce a resultant directed downward. The closest neighbours, however, exert a force of repulsion on the molecule which is therefore in equilibrium.

Owing to the forces of attraction and repulsion, the density of the liquid is smaller in the surface layer than inside. Indeed, molecule 1 (figure) is acted upon by the force of repulsion from molecule 2 and the forces of attraction from all the other molecules (3, 4, ...). Molecule 2 is acted upon by the forces of repulsion from 3 and 1 and the forces of attraction from the molecules in the deep layers. As a result, distance 1-2 should be greater than 2-3, etc.



This course of reasoning is quite approximate (thermal motion, etc. is disregarded), but nevertheless it gives a qualitatively correct result.

An increase in the surface of the liquid causes new sections of the rarefied surface layer to appear. Here work should be performed against the forces of attraction between the molecules. It is this work that constitutes the surface energy.

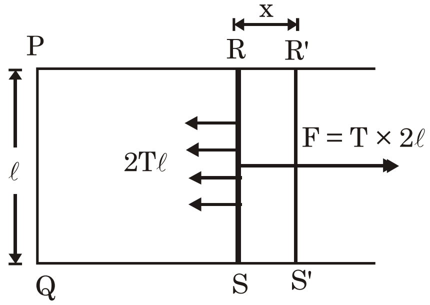
We know that the molecules on the liquid surface experience net downward force. So to bring a molecule from the interior of the liquid to the free surface, some work is required to the done against the intermolecular force of attraction, which will be stored as potential energy of the molecule on the surface. The potential energy of surface molecules per unit area of the surface is called surface is called surface energy. Unit of surface energy is erg cm^{-2} in C.G..S. system and Jm^{-2} in SI system. Dimensional formula of surface energy is $[ML^{\circ}T^{-2}]$ surface energy depends on number of surfaces e.g. a liquid drop is having one liquid air surface while bubble is having two liquid air surface.

Relation between surface tension and surface energy

Consider a rectangular frame PQRS of wire, whose arm RS can slide on the arms PR and QS. If this frame is dipped in a soap solution. then a soap film is produced in the frame PQRS in fig. Due to surface tension (T), the film exerts a force on the frame (towards the interior of the film). Let l be the length of the arm RS, then the force acting on the arm RS towards the film is $F = T \times 2l$ [Since soap film has two surfaces, that is way the length is taken twice.]

$\therefore$ work done, $W = Fx = 2Tlx$

Increase in potential energy of the soap film.



$= EA = 2Elx$ = work done in increasing the area (ΔW)
where E = surface energy of the soap film per unit area.
According the law of conservation of energy, the work done must be equal to the increase in the potential energy.

$$\therefore \quad 2Tlx = 2Elx \text{ or } T = E = \frac{\Delta W}{A}$$

Thus, surface tension is numerically equal to surface energy or work done per unit increase surface area.

Ex A mercury drop of radius 1 cm is sprayed into 10^6 droplets of equal size. Calculate the energy expanded if surface tension of mercury is 35×10^{-3} N/m.

Sol. If drop of radius R is sprayed into n droplets of equal radius r, then as a drop has only one surface, the initial surface area will be $4\pi R^2$ while final area is $n(4\pi r^2)$. So the increase in area

$$\Delta S = n(4\pi r^2) - 4\pi R^2$$

So energy expended in the process.

$$W = T\Delta S = 4\pi T[nr^2 - R^2] \qquad \text{... (1)}$$

Now since the total volume of n droplets is the same as that of initial drop, i.e.

$$\frac{4}{3}\pi R^3 = n[(4/3)\pi r^3] \text{ or } r = R/n^{1/3} \qquad \text{... (2)}$$

Putting the value of r from equation (2) in (1)

$$W = 4\pi R^2 T(n)^{1/3} - 1$$

Ex If a number of little droplets of water, each of radius r, coalesce to form a single drop of radius R, show that the rise in temperature will be given by

$$\frac{3T}{J}\left(\frac{1}{r} - \frac{1}{R}\right)$$

where T is the surface tension of water and J is the mechanical equivalent of heat.

Sol. Let n be the number of little droplets.

Since volume will remain constant, hence volume of n little droplets = volume of single drop

$$\therefore \quad n \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \quad \text{or} \quad nr^3 = R^3$$

Decrease in surface area = $n \times 4\pi^3 - 4\pi R^2$

$$\text{or} \quad \Delta A = 4\pi[nr^2 - R^2] = 4\pi\left[\frac{nr^3}{r} - R^2\right] = 4\pi\left[\frac{R^3}{r} - R^2\right] = 4\pi R^3\left[\frac{1}{r} - \frac{1}{R}\right]$$

Energy evolved W = T × decrease in surface area $= T \times 4\pi R^3\left[\frac{1}{r} - \frac{1}{R}\right]$

Heat produced, $Q = \frac{W}{J} = \frac{4\pi TR^3}{J}\left[\frac{1}{r} - \frac{1}{R}\right]$ But $Q = ms\,d\theta$

where m is the mass of big drop, s is the specific heat of water and $d\theta$ is the rise in temperature.

$$\therefore \quad \frac{4\pi TR^3}{J}\left[\frac{1}{r} - \frac{1}{R}\right] = \text{volume of big drop} \times \text{density of water} \times \text{sp. heat of water} \times d\theta$$

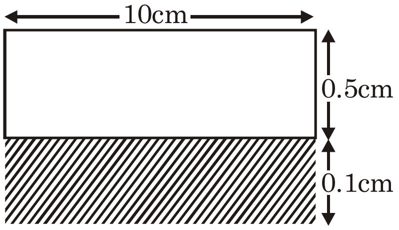
$$\text{or,} \quad \frac{4}{3}\pi R^3 \times 1 \times 1 \times d\theta = \frac{4\pi TR^3}{J}\left(\frac{1}{r} - \frac{1}{R}\right) \quad \text{or,} \quad d\theta = \frac{3T}{J}\left[\frac{1}{r} - \frac{1}{R}\right]$$

Ex A film of water is formed between two straight parallel wires each 10cm long and at a separation 0.5 cm. Calculate the work required to increase 1mm distance between them. Surface tension of water 72×10^{-3} N/m.

Sol. Here the increase in area is shown by shaded portion in the figure.

Since this a water film, it has two surface, therefore

increase in area, $\Delta S = 2 \times 10 \times 0.1 = 2\text{cm}^2$



$\therefore$ Work required to be done

$W = \Delta S \times T$

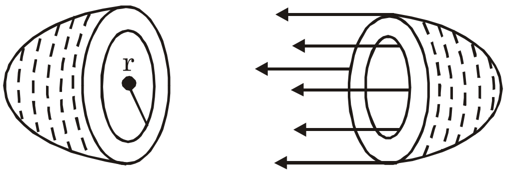
$= 2 \times 10^{-4} \times 72 \times 10^{-3}$

$= 144 \times 10^{-7}$ joule

$= 1.44 \times 10^{-5}$ joule

Excess pressure inside A liquid drop and a bubble

1. Inside a bubble : Consider a soap bubble of radius r. Let p be the pressure inside the bubble and p_a outside. The excess pressure $= p - p_a$. Imagine the bubble broken into two halves, and consider one half of it as shown in fig. Since there are two surface, inner and outer, so the force due to surface tension is



F = surface tension × length = T × 2 (circumference of the bubble) = $T \times 2\,(2T\,\pi r)$ ... (1)

The excess pressure $(p - p_a)$ acts on a cross-sectional area πr^2, so the force due to excess pressure is

$\Rightarrow \quad F = (p - p_a)\,\pi r^2$ ... (2)

The surface tension force given by equation (1) must balance the force due to excess pressure given by equation (2) to maintain the equilibrium , i.e. $(p - p_a)\,\pi r^2 = T \times 2(2\pi\, r)$

$$\text{or} \qquad (p - p_a) = \frac{4T}{r} = p_{\text{excess}}$$

above expression can also be obtained by equation of excess pressure of curve surface by putting $R_1 = R_2$.

2. Inside the drop : In a drop, there is only one surface and hence excess pressure can be written as

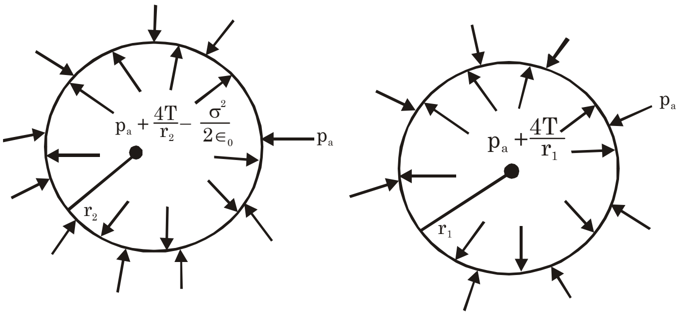
$$(p - p_a) = \frac{2T}{r} = p_{\text{excess}}$$

3. Inside air bubble in a liquid :

$$(p - p_a) = \frac{2T}{r} = p_{excess}$$

4. A charged bubble : If bubble is charged, it's radius increases. Bubble has pressure excess due to charge too. Initially pressure inside the bubble

$$= p_a + \frac{4T}{r_1}$$



for charge bubble, pressure inside = $p_a + \frac{4T}{r_2} - \frac{\sigma^2}{2\epsilon_0}$, where σ surface is surface charge density.

Taking temperature remains constant, then from Boyle's law

$$\left(p_a + \frac{4T}{r_1}\right)\frac{4}{3}\pi r_1^3 = \left[p_a + \frac{4T}{r_2} - \frac{\sigma^2}{2\epsilon_0}\right]\frac{4}{3}\pi r_2^3$$

From above expression the radius of charged drop may be calculated. It can conclude that radius of charged bubble increases, i.e. $r_2 > r_1$.

Solved example

Ex A minute spherical air bubble is rising slowly through a column of mercury contained in a deep jar. If the radius of the bubble at a depth of 100cm is 0.1 mm, calculate its depth where its radius is 0.126 mm, given that the surface tension of mercury is 567 dyne/cm. Assume that the atmospheric pressure is 76cm of mercury.

Sol. The total pressure inside the bubble at depth h_1 is (P atmospheric pressure)

$$= (P + h_1\rho g) + \frac{2T}{r_1} = P_1$$

and the total pressure inside the bubble at depth h_2 is = $(P + h_2\rho g) + \frac{2T}{r_2} = P_2$

Now, according to Boyl's Law

$$P_1V_1 = P_2V_2 \text{ where } V_1 = \frac{4}{3}\pi r_1^3, \text{ and } V_2 = \frac{4}{3}\pi r_2^3$$

Hence we get $\left[(P + h_1\rho g) + \frac{2T}{r_1}\right]\frac{4}{3}\pi r_1^3 = \left[(P + h_2\rho g) + \frac{2T}{r_2}\right]\frac{4}{3}\pi r_2^3$

or, $$\left[(P+h_1\rho g)+\frac{2T}{r_1}\right]r_1^{\,3}=\left[(P+h_2\rho g)+\frac{2T}{r_2}\right]r_2^3$$

Given that : h_1 = 100 cm, r_1 = 0.1 mm = 0.01 cm, r_2 = 0.126 cm, T = 567 dyne / cm, P = 76 cm of mercury. Substituting all the values, we get

h_2 = 9.48 cm.

THE FORCE OF COHESION

The force of attraction between the molecules of the same substance is called cohesion.

In case of solids, the force of cohesion is very large and due to this solids have definite shape and size. On the other hand, the force of cohesion in case of liquids is weaker than that of solids. Hence liquids do not have definite shape but have definite volume. The force of cohesion is negligible in case of gases. Because of this fact, gases have neither fixed shape nor volume.

Example

(i) Two drops of a liquid Two drops of a liquid coalesce into one when brought in mutual contact because of the cohesive force.

(ii) It is difficult to separate two sticky plates of glass wetted with water because a large force has to be applied against the cohesive force between the molecules of water.

(iii) It is very difficult to break a drop of mercury into small droplets because of large cohesive force between mercury molecules.

FORCE OF ADHESION

The force of attraction between molecules of different substance is called adhesion.

Example

(i) Adhesive force enables us to write on the black board with a chalk.

(ii) Adhesive force helps us to write on the paper with ink.

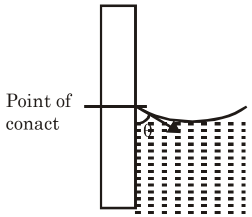
(iii) Large force of adhesion between cement and bricks helps us in construction work.

(iv) Due to force of adhesive, water wets the glass plate.

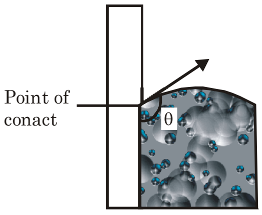
(v) Fevicol and gum are used in gluing two surfaces together because of adhesive force.

ANGLE OF CONTACT

The angle which the tangent to the liquid surface at the point of contact makes with the solid surface inside the liquid is called angle of contact. Those liquids which we the wall of the container (say in case of water and glass) have meniscus concave upwards and their value of angle of contact is less than 90° (also called acute angle). However, those liquids which don't wet the walls of the container (say in case of mercury and glass) have meniscus convex upwards and their value of angle of contact is greater than 90° (also called obtuse angle). The angle of contact of mercury with glass about 140°, whereas the angle of contact of water with glass is about 8°. But, for pure water, the angle of contact θ with glass is taken as 0°.



Water
(a)



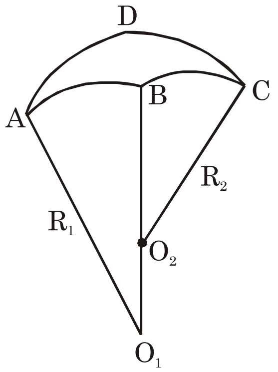
Mercury
(b)

RELATION BETWEEN SURFACE TENSION, RADII OF CURVATURE AND EXCESS PRESSURE ON A CURVED SURFACE.

Let us consider a small element ABCD (fig.) of a curved liquid surface which is convex on the upper side. R_1 and R_2 are the maximum and minimum radii of curvature respectively. They are called the 'principal radii of curvature' of the surface. Let p be the excess pressure on the concave side.

then $p = T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$. If instead of a liquid surface, we have a liquid film, the above expression will be

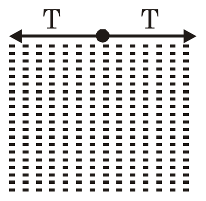
$p = 2T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$, because a film has two surface.



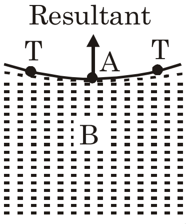
EXCESS OF PRESSURE INSIDE A CURVED SURFACE

1. Plane surface : If the surface of the liquid is plane [as shown in fig. (a)], the molecule on the liquid surface is attracted equally in all directions. The resultant force due to surface tension is zero. The pressure, therefore on the liquid surface is normal.

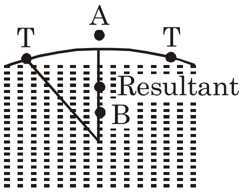
2. Concave surface : If the surface is concave upward [as shown in fig. (b)], there will be upward resultant force due to surface tension acting on the molecule. Since the molecule on the surface is in equilibrium, there must be an excess of pressure on the concave side in the downward direction to balance the resultant force of surface tension $p_A - p_B = \frac{2T}{r}$.



(a) Plane Surface



(a) Concave surface



(c) Convex surface

3. Convex surface : If the surface is convex [as shown fig.(c)], the resultant force due to surface tension acts in the downward direction. Since the molecule on the surface are in equilibrium, there must be an excess of pressure on the concave side of the surface acting in the upward direction to balance the downward resultant force of surface tension. Hence there is always in excess of pressure on concave side of a curved surface over that on the convex side.

$$p_B - p_A = \frac{2T}{r}$$

Solved Examples

Ex A barometer contains two uniform capillaries of radii 1.44×10^{-3} m and 7.2×10^{-4} m. if the height of the liquid in the tube is 0.2m more than that in the wide tube, calculate the true pressure difference. Density of liquid = 10^3 kg/m^3, surface tension = 72×10^{-3} N/m and g = 9.8 m/s^2.

Sol. Let the pressure in the wide and narrow capillaries of radii r_1 and r_2 respectively be P_1 and P_2. Then pressure just below the mensiscus in the wide and narrow tubes respectively are :

$$\left(P_1 - \frac{2T}{r_1}\right) \text{ and } \left(P_2 - \frac{2T}{r_2}\right) \qquad \left[\text{excess pressure} = \frac{2T}{r}\right]$$

Difference in these pressure = $\left(P_1 - \frac{2T}{r_1}\right) - \left(P_2 - \frac{2T}{r_2}\right) = h\rho g$

$\therefore$ True pressure difference = $P_1 - P_2$

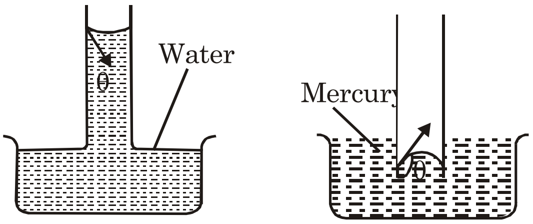
$$= h\rho g + 2T\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= 0.2 \times 10^3 \times 9.8 + 2 \times 72 \times 10^{-3}\left[\frac{1}{1.44 \times 10^{-3}} - \frac{1}{7.2 \times 10^{-4}}\right]$$

$$= 1.86 \times 10^3 = 1860 \text{ N/m}^2$$

Capillarity

A glass tube of very fine bore throughout the length of the tube is called capillary tube. If the capillary tube is dipped in water, the water wets the inner side of the tube and rises in it [shown in figure (a)]. If the same capillary tube is dipped in the mercury, then the mercury is depressed [shown in figure (b)]. The phenomenon of rise or fall of liquids in a capillary tube is called capillarity.

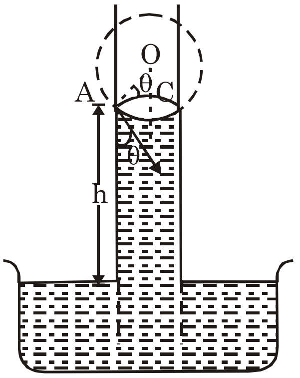


(a) (b)

Particle applications of capillarity

1. The oil in a lamp rises in the wick by capillary action.
2. The tip of nib of a pen is split up, to make a narrow capillary so that the ink rises upto the tin or nib continuously.
3. Sap and water rise upto the top of the leaves of the tree by capillary action.
4. If one end of the towel dips into a bucket of water and the Other end hangs over the bucket the towel soon becomes wet throughout due to capillary action.
5. Ink is absorbed by the blotter due to capillary action.
6. Sandy soil is more dry than clay. It is because the capillaries between sand particles are not so fine as to draw the water up by capillaries.
7. The moisture rises in the capillaries of soil to the surface, where it evaporates. To preserve the moisture m the soil, capillaries must be broken up. This is done by ploughing and leveling the fields.
8. Bricks are porous and behave like capillaries.

Capillary rise (height of a liquid in a capillary tube) ascent formula



consider the liquid which wets the wall of the tube, forms a concave meniscus shown in figure. Consider a capillary tube of radius r dipped in a liquid of surface tension T and density p. Let h be the height through which the liquid rises in the tube. Let p be the pressure on the concave side of the meniscus and p_a be the pressure on the convex side of the meniscus. The excess pressure

$(p - p_a)$ is given by $(p - p_a) = \dfrac{2T}{R}$

Where R is the radius of the meniscus. Due to this excess pressure, the liquid will rise in the capillary tube till it becomes equal to the hydrostatic pressure hpg. Thus in equilibrium state.

Excess pressure = Hydrostatic pressure or $\dfrac{2T}{R} = hpg$

Let θ be the angle of contact and r be the radius of the capillary tube shown in the fig.

From ΔOAC, $\dfrac{OC}{OA} = \cos\theta$ or $R = \dfrac{r}{\cos\theta} \Rightarrow h = \dfrac{2T\cos\theta}{r\rho g}$

The expression is called Ascent formula.

Discussion.

(i) For liquids which wet the glass tube or capillary tube, angle of contact $\theta < 90^\circ$. Hence $\cos\theta$ = positive ⇒ h = positive. It means that these liquids rise in the capillary tube. Hence, the liquids which wet capillary tube rise in the capillary tube. For example, water milk, kerosene oil, patrol etc.

Solved Examples

Ex. A liquid of specific gravity 1.5 is observed to rise 3.0 cm in a capillary tube of diameter 0.50 mm and the liquid wets the surface of the tube. Calculate th excess pressure inside a spherical bubble of 1.0 cm diameter blown from the same liquid. Angle of contact = 0°.

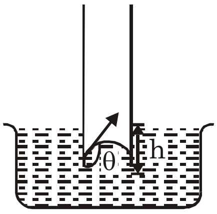
Sol. The surface tension of the liquid is

$$T = \frac{rh\rho g}{2} = \frac{(0.025\,\text{cm})(3.0\,\text{cm})(1.5\,\text{gm/cm}^3)(980\,\text{cm/sec}^2)}{2}$$

= 55 dyne/cm.

Hence excess pressure inside a spherical bubble

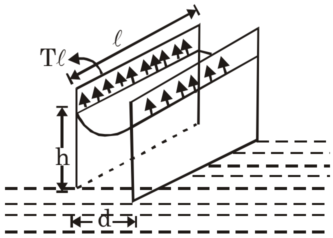
$$p = \frac{4T}{R} = \frac{4 \times 55\,\text{dyne/cm}}{(0.5\,\text{cm})} = 440\ \text{dyne/cm}^2.$$



(ii) For liquids which do not wet the glass tube or capillary tube, angle of contact $\theta > 90^\circ$. Hence $\cos\theta$ = negative ⇒ n = negative. Hence, the liquids which do not wet capillary tube are depressed in the capillary tube. For example, mercury.

(iii) T, θ, ρ and g are constant and hence $h \propto \dfrac{1}{r}$. Thus, the liquid rises more in a narrow tube and less in a wider tube. This is called Jurin's Law.

(iv) If two parallel plates with the spacing 'd' are placed in water reservoir, then height of rise



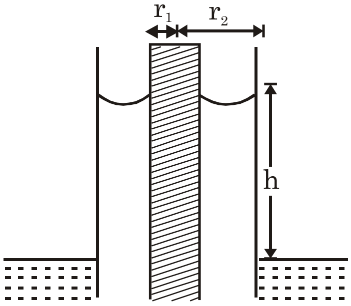
$$\Rightarrow \quad 3Tl = \rho l h d g$$

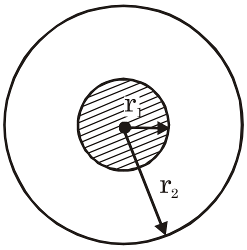
$$h = \frac{2T}{\rho d g}$$

(v) If two concentric tube of radius 'r_1' and 'r_2' (inner one is solid) are placed in water reservoir, then height of rise

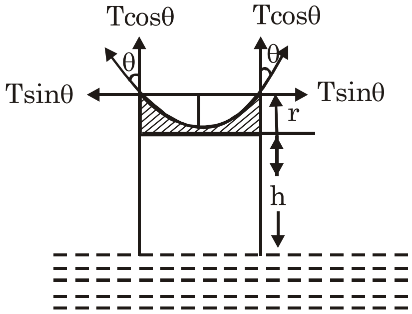
$$\Rightarrow \quad T\,[2\pi r_1 + 2\pi r_2] = [\pi r^2_2 h - \pi r_1^2 h]\,\rho g$$

$$h = \frac{2T}{(r_2 - r_1)\rho g}$$





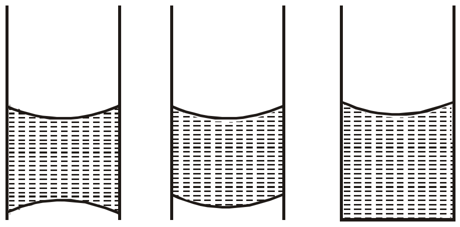
(vi) If weight of the liquid in the menscus is to be consider :



$$T\cos\theta \times 2\pi r = [\pi r^2 h + \frac{1}{3}\pi r^2 \times \pi r_1^2 h]\,\rho g$$

$$\left[h + \frac{r}{3}\right] = \frac{2T\cos\theta}{r\rho g}$$

(vii) When capillary tube (radius, 'r') is in vertical position, the upper meniscus is concave and pressure due to surface tension is directed vertically upward and is given by $p_1 = 2T / R_1$ where R_1 = radius of curvature of upper meniscus.



(a) (b) (c)

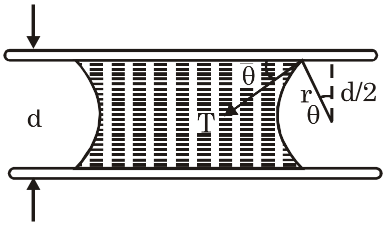
The hydrostatic pressure $p_2 = h\rho g$ is always directed downwards.
If $p_1 > p_2$ i.e. resulting pressure is directed upward. For equilibrium, the pressure due to lower meniscus should be downward. This makes lower meniscus concave downward (fig. (a)). The radius of lower meniscus R_2 can be given by $\frac{2T}{R_2} = (p_1 - p_2)$.

If $p_1 < p_2$ i.e. resulting pressure is directed downward for equilibrium, the pressure due to lower meniscus should be upward. This makes lower meniscus convex upward (fig. b).

The radius of lower meniscus can be given by $\frac{2T}{R_2} = p_2 - p_1$.

If $p_1 = p_2$, then is no resulting pressure, then $p_1 - p_2 = \frac{2T}{R_2} = 0$ or $R_2 = \infty$ i.e. lower surface will be FLAT. (fig.c).

(viii) **Liquid between two plates :** When a small drop of water is placed between two glass plates put face to face, it forms a thin film which is concave outward along its boundary. Let 'R' and 'r' be the radii of curvature of the enclosed film in
two perpendicular directions.



Hence the pressure inside the film is less than the atmospheric pressure outside it by an amount p given by $p = T\left(\frac{1}{r} + \frac{1}{r = \infty}\right)$ and we have, $p = \frac{T}{r}$.

If d be the distance between the two plates and θ the angle of contact for water and glass, then, from the figure, $\cos\theta = \frac{\frac{1}{2}d}{r}$ or $\frac{1}{r} = \frac{2\cos\theta}{d}$.

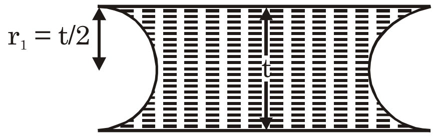
Substituting for $\frac{1}{r}$ in, we get $p = \frac{2T}{d}\cos\theta$.

θ can be taken zero water and glass, i.e. $\cos\theta = 1$. Thus the upper plate is pressed downward by the atmospheric pressure minus $\frac{2T}{d}$. Hence the resultant downward pressure acting on the upper plate is $\frac{2T}{d}$. If A be the are of the plate wetted by the film, the resultant force F pressing the upper plate downward is given by F = resultant pressure × area = $\frac{2TA}{d}$. For very nearly plane surface, d will be very small and hence the pressing force F very large. Therefore it will be difficult to separate the two plates normally.

Solved Examples

Ex A drop of water volume 0.05 cm^3 is pressed between two glass-plates, as a consequence of which, it spreads and occupies an area of 40 cm^2. If the surface tension of water is 70 dyne/cm, find the normal force required to separate out the two glass plates in newton.

Sol. Pressure inside the film is less than outside by an amount, $P = T\left[\frac{1}{r_1} + \frac{1}{r_2}\right]$, where r_1 and r_2 are the radii of curvature of the meniscus. Here $r_1 = 1/2$ and $r_2 = \infty$, then the force required to separate the two glass plates, between which a liquid film is enclosed (figure) is, $F = P \times A = \frac{2AT}{t}$, where t is the thickness of the film, A = area of film.



$$F = \frac{2A^2T}{At} = \frac{2A^2T}{V} = \frac{2\times(40\times10^{-4})^2\times(70\times10^{-3})}{0.05\times10^{-6}} = 45\text{ N}$$

Ex A glass plate of length 10cm, breadth 1.54 cm and thickness 0.20 cm weigh 8.2 gm in air. It is held vertically with the long side horizontal and the lower half under water. Find the apparent weight of the plate. Surface tension of water 73 dyne per cm, g = 980 cm/sec^2.

Sol. Volume of the portion of the plate immersed in water is

$$10 \times \frac{1}{2}(1.54) \times 0.2 = 1.54\text{ cm}^3.$$

Therefore, if the density of water is taken as 1, then upthrust

= wt. of the water displaced

= 1.54 × 1 × 980 = 1509.2 dynes.

Now, the total length of the plate in contact with the water surface is 2 (10 + 0.2) = 20.4 cm,

$\therefore$ downward pull upon the plate due to surface tension

= 20.4 × 73 = 1489.2 dynes

$\therefore$ resultant upthrust

= 1509.2 – 1489.2

$= 20.0\text{ dynes} = \frac{20}{980}$

= 0.0204 gm. wt.

$\therefore$ apparent weight of the plate in water

= weight of the plate in air – resultant upthrust

= 8.2 – 0.0204 = 8.1796 gm Ans.

Ex A glass tube of circular cross-section is closed at one end. This end is weighted and the tube floats vertically in water, heavy end down. How far below the water surface is the end of the tube? Give : Outer radius of the tube 0.14 cm, mass of weighted tube 0.2 gm, surface tension os water 73 dyne/cm and $g = 980\ \text{cm/sec}^2$.

Sol., Let l be the length of the tube inside water. The forces acting on the tube are :

(i) Upthrust of water acting upward

$$= \pi r^2 l \times 1 \times 980 \qquad = \frac{22}{7} \times (0.14)^2\, l \times 980 = 60.368\, l \text{ dyne.}$$

(ii) Weight of the system acting downward

$$= mg = 0.2 \times 980 = 196 \text{ dyne.}$$

(iii) Force of surface tension acting downward

$$= 2\pi r T$$

$$= 2 \times \frac{22}{7} \times 0.14 \times 73 = 64.24 \text{ dyne.}$$

Since the tube is in equilibrium, the upward force is balanced by the downward forces. That is,

$$60.368\, l = 196 + 64.24 = 260.24.$$

$$\therefore \qquad l = \frac{260.24}{60.368} = 4.31 \text{ cm.}$$

Ex A glass U-tube is such that the diametre of one limb is 3.0 mm and that of the other is 6.00 mm. The tube is inverted vertically with the open ends below the surface of water in a beaker. What is the difference between the heights to which water rises in the two limbs? Surface tension of water is $0.07\ \text{nm}^{-1}$. Assume that the angle of contact between water and glass is 0°.

Sol. Suppose pressure at the points A, B, C and D be P_A, P_B, P_C and P_0 respectively.

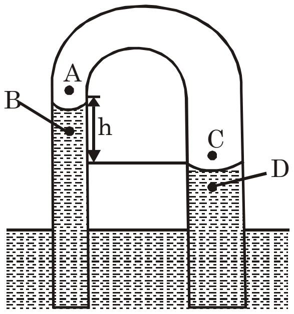
The pressure on the concave side of the liquid surface is greater than that on the other side by 2T/R.

Ang angle of contact θ is given to be 0°, hence $\qquad R \cos 0° = r$ or $R = r$

$$\therefore \qquad P_A = P_B + 2T / r_1 \quad \text{and} \quad P_C = P_0 + 2T / r_2$$

where r_1 and r_2 are the radii of the two limbs

But $\quad P_A = P_C$



$$\therefore \qquad P_B + \frac{2T}{r_1} = P_D + \frac{2T}{r_2}$$

$$\text{or} \qquad P_D - P_B = 2T\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

where h is the difference in water levels in the two limbs

$$\text{Now,} \qquad h = \frac{2T}{\rho g}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Given that $\qquad T = 0.07\ \text{Nm}^{-1}$, $\rho = 1000\ \text{kgm}^{-3}$

$$r_1 = \frac{3}{2}\text{ mm} = \frac{3}{20}\text{ cm} = \frac{3}{20 \times 100}\text{ m} = 1.5 \times 10^{-3}\text{ m},\ r_2 = 3 \times 10^{-3}\text{ m}$$

$$\therefore \qquad h = \frac{2 \times 0.07}{1000 \times 9.8}\left(\frac{1}{1.5 \times 10^{-3}} - \frac{1}{3 \times 10^{-3}}\right)\text{m} = 4.76 \times 10^{-3}\text{ m} = 4.76\text{ mm}$$

Ex Two narrow bores of diameters 3.0 mm and 6.0 mm are joined together to form a U-shaped tube open at both ends. If the U-tube contains water, what is the difference in its levels in the two limbs of the tube? Surface tension of water at the temperature of the experiment is 7.3×10^{-2} Nm^{-1}. Take the angle of contact to be zero.

Sol. Given that $r_1 = \frac{3.0}{2} = 1.5 \text{ mm} = 1.5 \times 10^{-3}$ m, $r_2 = \frac{6.0}{2} = 3.0 \times 10^{-3}$ m,

$T = 7.3 \times 10^{-3}\ \text{Nm}^{-1},\ \theta = 0°\ \ \rho = 1.0 \times 10^{3}\ \text{kg m}^{-1},\ g = 9.8\ \text{ms}^{-2}$

When angle of contact is zero degree, the radius of the meniscus equals radius of bore.

Excess pressure in the first bore, $P_1 = \frac{2T}{r_2} = \frac{2\times 7.3 \times 10^{-3}}{1.5 \times 10^{-3}} = 97.3$ Pascal

Excess pressure in the second before, $P_2 = \frac{2T}{r^2} = \frac{2\times 7.3\times 10^{-2}}{3\times 10^{-3}} = 48.7$ Pascal

Hence, pressure difference in the two limbs of the tube

$$\Delta P = P_1 - P_2 = h\rho g$$

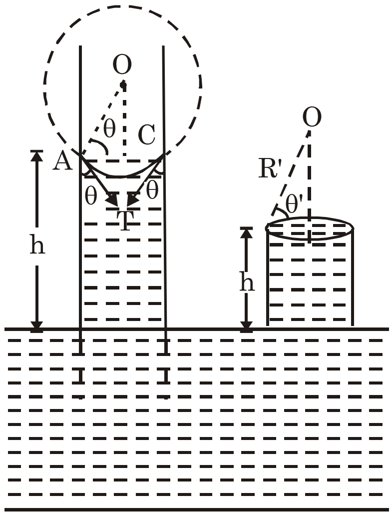
or $$h = \frac{P_1 - P_2}{\rho g} = \frac{97.3 - 48.7}{1.0 \times 10^{3} \times 9.8} = 5.0 \text{ mm.}$$

Capillary rise in a tube of insufficient length

We know, the height through which a liquid rises in the capillary tube of radius r is given by

$$\therefore \quad h = \frac{2T}{R\rho g} \quad \text{or} \quad h\ R = \frac{2T}{\rho g} = \text{constant}$$

When the capillary tube is cut and its length is less then h (i.e. h′). then the liquid rises upto the top of the tube and spreads in such a way that the radius (R′) of the liquid meniscus increases and it becomes more flat so that hR = h′R′ = Constant. Hence the liquid does not overflow.



If $\quad h' < h$ then $R' > R \quad$ or $\quad \frac{r}{\cos\theta'} > \frac{r}{\cos\theta}$

$\Rightarrow \quad \cos\theta' < \cos\theta \quad \Rightarrow \quad \theta' > \theta$

Ex If a 5cm long capillary tube with 0.1 mm internal diameter open at both ends is slightly dipped in water having surface tension 75 dyne cm^{-1}, state whether (i) water will rise half way in the capillary. (ii) Water will rise up to the upper end of capillary (iii) What will overflow out of the upper end of capillary. Explain your answer.

Sol. Given that surface tension of water, T = 75 dyne/cm

Radius $r = \frac{0.1}{2}$ mm = 0.05 mm = 0.005 cm

density $\rho = 1$ gm/cm^3, angle of contact, $\theta = 0°$

Let h be the height to which water rise in the capillary tube. Then

$$h = \frac{2T\cos\theta}{r\rho g} = \frac{2 \times 75 \times \cos 0°}{0.005 \times 1 \times 981} \text{ cm} = 30.58 \text{ cm}$$

But length of capillary tube, h′ = 5cm

(i) Because $h > \frac{h'}{2}$ therefore the first possibility does not exist.

(ii) Because the tube is of insufficient length therefore the water will rise upto the upper end of the tube.

(iii) The water will not overflow out of the upper end of the capillary. It will rise only upto the upper end of the capillary.

The liquid meniscus will adjust its radius of curvature R′ in such a way that

$$R'h' = Rh \qquad \left[\because\ hR = \frac{2T}{\rho g} = \text{constant}\right]$$

where R is the radius of curvature that the liquid meniscus would possess if the capillary tube were of sufficient length.

$$\therefore \qquad R' = \frac{Rh}{h'} = \frac{rh}{h'} \qquad \left[\because\ R = \frac{r}{\cos\theta} = \frac{r}{\cos 0^\circ} = r\right] = \frac{0.005 \times 30.58}{5} = 0.0306 \text{ cm}$$

Applications of surface tension

(i) The wetting property is made use of in detergents and waterproofing. When the detergent materials are added to liquids, the angle of contact decreases and hence the wettability increases. On the other hand, when water proofing material is added to a fabric, it increases the angle of contact, making the fabric water-repellant.

(ii) The antiseptics have very low value of surface tension. The low value of surface tension prevents the formation of drops that may otherwise block the entrance to skin or a wound. Due to low surface tension the antiseptics spreads properly over the wound. The lubricating oils and paints also have low surface tension. So they can spread properly.

(iii) Surface tension of all lubricating oils and paints is kept low so that they spread over a large area.

(iv) Oil spreads over the surface of water because the surface tension of oil is less than the surface tension of cold water.

(v) A rough sea can be calmed by pouring oil on its surface.

Effect of temperature and impurities on surface tension

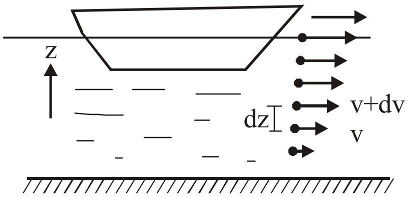
The surface tension of a liquid decreases with the rise in temperature and vice versa. According to Ferguson, $T = T_0\left(1 - \frac{\theta}{\theta_C}\right)^n$ where T_0 is surface tension at 0°C, θ is absolute temperature of the liquid, θ_C is the critical temperature and n is a constant varies slightly from liquid and has means value 1.21. This formula shows that the surface tension becomes zero at the critical temperature, where the while machinery parts get jammed in winter.

The surface tension of a liquid change appreciably with addition of impurities. For example, surface tension of water increases with addition of highly soluble substances like NaCl, $ZnSO_4$ etc. On the other hand surface tension of water gets reduced with addition of sparingly soluble substances like phenol, soap etc.

VISCOSITY

When a layer of a fluid slips or tens to slip on another layer in contact, the two layers exert tangential forces on each other. The directions are such that the relative motion between the layers is opposed. this property of a fluid to oppose relative motion between its layers is called viscosity. The forces between the layers opposing relative motion between them are known as the forces of viscosity. Thus, viscosity may be thought of as the internal friction of a fluid in motion.

If a solid surface is kept in contact with a fluid and is moved, forces of viscosity appear between the solid surface and the fluid layer in contact. the fluid in contact is dragged with the solid. If the viscosity is sufficient, the layer moves with the solid and there is no relative slipping. When a boat moves slowly on the water of a calm river, the water in contact with the boat is dragged with it, whereas the water in contact with the bed of the river remains at rest. Velocities of different layers are different. Let v be the velocity of the layer at a distance z from the bed and v +dv be the velocity at a distance z +dz (figure).



Thus, the velocity differs by dv in going through a distance dz perpendicular to it. The quantity dv/dz is called the velocity gradient.

The force of viscosity between two layers of a fluid is proportional to the velocity gradient in the direction perpendicular to the layers. Also the force is proportional to the area of the layer.

Thus, if F is the force exerted by a layer of area A on a layer in contact,

$$F \propto A \text{ and } F \propto dv/dz$$

or, $$F = -\eta A\, dv/dz$$

The negative sign is included as the force is frictional in nature and opposes relative motion. The constant of proportionality η is called the coefficient of viscosity.

The SI unit of viscosity can be easily worked out from equation. It is N-s/m^2 . However, the corresponding CGS unit dyne-s/cm^2 is in common use and is called a poise in honour of the French scientist Poiseuille. We have

$$1 \text{ poise} = 0.1 \text{ N-s/m}^2$$

TERMINAL VELOCITY

The viscous force on a solid moving through a fluid is proportional to its velocity. When a solid is dropped in a fluid, the forces acting on it are

(a) weight W acting vertically downward,

(b) the viscous force F acting vertically upward and

(c) the buoyancy force B acting vertically upward.

The weight W and the buoyancy B are constant but the force F is proportional to the velocity v, initially, the velocity and hence the viscous force F is zero and the solid is accelerated due to the force W-B. Because of the acceleration, the velocity increases. Accordingly, the viscous force also increases. At a certain instant the viscous force becomes equal to W-B. the net force then becomes zero and the solid falls with constant velocity. This constant velocity is known as the terminal velocity.

Consider a spherical body falling through a liquid. Suppose the density of the body = ρ, density of the liquid = σ, radius of the sphere = r and the terminal velocity = v_0 . The viscous force is

$$F = 6\pi\eta r v_0$$

the weight $$W = \frac{4}{3}\pi r^3 \rho g$$

and the buoyancy force $$B = \frac{4}{3}\pi r^3 \sigma g$$

We have

$$6\ 6\pi\eta r v_0 = W - B = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \sigma g$$

or $$v_0 = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

Q. A large wooden plate of area 10 m^2 floating on the surface of a river is made to move horizontally with a speed of 2 m/s by applying a tangential force. If the river is 1 m deep and the water in contact with the bed is stationary, find the tangential force needed to keep the plate moving. Coefficient of viscosity of water at the temperature of the river = 10^{-2} poise.

Sol. The velocity decreases from 2 m/s to zero in 1 m of perpendicular length. Hence, velocity gradient.

$$= dv/dx = 2\ s^{-1}$$

Now, $$\eta = \left|\frac{F/A}{dv/dx}\right|$$

or, $$10^{-3}\ \frac{N-s}{m^2} = \frac{F}{(10m)^2(2s^{-1})}$$

or, $$F = 0.02\ N.$$

Q. The velocity of water in a river is 18 km/hr near the surface. If the river is 5 m deep, find the shearing stress between the horizontal layers of water. The coefficient of viscosity of water = 10^{-2} poise.

Sol. The velocity gradient in vertical direction is

$$\frac{dv}{dx} = \frac{18\,km/hr}{5m} = 1.0\,s^{-1}$$

The magnitude of the force of viscosity is

$$F = \eta\ A\ \frac{dv}{dx}.$$

The shearing stress is

$$F/A = \eta\ \frac{dv}{dx} = (10^{-2}\ \text{poise})\ (1.0\ s^{-1}) = 10^{-3}\ N/m^2$$

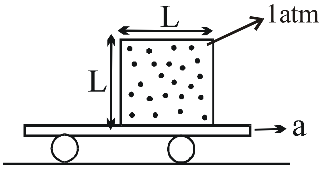
EXERCISE (S-1)

Fluid Statics

1. A spherical tank of 1.2 m radius is half filled with oil of relative density 0.8. If the tank is given a horizontal acceleration of 10 m/s^2. Calculate the inclination of the oil surface to horizontal and maximum pressure on the tank.

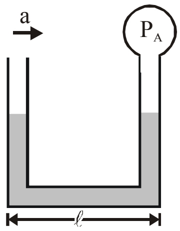
FM0001

2. A cubical sealed vessel with edge L is placed on a cart, which is moving horizontally with an acceleration 'a' as shown in figure. The cube is filled with an ideal fluid having density ρ. Find the gauge pressure at the centre of the cubical vessel.



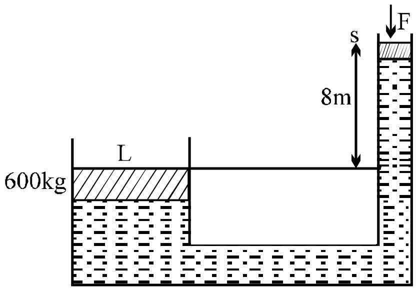
FM0002

3. A liquid of density ρ is filled in a U-tube, whose one end is open & at the other end a bulb is fitted whose pressure is P_A. Now this tube is moved horizontally with acceleration 'a' as shown in the figure. During motion it is found that liquid in both column is at same level at equilibrium. If atmospheric pressure is P_0, then find the value of P_A



FM0003

4. For the system shown in the figure, the cylinder on the left at L has a mass of 600 kg and a cross sectional area of 800 cm^2. The piston on the right, at S, has cross sectional area 25 cm^2 and negligible weight. If the apparatus is filled with oil.(ρ = 0.75 gm/cm^3) Find the force F required to hold the system in equilibrium.

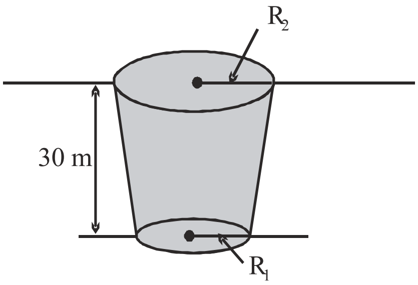


FM0004

5. Dams at two different locations are needed to form a lake. When the lake is filled, the water level will be at top of both dams. The second dam is twice as high and twice as wide as the first dam. The force of the water on the second dam is how much greater than the force on the first? (Ignore atmospheric pressure since it is pushing on both sides of the dams.)

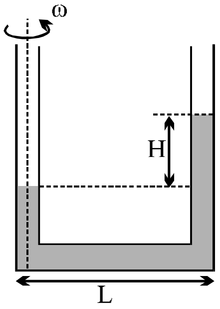
FM0005

6. As the drawing illustrates, a pond full of water has the shape of an inverted cone with the tip sliced off and has a depth of 30m. The atmospheric pressure above the pond is 1.0×10^5Pa. The circular top surface (radius = R_2) and circular bottom surface (radius = R_1) of the pond are both parallel to the ground. The magnitude of the force acting on the top surface by the liquid is the same as the magnitude of the force acting on the bottom surface by the liquid. Obtain $\frac{R_2}{R_1}$.



FM0006

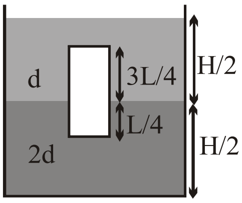
7. A U tube is rotated about one of it's limbs with an angular velocity ω. Find the difference in height H of the liquid (density ρ) level, where diameter of the tube d << L. **[IIT-JEE 2005]**



FM0007

Buoyancy

8. A container of a large uniform cross-sectional area A resting on a horizontal surface holds two immiscible, non viscous and incompressible liquids of densities d and 2d each of height $\frac{H}{2}$ as shown. The lower density liquid is open to atmosphere. A homogeneous solid cylinder of length $L\left(<\frac{H}{2}\right)$, cross-sectional area $\frac{A}{5}$ is immersed such that it floats with its axis vertical to the liquid -liquid interface with length $\frac{L}{4}$ in denser liquid. Find the density of the solid cylinder.



FM0008

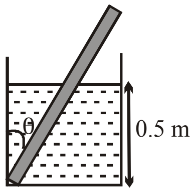
9. In air an object weighs 15N, when immersed completely in water the same object weighs 12N. When immersed in another liquid completely, it weighs 13N. Find

(a) the specific gravity of the object and

(b) the specific gravity of the other liquid.

FM0009

10. A wooden plank of length 1 m and uniform cross-section is hinged at one end to the bottom of a tank as shown. The tank is filled with water upto a height of 0.5 m. The specific gravity of the plank is 0.5. Find the angle θ made by the plank in equilibrium position



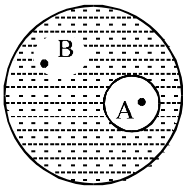
FM0010

11. The volume of an air bubble is doubled as it rises from the bottom of a lake to its surface. If the atmospheric pressure is H m of mercury & the density of mercury is n times that of lake water. Find the depth of the lake.

FM0011

Surface Tension

12. There is an air bubble of radius R inside a drop of water of radius 3R. Find the ratio of gauge pressure at point A to the gauge pressure at point B.

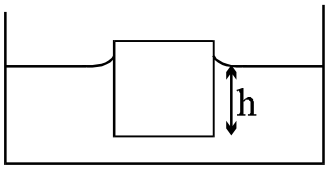


FM0012

13. Two arms of a U-tube have unequal diameters $d_1 = 1.0$ mm and $d_2 = 1.0$ cm. If water (surface tension 7×10^{-2} N/m) is poured into the tube held in the vertical position, find the difference of level of water in the U-tube. Assume the angle of contact to be zero.

FM0013

14. A cube with a mass 'm' completely wettable by water floats on the surface of water. Each side of the cube is 'a'. What is the distance h between the lower face of cube and the surface of the water if surface tension is S. Take density of water as ρ_w. Take angle of contact as zero.



FM0014

Fluid Dynamics

15. A jet of water having velocity = 10 m/s and stream cross-section = 2 cm^2 hits a flat plate perpendicularly, with the water splashing out parallel to plate. Find the force that the plate experiences.

FM0015

16. A laminar stream is flowing vertically down from a tap of cross-section area 1 cm^2. At a distance 10cm below the tap, the cross-section area of the stream has reduced to 1/2 cm^2. Find the volumetric flow rate of water from the tap.

FM0016

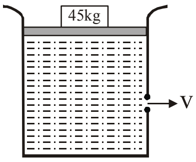
17. Calculate the rate of flow of glycerine of density 1.25 x 10^3 kg/m^3 through the conical section of a pipe if the radii of its ends are 0.1m & 0.04m and the pressure drop across its length is 10N/m^2.

FM0017

18. A cylindrical vessel open at the top is 20cm high and 10cm in diameter. A circular hole whose cross-sectional area 1 cm^2 is cut at the centre of the bottom of the vessel. Water flows from a tube above it into the vessel at the rate 100 cm^{3}s^{-1}. Find the height of water in the vessel under steady state.

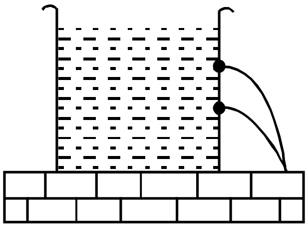
FM0018

19. A large cylindrical tank of cross-sectional area 1m^2 is filled with water. It has a small hole at a height of 1m from the bottom. A movable piston of mass 5 kg is fitted on the top of the tank such that it can slide in the tank freely. A load of 45 kg is applied on the top of water by piston, as shown in figure. Find the value of v when piston is 7m above the bottom (g = 10 m/s^2)



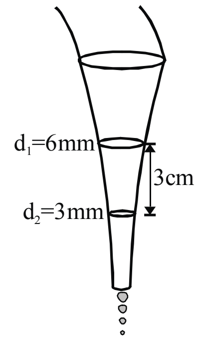
FM0019

20. In a cylindrical vessel containing liquid of density ρ, there are two holes in the side walls at heights of h_1 and h_2 respectively such that the range of efflux at the bottom of the vessel is same. Find the height of a hole, for which the range of efflux would be maximum.



FM0020

21. The tap in the garden was closed in appropriately resulting in the water flowing freely out of it which forms a downward narrowing beam. The beam of water has a circular cross-section, the diameter of the circle is 6 mm at one point and 3 cm below it is only 3 mm as shown in figure. If the rate of water wasted is (x × π) mL/minute then find the value of x. (Neglect the effect of viscosity and surface tension of the flowing water.)



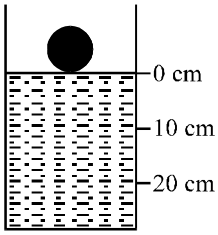
FM0021

Viscosity

22. A spherical ball of radius 1×10^{-4} m and density 10^4 kg/m^3 falls freely under gravity through a distance h before entering a tank of water. If after entering the water the velocity of the ball does not change, find h. The viscosity of water is 9.8×10^{-6} N-s/m^2.

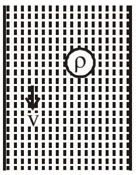
FM0022

23. A spherical ball of density ρ and radius 0.003m is dropped into a tube containing a viscous fluid filled up to the 0 cm mark as shown in the figure. Viscosity of the fluid = 1.260 N.m^{-2} and its density ρ_L= $\rho/2$ = 1260 kg.m^{-3}. Assume the ball reaches a terminal speed by the 10 cm mark. Find the time taken by the ball to traverse the distance between the 10 cm and 20 cm mark. (g = acceleration due to gravity = 10 ms^{-2})



FM0023

24. A liquid of density ρ = 1000 kg/m^3 and coefficient of viscosity η = 0.1 Ns/m^2 is flowing down in vertical pipe of large cross section. A small ball of density ρ_0 = 100 kg/m^3 and r = 5 cm will be at rest in flowing liquid, if velocity of flowing liquid is 10 k m/s. Then find the value of k.



FM0024

25. A small sphere falls from rest in a viscous liquid. Due to friction, heat is produced. Find the relation between the rate of production of heat and the radius of the sphere at terminal velocity.

[IIT-JEE 2004]

FM0025

EXERCISE (S-2)

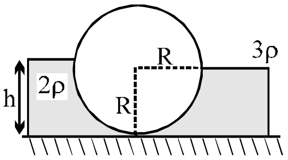
1. Place a glass beaker, partially filled with water, in a sink. The beaker has a mass 390 gm and an interior volume of 500cm^3. You now start to fill the sink with water and you find, by experiment, that if the beaker is less than half full, it will float; but if it is more than half full, it remains on the bottom of the sink as the water rises to its rim. What is the density of the material of which the beaker is made?

FM0026

2. A hollow cone floats with its axis vertical upto one-third of its height in a liquid of relative density 0.8 and with its vertex submerged. When another liquid of relative density ρ is filled in it upto one-third of its height, the cone floats upto half its vertical height. The height of the cone is 0.10 m and the radius of the circular base is 0.05 m. Find the specific gravity ρ.

FM0027

3. In the figure shown, the heavy cylinder (radius R) resting on a smooth surface separates two liquids of densities 2ρ and 3ρ. Find the height 'h' for the equilibrium of cylinder.

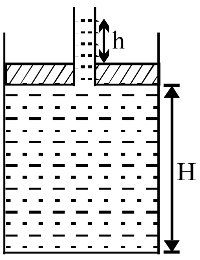


FM0028

4. A small wooden ball of density 'ρ' is immersed in water of density σ to depth h and then released. The height H above the surface of water up to which the ball jumps out of water is $\left(\frac{\sigma}{\rho}-\frac{n}{2}\right)h$. Value of n is (Neglect the effect of viscosity and surface tension and assume all quantities in SI unit)

FM0029

5. A piston of mass M = 3kg and radius R = 4cm has a hole into which a thin pipe of radius r = 1cm is inserted. The piston can enter a cylinder tightly and without friction, and initially it is at the bottom of the cylinder. 750gm of water is now poured into the pipe so that the piston & pipe are lifted up as shown. Find the height H of water in the cylinder and height h of water in the pipe.

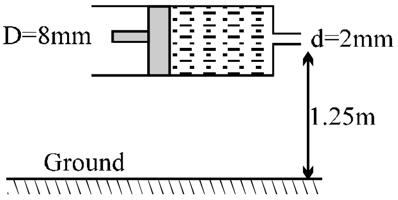


FM0030

6. A cylindrical wooden float whose base area is 4 m^2 and height 1 m drifts on the water surface in vertical position. Density of wood is 500 kg/m^3 and that of water is 1000 kg/m^3. What minimum work (in kJ) must be performed to take the float out of the water?

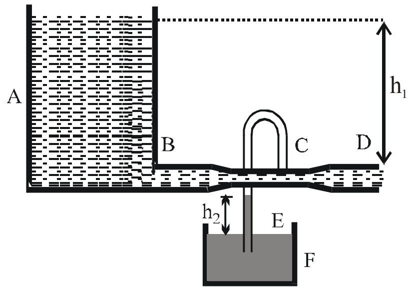
FM0031

7. Consider a horizontally oriented syringe containing water located at a height of 1.25 m above the ground. The diameter of the plunger is 8 mm and the diameter of the nozzle is 2 mm. The plunger is pushed with a constant speed of 0.25 m/s. Find the horizontal range of water stream on the ground. Take g = 10 m/s^2. **[IIT-JEE 2004]**



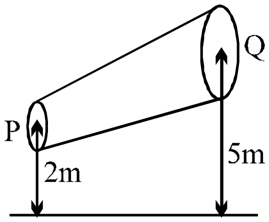
FM0032

8. Two very large open tanks A and F both contain the same liquid. A horizontal pipe BCD, having a constriction at C leads out of the bottom of tank A, and a vertical pipe E opens into the constriction at C and dips into the liquid in tank F. Assume streamline flow and no viscosity. If the cross section at C is one half that at D and if D is at a distance h_1 below the level of liquid in A, to what height h_2 (in terms of h_1) will liquid rise in pipe E ?



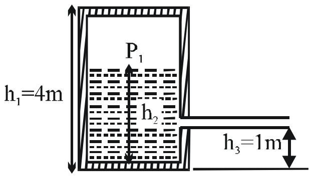
FM0033

9. A non-viscous liquid of constant density 1000 kg/m^3 flows in a streamline motion along a tube of variable cross section. The tube is kept inclined in the vertical plane as shown in the figure. The area of cross section of the tube at two points P and Q at heights of 2 meters and 5 meters are respectively 4×10^{-3}m^2 and 8×10^{-3} m^3. The velocity of the liquid at point P is 1 m/s. Find the work done per unit volume by the pressure and the gravity forces as the fluid flows from point P to Q.



FM0034

10. A large tank of (height h_1 = 4m) water has a hose connected to it, as shown in figure. The tank is sealed at the top and has compressed air between the water surface & the top. When the water height h_2 is 3m, the gauge pressure of air $P_1 = 1 \times 10^5 N/m^2$. Assume that the air above the water surface expands isothermally, the velocity of flow out of the hose when h_2 has decreased to 2m is given by α m/s ? Fill $\frac{\alpha^2}{5}$ in OMR sheet. (Assume ideal fluid flow & $P_{atm} = 10^5 N/m^2$)

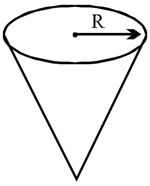


FM0035

11. Two large vertical parallel plates separated by a gap of d = 1mm have a highly viscous liquid of density 800 kg/m^3 and viscosity coefficient η = 5 poise flowing steadily under gravity in between the gap.Find the velocity gradient of flow near plates surface in sec^{-1}. (1 poise = 0.1 NS/m^2).

FM0036

12. An inverted hollow cone, of height H and radius at the top = R, is filled completely with a liquid of density d. Neglect atmosphere pressure.



(a) Find horizontal component of the force which the liquid exerts on half curved side wall of the cone.

(b) Find the vertical downward component of the thrush force due to water pressure which the liquid exerts on half side curved wall of the cone.

FM0037

13. A U-tube filled with a liquid of volumetric coefficient of 10^{-5}/°C lies in a vertical plane. The height of liquid column in the left vertical limb is 100 cm. The liquid in the left vertical limb is maintained at a temperature = 0°C while the liquid in the right limb is maintained at a temperature = 100°C. Find the difference in levels in the two limbs.

FM0038

14. There are three immiscible liquids of densities ρ_1, ρ_2 and ρ_3. If the first two liquids are taken in a U–tube then they stand like fig(i). When all the three liquids are taken in U–tube, they stand like fig(ii). Find the ratio of the densities of the three liquids.

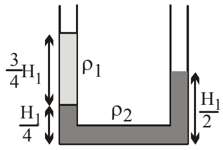


Fig.(i)

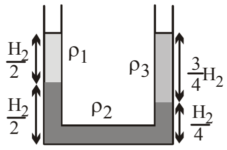
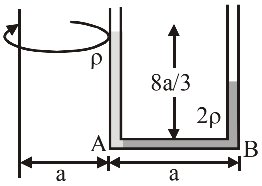


Fig.(ii)

FM0039

15. The interface of two liquids of densities ρ and 2ρ respectively lies at the point A in a U tube at rest. The height of liquid column above A is 8a/3 where AB = a. The cross sectional area of the tube is S. With what angular velocity the tube must be whirled about a vertical axis at a distance 'a' such that the interface of the liquids shifts towards B by 2a/3.

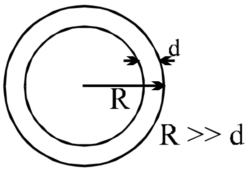


FM0040

16. An expansible balloon filled with air floats on the surface of a lake with 2/3 of its volume submerged. How deep must it be sunk in the water so that it is just inequilibrium neither sinking further nor rising? It is assumed that the temperature of the water is constant & that the height of the water barometer is 9 meters.

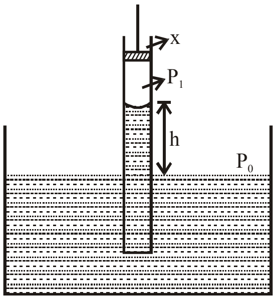
FM0041

17. A soap bubble has radius R and thickness d (<< R) as shown. It collapses into a spherical drop. Find the ratio of excess pressure in the drop to the excess pressure inside the bubble.



FM0042

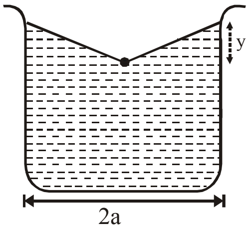
18. A capillary of radius r = 0.2 mm is dipped vertically in a liquid of density, ($\rho = 1\text{gm}/\text{cc}$). A small piston (x) is inserted inside the capillary which maintains the constant pressure of air ($P_1 = 2 \times 10^4$ dynes/cm^2) above the hemispherical meniscus. If ambient pressure is $P_0 = 10^5$ dynes/cm^2 and surface tension of liquid is T = 72 dynes/cm, then find the height h of liquid in cm. (weight of curved part of liquid is ignored)



FM0043

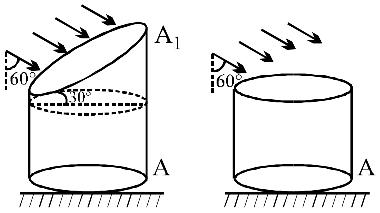
19. A container of width 2a is filled with a liquid. A thin wire of weight per unit length λ is gently placed over the liquid surface in the middle of the surface as shown in the figure. As a result, the liquid surface is depressed by a distance y (y << a). Determine the surface tension of the liquid.

[2004, 2M]



FM0044

20. A vertical cylindrical container of base area A and upper cross-section area A_1 making an angle 30° with the horizontal is placed in an open rainy field as shown near another cylindrical container having same base area A. Find the ratio of rates of collection of water in the two containers.

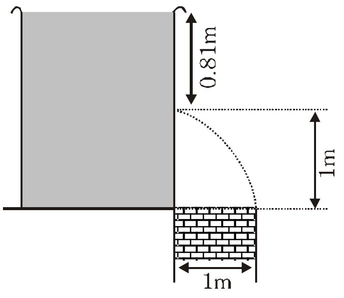


FM0045

21. A wide vessel with small hole in the bottom is filled with water and kerosene. Neglecting viscosity, find the velocity of water flow v, if the thickness of water layer is h_1 and that of kerosene layer is h_2 is (density of water ρ_1 gm/cc and that of kerosene is ρ_2 gm/cc.

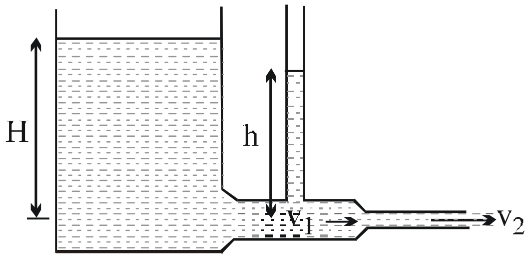
FM0046

22. For the arrangement shown in the figure. The time interval in seconds after which the water jet ceases to cross the wall is found to be $(\alpha)^3$. Area of the cross section of the tank $A = \sqrt{5}\ \text{m}^2$ and area of the orifice a = 32 cm^2. [Assume that the container remaining fixed]. Find the value of α. (Take $g = 10\ \text{m/s}^2$)



FM0047

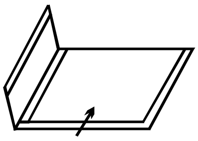
23. Water fills a reservoir, open to the atmosphere, to height H = 5m above the centerline of a horizontal exit pipe at the bottom of the reservoir. The first section of the pipe has radius r_1 = 2 cm, the unknown velocity there is v_1 and this section of the pipe has a manometer in which the water rises to an unknown height h above the centerline of the pipe. The water leaves the second section of radius r_2 = 1 cm, with speed v_2. Assume the flow is incompressible, frictionless, irrotational, and steady. Also, assume the reservoir is so large compared to the pipe, that the water level in the reservoir is almost constant.



(a) What is the speed, v_2, of the water leaving the exit pipe?
(b) What is the speed, v_1, in the first section of the pipe?
(c) What is the height, h, of the water in the manometer?
(d) What is the volume flow rate of the water flowing through the exit pipe?

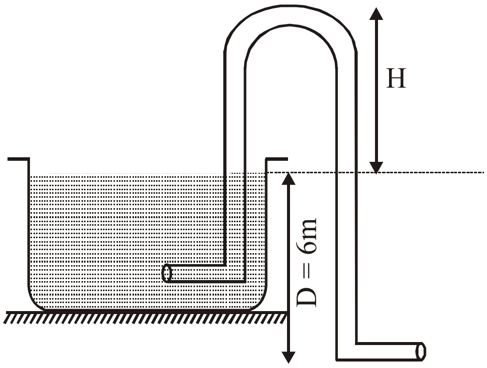
FM0048

24. A glass plate of area A and mass m is hinged along one of its sides. At what speed the air should be blown parallel to its upper surface to hold the plate horizontal? The density of air is ρ. Give the answer in term of $\sqrt{\dfrac{mg}{18\rho A}}$



FM0049

25. Figure shows a siphon, the vessel area is very large as compared to cross section of tube. Tube has a uniform cross section, its lower end is 6m below the surface of water. What is maximum height H (in m) of the upper end for siphon to work ? Take density of water = 10^3 kg/m^3, atmospheric pressure = 10^5 N/m^2.



FM0050

EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

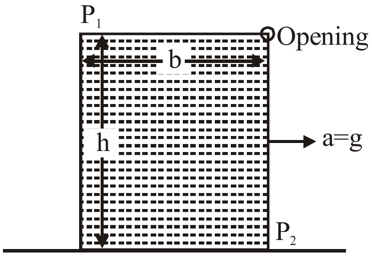
Fluid Statics

1. A vessel filled with a liquid of density ρ falls vertically downwards with an acceleration a(<g). The guage pressure P at depth h below the free surface of liquid is :

(A) $P = h\rho(g + a)$ (B) $P = h\rho g$ (C) $P = h\rho(g - a)$ (D) $P = h\rho a$

FM0051

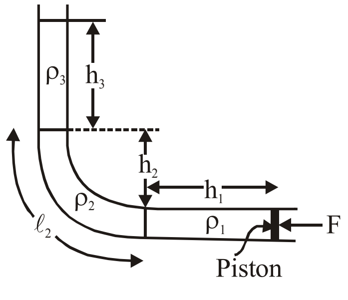
2. A closed rectangular vessel completely filled with a liquid of density ρ moves with an acceleration a = g. The value of the pressure difference $(P_1 - P_2)$ is :



(A) ρgb (B) $\frac{\rho g(b+h)}{2}$ (C) ρ(ab – gh) (D) ρgh

FM0052

3. A tube is bent into L shape and kept in a vertical plane. If these three liquids are kept in equilibrium by the piston of area A, the value of $\frac{F}{A}$ is :

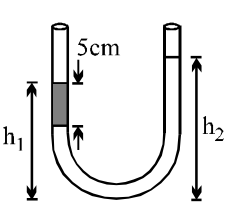


(A) $(\rho_1h_1 + \rho_2h_2 + \rho_3h_3)g$ (B) $(\rho_1h_1 + \rho_2\ell_2 + \rho_3h_3)g$

(C) $(\rho_2h_2 + \rho_3h_3)g$ (D) $(\rho_2\ell_2 + \rho_3h_3)g$

FM0053

4. An open-ended U-tube of uniform cross-sectional area contains water (density 1.0 gram/centimeter^3) standing initially 20 centimeters from the bottom in each arm. An immiscible liquid of density 4.0 grams/ centimeter^3 is added to one arm until a layer 5 centimeters high forms, as shown in the figure above. What is the ratio h_2/h_1 of the heights of the liquid in the two arms?



(A) 3/1 (B) 5/2 (C) 2/1 (D) 3/2

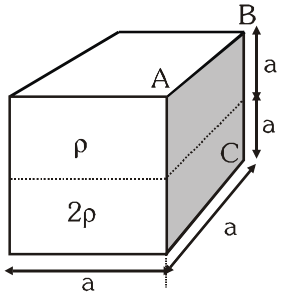
FM0054

5. A tube of length L is filled completely with incompressible liquid of mass M and closed at both the ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity ω. The force exerted by the liquid on the tube at other end is :-

(A) $\frac{M\omega^2L}{2}$ (B) $M\omega^2L$ (C) $\frac{M\omega^2L}{4}$ (D) $\frac{3M\omega^2L}{4}$

FM0055

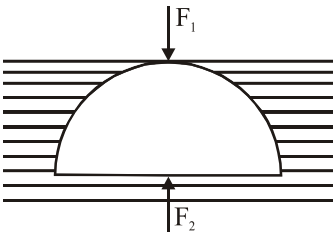
6. A cuboid (a × a × 2a) is filled with two immiscible liquids of density 2ρ & ρ as shown in the figure. Neglecting atmospheric pressure, ratio of force on base & side wall of the cuboid is :-



(A) 2 : 3 (B) 1 : 3 (C) 5 : 6 (D) 6 : 5

FM0056

7. A solid hemisphere is just pressed below the liquid, the value of $\frac{F_1}{F_2}$ is (where F_1 and F_2 are the hydrostatic forces acting on the curved and flat surfaces of the hemisphere) (Neglect atmospheric pressure).



(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{1}{3}$ (D) none of these

FM0057

8. A thin walled cylindrical metal vessel of linear coefficient of expansion 10^{-3} °C^{-1} contains benzene of volume expansion coefficient 10^{-3}°C^{-1}. If the vessel and its contents are now heated by 10°C, the pressure due to the liquid at the bottom.

(A) increases by 2% (B) decreases by 1%

(C) decreases by 2% (D) remains unchanged

FM0058

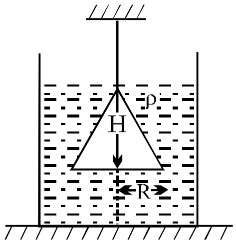
9. A cork of density $0.5 gcm^{-3}$ floats on a calm swimming pool. The fraction of the cork's volume which is under water is :-

(A) 0% (B) 25% (C) 10% (D) 50%

FM0059

Buoyancy

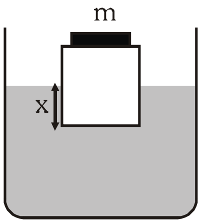
10. A cone of radius R and height H, is hanging inside a liquid of density ρ by means of a string as shown in the figure. The force, due to the liquid acting on the slant surface of the cone is (Neglect atmosphere pressure)



(A) $\rho\pi gHR^2$ (B) $\pi\rho HR^2$ (C) $\frac{4}{3}\pi\rho gHR^2$ (D) $\frac{2}{3}\pi\rho gHR^2$

FM0060

11. A cubical block (a × a × a), with a coin of mass 'm' over it is floating over a liquid of density ρ. In this case x_1 depth of the block is immersed. Now the coin is removed & it is found that x_2 depth is immersed in liquid. Value of $(x_1–x_2)$ is :-

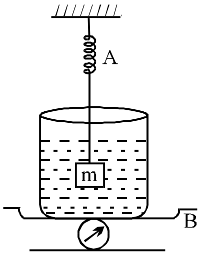


(A) $\frac{m}{\rho a^2}$ (B) $\frac{\rho a^4}{m}$

(C) $\frac{m}{2\rho a^2}$ (D) data insufficient

FM0061

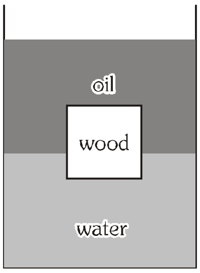
12. The spring balance A reads 2 kg with a block m suspended from it. A balance B reads 5 kg when a beaker with liquid is put on the pan of the balance. The two balances are now so arranged that the hanging mass is inside the liquid in the beaker as shown in the figure in this situation:



(A) the balance A will read more than 2 kg
(B) the balance B will read more than 5 kg
(C) the balance A will read less than 2 kg and B will read less than 5 kg
(D) the balances A and B will read 2 kg and 5 kg respectively.

FM0062

13. A cubical block of wood of side of 10 cm, floats at the interface between oil and water as shown in figure with its lower face 2 cm below the interface. The density of oil is 0.6 g/cm^3. The mass of the block is



(A) 600 g (B) 680 g (C) 800 g (D) 200 g

FM0063

14. A solid floats with 2/3 of its volume immersed in a liquid and with 3/4 of its volume immersed in another liquid. What fraction of its volume will be immersed if it floats in a homogenous mixture formed of equal volumes of the liquids?

(A) 6/7 (B) 8/11 (C) 11/16 (D) 12/17

FM0064

15. A small ball of relative density 0.8 falls into water from a height of 2m. The depth to which the ball will sink is (neglect viscous forces):

(A) 8 m (B) 2 m (C) 6 m (D) 4 m

FM0065

16. An object of density d_o kept deep inside water of density d_w and released. During the time it moves a vertical distance h with in the water :-

(A) The gravitational potential energy of the water in the vessel increases if $d_0 < d_w$

(B) The gravitational potential energy of the water in the vessel decreases if $d_0 < d_w$

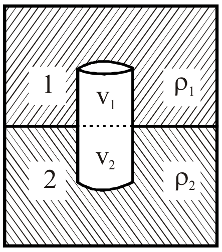
(C) The gravitational potential energy of the object increases if $d_0 > d_w$

(D) The gravitational potential energy of the object decreases if $d_0 < d_w$

FM0066

17. **Statement-1 :** When a body floats such that it's parts are immersed into two immersible liquids then force exerted by liquid-1 is of magnitude $\rho_1 v_1 g$.

Statement-2 : Total Buogyant force = $\rho_1 v_1 g + \rho_2 v_2 g$



(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

FM0067

18. **Statement-1 :** Submarine sailors are advised that they should not allow it to rest on floor of the ocean.

Statement-2 : The force exerted by a liquid on a submerged body may be downwards.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

FM0068

Bernoulli

19. Water is flowing steadily through a horizontal tube of non uniform cross-section. If the pressure of water is 4×10^4 N/m^2 at a point where cross-section is 0.02 m^2 and velocity of flow is 2 m/s, what is pressure at a point where cross-section reduces to 0.01 m^2.

(A) 1.4×10^4 N/m^2 (B) 3.4×10^4 N/m^2 (C) 2.4×10^{-4} N/m^2 (D) none of these

FM0069

20. In the case of a fluid, Bernoulli's theorem expresses the application of the principle of conservation of :-

(A) linear momentum (B) energy (C) mass (D) angular momentum

FM0070

21. Two water pipes P and Q having diameters 2×10^{-2} m and 4×10^{-2}m, respectively, are joined in series with the main supply line of water. The velocity of water flowing in pipe P is :-

(A) 4 times that of Q (B) 2 times that of Q

(C) 1/2 times of that of Q (D) 1/4 times that of Q

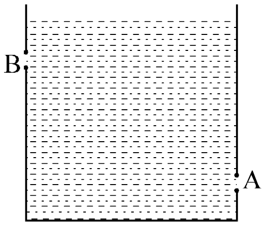
FM0071

22. A cylinder of height 20 m is completely filled with water. The velocity of efflux of water (in ms^{-1}) through a small hole on the side wall of the cylinder near its bottom, is- **[AIEEE - 2002]**

(A) 10 (B) 20 (C) 25.5 (D) 5

FM0072

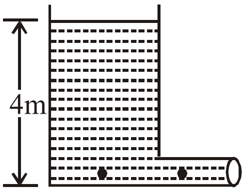
23. Two identical holes each of cross-sectional area 10^{-3} m^2 are made on the opposite sides of a tank containing water as shown in the figure. As the water comes out of the holes, the tank will experience a net horizontal force of 20 N. The difference in height between the holes A and B is.



(A) 1 m (B) 0.5 m (C) 2 m (D) 0.25 m

FM0073

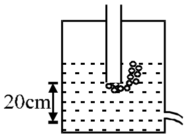
24. A vent tank of large cross-sectional area has a horizontal pipe 0.12 m in diameter at the bottom. This holds a liquid whose density is 1500 kg/m^3 to a height of 4.0 m. Assume the liquid is an ideal fluid in laminar flow. In figure, the velocity with which fluid flows out is **:-**



(A) $2\sqrt{5}$ m/s (B) $\sqrt{5}$ m/s (C) $4\sqrt{5}$ m/s (D) $\sqrt{10}$ m/s

FM0074

25. A tube is attached as shown in closed vessel containing water. The velocity of water coming out from a small hole is :



(A) $\sqrt{2}$ m/s

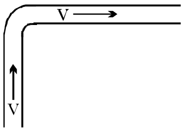
(B) 2 m/s

(C) depends on pressure of air inside vessel

(D) None of these

FM0075

26. A fire hydrant delivers water of density ρ at a volume rate L. The water travels vertically upward through the hydrant and then does 90° turn to emerge horizontally at speed V. The pipe and nozzle have uniform cross-section throughout. The force exerted by the water on the corner of the hydrant is :-



(A) ρVL (B) zero (C) 2ρVL (D) $\sqrt{2}\rho VL$

FM0076

27. A large tank is filled with water to a height H. A small hole is made at the base of the tank. It takes T_1 time to decrease the height of water to H/η, $(\eta > 1)$ and it takes T_2 time to take out the rest of water. If $T_1 = T_2$, then the value of η is :

(A) 2 (B) 3 (C) 4 (D) $2\sqrt{2}$

FM0077

28. Water is filled in a container upto height 3m. A small hole of area 'a' is punched in the wall of the container at a height 52.5 cm from the bottom. The cross sectional area of the container is A. If $\frac{a}{A} = 0.1$ then v^2 is (where v is the velocity of water coming out of the hole) **[IIT-JEE' 2005 (Scr)]**

(A) 48 (B) 51 (C) 50 (D) 51.5

FM0078

Surface Tension

29. If two soap bubbles of different radii are connected by a tube,

(A) air flows from the bigger bubble to the smaller bubble till the sizes become equal

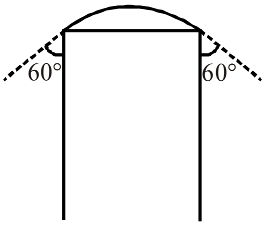
(B) air flows from bigger bubble to the smaller bubble till the sizes are interchanged

(C) air flows from the smaller bubble to the bigger

(D) there is no flow of air.

FM0079

30. A soap bubble is being blown on a tube of radius 1 cm. The surface tension of the soap solution is 0.05 N/m and the bubble makes an angle of 60° with the tube as shown. The excess of pressure over the atmospheric pressure in the tube is :



(A) 5 Pa (B) 1 Pa (C) 10 Pa (D) 20 Pa

FM0080

31. When an air bubble rises from the bottom of a deep lake to a point just below the water surface, the pressure of air inside the bubble :-

(A) is greater than the pressure outside it (B) is less than the pressure outside it

(C) increases as the bubble moves up (D) remains same as the bubble moves up

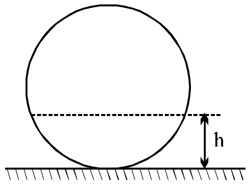
FM0081

32. Two merucry drops (each of radius 'r') merge to form a bigger drop. The surface energy of the bigger drop, if $\frac{1}{\pi}$ is the surface tension (in SI unit), is :

(A) $2^{5/3}r^2$ (B) $4r^2$ (C) $2r^2$ (D) $2^{8/3}r^2$

FM0082

33. A liquid is filled in a spherical container of radius R till a height h. At this position the liquid surface at the edges is also horizontal. The contact angle is :-



(A) 0 (B) $\cos^{-1}\left(\frac{R-h}{R}\right)$

(C) $\cos^{-1}\left(\frac{h-R}{R}\right)$ (D) $\sin^{-1}\left(\frac{R-h}{R}\right)$

FM0083

34. An open capillary tube is lowered in a vessel with mercury. The difference between the levels of the mercury in the vessel and in the capillary tube $\Delta h = 4.6$mm. What is the radius of curvature of the mercury meniscus in the capillary tube? Surface tension of mercury is 0.46 N/m, density of mercury is 13.6 gm/cc.

(A) $\frac{1}{340}$ m (B) $\frac{1}{680}$ m

(C) $\frac{1}{1020}$ m (D) Information insufficient

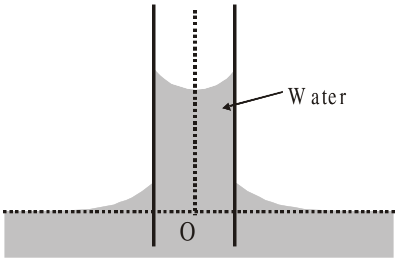
FM0084

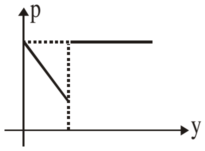
35. A container, whose bottom has round holes with diameter 0.1 mm is filled with water. The maximum height in cm upto which water can be filled without leakage will be what?
Surface tension = 75×10^{-3} N/m and g = 10 m/s^2:

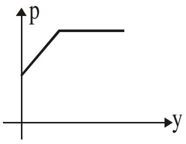
(A) 20 cm (B) 40 cm (C) 30 cm (D) 60 cm

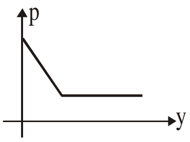
FM0085

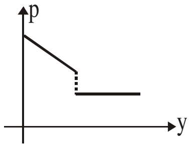
36. Water rises in a capillary as shown. The correct graph of pressure (p) vs height (y) above point O along the axis of capillary is :-



(A)


(B)


(C)


(D)


FM0086

37. If two soap bubbles of different radii are connected by a tube- **[AIEEE - 2004]**

(A) air flows from the bigger bubble to the smaller bubble till the sizes become equal
(B) air flows from bigger bubble to the smaller bubble till the sizes are interchanged
(C) air flows from the smaller bubble to the bigger bubble
(D) there is no flow of air

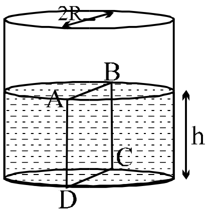
FM0087

38. A 20 cm long capillary tube is dipped in water. The water rises upto 8 cm. If the entire arrangement is put in a freely falling elevator, the length of water column in the capillary tube will be- **[AIEEE - 2005]**

(A) 8 cm (B) 10 cm (C) 4 cm (D) 20 cm

FM0088

39. Water is filled up to a height h in a beaker of radius R as shown in the figure. The density of water is ρ, the surface tension of water is T and the atmospheric pressure is P_0. Consider a vertical section ABCD of the water column through a diameter of the beaker. The force on water on one side of this section by water on the other side of this section has magnitude **[IIT-JEE 2007]**



(A) $|2P_0Rh + \pi R^2\rho gh - 2RT|$ (B) $|2P_0Rh + R\rho gh^2 - 2RT|$
(C) $|P_0\pi R^2 + R\rho gh^2 - 2RT|$ (D) $|P_0\pi R^2 + R\rho gh^2 + 2RT|$

FM0089

Viscosity

40. Two drops of same radius are falling through air with steady velocity of v cm/s. If the two drops coalesce, what would be the terminal velocity?

(A) 4 v (B) $(4)^{1/3}$ v (C) 2 v (D) 64 v

FM0090

41. Spherical balls of radius R are falling in a viscous fluid of viscosity η with a velocity v. the retarding viscous force acting on the spherical ball is- **[AIEEE - 2004]**

(A) directly proportional to R but inversely proportional to v

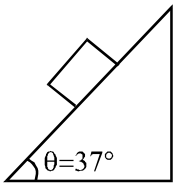
(B) directly proportional to both radius R and velocity v

(C) inversely proportional to both radius R and velocity v

(D) inversely proportional to R but directly proportional to velocity v

FM0091

42. A cubical block of side 'a' and density 'ρ' slides over a fixed inclined plane with constant velocity 'v'. There is a thin film of viscous fluid of thickness 't' between the plane and the block. Then the coefficient of viscosity of the thin film will be:



(A) $\frac{3\rho agt}{5v}$ (B) $\frac{4\rho agt}{5v}$ (C) $\frac{\rho agt}{v}$ (D) none of these

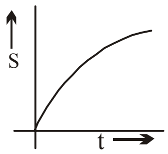
FM0092

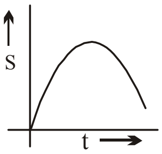
43. There is a 1mm thick layer of glycerine between a flat plate of area 100 cm^2 & a big fixed plate. If the coefficient of viscosity of glycerine is 1.0 kg/m-s then how much force is required to move the plate with a velocity of 7 cm/s?

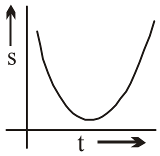
(A) 3.5 N (B) 0.7 N (C) 1.4 N (D) None

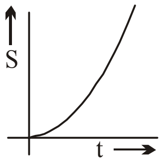
FM0093

44. The displacement of a ball falling from rest in a viscous medium is platted against time. Choose a possible option

(A)


(B)


(C)


(D)


FM0094

45. An air bubble of radius 1cm is found to rise in a cylindrical vessel of large radius at a steady rate of 0.2 cm per second. If the density of the liquid is 1470 kg m^{-3}, then coefficient of viscosity of liquid is approximately equal to

(A) 163 poise (B) 163 centi-poise (C) 140 poise (D) 140 centi-poise

FM0095

46. **Statement-1 :** The free surface of a liquid at rest with respect to stationary container is always normal to the $\vec{g}$.

Statement-2 : Liquids at rest cannot have shear stress.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

FM0096

47. If the terminal speed of a sphere of gold (density = 19.5 kg/m^3) is 0.2 m/s in a viscous liquid (density = 1.5 kg/m^3), find the terminal speed of a sphere of silver (density=10.5 kg/m^3) of the same size in the same liquid. **[AIEEE- 2006]**

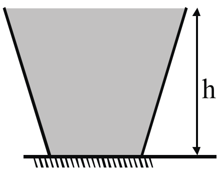
(A) 0.4 m/s (B) 0.133 m/s (C) 0.1 m/s (D) 0.2 m/s

FM0097

MULTIPLE CORRECT TYPE QUESTIONS

Fluid Statics

48. M gm of a liquid of density ρ is filled in a light beaker and kept on a horizontal table as shown in the figure. The height of the liquid in the beaker is h. The beaker is wider on top than at its base and the cross-sectional area of the base is A. Neglect the effect of atmospheric pressure. Now, choose the **CORRECT** statement(s) from the following.



(A) The pressure of liquid at the bottom surface is ρgh.

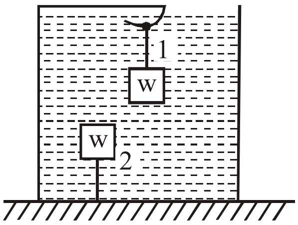
(B) The normal reaction exerted by the table on the beaker is ρghA.

(C) The pressure of the liquid at the bottom surface is $\frac{Mg}{A}$.

(D) The normal reaction exerted by the table on the beaker is Mg.

FM0098

49. An iron block and a wooden block are positioned in a vessel containing water as shown in the figure. The iron block (1) hangs from a massless string with a rigid support from the top while the wooden block (2) floats being tied to the bottom through a massless string. If now the vessel starts acceleration towards right.



(A) iron block gets deflected towards right.

(B) wooden block gets deflected towards right.

(C) iron block gets deflected towards left.

(D) wooden block gets deflected towards left.

FM0099

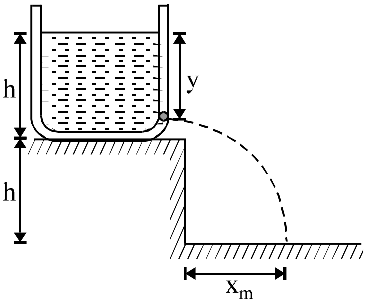
50. A beaker is filled in with water is accelerated a m/s^2 in +x direction. The surface of water shall make an angle

(A) $\tan^{-1}(a/g)$ backwards (B) $\tan^{-1}(a/g)$ forwards

(C) $\cot^{-1}(g/a)$ backwards (D) $\cot^{-1}(g/a)$ forwards

FM0100

Fluid Dynamics



51. A tank is filled upto a height h with a liquid and is placed on a platform of height h from the ground. To get maximum range x_m a small hole is punched at a distance of y from the free surface of the liquid. Then

(A) $x_m = 2h$ (B) $x_m = 1.5$ h

(C) y = h (D) y = 0.75 h

FM0101

MATRIX MATCH TYPE QUESTION

Fluid Statics

52. Bucket A contains only water ; an identical bucket B contains water, but also contains a solid object in the water. Consider the following four situations. Which bucket weighs more ?

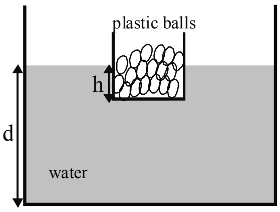
	Column-I		**Column-II**
(A)	The object floats in bucket B, and the buckets have the same water level	(P)	Bucket A
(B)	The object floats in bucket B, and the buckets have the same volume of water	(Q)	Bucket B
(C)	The object sinks completely in bucket B, and the buckets have the same water level	(R)	Both buckets have the same weight
(D)	The object sinks completely in bucket B, and the buckets have the same volume of water	(S)	The answer cannot be determined from the information given.

FM0102

53. Column I shows different system as describe, with some parameter while column-II gives the change in the parameter.

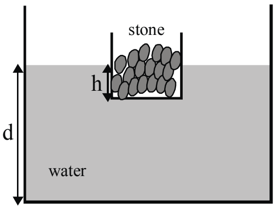
Column-I **Column-II**

(A)



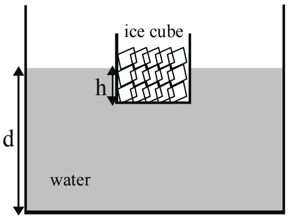
Plastic ball is thrown from the container in the water.
[d is the height of water level while h is the depth to which the container is submerged]

(B)



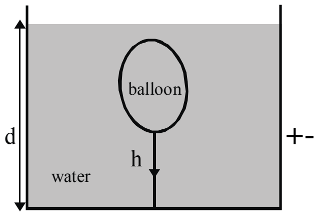
Stone is thrown from the container in the water.
[d is the height of water level while h is the depth to which the container is submerged]

(C)



Ice melts and remain in the container.
[d is the height of water level while h is the depth to which the container is submerged]

(D)



The water is heated slowly.
[d is the height of water level while h is the tension in the string. $\gamma_{air} > \gamma_{water}$]

(P) h decreases

(Q) h remain same

(R) h increases

(S) d decreases

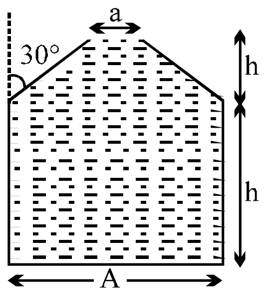
(T) d remain same

FM0103

EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

1. The vessel shown in the figure has two sections. The lower part is a rectangular vessel with area of cross-section A and height h. The upper part is a conical vessel of height h with base area 'A' and top area 'a' and the walls of the vessel are inclined at an angle 30° with the vertical. A liquid of density ρ fills both the sections upto a height 2h. Neglecting atmospheric pressure.



(A) The force F exerted by the liquid on the base of the vessel is $2h\rho g\,\frac{(A+a)}{2}$

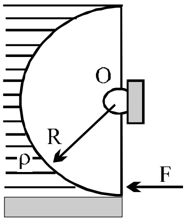
(B) the pressure P at the base of the vessel is $2h\rho g\,\frac{A}{a}$

(C) the weight of the liquid W is greater than the force exerted by the liquid on the base

(D) the walls of the vessel exert a downward force (F–W) on the liquid.

FM0104

2. A light semi cylindrical gate of radius R is pivoted at its mid point O, of the diameter as shown in the figure holding liquid of density ρ. The force F required to prevent the rotation of the gate is equal to



(A) $2\pi R^3\rho g$ (B) $2\rho g R^3 l$ (C) $\frac{2R^2 l\rho g}{3}$ (D) none of these

FM0105

3. A heavy hollow cone of radius R and height h is placed on a horizontal table surface, with its flat base on the table. The whole volume inside the cone is filled with water of density ρ. The circular rim of the cone's base has a watertight seal with the table's surface and the top apex of the cone has a small hole. Neglecting atmospheric pressure find the total upward force exerted by water on the cone is

(A) $(2/3)\pi R^2 h\rho g$ (B) $(1/3)\pi R^2 h\rho g$ (C) $\pi R^2 h\rho g$ (D) None

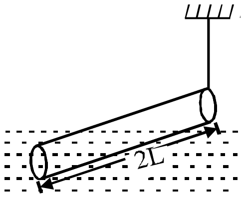
FM0106

4. Two cubes of size 1.0 m sides, one of relative density 0.60 and another of relative density = 1.15 are connected by weightless wire and placed in a large tank of water. Under equilibrium the lighter cube will project above the water surface to a height of

(A) 50 cm (B) 25 cm (C) 10 cm (D) zero

FM0107

5. A slender homogeneous rod of length 2*L* floats partly immersed in water, being supported by a string fastened to one of its ends, as shown. The specific gravity of the rod is 0.75. The length of rod that extends out of water is :



(A) L (B) $\frac{1}{2}$ L (C) $\frac{1}{4}$ L (D) 3 L

FM0108

6. Mercury is poured into a uniform vertical U-tube and water is poured in above it. The level of water is the same in both arms. A piece of wood is dropped into one arm and some water equal in weight to the piece of wood is added to the other. Then, consider the statements :

(I) The level of mercury will be same in both the arms.

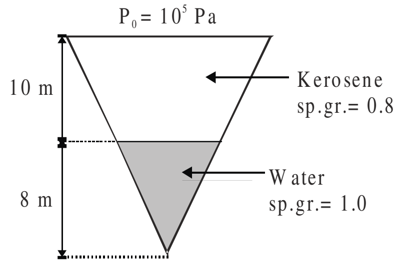
(II) The level of water will be same in both the arms.

(A) Both I and II are true (B) I is true but II is false

(C) I is false but II is true (D) Both I and II are false.

FM0109

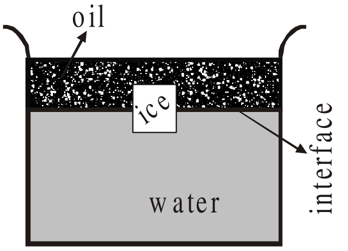
7. Shown in figure, a conical container of half-apex angle 37° filled with certain quantities of kerosene and water. The force exerted by the water on the kerosene is approximately, (Take atmospheric pressure = 10^5 Pa)



(A) 3×10^7 N (B) 4×10^7 N (C) 2×10^7 N (D) 5×10^7 N

FM0110

8. An ice cube is floating in water above which a layer of a lighter oil is poured. As the ice melts completely, the level of interface and the upper most level of oil will respectively :-

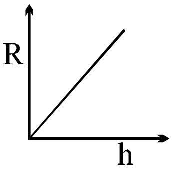


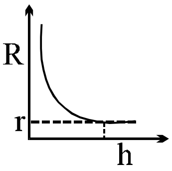
(A) rise and fall (B) fall and rise (C) rise and not change (D) not change and fall

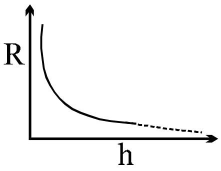
FM0111

Statics

9. A long capillary tube of radius ‘r’ is initially just vertically completely immerged inside a liquid of angle of contact 0^0. If the tube is slowly raised then relation between radius of curvature of meniscus inside the capillary tube and displacement (h) of tube can be represented by

(A)


(B)


(C)


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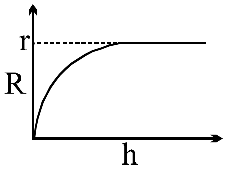
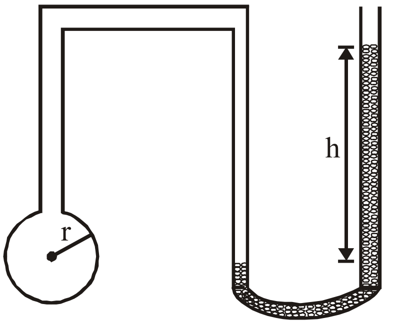


FM0112

10. If the radius of the Soap-bubble on one side of tube is r and difference in height of liquid of density ρ in manometer is h, then surface tension of liquid used to make the bubble is :-

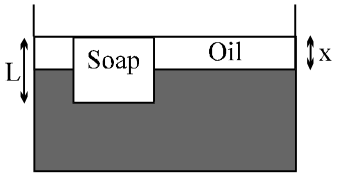


(A) $T = 2r\rho hg$ (B) $T = \frac{rh\rho g}{4}$ (C) $T = \frac{2\pi rh\rho g}{2}$ (D) $T = \frac{rh\rho g}{2}$

FM0113

Surface tension

11. A rectangular bar of soap has density 800 kg/m^3 floats in water of density 1000 kg/m^3. Oil of density 300 kg/m^3 is slowly added, forming a layer that does not mix with the water. When the top surface of the oil is at the some level as the top surface of the soap. What is the ratio of the oil layer thickness to the soap's thickness, x/L ?



(A) $\frac{2}{10}$ (B) $\frac{2}{7}$ (C) $\frac{3}{10}$ (D) $\frac{3}{8}$

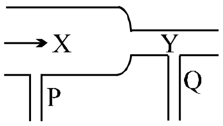
FM0114

12. Water coming out of a horizontal tube at a speed v strikes normally a vertically wall close to the mouth of the tube and falls down vertically after impact. When the speed of water is increased to 2v.

(A) the thrust exerted by the water on the wall will be doubled
(B) the thrust exerted by the water on the wall will be unchanged
(C) the energy lost per second by water striking the wall will also be four times
(D) the energy lost per second by water striking the wall be increased eight times.

FM0115

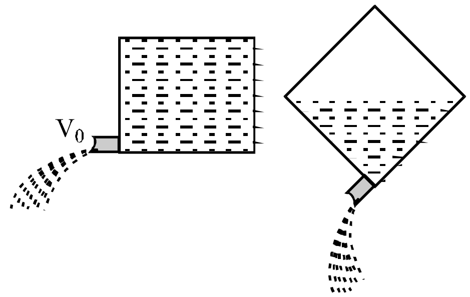
13. A steady flow of water passes along a horizontal tube from a wide section X to the narrower section Y, see figure. Manometers are placed at P and Q at the sections. Which of the statements A, B, C, D, is most correct?



(A) water velocity at X is greater than at Y
(B) the manometer at P shows lower pressure than at Q
(C) kinetic energy per m^3 of water at X = kinetic energy per m^3 at Y
(D) the manometer at P shows greater pressure than at Q

FM0116

14. A cubical box of wine has a small spout located in one of the bottom corners. When the box is full and placed on a level surface, opening the spout results in a flow of wine with a initial speed of v_0 (see figure). When the box is half empty, someone tilts it at 45° so that the spout is at the lowest point (see figure). When the spout is opened the wine will flow out with a speed of

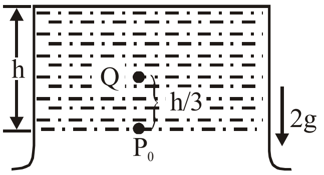


(A) v_0 (B) $v_0/2$ (C) $v_0/\sqrt{2}$ (D) $v_0/\sqrt[4]{2}$

FM0117

Bernoulli's

15. An inverted test tube having a liquid of density ρ accelerates down with a = 2g then, $P_Q - P_0$ is:



(A) $-\frac{\rho gh}{3}$ (B) $\frac{2\rho gh}{3}$ (C) $\frac{\rho gh}{3}$ (D) indeterminate

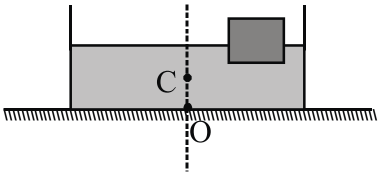
FM0118

16. Some liquid is filled in a cylindrical vessel of radius R. Let F_1 be the force applied by the liquid on the bottom of the cylinder. Now the same liquid is poured into a vessel of uniform square crss-section of side R. Let F_2 be the force applied by the liquid on the bottom of this new vessel.(Neglect atmosphere pressure) Then:

(A) $F_1 = \pi F_2$ (B) $F_1 = \frac{F_2}{\pi}$ (C) $F_1 = \sqrt{\pi}F_2$ (D) $F_1 = F_2$

FM0119

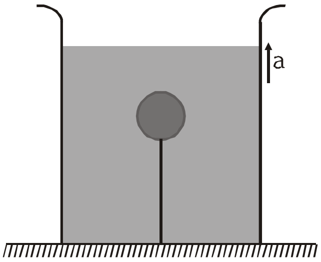
17. There is water in container with center of mass at C. Now a small wooden piece is place towards right as shown in the figure. After putting the wooden piece.



(A) Pressure at base remains same and centre of mass of water and wooden piece will be right of line OC.
(B) Pressure at base remains same and centre of mass of water and wooden piece will be on line OC.
(C) Pressure at base changes and centre of mass of water and wooden piece will be right of line OC.
(D) Pressure at base changes and centre of mass of water and wooden piece will be on line OC.

FM0120

18. The tension in a string holding a solid block below the surface of a liquid (of density greater than that of solid) as shown in figure is T_0 when the system is at rest. What will be the tension in the string if the system has an upward acceleration a?



(A) $T_0\left(1+\frac{a}{g}\right)$ (B) $T_0\left(1-\frac{a}{g}\right)$ (C) $2T_0\left(1-\frac{a}{g}\right)$ (D) $3T_0\left(1+\frac{2a}{g}\right)$

FM0121

19. A wooden block floats in water in a sealed container. When the container is at rest, 25% of the block is above the water. Consider the following five situation :

(i) the container is lifted up at constant speed.

(ii) the container is lowered at constant speed.

(iii) the container is lifted up at an increasing speed.

(iv) the container is lowered at a decreasing speed.

(v) the air pressure above the water in the container is increased.

What happens in each situation?

(A) 75% of block is submerged in siutation (i) & (ii) and higher fraction is submerged in situations (iii), (iv) & (v)

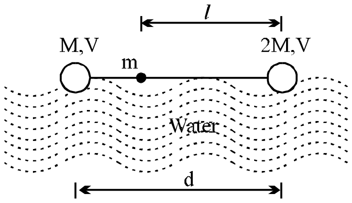
(B) 75% of block is submerged in situation (i), (ii) & (v) and higher fraction is submerged in situations (iii), (iv).

(C) 75% of block is submerged in situation (i), (ii), (iii) & (iv) and higher fraction is submerged in situations (v).

(D) 75% of block is submerged in all situation.

FM0122

20. A dumbbell is placed in water of density ρ. It is observed that by attaching a mass m to the rod, the dumbbell floats with the rod horizontal on the surface of water and each sphere exactly half submerged as shown in the figure. The volume of the mass m is negligible. The value of length *l* is :-



(A) $\frac{d(V\rho-3M)}{2(V\rho-2M)}$ (B) $\frac{d(V\rho-2M)}{2(V\rho-3M)}$ (C) $\frac{d(V\rho+2M)}{2(V\rho-3M)}$ (D) $\frac{d(V\rho-2M)}{2(V\rho+3M)}$

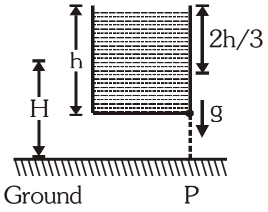
FM0123

21. Fountains usually seen in gardens are generated by a wide pipe with an enclosure at one end having many small holes. Consider one such fountain which is produced by a pipe of internal diameter 2 cm in which water flows at a rate 3 ms^{-1}. The enclosure has 100 holes each of diameter 0.05 cm. The velocity of water coming out of the holes is (in ms^{-1}):

(A) 0.48 (B) 96 (C) 24 (D) 48

FM0124

22. An open vessel full of water is falling freely under gravity. There is a small hole in one face of the vessel, as shown in the figure. The water which comes out from the hole at the instant when hole is at height H above the ground, strikes the ground at a distance of x from P. Which of the following is correct for the situation described ?



(A) The value of x is $2\sqrt{\dfrac{2hH}{3}}$

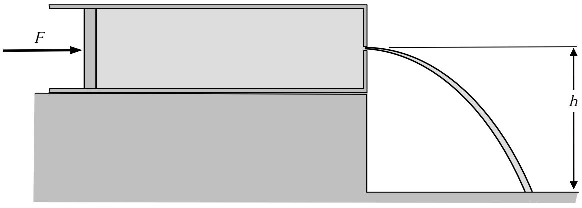
(B) The value of x is $\sqrt{\dfrac{4hH}{3}}$

(C) The value of x can't be computed from information provided.

(D) The question is irrevalent as no water comes out from the hole.

FM0125

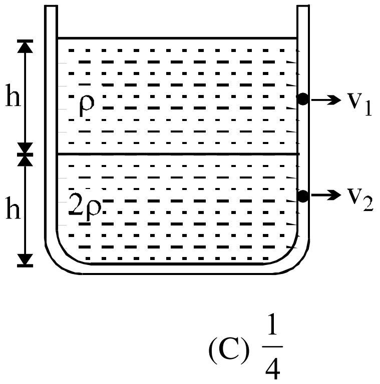
23. An ideal liquid of density ρ is filled in a horizontally fixed syringe fitted with piston. There is no friction between the piston and the inner surface of the syringe. Cross-section area of the syringe is A. At one end of the syringe, an orifice of negligible cross-section area is made. When the piston is pushed into the syringe, the liquid comes out of the orifice following parabolic path and falls on the ground. With what speed the liquid strikes the ground? Neglect the air drag :-



(A) $\sqrt{\dfrac{F+\rho ghA}{\rho A}}$ (B) $\sqrt{\dfrac{F+2\rho ghA}{\rho A}}$ (C) $\sqrt{\dfrac{2F+\rho ghA}{\rho A}}$ (D) $\sqrt{\dfrac{2(F+\rho ghA)}{\rho A}}$

FM0126

24. Equal volumes of two immiscible liquids of densities ρ and 2ρ are filled in a vessel as shown in figure. Two small holes are punched at depth h/2 and 3h/2 from the surface of lighter liquid. If v_1 and v_2 are the velocities of a flux at these two holes, then v_1/v_2 is :



(A) $\dfrac{1}{2\sqrt{2}}$ (B) $\dfrac{1}{2}$ (C) $\dfrac{1}{4}$ (D) $\dfrac{1}{\sqrt{2}}$

FM0127

25. A vertical tank, open at the top, is filled with a liquid and rests on a smooth horizontal surface. A small hole is opened at the centre of one side of the tank. The area of cross-section of the tank is N times the area of the hole, where N is a large number. Neglect mass of the tank itself. The initial acceleration of the tank is :-

(A) $\frac{g}{2N}$ (B) $\frac{g}{\sqrt{2N}}$ (C) $\frac{g}{N}$ (D) $\frac{g}{2\sqrt{N}}$

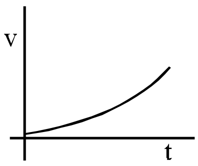
FM0128

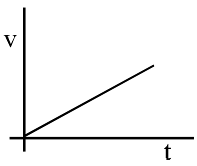
26. A solid metallic sphere of radius r is allowed to fall freely through air. If the frictional resistance due to air is proportional to the cross-sectional area and to the square of the velocity, then the terminal velocity of the sphere is proportional to which of the following?

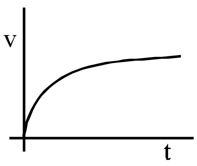
(A) r^2 (B) r (C) $r^{3/2}$ (D) $r^{1/2}$

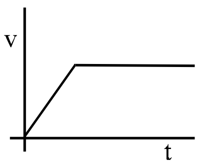
FM0129

27. Which of the following graphs best represents the motion of a raindrop?

(A)


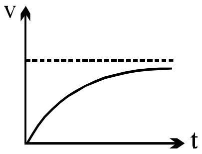
(B)


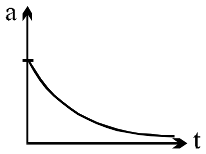
(C)


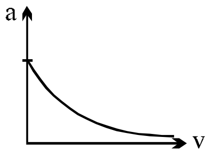
(D)


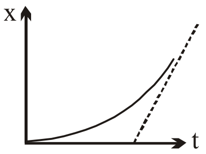
FM0130

28. Which of the following is the ***incorrect*** graph for a sphere falling in a viscous liquid? (Given at t = 0, velocity v = 0 and displacement x = 0.)

(A)


(B)


(C)


(D)


FM0131

MULTIPLE CORRECT TYPE QUESTIONS

29. A cubical block is floating in a liquid with one third of its volume immersed in the liquid. When the whole system accelerates upwards with acceleration of g/2 :-

(A) the fraction of volume immersed in the liquid will change.

(B) the buoyancy force on the block will change.

(C) the buoyancy force will increase by 50 percent.

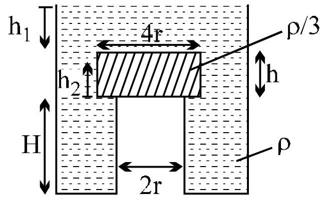
(D) the pressure in the liquid will increased.

FM0132

COMPREHENSION TYPE QUESTIONS

Paragraph for Question no. 30 to 32

A wooden cylinder of diameter 4r, height h and density ρ/3 is kept on a hole of diameter 2r of a tank, filled with water of density ρ as shown in the figure. The height of the base of cylinder from the base of tank is H. **[IIT-JEE 2006]**



30. If level of liquid starts decreasing slowly when the level of liquid is at a height h_1 above the cylinder, the block just starts moving up. Then, value of h_1 is :-

(A) $\frac{2h}{3}$ (B) $\frac{5h}{4}$ (C) $\frac{5h}{3}$ (D) $\frac{5h}{2}$

FM0133

31. Let the cylinder is prevented from moving up, by applying a force and water level is further decreased. Then, height of water level (h_2 in figure) for which the cylinder remains in original position without application of force is

(A) $\frac{h}{3}$ (B) $\frac{4h}{9}$ (C) $\frac{2h}{3}$ (D) h

FM0133

32. If height h_2 of water level is further decreased, then

(A) cylinder will not move up and remains at its original position.

(B) for $h_2 = h/3$, cylinder again starts moving up

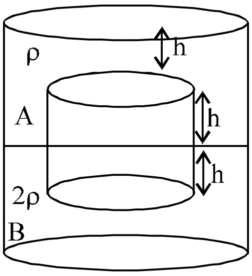
(C) for $h_2 = h/4$, cylinder again starts moving up

(D) for $h_2 = h/5$ cylinder again starts moving up

FM0133

MATRIX MATCH TYPE QUESTION

33. Shown below is a cylinder of radius R floating in vessel containing liquids A and B. Neglecting atmospheric pressure match the quantities mentioned in column-I with corresponding expression in column-II.



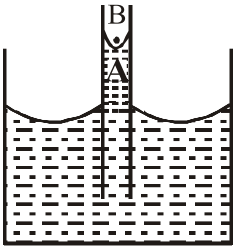
Column-I	**Column-II**
(A) Net force exerted by liquid A of density ρ on the cylinder.	(P) 9 $\rho g R h^2$
(B) Net force exerted by liquid B of density 2ρ on the cylinder.	(Q) $\pi \rho g R^2 h$
(C) Net force exerted by liquids A and B on the left half of the curved part of cylinder.	(R) $4\pi \rho g R^2 h$
(D) Net force exerted by liquid A and B on the cylinder.	(S) $3\pi \rho g R^2 h$

FM0134

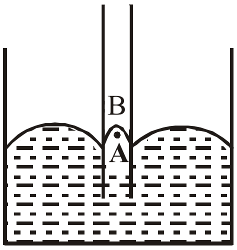
34. Capillary rise and shape of droplets on a plate due to surface tension are shown in column-II. Point A and B are just inside and outside the surface. Match the following

Column-I **Column-II**

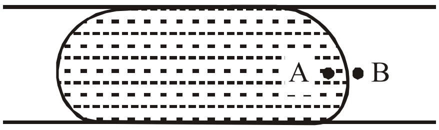
(A) liquid is wetting the solid (P)



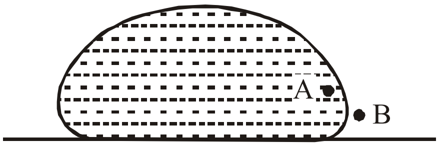
(B) liquid is not wetting the solid (Q)



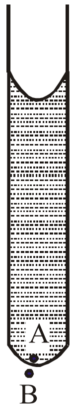
(C) Pressure at A > pressure at B (R) A liquid drop is pressed between two parallel glass plates.



(D) Pressure at B > Pressure at A (S)



(T)



both end open cylindrical pipe

FM0166

EXERCISE (JM)

1. A spherical solid ball of volume V is made of a material of density ρ_1. It is falling through a liquid of density $\rho_2\left(\rho_2 < \rho_1\right)$. Assume that the liquid applies a viscous force on the ball that is propoertional to the square of its speed v, i.e., $F_{\text{viscous}} = -kv^2$ (k > 0). Then terminal speed of the ball is

[AIEEE - 2008]

(1) $\sqrt{\dfrac{Vg\left(\rho_1-\rho_2\right)}{k}}$ (2) $\dfrac{Vg\rho_1}{k}$ (3) $\sqrt{\dfrac{Vg\rho_1}{k}}$ (4) $\dfrac{Vg\left(\rho_1-\rho_2\right)}{k}$

FM0135

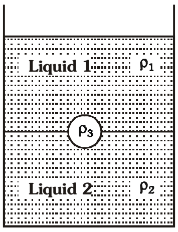
2. A jar is filled with two non-mixing liqudis 1 and 2 having densities ρ_1 and ρ_2, respectively. A solid ball, made of a material of density ρ_3, is dropped in the jar. It comes to equilibrium in the position shown in the figure. Which of the following is true for ρ_1, ρ_2 and ρ_3 **[AIEEE - 2008]**

(1) $\rho_3 < \rho_1 < \rho_2$

(2) $\rho_1 > \rho_3 > \rho_2$

(3) $\rho_1 < \rho_2 < \rho_3$

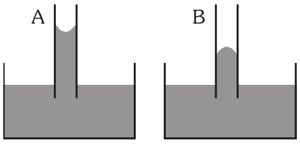
(4) $\rho_1 < \rho_3 < \rho_2$

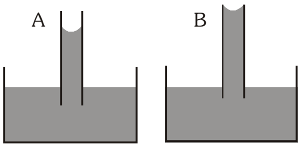


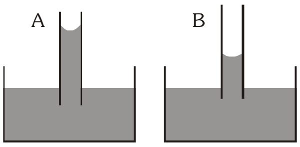
FM0136

3. A capillary tube (1) is dipped in water. Another identical tube (2) is dipped in a soap -water solution. Which of the following shows the relative nature of the liquid columns in the two tubes?

[AIEEE - 2008]

(1)


(2)


(3)


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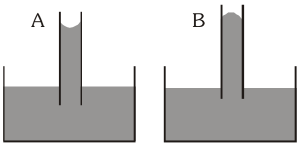
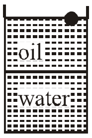
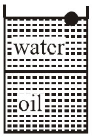
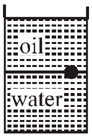


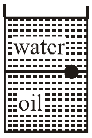
FM0137

4. A ball is made of a material of density ρ where $\rho_{oil} < \rho < \rho_{water}$ with ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a mixture of this oil and water, which of the following pictures represents its equilibrium position ? **[AIEEE-2010]**

(1)


(2)


(3)


(4)


FM0138

5. Water is flowing continuously from a tap having an internal diameter 8×10^{-3} m. The water velocity as it leaves the tap is 0.4 ms^{-1}. The diameter of the water stream at a distance 2×10^{-1} m below the tap is close to :- **[AIEEE-2011]**

(1) 9.6×10^{-3} m (2) 3.6×10^{-3} m (3) 5.0×10^{-3} m (4) 7.5×10^{-3} m

FM0139

6. Work done in increasing the size of a soap bubble from a radius of 3 cm to 5cm is nearly (Surface tension of soap solution = 0.03 Nm^{-1}) :- **[AIEEE-2011]**

(1) 2π mJ (2) $0.4\ \pi$ mJ (3) 4π mJ (4) $0.2\ \pi$ mJ

FM0140

7. Two mercury drops (each of radius 'r') merge to form a bigger drop. The surface energy of the bigger drop, ifs T is the surface tension, is : **[AIEEE-2011]**

(1) $2^{5/3}\pi r^2T$ (2) $4\pi r^2T$ (3) $2\pi r^2T$ (4) $2^{8/3}\pi r^2T$

FM0141

8. If a ball of steel (density $\rho = 7.8$ g cm^{-3}) attains a terminal velocity of 10 cm s^{-1} when falling in a tank of water (coefficient of viscosity $\eta_{water} = 8.5 \times 10^{-4}$ Pa.s) then its terminal velocity in glycerine ($\rho = 1.2$ g cm^{-3}, $\eta = 13.2$ Pa.s) would be nearly :- **[AIEEE-2011]**

(1) 1.6×10^{-5} cm s^{-1} (2) 6.25×10^{-4} cm s^{-1}

(3) 6.45×10^{-4} cm s^{-1} (4) 1.5×10^{-5} cm s^{-1}

FM0142

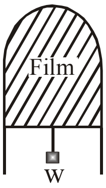
9. A thin liquid film formed between a U-shaped wire and a light slider supports a weight of 1.5×10^{-2} N (see figure). The length of the slider is 30 cm and its weight negligible. The surface tension of the liquid film is :- **[AIEEE-2012]**

(1) 0.025 Nm^{-1}

(2) 0.0125 Nm^{-1}

(3) 0.1 Nm^{-1}

(4) 0.05 Nm^{-1}



FM0143

10. A uniform cylinder of length L and mass M having cross- sectional area A is suspended, with its length vertical, form a fixed point by a massless spring, such that it is half submerged in a liquid of density σ at equilibrium position. The extension x_0 of the spring when it is in equilibrium is : **[AIEEE-2013]**

(1) $\frac{Mg}{k}$ (2) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{M}\right)$ (3) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{2M}\right)$ (4) $\frac{Mg}{k}\left(1+\frac{LA\sigma}{M}\right)$

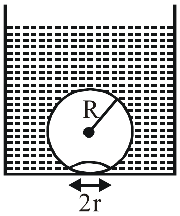
(Here k is spring constant)

FM0144

11. Assume that a drop of liquid evaporates by decrease in its surface energy, so that its temperature remains unchanged. What should be the minimum radius of the drop for this to be possible? The surface tension is T, density of liquid is ρ and L is its latent heat of vaporization. **[AIEEE-2013]**

(1) $\frac{\rho L}{T}$ (2) $\sqrt{\frac{T}{\rho L}}$ (3) $\frac{T}{\rho L}$ (4) $\frac{2T}{\rho L}$

FM0145

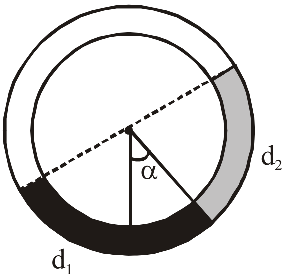


12. On heating water, bubbles being formed at the bottom of the vessel detatch and rise. Take the bubbles to be spheres of radius R and making a circular contact of radius r with the bottom of the vessel. If r << R, and the surface tension of water is T, value of r just before bubbles detatch is:-
(density of water is ρ_w) **[JEE Mains-2014]**

(1) $R^2\sqrt{\frac{\rho_w g}{T}}$ (2) $R^2\sqrt{\frac{3\rho_w g}{T}}$ (3) $R^2\sqrt{\frac{\rho_w g}{3T}}$ (4) $R^2\sqrt{\frac{\rho_w g}{6T}}$

FM0146

13. There is a circular tube in a vertical plane. Two liquids which do not mix and of densities d_1 and d_2 are filled in the tube. Each liquid subtends 90° angle at centre. Radius joining their interface makes an angle α with vertical. Ratio $\frac{d_1}{d_2}$ is : **[JEE Mains-2014]**



(1) $\frac{1+\tan\alpha}{1-\tan\alpha}$ (2) $\frac{1+\sin\alpha}{1-\cos\alpha}$ (3) $\frac{1+\sin\alpha}{1-\sin\alpha}$ (4) $\frac{1+\cos\alpha}{1-\cos\alpha}$

FM0147

EXERCISE (JA)

1. **Statement-1 :** The stream of water flowing at high speed from a garden hose pipe tends to spread like a fountain when held vertically up, but tends to narrow down when held vertically down.

and

Statement-2 : In any steady flow of an incompressible fluid, the volume flow rate of the fluid remains constant. **[IIT-JEE 2008]**

(A) Statement-1 is True, Statement-2 is True ; statement-2 is a correct explanation for statement-1

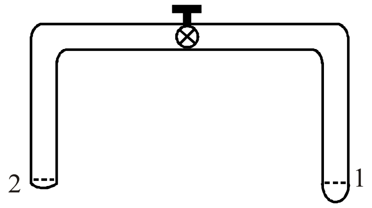
(B) Statement-1 is True, Statement-2 is True ; statement-2 is NOT a correct explanation for statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

FM0148

2. A glass tube of uniform internal radius (r) has a valve separating the two identical ends. Initially, the valve is in a tightly closed position. End 1 has a hemispherical soap bubble of radius r. End 2 has sub-hemispherical soap bubble as shown in figure. Just after opening the valve, **[IIT-JEE 2008]**



(A) air from end 1 flows towards end 2. No change in the volume of the soap bubbles

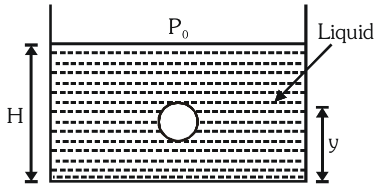
(B) air from end 1 flows towards end 2. Volume of the soap bubble at end 1 decreases

(C) no change occurs

(D) air from end 2 flows towards end 1. Volume of the soap bubble at end 1 increases

Paragraph for Question Nos. 3 to 5

A small spherical monoatomic ideal gas bubble $\left(\gamma = \frac{5}{3}\right)$ is trapped inside a liquid of density ρ_ℓ (see figure). Assume that the bubble does not exchange any heat with the liquid. The bubble contains n moles of gas. The temperature of the gas when the bubble is at the bottom is T_0, the height of the liquid is H and the atmospheric pressure is P_0 (Neglect surface tension). **[IIT-JEE 2008]**



FM0149

3. As the bubble moves upwards, besides the buoyancy force the following forces are acting on it.

(A) Only the force of gravity

(B) The force due to gravity and the force due to the pressure of the liquid

(C) The force due to gravity, the force due to the pressure of the liquid and the force due to viscosity of the liquid

(D) The force due to gravity and the force due to viscosity of the liquid.

FM0150

4. When the gas bubble is at a height y from the bottom, its temperature is :-

(A) $T_0\left(\frac{P_0+\rho_\ell gH}{P_0+\rho_\ell gy}\right)^{2/5}$ (B) $T_0\left(\frac{P_0+\rho_\ell g(H-y)}{P_0+\rho_\ell gH}\right)^{2/5}$

(C) $T_0\left(\frac{P_0+\rho_\ell gH}{P_0+\rho_\ell gy}\right)^{3/5}$ (D) $T_0\left(\frac{P_0+\rho_\ell g(H-y)}{P_0+\rho_\ell gH}\right)^{3/5}$

FM0150

5. The buoyancy force acting on the gas bubble is (Assume R is the universal gas constant)

(A) $\rho_\ell nRgT_0\frac{(P_0+\rho_\ell gH)^{2/5}}{(P_0+\rho_\ell gy)^{7/5}}$ (B) $\frac{\rho_\ell nRgT_0}{(P_0+\rho_\ell gH)^{2/5}[P_0+\rho_\ell g(H-y)]^{3/5}}$

(C) $\rho_\ell nRgT_0\frac{(P_0+\rho_\ell gH)^{3/5}}{(P_0+\rho_\ell gy)^{8/5}}$ (D) $\frac{\rho_\ell nRgT_0}{(P_0+\rho_\ell gH)^{3/5}[P_0+\rho_\ell g(H-y)]^{2/5}}$

FM0150

6. A cylindrical vessel of height 500 mm has an orifice (small hole) at its bottom. The orifice is initially closed and water is filled in it up to height H. Now the top is completely sealed with a cap and the orifice at the bottom is opened. Some water comes out from the orifice and the water level in the vessel becomes steady with height of water column being 200 mm. Find the fall in height (in mm) of water level due to opening of the orifice. [Take atmospheric pressure = 1.0×10^5 N/m^2, density of water = 1000 kg/m^3 and g = 10 m/s^2. Neglect any effect of surface tension.] **[IIT-JEE-2009]**

FM0151

7. Two soap bubbles A and B are kept in a closed chamber where the air is maintained at pressure 8N/m^2. The radii of bubbles A and B are 2 cm and 4cm, respectively. Surface tension of the soap-water used to make bubbles is 0.04 N/m. Find the ratio n_B/n_A, where n_A and n_B are the number of moles of air in bubbles A and B, respectively. [Neglect the effect of gravity] **[IIT-JEE-2009]**

FM0152

Paragraph for Questions no. 8 to 10

When liquid medicine of density ρ is to be put in the eye, it is done with the help of a dropper. As the bulb on the top of the dropper is pressed, a drop forms at the opening of the dropper. We wish to estimate the size of the drop. We first assume that the drop formed at the opening is spherical because that requires a minimum increase in its surface energy. To determine the size, we calculate the net vertical force due to the surface tension T when the radius of the drop is R. When this force becomes smaller than the weight of the drop, the drop gets detached from the dropper.

[IIT-JEE-2010]

8. If the radius of the opening of the dropper is r, the vertical force due to the surface tension on the drop of radius R (assuming r <<R) is

(A) $2\pi rT$ (B) $2\pi RT$ (C) $\frac{2\pi r^2 T}{R}$ (D) $\frac{2\pi R^2 T}{r}$

FM0153

9. If $r = 5 \times 10^{-4}$ m, $\rho = 10^3$ kgm^{-3}, $g = 10$ ms^{-2}, $T = 0.11$ Nm^{-1}, the radius of the drop when it detaches from the dropper is approximately

(A) 1.4×10^{-3} m (B) 3.3×10^{-3} m (C) 2.0×10^{-3} m (D) 4.1×10^{-3} m

FM0153

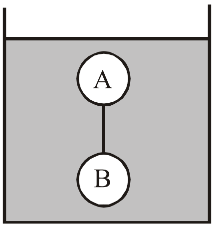
10. After the drop detaches, its surface energy is

(A) 1.4×10^{-6} J (B) 2.7×10^{-6} J (C) 5.4×10^{-6} J (D) 8.1×10^{-6} J

FM0153

11. Two solid spheres A and B of equal volumes but of different densities d_A and d_B are connected by a string. They are fully immersed in a fluid of density d_F. They get arranged into an equilibrium state as shown in the figure with a tension in the string. The arrangement is possible only if

[IIT-JEE-2011]



(A) $d_A < d_F$ (B) $d_B > d_F$ (C) $d_A > d_F$ (D) $d_A + d_B = 2d_F$

FM0154

12. A thin uniform cylindrical shell, closed at both ends, is partially filled with water. It is floating vertically in water in half-submerged state. If ρ_C is the relative density of the material of the shell with respect to water, then the correct statement is that the shell is **[IIT-JEE-2012]**

(A) more than half-filled if ρ_C is less than 0.5

(B) more than half-filled if ρ_C is more than 1.0

(C) half-filled if ρ_C is more than 0.5

(D) less than half-filled if ρ_c is less than 0.5

FM0155

13. A solid sphere of radius R and density ρ is attached to one end of a mass-less spring of force constant k. The other end of the spring is connected to another solid sphere of radius R and density 3ρ. The complete arrangement is placed in a liquid of density 2ρ and is allowed to reach equilibrium. The correct statement(s) is (are) **[IIT-JEE-2013]**

(A) the net elongation of the spring is $\frac{4\pi R^3 \rho g}{3k}$

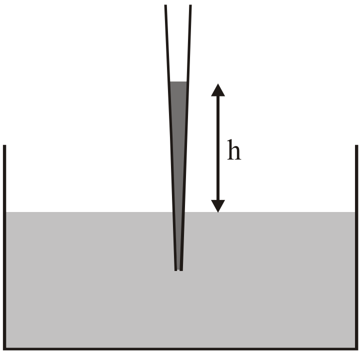
(B) the net elongation of the spring is $\frac{8\pi R^3 \rho g}{3k}$

(C) the light sphere is partially submerged.

(D) the light sphere is completely submerged.

FM0156

14. A glass capillary tube is of the shape of truncated cone with an apex angle α so that its two ends have cross sections of different radii. When dipped in water vertically, water rises in it to height h, where the radius of its cross section is b. If the surface tension of water is S, its density is ρ, and its contact angle with glass is θ, the value of h will be (g is the acceleration due to gravity) **[JEE Advanced-2014]**



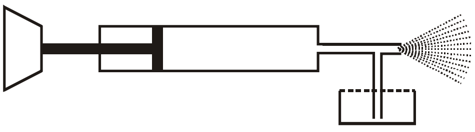
(A) $\frac{2S}{b\rho g}\cos(\theta-\alpha)$ (B) $\frac{2S}{b\rho g}\cos(\theta+\alpha)$

(C) $\frac{2S}{b\rho g}\cos(\theta-\alpha/2)$ (D) $\frac{2S}{b\rho g}\cos(\theta+\alpha/2)$

FM0157

Paragraph for Questions 15 to 17

A spray gun is shown in the figure where a piston pushes air out of a nozzle. A thin tube of uniform cross section is connected to the nozzle. The other end of the tube is in a small liquid container. As the piston pushes air through the nozzle, the liquid from the container rises into the nozzle and is sprayed out. For the spray gun shown, the radii of the piston and the nozzle are 20 mm and 1 mm respectively. The upper end of the container is open to the atmosphere. **[JEE Advanced-2014]**



15. If the piston is pushed at a speed of 5 mms^{-1}, the air comes out of the nozzle with a speed of
(A) 0.1 ms^{-1} (B) 1 ms^{-1} (C) 2 ms^{-1} (D) 8 ms^{-1}

FM0158

16. If the density of air is ρ_a and that of the liquid ρ_ℓ, then for a given piston speed the rate (volume per unit time) at which the liquid is sprayed will be proportional to

(A) $\sqrt{\frac{\rho_a}{\rho_\ell}}$ (B) $\sqrt{\rho_a \rho_\ell}$ (C) $\sqrt{\frac{\rho_\ell}{\rho_a}}$ (D) $\rho\ell$

FM0158

17. A person in a lift is holding a water jar, which has a small hole at the lower end of its side. When the lift is at rest, the water jet coming out of the hole hits the floor of the lift at a distance d of 1.2 m from the person. In the following, state of the lift's motion is given in List I and the distance where the water jet hits the floor of the lift is given in List II. Match the statements from List I with those in List II and select the correct answer using the code given below the lists.

[JEE Advanced-2014]

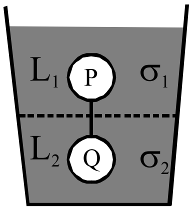
List - I	**List-II**
(P) Lift is accelerating vertically up.	(1) d = 1.2 m
(Q) Lift is accelerating vertically down with an acceleration less than the gravitational acceleration.	(2) d > 1.2 m
(R) Lift is moving vertically up with constant speed.	(3) d < 1.2 m
(S) Lift is falling freely.	(4) No water leaks out of the jar

Code :

(A) P-2, Q-3, R-2, S-4 (B) P-2, Q-3, R-1, S-4
(C) P-1, Q-1, R-1, S-4 (D) P-2, Q-3, R-1, S-1

FM0159

18. Two spheres P and Q of equal radii have densities ρ_1 and ρ_2, respectively. The spheres are connected by a massless string and placed in liquids L_1 and L_2 of densities σ_1 and σ_2 and viscosities η_1 and η_2, respectively. They float in equilibrium with the sphere P in L_1 and sphere Q in L_2 and the string being taut (see figure). If sphere P alone in L_2 has terminal velocity $\vec{V}_P$ and Q alone in L_1 has terminal velocity $\vec{V}_Q$, then **[JEE Advanced-2015]**



(A) $\frac{|\vec{V}_P|}{|\vec{V}_Q|} = \frac{\eta_1}{\eta_2}$ (B) $\frac{|\vec{V}_P|}{|\vec{V}_Q|} = \frac{\eta_2}{\eta_1}$ (C) $\vec{V}_P \cdot \vec{V}_Q > 0$ (D) $\vec{V}_P \cdot \vec{V}_Q < 0$

FM0160

19. Consider two solid spheres P and Q each of density 8 gm cm^{-3} and diameters 1 cm and 0.5 cm, respectively. Sphere P is dropped into a liquid of density 0.8 gm cm^{-3} and viscosity $\eta = 3$ poiseulles. Sphere Q is dropped into a liquid of density 1.6 gm cm^{-3} and viscosity $\eta = 2$ poiseulles. The ratio of the terminal velocities of P and Q is. **[JEE Advanced-2016]**

FM0161

20. A drop of liquid of radius $R = 10^{-2}$ m having surface tension $S = \frac{0.1}{4\pi} Nm^{-1}$ divides itself into K identical drops. In this process the total change in the surface energy $\Delta U = 10^{-3}$ J. If $K = 10^{\alpha}$ then the value of α is **[JEE Advanced-2017]**

FM0162

21. A uniform capillary tube of inner radius r is dipped vertically into a beaker filled with water. The water rises to a height h in the capillary tube above the water surface in the beaker. The surface tension of water is σ. The angle of contact between water and the wall of the capillary tube is θ. Ignore the mass of water in the meniscus. Which of the following statements is (are) true? **[JEE Advanced-2018]**

(A) For a given material of the capillary tube, h decreases with increase in r

(B) For a given material of the capillary tube, h is independent of σ.

(C) If this experiment is performed in a lift going up with a constant acceleration, then h decreases.,

(D) h is proportional to contact angle θ.

FM0163

22. Consider a thin square plate floating on a viscous liquid in a large tank. The height h of the liquid in the tank is much less than the width of the tank. The floating plate is pulled horizontally with a constant velocity u_0. Which of the following statements is (are) true ? **[JEE Advanced-2018]**

(A) The resistive force of liquid on the plate is inversely proportional to h

(B) The resistive force of liquid on the plate is independent of the area of the plate

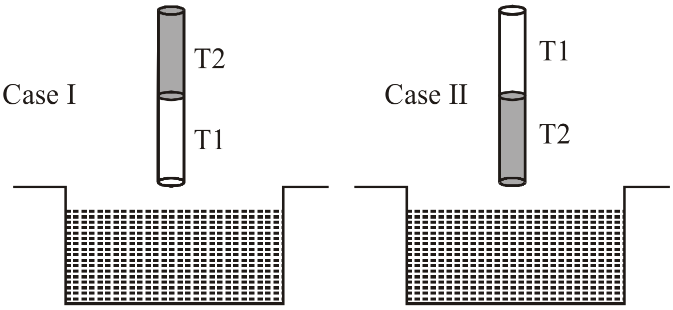
(C) The tangential (shear) stress on the floor of the tank increases with u_0.

(D) The tangential (shear) stress on the plate varies linearly with the viscosity η of the liquid.

FM0164

23. A cylindrical capillary tube of 0.2 mm radius is made by joining two capillaries T1 and T2 of different materials having water contact angles of 0° and 60°, respectively. The capillary tube is dipped vertically in water in two different configurations, case I and II as shown in figure. Which of the following option(s) is(are) correct ? **[JEE Advanced-2019]**

(Surface tension of water = 0.075 N/m, density of water = 1000 kg/m^3, take g = 10 m/s^2)



(1) The correction in the height of water column raised in the tube, due to weight of water contained in the meniscus, will be different for both cases.

(2) For case I, if the capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be more than 8.75 cm. (Neglect the weight of the water in the meniscus)

(3) For case I, if the joint is kept at 8 cm above the water surface, the height of water column in the tube will be 7.5 cm. (Neglect the weight of the water in the meniscus)

(4) For case II, if the capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be 3.75 cm. (Neglect the weight of the water in the meniscus)

FM0165

ANSWER KEY

EXERCISE (S-1)

1. Ans. 45°, $9600\sqrt{2}$ Pa **2. Ans.** $\frac{L}{2}\rho(g+a)$ **3. Ans.** $P_0 - \rho a\ell$ **4. Ans.** 37.5 N

5. Ans. 8 **6. Ans.** 2 **7. Ans.** $H = \frac{L^2\omega^2}{2g}$ **8. Ans.** $\frac{5d}{4}$ **9. Ans.** (a) 5, (b) 2/3

10. Ans. 45° **11. Ans.** nH **12. Ans.** 4 **13. Ans.** 2.52 cm **14. Ans.** $h = \frac{mg + 4sa}{\rho_w a^2 g}$

15. Ans. 20 N **16. Ans.** 4.9 litre/min **17. Ans.** 6.43×10^{-4} m³/s **18. Ans.** 5 cm

19. Ans. 11 m/s **20. Ans.** $\frac{h_2 + h_1}{2}$ **21. Ans.** 108 **22. Ans.** 20.4 m **23. Ans.** 5 s **24. Ans.** 5

25. Ans. $\frac{dQ}{dt} \propto r^5$

EXERCISE (S-2)

1. Ans. 2.79 gm/cc **2. Ans.** 1.9 **3. Ans.** $R\sqrt{\frac{3}{2}}$ **4. Ans.** 2

5. Ans. $h = \frac{2m}{\pi}$, $H = \frac{11}{32\pi}$ m **6. Ans.** 5 **7. Ans.** x = 2 m

8. Ans. $h_2 = 3h_1$ **9. Ans.** + 29625 J/m³, – 30000 J/m³ **10. Ans.** 4

11. Ans. 8 **12. Ans.** (a) $\left[\frac{dgRH^2}{3}\right]$, (b) $\frac{dg\pi R^2 H}{6}$ **13. Ans.** 0.1 cm

14. Ans. 3 : 9 : 5 **15. Ans.** $\sqrt{\frac{18g}{19a}}$ **16. Ans.** 4.5m **17. Ans.** $\left(\frac{R}{24d}\right)^{\frac{1}{3}}$

18. Ans. 80.72 **19. Ans.** $\frac{\lambda a}{2y}g$ **20. Ans.** 2 **21. Ans.** $v = \sqrt{2g\left[h_1 + h_2\left(\frac{\rho_2}{\rho_1}\right)\right]}$

22. Ans. 5

23.Ans. (a) 10 m/s, (b) 2.5 m/s, (c) 4.688 m , (d) 3.14×10^{-3} m³/s **24. Ans.** 6

25. Ans. 4

EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

1. Ans. (C) 2. Ans. (C) 3. Ans. (C) 4. Ans. (C) 5. Ans. (A) 6. Ans. (D)

7. Ans. (C) 8. Ans. (C) 9. Ans. (D) 10. Ans. (D) 11. Ans. (A) 12. Ans. (B)

13. Ans. (B) 14. Ans. (D) 15. Ans. (A) 16. Ans. (B) 17. Ans. (D) 18. Ans. (A)

19. Ans. (B) 20. Ans. (B) 21. Ans. (A) 22. Ans. (B) 23. Ans. (A) 24. Ans. (C)

25. Ans. (B) 26. Ans. (D) 27. Ans. (C) 28. Ans. (C) 29. Ans. (C) 30. Ans. (C)

31. Ans. (A) 32. Ans. (D) 33. Ans. (B) 34. Ans. (B) 35. Ans. (C) 36. Ans. (A)

37. Ans. (C) 38. Ans. (D) 39. Ans. (B) 40. Ans. (B) 41. Ans. (B) 42. Ans. (A)

43. Ans. (B) 44. Ans. (D) 45. Ans. (A) 46. Ans. (A) 47. Ans. (C)

MULTIPLE CORRECT TYPE QUESTIONS

48. Ans. (A,D) 49. Ans. (B, C) 50. Ans. (A,C) 51. Ans. (A,C)

MATRIX MATCH TYPE QUESTION

52. Ans. (A)-R; (B)-Q; (C)-Q; (D)-Q 53. Ans. (A)-P,T; (B)-P,S; (C)-Q,T; (D)-R

EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

1. Ans. (D) 2. Ans. (D) 3. Ans. (A) 4. Ans. (B) 5. Ans. (A) 6. Ans. (A)

7. Ans. (C) 8. Ans. (A) 9. Ans. (B) 10. Ans. (B) 11. Ans. (B) 12. Ans. (D)

13. Ans. (D) 14. Ans. (D) 15. Ans. (C) 16. Ans. (D) 17. Ans. (D) 18. Ans. (A)

19. Ans. (D) 20. Ans. (B) 21. Ans. (D) 22. Ans. (D) 23. Ans. (D) 24. Ans. (D)

25. Ans. (C) 26. Ans. (D) 27. Ans. (C) 28. Ans. (C)

MULTIPLE CORRECT TYPE QUESTIONS

29. Ans. (B, C, D)

COMPREHENSION TYPE QUESTIONS

30. Ans. (C) 31. Ans. (B) 32. Ans. (A)

MATRIX MATCH TYPE QUESTION

33. Ans. (A)-Q, (B)-R, (C)-P, (D)-S 34. Ans. (A)-P,T; (B)-Q,R,S; (C)-Q,R,S,T; (D)-P

EXERCISE (JM)

1. Ans. (1) 2. Ans. (4) 3. Ans. (3) 4. Ans. (3) 5. Ans. (2) 6. Ans. (2)

7. Ans. (4) 8. Ans. (2) 9. Ans. (1) 10. Ans. (3) 11. Ans. (4) 12. Ans. (3)

13. Ans. (1)

EXERCISE (JA)

1. Ans. (A) 2. Ans. (B) 3. Ans. (D) 4. Ans. (B) 5. Ans. (B) 6. Ans. 6

7. Ans. 6 8. Ans. (C) 9. Ans. (A) 10. Ans. (B) 11. Ans. (A,B,D)

12. Ans. (A) 13. Ans. (A, D) 14. Ans. (D) 15. Ans. (C) 16. Ans. (A)

17. Ans. (C) 18. Ans. (A, D) 19. Ans. 3 20. Ans. 6 21. Ans. (A,C)

22. Ans. (A,C,D) 23. Ans. (1,3,4)

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Chapter Contents

Simple Harmonic Motion

SIMPLE HARMONIC MOTION

KEY CONCEPTS

PERIODIC MOTION

- Any motion which repeats itself after regular interval of time is called periodic motion.
- The constant interval of time after which the motion is repeated is called time period.

Examples : (i) Motion of planets around the sun. (ii) Motion of the pendulum of wall clock.

OSCILLATORY MOTION

- The motion of body is said to be oscillatory if it moves back and forth (to and fro) about a fixed point. Oscillation of very high frequency & small amplitude is called vibration.
- The fixed point about which the body oscillates is called mean position or equilibrium position.

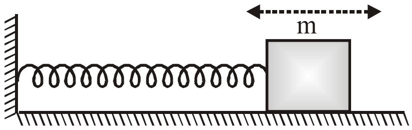
Examples : (i) Vibration of the wire of 'Sitar'. (ii) Oscillation of the mass suspended from spring.

SIMPLE HARMONIC MOTION (S.H.M.)

Simple harmonic motion is the simplest form of oscillatory motion.

(i) S.H.M. are of two types

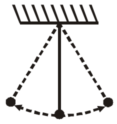
- **Linear S.H.M. :** When a particle moves to and fro about a fixed point (called equilibrium position) along a straight line then its motion is called linear simple harmonic motion.



Example : Motion of a mass connected to spring.

- **Angular S.H.M.**

When a system oscillates angularly with respect to a fixed axis then its motion is called angular simple harmonic motion.
Example :- Motion of a bob of simple pendulum.



(ii) Necessary Condition to execute S.H.M.

- **In linear S.H.M. :** The restoring force (or acceleration) acting on the particle should always be proportional to the displacement of the particle and directed towards the equilibrium position

$$\therefore \quad F \propto -x \quad \text{or} \quad a \propto -x$$

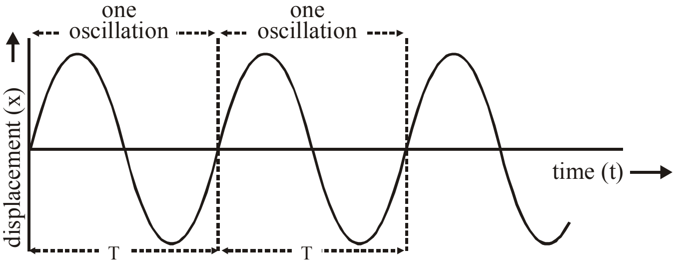
Negative sign shows that direction of force and acceleration is towards equilibrium position and x is displacement of particle from equilibrium position.

- **In angular S.H.M. :** The restoring torque (or angular acceleration) acting on the particle should always be proportional to the angular displacement of the particle and directed towards the equilibrium position

$$\therefore \quad \tau \propto -\theta \quad \text{or} \quad \alpha \propto -\theta$$

SOME BASIC TERMS

- **Mean Position :** The point at which the restoring force on the particle is zero and potential energy is minimum, is known as its mean position.
- **Restoring Force**
  - The force acting on the particle which tends to bring the particle towards its mean position, is known as restoring force.
  - Restoring force always acts in a direction opposite to that of displacement. Displacement is measured from the mean position.
- **Amplitude :** The maximum value of displacement of particle from mean position is define as amplitude.
- **Time period (T)**
  - The minimum time after which the particle keeps on repeating its motion is known as time period.
  - The smallest time taken to complete one oscillation or vibration is also define as time period.
  - It is given by $T = \frac{2\pi}{\omega} = \frac{1}{n}$ where ω is angular frequency and n is frequency.
- **Oscillation :** When a particle goes on one side from mean position and returns back and then it goes to other side and again returns back to mean position, then this process is known as one oscillation.



- **Frequency (n or f)**
  - The number of oscillations per second is define as frequency.
  - It is given by $n = \frac{1}{T}$, $n = \frac{\omega}{2\pi}$
  - **SI UNIT :** hertz (Hz), 1 hertz = 1 cycle per second.
  - **Dimensions :** $M^0 L^0 T^{-1}$.
- **Phase** :
  - Phase of a vibrating particle at any instant is the state of the vibrating particle regarding its displacement and direction of vibration at that particular instant.
  - In the equation $x = A \sin(\omega t + \theta)$, $(\omega t + \theta)$ is the phase of the particle.
  - The phase angle at time t = 0 is known as initial phase.
  - The difference of total phase angles of two particles executing S.H.M. with respect to the mean position is known as phase difference.
  - Two vibrating particles are said to be in same phase if the phase difference between them is an even multiple of π, i.e., $\Delta\phi = 2n\pi$ Where n = 0, 1, 2, 3,....
  - Two vibrating particle are said to be in opposite phase if the phase difference between them is an odd multiple of π i.e., $\Delta\phi = (2n + 1)\pi$ Where n = 0, 1, 2, 3,....

- **Angular frequency (ω)** :
  The rate of change of phase angle of a particle with respect to time is define as its angular frequency.
  SI unit **:** radian/second, *Dimensions* **:** $M^0 L^0 T^{-1}$,

DISPLACEMENT IN S.H.M.

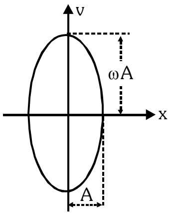
(i) The displacement of a particle executing linear S.H.M. at any instant is defined as the distance of the particle from the mean position at that instant.

(ii) It can be given by relation $x = A\sin\omega t$ or $x = A\cos\omega t$.
The first relation is valid when the time is measured from the mean position and the second relation is valid when the time is measured from the extreme position of the particle executing S.H.M. along a straight line path.

VELOCITY IN SHM

(i) It is define as the time rate of change of the displacement of the particle at the given instant.

(ii) Velocity in S.H.M. is given by $v = \frac{dx}{dt} = \frac{d}{dt}(A\sin\omega t) \Rightarrow v = A\omega\cos\omega t$



$$v = \pm A\omega\sqrt{1-\sin^2\omega t} \Rightarrow v = \pm A\omega\sqrt{1-\frac{x^2}{A^2}} = \pm\omega\sqrt{(A^2-x^2)}\ [\because\ x = A\sin\omega t]$$

Squaring both the sides $v^2 = \omega^2 (A^2 - x^2) \Rightarrow \frac{v^2}{\omega^2} = A^2 - x^2 \Rightarrow \frac{v^2}{\omega^2 A^2} = 1 - \frac{x^2}{A^2} \Rightarrow \frac{x^2}{A^2} + \frac{v^2}{A^2\omega^2} = 1$

This is equation of ellipse. So curve between displacement and velocity of particle executing S.H.M. is ellipse.

(iii) The graph between velocity and displacement is shown in figure. If particle oscillates with unit angular frequency ($\omega = 1$) then curve between v and x will be circular.

Note :

(i) The direction of velocity of a particle in S.H.M. is either towards or away from the mean position.

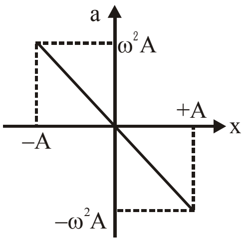
(ii) At mean position ($x = 0$), velocity is maximum ($=A\omega$) and at extreme position ($x = \pm A$), the velocity of particle executing S.H.M. is zero (minimum).

ACCELERATION IN SHM

(i) It is define as the time rate of change of the velocity of the particle at given instant.

(ii) Acceleration in S.H.M. is given by $a = \frac{dv}{dt} = \frac{d}{dt}(A\omega\cos\omega t)$

$a = -\omega^2 A \sin\omega t \Rightarrow \quad a = -\omega^2 x$



(iii) The graph between acceleration and displacement as shown in figure

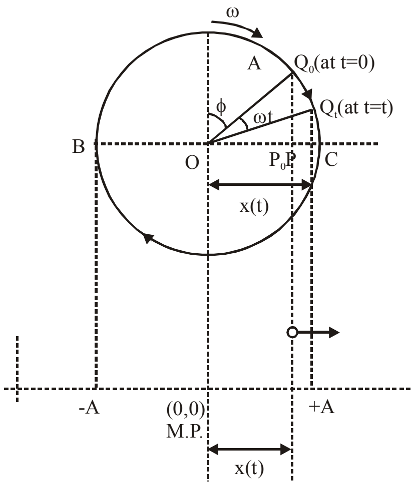
Note

(i) The acceleration of a particle executing S.H.M. is always directed towards the mean position.

(ii) The acceleration of the particle executing S.H.M. is maximum at extreme position ($= \omega^2 A$) and minimum at mean position (= zero)

SHM AS A PROJECTION OF UNIFORM CIRCULAR MOTION

Consider a particle Q, moving on a circle of radius A with constant angular velocity ω. The projection of Q on a diameter BC is P. It is clear from the figure that as Q moves around the circle the projection P executes a simple harmonic motion on the x-axis between B and C. The angle that the radius OQ makes with the +ve vertical in clockwise direction in at t = 0 is equal to phase constant (ϕ).



Let the radius OQ_0 makes an angle ωt with the OQ_t at time t. Then $x(t)=A\sin(\omega t+\phi)$

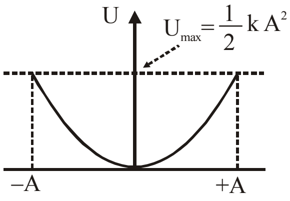
In the above discussion the foot of projection is x-axis so it is called horizontal phasor. Similarly the foot of perpendiuclar on y-axis will also executes SHM of amplitude A and angular frequency ω[$y(t)=A\cos\omega t$]. This is called vertical phasor. The phaser of the two SHM differ by π/2.

ENERGY OF PARTICLE IN S.H.M.

- **Potential Energy (U or P.E.)**

(i) In terms of displacement

The potential energy is related to force by the relation $F=-\dfrac{dU}{dx} \Rightarrow \int dU=-\int F dx$

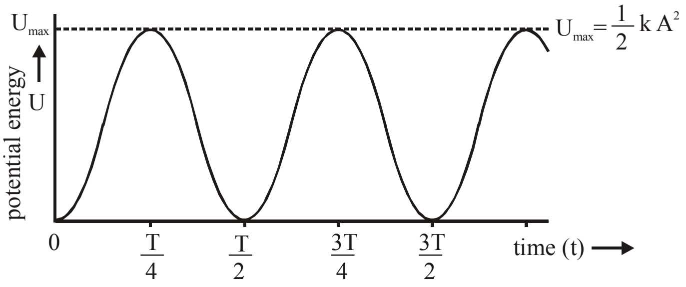


For S.H.M. F = – kx so $\int dU=-\int(-kx)dx=\int kx\,dx \Rightarrow U=\frac{1}{2}kx^2+C$

At x = 0, $U=U_0 \Rightarrow C=U_0$ So $U=\frac{1}{2}kx^2+U_0$

Where the potenital energy at equilibrium position = U_0 when $U_0=0$ then $U=\frac{1}{2}kx^2$

(ii) In terms of time



Since $x = A\sin(\omega t + \phi)$, $U = \frac{1}{2}kA^2\sin^2(\omega t + \phi)$

If initial phase (ϕ) is zero then $U = \frac{1}{2}kA^2\sin^2\omega t = \frac{1}{2}m\omega^2A^2\sin^2\omega t$

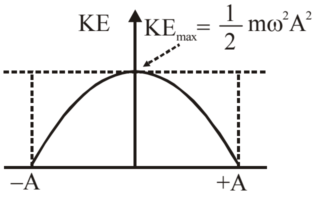
Note :

(i) In S.H.M. the potential energy is a parabolic function of displacement, the potential energy is minimum at the mean position ($x = 0$) and maximum at extreme position ($x = \pm A$)

(ii) The potential energy is the periodic function of time. For $x = A \sin(\omega t)$, it is minimum at $t = 0, \frac{T}{2}, T, \frac{3T}{2}...$ and maximum at $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}...$

- **Kinetic Energy (K)**

(i) In terms of displacement



If mass of the particle executing S.H.M. is m and its velocity is v then kinetic energy at any instant.

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{2}k(A^2 - x^2)$$

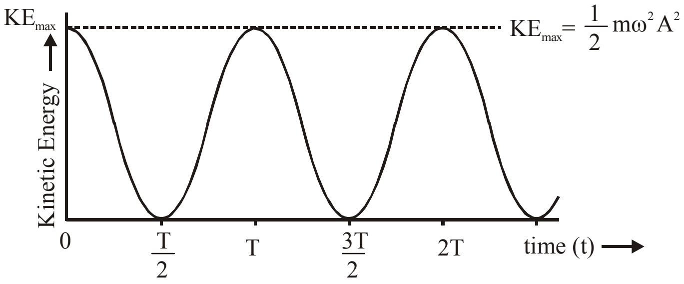
(ii) In terms of time

$v = A\omega\cos(\omega t + \phi)$

$K = \frac{1}{2}m\omega^2A^2\cos^2(\omega t + \phi)$

If initial phase ϕ is zero

$K = \frac{1}{2}m\omega^2A^2\cos^2\omega t$



Note :

(i) In S.H.M. the kinetic energy is a inverted parabolic function of displacement. The kinetic energy is maximum ($\frac{1}{2}kA^2$) at mean position ($x = 0$) and minimum (zero) at extreme position ($x = \pm A$)

(ii) The kinetic energy is the periodic function of time. For $x = A \sin(\omega t)$, it is maximum at $t = 0$, T, 2T, 3T.............and minimum at $t = \frac{T}{2}, \frac{3T}{2}, \frac{5T}{2}...$

- **Total energy (E)**

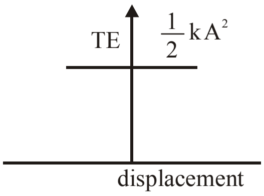
Total energy in S.H.M. is given by ; E = potential energy + kinetic energy = U + K

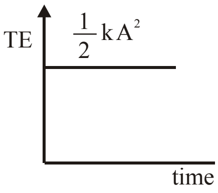
(i) w.r.t. position $E = \frac{1}{2}kx^2 + \frac{1}{2}k(A^2 - x^2) \Rightarrow E = \frac{1}{2}kA^2 = \text{constant}$

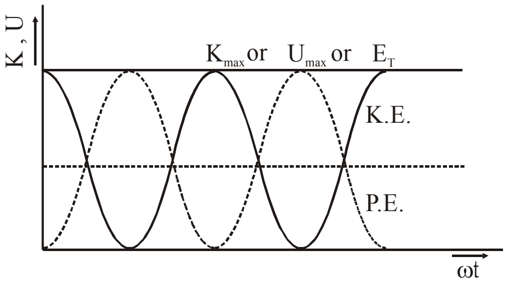
(ii) w.r.t. time

$$E = \frac{1}{2}m\omega^2A^2\sin^2\omega t + \frac{1}{2}m\omega^2A^2\cos^2\omega t = \frac{1}{2}m\omega^2A^2\left(\sin^2\omega t + \cos^2\omega t\right) = \frac{1}{2}m\omega^2A^2$$

$$= \frac{1}{2}kA^2 = \text{constant}$$







Note :

(i) Total energy of a particle in S.H.M. is same at all instant and at all displacement.

(ii) Total energy depends upon mass, amplitude and frequency of vibration of the particle executing S.H.M.

SIMPLE PENDULUM

If a heavy point mass is suspended by a weightless, inextensible string from a rigid support, then this arrangement is called a simple pendulum

- **Second's pendulum**

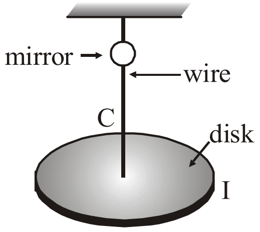
If the time period of a simple pendulum is 2 second then it is called second's pendulum. Second's pendulum take one second to go from one extreme position to other extreme position.

COMPOUND PENDULUM

Any rigid body which is free to oscillate in a vertical plane about a horizontal axis passing through a point, is define compound pendulum

- **Torsional Oscillator :** (Angular SHM)

$T = 2\pi\sqrt{\frac{I}{C}}$ where $C = \frac{\eta\pi r^4}{2\ell}$



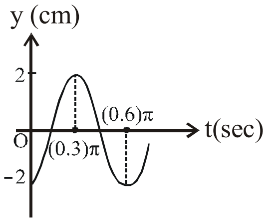
η = modulus of elasticity of the wire ; r = radius of the wire

L = length of the wire ; I = Moment of inertia of the disc

EXERCISE (S-1)

Kinematics of SHM :

1. Part of a simple harmonic motion is graphed in the figure, where y is the displacement from the mean position. The correct equation describing this S.H.M is :-



SH0001

2. The displacement of a body executing SHM is given by $x = A \sin(2\pi t + \pi/3)$. The first time from $t = 0$ when the velocity is maximum is.

SH0002

3. A body undergoing SHM about the origin has its equation given by $x = 0.2 \cos 5\pi t$. Find its average speed from $t = 0$ to $t = 0.7$ sec.

SH0003

Energy of SHM :

4. An object of mass 0.2 kg executes SHM along the x-axis with frequency of $(25/\pi)$ Hz. At the point $x = 0.04$m the object has KE 0.5 J and PE 0.4 J. The amplitude of oscillation is ________.

SH0004

5. A point particle of mass 0.1kg is executing SHM with amplitude of 0.1m. When the particle passes through the mean position, its K.E. is 8×10^{-3}J. Obtain the equation of motion of this particle if the initial phase of oscillation is 45°.

SH0005

6. Potential Energy (U) of a body of unit mass moving in a one-dimension conservative force field is given by $U = (x^2 - 4x + 3)$. All units are in S.I.

(i) Find the equilibrium position of the body.

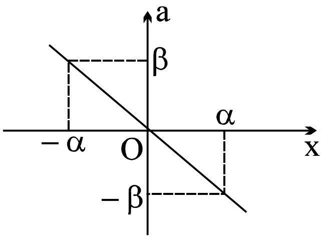
(ii) Show that oscillations of the body about this equilibrium position is simple harmonic motion & find its time period.

(iii) Find the amplitude of oscillations if speed of the body at equilibrium position is $2\sqrt{6}$ m/s.

SH0006

Time Period :

7. The acceleration-displacement (a – x) graph of a particle executing simple harmonic motion is shown in the figure. Find the frequency of oscillation.



SH0007

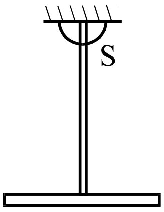
8. A small body of mass m is fixed to the middle of a stretched string of length 2ℓ. In the equilibrium position the string tension is equal to T_0. Find the angular frequency of small oscillations of the body in the transverse direction. The mass of the string is negligible, the gravitational field is absent. Assume tension in string to be constant.

SH0008

9. A body is in SHM with period T when oscillated from a freely suspended spring. If this spring is cut in two parts of length ratio 1 : 3 & again oscillated from the two parts separately, then the periods are T_1 & T_2 then find T_1/T_2.

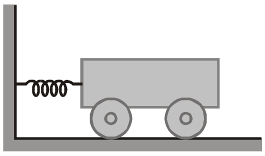
SH0009

10. Two identical rods each of mass m and length L, are rigidly joined and then suspended in a vertical plane so as to oscillate freely about an axis normal to the plane of paper passing through 'S' (point of suspension). Find the time period of such small oscillations.



SH0010

11. A cart consists of a body and four wheels on frictionless axles. The body has a mass m. The wheels are uniform disks of mass M and radius R. The cart rolls, without slipping, back and forth on a horizontal plane under the influence of a spring attached to one end of the cart (figure). The spring constant is k. Taking into account the moment of inertia of the wheels, find a formula for the frequency of the back and forth motion of the cart.



SH0011

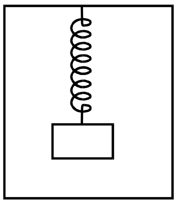
12. A mass M attached to a spring, oscillates with a period of 2 sec. If the mass is increased by 2 kg the period increases by one sec. Find the initial mass M assuming that Hooke's Law is obeyed.

SH0012

Complex situations :

13. A spring mass system is hanging from the ceiling of an elevator in equilibrium. Elongation of spring is l. The elevator suddenly starts accelerating downwards with acceleration g/3, find

(a) the frequency and (b) the amplitude of the resulting SHM.



SH0013

14. (a) Find the time period of oscillations of a torsional pendulum, if the torsional constant of the wire is $K = 10\pi^2$ J/rad. The moment of inertia of rigid body is 10 kg m^2 about the axis of rotation.

(b) A simple pendulum of length $l = 0.5$ m is hanging from ceiling of a car. The car is kept on a horizontal plane. The car starts accelerating on the horizontal road with acceleration of 5 m/s^2. Find the time period of oscillations of the pendulum for small amplitudes about the mean position.

SH0014

15. The motion of a simple pendulum is given by

$$\theta = A\cos\left(\sqrt{\frac{g}{\ell}}\, t\right) \text{ (symbols have their usual meaning)}$$

(a) Find the tension in the string of this pendulum as a function of time assume that $\theta << \ell$.

(b) At what time is the tension maximum? What is the value of this maximum tension?

SH0015

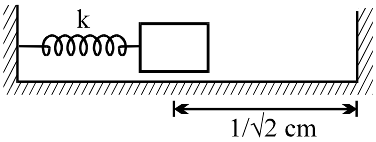
16. A physical pendulum has the shape of a disk of radius R. The pendulum swings about an axis perpendicular to the plane of the disk and at distance ℓ from the center of the disk.

(a) Show that the frequency of the oscillations of this pendulum is $\omega = \sqrt{\dfrac{g\ell}{\frac{1}{2}R^2 + \ell^2}}$

(b) For what value of ℓ is this frequency at a maximum ?

SH0016

17. A block of mass 0.9 kg attached to a spring of force constant k is lying on a frictionless floor. The spring is compressed to $\sqrt{2}$ cm and the block is at a distance $1/\sqrt{2}$ cm from the wall as shown in the figure. When the block is released, it makes elastic collision with the wall and its period of motion is 0.2 sec. Find the approximate value of k.

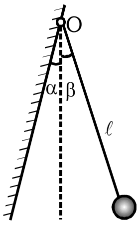


SH0017

18. A body of mass 1 kg is suspended from a weightless spring having force constant 600N/m. Another body of mass 0.5 kg moving vertically upwards hits the suspended body with a velocity of 3.0m/s and get embedded in it. Find the frequency of oscillations and amplitude of motion.

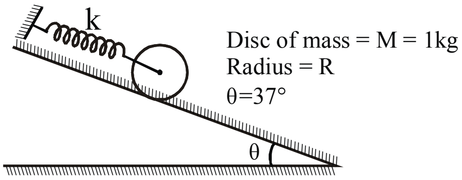
SH0018

19. A ball is suspended by a thread of length ℓ at the point O on the wall, forming a small angle α with the vertical as shown in figure. Then the thread with the ball was deviated through a small angle β ($\beta > \alpha$) and set free. Assuming the collision of the ball against the wall to be perfectly elastic, find the oscillation period of such a pendulum.



SH0019

20. A disc of mass m is connected to an ideal spring of force constant 'k'. If disc is released from rest, then what is maximum friction force on disc (in N). Assuming friction is sufficient for pure rolling

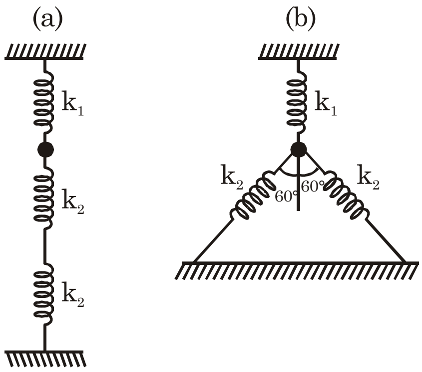


SH0020

21. A particle of mass 5×10^{-5} kg is placed at the lowest point of a smooth parabola having the equation $20x^2 = y$ (x,y in m). Here y is the vertical height. If it is displaced slightly and it moves such that it is constrained to move along the parabola, the angular frequency of oscillation will be, (in rad/s). If your answer is N fill value N/4.

SH0021

22. Mass m is suspended by ideal massless springs in two different ways, indicated by (a) and (b) in the figure. The mass is displaced upwards by a small amount from equilibrium position and is then released resulting in a SHM of the mass in the vertical direction. We denote the oscillation frequencies associated with the two cases (a) and (b) by f_a and f_b respectively. Find $\frac{f_a}{f_b}$. Given $k_2 = 2k_1$.



SH0022

23. A solid sphere of radius R is floating in a liquid of density ρ with half of its volume submerged. If the sphere is slightly pushed and released, it starts performing simple harmonic motion. Find the frequency of these oscillations. **[IIT-JEE 2004]**

SH0023

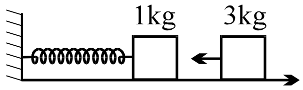
EXERCISE (S-2)

1. One end of an ideal spring is fixed to a wall at origin O and the axis of spring is parallel to x-axis. A block of mass m = 1 kg is attached to free end of the spring and it is performing SHM. Equation of position of block in coordinate system shown is $x = 10 + 3\sin 10t$, t is in second and x in cm. Another block of mass M = 3kg, moving towards the origin with velocity 30cm/s collides with the block performing SHM at t = 0 and gets struck to it, calculate :

(i) new amplitude of oscillations.

(ii) new equation for position of the combined body.

(iii) loss of energy during collision. Neglect friction.



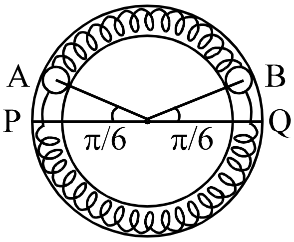
SH0024

2. Two identical balls A and B each of mass 0.1 kg are attached to two identical massless springs. The spring mass system is constrained to move inside a rigid smooth pipe in the form of a circle as in fig. The pipe is fixed in a horizontal plane. The centres of the ball can move in a circle of radius 0.06m. Each spring has a natural length 0.06π m and force constant 0.1N/m. Initially both the balls are displaced by an angle of $\theta = \pi/6$ radian with respect to diameter PQ of the circle and released from rest

(a) Calculate the frequency of oscillation of the ball B.

(b) What is the total energy of the system.

(c) Find the speed of the ball A when A and B are at the two ends of the diameter PQ.



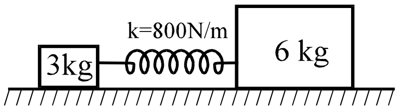
SH0025

3. The system shown in the figure can move on a smooth surface. The spring is initially compressed by 6 cm and then released. Find

(a) time period;

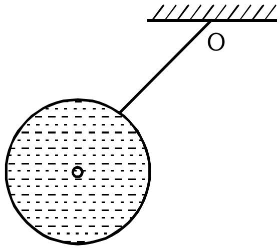
(b) amplitude of 3 kg block and

(c) maximum momentum of 6 kg block



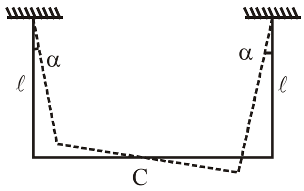
SH0026

4. A pendulum is constructed as a light thin-walled sphere of radius R filled up with water and suspended at the point O from a light rigid rod. The distance between the point O and the centre of the sphere is equal to ℓ. How many times will the small oscillations of such a pendulum change after the water freezes? The viscosity of water and the change of its volume on freezing are to be neglected.



SH0027

5. A uniform rod of mass m = 1.5 kg suspended by two identical threads ℓ = 90 cm in length was turned through a small angle about the vertical axis passing through its middle point C. The threads deviated in the process through an angle α = 5.0°. Then the rod was released to start performing small oscillations. Find:
(a) the oscillation period;
(b) the rod's oscillation energy.

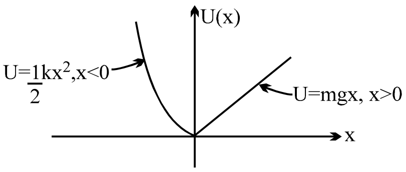


SH0028

6. Two physical pendulums perform small oscillations about the same horizontal axis with frequencies ω_1 and ω_2. Their moments of inertia relative to the given axis are equal to I_1 and I_2 respectively. In a state of stable equilibrium the pendulums were fastened rigidly together. What will be the frequency of small oscillations of the compound pendulum?

SH0029

7. A particle of mass *m* moves in the potential energy U shown above. Find the period of the motion when the particle has total energy E.



SH0030

8. A particle of mass 4 kg moves between two points A and B on a smooth horizontal surface under the action of two forces such that when it is at a point P, the forces are $2\overrightarrow{PA}$ N and $2\overrightarrow{PB}$ N. If the particle is released from rest at A, find the time it takes to travel a quarter of the way from A to B.

SH0031

9. Two masses m_1 and m_2 connected by a light spring of natural length ℓ_0 is compressed completely and tied by a string. This system while moving with a velocity v_0 along +ve *x*-axis pass through the origin at $t = 0$. At this position the string snaps. Position of mass m_1 at time *t* is given by the equation

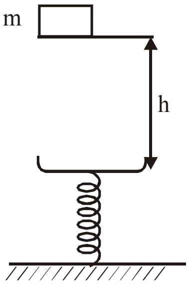
$$x_1(t) = v_0 t - A\,(1 - \cos\omega t)$$

Calculate : **[IIT JEE 2003]**

(i) Position of the particle m_2 as a function of time. (ii) ℓ_0 in terms of A.

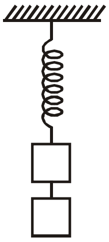
SH0032

10. A body of mass m fell from a height h onto the pan of a spring balance. The masses of the pan and the spring are negligible, the stiffness of the latter is x. Having stuck to the pan, the body starts performing harmonic oscillations in the vertical direction. Find the amplitude and the energy of these oscillations.



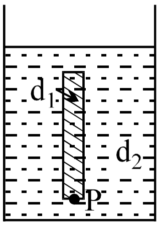
SH0033

11. Two small bodies of mass 2kg each hang on a spring of force constant of 200 N/m, attached to each other using a thread with a length of 10 cm, as shown in the figure. We burn the thread. Find the distance between the two bodies (in cm) when the top body first arrives at its highest position? If the distance comes out to be 16 × n. Fill n [Take: $g = 10\ m/s^2$ and $\pi^2 = 10$]



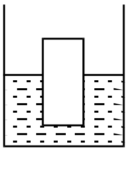
SH0034

12. A thin rod of length L & area of cross-section S is pivoted at its lowest point P inside a stationary, homogeneous & non-viscous liquid (Figure). The rod is free to rotate in a vertical plane about a horizontal axis passing through P. The density d_1 of the material of the rod is smaller than the entity d_2 of the liquid. The rod is displaced by a small angle θ from its equilibrium position and then released. Show that the motion of the rod is simple harmonic and determine its angular frequency in terms of the given parameters.



SH0035

13. A liquid of density 2ρ is filled in a cylindrical vessel, whose cross-sectional area is 2A. A wooden cylinder of height H, cross-sectional area A and density ρ is floating in the liquid at equilibrium with its axis vertical. The cylinder is pushed down by a small distance x from its equilibrium position and released. Find its initial acceleration.



SH0036

EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

Kinematics of SHM :

1. The equation of motion of a particle is $x = a\cos(\alpha t)^2$. The motion is

(A) periodic but not oscillatory. (B) periodic and oscillatory.

(C) oscillatory but not periodic. (D) neither periodic nor oscillatory.

SH0037

2. **Statement 1** : Position–time equation of a particle moving along x–axis is $x = 4 + 6\sin\omega t$. Under this situation, motion of particle is not simple harmonic.

and

Statement 2 : $\frac{d^2x}{dt^2}$ for the given equation is not proportional to $-x$.

(A) Statement–1 is True, Statement–2 is True ; Statement–2 is a correct explanation for Statement–1.

(B) Statement–1 is True, Statement–2 is True ; Statement–2 is not a correct explanation for Statement–1.

(C) Statement–1 is True, Statement–2 is False.

(D) Statement–1 is False, Statement–2 is True.

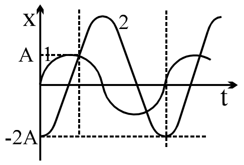
SH0038

3. A simple harmonic motion having an amplitude A and time period T is represented by the equation : $y = 5\sin\pi(t + 4)$ m. Then the values of A (in m) and T (in sec) are :

(A) A = 5; T = 2 (B) A = 10 ; T = 1 (C) A = 5 ; T = 1 (D) A = 10 ; T = 2

SH0039

4. The oscillations represented by curve 1 in the graph are expressed by equation $x = A\sin\omega t$. The equation for the oscillations represented by curve 2 is expressed as:

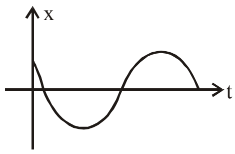


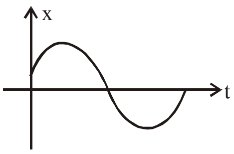
(A) $x = 2A\sin(\omega t - \pi/2)$ (B) $x = 2A\sin(\omega t + \pi/2)$

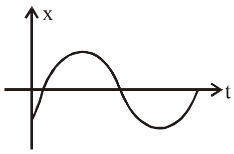
(C) $x = -2A\sin(\omega t - \pi/2)$ (D) $x = A\sin(\omega t - \pi/2)$

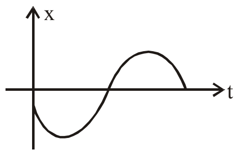
SH0040

5. A particle performing S.H.M. about mean position x = 0 and at t = 0 it is at position $x = \frac{A}{\sqrt{2}}$ and moving towards the origin. Then which of the following is its possible graph between position (x) and time (t)

(A)


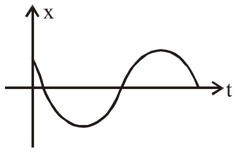
(B)


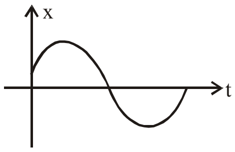
(C)


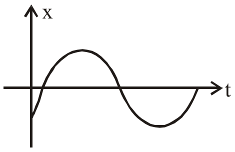
(D)


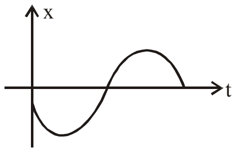
SH0041

6. A particle is performing S.H.M. and at $t = \frac{3T}{4}$, is at position $= \frac{A}{\sqrt{2}}$ and moving towards the origin. Equilibrium position of the particle is at $x = 0$. After $t = \frac{3T}{2}$ what will be the graph of the particle :-

(A) 

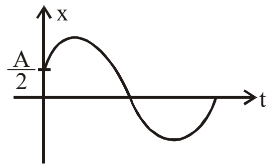
(B) 

(C) 

(D) 

SH0042

7. Following graph shows a particle performing S.H.M. about mean position x = 0. The equation of particle if $t = \frac{T}{4}$ is taken as starting time is (Notations have usual meanings)



(A) $A\sin\left(\omega t + \frac{2\pi}{3}\right)$ (B) $A\sin\left(\omega t + \frac{\pi}{3}\right)$ (C) $A\sin\left(\omega t + \frac{\pi}{6}\right)$ (D) $A\cos\left(\omega t + \frac{2\pi}{3}\right)$

SH0043

8. A particle is performing S.H.M. and at $t = \frac{3T}{4}$, is at position $\frac{\sqrt{3}A}{2}$ and moving towards the origin. Equilibrium position of the particle is at $x = 0$. Then what was the position and direction of particle at $t = 0$?

(A) $-\frac{A}{2}$, away from mean position (B) $\frac{A}{2}$, away from mean position

(C) $\frac{A}{2}$, towards mean position (D) $-\frac{A}{2}$, towards mean position

SH0044

9. The time taken by a particle performing SHM to pass from point A to B where its velocities are same is 2 seconds. After another 2 seconds it returns to B. The time period of oscillation is (in seconds)

(A) 2 (B) 8 (C) 6 (D) 4

SH0045

10. A bob is attached to a long, light string. The string is deflected by 3° initially with respect to vertical. The length of the string is 1 m . The value of θ at any time t after the bob released can be approximately written as (Use : $g = \pi^2$)

(A) $3° \cos \pi t$ (B) $3° \sin \pi t$ (C) $3° \sin (\pi t + \frac{\pi}{6})$ (D) $3° \cos (\pi t + \frac{\pi}{6})$

SH0046

11. A particle performing SHM is found at its equilibrium at t = 1sec. and it is found to have a speed of 0.25 m/s at t = 2 sec. If the period of oscillation is 6 sec. Calculate amplitude of oscillation

(A) $\frac{3}{2\pi}$ m (B) $\frac{3}{4\pi}$ m (C) $\frac{6}{\pi}$ m (D) $\frac{3}{8\pi}$

SH0047

12. A particle performs SHM with a period T and amplitude a. The mean velocity of the particle over the time interval during which it travels a distance a/2 from the extreme position is

(A) a/T (B) 2a/T (C) 3a/T (D) a/2T

SH0048

13. Two particles execute SHM of same amplitude of 20 cm with same period along the same line about the same equilibrium position. The maximum distance between the two is 20 cm. Their phase difference in radians is :-

(A) $\frac{2\pi}{3}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$

SH0049

14. Two particles are in SHM on same straight line with amplitude A and 2A and with same angular frequency ω. It is observed that when first particle is at a distance $A/\sqrt{2}$ from origin and going toward mean position, other particle is at extreme position on other side of mean position. Find phase difference between the two particles

(A) 45° (B) 90° (C) 135° (D) 180°

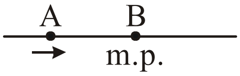
SH0050

15. Two pendulums have time periods T and 5T/4. They start SHM at the same time from the mean position. After how many oscillations of the smaller pendulum they will be again in the same phase:

(A) 5 (B) 4 (C) 11 (D) 9

SH0051

16. A particle is oscillating simple harmonically with angular frequency ω and amplitude A . It is at a point (A) at certain instant (shown in figure). At this instant it is moving towards mean position (B). It takes time t to reach mean position (B). If time period of oscillation is T, the average speed between A and B is :-



(A) $\frac{A\sin\omega t}{t}$ (B) $\frac{A\cos\omega t}{t}$ (C) $\frac{A\sin\omega t}{T}$ (D) $\frac{A\cos\omega t}{T}$

SH0052

17. A particle executes SHM on a straight line path. The amplitude of oscillation is 2 cm. When the displacement of the particle from the mean position is 1 cm, the numerical value of magnitude of acceleration is equal to the numerical value of magnitude of velocity. The frequency of SHM (in second^{-1}) is:

(A) $2\pi\sqrt{3}$ (B) $\frac{2\pi}{\sqrt{3}}$ (C) $\frac{\sqrt{3}}{2\pi}$ (D) $\frac{1}{2\pi\sqrt{3}}$

SH0053

18. Speed v of a particle moving along a straight line, when it is at a distance x from a fixed point on the line is given by $v^2 = 108 - 9x^2$ (all quantities in S.I. unit). Then

(A) The motion is uniformly accelerated along the straight line

(B) The magnitude of the acceleration at a distance 3 cm from the fixed point is 0.27 m/s^2.

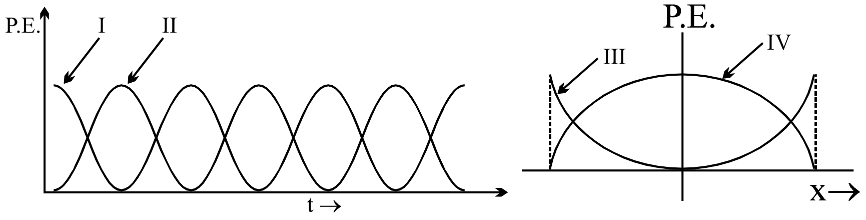
(C) The motion is simple harmonic about $x = \sqrt{12}$ m.

(D) The maximum displacement from the fixed point is 4 cm.

SH0054

Energy of SHM :

19. A particle is executing SHM according to $x = a \cos \omega t$. Then which of the graphs represents variations of potential energy : **[IIT JEE (Scr) 2003]**



(A) (I) & (III) (B) (II) & (IV) (C) (I) & (IV) (D) (II) & (III)

SH0055

20. Potential energy of a particle is given as $U(x) = 2x^3 - 9x^2 + 12x$ where U is in joule and x is in metre. If the motion of a particle is S.H.M., then find the potential energy of the particle at mean position :-

(A) – 36 J (B) 4 J (C) 5 J (D) None of these

SH0056

21. If the potential energy of a harmonic oscillator of mass 2 kg on its equilibrium position is 5 joules and the total energy is 9 joules. If the amplitude is one meter then period of the oscillator (in sec) is:

(A) 1.5 (B) 3.14 (C) 6.28 (D) 4.67

SH0057

22. Kinetic energy of a particle executing simple harmonic motion in straight line is pv^2 and potential energy is qx^2, where v is speed at distance x from the mean position. It time period is given by the expression :-

(A) $2\pi\sqrt{\frac{q}{p}}$ (B) $2\pi\sqrt{\frac{p}{q}}$ (C) $2\pi\sqrt{\frac{q}{p+q}}$ (D) $2\pi\sqrt{\frac{p}{p+q}}$

SH0058

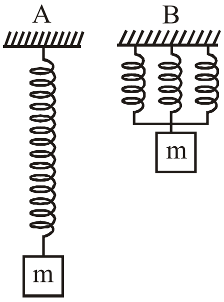
Time Period :

23. The angular frequency of motion whose equation is $4\frac{d^2y}{dt^2} + 9y = 0$ is (y = displacement and t = time)

(A) $\frac{9}{4}$ (B) $\frac{4}{9}$ (C) $\frac{3}{2}$ (D) $\frac{2}{3}$

SH0059

24. The springs in figure A and B are identical but length of spring in A is three times than the length of each spring in B. The ratio of period T_A/T_B is :-



(A) √3 (B) 1/3 (C) 3 (D) 1/√3

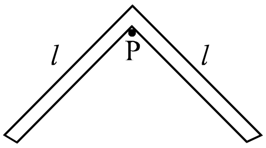
SH0060

25. A rod whose ends are A & B of length 25 cm is hanged in vertical plane. When hanged from point A and point B the time periods calculated are 3 sec & 4 sec respectively. Given the moment of inertia of rod about axis perpendicular to the rod is in ratio 9 : 4 at points A and B. Find the distance of the centre of mass from point A.

(A) 9 cm (B) 5 cm (C) 25 cm (D) 20 cm

SH0061

26. A system of two identical rods (L-shaped) of mass *m* and length *l* are resting on a peg P as shown in the figure. If the system is displaced in its plane by a small angle θ, find the period of oscillations:



(A) $2\pi\sqrt{\frac{\sqrt{2}l}{3g}}$ (B) $2\pi\sqrt{\frac{2\sqrt{2}l}{3g}}$ (C) $2\pi\sqrt{\frac{2l}{3g}}$ (D) $3\pi\sqrt{\frac{l}{3g}}$

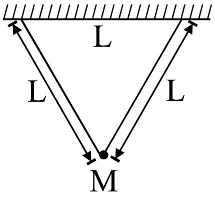
SH0062

27. A ring of diameter 2m oscillates as a compound pendulum about a horizontal axis passing through a point at its rim. It oscillates such that its centre move in a plane which is perpendicular to the plane of the ring. The equivalent length of the simple pendulum is :-

(A) 2m (B) 4m (C) 1.5m (D) 3m

SH0063

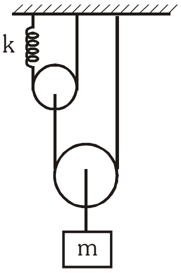
28. A man is swinging on a swing made of 2 ropes of equal length L and in direction perpendicular to the plane of paper. The time period of the small oscillations about the mean position is :-



(A) $2\pi\sqrt{\frac{L}{2g}}$ (B) $2\pi\sqrt{\frac{\sqrt{3}L}{2g}}$ (C) $2\pi\sqrt{\frac{L}{2\sqrt{3}g}}$ (D) $\pi\sqrt{\frac{L}{g}}$

SH0064

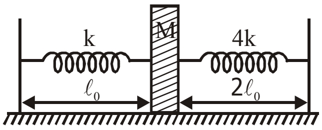
29. What is the period of small oscillations of the block of mass m if the springs are ideal and pulleys are massless ?



(A) $\frac{\pi}{2}\sqrt{\frac{m}{k}}$ (B) $\frac{\pi}{2}\sqrt{\frac{m}{2k}}$ (C) $\frac{\pi}{2}\sqrt{\frac{2m}{k}}$ (D) $\pi\sqrt{\frac{m}{2k}}$

SH0065

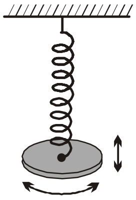
30. A block of mass M is kept in gravity free space and touches the two springs as shown in the figure. Initially springs are in their natural lengths. Now, the block is shifted ($\ell_0/2$) from the given position in such a way that it compresses a spring and is released. The time-period of oscillation of mass will be:-



(A) $\frac{\pi}{2}\sqrt{\frac{M}{K}}$ (B) $2\pi\sqrt{\frac{m}{5K}}$ (C) $\frac{3\pi}{2}\sqrt{\frac{M}{K}}$ (D) $\pi\sqrt{\frac{M}{2K}}$

SH0066

31. A solid disk of radius R is suspended from a spring of linear spring constant k and torsional constant c, as shown in figure. In terms of k and c, what value of R will give the same period for the vertical and torsional oscillations of this system?



(A) $\sqrt{\frac{2c}{k}}$ (B) $\sqrt{\frac{c}{2k}}$ (C) $2\sqrt{\frac{c}{k}}$ (D) $\frac{1}{2}\sqrt{\frac{c}{k}}$

SH0067

Superposition :

32. The amplitude of the vibrating particle due to superposition of two SHMs,

$y_1 = \sin\left(\omega t + \frac{\pi}{3}\right)$ and $y_2 = \sin\omega t$ is :

(A) 1 (B) $\sqrt{2}$ (C) $\sqrt{3}$ (D) 2

SH0068

33. Two simple harmonic motions $y_1 = A \sin \omega t$ and $y_2 = A \cos \omega t$ are superimposed on a particle of mass m. The total mechanical energy of the particle is:

(A) $\frac{1}{2}m\omega^2A^2$ (B) $m\omega^2A^2$ (C) $\frac{1}{4}m\omega^2A^2$ (D) zero

SH0069

34. The displacement of a particle varies with time according to the relation $y = a \sin \omega t + b \cos \omega t$.

(A) The motion is oscillatory but not S.H.M.

(B) The motion is S.H.M. with amplitude $a + b$.

(C) The motion is S.H.M. with amplitude $a^2 + b^2$.

(D) The motion is S.H.M. with amplitude $\sqrt{a^2+b^2}$

SH0070

35. Equations $y = 2A \cos^2 \omega t$ and $y = A(\sin \omega t + \sqrt{3} \cos \omega t)$ represent the motion of two particles.

(A) Only one of these is S.H.M. (B) Ratio of maximum speeds is 2 : 1

(C) Ratio of maximum speeds is 1 : 1 (D) Ratio of maximum accelerations is 1 : 4

SH0071

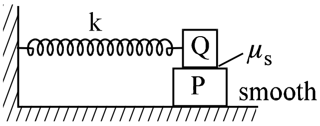
Complex situations :

36. A plank with a small block on top of it is under going vertical SHM. Its period is 2 sec. The minimum amplitude at which the block will separate from plank is :

(A) $\frac{10}{\pi^2}$ (B) $\frac{\pi^2}{10}$ (C) $\frac{20}{\pi^2}$ (D) $\frac{\pi}{10}$

SH0072

37. A block P of mass m is placed on a frictionless horizontal surface. Another block *Q* of same mass is kept on P and connected to the wall with the help of a spring of spring constant k as shown in the figure. μ_s is the coefficient of friction between *P* and *Q*. The blocks move together performing SHM of amplitude *A*. The maximum value of the friction force between *P* and *Q* is :- **[IIT JEE 2004]**



(A) kA (B) $\frac{kA}{2}$ (C) zero (D) $\mu_s mg$

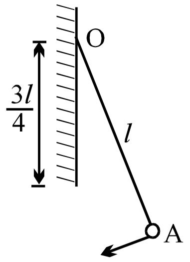
SH0073

38. Two bodies P & Q of equal mass are suspended from two separate massless springs of force constants k_1 & k_2 respectively. If the maximum velocity of them are equal during their motion, the ratio of amplitude of P to Q is :

(A) $\frac{k_1}{k_2}$ (B) $\sqrt{\frac{k_2}{k_1}}$ (C) $\frac{k_2}{k_1}$ (D) $\sqrt{\frac{k_1}{k_2}}$

SH0074

39. A small bob attached to a light inextensible thread of length l has a periodic time T when allowed to vibrate as a simple pendulum. The thread is now suspended from a fixed end O of a vertical rigid rod of length $\frac{3l}{4}$ (as in figure). If now the pendulum performs periodic oscillations in this arrangement, the periodic time will be



(A) $\frac{3T}{4}$ (B) $\frac{T}{2}$ (C) T (D) 2T

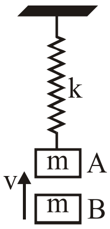
SH0075

40. Vertical displacement of a plank with a body of mass 'm' on it is varying according to law $y = \sin \omega t + \sqrt{3} \cos \omega t$. The minimum value of ω for which the mass just breaks off the plank and the moment it occurs first after t = 0 are given by: (y is positive vertically upwards)

(A) $\sqrt{\frac{g}{2}}, \frac{\sqrt{2}}{6}\frac{\pi}{\sqrt{g}}$ (B) $\frac{g}{\sqrt{2}}, \frac{2}{3}\sqrt{\frac{\pi}{g}}$ (C) $\sqrt{\frac{g}{2}}, \frac{\pi}{3}\sqrt{\frac{2}{g}}$ (D) $\sqrt{2g}, \sqrt{\frac{2\pi}{3g}}$

SH0076

41. Block A is hanging from a vertical spring and is at rest. Block B strikes the block A with velocity v and sticks to it. Then the value of v for which the spring just attains natural length is



(A) $\sqrt{\frac{60mg^2}{k}}$ (B) $\sqrt{\frac{6mg^2}{k}}$ (C) $\sqrt{\frac{10mg^2}{k}}$ (D) $\sqrt{\frac{8mg^2}{k}}$

SH0077

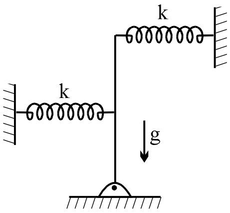
42. A particle of mass 10 gm moves in a field where potential energy per unit mass is given by expression $v = 8 \times 10^4 x^2$ erg/gm . If the total energy of the particle is 8×10^7 erg then the relation between x and time t is :[ϕ = constant]

(A) $x = 10 \sin (400\,t + \phi)$ cm (B) $x = \sin (400\,t + \phi)$ m

(C) $x = 10 \sin (40\,t + \phi)$ cm (D) $x = 100 \sin (4\,t + \phi)$ m

SH0078

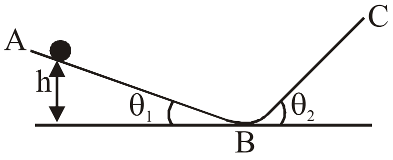
43. In the figure shown, the spring are connected to the rod at one end and at the midpoint. The rod is hinged at its lower end. Rotational SHM of the rod (Mass m, length L) will occur only if



(A) $k > mg/3L$ (B) $k > 2mg/3L$ (C) $k > 2mg/5L$ (D) $k > 0$

SH0079

44. A small glass bead of mass m initially at rest starts from a point at height h above the horizontal and rolls down the inclined plane AB as shown. Then it rises along the inclined plane BC. Assuming no loss of energy and sufficient friction for pure rolling, the time period of oscillation of the glass bead is:



(A) $\sqrt{\frac{8h}{g}}(\sin\theta_1+\sin\theta_2)$

(B) $2\sqrt{\frac{14h}{5g}}\left(\frac{1}{\sin\theta_1}+\frac{1}{\sin\theta_2}\right)$

(C) $\sqrt{\frac{8h}{g}}\left(\frac{1}{\sin\theta_1}+\frac{1}{\sin\theta_2}\right)$

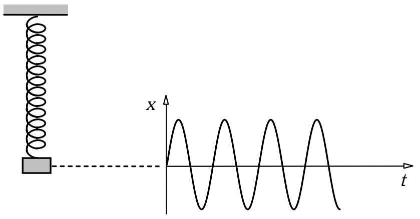
(D) $\sqrt{\frac{8h}{5g}}\left(\frac{1}{\sin\theta_1}+\frac{1}{\sin\theta_2}\right)$

SH0080

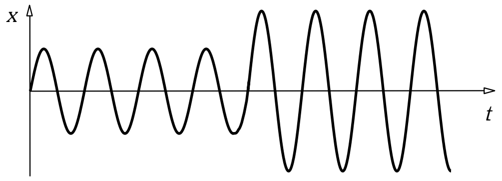
EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

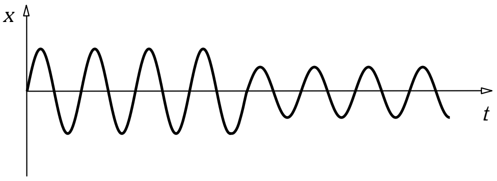
1. In the figure is shown a spring-mass system oscillating in uniform gravity. If we neglect all dissipative forces, it will keep on oscillating endlessly with constant amplitude and frequency. Accompanying graph shows how displacement x of the block from the equilibrium position varies with time t.
Now at a certain instant $t = t_o$ when the block reaches its lowest position, gravity is switched off by some unknown mechanism. Which of the following graphs would correctly describes the changes taking place due to switching off the gravity?



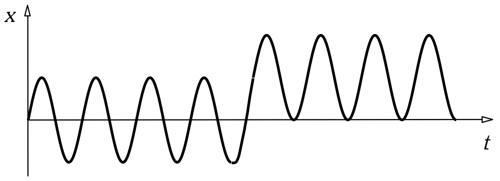
(A)



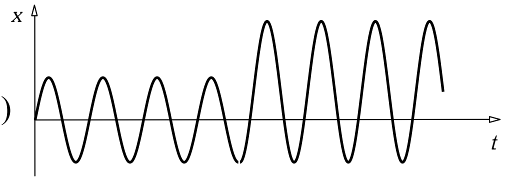
(B)



(C)



(D)



SH0081

2. Time period of a particle executing SHM is 8 sec. At t = 0 it is at the mean position. The ratio of the distance covered by the particle in the 1st second to the 2nd second is :

(A) $\frac{1}{\sqrt{2}+1}$ (B) $\sqrt{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{2}+1$

SH0082

3. A particle executes SHM with time period T and amplitude A. The maximum possible average velocity in time $\frac{T}{4}$ is

(A) $\frac{2A}{T}$ (B) $\frac{4A}{T}$ (C) $\frac{8A}{T}$ (D) $\frac{4\sqrt{2}A}{T}$

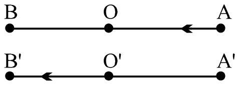
SH0083

4. A particle moves along the x-axis according to : $x = A.[1 + \sin \omega t]$. What distance does it travel between $t = 0$ and $t = 2.5\pi/\omega$?

(A) 4A (B) 6A (C) 5A (D) None

SH0084

5. Two particles undergo SHM along parallel lines with the same time period (T) and equal amplitudes. At a particular instant, one particle is at its extreme position while the other is at its mean position. They move in the same direction. They will cross each other after a further time



(A) T/8 (B) 3T/8 (C) T/6 (D) 4T/3

SH0085

6. Two particles are in SHM in a straight line about same equilibrium position. Amplitude A and time period T of both the particles are equal. At time t=0, one particle is at displacement $y_1 = +A$ and the other at $y_2 = -A/2$, and they are approaching towards each other. After what time they cross each other ?

(A) T/3 (B) T/4 (C) 5T/6 (D) T/6

SH0086

7. Two particles A and B perform SHM along the same straight line with the same amplitude 'a', same frequency 'f' and same equilibrium position 'O'. The greatest distance between them is found to be $\frac{3a}{2}$. At some instant of time they have the same displacement from mean position. What is the displacement?

(A) a/2 (B) $a\sqrt{7}/4$ (C) $\sqrt{3}\,a/2$ (D) $3a/4$

SH0087

8. A wire frame in the shape of an equilateral triangle is hinged at one vertex so that it can swing freely in a vertical plane, with the plane of the Δ always remaining vertical. The side of the frame is $1/\sqrt{3}$ m. The time period in seconds of small oscillations of the frame will be

(A) $\frac{\pi}{\sqrt{2}}$ (B) $\pi\sqrt{2}$ (C) $\frac{\pi}{\sqrt{6}}$ (D) $\frac{\pi}{\sqrt{5}}$

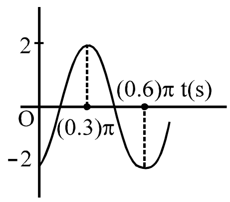
SH0088

9. For a particle acceleration is defined as $\vec{a} = \frac{-5x\hat{i}}{|x|}$ for $x \neq 0$ and $\vec{a} = 0$ for $x = 0$. If the particle is initially at rest at (a, 0), what is period of motion of the particle.

(A) $4\sqrt{2a/5}$ sec. (B) $8\sqrt{2a/5}$ sec. (C) $2\sqrt{2a/5}$ sec. (D) cannot be determined

SH0089

10. Part of a simple harmonic motion is graphed in the figure, where y is the displacement from the mean position. The correct equation describing this S.H.M is



(A) $y = 4\cos(0.6t)$ (B) $y = 2\sin\left(\frac{10}{3}t - \frac{\pi}{2}\right)$

(C) $y = 4\sin\left(\frac{10}{3}t + \frac{\pi}{2}\right)$ (D) $y = 2\cos\left(\frac{10}{3}t + \frac{\pi}{2}\right)$

SH0090

11. The angular frequency of a spring block system is ω_0. This system is suspended from the ceiling of an elevator moving downwards with a constant speed v_0. The block is at rest relative to the elevator. Lift is suddenly stopped. Assuming the downwards as a positive direction, choose the wrong statement:

(A) The amplitude of the block is $\frac{v_0}{\omega_0}$

(B) The initial phase of the block is π.

(C) The equation of motion for the block is $\frac{v_0}{\omega_0}\sin\omega_0 t$.

(D) The maximum speed of the block is v_0.

SH0091

12. Two particles are performing SHM with same angular frequency and amplitudes A and 2A respectively along same straight line with same mean position. They cross each other at position A/2 distance from mean position in opposite direction. The phase between them is :

(A) $\frac{5\pi}{6} - \sin^{-1}\left(\frac{1}{4}\right)$ (B) $\frac{\pi}{6} - \sin^{-1}\left(\frac{1}{4}\right)$ (C) $\frac{5\pi}{6} - \cos^{-1}\left(\frac{1}{4}\right)$ (D) $\frac{\pi}{6} - \cos^{-1}\left(\frac{1}{4}\right)$

SH0092

13. A particle executing a simple harmonic motion of period 2s. When it is at its extreme displacement from its mean position, it receives an additional energy equal to what it had in its mean position. Due to this, in its subsequent motion,

(A) its amplitude will change and become equal to $\sqrt{2}$ times its previous amplitude

(B) its periodic time will become doubled i.e. 4s

(C) its potential energy will be decreased

(D) it will continue to execute simple harmonic motion of the same amplitude and period as before receiving the additional energy.

SH0093

14. A particle is executing SHM of amplitude A, about the mean position x = 0. Which of the following cannot be a possible phase difference between the positions of the particle at $x = +A/2$ and $x = -A/\sqrt{2}$.

(A) 75° (B) 165° (C) 135° (D) 195°

SH0094

15. A particle free to move along the x-axis has potential energy given by $U(x) = k[1-\exp(-x^2)]$ for $-\infty < x < +\infty$, where k is a positive constant of appropriate dimensions. Then

(A) at point away from the origin, the particle is in unstable equilibrium.

(B) for any finite non-zero value of x, there is a force directed away from the origin.

(C) if its total mechanical energy is k/2, it has its minimum kinetic energy at the origin.

(D) for small displacements from x = 0, the motion is simple harmonic.

SH0095

16. The time period of a bar pendulum when suspended at distances 30 cm and 50 cm from its centre of gravity comes out to be the same. If the mass of the body is 2kg. Find out its moment of inertia about an axis passing through first point.

(A) 0.24 kg-m^2 (B) 0.72 kg-m^2 (C) 0.48 kg-m^2 (D) Data insufficient

SH0096

MULTIPLE CORRECT TYPE QUESTIONS

17. For a SHM with given angular frequency, two arbitrary initial conditions are necessary and sufficient to determine the motion completely. These initial conditions may be-

(A) Amplitude and initial phase

(B) Amplitude and total energy of oscillation

(C) Initial phase and total energy of oscillation

(D) Initial position and initial velocity

SH0097

18. A particle is executing SHM between points $-X_m$ and X_m, as shown in figure-I. The velocity V(t) of the particle is partially graphed and shown in figure-II. Two points A and B corresponding to time t_1 and time t_2 respectively are marked on the V(t) curve.

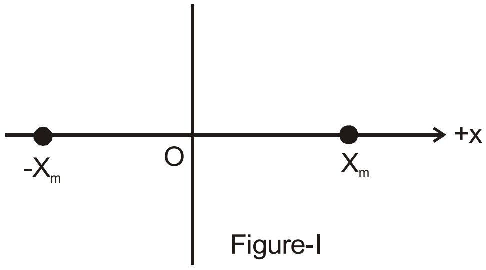


Figure-I

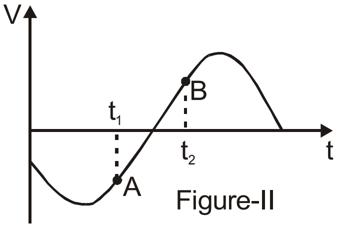


Figure-II

(A) At time t_1 , it is going towards X_m.

(B) At time t_1, its speed is decreasing.

(C) At time t_2, its position lies in between $-X_m$ and O.

(D) The phase difference $\Delta\phi$ between points A and B must be expressed as $90° < \Delta\phi < 180°$.

SH0098

19. For a body executing SHM with amplitudes A, time period T, max velocity v_{max} and phase constant zero, which of the following statements are correct?

(A) At $y = (A/2)$, $v > (v_{max} / 2)$ (B) $v = (v_{max}/2)$ for $|y| > (A/2)$

(C) For $t = (T/8)$, $y > (A/2)$ (D) For $y = (A/2)$, $t < (T/8)$

SH0099

20. The amplitude of a particle executing SHM about O is 10 cm. Then :

(A) When the K.E. is 0.64 of its max. K.E. its displacement is 6cm from O.

(B) When the displacement is 5 cm from O its K.E. is 0.75 of its max.P.E.

(C) Its total energy at any point is equal to its maximum K.E.

(D) Its velocity is half the maximum velocity when its displacement is half the maximum displacement.

SH0100

21. The position vector of a particle that is moving in space is given by

$\vec{r}=(1+2\cos 2\omega t)\hat{i}+(3\sin^2 \omega t)\hat{j}+(3)\hat{k}$

in the ground frame. All units are in SI. Choose the correct statement (s) :

(A) The particle executes SHM in the ground frame about the mean position $\left(1,\frac{3}{2},3\right)$

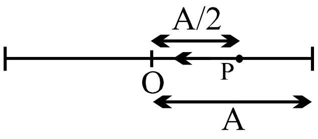
(B) The particle executes SHM in a frame moving along the z-axis with a velocity of 3 m/s.

(C) The amplitude of the SHM of the particle is $\frac{5}{2}$ m.

(D) The direction of the SHM of the particle is given by the vector $\left(\frac{4}{5}\hat{i}-\frac{3}{5}\hat{j}\right)$

SH0101

22. A particle starts from a point P at a distance of A/2 from the mean position O & travels towards left as shown in the figure. If the time period of SHM, executed about O is T and amplitude A then the equation of motion of particle is :



(A) $x = A\sin\left(\frac{2\pi}{T}t+\frac{\pi}{6}\right)$

(B) $x = A\sin\left(\frac{2\pi}{T}t+\frac{5\pi}{6}\right)$

(C) $x = A\cos\left(\frac{2\pi}{T}t+\frac{\pi}{6}\right)$

(D) $x = A\cos\left(\frac{2\pi}{T}t+\frac{\pi}{3}\right)$

SH0102

23. A spring has natural length 40 cm and spring constant 500 N/m. A block of mass 1 kg is attached at one end of the spring and other end of the spring is attached to ceiling. The block released from the position, where the spring has length 45 cm.

(A) the block will perform SHM of amplitude 5 cm.

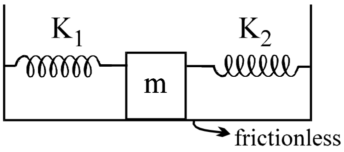
(B) the block will have maximum velocity $30\sqrt{5}$ cm/sec.

(C) the block will have maximum acceleration 15 m/s^2

(D) the minimum potential energy of the spring will be zero.

SH0103

24. Two springs with negligible masses and force constant of $K_1 = 200\ \text{Nm}^{-1}$ and $K_2 = 160\ \text{Nm}^{-1}$ are attached to the block of mass m = 10 kg as shown in the figure. Initially the block is at rest, at the equilibrium position in which both springs are neither stretched nor compressed. At time t = 0, a sharp impulse of 50 Ns is given to the block with a hammer.



(A) Period of oscillations for the mass m is $\frac{\pi}{3}$ s.

(B) Maximum velocity of the mass m during its oscillation is 5 ms^{-1}.

(C) Data are insufficient to determine maximum velocity.

(D) Amplitude of oscillation is 0.42 m.

SH0104

25. A mass of 0.2kg is attached to the lower end of a massless spring of force-constant 200 N/m, the upper end of which is fixed to a rigid support. Which of the following statements is/are true?

(A) In equilibrium, the spring will be stretched by 1cm.

(B) If the mass is raised till the spring is unstretched state and then released, it will go down by 2cm before moving upwards.

(C) The frequency of oscillation will be nearly 5 Hz.

(D) If the system is taken to the moon, the frequency of oscillation will be the same as on the earth.

SH0105

26. A particle moves in x-y plane according to equation $\vec{S} = (2\hat{i} + \hat{j})\cos\omega t$. The motion of particle is:-

(A) On straight line (B) On ellipse (C) Periodic (D) SHM

SH0106

27. Two particles are in SHM with same amplitude A and same angular frequency ω. At time t = 0, one is at $x = +\frac{A}{2}$ and other is at $x = -\frac{A}{2}$. Both are moving in same direction.

(A) Phase difference between the two particle is $\frac{\pi}{3}$

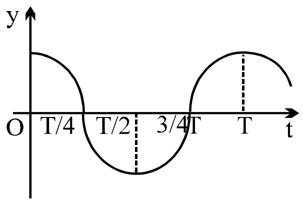
(B) Phase difference between the two particle is $\frac{2\pi}{3}$

(C) They will collide after time $t = \frac{\pi}{2\omega}$

(D) They will collide after time $t = \frac{3\pi}{\omega}$

SH0107

28. The displacement-time graph of a particle executing SHM is shown. Which of the following statements is/are true?



(A) The velocity is maximum at t = T/2

(B) The acceleration is maximum at t = T

(C) The force is zero at t = 3T/4

(D) The potential energy equals the oscillation energy at t = T/2.

SH0108

29. A particle is executing SHM with amplitude A, time period T, maximum acceleration a_o and maximum velocity v_0. Its starts from mean position at t = 0 and at time t, it has the displacement A/2, acceleration a and velocity v then :-

(A) t = T/12 (B) $a = a_o/2$ (C) $v = v_o/2$ (D) t = T/8

SH0109

30. Three simple harmonic motions in the same direction having the same amplitude a and same period are superposed. If each differs in phase from the next by 45°, then :- **(1999S)**

(A) the resultant amplitude is $(1 + \sqrt{2})$ a

(B) the phase of the resultant motion relative to the first is 90°

(C) the energy associated with the resulting motion is $(3 + 2\sqrt{2})$ times the energy associated with any single motion

(D) the resulting motion is not simple harmonic

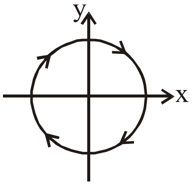
SH0110

COMPREHENSION TYPE QUESTIONS

Paragraph for question no. 31 and 32

Lissajous figures are produced by superposition of 2 SHM's in mutually perpendicular directions. If both SHM have same frequency, the lissajous figure is simple. It may be a circle, ellipse or a straight line depending on the phase difference between two SHMs. Interesting case occurs when one of the SHM is at double frequency of another. Suppose a body executes SHM vertically with frequency f_0, but horizontally with a frequency $2f_0$ and is initially at A, mean position of vertical as well as horizontal SHM. It can be seen to trace out a figure of 8 in space as shown. Since horizontal SHM has half the time period, it executes two horizontal oscillations by the time it completes a vertical oscillation.

31. A lissajous figure as shown here can be produced by

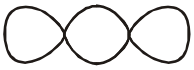


(A) x = A sin ωt ; y = A cos ωt (B) x = A cos ωt ; y = A sin ωt

(C) x = A sin ωt ; $y = A \sin\left(\omega t + \frac{\pi}{4}\right)$ (D) x = A sin ωt ; $y = A \sin\left(\omega t + \frac{3\pi}{4}\right)$

SH0111

32. For the Lissajous figure shown here, the frequency in vertical direction is _______ times the frequency in horizontal direction :-

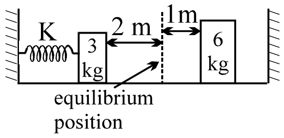


(A) 3 (B) $\frac{1}{3}$ (C) 2 (D) $\frac{1}{2}$

SH0111

Paragraph for Question No. 33 and 34

Two blocks of masses 3 kg and 6 kg rest on on a horizontal frictionless surface. The 3 kg block is attached to a spring with a force constant K = 900 N/m which is compressed 2 m initially from its equilibrium position. When 3 kg mass is released, it strikes the 6 kg mass and the two stick together.



33. The common velocity of the blocks after collision is :-

(A) 10 m/s (B) 30 m/s (C) 15 m/s (D) 2 m/s

SH0112

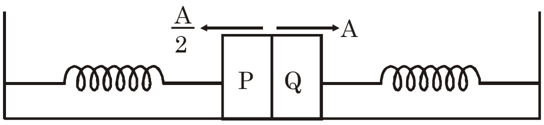
34. The amplitude of resulting oscillation after the collision is :-

(A) $\frac{1}{\sqrt{2}}$ m (B) $\frac{1}{\sqrt{3}}$ m (C) $\sqrt{2}$ m (D) $\sqrt{3}$ m

SH0112

Paragraph for Question no. 35 and 36

Two identical blocks P and Q have mass m each. They are attached to two identical springs initially unstretched. Now the left spring (alongwith P) is compressed by $\frac{A}{2}$ and the right spring (alongwith Q) is compressed by A. Both the blocks are released simultaneously. They collide perfectly inelastically. Initially time period of both the block was T.



35. The time period of oscillation of combined mass is :

(A) $\frac{T}{\sqrt{2}}$ (B) $\sqrt{2}T$ (C) T (D) $\frac{T}{2}$

SH0113

36. The amplitude of combined mass is :

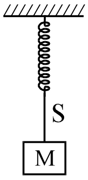
(A) $\frac{A}{4}$ (B) $\frac{A}{2}$ (C) $\frac{2A}{3}$ (D) $\frac{3A}{4}$

SH0113

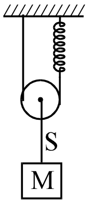
MATRIX MATCH TYPE QUESTION

37. In the following four situations, mass M of 1kg is kept in equilibrium. k = 100 N/m in all cases. What speed can be given to mass M vertically so that inextensible string S does not become slack in subsequent motion. Consider that pulley is ideal and string is massless :

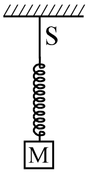
Column-I **Column-II**

(A) 

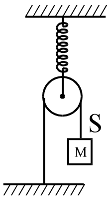
(the block is attached to spring by an inextensible thread) (P) 1 ms^{-1}

(B) 

(Q) 0.5 ms^{-1}

(C) 

(block is attached to the spring) (R) 0.25 ms^{-1}

(D) 

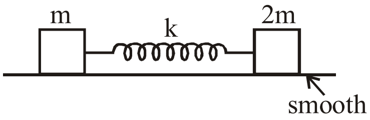
(S) 2 ms^{-1}

SH0114

38. In list-I, the systems are performing SHM and in list-II, the time period of SHM is shown then match list-I with list-II.

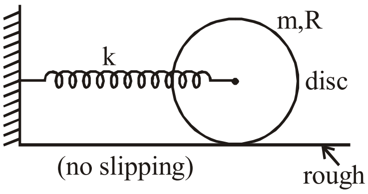
List-I **List-II**

(P)



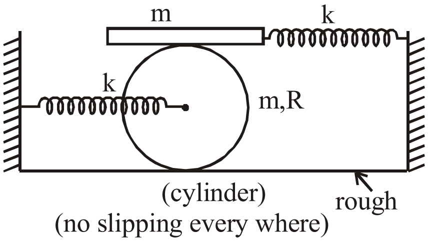
(1) $\left[2\pi\sqrt{\frac{11m}{10k}}\right]$

(Q)



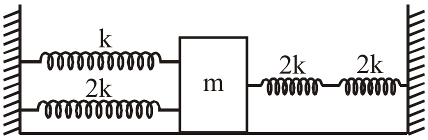
(2) $2\pi\sqrt{\frac{m}{4k}}$

(R)



(3) $2\pi\sqrt{\frac{3m}{2k}}$

(S)



(4) $2\pi\sqrt{\frac{2m}{3k}}$

Codes :

	P	Q	R	S
(A)	3	2	1	4
(B)	4	3	2	1
(C)	3	4	2	1
(D)	4	3	1	2

SH0115

EXERCISE (JM)

1. A mass M, attached to a horizontal spring, executes S.H.M. with amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\frac{A_1}{A_2}\right)$ is :- **[AIEEE-2011]**

(1) $\left(\frac{M}{M+m}\right)^{1/2}$ (2) $\left(\frac{M+m}{M}\right)^{1/2}$ (3) $\frac{M}{M+m}$ (4) $\frac{M+m}{M}$

SH0116

2. Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x-axis. Their mean position is separated by distance $X_0(X_0 > A)$. If the maximum separation between them is $(X_0 + A)$, the phase difference between their motion is :- **[AIEEE-2011]**

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{3}$

SH0117

3. A wooden cube (density of wood 'd') of side 'ℓ' floats in a liquid of density 'ρ' with its upper and lower surfaces horizontal. If the cube is pushed slightly down and released, it performs simple harmonic motion of period 'T'. Then, 'T' is equal to :- **[AIEEE-2011]**

(1) $2\pi\sqrt{\frac{\ell\rho}{(\rho-d)g}}$ (2) $2\pi\sqrt{\frac{\ell d}{\rho g}}$

(3) $2\pi\sqrt{\frac{\ell\rho}{dg}}$ (4) $2\pi\sqrt{\frac{\ell d}{(\rho-d)g}}$

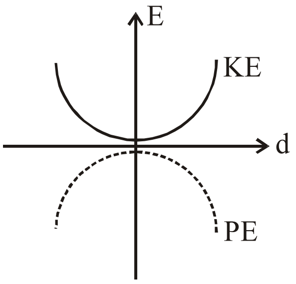
SH0118

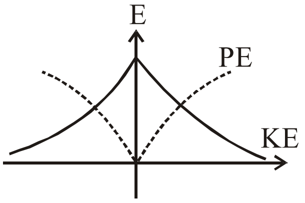
4. A particle moves with simple harmonic motion in a straight line. In first τ s, after starting from rest it travels a distance a, and in next τ s it travels 2a, in same direction, then : **[JEE Mains-2014]**

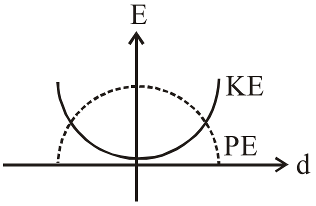
(1) Amplitude of motion is 4a
(2) Time period of oscillation is 6τ
(3) Amplitude of motion is 3a
(4) Time period of oscillation is 8τ

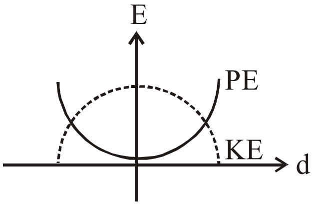
SH0119

5. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly ? (graphs are schematic and not drawn to scale) **[JEE Mains-2015]**

(1)


(2)


(3)


(4)


SH0120

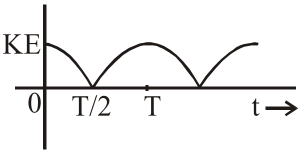
6. A particle performs simple harmonic motion with amplitude A. Its speed is trebled at the instant that it is at a distance $\frac{2A}{3}$ from equilibrium position. The new amplitude of the motion is :-

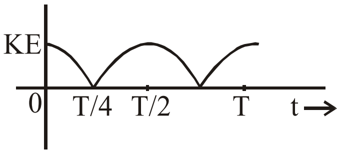
[JEE Mains-2016]

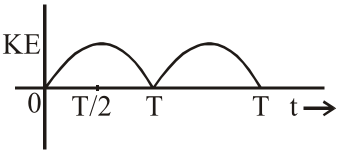
(1) $\frac{7A}{3}$ (2) $\frac{A}{3}\sqrt{41}$ (3) 3A (4) $A\sqrt{3}$

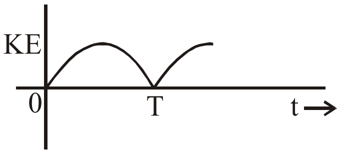
SH0121

7. A particle is executing simple harmonic motion with a time period T. AT time t = 0, it is at its position of equilibrium. The kinetic energy-time graph of the particle will look like **[JEE Main-2017]**

(1)


(2)


(3)


(4)


SH0122

8. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12}/sec. What is the force constant of the bonds connecting one atom with the other ? (Mole wt. of silver =108 and Avagadro number = 6.02×10^{23} gm mole^{-1}) **[JEE Main-2018]**

(1) 7.1 N/m (2) 2.2 N/m (3) 5.5 N/m (4) 6.4 N/m

SH0123

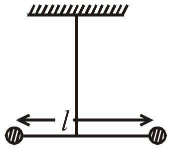
SELECTIVE PROBLEMS FROM JEE-MAINS ONLINE PAPERS

9. A rod of mass 'M' and length '2L' is suspended at its middle by a wire. It exhibits torsional oscillations; If two masses each of 'm' are attached at distance 'L/2' from its centre on both sides, it reduces the oscillation frequency by 20%. The value of ratio m/M is close to : **[JEE-Main-2019_Jan]**

(1) 0.17 (2) 0.37 (3) 0.57 (4) 0.77

SH0142

10. Two masses m and $\frac{m}{2}$ are connected at the two ends of a massless rigid rod of length l. The rod is suspended by a thin wire of torsional constant k at the centre of mass of the rod-mass system(see figure). Because of torsional constant k, the restoring torque is $\tau = k\theta$ for angular displacement θ. If the rod is rotated by θ_0 and released, the tension in it when it passes through its mean position will be: **[JEE-Main-2019_Jan]**



(1) $\frac{3k\theta_0^2}{l}$ (2) $\frac{k\theta_0^2}{2l}$ (3) $\frac{2k\theta_0^2}{l}$ (4) $\frac{k\theta_0^2}{l}$

SH0143

11. A particle executes simple harmonic motion with an amplitude of 5 cm. When the particle is at 4 cm from the mean position, the magnitude of its velocity in SI units is equal to that of its acceleration. Then, its periodic time in seconds is : **[JEE-Main-2019_Jan]**

(1) $\frac{7}{3}\pi$ (2) $\frac{3}{8}\pi$ (3) $\frac{4\pi}{3}$ (4) $\frac{8\pi}{3}$

SH0144

12. A simple pendulum of length 1 m is oscillating with an angular frequency 10 rad/s. The support of the pendulum starts oscillating up and down with a small angular frequency of 1 rad/s and an amplitude of 10^{-2} m. The relative change in the angular frequency of the pendulum is best given by :- **[JEE-Main-2019_Jan]**

(1) 10^{-3} rad/s (2) 10^{-1} rad/s (3) 1 rad/s (4) 10^{-5} rad/s

SH0145

13. A pendulum is executing simple harmonic motion and its maximum kinetic energy is K_1. If the length of the pendulum is doubled and it performs simple harmonic motion with the same amplitude as in the first case, its maximum kinetic energy is K_2. Then :- **[JEE-Main-2019_Jan]**

(1) $K_2 = \frac{K_1}{4}$ (2) $K_2 = \frac{K_1}{2}$ (3) $K_2 = 2K_1$ (4) $K_2 = K_1$

SH0146

14. A simple harmonic motion is represented by:

$y = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$ cm

The amplitude and time period of the motion are: **[JEE-Main-2019_Jan]**

(1) 5cm, $\frac{3}{2}$s (2) 5cm, $\frac{2}{3}$s (3) 10cm, $\frac{3}{2}$s (4) 10cm, $\frac{2}{3}$s

SH0147

15. A simple pendulum oscillating in air has period T. The bob of the pendulum is completely immersed in a non-viscous liquid. The density of the liquid is $\frac{1}{16}$th of the material of the bob. If the bob is inside liquid all the time, its period of oscillation in this liquid is : **[JEE-Main-2019_April]**

(1) $4T\sqrt{\frac{1}{15}}$ (2) $2T\sqrt{\frac{1}{10}}$ (3) $4T\sqrt{\frac{1}{14}}$ (4) $2T\sqrt{\frac{1}{14}}$

SH0148

16. A ring is hung on a nail. It can oscillate, without slipping or sliding (i) in its plane with a time period T_1 and, (ii) back and forth in a direction perpendicular to its plane, with a period T_2. the ratio $\frac{T_1}{T_2}$ will be :

[JEE-Main-2020_Sep]

(1) $\frac{2}{\sqrt{3}}$ (2) $\frac{\sqrt{2}}{3}$ (3) $\frac{2}{3}$ (4) $\frac{3}{\sqrt{2}}$

SH0149

17. When a particle of mass m is attached to a vertical spring of spring constant k and released, its motion is described by $y(t) = y_0\sin^2\omega t$, where 'y' is measured from the lower end of unstretched spring. Then ω is : **[JEE-Main-2020_Sep]**

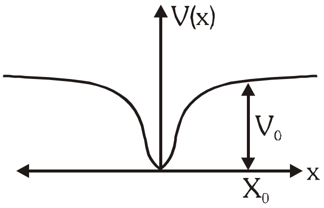
(1) $\sqrt{\frac{g}{y_0}}$ (2) $\sqrt{\frac{g}{2y_0}}$ (3) $\frac{1}{2}\sqrt{\frac{g}{y_0}}$ (4) $\sqrt{\frac{2g}{y_0}}$

SH0150

EXERCISE (JA)

Paragraph for Question No. 1 to 3

When a particle of mass *m* moves on the x–axis in a potential of the form $V(x) = kx^2$, it performs simple harmonic motion. The corresponding time period is proportional to $\sqrt{\frac{m}{k}}$, as can be seen easily using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of $x = 0$ in a way different from kx^2 and its total energy is such that the particle does not escape to infinity. Consider a particle of mass *m* moving on the *x*–axis. Its potential energy is $V(x) = \alpha x^4$ $(\alpha > 0)$ for $|x|$ near the origin and becomes a constant equal to V_0 for $|x| \geq X_0$ (see figure) **[IIT-JEE 2010]**



1. If the total energy of the particle is E, it will perform periodic motion only if :-

(A) $E < 0$ (B) $E > 0$ (C) $V_0 > E > 0$ (D) $E > V_0$

SH0124

2. For periodic motion of small amplitude *A*, the time period *T* of this particle is proportional to :-

(A) $A\sqrt{\frac{m}{\alpha}}$ (B) $\frac{1}{A}\sqrt{\frac{m}{\alpha}}$ (C) $A\sqrt{\frac{\alpha}{m}}$ (D) $\frac{1}{A}\sqrt{\frac{\alpha}{m}}$

SH0124

3. The acceleration of this particle for $|x| > X_0$ is :-

(A) proportional to V_0

(B) proportional to $\frac{V_0}{mX_0}$

(C) proportional to $\sqrt{\frac{V_0}{mX_0}}$

(D) Zero

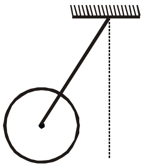
SH0124

4. A point mass is subjected to two simultaneous sinusoidal displacements in x-direction, $x_1(t) = A\sin\omega t$ and $x_2(t) = A\sin\left(\omega t + \frac{2\pi}{3}\right)$. Adding a third sinusoidal displacement $x_3(t) = B\sin(\omega t + \phi)$ brings the mass to a complete rest. The values of B and ϕ are :- **[IIT-JEE 2011]**

(A) $\sqrt{2}A, \frac{3\pi}{4}$ (B) $A, \frac{4\pi}{3}$ (C) $\sqrt{3}A, \frac{5\pi}{6}$ (D) $A, \frac{\pi}{3}$

SH0125

5. A metal rod of length 'L' and mass 'm' is pivoted at one end. A thin disk of mass 'M' and radius 'R' (< L) is attached at its center to the free end of the rod. Consider two ways the disc is attached: (case A). The disc is not free to rotate about its center and (case B) the disc is free to rotated about its center. The rod-disc system performs SHM in vertical plane after being released from the same displaced position. Which of the following statement(s) is(are) true? **[IIT-JEE 2011]**



(A) Restoring torque in case A = Restoring torque in case B

(B) Restoring torque in case A < Restoring torque in case B

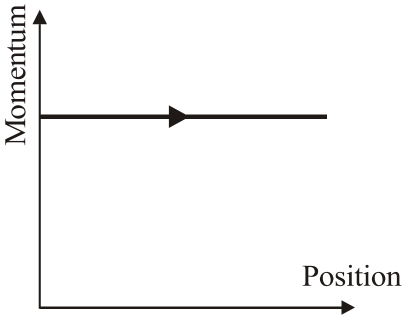
(C) Angular frequency for case A> Angular frequency for case B

(D) Angular frequency for case A< Angular frequency for case B

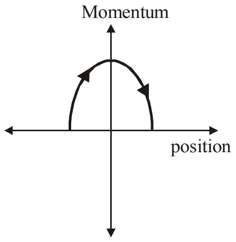
SH0126

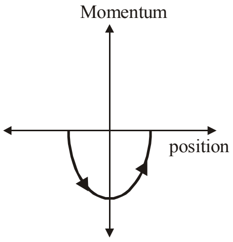
Paragraph for Questions Nos. 6 to 8

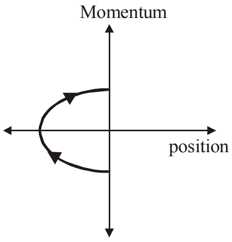
Phase space diagrams are useful tools in analyzing all kinds of dynamical problems. They are especially useful in studying the changes in motion as initial position and momentum are changed. Here we consider some simple dynamical systems in one-dimension. For such systems, phase space is a plane in which position is plotted along horizontal axis and momentum is plotted along vertical axis. The phase space diagram is x(t) vs. p(t) curve in this plane. The arrow on the curve indicates the time flow. For example, the phase space diagram for a particle moving with constant velocity is a straight line as shown in the figure. We use the sign convention in which position or momentum upwards (or to right) is positive and downwards (or to left) is negative. **[IIT-JEE 2011]**

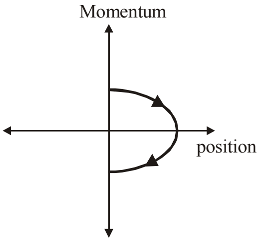


6. The phase space diagram for a ball thrown vertically up from ground is

(A)


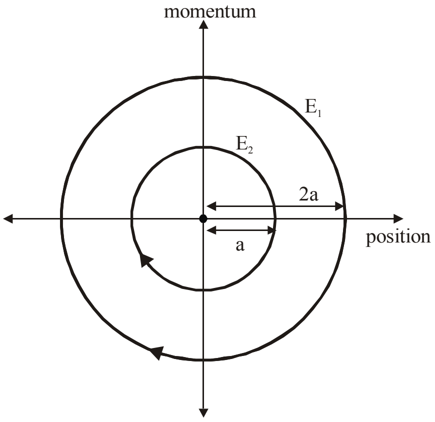
(B)


(C)


(D)


SH0127

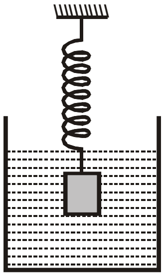
7. The phase space diagram for simple harmonic motion is a circle centered at the origin. In the figure, the two circles represent the same oscillator but for different initial conditions, and E_1 and E_2 are the total mechanical energies respectively. Then

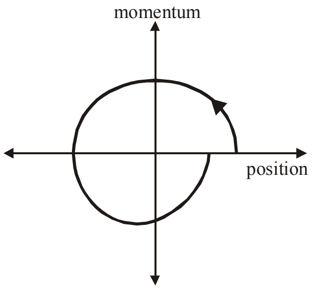


(A) $E_1 = \sqrt{2}E_2$ (B) $E_1=2E_2$ (C) $E_1=4E_2$ (D) $E_1=16E_2$

SH0127

8. Consider the spring-mass system, with the mass submerged in water, as shown in the figure. The phase space diagram for one cycle of this system is



(A)


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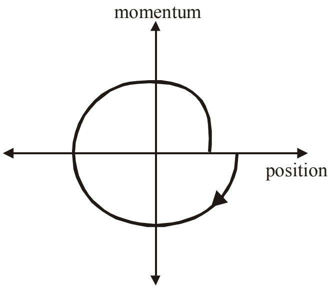
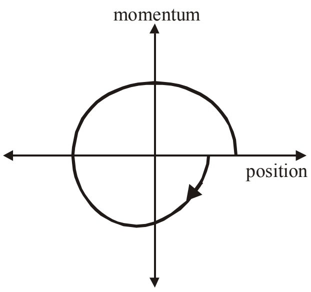
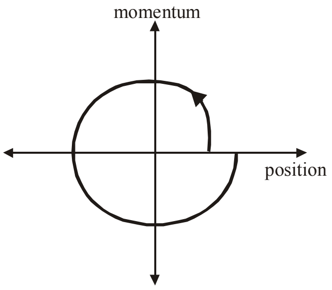
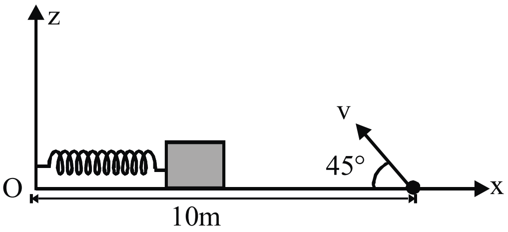


(C)


(D)


SH0127

9. A small block is connected to one end of a massless spring of un-stretched length 4.9 m. The other end of the spring (see the figure) is fixed. They system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at t = 0. It then executes simple harmonic motion with angular frequency $\omega = \frac{\pi}{3}$ rad/s. Simultaneously at t = 0, a small pebble is projected with speed v from point P at an angle of 45° as shown in the figure. Point P is at a horizontal distance of 10m from O. If the pebble hits the block at t = 1s, the value of v is:- (take g = 10 m/s^2) **[IIT-JEE 2012]**



(A) $\sqrt{50}$ m/s (B) $\sqrt{51}$ m/s

(C) $\sqrt{52}$ m/s (D) $\sqrt{53}$ m/s

SH0128

10. A particle of mass m is attached to one end of a mass-less spring of force constant k, lying on a frictionless horizontal plane. The other end of the spring is fixed. The particle starts moving horizontally from its equilibrium position at time t = 0 with an initial velocity u_0. When the speed of the particle is 0.5 u_0, it collides elastically with a rigid wall. After this collision :- **[JEE-Advanced-2013]**

(A) the speed of the particle when it returns to its equilibrium position is u_0

(B) the time at which the particle passes through the equilibrium position for the first time is $t = \pi\sqrt{\frac{m}{k}}$

(C) the time at which the maximum compression of the spring occurs is $t = \frac{4\pi}{3}\sqrt{\frac{m}{k}}$

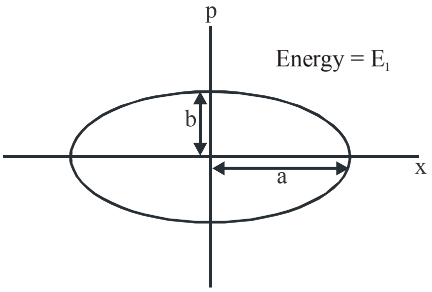
(D) the time at which the particle passes through the equilibrium position for the second time is

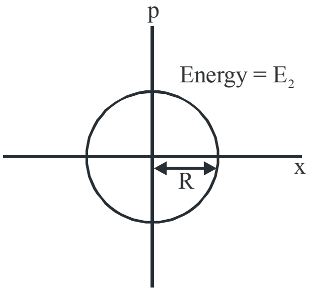
$t = \frac{5\pi}{3}\sqrt{\frac{m}{k}}$

SH0129

11. Two independent harmonic oscillators of equal mass are oscillating about the origin with angular frequencies ω_1 and ω_2 and have total energies E_1 and E_2, respectively. The variations of their momenta p with positions x are shown in the figures. If $\frac{a}{b} = n^2$ and $\frac{a}{R} = n$, then the correct equation (s) is (are)

[JEE-Advanced-2015]





(A) $E_1\omega_1 = E_2\omega_2$ (B) $\frac{\omega_2}{\omega_1} = n^2$ (C) $\omega_1\omega_2 = n^2$ (D) $\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$

SH0130

12. A particle of unit mass is moving along the x-axis under the influence of a force and its total enegy is conserved. Four possible forms of the potential energy of the particle are given in column I (a and U_0 are constants). Match the potential energies in column I to the corresponding statement(s) in column-II **[JEE-Advanced-2015]**

Column-I	Column-II
(A) $U_1(x) = \frac{U_0}{2}\left[1-\left(\frac{x}{a}\right)^2\right]^2$	(P) The force acting on the particle is zero at $x = a$.
(B) $U_2(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2$	(Q) The force acting on the particle is zero at $x = 0$.
(C) $U_3(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2 \exp\left[-\left(\frac{x}{a}\right)^2\right]$	(R) The force acting on the particle is zero at $x = -a$.
(D) $U_4(x) = \frac{U_0}{2}\left[\frac{x}{a} - \frac{1}{3}\left(\frac{x}{a}\right)^3\right]$	(S) The particle experiences an attractive force towards $x = 0$ in the region $\|x\| < a$.
	(T) The particle with total enegy $\frac{U_0}{4}$ can oscillate about the point $x = -a$.

SH0131

13. A block with mass M is connected by a massless spring with stiffness constant k to a rigid wall and moves without friction on a horizontal surface. The block oscillates with small amplitude A about an equilibrium position x_0. Consider two cases : (i) when the block is at x_0; and (ii) when the block is at $x = x_0 + A$. In both the cases, a particle with mass m (<M) is softly placed on the block after which they stick to each other. Which of the following statement(s) is(are) true about the motion after the mass m is placed on the mass M? **[JEE-Advanced-2016]**

(A) The amplitude of oscillation in the first case changes by a factor of $\sqrt{\frac{M}{m+M}}$, whereas in the second case it remains unchanged

(B) The final time period of oscillation in both the cases is same

(C) The total energy decreases in both the cases

(D) The instantaneous speed at x_0 of the combined masses decreases in both the cases.

SH0132

ANSWER KEY

EXERCISE (S-1)

1. Ans. $y = 2\sin\left(\frac{10}{3}t - \frac{\pi}{2}\right)$ **2. Ans.** 0.33 sec **3. Ans.** 2 m/s **4. Ans.** 0.06m

5. Ans. $y = 0.1\sin(4t + \pi/4)$ **6. Ans.** (i) $x_0 = 2m$; (ii) $T = \sqrt{2}\,\pi$ sec.; (iii) $2\sqrt{3}$

7. Ans. $\frac{1}{2\pi}\sqrt{\frac{\beta}{\alpha}}$ **8. Ans.** $\omega = \sqrt{\frac{2T_0}{m\ell}}$ **9. Ans.** $1/\sqrt{3}$ **10. Ans.** $2\pi\sqrt{\frac{17L}{18g}}$

11. Ans. $\omega = \sqrt{\frac{k}{(m+6M)}}$ **12. Ans.** M = 1.6 kg **13. Ans.** (a) $\frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$, (b) $\frac{L}{3}$

14. Ans. (a) 2 sec, (b) $T = \frac{2}{5^{1/4}}$ sec

15. Ans. (a) $T = mg + mgA^2\sin^2\sqrt{\frac{g}{\ell}}\,t$ (b) $t = (2n+1)\frac{\pi}{2}\sqrt{\frac{\ell}{g}}$; $n \in I$ $T_{max} = mg + mgA^2$

16. Ans. (b) $\frac{R}{\sqrt{2}}$ **17. Ans.** 100 Nm^{-1} **18. Ans.** $10/\pi$ Hz, $\frac{5\sqrt{37}}{6}$ cm

19. Ans. $T = 2\sqrt{\ell/g}\,[\pi/2 + \sin^{-1}(\alpha/\beta)]$ **20. Ans.** 2 **21. Ans.** 5

22. Ans. 1 **23. Ans.** $f = \frac{1}{2\pi}\sqrt{\frac{3g}{2R}}$

EXERCISE (S-2)

1. Ans. 3cm, $x = 10 - 3\sin 5t$; $\Delta E = 0.135J$ **2. Ans.** $f = \frac{1}{\pi}$; $E = 4\pi^2 \times 10^{-5}J$; $v = 2\pi \times 10^{-2}m/s$

3. Ans. (a) $\frac{\pi}{10}$ sec, (b) 4 cm, (c) 2.40 kg m/s. **4. Ans.** Will increase $\sqrt{1+\frac{2}{5}\left(\frac{R}{\ell}\right)^2}$ times.

5. Ans. (a) $T = 2\pi\sqrt{\ell/3g} = 1.1$ s; (b) $E = \frac{1}{2}mg\ell\alpha^2 = 0.05$ J

6. Ans. $\omega = \sqrt{(I_1\omega_1^2 + I_2\omega_2^2)/(I_1+I_2)}$ **7. Ans.** $\pi\sqrt{m/k} + 2\sqrt{2E/mg^2}$

8. Ans. $\frac{\pi}{3}$ s **9. Ans.** (i) $x_2 = v_0 t + \frac{m_1}{m_2}A(1-\cos\omega t)$ (ii) $\ell_0 = \left(\frac{m_1}{m_2}+1\right)A$

10. Ans. $a = (mg/x)\sqrt{1+2hx/mg}$, $\frac{1}{2}xA^2$ **11. Ans.** 5

12. Ans. $\omega = \sqrt{\frac{3g}{2L}\left(\frac{d_2-d_1}{d_1}\right)}$ **13. Ans.** $\left(\frac{4g}{H}\right)x$

EXERCISE (O-1)

1. Ans. (C) **2. Ans. (D)** **3. Ans. (A)** **4. Ans. (A)** **5. Ans. (A)** **6. Ans. (B)**
7. Ans. (A) **8. Ans. (A)** **9. Ans. (B)** **10. Ans. (A)** **11. Ans. (A)** **12. Ans. (C)**
13. Ans. (C) **14. Ans. (C)** **15. Ans. (A)** **16. Ans. (A)** **17. Ans. (C)** **18. Ans. (B)**
19. Ans. (A) **20. Ans. (B)** **21. Ans. (B)** **22. Ans. (B)** **23. Ans. (C)** **24. Ans. (C)**
25. Ans. (D) **26. Ans. (B)** **27. Ans. (C)** **28. Ans. (B)** **29. Ans. (A)** **30. Ans. (C)**
31. Ans. (A) **32. Ans. (C)** **33. Ans. (B)** **34. Ans. (D)** **35. Ans. (C)** **36. Ans. (A)**
37. Ans. (B) **38. Ans. (B)** **39. Ans. (A)** **40. Ans. (A)** **41. Ans. (B)** **42. Ans. (A)**
43. Ans. (C) **44. Ans. (B)**

EXERCISE (O-2)

1. Ans. (D) **2. Ans. (D)** **3. Ans. (D)** **4. Ans. (C)** **5. Ans. (B)** **6. Ans. (D)**
7. Ans. (B) **8. Ans. (D)** **9. Ans. (A)** **10. Ans. (B)** **11. Ans. (B)** **12. Ans. (A)**
13. Ans. (A) **14. Ans. (C)** **15. Ans. (D)** **16. Ans. (C)** **17. Ans. (A,D)** **18. Ans. (B, C)**
19. Ans. (A,B,C) **20. Ans. (A,B,C)** **21. Ans. (A,C,D)** **22. Ans. (B,D)**
23. Ans. (B,C,D) **24. Ans. (A,B)** **25. Ans. (A,B,C,D)** **26. Ans. (A,C,D)**
27. Ans. (A, C) **28. Ans. (B,C,D)** **29. Ans. (A,B)** **30. Ans. (A, C)**
31. Ans. (A) **32. Ans. (A)** **33. Ans. (A)** **34. Ans. (C)** **35. Ans. (C)** **36. Ans. (A)**
37. Ans. (A)-Q,R ; (B)-R ; (C)-Q,R ; (D)-P,Q,R **38. Ans. (D)**

EXERCISE (JM)

1. Ans. (2) **2. Ans. (4)** **3. Ans. (2)** **4. Ans. (2)** **5. Ans. (4)** **6. Ans. (1)**
7. Ans. (2) **8. Ans. (1)** **9. Ans. (2)** **10. Ans. (4)** **11. Ans. (4)** **12. Ans. (1)**
13. Ans. (2) **14. Ans. (4)** **15. Ans. (1)** **16. Ans. (1)** **17. Ans. (2)**

EXERCISE (JA)

1. Ans. (C) **2. Ans. (B)** **3. Ans. (D)** **4. Ans. (B)** **5. Ans. (A,D)** **6. Ans. (D)**
7. Ans. (C) **8. Ans. (B)** **9. Ans. (A)** **10. Ans. (A,D)** **11. Ans. (B,D)**
12. Ans. (A)-P,Q,R,T; (B)-Q,S; (C)-P,Q,R,S; (D)-P,R,T **13. Ans. (A,B,D)**

ALLEN CAREER INSTITUTE
KOTA (RAJASTHAN)

GEOMETRICAL OPTICS

Chapter Contents

GEOMETRICAL OPTICS

KEY CONCEPTS

RAY OPTICS

INTRODUCTION

The branch of Physics called optics deals with the behavior of light and other electromagnetic waves. Under many circumstances, the wavelength of light is negligible compared with the dimensions of the device as in the case of ordinary mirrors and lenses. A light beam can then be treated as a ray whose propagation is governed by simple geometric rules. The part of optics that deals with such phenomenon is known as geometric optics.

PROPAGATION OF LIGHT

Light travels along straight line in a medium or in vacuum. The path of light changes only when there is an object in its path or where the medium changes. We call this rectilinear (straight–line) propagation of light. Light that starts from a point A and passes through another point B in the same medium actually passes through all the points on the straight line AB. Such a straight line path of light is called a ray of light. Light rays start from each point of a source and travel along straight lines till they fall on an object or a surface separating two media (mediums). A bundle of light rays is called a beam of light.

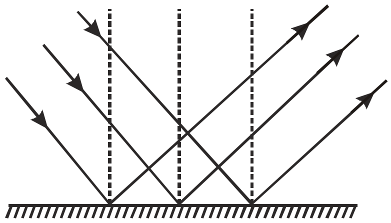
Apart from vacuum and gases, light can travel through some liquids and solids. A medium in which light can travel freely over large distances is called a transparent medium. Water, glycerine, glass and clear plastics are transparent. A medium in which light cannot travel is called opaque. Wood, metals, bricks, etc., are opaque. In materials like oil, light can travel some distance, but its intensity reduces rapidly. Such materials are called translucent.

REFLECTION OF LIGHT

When light rays strike the boundary of two media such as air and glass, a part of light is bounced back into the same medium. This is called **Reflection of light**.

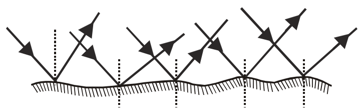
(i) **Regular / Specular reflection :**

When the reflection takes place from a perfect plane surface then after reflection rays remain parallel.
It is called **Regular reflection**.



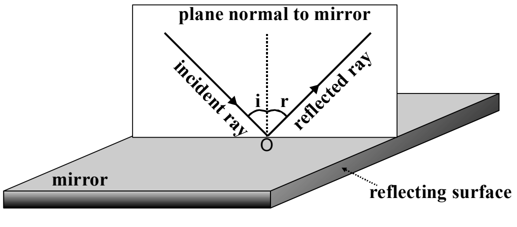
(ii) **Diffused reflection**

When the surface is rough, light is reflected from the surface from bits of its plane surfaces in irregular directions. This is called **diffused reflection**. This process enables us to see an object from any position.

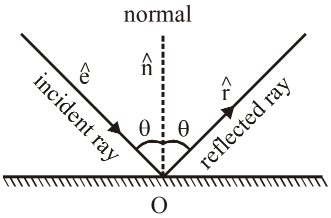


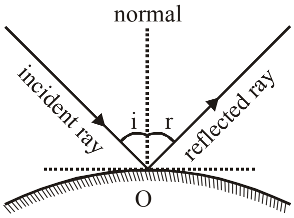
LAWS OF REFLECTION

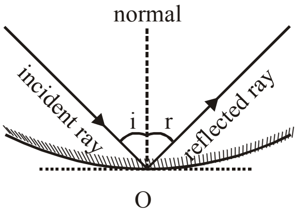
(1) Incident ray, reflected ray and normal lies in the same plane.



(2) The angle of reflection is equal to the angle of incident i.e. ∠ i = ∠r.



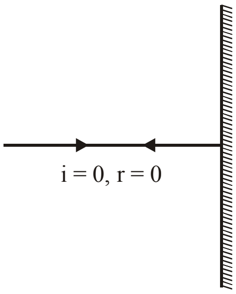




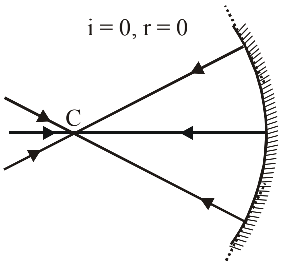
In vector form $\boxed{\hat{r} = \hat{e} - 2(\hat{e}.\hat{n})\hat{n}}$

KEY POINTS

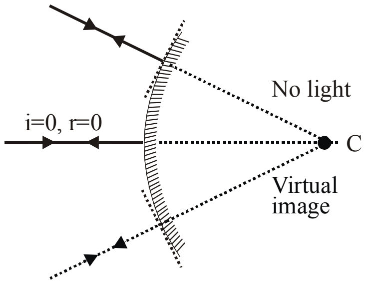
- **Rectilinear propagation of light** : In a homogeneous transparent medium light travels in straight line.
- When a ray is incident normally on a boundary after reflection it retraces its path.



plane mirror



concave mirror



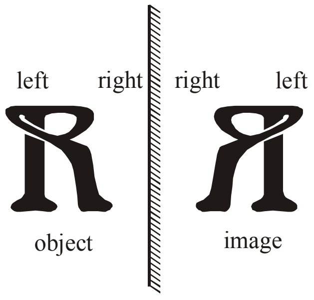
convex mirror

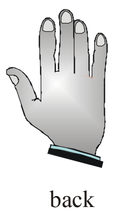
- The frequency, wavelength and speed does not change on reflection.
- Eye is mostly sensitive for yellow colour and least sensitive for violet and red colour.
  Due to this reason :
  • Commercial vehicle's are painted with yellow colour.
  • Sodium lamps (yellow colour) are used in road lights.

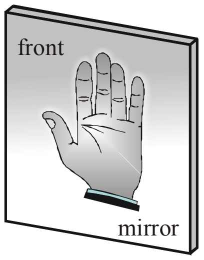
REFLECTION FROM PLANE MIRROR

Plane mirror is the perpendicular bisector of the line joining object and image.

- The image formed by a plane mirror suffers **lateral–inversion,** i.e., in the image formed by a plane mirror left is turned into right and vice–versa with respect to object.







When a watch placed in front of a plane mirror then watch is object and its time is object time and image of watch observed by a person standing in front of mirror then time seen by person.

(i) Object Time = A^H

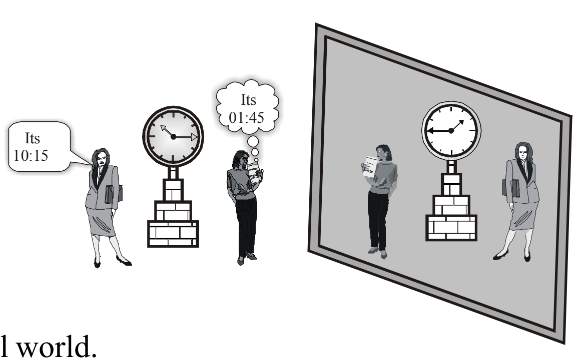
Image Time = $12 - A^H$

(ii) Object Time = $A^H B^M$

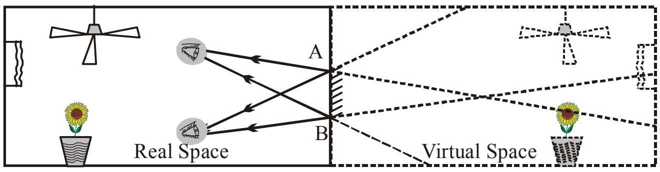
Image Time = $11 - 60' - A^H B^M$

(iii) Object Time = $A^H B^M C^S$

Image Time = $11 - 59' - 60'' - A^H B^M C^S$



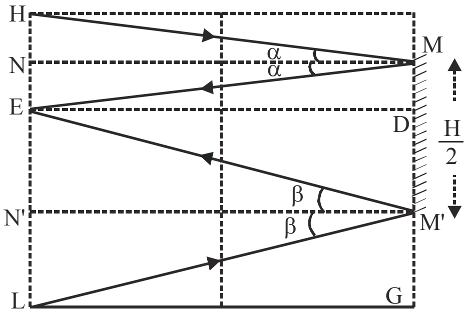
- A plane mirror behaves like a window to virtual world.



AB → Plane mirror

- To see complete image in a plane mirror the minimum length of plane mirror should be half the height of a person.

From figure. ΔHNM and ΔENM are congruent



$\therefore$ EN = HN $\qquad \therefore$ $MD = EN = \frac{1}{2}HE$

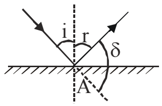
Similarly $\Delta EN'M'$ and $\Delta LN'M'$ are congruent

Length of the mirror $MM' = MD + M'D = \frac{1}{2}HE + \frac{1}{2}EL$

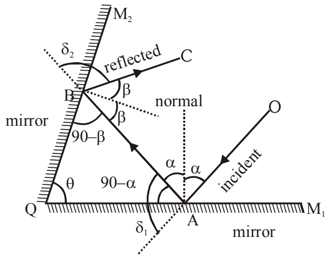
$$= \frac{1}{2}(HE + EL) = \frac{1}{2}HL$$

$\therefore$ Minimum of length of mirror is just half of the person.

- This result does not depend on position of eye (height of the eye from ground).
- This result is independent of distance of person in front of mirror.

- Deviation for a single mirror  $\delta = 180 - (i + r)$; $\angle i = \angle r$; $\delta = 180 - 2i$

- Total deviation produced by the combination of two plane mirrors which are inclined at an angle θ from each other.



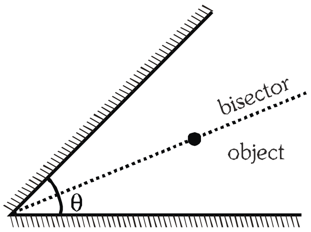
$\delta = \delta_1 + \delta_2 = 180 - 2\alpha + 180 - 2\beta = 360 - 2(\alpha + \beta)$ ...(i)

From ΔQAB, $\theta + 90 - \alpha + 90 - \beta = 180 \Rightarrow \theta = \alpha + \beta$ ...(ii)

Putting the value of θ in (i) from (ii), $\delta = 360 - 2\theta$

- If there are two plane mirror inclined to each other at an angle θ the number of image (n) of a point object formed are determined as follows.

(a) If $\frac{360°}{\theta} = m$ is even then number of images n = m – 1



(b) If $\frac{360°}{\theta} = m$ is odd. There will be two case.

(i) When object is not on bisector, then number of images n = m

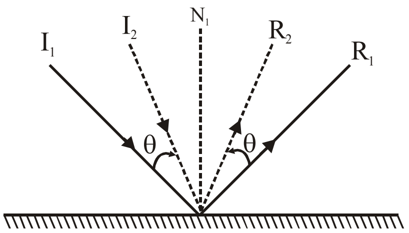
(ii) When object is at bisector, then number of images n = m – 1

(c) If $\frac{360°}{\theta} = m$ is a fraction, and the object is placed symmetrically then no. of images n =nearest even integer

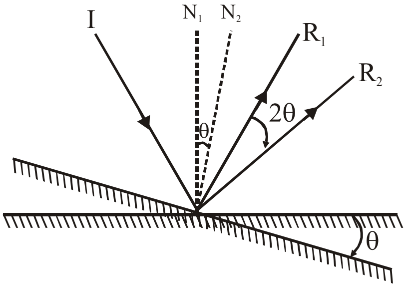
S.No.	**θ in degree**	$m = \frac{360°}{\theta}$	**Number of images formed if object is placed**	
			asymmetrically	**symmetrically**
1.	0	∞	∞	∞
2.	30	12	11	11
3.	45	8	7	7
4.	60	6	5	5
5.	72	5	5	4
6.	75	4.8	–	4
7.	90	4	3	3
8.	112.5	3.2	–	4
9.	120	3	3	2

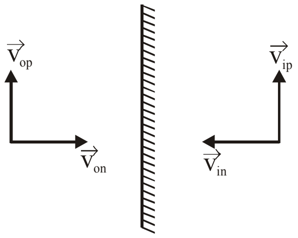
- If the object is placed between two plane mirrors then images are formed due to multiple reflections. At each reflection, a part of light energy is absorbed. Therefore, distant images get fainter.
- Keeping the mirror fixed if the incident ray is rotated by some angle, the reflected ray is also rotated by the same angle but in opposite sense. (See Fig. 1)

(Fig. 1)



(Fig. 2)



- Keeping the incident ray fixed, if the mirror is rotated by some angle, then the reflected ray rotates by double the angle in the same sense. (See Fig. 2)
- 

Stationary Plane Mirror

$\vec{v}_{on} = -\vec{v}_{in}$, $\vec{v}_{op} = \vec{v}_{ip}$

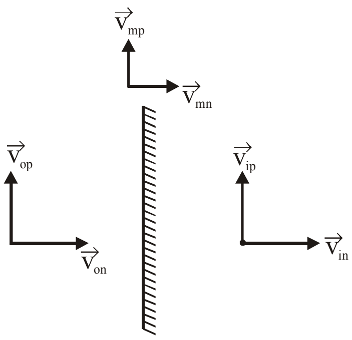
though speed of object and image are the same

v_{op} = component of velocity of object along parallel to mirror.

v_{on} = component of velocity of object along normal to mirror.

v_{ip} = component of velocity of image along parallel to mirror.

v_{in} = component of velocity of image along normal to mirror.

- 

If mirror is moving $\vec{v}_{ip} = \vec{v}_{op}$ and $\left(\vec{v}_{im}\right)_n = -\left(\vec{v}_{om}\right)_n$

$\Rightarrow \vec{v}_{in} - \vec{v}_{mn} = -\left(\vec{v}_{on} - \vec{v}_{mn}\right) \Rightarrow \vec{v}_{in} = 2\vec{v}_{mn} - \vec{v}_{on}$

$\vec{v}_{mn}$ = component of velocity of mirror along normal.

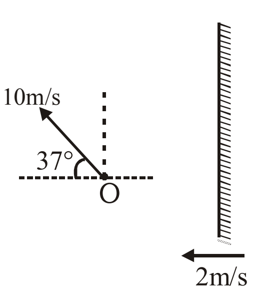
$\vec{v}_{op}$ = component of velocity of object along mirror.

$\vec{v}_{on}$ = component of velocity of object along normal

$\vec{v}_{ip}$ = component of velocity of image along mirror.

$\vec{v}_{in}$ = component of velocity of image along normal.

Ex. Find the velocity of the image.



Sol. $\vec{v}_{ox} = (-10\cos 37°)\hat{i} = -8\hat{i}$ and

$\vec{v}_{oy} = (10\sin 37°)\hat{j} = 6\hat{j}$

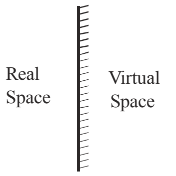
$\vec{v}_{ix} = 2\vec{v}_{mx} - \vec{v}_{ox} = 2(-2i) - \left(-8\hat{i}\right) = 4\hat{i}$ and $\vec{v}_{iy} = \vec{v}_{oy} = 6\hat{j}$

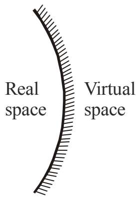
REAL AND VIRTUAL SPACES

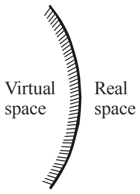
A mirror, plane or spherical divides the space into two ;

(a) Real space, a side where the reflected rays exist.

(b) Virtual space is on the other side where the reflected rays do not exist.

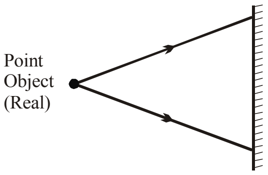


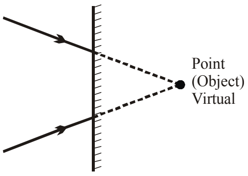


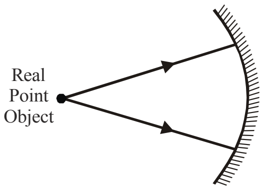


OBJECT

Object is decided by incident rays only. The point object is that point from which the incident rays actually diverge (Real object) or towards which the incident rays appear to converge (virtual object).

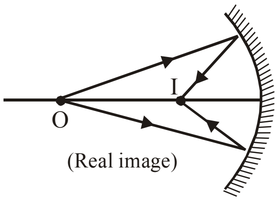


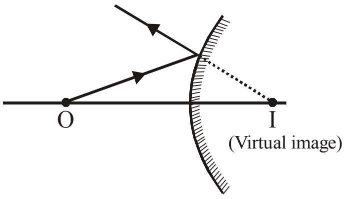




IMAGE

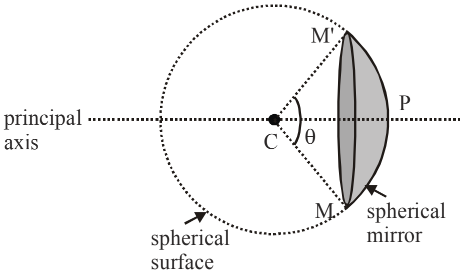
Image is decided by reflected or refracted rays only. The point image is that point at which the refracted / reflected rays reflected from the mirror, actually converge (real image) or from which the refracted /reflected rays appear to diverge (virtual image).

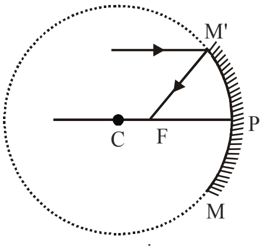




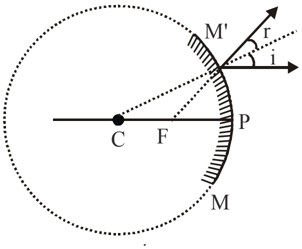
SPHERICAL (CURVED) MIRROR

Curved mirror is part of a hollow sphere. If reflection takes place from the inner surface then The mirror is called concave and if its outer surface acts as reflector it is convex.





concave mirror



convex mirror

DEFINITIONS FOR THIN SPHERICAL MIRRORS

(i) **Pole** is any point on the reflecting surface of the mirror. For convenience we take it as the midpoint P of the mirror (as shown).

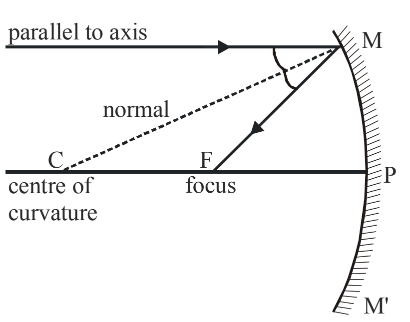
(ii) **Principal–section** is any section of the mirror such as MM' passing through the pole is called principal–section.

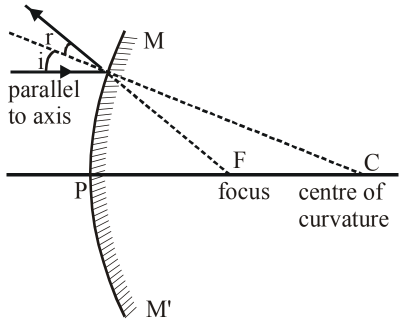
(iii) **Centre of curvature** is the centre C of the sphere of which the mirror is a part.

(iv) **Radius of curvature** is the radius R of the sphere of which the mirror is a part.

(v) **Principal–axis** is the line CP, joining the pole and centre of curvature of the mirror.

(vi) **Principal– focus** is an image point F on principal axis for which object is at infinity.

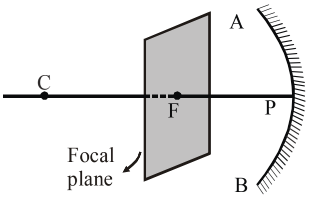


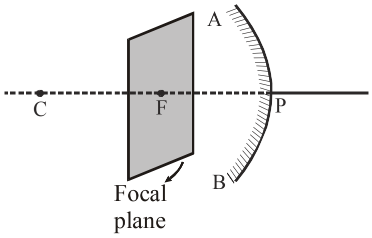


(vii) **Focal–length** is the distance PF between pole P and focus F along principal axis.

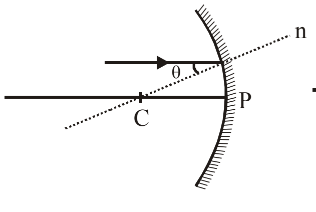
(viii) **Aperture,** in reference to a mirror, means the effective diameter of the light reflecting area of the mirror.

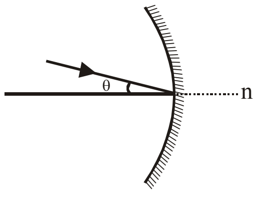
(ix) **Focal Plane** is the plane passing through focus and perpendicular to principal axis.



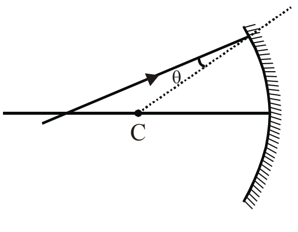


(x) **Paraxial Rays** Those rays which make small angle with normal at point of incidence and hence are close to principal axis.



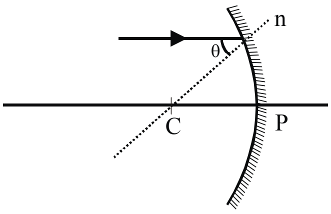


(θ is very small)



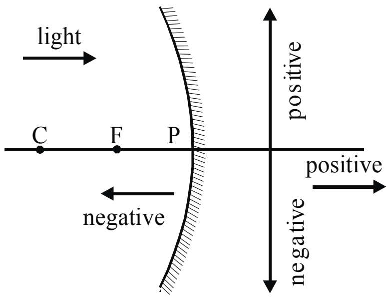
(xi) **Marginal rays :**

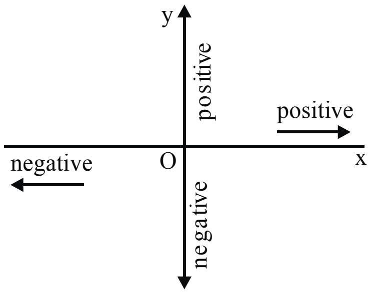
Rays having a large angle of incidence

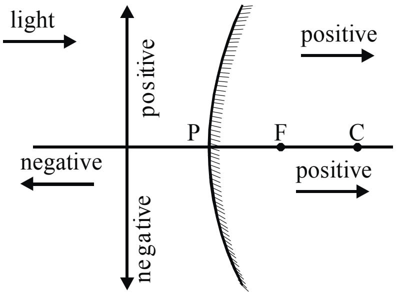


(θ is large)

SIGN– CONVENTION





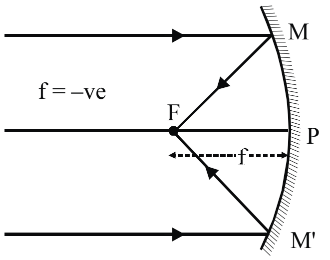


- Along principal axis, distances are measured from the pole (pole is taken as the origin).
- Distance in the direction of light are taken to be positive while opposite to be negative.
- The distances above principal axis are taken to be positive while below it negative.
- Whenever and wherever possible the ray of light is taken to travel from left to right.

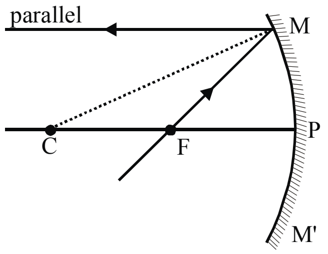
RULES FOR IMAGE FORMATION (FOR PARAXIAL RAYS ONLY)

(These rules are based on the laws of reflection ∠i = ∠r)

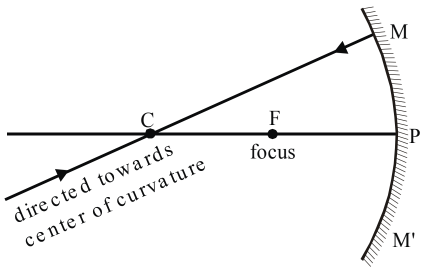
- A ray parallel to principal axis after reflection from the mirror passes or appears to pass through its focus (by definition of focus).



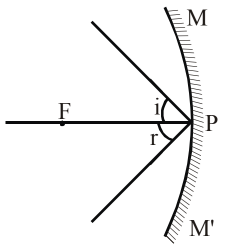
- A ray passing through or directed towards focus, after reflection from the mirror, becomes parallel to the principal axis.



- A ray passing through or directed towards centre of curvature, after reflection from the mirror, retraces its path (as for it ∠ i =0 and so ∠r =0).



- Incident and reflected rays at the pole of a mirror are symmetrical about the principal axis ∠i = ∠r.



RELATIONS FOR SPHERICAL MIRRORS

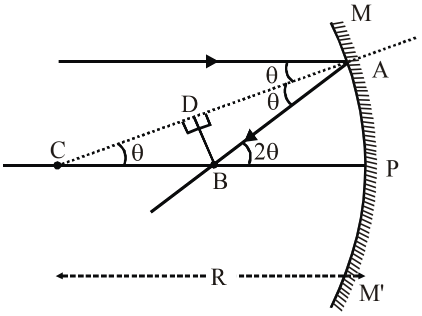
Relation between f and R for the spherical mirror

- **For Marginal rays**

In ΔABC, AB = BC

AC = CD + DA = 2BCcosθ ⇒ R = 2BCcosθ

$$\Rightarrow BC = \frac{R}{2\cos\theta} \text{ and } BP = PC - BC = R - \frac{R}{2\cos\theta}$$



Note : B is not the focus ; it is just a point where a marginal ray after reflection meets.

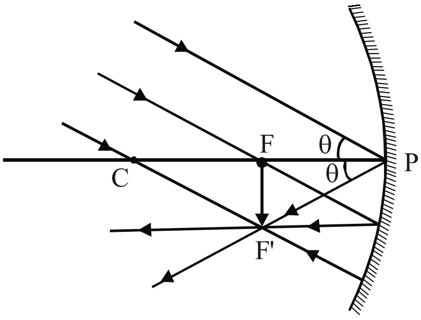
- **For paraxial rays (parallel to principal axis)**

  (θ small so $\sin\theta \simeq \theta$, $\cos\theta \simeq 1$, $\tan\theta \simeq \theta$) Hence $BC = \frac{R}{2}$ and $BP = \frac{R}{2}$

Thus, point B is the midpoint of PC (i.e. radius of curvature) and is defined as FOCUS so $BP = f = \frac{R}{2}$

(**Definition :** Paraxial rays parallel to the principal axis after reflection from the mirror meet the principal axis at focus)

- BP is the focal length (f) $\boxed{\mathbf{f = \frac{R}{2}}}$

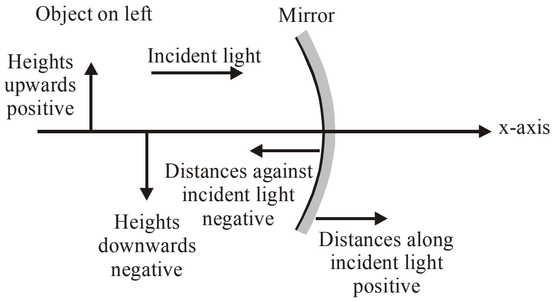


- **Paraxial rays (not parallel to principal axis)**

Such rays after reflection meet at a point in the focal plane (F'), such that

$$\frac{FF'}{FP} = \tan\theta \approx \theta \Rightarrow \frac{FF'}{f} = \theta \Rightarrow FF' = f\theta$$

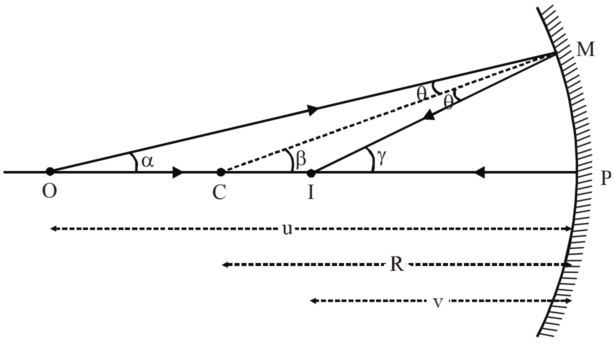
Sign convention



<u>Relation between u,v and f for curved mirror</u>

An object is placed at a distance u from the pole of a mirror and its image is formed at a distance v (from the pole)

If angle is very small : $\alpha = \frac{MP}{u}$, $\beta = \frac{MP}{R}$, $\gamma = \frac{MP}{v}$



from ΔCMO, $\beta = \alpha + \theta \Rightarrow \theta = \beta - \alpha$

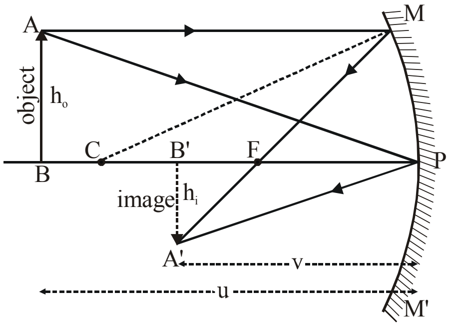
from ΔCMI, $\gamma = \beta + \theta \Rightarrow \theta = \gamma - \beta$

so we can write $\beta - \alpha = \gamma - \beta \Rightarrow 2\beta = \gamma + \alpha$

$$\therefore \frac{2}{R} = \frac{1}{v} + \frac{1}{u} \Rightarrow \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

MAGNIFICATION

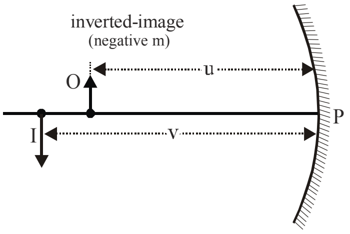
Transverse or lateral magnification

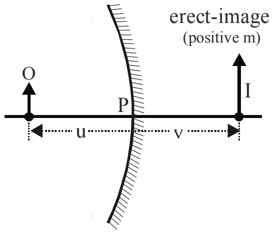


Linear magnification $m = \dfrac{\text{height of image}}{\text{height of object}} = \dfrac{h_i}{h_o}$

ΔABP and ΔA'B'P are similar so $\dfrac{-h_i}{h_o} = \dfrac{-v}{-u} \Rightarrow \dfrac{h_i}{h_o} = -\dfrac{v}{u}$

Magnification $m = -\dfrac{v}{u}$; $m = -\dfrac{v}{u} = \dfrac{f}{f-u} = \dfrac{f-v}{f} = \dfrac{h_i}{h_o}$



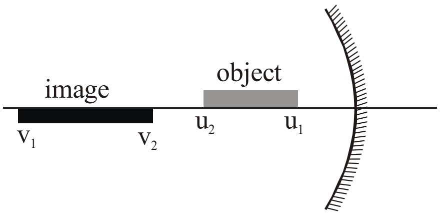


If one dimensional object is placed perpendicular to the principal axis then linear magnification is called transverse or lateral magnification. $\boxed{m = \dfrac{h_i}{h_o} = -\dfrac{v}{u}}$

Magnification	**Image**	**Magnification**	**Image**
$\lvert m\rvert > 1$	enlarged	$\lvert m\rvert < 1$	diminished
$m < 0$	inverted	$m > 0$	erect

- **Longitudinal magnification**

If one dimensional object is placed with its length along the principal axis then linear magnification is called longitudinal magnification.



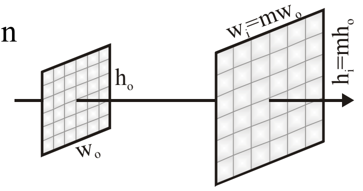
Longitudinal magnification $m_L = \dfrac{\text{length of image}}{\text{length of object}} = \dfrac{|v_2 - v_1|}{|u_2 - u_1|}$

For small objects only : $m_L = -\dfrac{dv}{du}$

differentiation of $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$ gives us $-\dfrac{dv}{v^2} - \dfrac{du}{u^2} = 0 \Rightarrow -\dfrac{dv}{du} = \left[\dfrac{v}{u}\right]^2$ so $m_L = -\dfrac{dv}{du} = \left[\dfrac{v}{u}\right]^2 = m^2$

- **Superficial magnification**

If two dimensional object placed with its plane perpendicular to principal axis its magnification is known as superficial magnification



Linear magnification $m = \dfrac{h_i}{h_o} = \dfrac{w_i}{w_o}$

$h_i = mh_o$, $w_i = mw_o$ and $A_{obj} = h_o \times w_o$

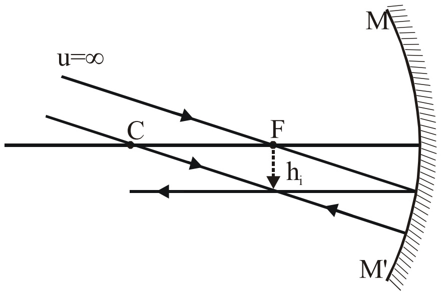
Area of image : $A_{image} = h_i \times w_i = mh_o \times mw_o = m^2 A_{obj}$

superficial magnification $m_s = \dfrac{\text{area of image}}{\text{area of object}} = \dfrac{(ma)\times(mb)}{(a\times b)} = m^2$

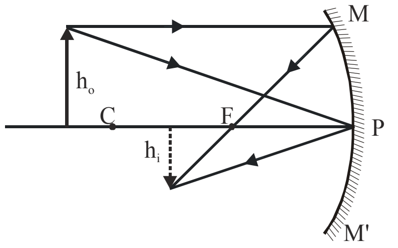
IMAGE FORMATION BY SPHERICAL MIRRORS

Concave mirror

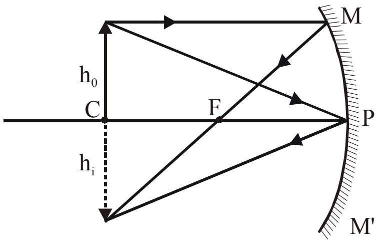
(i) **Object :** Placed at infinity
Image : real, inverted, diminished at F
$|m| << 1$ & $m<0$



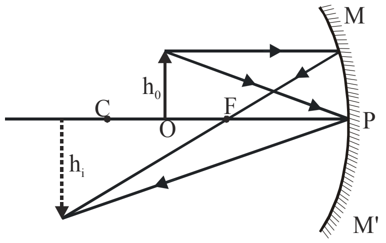
(ii) **Object :** Placed in between infinity and C
Image : real, inverted, diminished in between C and F
$|m| < 1$ & $m<0$



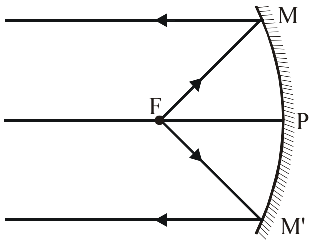
(iii) **Object :** Placed at C
Image : real, inverted, equal at C
$(m = -1)$



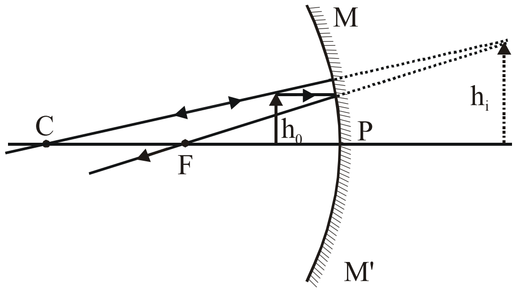
(iv) **Object :** Placed in between F and C
Image : real, inverted, enlarged beyond C
$|m| > 1$ & $m<0$



(v) **Object :** Placed at F
Image : real, inverted, very large (assumed) at infinity $(m << -1)$



(vi) **Object :** Placed between F and P
Image : virtual, erect, enlarged and behind the mirror $(m > +1)$

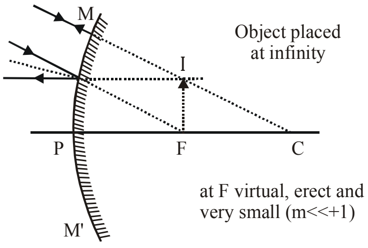


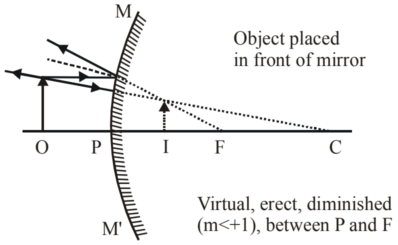
For concave mirror

Object	Image	Magnification
$-\infty$	F	$\lvert m\rvert << 1$ & $m<0$
$-\infty - C$	$C - F$	$\lvert m\rvert < 1$ & $m<0$
C	C	$m = -1$
$C - F$	$-\infty - C$	$\lvert m\rvert > 1$ & $m<0$
Just before F towards C	$-\infty$	$m << -1$
Just after F towards P	$+\infty$	$m >> 1$

Convex mirror

Image is always virtual and erect, whatever be the position of the object and m is always positive.



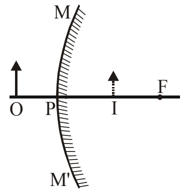
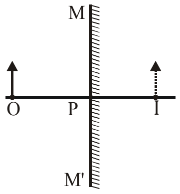
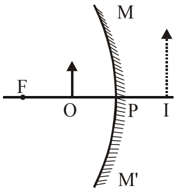


KEY POINTS

- **Differences in real & virtual image for spherical mirror**

Real Image	Virtual Image
(i) Inverted w.r.t. object	(i) Erect w.r.t. object
(ii) Can be obtained on screen	(ii) Can not be obtained on screen
(iii) Its magnification is negative	(iii) Its magnification is positive
(iv) Forms in front of mirror	(iv) Forms behind the mirror

- For real extended object, if the image formed by a single mirror is erect it is always virtual (i.e.,m is + ve) and in this situation if the size of image is :

Smaller than object the mirror is convex	Equal to object the mirror is plane	Larger than object the mirror is concave
m<+1 	m=+1 	m>+1 

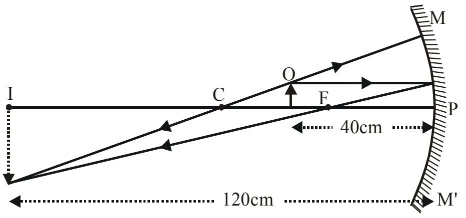
Ex. The focal length of a concave mirror is 30cm. Find the position of the object in front of the mirror, so that the image is three times the size of the object.

Sol. As the object is in front of the mirror it is real and for real object the magnified image formed by concave mirror can be inverted (i.e.,real) or erect (i.e.,virtual), so there are two possibilities.

(a) If the image is inverted (i.e., real)

$$m = \frac{f}{f-u} \Rightarrow -3 = \frac{-30}{-30-u} \Rightarrow u = -40 \text{ cm}$$

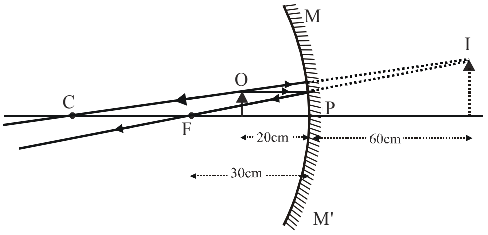
Object must be at a distance of 40 cm in front of the mirror (in between C and F).



(b) If the image is erect (i.e., virtual)

$$m = \frac{f}{f-u} \Rightarrow 3 = \frac{-30}{-30-u} \Rightarrow u = -20 \text{ cm}$$

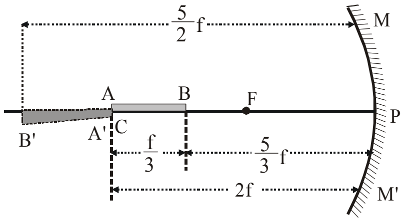
Object must be at a distance of 20 cm in front of the mirror (in between F and P).



Ex. A thin rod of length $\frac{f}{3}$ is placed along the principal axis of a concave mirror of focal length f such that its image which is real and elongated, just touches the rod. What is magnification ?

Sol. Image is real and enlarged, the object must be between C and F. One end A' of the image coincides with the end A of rod itself, so

$$v_A = u_A,\ \frac{1}{v_A}+\frac{1}{v_A}=\frac{1}{-f}\ \text{ i.e., } v_A = u_A = -2f$$



so it clear that the end A is at C. $\because$ the length of rod is $\frac{f}{3}$

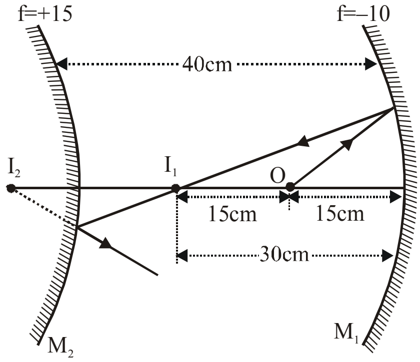
$\therefore$ distance of the other end B from P is $u_B = 2f-\frac{f}{3}=\frac{5}{3}f$

if the distance of image of end B from P is v_B then $\frac{1}{v_B}+\frac{1}{-\frac{5}{3}f}=\frac{1}{-f}\Rightarrow v_B=-\frac{5}{2}f$

$\therefore$ the length of the image $|v_B|-|v_A|=\frac{5}{2}f-2f=\frac{1}{2}f$ and magnification $m=\frac{|v_B|-|v_A|}{|u_B|-|u_A|}=\frac{\frac{1}{2}f}{-\frac{1}{3}f}=-\frac{3}{2}$

Negative sign implies that image is inverted with respect to object and so it is real.

Ex A concave mirror of focal length 10 cm and convex mirror of focal length 15 cm are placed facing each other 40 cm apart. A point object is placed between the mirror on their common axis and 15 cm from the concave mirror. Find the position of image produced by the reflection first at concave mirror and then at convex mirror.



Sol. For M_1 mirror O act as a object, let its image is I_1 then,

$$u=-15\text{ cm},\ f=-10\text{ cm}\ \Rightarrow \frac{1}{v}+\frac{1}{-15}=\frac{1}{-10}\Rightarrow v=-30\text{ cm}$$

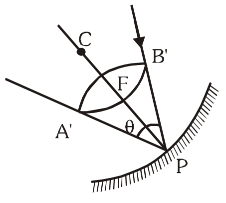
Image I_1 will act as a object for mirror M_2 its distance from mirror M_2.

$u_1 = -(40-30)\text{ cm} = -10\text{ cm}$

so $\frac{1}{v_1}+\frac{1}{u_1}=\frac{1}{f}\Rightarrow\frac{1}{v_1}-\frac{1}{10}=\frac{1}{15}\Rightarrow v_1=+6\text{cm}$

So final image I_2 is formed at a distance 6 cm behind the convex mirror and is virtual.

Ex. The sun subtends an angle θ radians at the pole of a concave mirror of focal length f. What is the diameter of the image of the sun formed by the mirror.



Sol. Since the sun is at large distance very distant, u is very large and so $\frac{1}{u}\approx 0$

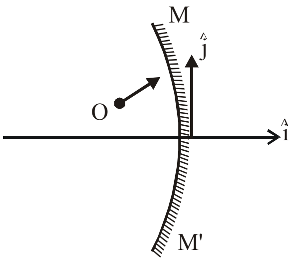
$$\therefore\ \frac{1}{v}+\frac{1}{u}=\frac{1}{f}\ \Rightarrow\frac{1}{v}=-\frac{1}{f}\ \Rightarrow\ v=-f$$

The image of sun will be formed at the focus and will be real, inverted and diminished

A'B' = height of image and $\theta=\frac{\text{Arc}}{\text{Radius}}=\frac{A'B'}{FP}\Rightarrow\ \theta=\frac{d}{f}\Rightarrow d=f\theta$

VELOCITY OF IMAGE OF MOVING OBJECT (SPHERICAL MIRROR)

(a) Velocity component along axis (Longitudinal velocity)



When an object is coming from infinite towards the focus of concave mirror

$$\because \frac{1}{v}+\frac{1}{u}=\frac{1}{f} \Rightarrow -\frac{1}{v^2}\frac{dv}{dt}-\frac{1}{u^2}\frac{du}{dt}=0 \Rightarrow \vec{v}_{ix}=-\frac{v^2}{u^2}\vec{v}_{ox}=-m^2\,\vec{v}_{ox}$$

$$\left[v_{ix}=\frac{dv}{dt}=\text{velocity of image along principal-axis; } v_{ox}=\frac{du}{dt}=\text{velocity of object along principal-axis}\right]$$

(b) Velocity component perpendicular to axis (Transverse velocity)

$$m=\frac{h_I}{h_0}=-\frac{v}{u}=\frac{f}{f-u} \Rightarrow h_I=\left(\frac{f}{f-u}\right)h_0$$

$$\frac{dh_I}{dt}=\left(\frac{f}{f-u}\right)\frac{dh_0}{dt}+\frac{f\,h_0}{(f-u)^2}\frac{du}{dt}\ ;\ \vec{v}_{iy}=\left[m\vec{v}_{oy}+\frac{m^2h_0}{f}\vec{v}_{ox}\right]\hat{j}$$

$$\begin{bmatrix}\dfrac{dh_I}{dt}=\text{velocity of image}\perp^r\text{ to principal-axis}\\ \dfrac{dh_o}{dt}=\text{velocity of object }\perp^r\text{ to principal-axis}\end{bmatrix}$$

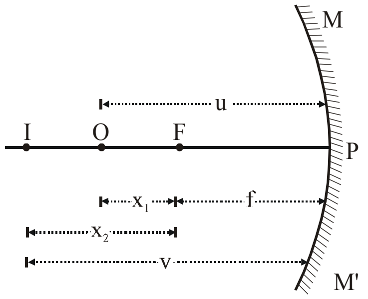
Note : Here principal axis has been taken to be along x–axis.

POWER OF A MIRROR

The power of a mirror is defined as $P=-\frac{1}{f(m)}=-\frac{100}{f(cm)}$

NEWTON'S FORMULA

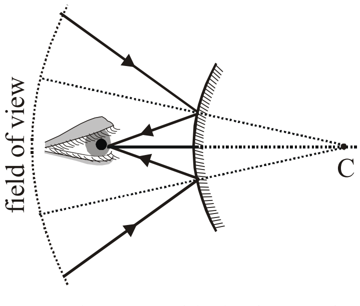
In case if spherical mirrors if object distance (x_1) and image distance (x_2) are measured from focus instead of pole, $u=-(f+x_1)$ and $v=-(f+x_2)$,



by $\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \Rightarrow -\frac{1}{(f+x_2)}-\frac{1}{(f+x_1)}=-\frac{1}{f}$

on solving $\boxed{x_1x_2=f^2}$ This is Newton's formula.

POINTS

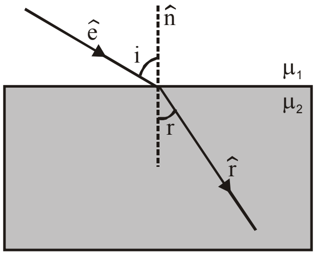


- **Convex mirrors** gives erect, virtual and diminished image. In convex mirror the field of view is increased as compared to plane mirror. It is used as rear–view mirror in vehicles.
- **Concave mirrors** give enlarged, erect and virtual image, so these are used by dentists for examining teeth. Due to their converging property concave mirrors are also used as reflectors in automobile head lights and search lights
- As focal length of a spherical mirror f = R/2 depends only on the radius of mirror and is independent of wavelength of light and refractive index of medium so the focal length of a spherical mirror in air or water and for red or blue light is same.

REFRACTION

Refraction is the phenomenon in which direction of propagation of light changes at the boundary when it passes from one medium to the other. In case of refraction frequency does not change.

◎ Laws of Refraction



(i) Incident ray, refracted ray and normal always lie in the same plane.

In vector form $\boxed{(\hat{e}\times\hat{n}).\hat{r}=0}$

(ii) The product of refractive index and sine of angle of incidence at a point in a medium is constant. $\mu_1 \sin i = \mu_2 \sin r$ (Snell's law)

In vector form $\boxed{\mu_1\left|\hat{e}\times\hat{n}\right| = \mu_2\left|\hat{r}\times\hat{n}\right|}$

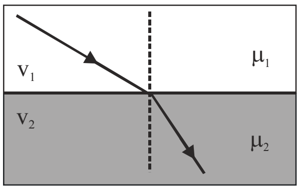
Absolute refractive index

It is defined as the ratio of speed of light in free space 'c' to that in a given medium v. μ or $n = \frac{c}{v}$

Denser is the medium, lesser will be the speed of light and so greater will be the refractive index,

$$\because v_{glass} < v_{water}, \qquad \therefore \mu_G > \mu_W$$

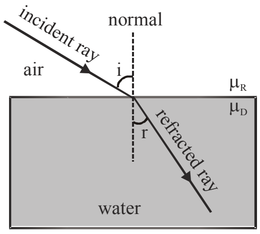
Relative refractive index



When light passes from one medium to the other, the refractive index of medium 2 relative to 1 is written as ${}_1\mu_2$ and is defined as

$${}_1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{(c/v_2)}{(c/v_1)} = \frac{v_1}{v_2}$$

◎ Bending of light ray

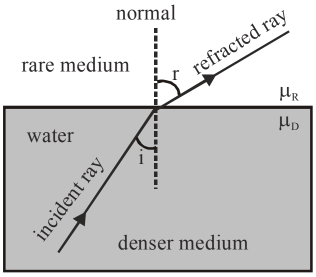


According to Snell's law, $\mu_1 \sin i = \mu_2 \sin r$

(i) If light passes from rarer to denser medium $\mu_1 = \mu_R$ and $\mu_2 = \mu_D$

so that $\frac{\sin i}{\sin r} = \frac{\mu_D}{\mu_R} > 1 \Rightarrow \angle i > \angle r$

In passing from rarer to denser medium, the ray bends towards the normal.



(ii) If light passes from denser to rarer medium $\mu_1 = \mu_D$ and $\mu_2 = \mu_R$

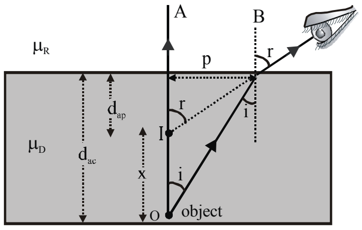
$$\frac{\sin i}{\sin r} = \frac{\mu_R}{\mu_D} < 1 \Rightarrow \angle i < \angle r$$

In passing from denser to rarer medium, the ray bends away from the normal

APPARENT DEPTH AND NORMAL SHIFT

If a point object in denser medium is observed from rarer medium and boundary is plane, then from Snell's law we have μ_D sin i = μ_R sin r...(i)

If the rays OA and OB are close enough to reach the eye.



$$\sin i \simeq \tan i = \frac{p}{d_{ac}} \text{ and } \sin r \simeq \tan r = \frac{p}{d_{ap}}$$

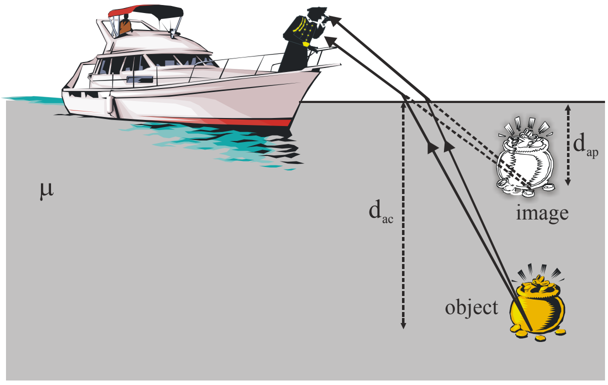
here d_{ac} = actual depth, d_{ap} = apparent depth

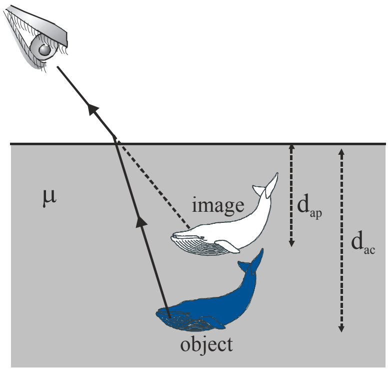
So that equation (i) becomes $\mu_D = \frac{p}{d_{ac}} = \mu_R \frac{p}{d_{ap}} \Rightarrow \frac{d_{ac}}{d_{ap}} = \frac{\mu_D}{\mu_R} = \frac{\mu_1}{\mu_2}$

(If $\mu_R = 1, \mu_D = \mu$) then $d_{ap} = \frac{d_{ac}}{\mu}$ so $d_{ap} < d_{ac}$ ...(ii)

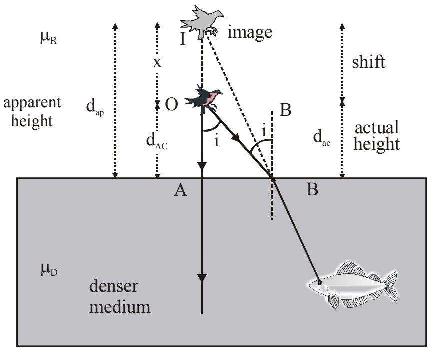
The distance between object and its image, called normal shift (x)

$$x = d_{ac} - d_{ap}\left[\because d_{ap} = \frac{d_{ac}}{\mu}\right];\ x = d_{ac} - \frac{d_{ac}}{\mu} = d_{ac}\left[1 - \frac{1}{\mu}\right] \text{...(iii)} \qquad \text{If } d_{ac} = d \text{ then } \boxed{x = d\left[1 - \frac{1}{\mu}\right]}$$





Object in a rarer medium is seen from a denser medium



$$\frac{d_{ac}}{d_{ap}} = \frac{\mu_1}{\mu_2} = \frac{\mu_R}{\mu_D} = \frac{1}{\mu}(<1)$$

$d_{ap} = \mu\, d_{ac}$ i.e., $d_{ap} > d_{ac}$

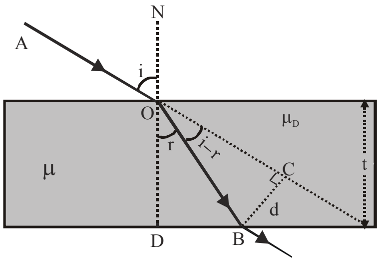
A high flying object appears to be higher than in reality.

$x = d_{ap} - d_{ac} \Rightarrow x = [\mu - 1]\, d_{ac}$

LATERAL SHIFT

The perpendicular distance between incident and emergent ray is known as lateral shift.

Lateral shift d = BC and t = thickness of slab

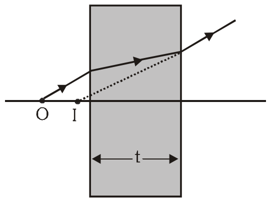


In Δ BOC $\sin(i-r)\dfrac{BC}{OB} = \dfrac{d}{OB} \Rightarrow d = OB\sin(i-r)$ ...(i)

In Δ OBD $\cos r = \dfrac{OD}{OB} = \dfrac{t}{OB} \Rightarrow OB = \dfrac{t}{\cos r}$ ...(ii)

From (i) and (ii) $d = \dfrac{t}{\cos r}\sin(i-r)$

TRANSPARENT GLASS SLAB (Normal shift)



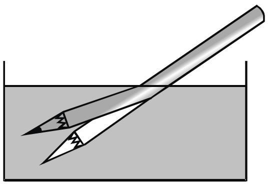
When an object is placed infront of a glass slab, it shift the object in the direction of incident light and form a image at a distance x.

$$x = t\left[1-\frac{1}{\mu}\right]$$

SOME ILLUSTRATIONS OF REFRACTION

- **Bending of an object**

  When a point object in a denser medium is seen from a rarer medium it appears to bend by $\frac{d}{\mu}$



- **Twinkling of stars**

  Due to fluctuations in refractive index of atmosphere the refraction becomes irregular and the light sometimes reaches the eye and sometimes it does not. This gives rise to twinkling of stars.

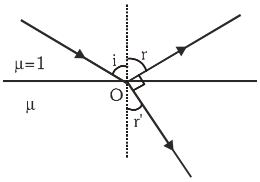
KEY POINTS

- μ is a scalar and has no units and dimensions.
- If ε_0 and μ_0 are electric permittivity and magnetic permeability respectively of free space while ε and μ those of a given medium, then according to electromagnetic theory.

$$c = \frac{1}{\sqrt{\varepsilon_0\mu_0}} \text{ and } v_m = \frac{1}{\sqrt{\varepsilon\mu}} \Rightarrow n_m = \frac{c}{v} = \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}} = \sqrt{\varepsilon_r\mu_r}$$

- As in vacuum or free space, speed of light of all wavelengths is maximum and equal to c so for all wavelengths the refractive index of free space is minimum and is $\mu = \frac{c}{v_m} = \frac{c}{c} = 1$

Ex. A ray of light is incident on a transparent glass slab of refractive index 1.62. If the reflected and refracted rays are mutually perpendicular, what is the angle of incidence ? [$\tan^{-1}(1.62) = 58.3°$]



Sol. According to given problem : $r + 90° + r' = 180°$

i.e, $r' = 90° - r \Rightarrow r' = (90° - i)$ [$\because \angle i = \angle r$]

And as according to Snell's law: $1 \sin i = \mu \sin r'$

$\sin i = \mu \sin(90 - i) \Rightarrow \sin i = \mu \cos i$ [$\because \sin(90 - i) = \cos i$]

$\Rightarrow \tan i = \mu \Rightarrow i = \tan^{-1}\mu = \tan^{-1}(1.62) = 58.3°$

Ex. A 20 cm thick glass slab of refractive index 1.5 is kept infront of a plane mirror. An object is kept in air at a distance 40 cm from the mirror. Find the position of image w.r.t an observer near the object. What is effect of separation between glass slab and the mirror on image.

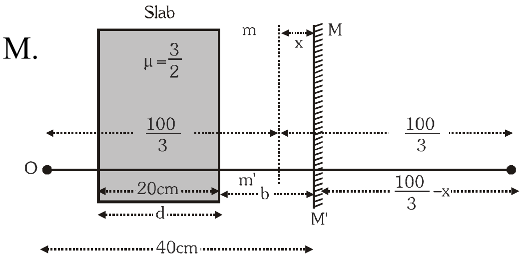
Sol. Shifting in object due to glass slab $x = d\left(1-\frac{1}{\mu}\right) = 20\left[1-\frac{1}{1.5}\right] = \frac{20}{3}$ cm

Distance of object from mirror (as seen by mirror) $= 40 - \frac{20}{3} = \frac{100}{3}$ cm

Image will be formed at a distance $\frac{100}{3}$ cm from mirror M.

Shifting in image due to glass slab $= \frac{20}{3}$ cm

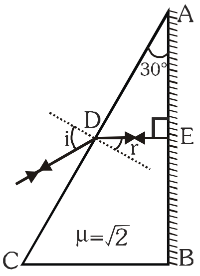
So distance of image from mirror $= \frac{100}{3} - \frac{20}{3} = \frac{80}{3}$ cm



Distance of image from the actual plane mirror is independent of separation b between glass slab and the mirror. If the distance is more then brightness of image will be less.

Ex. If one face of a prism angle 30° and $\mu = \sqrt{2}$ is silvered, the incident ray retraces its initial path. What is the angle of incidence ?

Sol. As incident ray retraces its path the ray is incident normally on the silvere face of the prism as shown in figure.



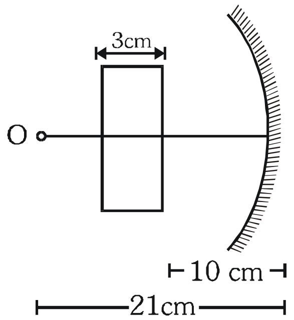
Further, as in ΔAED $30° + 90° + \angle D = 180° \Rightarrow \angle D = 60°$

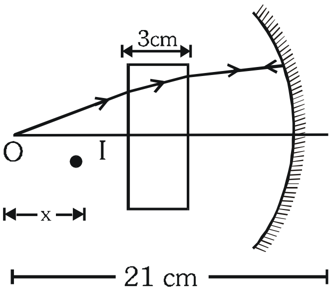
Now as by construction, $\angle D + \angle r = 90° \Rightarrow \angle r = 90° - 60° = 30°$

$\therefore$ from Snell's law at surface AC, $1 \sin i = \sqrt{2} \sin 30° = \sqrt{2} \times \frac{1}{2} = \frac{1}{\sqrt{2}}$

$\therefore \sin i = \frac{1}{\sqrt{2}}$ so $i = 45°$

Ex. An object is placed 21 cm infront of a concave mirror of radius of curvature 20 cm. A glass slab of thickness 3 cm and refractive index 1.5 is placed closed to the mirror in space between the object and the mirror. Find the position of final image formed if distance of nearer surface of the slab from the mirror is 10 cm.





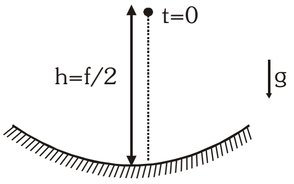
Sol. shift by slab $x = d\left(1 - \frac{1}{\mu}\right) = 3\left(1 - \frac{1}{1.5}\right) = 1\text{cm}$

for image formed by mirror $u = -(21 - 1)\text{ cm} = -20\text{ cm}$.

$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \Rightarrow \frac{1}{-20} + \frac{1}{v} = \frac{1}{-10} \Rightarrow v = -20\text{ cm}$

shift in the direction of light $v = -(20 + 1) = -21$ cm.

Ex. A particle is dropped along the axis from a height $\frac{f}{2}$ on a concave mirror of focal length f as shown in figure. Find the maximum speed of image.



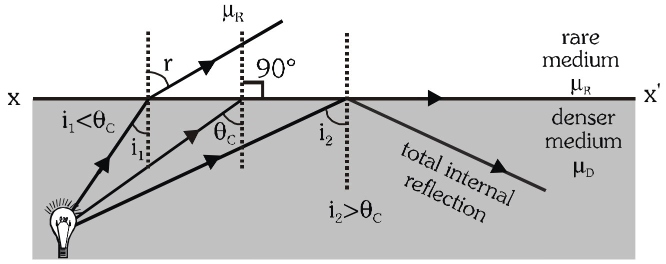
Sol. $v_{IM} = -m^2 v_{OM} = -m^2(gt)$ where

$$m = \frac{f}{f-u} = \frac{-f}{-f + \left(\frac{f}{2} - \frac{gt^2}{2}\right)} = \frac{2f}{f + gt^2} \Rightarrow v_1 = -\left(\frac{2f}{f+gt^2}\right)^2 (gt) = \frac{-4f^2 gt}{(f+gt^2)^2}$$

For maximum speed $\frac{dv_I}{dt} = 0 \Rightarrow t = \sqrt{\frac{f}{3g}} \Rightarrow v_{I\max} = \frac{3}{4}\sqrt{3fg}$

TOTAL INTERNAL REFLECTION

When light ray travel from denser to rarer medium it bend away from the normal if the angle of incident is increased, angle of refraction will also increased. At a particular value of angle the refracted ray subtend 90^0 angle with the normal, this angle of incident is known as critical angle (θ_C). If angle of incident further increase the ray come back in the same medium this phenomenon is known as total internal reflection.



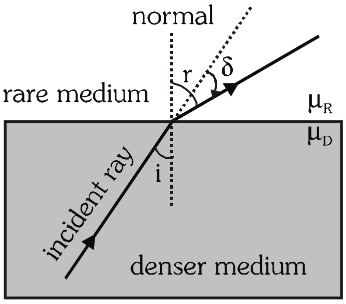
CONDITIONS

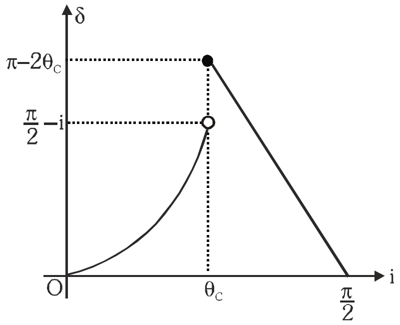
- Angle of incident > critical angle [$i > \theta_c$]
- Light should travel from denser to rare medium ⇒ Glass to air, water to air, Glass to water

Snell's Law at boundary xx', $\mu_D \sin\theta_C = \mu_R \sin 90°$ ⇒ $\sin\theta_C = \frac{\mu_R}{\mu_D}$

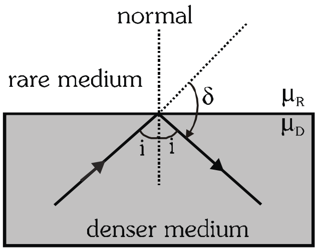
Graph between angle of deviation (δ) and angle of incidence (i) as rays goes from denser to rare medium

- If $i < \theta_c$ $\mu_D \sin i = \mu_R \sin r$; $r = \sin^{-1}\left(\frac{\mu_D}{\mu_R}\sin i\right)$ so $\delta = r - i = \sin^{-1}\left(\frac{\mu_D}{\mu_R}\sin i\right) - i$

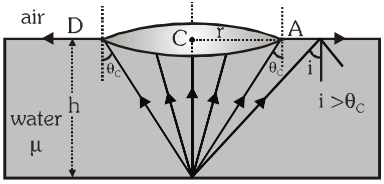


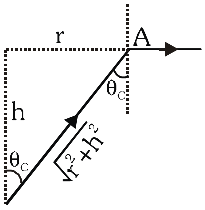


- If $i > \theta_c$; $\delta = \pi - 2i$



- A point object is situated at the bottom of tank filled with a liquid of refractive index μ upto height h. It is found light from the source come out of liquid surface through a circular portion above the object





$$\sin\theta_C = \frac{r}{\sqrt{r^2+h^2}} \ \& \ \sin\theta_C = \frac{1}{\mu} \Rightarrow \frac{1}{\mu} = \frac{r}{\sqrt{r^2+h^2}} \Rightarrow \frac{1}{\mu^2} = \frac{r^2}{r^2+h^2}$$

$$\Rightarrow \mu^2 r^2 = r^2 + h^2 \Rightarrow (\mu^2 - 1) r^2 = h^2 \ \Rightarrow \text{radius of circular portion}$$

$$r = \frac{h}{\sqrt{\mu^2 - 1}} \quad \text{and} \quad \text{area} = \pi r^2$$

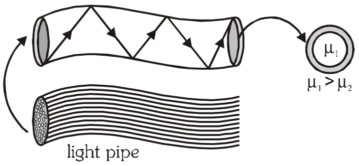
SOME ILLUSTRATIONS OF TOTAL INTERNAL REFLECTION

- **Sparkling of diamond**

  The sparkling of diamond is due to total internal reflection inside it. As refractive index for diamond is 2.5 so $\theta_c = 24°$. Now the cutting of diamond are such that $i > \theta_C$. So TIR will take place again and again inside it. The light which beams out from a few places in some specific directions makes it sparkle.

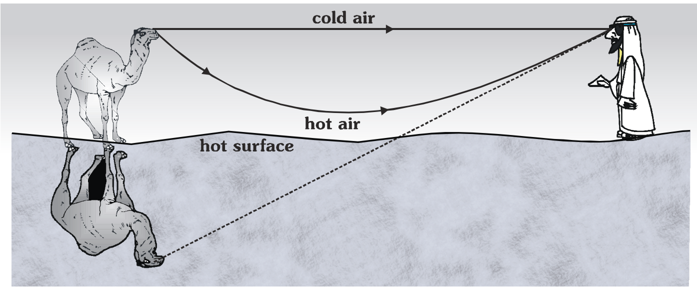
- **Optical Fibre**

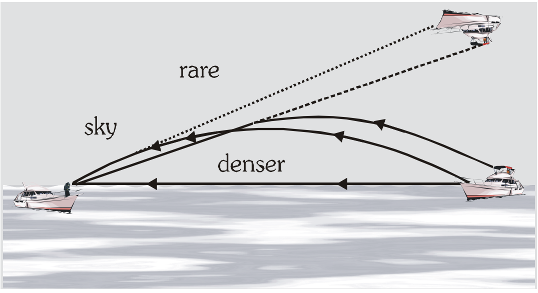
  In it light through multiple total internal reflections is propagated along the axis of a glass fibre of radius of few microns in which index of refraction of core is greater than that of surroundings.



- **Mirage and looming**

  Mirage is caused by total internal reflection in deserts where due to heating of the earth, refractive index of air near the surface of earth becomes lesser than above it. Light from distant objects reaches the surface of earth with $i > \theta_C$ so that TIR will take place and we see the image of an object along with the object as shown in figure.





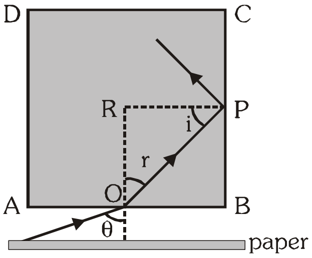
Similar to 'mirage' in deserts, in polar regions 'looming' takes place due to TIR. Here μ decreases with height and so the image of an object is formed in air if $(i > \theta_C)$ as shown in figure.

KEY POINTS

- A diver in water at a depth d sees the world outside through a horizontal circle of radius. $r = d \tan\theta_c$.
- In case of total internal reflection, as all (i.e. 100%) incident light is reflected back into the same medium there is no loss of intensity while in case of reflection from mirror or refraction from lenses there is some loss of intensity as all light can never be reflected or refracted. This is why images formed by TIR are much brighter than formed by mirrors or lenses.

Ex. A rectangular block of glass is placed on a printed page laying on a horizontal surface. Find the minimum value of the refractive index of glass for which the letters on the page are not visible from any of the vertical faces of the block.

Sol. The situation is depicted in figure. Light will not emerge out from the vertical face BC if at it



$i > \theta_C$ or $\sin i > \sin\theta_C \Rightarrow \sin i > \frac{1}{\mu}\left[\text{as } \sin\theta_C = \frac{1}{\mu}\right]$ ... (i)

But from Snell's law at O $1 \times \sin\theta = \mu \sin r$

And in Δ OPR, $r + 90 + i = 180 \Rightarrow r + i = 90° \Rightarrow r = 90 - i$

So $\sin\theta = \mu\sin(90 - i) = \mu\cos i \Rightarrow \cos i = \frac{\sin\theta}{\mu}$

so $\sin i = \sqrt{1 - \cos^2 i} = \sqrt{1 - \left[\frac{\sin\theta}{\mu}\right]^2}$ ... (ii)

so substituting the value of sin i from equation (ii) in (i),

$\sqrt{1 - \frac{\sin^2\theta}{\mu^2}} > \frac{1}{\mu}$ i.e., $\mu^2 > 1 + \sin^2\theta \because (\sin^2\theta)_{max} = 1 \therefore \mu^2 > 2 \Rightarrow \mu > \sqrt{2} \therefore$

$\mu_{min} = \sqrt{2}$

REFRACTION AT TRANSPARENT CURVED SURFACE

μ_1 = refractive index of the medium in which actual incident ray lies.

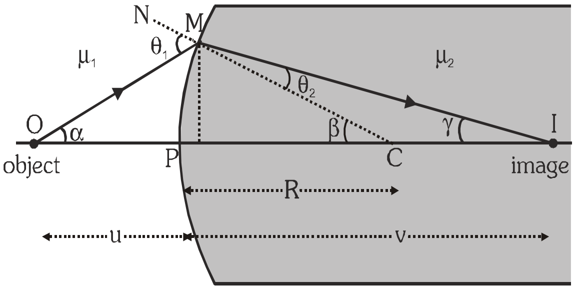
m_2 = refractive index of the medium in which actual refractive ray lies.

O = Object

P= pole

C= centre of curvature

R = PC = radius of curvature



Refraction from curved surface

$\mu_1\sin\theta_1 = \mu_2\sin\theta_2$

if angle is very small : $\mu_1\theta_1 = \mu_2\theta_2$...(i)

But $\theta_1 = \alpha + \beta$ ...(ii)

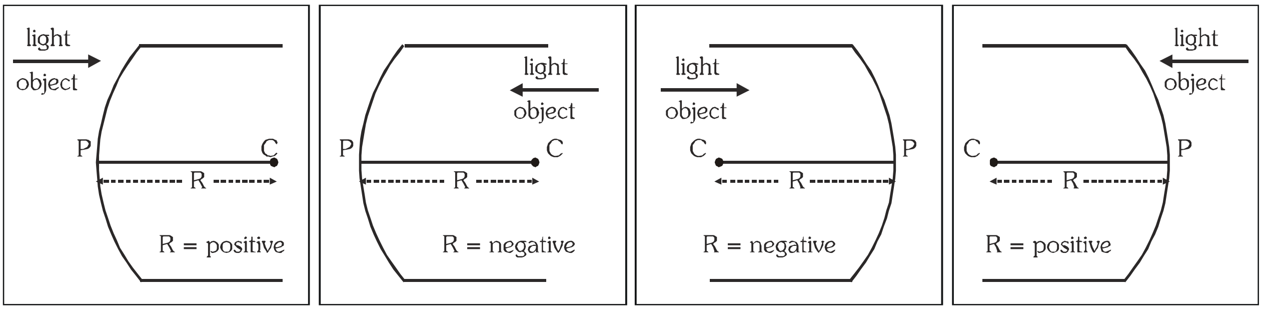
$\beta = \theta_2 + \gamma$ ...(iii)

from (i), (ii) and (iii) $\mu_1(\alpha + \beta) = \mu_2(\beta - \gamma)$

$\Rightarrow \mu_1\alpha + \mu_1\beta = \mu_2\beta - \mu_2\gamma \Rightarrow \mu_1\alpha + \mu_2\gamma = (\mu_2 - \mu_1)\beta$

$\Rightarrow \frac{\mu_1 PM}{-u} + \frac{\mu_2 PM}{v} = \frac{(\mu_2 - \mu_1)PM}{R} \Rightarrow \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

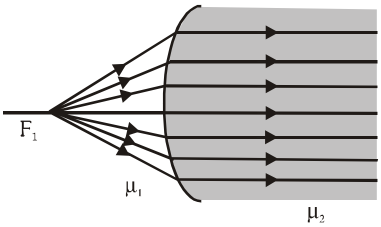
SIGN CONVENTION FOR RADIUS OF CURVATURE



These are valid for all single refraction surfaces – convex, concave or plane. In case of plane refracting surface $R \rightarrow \infty$, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \Rightarrow \frac{\mu_2}{v} - \frac{\mu_1}{u} = 0$ i.e. $\frac{u}{v} = \frac{\mu_1}{\mu_2}$ or $\frac{d_{Ac}}{d_{Ap}} = \frac{\mu_1}{\mu_2}$

FOCAL LENGTH OF A SINGLE SPHERICAL SURFACE

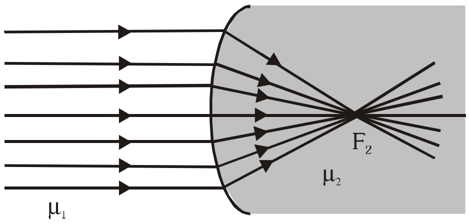
A single spherical surface as two principal focus points which are as follows–



(i) **First focus:** The first principal focus is the point on the axis where when an object is placed, the image is formed at infinity. That is when

u = f_1, v = ∞, then from $-\frac{\mu_1}{u}+\frac{\mu_2}{v}=\left(\frac{\mu_2-\mu_1}{R}\right)$

We get $-\frac{\mu_1}{f_1}=\frac{\mu_2-\mu_1}{R}\Rightarrow f_1=\frac{-\mu_1 R}{(\mu_2-\mu_1)}$

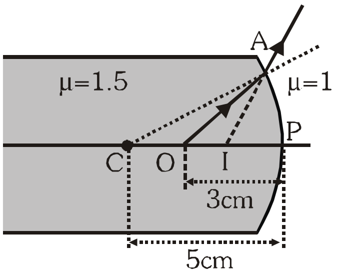


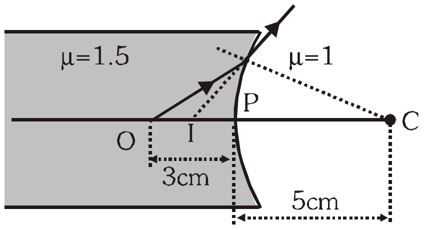
(ii) **Second focus:** Similarly, the second principal focus is the point where parallel rays focus. That is $u_1 = -\infty$, $v_1 = f_2$, then

$\frac{\mu_2}{f_2}=\frac{\mu_2-\mu_1}{R}$; $f_2=\frac{\mu_2 R}{(\mu_2-\mu_1)}$

(iii) **Ratio of Focal length:** $\boxed{\frac{f_1}{f_2}=-\frac{\mu_1}{\mu_2}}$

Ex An air bubble in glass (μ = 1.5) is situated at a distance 3 cm from a spherical surface of diameter 10 cm as shown in Figure. At what distance from the surface will the bubble appear if the surface is (a) convex (b) concave.





Sol. In case of refraction from curved surface $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{(\mu_2-\mu_1)}{R}$

(a) $\mu_1 = 1.5$, $\mu_2 = 1$, R = – 5 cm and u = –3 cm $\Rightarrow \frac{1}{v}-\frac{(1.5)}{(-3)}=\frac{1-1.5}{(-5)}\Rightarrow v=-2.5$ cm

the bubble will appear at a distance 2.5 cm from the convex curved surface inside the glass.

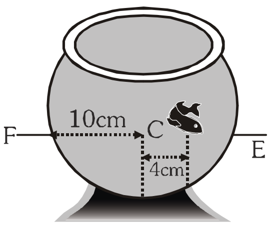
(b) $\mu_1 = 1.5$, $\mu_2 = 1$, R = 5 cm and u = –3 cm $\Rightarrow \frac{1}{v}-\frac{(1.5)}{(-3)}=\frac{1-1.5}{(5)}\Rightarrow v=-1.66$ cm

the bubble will appear at a distance 1.66 cm from the concave curved surface inside the glass.

Note : If the surface is plane then R → ∞

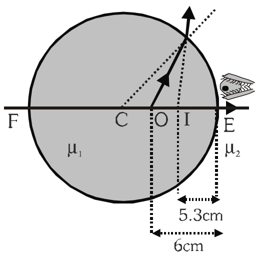
case (a) or (b) would yield $\frac{1}{v}-\frac{(1.5)}{(-3)}=\frac{(1-1.5)}{\infty}\Rightarrow v=-2$cm

Ex. In a thin spherical fish bowl of radius 10 cm filled with water of refractive index (4/3), there is a small fish at a distance 4 cm from the centre C as shown in Figure. Where will the fish appear to be, if seen from (a) E and (b) F (neglect the thickness of glass) ?



Sol. In the case of refraction from curved surface $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{(\mu_2-\mu_1)}{R}$

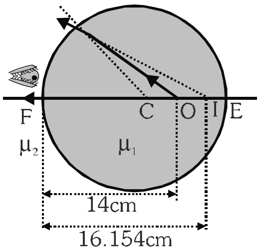
(a) Seen from E $\mu_1=\frac{4}{3}$, $\mu_2=1$, R = – 10 cm & u = –(10 – 4) = –6 cm



$$\Rightarrow \frac{1}{v}-\frac{\frac{4}{3}}{-6}=\frac{1-\frac{4}{3}}{-10}\Rightarrow v==\frac{90}{17}=-5.3\text{cm}$$

i.e., fish will appear at a distance 5.3 cm from E towards F (lesser than actual distance, i.e., 6 cm)

(b) Seen from F $\mu_1=\frac{4}{3}$, $\mu_2=1$, R = – 10 cm and u= –(10 + 4) = –14 cm



$$\Rightarrow \frac{1}{v}-\frac{\frac{4}{3}}{-14}=\frac{1-\frac{4}{3}}{-10}\quad\Rightarrow v=\frac{-210}{13}=-16.154\text{ cm}$$

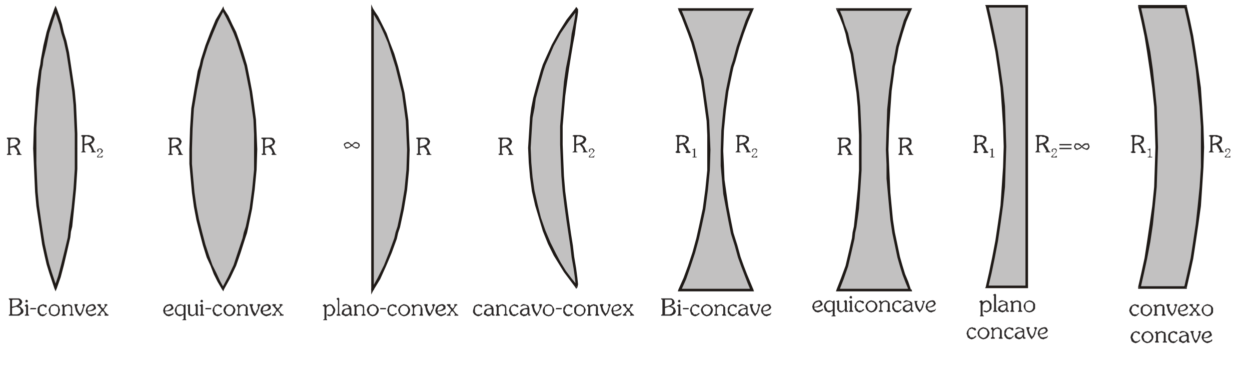
so fish will appear at a distance 16.154 cm from F toward E (more than actual distance, i.e., 14 cm)

LENS

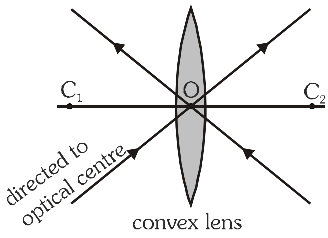
A lens is a piece of transparent material with two refracting surfaces such that at least one is curved and refractive index of its material is different from that of the surroundings.

A thin spherical lens with refractive index greater than that of surroundings behaves as a convergent or convex lens, i.e., converges parallel rays if its central (i.e. paraxial) portion is thicker than marginal one.

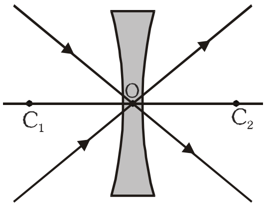
However if the central portion of a lens is thinner than marginal, it diverges parallel rays and behaves as divergent or concave lens. This is how wse and classify identify convergent and divergent lenses.



- **Optical Centre :** O is a point for a given lens through which any ray passes undeviated

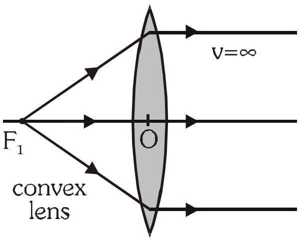


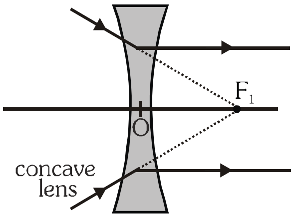
convex lens



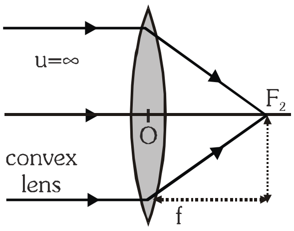
concave lens

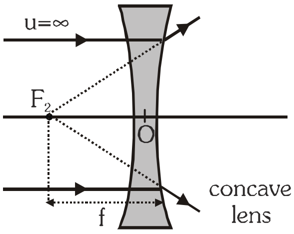
- **Principal Axis : C_1C_2** is a line passing through optical centre and perpendicular to the lens.
- **Principal Focus :** A lens has two surfaces and hence two focal points. First focal point is an object point on the principal axis for which image is formed at infinity.





While second focal point is an image point on the principal axis for which object lies at infinity





- **Focal Length f** is defined as the distance between optical centre of a lens and the point where the parallel beam of light converges or appears to converge.
- **Aperture :** In reference to a lens, aperture means the effective diameter. Intensity of image formed by a lens which depends on the light passing through the lens will depend on the square of aperture, i.e., Intensity ∝ $(\text{Aperture})^2$

LENS–MAKER'S FORMULA

In case of image formation by a lens

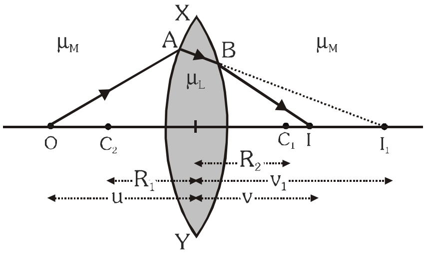


Image formed by first surface acts as object for the second.
So, from the formula of refraction at curved surface.

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

For first surface A $\frac{\mu_L}{v_1}-\frac{\mu_M}{u}=\frac{\mu_L-\mu_M}{R_1}$ ...(i) $[\because\ \mu_2=\mu_L,\ \mu_1=\mu_M]$

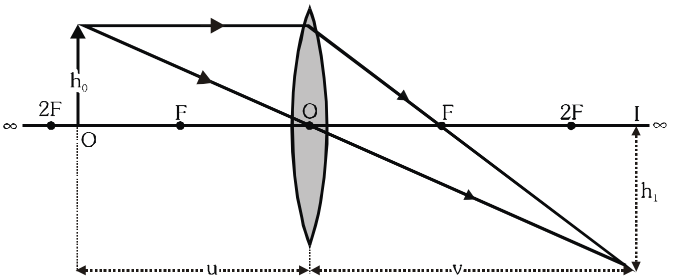
For second surface B $\frac{\mu_M}{v}-\frac{\mu_L}{v_1}=\frac{\mu_M-\mu_L}{R_2}=-\frac{\mu_L-\mu_M}{R_2}$ ...(ii) $[\because\ \mu_2=\mu_M,\ \mu_1=\mu_L,\ \mu_1=\mu_2,\ u\rightarrow v_1]$

By adding (i) and (ii)

$$\mu_M\left[\frac{1}{v}-\frac{1}{u}\right]=(\mu_L-\mu_M)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]\Rightarrow\frac{1}{v}-\frac{1}{u}=\frac{\mu_L-\mu_M}{\mu_M}\left[\frac{1}{R_1}-\frac{1}{R_2}\right]=(\mu-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]\text{...(iii)}\ \left(\because\ \mu=\frac{\mu_L}{\mu_M}\right)$$

Now if object is at infinity, Image will be formed at the focus, $u=-\infty,\ v=f$

So $\boxed{\frac{1}{f}=(\mu-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]}$ ...(iv)



This is known as lens makers formula by equating (iii) and (iv) $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

this is known as lens formula

Magnification : $m=\frac{\text{height of image}}{\text{height of object}}=\frac{h_1}{h_0}=\frac{v}{u}=\frac{f}{f+u}=\frac{f-v}{f}$

RULES FOR IMAGE FORMATION

- A ray passing through optical centre proceeds undeviated through the lens
- A ray passing through first focus or directed towards it, after refraction from the lens, becomes parallel to the principal axis.
- A ray passing parallel to the principal axis after refraction through the lens passes or appears to pass through F_2

For Convergent or convex lens

Object	Image	Magnification
– ∞	F	$\|m\| << 1$ & $m<0$
– ∞ – 2F	F–2F	$\|m\| < 1$ & $m<0$
2F	2F	$m = -1$
F – 2F	∞–2F	$\|m\| > 1$ & $m<0$
Just before F towards C	+ ∞	$m <<-1$
Just before F towards P	– ∞	$m >>1$
F – O	In front of lens	$m >1$

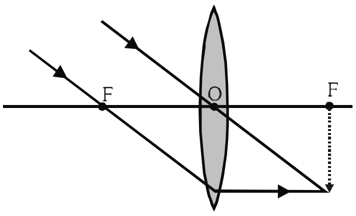
IMAGE FORMATION FOR CONVEX LENS (CONVERGENT LENS)

(i) Object is placed at infinity

Image :

at F real inverted very small in size

$|m| << 1$ & $m<0$

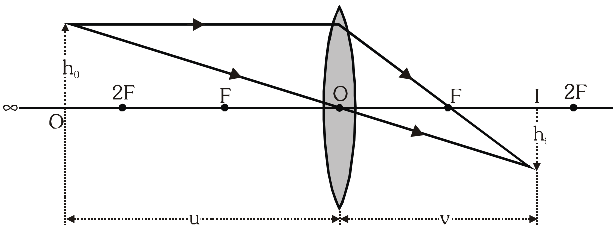


(ii) Object is placed in between ∞ – 2F

Image :

real (F – 2F) inverted small in size (diminished)

$|m| < 1$ & $m<0$

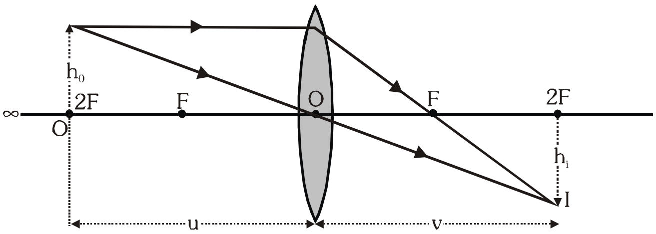


(iii) Object is placed at 2F

Image :

real (at 2F) inverted equal (of same size)

($m = -1$)

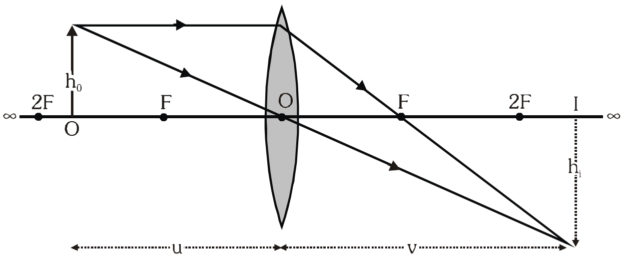


(iv) Object is placed in between 2F – F

Image :

real (2F – ∞) inverted enlarged

$|m| > 1$ & $m<0$



(v) Object is placed in between F – O

Image :

virtual (in front of lens) erected enlarge

($m > +1$)

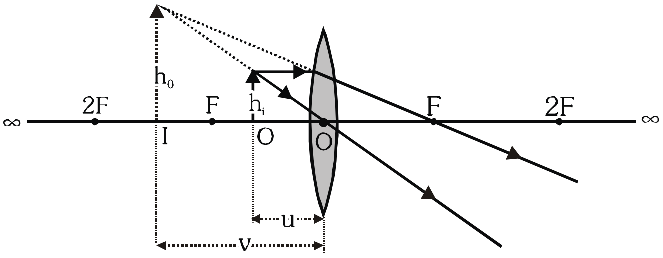


IMAGE FORMATION FOR CONCAVE LENS (DIVERGENT LENS)

Imge is virtual, diminished, erect, towards the object, m = **+ve**

(i) Object is placed at infinity

Image :

At F virtual erected

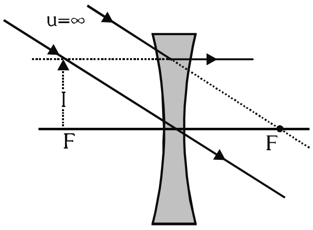
diminished ($m << +1$)

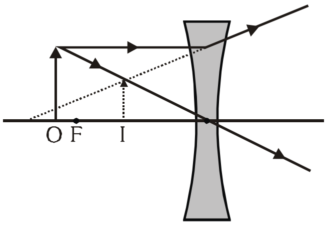
(ii) Object is placed infront of lens

Image :

between F and optical centre

virtual erected diminished ($m < +1$)





Sign convention for object/image for lens

Real object	u – ve
Real image	v + ve
Virtual object	u + ve
Virtual image	v – ve

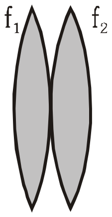
POWER OF LENS

Reciprocal of focal length in meter is known as power of lens.

SI UNIT : dioptre (D) **Power of lens :** $P = \frac{1}{f(m)} = \frac{100}{f(cm)}$ dioptre [in air]

COMBINATION OF LENSES

Two thin lens are placed in contact to each other

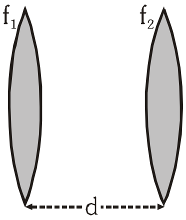


power of combination. $P = P_1 + P_2 \Rightarrow \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$

Use sign convention when solve numericals

Two thin lens are placed in at a small distance d

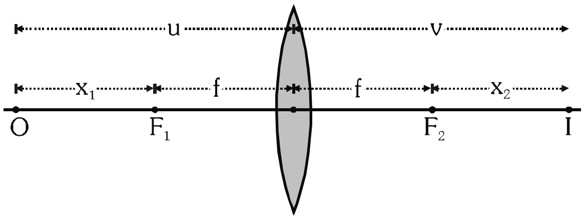
(provided incident rays are parallel to principal axis).



$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \Rightarrow P = P_1 + P_2 - d\ P_1 P_2$$

Use sign convention when solving numericals

- **Newton's Formula**



$$f = \sqrt{x_1 x_2}$$

x_1 = distance of object from focus.

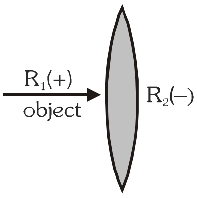
x_2 = distance of image from focus.

SOME SPECIAL CASES

(i) The focal length of equiconvex lens placed in air

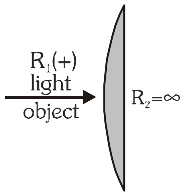
refractive index of lens $\mu_L = \mu$ refractive index of medium $\mu_M = 1$

$R_1 = +R$, $R_2 = -R$



$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R} - \left(-\frac{1}{R}\right)\right] \Rightarrow \text{Focal length } f = \frac{R}{2(\mu - 1)}$$

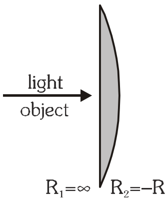
(ii) Focal length of planoconvex lens placed in air



$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R} - \frac{1}{\infty}\right]$$

$$\Rightarrow \text{Focal length } f = \frac{R}{(\mu - 1)}$$

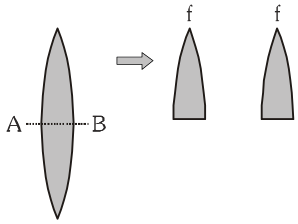
If object is placed towards plane surface



$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{\infty} - \left(-\frac{1}{R}\right)\right]$$

$$\Rightarrow \text{Focal length } f = \frac{R}{(\mu - 1)}$$

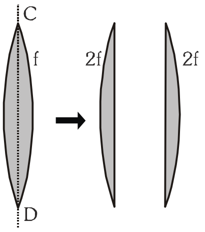
(iii) If an equiconvex lens of focal length f is cut into equal parts by a horizontal plane AB then the focal length of each part will be equal to that of initial lens. Because μ, R_1 and R_2 will remain unchanged. Only intensity will be reduced.



$\because$ intensity $I \propto (\text{apertures})^2$

$\therefore$ intensity through a single part will be reduced

(iv) If the same lens is cut into equal parts by a vertical plane CD the focal length of each part will be double of initial value but intensity will remain unchanged.



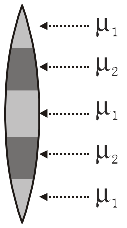
focal length of each part become 2f

For equiconvex lens $\frac{1}{f}=\frac{(\mu-1)2}{R}$ For plano convex lens $\frac{1}{f_1}=\frac{\mu-1}{R}$

So $\frac{1}{f}=\frac{2}{f_1}\Rightarrow f_1=2f\Rightarrow$ Focal length of each part = 2

(focal length of original lens)

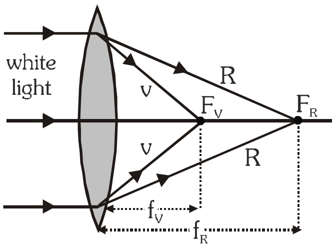
(v) If a lens is made of number of layers of different refractive index for a given wavelength then no. of images is equal to number of refractive index, as

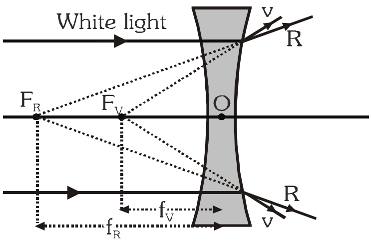


$\frac{1}{f}\propto(\mu-1)$

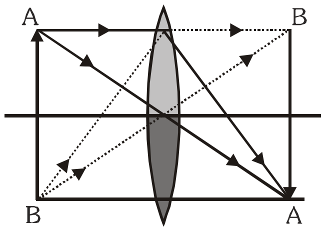
In figure number of images = 2

(vi) Focal length of lens depends on wavelength. $\because\frac{1}{f}\propto(\mu-1)\propto\frac{1}{\lambda}\ \left[f\propto\lambda\right]\ f_R>f_V$



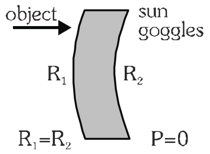


(vii) If half portion of lens is covered by black paper then intensity of image will be reduced but complete image will be formed.



(viii) Sun–goggles :

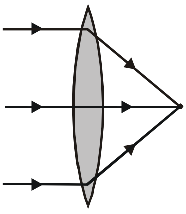
radius of curvature of two surfaces is equal with centre on the same side

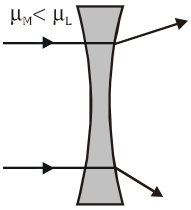


$R_1=R_2=+R$ so $\frac{1}{f}=(\mu-1)\left[\frac{1}{R}-\frac{1}{R}\right]$

$\Rightarrow\ \frac{1}{f}=0\Rightarrow f=\infty$ and $P=0$ $\Rightarrow$ sun goggles have no power

(ix) If refractive index of medium < Refractive index of lens

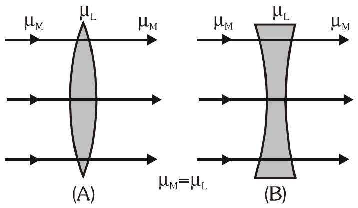




If $\mu_M < \mu_L$ then f = +ve

Convex lens behave as convex lens. While concave lens behave as concave lens.

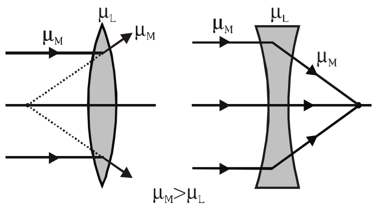
(x) Refractive index of medium = Refractive index of lens ($\mu_M = \mu_L$)



$$\frac{1}{f}=\left(\frac{\mu_L}{\mu_M}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right);\ \frac{1}{f}=0 \Rightarrow f=\infty\ \&\ P=0$$

Lens will behave as plane glass plate

(xi) Refractive index of medium > Refractive index of lens



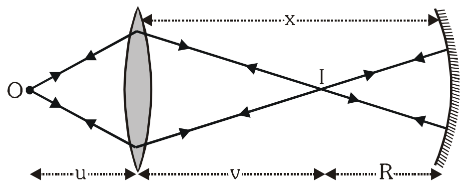
$$\mu_M > \mu_L \Rightarrow \frac{\mu_L}{\mu_M} < 1 \text{ [f will be negative]}$$

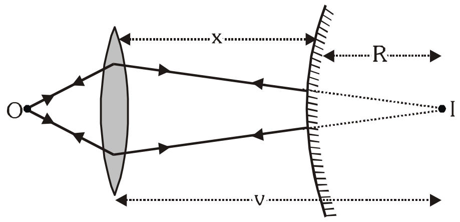
convex lens will behave as concave lens and concave lens will behave as convex lens. If a air bubble is formed in water it behaves as concave lens.

Ex. A point source S is placed at distance of 15 cm from a converging lens of focal length 10 cm. Where should a (i) concave mirror (ii) convex mirror of focal length 12 cm be placed so that real image is formed on object itself.

Sol. u = –15cm, f = +10cm; $\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}-\frac{1}{(-15)}=\frac{1}{10} \Rightarrow v = 30$ cm

(i) $x = v + 2f \Rightarrow 30 + 2 \times 12 = 54$ cm (ii) $x = v - 2f = 30 - 2 \times 12 = 6$ cm





Ex. A convex lens of focal length f is producing real image which is $\frac{1}{n}$ times of the size of the object. Find out position of the object.

Sol. Image is real so $m=\frac{v}{u}=-\frac{1}{n} \Rightarrow v=-\frac{u}{n}$

from lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{-\frac{u}{n}}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{-(n+1)}{u}=\frac{1}{f} \Rightarrow u=-f(1+n)$

Ex. (a) If f = +0.5m, what is the power of the lens ?

(b) The radii of curvature of the faces of a double convex lens are 10 cm and 15 cm. Its focal length is 12 cm. What is the refractive index of glass ?

(c) A convex lens has 20 cm focal length in air. What is the focal length in water ? (Refractive index of air–water = 1.33, refractive index for air glass is 1.5)

Sol. (a) $P=\frac{1}{f(m)}=\frac{1}{0.5}=+2D$

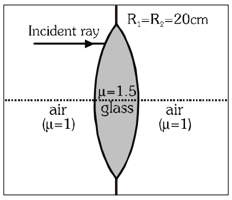
(b) $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \Rightarrow \frac{1}{12}=(\mu-1)\left(\frac{1}{10}-\frac{1}{-15}\right) \Rightarrow \mu = 1.5$

(c) $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ so $\frac{1}{f}\propto(\mu-1)$

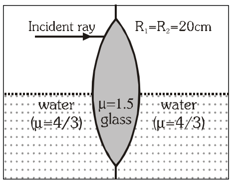
$$\frac{f_w}{f_a}=\frac{(\mu_\ell-1)}{\left({}_w\mu_\ell-1\right)}\Rightarrow f_w=\frac{(1.5-1)}{\left(\frac{1.5}{1.33}-1\right)}\times 20=78.2\text{cm}$$

Ex. Column I (optical system)

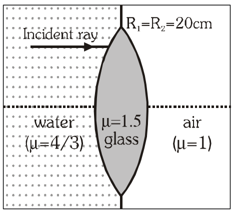
(A)



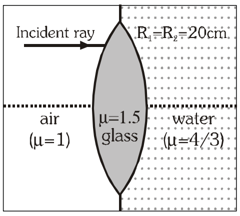
(B)



(C)



(D)



Column II (focal length)

(P) 80 cm (Q) 40 cm (R) 30 cm (S) 20 cm

Sol. Ans. (A) –S (B) –P (C) –R

For (A) : $\frac{1}{f}=(\mu-1)\left(\frac{1}{p_1}+\frac{1}{p_2}\right)=(1.5-1)\left(\frac{1}{20}+\frac{1}{20}\right)=\frac{1}{20}\Rightarrow f=20\text{cm}$

For (B) : $\frac{1}{f}=\left(\frac{1.5}{4/3}-1\right)\left(\frac{1}{20}+\frac{1}{20}\right)=\frac{1}{80}\Rightarrow f=80\text{cm}$

For (C) : $\frac{1.5}{v_1}-\frac{4/3}{\infty}=\left(1.5-\frac{4}{3}\right)\left(\frac{1}{20}\right)$ & $\frac{1}{f}-\frac{1.5}{v_1}=(1-1.5)\left(\frac{1}{-20}\right)\Rightarrow f=30\text{cm}$

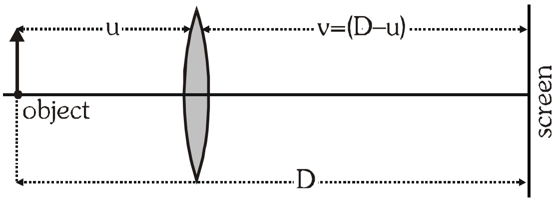
For (D) $\frac{1.5}{v_1}-\frac{1}{\infty}=(1.5-1)\left(\frac{1}{20}\right)$ & $\frac{4/3}{f}-\frac{1.5}{v_1}=\left(\frac{4}{3}-1.5\right)\left(-\frac{1}{20}\right)\Rightarrow f=40\text{cm}$

DISPLACEMENT METHOD

It is used for determination of focal length of convex lens in laboratory.

A thin convex lens of focal length f is placed between an object and a screen fixed at a distance D apart.

If D > 4f there are two position of lens at which a sharp image of the object is formed on the screen



By lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{D-u}-\frac{1}{-u}=\frac{1}{f}$

$\Rightarrow u^2-Du+Df=0\Rightarrow u=\frac{D\pm\sqrt{D(D-4f)}}{2}$

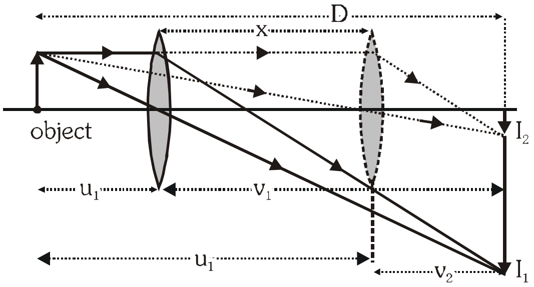
there are three possibilities

(i) for $D<4f$ u will be imaginary hence physically no position of lens is possible

(ii) for $D=4f$ $u=\frac{D}{2}=2f$ so only one position of lens is possible

and since $v=D-u=4f-2f=u=2f$

(iii) for $D>4f$ $u_1=\frac{D-\sqrt{D(D-4f)}}{2}$ and $u_2=\frac{D+\sqrt{D(D-4f)}}{2}$



So there are two positions of lens for which real image will be formed on the screen.
(for two distances u_1 and u_2 of the object from lens)

If the distance between two positions of lens is x

then $x = u_2 - u_1 = \frac{D+\sqrt{D(D-4f)}}{2} - \frac{D-\sqrt{D(D-4f)}}{2}$

$= \sqrt{D(D-4f)} \Rightarrow x^2 = D^2 - 4\,Df \Rightarrow f = \frac{D^2-x^2}{4D}$

Distance of image corresponds to two positions of the lens :

$$v_1 = D - u_1 = D - \frac{1}{2}[D - \sqrt{D(D-4f)}] = \frac{1}{2}[D + \sqrt{D(D-4f)}] = u_2 \Rightarrow v_1 = u_2$$

$$v_2 = D - u_2 = D - \frac{1}{2}[D + \sqrt{D(D-4f)}] = \frac{1}{2}[D - \sqrt{D(D-4f)}] = u_1 \Rightarrow v_2 = u_1$$

for two positions of the lens distances of object and image are interchangeable.
Now $x = u_2 - u_1$ and $D = v_1 + u_1 = u_2 + u_1$ $\quad [\because\ v_1 = u_2]$

so $u_1 (= v_2) = \frac{D-x}{2}$ and $v_1 = \frac{D+x}{2} = u_2$; $m_1 = \frac{I_1}{O} = \frac{v_1}{u_1} = \frac{D+x}{D-x}$ and $m_2 = \frac{I_2}{O} = \frac{v_2}{u_2} = \frac{D-x}{D+x}$

Now $m_1 \times m_2 = \frac{D+x}{D-x} \times \frac{D-x}{D+x} \Rightarrow \frac{I_1 I_2}{O^2} = 1 \Rightarrow O = \sqrt{I_1 I_2}$

Ex. A convex lens is placed between an object and a screen which are at a fixed distance apart for one position of the lens. The magnification of the image obtained on the screen is m_1. When the lens is moved by a distance d the magnification of the image obtained on the same screen is m_2, Find the focal length of the lens.

Sol. If D is the distance between the object and the screen, d the separation of the two position of lens throwing two images on the screen then

$m_1 = \frac{(D+d)}{(D-d)}$ and $m_2 = \frac{(D-d)}{(D+d)}$ $\therefore\ m_1 - m_2 = \frac{4Dd}{D^2-d^2}$ but $\frac{D^2-d^2}{4D} = f$ so $m_1 - m_2 = \frac{d}{f}$ $\Rightarrow f = \frac{d}{m_1 - m_2}$

Ex. In a displacement method using lens, we obtain two images for separation of the lens d. One image is magnified as much as the other is diminished. If m is the magnifications of one image, find the focal length of the lens.

Sol. From above question $f = \frac{d}{m_1 - m_2}$ here if m_1 is taken as m, $m_2 = \frac{1}{m}$, so f becomes $\frac{md}{(m^2-1)}$

Ex. In the displacement method the distance between the object and the screen is 70 cm and the focal length of the lens is 16 cm, find the separations of the magnified and diminished image position of the lens.

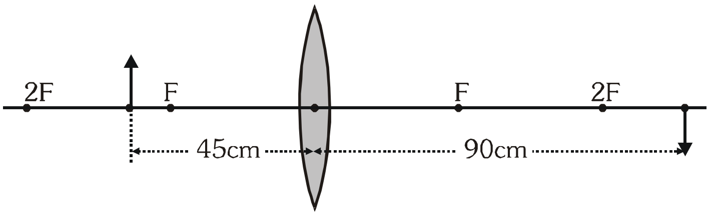
Sol. $d = \sqrt{D^2 - 4fd} = \sqrt{(70)^2 - 4 \times 16 \times 70} = \sqrt{420} = 20.5\text{cm}$

Ex. An object 25 cm high is placed in front of a convex lens of focal length 30 cm. If the height of image formed is 50 cm, find the distance between the object and the image (real and virtual) ?

Sol. As object is in front of the lens, it is real and as $h_1 = 25$ cm, $f = 30$ cm, $h_2 = -50$ cm ;

$$m = \frac{h_2}{h_1} = \frac{-50}{25} = -2$$

$$m = \frac{f}{f+u} \Rightarrow -2 = \frac{30}{30+u} \Rightarrow \ u = -45 \text{ cm} \Rightarrow m = \frac{v}{u} \Rightarrow -2 = \frac{v}{-45} \Rightarrow v = 90 \text{ cm}$$



As in this situation object and image are on opposite sides of lens, the distance between object and image $d_1 = u + v = 45 + 90 = 135$ cm. If the image is erect (i.e., virtual)

$$m = \frac{f}{f+u} \Rightarrow 2 = \frac{30}{30+u} \quad \Rightarrow \quad u = -15 \text{ cm} \Rightarrow m = -\frac{v}{u} \Rightarrow 2 = \frac{-v}{-15} \quad \Rightarrow \quad v = 30 \text{ cm}$$

As in the situation both image and object are in front of the lens, the distance between object and image

$d_2 = v - u = 30 - 15 = 15$ cm.

COMBINATION OF LENSES AND MIRRORS

When several lenses or mirrors are used, the image formation is considered one after another in steps, The image formed by the lens facing the object serves as an object for the next lens or mirror, the image formed by the second lens acts as an object for the third, and so on, The total magnification in such situations will be given by

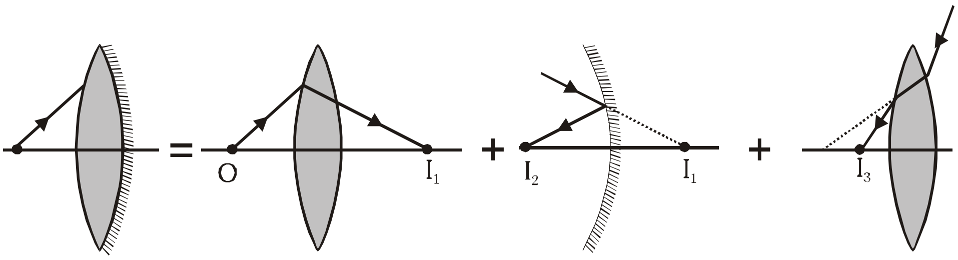
$$m = \frac{I}{O} = \frac{I_1}{O} \times \frac{I_2}{I_1} \times \dots \Rightarrow m = m_1 \times \ m_2 \times \ \dots$$

Power of Lens [in air] $P_L = \frac{1}{f_L}$ Converging lens $P_L = +ve$ Diverging lens $P_L = -ve$

Power For mirror $P_L = -\frac{1}{f_m}$ Convex mirror $P_M = -ve$ Concave mirror $P_M = +ve$

SILVERING OF LENS

Calculate equivalent focal length of a equiconvex lens silvered at one side.

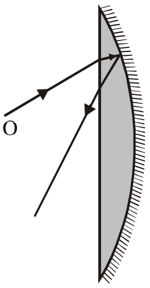


$$P = P_L + P_M + P_L = 2P_L + P_M$$

$$\frac{1}{F} = \frac{1}{f_\ell} + \frac{1}{f_m} + \frac{1}{f_\ell} == \frac{2}{f_\ell} + \frac{1}{f_m} = \frac{2(\mu-1)\times 2}{R} + \frac{2}{R} = \frac{4\mu - 4 + 2}{R} \Rightarrow F = \frac{R}{4\mu - 2}$$

Ex. Calculate equivalent focal length of plano convex lens for following case :–
(i) When curved surface is silvered. (ii) When plane surface is silvered.

Sol. (i)

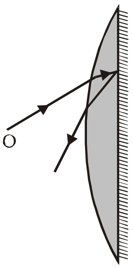


$$P = 2P_L + P_M$$

$$\Rightarrow \frac{1}{F} = \frac{2}{f_\ell} + \frac{1}{f_m} \Rightarrow \frac{1}{F} = \frac{2(\mu-1)}{R} + \frac{2}{R}$$

$$\Rightarrow \frac{1}{F} = \frac{2\mu}{R} \Rightarrow F = \frac{R}{2\mu}$$

(ii)



$$P = 2P_L + P_M$$

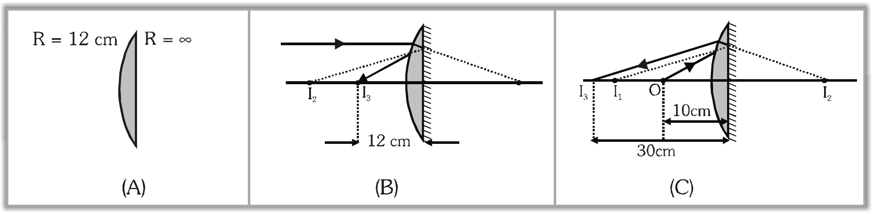
$$\Rightarrow \frac{1}{F} = \frac{2}{f_L} + \frac{1}{f_m} \Rightarrow \frac{1}{F} = \frac{2(\mu-1)}{R} + \frac{1}{\infty}$$

$$\Rightarrow F = \frac{R}{2(\mu-1)}$$

Ex. The radius of curvature of the convex face of a plano–convex lens is 12 cm and its refractive index is 1.5. (a) Find the focal length of this lens. The plane surface of the lens is now silvered. (b) At what distance from the lens will parallel rays incident on the convex face converge ? (c) Sketch the ray diagram to locate the image, when a point object is placed on the axis 20 cm from the lens. (d) Calculate the image distance when the object is placed as in (c).

Sol. (a) As for a lens, by lens–maker's formula $\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$ Here $\mu = 1.5$; $R_1 = 12$ cm and $R_2 = \infty$

So $\frac{1}{f} = (1.5 - 1)\left[\frac{1}{12} - \frac{1}{\infty}\right]$ i.e. f = 24 cm i.e., the lens as convergent with focal length 24 cm.



(A) (B) (C)

(b) As light after passing through the lens will be incident on the mirror which will reflect it back through the lens again, so $P = P_L + P_M + P_L = 2P_L + P_M$ But $P_L = \frac{1}{f_L} = \frac{1}{0.24}$ and $P_M = -\frac{1}{\infty} = 0 \left[\text{as } f_M = \frac{R}{2} = \infty\right]$

So $P = 2 \times \frac{1}{0.24} + 0 = \frac{1}{0.12}$ D. The system is equivalent to a concave mirror of focal length F,

$P = -\frac{1}{F}$

i.e., $F = -\frac{1}{P} = -0.12$ m $= -12$ cm i.e., the rays will behave as a concave mirror of focal length 12 cm.

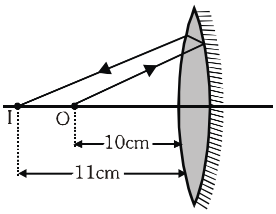
So as for parallel incident rays $u = -\infty$, from mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ we have $\frac{1}{v} + \frac{1}{-\infty} = \frac{1}{-12}$

$\Rightarrow$ v = – 12 cm i.e., parallel incident rays will focus will at a distance of 12 cm in front of the lens as shown in Figure (c) and (d) When object is at 20 cm in front of the given silvered lens which behaves as a concave mirror of focal length 12 cm, from mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ we have $\frac{1}{v} + \frac{1}{-20} = \frac{1}{-12}$

$\Rightarrow$ v = – 30 cm i.e., the silvered lens will form image at a distance of 30 cm in front of it as shown in fig. (C)

Ex. A pin is placed 10 cm in front of a convex lens of focal length 20 cm, made of material having refractive index 1.5. The surface of the lens farther away from the pin is silvered and has a radius of curvature 22 cm. Determine the position of the final image. Is the image real or virtual ?

Sol. As radius of curvature of silvered surface is 22 cm, so



$f_M = \frac{R}{2} = \frac{-22}{2} = -11$ cm $= -0.11$ m and hence,

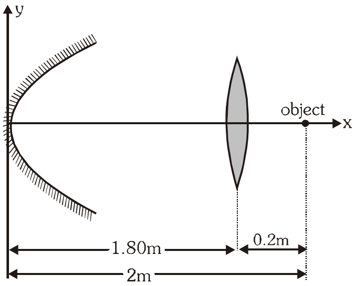
$$P_M = -\frac{1}{f_M} = -\frac{1}{-0.11} = \frac{1}{0.11}\text{D}$$

Further as the focal length of lens is 20 cm, i.e., 0.20 m its power will be given by :

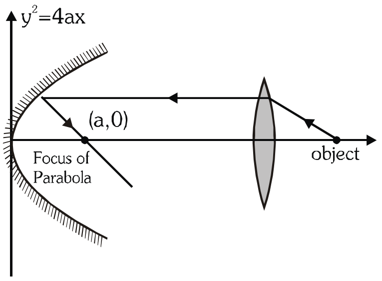
$P_L = \frac{1}{f_L} = \frac{1}{0.20}$ D. Now as in image formation, light after passing through the lens will be reflected back by the curved mirror through the lens again $P = P_L + P_M + P_L = 2P_L + P_M$ i.e. $P = \frac{2}{0.20} + \frac{1}{0.11} = \frac{210}{11}\text{D}$.

So the focal length of equivalent mirror $F = -\frac{1}{P} = -\frac{11}{210}\text{m} = -\frac{110}{21}\text{cm}$ i.e., the silvered lens behave as a concave mirror of focal length (110/21) cm. So for object at a distance 10 cm in front of it, $\frac{1}{v} + \frac{1}{-10} = -\frac{21}{110}$ i.e., $v = -11$cm i.e., image will be 11 cm in front of the silvered lens and will be real as shown in Figure.

Ex. A point object is kept at a distance of 2m from a parabolic reflecting surface $y^2 = 2x$. An equiconvex lens is kept at a distance of 1.80 m from the parabolic surface. The focal length of the lens is 20 cm. Find the position from origin of the image in cm, after reflection from the surface.



Sol.



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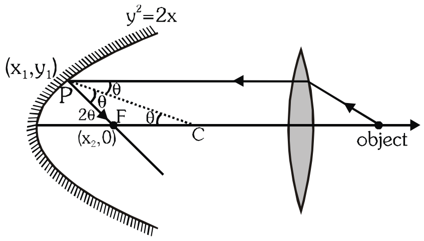


Comparing with $y^2 = 4ax \Rightarrow a = 0.5$

$\Rightarrow$ final position of image = 0.5 m = 50 cm

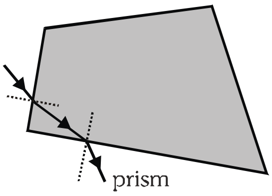
PC is a normal so $\tan(\pi - \theta) = \frac{-1}{(dy/dx)_{x_1,y_1}} = -y_1$

But $\tan 2\theta = \frac{y_1 - 0}{x_2 - x_1}$ & $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

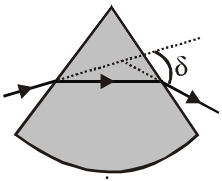
$$\Rightarrow \frac{y_1}{x_2 - x_1} = \frac{2(y_1)}{1-y_1^2}\, x_2 = \frac{1}{2}\text{m}$$

PRISM

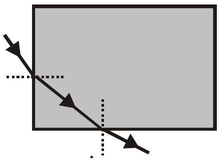
A prism is a homogeneous, transparent medium (such as glass) enclosed by two plane surfaces inclined at an angle. These surfaces are called the 'refracting surfaces' and the angle between them is called the 'refracting angle' or the 'angle of prism'. The section cut by a plane perpendicular to the refracting surfaces is called the 'principal section' of the prism.



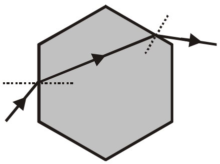
prism



prism

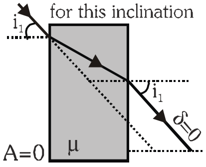


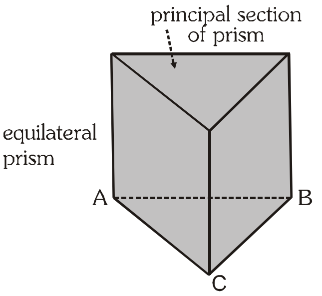
prism

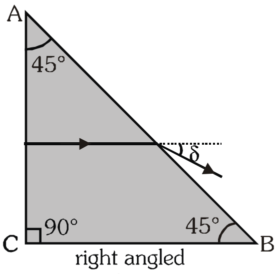


prism

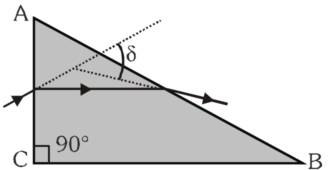
It is not a prism for this inclination





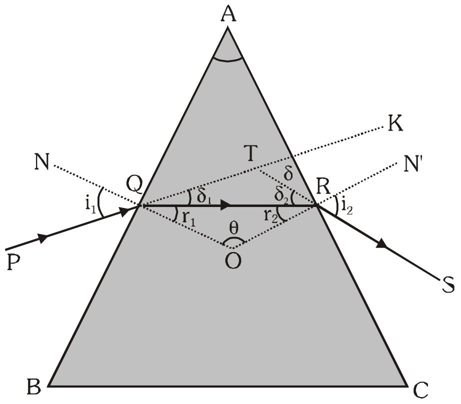


right angled isosceles prism



right angled prism

DEVIATION



PQ = incident ray

QR = Refracted ray

RS = emergent ray

A = Prism angle

i_1 = incident angle on face AB

i_2 = emergent angle on face AC

r_1 = refracted angle on face AB

r_2 = incident angle on face AC

Angle of deviation on face AB. $\delta_1 = i_1 - r_1$

Angle of deviation on face AC $\delta_2 = i_2 - r_2$

Total angle of deviation

$\delta = \delta_1 + \delta_2 \Rightarrow \delta = (i_1 - r_1) + (i_2 - r_2) = i_1 + i_2 - (r_1 + r_2)$ .....(i)

In ΔQOR $r_1 + r_2 + \theta = 180°$ ...(ii)

In AQOR $A + \theta = 180°$ ...(iii)

from (ii) and (iii) $r_1 + r_2 = A$ ...(iv)

from (i) and (iv) Total angle of deviation $\delta = i_1 + i_2 - A$

from Snell's law at surface AB $\mu_1 \sin i_1 = \mu_2 \sin r_1$

and at surface AC $\mu_2 \sin r_2 = \mu_1 \sin i_2$

CONDITION OF MINIMUM DEVIATION

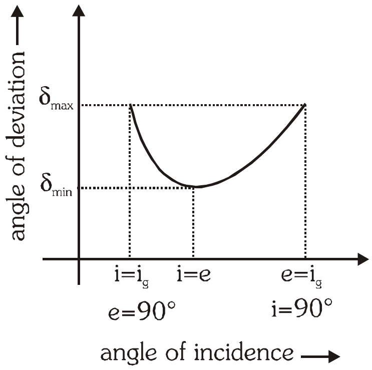
For minimum deviation

In this condition $i_1 = i_2 = i \Rightarrow r_1 = r_2 = r$ and since $r_1 + r_2 = A \;\therefore\; r + r = A \Rightarrow 2r = A \Rightarrow r = \frac{A}{2}$

Minimum deviation $\delta_{min} = 2i - A;\; i = \frac{A+\delta_{min}}{2}\;,\; r = \frac{A}{2}$

if prism is placed in air $\mu_1 = 1;\; 1 \times \sin i = \mu \sin r$

$$\sin\left[\frac{A+\delta_{min}}{2}\right] = \mu \sin\frac{A}{2} \Rightarrow \mu = \frac{\sin\left[\frac{A+\delta_{min}}{2}\right]}{\sin\frac{A}{2}}$$



if angle of prism is small $A < 10°$ then $\sin\theta \approx \theta$

$$\mu = \frac{\frac{A+\delta_{min}}{2}}{\frac{A}{2}} = \frac{A+\delta_{min}}{A} \Rightarrow A + \delta_{min} = \mu A \Rightarrow \delta_{min} = (\mu - 1)A$$

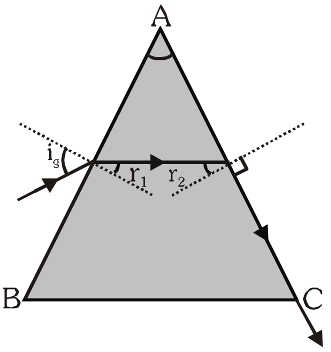
CONDITION FOR MAXIMUM DEVIATION/GRAZING EMERGENCE

- **Angle of incidence (i_g)for grazing emergence**

For i_g, $e = 90°$

Applying Snell's law at face AC

$$\mu \sin r_2 = 1 \times 1 \Rightarrow \sin r_2 = \frac{1}{\mu};\; r_2 = \sin^{-1}\left(\frac{1}{\mu}\right) = \theta_c$$



$r_1 + r_2 = A \Rightarrow r_1 = A - \theta_c$

Again, Applying Snell's law at face AB

$1 \times \sin i_g = \mu \sin r_1;\; 1 \times \sin i_g = \mu \sin(A - \theta_c)$

$\sin i_g = \mu[\sin A \cos\theta_c - \cos A \sin\theta_c]$

$$i_g = \sin^{-1}\left[\sqrt{\mu^2 - 1}\sin A - \cos A\right] \qquad \left[\left(\text{as } \sin\theta_c = \frac{1}{\mu},\; \cos\theta_c = \frac{\sqrt{\mu^2-1}}{\mu}\right)\right]$$

If i increases beyond i_g, r_1 increases thus r_2 decreases and becomes less than θ_c and ray emerges.

Thus $i \geq i_g \Rightarrow$ ray emerges, otherwise TIR. $\delta_{max} = i_g + 90° - A$

NO EMERGENCE CONDITION

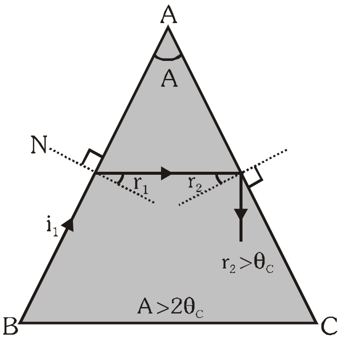
Let maximum incident angle on the face AB $i_{max} = 90°$

$1 \times \sin 90° = \mu \sin r_1;\; \sin r_1 = \frac{1}{\mu} = \sin\theta_C;\; r_1 = \theta_C$ ...(i)

if TIR occur at face AC then $r_2 > \theta_C$ ...(ii)

$r_1 + r_2 = A$ ...(iii)

from (i) and (ii) $r_1 + r_2 > \theta_C + \theta_C \Rightarrow r_1 + r_2 > 2\theta_C$ ...(iv)



from (iii) and (iv) $A > 2\theta_C \Rightarrow \frac{A}{2} > \theta_C \Rightarrow \sin\frac{A}{2} > \sin\theta_C \Rightarrow \sin\frac{A}{2} > \frac{1}{\mu} \Rightarrow \frac{1}{\sin\frac{A}{2}} < \mu$

Ex. A ray of light passes through an equilateral prism such that angle of incidence is equal of emergence and the later is equal to 3/4th of the angle of prism. Calculate the angle of deviation. Refractive index of prism is 1.5.

Sol. $A = 60°, \mu = 1.5$; $i_1 = i_2 = \frac{3}{4}A = 45°$, $\delta = ?$

$\because A + \delta = i_1 + i_2$ $\therefore 60° + \delta = 45° + 45° \Rightarrow \delta = 90° - 60° = 30°$

Ex. A prism of refractive index 1.53 is placed in water of refractive index 1.33. If the angle of prism is 60°, calculate the angle of minimum deviation in water. (sin 35.1° = 0.575)

Sol. Here, ${}^a\mu_g = 1.33$, ${}^a\mu_w = 1.53$, $A = 60°$, $\delta_m = ?$ ${}^w\mu_g = \frac{{}^a\mu_g}{{}^a\mu_w} = \frac{1.53}{1.33} = 1.15 \because {}^w\mu_g = \frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}}$

$\therefore \frac{\sin(A+\delta_m)}{2} = {}^w\mu_g \times \sin\frac{A}{2} = 1.15\sin\frac{60^\circ}{2} = 0.575 \Rightarrow \frac{A+\delta_m}{2} = \sin^{-1}(0.575) = 35.1°$

$\therefore \delta_m = 35.1 \times 2 - 60 = 10.2°$

KEY POINTS

- Angle of prism or refracting angle of prism means the angle between the faces on which light is incident and from which it emerges.
- If the faces of a prism on which light is incident and from which it emerges are parallel then the angle of prism will be zero and as incident ray will emerge parallel to itself, deviation will also be zero, i.e., the prism will act as a transparent plate.
- If μ of the material of the prism is equal to that of surroundings, no refraction at its faces will take place and light will pass through it undeviated, i.e., $\delta = 0$.

DISPERSION OF LIGHT

When white light is incident on a prism then it is splitted into seven colours. This phenomenon is known as dispersion. Prism introduces different refractive index with different wavelength

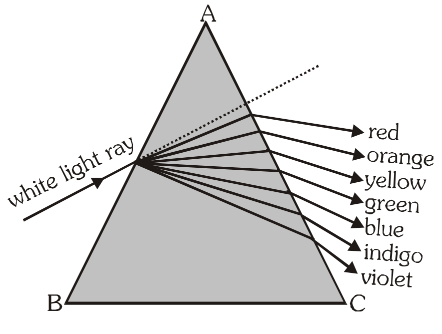
As $\delta_{min} = (\mu - 1)A \because \lambda_R > \lambda_V$ So $\mu_V > \mu_R \Rightarrow \delta_{m(violet)} > \delta_{m(red)}$

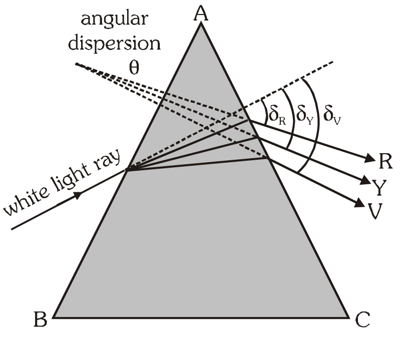
ANGULAR DISPERSION

It is the difference of angle of deviation for violet colour and red colour

Angular dispersion $\theta = \delta_V - \delta_R = (\mu_V - 1)A - (\mu_R - 1)A = (\mu_V - \mu_R)A$

It depends on prism material and on the angle of prism $\theta = (\mu_V - \mu_R)A$





DISPERSIVE POWER (ω)

It is ratio of angular dispersion (θ) to mean colour deviation (δ_y)

Dispersive power $\omega = \dfrac{\theta}{\delta_y} \Rightarrow \omega = \dfrac{(\mu_V - \mu_R)A}{(\mu_y - 1)A} = \dfrac{\mu_V - \mu_R}{\mu_y - 1} \Rightarrow \omega = \dfrac{\mu_V - \mu_R}{\mu_y - 1}$

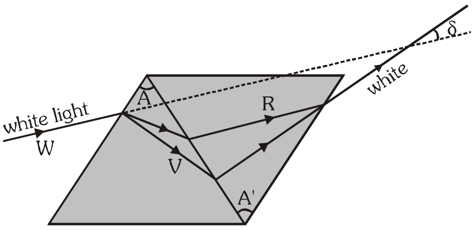
Refractive index of mean colour $\mu_y = \dfrac{\mu_V + \mu_R}{2}$

Dispersive power depends only on the material of the prism.

COMBINATION OF PRISM

Deviation without dispersion ($\theta = 0°$)

Two or more than two thin prism are combined in such a way that deviation occurs i.e. emergent light ray makes angle with incident light ray but dispersion does not occur i.e., light is not splitted into seven colours.



Total dispersion $= \theta = \theta_1 + \theta_2 = (\mu_V - \mu_R)A + (\mu'_V - \mu'_R)A'$

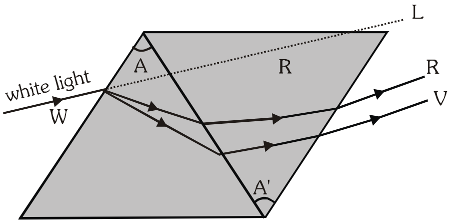
For no dispersion $\theta = 0$; $(\mu_V - \mu_R)A + (\mu'_V - \mu'_R)A' = 0$

Therefore, $A' = -\dfrac{(\mu_V - \mu_R)A}{\mu'_V - \mu'_R}$

–ve sign indicates that prism angles are in opposite direction.

Dispersion without deviation ($\delta = 0°$)

Two or more than two prisms combine in such a way that dispersion occurs i.e., light is splitted into seven colours but deviation do not occur i.e., emergent light ray becomes parallel to incident light ray.



Total deviation $\delta = \delta_1 + \delta_2$

$\Rightarrow \delta = 0; (\mu - 1) A + (\mu' - 1)A' = 0 \Rightarrow A' = -\dfrac{(\mu - 1)A}{\mu' - 1}$

–ve sign indicates that prism angles are in opposite direction.

KEY POINTS

- Dispersive power like refractive index has no units and dimensions and depends on the material of the prism and is always positive.
- As for a given prism dispersive power is constant, i.e., dispersion of different wavelengths will be different and will be maximum for violet and minimum for red (as deviation is maximum for violet and minimum for red).
- As for a given prism $\theta \propto \delta$ *a* single prism produces both deviation and dispersion simultaneously, i.e., a single prism cannot give deviation without dispersion or dispersion without deviation.

Ex. White light is passed through a prism of angle 5°. If the refractive indices for red and blue colours are 1.641

and 1.659 respectively, calculate the angle of dispersion between them.

Sol. As for small angle of prism $\delta = (\mu - 1)A$,

$\delta_B = (1.659 - 1) \times 5° = 3.295°$ and $\delta_R = (1.641 - 1) \times 5° = 3.205°$

so $\theta = \delta_B - \delta_R = 3.295° - 3.205° = 0.090°$

Ex. Prism angle of a prism is 10°. Their refractive index for red and violet color is 1.51 and 1.52 respectively. Then find the dispersive power.

Sol. Dispersive power of prism $\omega = \left(\frac{\mu_v - \mu_r}{\mu_y - 1}\right)$ but $\mu_y = \frac{\mu_v + \mu_r}{2} = \frac{1.52 + 1.51}{2} = 1.515$

Therefore $\omega = \frac{1.52 - 1.51}{1.515 - 1} = \frac{0.01}{1.515} = 0.019$

Ex. The refractive indices of flint glass for red and violet colours are 1.644 and 1.664. Calculate its dispersive power.

Sol. Here, $\mu_r = 1.644$, $\mu_v = 1.664$, $\omega = ?$

Now $\mu_y = \frac{\mu_v + \mu_r}{2} = \frac{1.664 + 1.644}{2} = 1.654$ $\because$ $\omega = \frac{\mu_v - \mu_r}{\mu_y - 1} = \frac{1.664 - 1.644}{1.654 - 1} = 0.0305$

Ex. In a certain spectrum produced by a glass prism of dispersive power 0.031, it was found that μ_r = 1.645 and μ_v = 1.665. What is the refractive index for yellow colour ?

Sol. Here, $\omega = 0.031$, $\mu_r = 1.645$ $\mu_v = 1.665$, $\mu_y = ?$

$\because \omega = \frac{\mu_v - \mu_r}{\mu_y - 1}$ $\therefore \mu_y - 1 = \frac{\mu_v - \mu_r}{\omega} = \frac{1.665 - 1.645}{0.031} = \frac{0.020}{0.31} = 0.645$ $\therefore \mu_y = 0.645 + 1 = 1.645$

Ex. A combination of two prisms, one of flint and other of crown glass produces dispersion without deviation. The angle of flint glass prism is 15°. Calculate the angle of crown glass prism and angular dispersion of red and violet. (μ for crown glass = 1.52, μ for flint glass = 1.65, ω for crown glass 0.20, ω for flint glass = 0.03).

Sol. Here, A = 15°, A' = ?, $\omega = 0.03$, $\omega' = 0.02$, $\mu = 1.65$, $\mu' = 1.52$,

For no deviation, $\delta + \delta' = 0$

$(\mu - 1)A + (\mu' - 1)A' = 0 \Rightarrow (1.65 - 1)15° + (1.52 - 1)A' = 0 \Rightarrow A' = \frac{-0.65 \times 15}{0.52} = -18.75°$

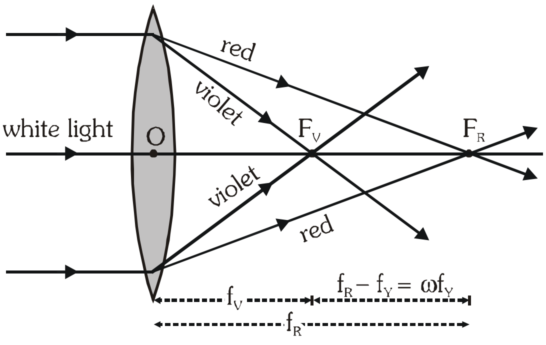
Negative sign indicates that two prisms must be joined in opposition. Net angular dispersion

$(\mu_v - \mu_r)A + (\mu'_v - \mu'_r)A' = \omega(\mu - 1)A + \omega'(\mu' - 1)A' = 0.03\,(1.65 - 1)15° + 0.02\,(1.52 - 1)\,(-18.75°)$

$= 0.2925 - 0.195 = 0.0975°$

CHROMATIC ABERRATION

The image of a object in white light formed by a lens is usually colored and blurred. This defect of image is called chromatic aberration and arises due to the fact that focal length of a lens is different for different colors.



For a single lens $\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$ and as μ of lens is maximum for violet while minimum for red, violet is focused nearest to the lens while red farthest from it. It is defect of lens.

Longitudinal or Axial Chromatic Aberration

When a white object O is situated on the axis of a lens, then images of different colors are formed at different points along the axis. The formation of images of different colors at different positions is called 'axial' or longitudinal chromatic aberration. The axial distance between the red and the violet images $I_R - I_V$ is known as longitudinal aberration. When white light is incident on lens, image is obtained at different point on the axis because focal length of lens depend on wavelength. $f \propto \lambda \Rightarrow f_R > f_V$

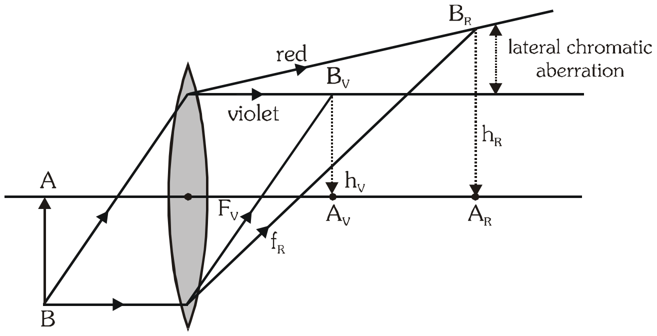
$$\mathbf{f_R - f_V = \omega f_y \Rightarrow}\ \textbf{Axial or longitudinal chromatic aberration}$$

If the object is at infinity, then the longitudinal chromatic aberration is equal to the difference in focal–lengths $(f_R f_V)$ for the red and the violet rays.

LATERAL CHROMATIC ABERRATION

As the focal–length of the lens varies from color to color, the magnification $m = \left[\frac{f}{u+f}\right]$ produced by the lens also varies from color to color.

Therefore, for a finite–size white object AB, the images of different colors formed by the lens are of different sizes.



The formation of images of different colors in different sizes is called lateral chromatic aberration. The difference in the height of the red image $B_R A_R$ and the violet image $B_V A_V$ is known as lateral chromatic aberration. $LCA = h_R - h_V$

ACHROMATISM

If two or more lens combined together in such a way that this combination produce image at a same point then this combination is known as achromatic combination of lenses.

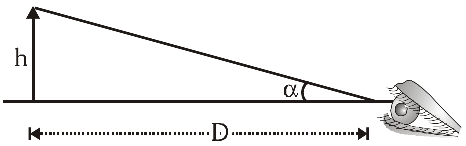
$$\frac{\omega}{f_y} + \frac{\omega'}{f'_y} = 0 \Rightarrow \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0 \Rightarrow \frac{\omega_1}{\omega_2} = -\frac{f_1}{f_2}$$

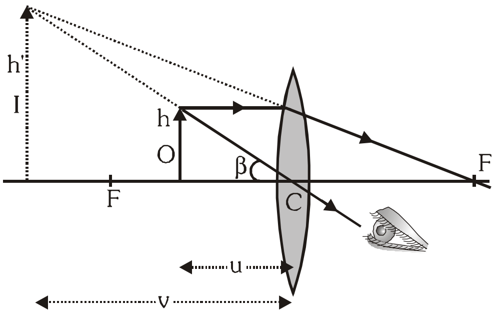
For combination of lens. $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$ (Apply sign convention in numerical)

OPTICAL INSTRUMENTS

Simple microscope

When object is placed between focus and optical centre a virtual, magnified and erect image is formed





Magnifying power (MP) = $\dfrac{\text{visual angle with instrument } (\beta)}{\text{maximum visual angle for unaided eye } (\alpha)} \Rightarrow \text{MP} = \dfrac{\frac{h}{-u}}{\frac{h}{-D}} = \dfrac{D}{u}$

(i) When the image is formed at infinity :

by lens equation $\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{-\infty}-\frac{1}{-u}=\frac{1}{f} \Rightarrow u=f$ So $\text{MP}=\frac{D}{u}=\frac{D}{f}$

(ii) If the image is at minimum distance of clear vision D :

$\frac{1}{-D}-\frac{1}{-u}=\frac{1}{f} \Rightarrow \frac{1}{u}=\frac{1}{D}+\frac{1}{f}$ [v = –D and u = –ve]

Multiplying by D both the sides $\frac{D}{u}=1+\frac{D}{f} \Rightarrow \text{MP}=\frac{D}{u}=1+\frac{D}{f}$

Ex. A man with normal near point 25 cm reads a book with small print using a magnifying glass, a thin convex lens of focal length 5 cm.

(a) What is the closest and farthest distance at which he can read the book when viewing through the magnifying glass ?

(b) What is the maximum and minimum MP possible using the above simple microscope ?

Sol. (a) As for normal eye far and near point are ∞ and 25 cm respectively, so for magnifier $v_{max} = -\infty$ and $v_{min} = -25$ cm. However, for a lens as $\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow u=\frac{f}{(f/v)-1}$

So u will be minimum when v = minimum = –25 cm i.e. $(u)_{min} = \frac{5}{-(5/25)-1} = -\frac{25}{6} = -4.17\text{cm}$

Ans u will be maximum when v = maximum = ∞ i.e., $u_{max} = \frac{5}{\left(\frac{5}{\infty}-1\right)} = -5$ cm

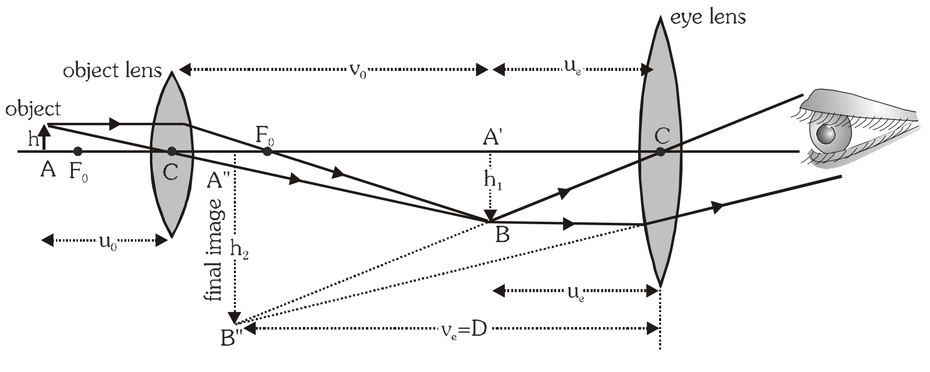
So the closest and farthest distance of the book from the magnifier (or eye) for clear viewing are 4.17 cm and 5 cm respectively.

(b) As in case of simple magnifier MP = (D/u). So MP will be minimum when u = max = 5 cm

$\Rightarrow (\text{MP})_{min} = \frac{-25}{-5} = 5\left[=\frac{D}{f}\right]$ and MP will be maximum when u = min = (25/6) cm

$\Rightarrow (\text{MP})_{max} = \frac{-25}{-(25/6)} = 6\left[=1+\frac{D}{f}\right]$

COMPOUND MICROSCOPE



Compound microscope is used to get more magnified image. Object is placed infront of objective lens and image is seen through eye piece. The aperture of objective lens is less as compare to eye piece because object is very near so collection of more light is not required. Generally object is placed between F – 2F due to this a real inverted and magnified image is formed between 2F – ∞. It is known as intermediate image A'B'. The intermediate image act as a object for eye piece. Now the distance between both the lens are adjusted in such a way that intermediate image falls between the optical centre of eye piece and its focus. In this condition, the final image is virtual, inverted and magnified.

Total magnifying power = Linear magnification × angular magnification $MP = m_0 m_e = \frac{v_0}{u_0}\frac{D}{u_e}$

(i) When final image is formed at distance of distinct vision.

$$MP = \frac{v_0}{u_0}\left[1+\frac{D}{f_e}\right] = \frac{f_0}{f_0+u_0}\left[1+\frac{D}{f_e}\right] = \frac{f_0-v_0}{f_0}\left[1+\frac{D}{f_e}\right] = \frac{h_2}{h_1}\left(1+\frac{D}{f_e}\right)$$

Length of the tube = $v_0 + |u_e|$

(ii) When final image is formed at infinity $\frac{1}{v_e}-\frac{1}{u_e}=\frac{1}{f_e} \Rightarrow \frac{1}{\infty}+\frac{1}{u_e}=\frac{1}{f_e} \Rightarrow u_e = f_e$

$$MP = \frac{v_0}{u_0}\left[\frac{D}{f_e}\right] = \frac{f_0}{f_0+u_0}\left[\frac{D}{f_e}\right] = \frac{f_0-v_0}{f_0}\left[\frac{D}{f_e}\right] = \frac{h_2}{h_1}\left[\frac{D}{f_e}\right]$$. Length of the tube $L = v_0 + f_e$

Sign convention for solving numerical u_0 = –ve, v_0 = +ve, f_0 = +ve,

u_e = –ve, v_e = –ve, f_e = +ve, m_0 = –ve, m_e = +ve, M = –ve

Ex. A thin convex lens of focal length 5 cm is used as a simple microscope by a person with normal near point (25 cm). What is the magnifying power of the microscope ?

Sol. Here, f = 5 cm; D = 25 cm, M = ? $MP = 1+\frac{D}{f} = 1+\frac{25}{5} = 6$

Ex. A compound microscope consists of an objective lens of focal length 2.0 cm and an eye piece of focal length 6.25 cm, separated by a distance of 15 cm. How far from the objective should an object be placed in order to obtain the final image at (a) the least distance of distinct vision (25 cm) (b) infinity?

Sol. Here, f_0 = 2.0 cm; f_e = 6.25 cm, u_0 = ?

(a) v_e = – 25 cm $\because \frac{1}{v_e}-\frac{1}{u_e}=\frac{1}{f_e} \therefore \frac{1}{u_e}=\frac{1}{v_e}-\frac{1}{f_e}=\frac{1}{-25}-\frac{1}{6.25}=\frac{-1-4}{25}=\frac{-5}{25} \Rightarrow u_e = -5$ cm

As distance between objective and eye piece = 15 cm; v_0 = 15 – 5 = 10 cm

$$\because \frac{1}{v_0}-\frac{1}{u_0}=\frac{1}{f_0} \therefore \frac{1}{u_0}=\frac{1}{v_0}-\frac{1}{f_0}=\frac{1}{10}-\frac{1}{2}=\frac{1-5}{10} \Rightarrow u_0 = \frac{-10}{4} = -2.5 \text{ cm}$$

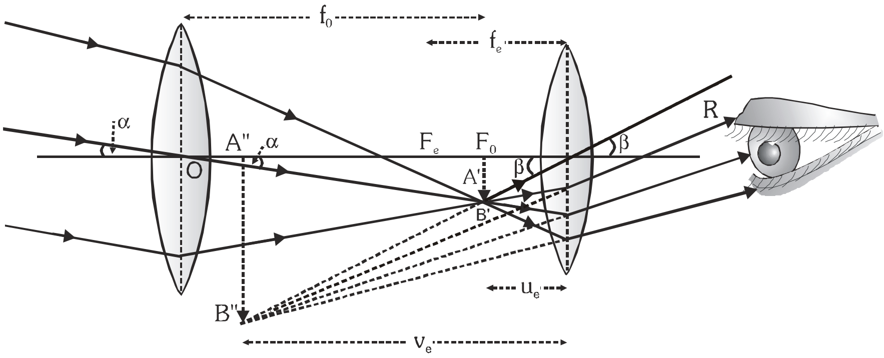
$$\text{Magnifying power} = \frac{v_0}{|u_0|}\left[1+\frac{D}{f_e}\right] = \frac{10}{2.5}\left[1+\frac{25}{6.25}\right] = 20$$

(b) $\because v_e = \infty, u_e = f_e = 6.25$ cm $\therefore v_0 = 15 - 6.25 = 8.75$ cm.

$$\because \frac{1}{v_0}-\frac{1}{u_0}=\frac{1}{f_0} \Rightarrow \frac{1}{u_0}=\frac{1}{v_0}-\frac{1}{f_0}=\frac{1}{8.75}-\frac{1}{20}=\frac{2-8.75}{17.5} \Rightarrow u_0 = \frac{-17.5}{6.75} = -2.59\text{cm}$$

$$\text{Magnifying power} = \frac{v_0}{|u_0|}\times\left[1+\frac{D}{f_e}\right] = \frac{v_0}{|u_0|}\times\frac{D}{|u_e|} = \frac{8.75}{2.59}\times\frac{25}{6.25} = 13.51$$

ASTRONOMICAL TELESCOPE



A telescope is used to see distant object, objective lens forms the image A'B' at its focus. This image A'B' acts as a object for eyepiece and it forms final image A"B".

$$\text{MP} = \frac{\text{visual angle with instrument } (\beta)}{\text{visual angle for unaided eye } (\alpha)} \Rightarrow \text{MP} = \frac{\frac{h'}{f_0}}{\frac{h'}{-u_e}} = -\frac{f_0}{u_e}[A'B' = h']$$

(i) If the final image is at infinity $v_e = -\infty$, $u_e = -\text{ve}$

$$\frac{1}{-\infty} - \frac{1}{-u_e} = \frac{1}{f_e} \Rightarrow u_e = f_e. \text{ So } \text{MP} = -\frac{f_0}{f_e} \text{ and length of the tube } L = f_0 + f_e$$

(ii) If the final image is at D : $v_e = -D$ $u_e = -\text{ve}$

$$\frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e} \Rightarrow \frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D} = \frac{1}{f_e}\left[1 + \frac{f_e}{D}\right] \text{ So } \text{MP} = -\frac{f_0}{u_e} = -\frac{f_0}{f_e}\left[1 + \frac{f_e}{D}\right]$$

Length of the tube is $L = f_0 + u_e$

Ex. A small telescope has an objective lens of focal length 144 cm and an eyepiece of focal length 6.0 cm. What is the magnifying power of the telescope ? What is the separation between the objective and the eyepiece? When final image is formed at infinity.

Sol. Here, $f_0 = 144$ cm; $f_e = 6.0$ cm, MP = ?, L = ?

$$\text{MP} = \frac{-f_0}{f_e} = \frac{-144}{6.0} = -24 \text{ and } L = f_0 + f_e = 144 + 6.0 = 150.0 \text{ cm}$$

Ex. Diameter of the moon is 3.5×10^3 km and its distance from earth is 3.8×10^5 km. It is seen by a telescope whose objective and eyepiece have focal lengths 4m and 10cm respectively. What will the angular diameter of the image of the moon.

Sol. $\text{MP} = -\frac{f_0}{f_e} = -\frac{400}{10} = -40$. Angle subtended by the moon at the objective $= \frac{3.5 \times 10^3}{3.8 \times 10^5} = 0.009$ radian.

Thus angular diameter of the image = MP × visual angle $= 40 \times 0.009 = 0.36$ radian $= \frac{0.36 \times 180}{3.14} \simeq 21°$

Ex. A telescope consisting of an objective of focal length 60 cm and a single–lens eyepiece of focal length 5 cm is focussed at a distant object in such a way that parallel rays emerge from the eye piece. If the object subtends an angle of 2° at the objective, then find the angular width of the image.

Sol. $\text{MP} = \frac{f_0}{f_e} = \frac{\beta}{\alpha} \Rightarrow \beta = \alpha\frac{f_0}{f_e} = 2° \times \frac{60}{5} = 24°$

Ex. The focal lengths of the objective and the eye piece of an astronomical telescope are 60 cm and 5 cm respectively. Calculate the magnifying power and the length of the telescope when the final image is formed at (i) infinity, (ii) least distance of distinct vision (25 cm)

Sol. (i) When the final image is at infinity, then :

$$MP = -\frac{f_0}{f_e} = -\frac{60}{5} = -12 \text{ and length of the telescope is } L = f_0 + f_e = 60 + 5 = 65 \text{ cm}$$

(ii) For least distance of distinct vision, the magnifying power is :

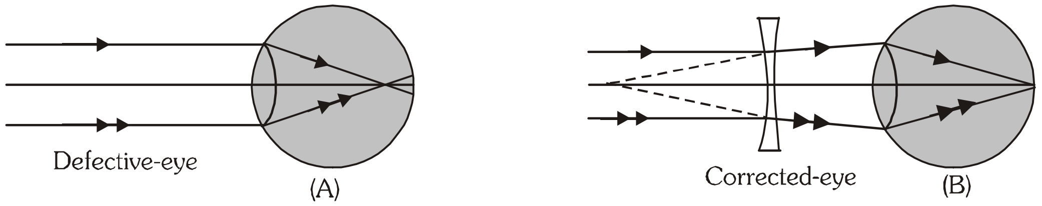
$$MP = -\frac{f_0}{f_e}\left(1+\frac{f_e}{D}\right) = -\frac{60}{5}\left(1+\frac{5}{25}\right) = -\frac{12\times 6}{5} = -14.4$$

$$\text{Now } \frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e} \Rightarrow \frac{1}{5} = -\frac{1}{25} - \frac{1}{u_e} \Rightarrow \frac{-1}{u_e} = \frac{1}{25} + \frac{1}{5} \Rightarrow u_e = -4.17 \text{ cm} \Rightarrow |u_e| = 4.17 \text{ cm}$$

The length of telescope in this position is $L = f_0 + |u_e| = 60 + 4.17 = 64.17$ cm

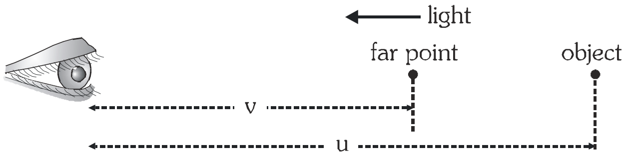
DEFECTS OF EYES

MYOPIA [or Short–sightedness or Near– sightedness]



(i) Distant object are not clearly visible, but near object are clearly visible because image is formed before the retina.

(ii) To remove the defect concave lens is used.



◎ The maximum distance. Which a person can see without help of spectacles is known as far point.

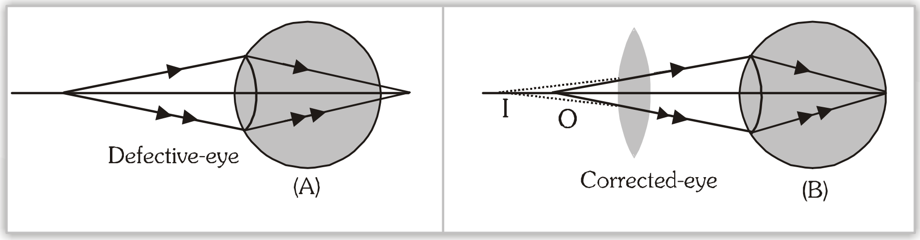
◎ If the reference of object is not given then it is taken as infinity.

◎ In this case image of the object is formed at the far point of person.

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} = P \Rightarrow \frac{1}{\text{distance of far point (in m)}} - \frac{1}{\text{distance of object (in m)}} = \frac{1}{f} = P$$

$$\frac{100}{\text{distance of far point (in cm)}} - \frac{100}{\text{distance of object (in cm)}} = P$$

LONG–SIGHTEDNESS OR HYPERMETROPIA



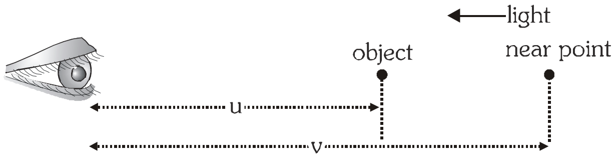
(i) Near object are not clearly visible but far object are clearly visible.

(ii) The image of near object is formed behind the retina.

(iii) To remove this defect convex lens is used.

Near Point :–

The minimum distance which a person can see without help of spectacles.



- In this case image of the object is formed at the near point.
- If reference of object is not given it is taken as 25 cm.

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}=P \Rightarrow \frac{1}{\text{distance of near point (in m)}}-\frac{1}{\text{distance of object (in m)}}=\frac{1}{f}=P$$

distance of near point = –ve, distance of object = –ve, P = +ve

PRESBYOPIA

In this case both near and far object are not clearly visible. To remove this defect two separate spectacles one for myopia and other for hypermetropia are used or bifocal lenses are used.

ASTIGMATISM

In this defect eye can not see object in two orthogonal direction clearly. It can be removed by using cylindrical lens in particular direction.

Ex. A person can not see clearly an object kept at a distance beyond of 100 cm. Find the nature and the power of lens to be used for seeing clearly the object at infinity.

Sol. For lens u = – ∞ and and v = – 100 cm

$$\therefore \frac{1}{v}-\frac{1}{u}=\frac{1}{f}=\frac{1}{v}=\frac{1}{f} \Rightarrow f=v=-100\text{cm}(\text{concave}) \quad \therefore \text{ Power of lens } P=\frac{1}{f}=-\frac{1}{1}=-1D$$

Ex. A far sighted person has a near point of 60 cm. What power lens should be used for eye glasses such that the person can read this book at a distance of 25 cm.

Sol. Here v = – 60 cm, u = –25 cm

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}=-\frac{1}{60}+\frac{1}{25} \Rightarrow f=\frac{300}{7}\text{cm} \quad \therefore \text{Power}=\frac{1}{f(\text{in m})}=\frac{1}{(3/7)}=+2.33D$$

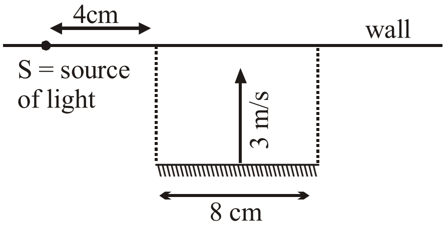
EXERCISE (S-1)

Plane mirror :

1. A fluorescent lamp of length 1 m is placed horizontally at a depth of 1.2 m below a ceiling. A plane mirror of length 0.6 m is placed below the lamp parallel to and symmetric to the lamp at a distance 2.4m from it. The length of the reflected patch of length on the ceiling ______ .

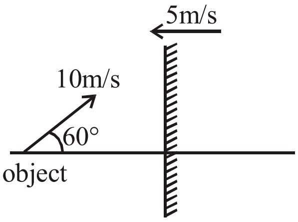
GO0001

2. A plane mirror of length 8 cm is moving with speed 3 m/s towards a wall in situation as shown in figure. Size of spot formed on the wall is 32/x cm. Find the value of x.



GO0002

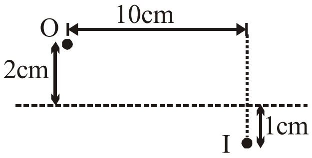
3. The angle between the velocity vector of object and image is $\frac{\pi}{\alpha}$. Velocity are shown in the figure. Then the value of α is.



GO0003

Spherical mirror :

4. The principal axis of a spherical mirror is shown by dotted line. O is the point object whose real image is I. Find the distance of the pole and centre of curvature of the mirror from object measured along principal axis by drawing ray diagram.



GO0004

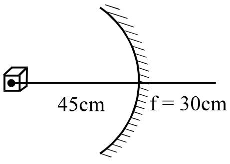
5. An experimentalist devises a method for finding the radius of curvature of a convex mirror. He uses a plane mirror strip between the object and the convex mirror and adjusts it till the two virtual images formed by reflection at both the mirrors coincide without parallax. In his observations, the object distance from the convex mirror is 0.5 m while it is 0.30 m in front of the plane mirror. Find the radius of curvature (in cm) of the convex mirror.

GO0005

6. A thin rod of length d/3 is placed along the principal axis of a concave mirror of focal length = d such that its image, which is real and elongated, just touches the rod. Find the length of the image?

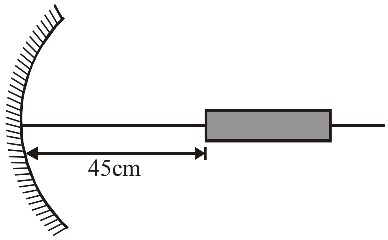
GO0006

7. A cube of side length 1mm is placed on the axis of a concave mirror at a distance of 45 cm from the pole as shown in the figure. One edge of the cube is parallel to the axis. The focal length of the mirror is 30 cm. Find approximate volume of the image.



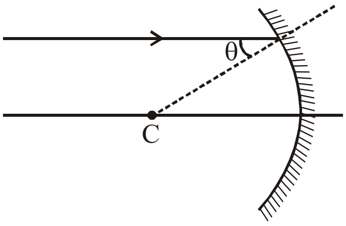
GO0007

8. An object of length 30 cm is placed on principal axis of a concave mirror of focal length 30 cm. Its one end at a distance of 45 cm as shown. If length of image is 10x (in cm) find the value of x.



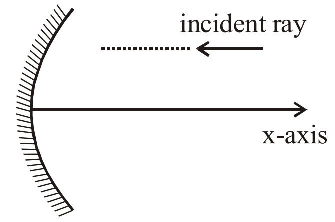
GO0008

9. A ray incident parallel to principal axis on a concave mirror at an angle θ after two reflection, reflected ray becomes parallel to incident ray. If $\theta = \frac{\pi}{N}$ (in radian) then find the value of N.



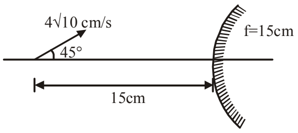
GO0009

10. A parabolic reflecting surface given by $x = \frac{y^2}{2}$, is placed at oirign, as shown. An incidnet ray moving along y = 1 is incident on it. The ray gets reflected by the surface twice. The deviation suffered by the ray is $\frac{\pi}{n}$ radians. Find n.



GO0010

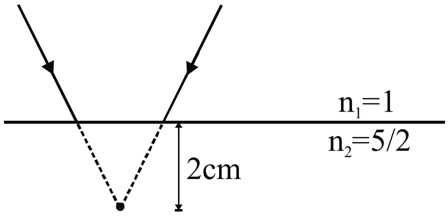
11. In the figure shown , the speed cm/s of image with respect to mirror is :



GO0011

Refraction on plane surface :

12. In the given figure rays incident on an interface would converge 2 cm below the interface if they continued to move in straight lines without bending. But due to refraction, the rays will bend and meet somewhere else. Find the distance of meeting point of refracted rays below the interface (in cm). (Assuming the rays to be making small angles with the normal to the interface)



GO0012

13. A point object is placed 33 cm from a convex mirror of curvature radius = 40 cm. A glass plate of thickness 6 cm and index 2.0 is placed between the object and mirror, close to the mirror. Find the distance of final image from the object?

GO0013

14. An opaque cylindrical tank with an open top has a diameter of 3.00 m and is completely filled with water. When the setting sun reaches an angle of 37° above the horizon, sunlight ceases to illuminate any part of the bottom of the tank. How deep is tank in meter?

GO0014

15. A cylindrical bucket of depth 60 cm is partly filled with a liquid of refractive index 1.5 and with oil (on top of liquid) of refractive index 2. It appears that the volume of air, volume of liquid and volume of oil are equal, to an observer who views from top of the bucket. The apparent depth of the bucket as seen by the observer is given as α cm. Fill $\left(\frac{\alpha}{5}\right)$ in OMR sheet.

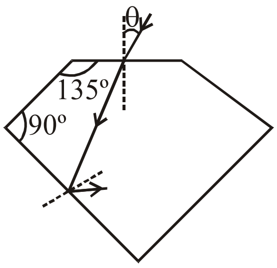
GO0015

Total internal reflection :

16. A room contains air in which the speed of sound is 340 m/s. The walls of the room are made of concrete, in which the speed of sound is 1700 m/s. (a) Find the critical angle for total internal reflection of sound at the concrete–air boundary. (b) In which medium must the sound be traveling to undergo total internal reflection? (Assume sound waves follow snell's law)

GO0016

17. A ray of light enters a diamond (n = 2) from air and is being internally reflected near the bottom as shown in the figure. Find maximum value of angle θ possible ?

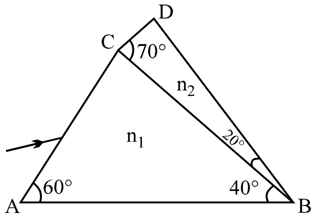


GO0017

18. A prism of refractive index n_1 & another prism of refractive index n_2 are stuck together without a gap as shown in the figure. The angles of the prisms are as shown. n_1 & n_2 depend on λ, the wavelength of light according to $n_1 = 1.20 + \dfrac{10.8\times10^4}{\lambda^2}$ & $n_2 = 1.45 + \dfrac{1.80\times10^4}{\lambda^2}$ where λ is in nm. **[JEE 1998]**

(i) Calculate the wavelength λ_0 for which rays incident at any angle on the interface BC pass through without bending at that interface .

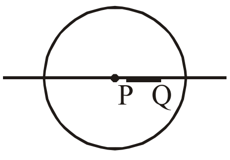
(ii) For light of wavelength λ_0, find the angle of incidence i on the face AC such that the deviation produced by the combination of prisms is minimum .



GO0018

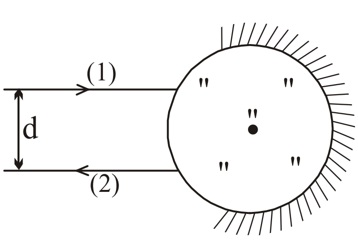
Refraction at curved surface :

19. A small object of length 1 mm lies along the principal axis of a spherical glass of radius R = 10 cm and refractive index is 3/2. The object is seen from air along the principal axis from left. The distance of object from the centre is 5 cm. Find the size of the image. Is real, inverted ?



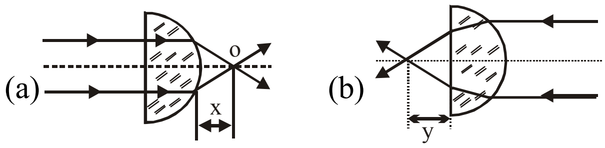
GO0019

20. A transparent sphere of radius R = 2.0 m has a mirrored surface on its right half as shown in figure. A light ray travelling in air is incident on the left side of the sphere. The incident light ray (1) & exiting light ray (2) are parallel & separated by distance d = 2.0 m. Then find the refractive index of the material. (take : sin15°= 0.25)



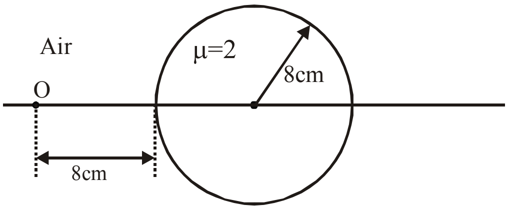
GO0020

21. A narrow beam of light passing through the hemisphere of material with refractive index n, intersects at point O. Where does the beam converge (i.e. y in cm) if beam were to travel in the opposite direction as shown in figure (b)? The value of x is given to be 10 cm. Radius of the hemisphere is also 10 cm :-



GO0021

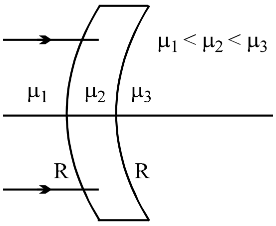
22. On the axis of a transparent sphere of refractive index (n=2) & radius 8 cm, an object is kept at the distance 8 cm from the surface of the sphere. Find the minimum distance of the image after all possible refraction from the surface of the sphere in cm.



GO0022

Lens :

23. A meniscus lens is made of a material of refractive index μ_2. Both its surfaces have radii of curvature R. It has two different media of refractive indices μ_1 and μ_3 respectively, on its two sides (see figure). Calculate its focal length for $\mu_1 < \mu_2 < \mu_3$, when light is incident on it as shown. **[IIT-JEE 2003]**

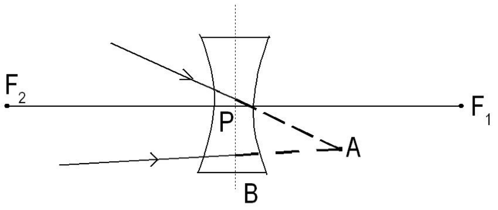


GO0023

24. A converging beam of rays is incident on a diverging lens. Having passed through the lens the rays intersect at a point 15 cm from the lens. If the lens is removed, the point where the rays meet will move 5cm closer to the mounting that holds. Find the focal length (in cm) of the lens without sign.

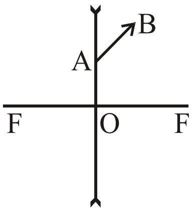
GO0024

25. The rays of a converging beam meet at a point A. A diverging lens is placed in their path in the plane B. Plot the position of the point where the rays meet after passing through the lens. The position of the principal foci FF is known.



GO0025

26. Figure shows ray AB that has passed through a divergent lens. Construct the path of the ray up to the lens if the position of its foci F is known.



GO0026

27. A plano convex lens ($\mu = 1.5$) has a maximum thickness of 1 mm. If diameter of its aperture is 4 cm. Find (i) Radius of curvature of curved surface; (ii) its focal length in air.

GO0027

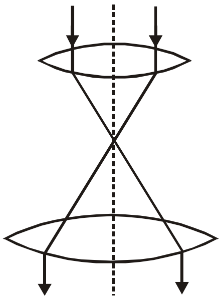
28. A point source of light is kept at a distance of 150 cm from a converging lens, on its optical axis. The focal length of the lens is 100 cm and its diameter is 3 cm. A screen is placed on the other side of the lens, perpendicular to the axis of lens, at a distance 200 cm from it. Then find the area of the illuminated part of the screen? (Assume all rays incident on lens as paraxial)

GO0028

29. There are two thin symmetrical lenses, one is converging, with refractive index $n_1 = 1.70$, and the other is diverging with refractive index $n_2 = 1.50$. Both lenses have the same curvature radius of their surface equal to R = 10 cm. The lenses are put close together and submerged into water. The focal length of lens system is found to be $\frac{100}{x}$ cm in water. What is the value of x. (R.I. of water = 4/3)

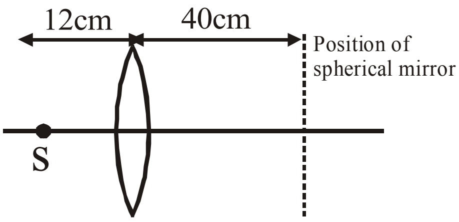
GO0029

30. Consider a 'beam expander' which consists of two converging lenses of focal lengths 40 cm and 100cm having a common optical axis. A laser beam of diameter 2 mm is incident on the 40 cm focal length lens. Then what is the diameter (in mm) of the final beam (see figure)?



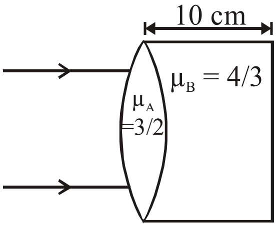
GO0030

31. A converging lens and a spherical mirror are placed with their principal axis coinciding. Their separation equals 40 cm. A point source S is placed on the principal axis at a distance of 12 cm from the lens as shown in the figure. It is found that the final beam comes out parallel to the principal axis. Focal length of the lens equals 15 cm. Find the focal length of the mirror.



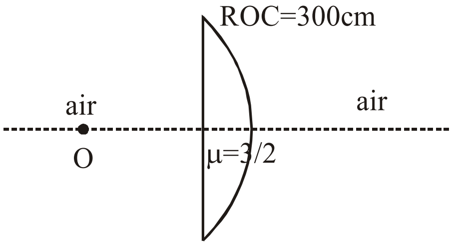
GO0031

32. A parallel paraxial beam of light is incident on the arrangement as shown $\mu_A = 3/2$, $\mu_B = 4/3$, the two spherical surfaces are very close and each has radius of curvature 10 cm. Find the point where the rays are focussed. (w.r.t. point of entry)



GO0032

33. Plane surface of a thin planoconvex lens reflects 50% of light, while the curved surface is completely transparent, if final image of 'O' after refraction through thin lens coincides with the image formed due to partial reflection from plane surface. If distance between O and lens is x (in m) then find the value of x/4.

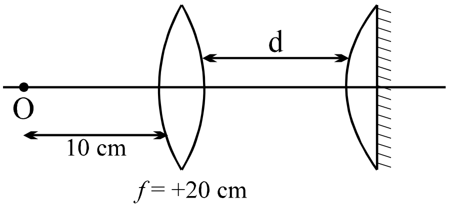


GO0033

34. A lens is placed at origin, with x-axis as its principal axis. A ray of light is incident on it from the –ve side of x-axis along the line $y = \frac{x}{400} + 0.1$, where x, y are in cm. Focal length of lens is 30 cm. Find the equation of the ray after passing through the lens.

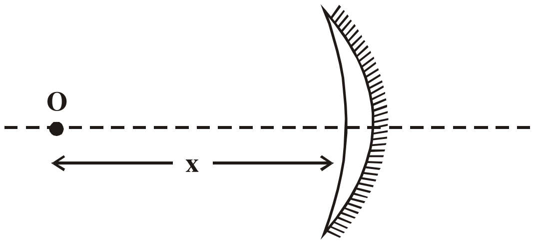
GO0034

35. A convex lens of focal length 20 cm and another plano convex lens of focal length 40 cm are placed co-axially (see fig.). The plano convex lens is silvered on plane surface. What should be the 5d (in m) so that final image of the object 'O' is formed on O itself.



GO0035

36. Radii of curvature of a concavo-convex lens (refractive index = 1.5) are 40 cm and 20 cm as shown. The convex side is silvered. The distance x (in cm) on the principal axis where an object is placed so that its image is created on the object itself, is given as 4β. Find the value of β.



GO0036

Dispersion :

37. A flint glass convex lens of focal length 16 cm is placed in contact with crown glass lens. The refractive indices of crown glass for violet and red colour are $\mu_v = 1.525$, $\mu_r = 1.515$ and for flint glass $\mu_{v'} = 1.655$ and $\mu_{r'} = 1.645$.

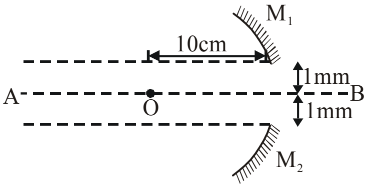
(a) Find the nature and focal length of the crown glass lens which will form an achromatic combination with the flint glass lens.

(b) What is the focal length of the combination?

GO0037

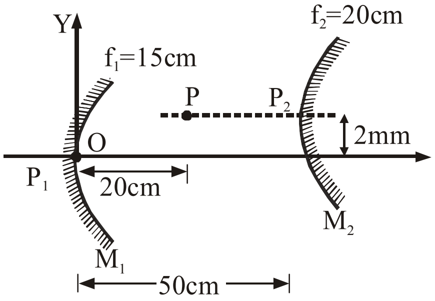
EXERCISE (S-2)

1. A concave mirror of focal length 20 cm is cut into two parts from the middle and the two parts are moved perpendicularly by a distance 1 mm from the previous principal axis AB. If an object is placed of a distance of 10 cm from the mirror on the line AB then, find the distance between the images formed by the two parts?



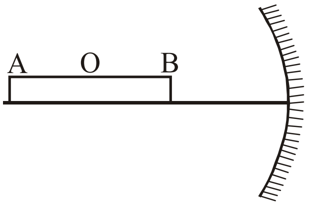
GO0038

2. Find the co-ordinates of image of point object P formed after two successive reflection in situation as shown in figure considering first reflection at concave mirror and then at convex.



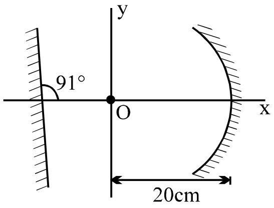
GO0039

3. A rod of length ℓ is moving with constant speed 1 m/s towards the pole of a concave mirror of focal length ℓ. Find the rate of change in length of the image of the rod (in m/s) at the moment centre of the rod coincides with the centre of curvature of the mirror.



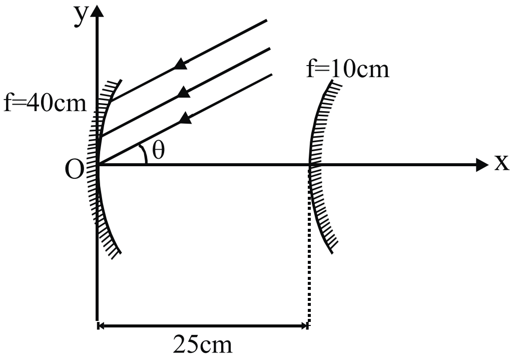
GO0040

4. A point object is placed at the centre of curvature of a concave mirror (taken as origin). A plane mirror is also placed at a distance of 10 cm from the object as shown. Consider two reflection first at plane mirror and then at concave mirror (x_0, y_0). Find $\frac{\pi x_0}{y_0}$ the coordinates of the image thus formed are.



GO0041

5. Consider a concave mirror kept at origin with focal length 40 cm. Parallel rays which subtend an angle $\theta = \frac{1}{40}$ radian are incident on it. A convex mirror is kept at a distance 25 cm from the first mirror as shown. Find the y-coordinate of the image (in cm) formed by the system of mirrors after two reflections.

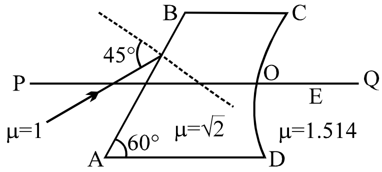


GO0042

6. The x-y plane is the boundary between two transparent media. Medium-1 with $z > 0$ has refractive index $\sqrt{2}$ and medium – 2 with $z < 0$ has a refractive index $\sqrt{3}$. A ray of light in medium –1 given by the vector $A = 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\hat{k}$ is incident on the plane of separation. Find the unit vector in the direction of refracted ray in medium -2. **[1999, 10M]**

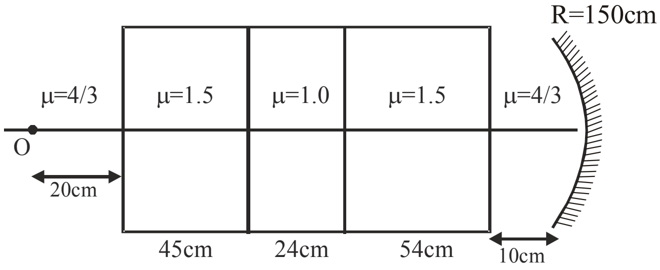
GO0043

7. Figure shows an irregular block of material of refractive index $\sqrt{2}$. A ray of light strikes the face AB as shown in the figure. After refraction it is incident on a spherical surface CD of radius of curvature 0.4 m and enters a medium of refractive index 1.514 to meet PQ at E. Find the distance OE upto two places of decimal. **[IIT-JEE 2004]**



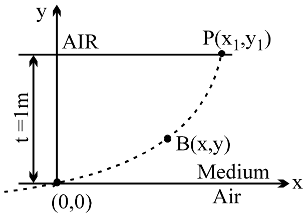
GO0044

8. A composite slab consisting of different media is placed infront of a concave mirror of radius of curvature 150 cm.The whole arrangement is placed in water. An object O is placed at a distance 20cm from the slab. The R.I. of different media are given in the diagram. Find the position of the final image formed by the system.



GO0045

9. A ray of light travelling in air is incident at grazing angle (incident angle = 90°) on a long rectangular slab of a transparent medium of thickness t = 1.0 (see figure). The point of incidence is the origin A (O, O). The medium has a variable index of refraction n(y) given by: $n(y) = [ky^{3/2} + 1]^{1/2}$, where $k=1.0\ m^{-3/2}$. The refractive index of air is 1.0.



(i) Obtain a relation between the slope (dy/dx) of the trajectory of the ray at a point B (x , y) in the medium and the incident angle (*i*) at that point. **[JEE-1995]**

(ii) Find the value of n sin *i*.

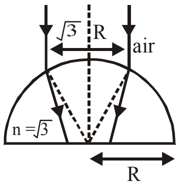
(iii) Obtain an equation for the trajectory y (x) of the ray in the medium.

(iv) Determine the coordinates (x_1, y_1) of the point P, where the ray intersects the upper surface of the slab-air boundary .

(v) Indicate the path of the ray subsequently.

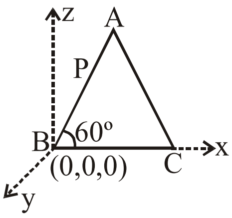
GO0046

10. A light beam of diameter $\sqrt{3}$ R is incident symmetrically on a glass hemisphere of radius R and of refractive index $n = \sqrt{3}$. Find the radius of the beam at the base of hemisphere.



GO0047

11. An equilateral prism ABC is placed in air with its base side C lying horizontally along X-axis as shown in the figure. A ray given by $\sqrt{3}\,z + x = 10$ is incident at a point P on face AB of prism



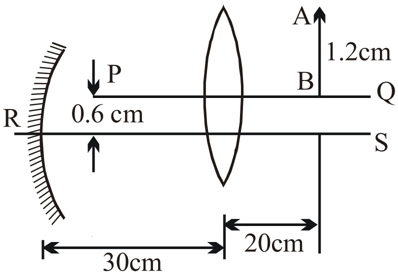
(a) Find the value of μ for which the ray grazes the faces AC.
(b) Find direction of the finally refracted ray if μ = 3/2.
(c) Find the equation of ray coming out of prism if bottom BC is silvered ?

GO0048

12. An object is approaching a thin convex lens of focal length 0.3 m with a speed of 0.01 m/s. Find the magnitudes of the rates of change of position and lateral magnification of image when the object is at a distance of 0.4 m from the lens. **[IIT-JEE 2004]**

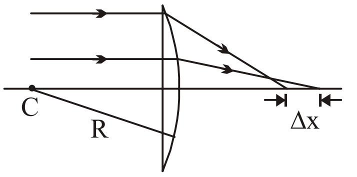
GO0049

13. A convex lens of focal length 15 cm and a concave mirror of focal length 30 cm are kept with their optic axes *PQ* and *RS* parallel but separated in vertical direction by 0.6 cm as shown. The distance between the lens and mirror is 30 cm. An upright object *AB* of height 1.2 cm is placed on the optic axis *PQ* of the lens at a distance of 20 cm from the lens . If *A'B'* is the image after refraction from the lens and reflection from the mirror, find the distance of *A'B'* from the pole of the mirror and obtain its magnification. Also locate positions of *A'* and *B'* with respect to the optic axis *RS*. **[IIT-JEE 2000]**



GO0050

14. Two rays travelling parallel to the principal axis strike a large plano–convex lens having a refractive index of 1.60. If the convex face is spherical, a ray near the edge does not pass through the focal point (spherical aberration occurs). If this face has a radius of curvature of magnitude 20.0 cm and the two rays are $h_1 = 0.500$ cm and $h_2 = 12.0$ cm from the principal axis, find the difference in the positions where they cross the principal axis.

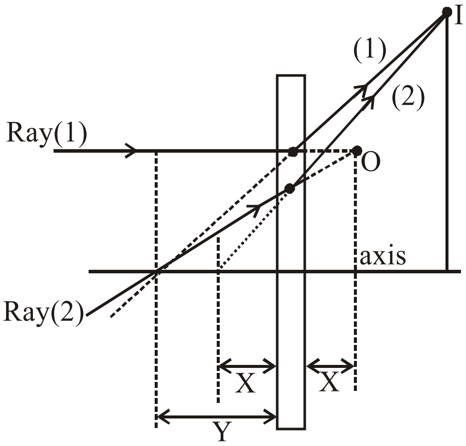


GO0051

15. The rectangular box shown is the place of lens. By looking at the ray diagram, answer the following questions :

(i) If X is 5 cm then what is the focal length of the lens ?

(ii) If the point O is 1 cm above the axis then what is the position of the image ? Consider the optical center of the lens to be the origin.

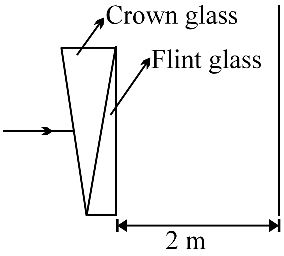


GO0052

16. The refractive indices of the crown glass for violet and red lights are 1.51 and 1.49 respectively and those of the flint glass are 1.77 and 1.73 respectively. A prism of angle 6° is made of crown glass. A beam of white light is incident at a small angle on this prism. The other thin flint glass prism is combined with the crown glass prism such that the net mean deviation is 1.5° anticlockwise.

(i) Determine the angle of the flint glass prism.

(ii) A screen is placed normal to the emerging beam at a distance of 2m from the prism combination. Find the distance between red and violet spot on the screen. Which is the topmost colour on screen.



GO0053

17. A large temple has a depression in one wall. On the floor plan it appears as a indentation having spherical shape of radius 2.50 m. A worshiper stands on the center line of the depression, 2.00 m out from its deepest point, and whispers a prayer. Where is the sound concentrated after reflection from the back wall of the depression?

GO0054

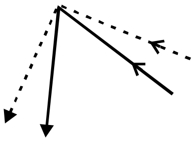
EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

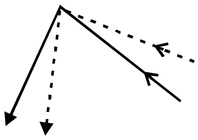
Plane mirror :

1. Each of these diagrams is supposed to show two different rays being reflected from the same point on the same mirror. Which option is correct :-

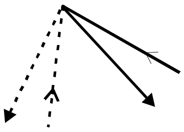
(I)



(II)



(III)



(A) Only I (B) Only II (C) Only III (D) All

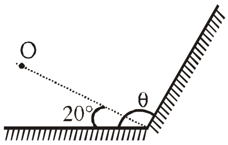
GO0055

2. If two mirrors are kept at 60° to each other, then the number of images formed by them is :- **[AIEEE- 2002]**

(A) 5 (B) 6 (C) 7 (D) 8

GO0056

3. If $\theta = 110°$ then total number of images formed by the mirror system will be :-



(A) 2 (B) 3 (C) 4 (D) 5

GO0057

4. A point object approaches a plane mirror with a speed of 10 ms^{-1}, while the image recedes away from the mirror with a speed of 6 ms^{-1} (as seen by stationary observer). The direction and magnitude of the velocity of mirror is :-

(A) towards the object, 8 ms^{-1} (B) towards the image, 6 ms^{-1}

(C) away from the object, 8 ms^{-1} (D) away from the object, 2 ms^{-1}

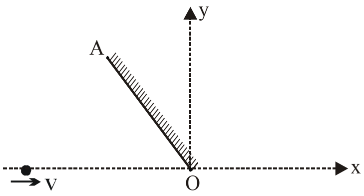
GO0058

5. A boy of TOA3 batch is 1.8m tall and can see his image in a plane mirror fixed on a wall. His eyes are 1.6m from the floor level. The minimum length of the mirror to see his full image is :-

(A) 0.9 m (B) 0.85 m (C) 0.8 m (D) Can't be determined

GO0059

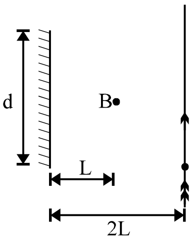
6. A plane mirror OA of length 2 m is kept along the line y = –x as shown in the figure. An insect having velocity of $4\hat{i}$ cm/s is moving along the x-axis from far away. The time span for which the insect can see its image will be :-



(A) 50 sec (B) 25 sec (C) $25\sqrt{2}$ sec (D) $50\sqrt{2}$ sec

GO0060

7. A point source of light *B* is placed at a distance *L* in front of the centre of a mirror of width *d* hung vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance 2*L* from it as shown. The greatest distance over which he can see the image of the light source in the mirror is :- **[IIT-JEE '2000 (Scr)]**

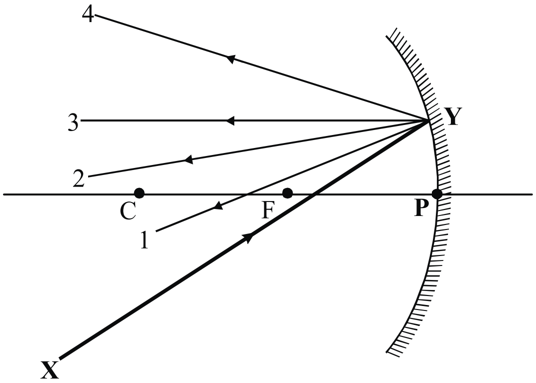


(A) d/2 (B) d (C) 2d (D) 3d

GO0061

Spherical mirror :

8. Figure shows a small concave mirror with *CP* as its principal axis. A ray *XY* is incident on the mirror. Which of the four rays can be the reflected ray.



(A) 1 (B) 2 (C) 3 (D) 4

GO0062

9. The distance of an object from a spherical mirror is equal to the focal length of the mirror. Then the image:

(A) must be at infinity
(B) may be at infinity
(C) may be at the focus
(D) none

GO0063

10. An object is placed in front of a spherical mirror whose 2 times magnified image is formed on screen. Then choose **CORRECT** option :-

(A) Mirror is concave m = +2
(B) Mirror is concave m = –2
(C) Mirror is convex m = +2
(D) Mirror is convex m = –2

GO0064

11. The table below lists object and image positions for four objects placed in front of mirrors, using Cartesian sign convention with pole of the mirror as origin and direction of incident rays as positive. In the following cases, the case in which images is formed by a convex spherical mirror, is :-

	Object position	**Image position**
(A)	– 25.0 cm	– 16.7 cm
(B)	– 5.0 cm	10.0 cm
(C)	– 20.0 cm	5.71 cm
(D)	– 40.0 cm	80.0 cm

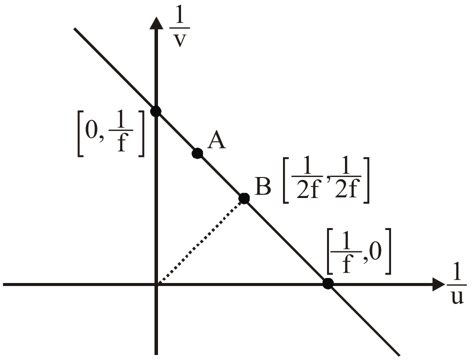
GO0065

12. A particle approaches from very large distance towards concave mirror along the principal axis. By the time the particle reaches the mirror the distance between the particle and its image

(A) first decreases then increases
(B) first increases then decreases
(C) first increases then decreases and then again increases
(D) first decreases then increases and then again decreases

GO0066

13. Figure shows $\frac{1}{v}$ vs $\frac{1}{u}$ curve for convex mirror. Nature of image at point A is :-



(A) Real, erect, magnified
(B) virtual, inverted, magnified
(C) virtual, inverted, diminished
(D) virtual, erect, diminished

GO0067

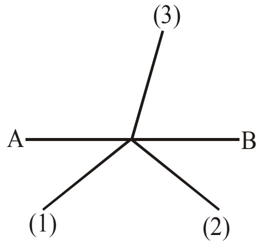
14. A boy of height 1 m stands in front of a convex mirror. His distance from the mirror is equal to its focal length. The height of his image is :- (Assume paraxial ray approximation holds)

(A) 0.25 m (B) 0.33 m (C) 0.5 m (D) 0.67 m

GO0068

Refraction at plane surface :

15. AB is a boundary separating two media of different refractive indices. A ray is incident on the boundary is partially reflected and partially transmitted. Choose the **CORRECT** statement.



(A) 3 is incident ray and 1 is refracted ray

(B) 2 is incident ray and 1 is partially reflected ray

(C) 1 is incident ray and 3 is refracted ray

(D) 3 is incident ray and 2 is partially reflected ray

GO0069

16. **Statement-1:** You see a geostationary satellite above the horizon. You desire to communicate with the satellite by sending a beam of laser light. You should aim your laser slightly higher than the line of sight of the satellite.

Statement-2: Light bends away from the normal while moving from denser to rarer medium.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

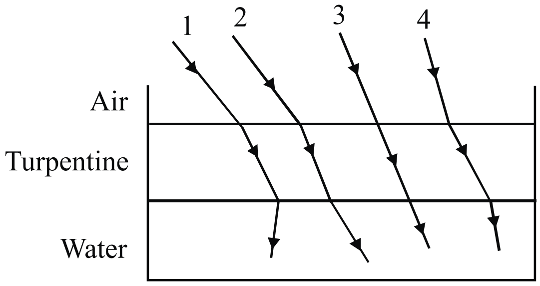
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

GO0070

17. The optical density of turpentine is higher than that of water while its mass density is lower. Figure shows a layer of turpentine floating over water in a container. For which one of the four rays incident on turpentine in figure, the path shown is **CORRECT** ?



(A) 1 (B) 2 (C) 3 (D) 4

GO0071

18. When a ray of light of frequency 6×10^{14} Hz travels from water of refractive index 4/3 to the glass of refractive index 8/5, its :-

(A) frequency decreases to 5/6 of its initial value

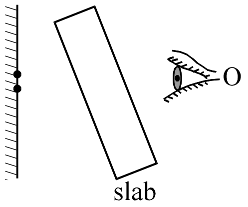
(B) speed decreases to 5/6 of its initial value

(C) wavelength decreases to 6/5 of its initial value

(D) speed increases to 6/5 of its initial value

GO0072

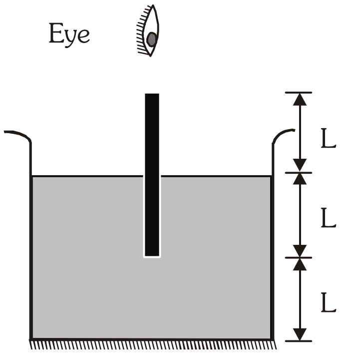
19. The observer at O views two closely spaced spots on a vertical wall through an angled glass slab as shown. As seen by observer, the spots appear.



(A) shifted upward
(B) shifted downward
(C) spaced farther apart
(D) spaced closer together

GO0073

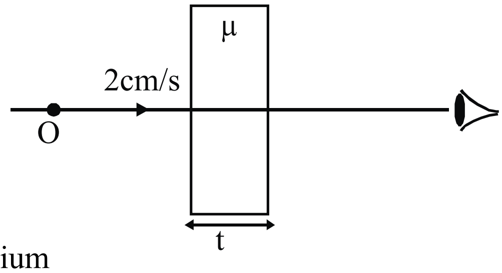
20. What is the length of the image of the rod in mirror, according to the observer in air? (refractive index of the liquid is μ)



(A) $\mu L + L$
(B) $L + \frac{L}{\mu}$
(C) $L\mu + \frac{L}{\mu}$
(D) None of these

GO0074

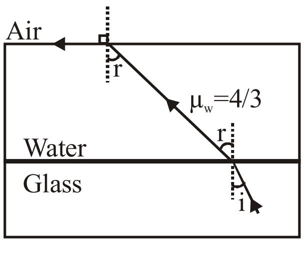
21. A glass slab of width 't', refractive index 'μ' is placed as shown in the figure. If the point object, moves with a speed 2 cm/s towards the slab the speed observered will be



(A) 2 cm/s
(B) less than 2 cm/s
(C) greater than 2 cm/s
(D) dependent on the refractive index of surrounding medium

GO0075

22. A ray of light is incident at the glass–water interface at an angle i, it emerges finally parallel to the surface of water, then the value of μ_g would be :– **[IIT-JEE 2003]**



(A) (4/3) sin i
(B) 1/sin i
(C) 4/3
(D) 1

GO0076

Total internal reflection :

23. **Statement-1**: A point source of light is placed inside water. A light detector present out side, in air can detect light only in a conical region, with the apex at the source and circumscribing the circle of illuminance.

Statement-2: Ray incident from denser to rarer medium undergoes total internal reflection when their angle of incidence become more than the critical angle, this situation create a circular region from which light escapes and is called circle of illuminance.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

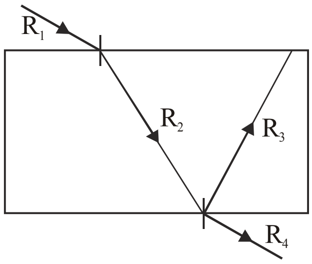
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

GO0077

24. A ray R_1 is incident on the plane surface of the glass slab (kept in air) of refractive index $\sqrt{2}$ at an angle of incidence equal to the critical angle for this air glass system. The refracted ray R_2 undergoes partial reflection and refraction at the other surface. The angle between reflected ray R_3 and the refracted ray R_4 at that surface is :

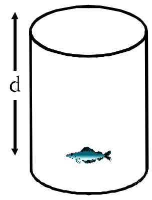


(A) 45° (B) 135°

(C) 105° (D) 75°

GO0078

25. A fish floats in liquid with its eye at the centre of an opaque walled full tank of liquid of circular cross section. When the fish look upwards, it can see a fish-eye view of the surrounding scene i.e. it is able to view the entire space above the liquid surface. The diameter of the tank is 30 cm, and the critical angle for liquid is 37°. At what maximum depth below the surface of the liquid, d, must the fish be floating? $\left(\sin 37° = \frac{3}{5}\right)$



(A) 16 cm (B) 20 cm (C) 11.25 cm (D) 25 cm

GO0079

26. **Statement-1**: When light falls on a sphere made of diamond total internal reflection takes place which makes it shine more than a similar sphere made of common glass.

Statement-2: Refractive index for diamond is more than refractive index of cheap glass.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

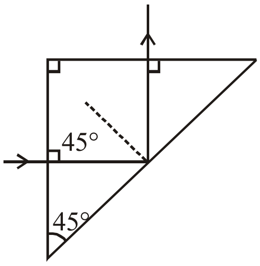
(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

GO0080

27. A light ray is incident perpendicular to one face of a 90° prism and is totally internally reflected at the glass-air interface. If the angle of reflection is 45°, we conclude that the refractive index n :

[AIEEE-2004]



(A) $n < \frac{1}{\sqrt{2}}$ (B) $n > \sqrt{2}$ (C) $n > \frac{1}{\sqrt{2}}$ (D) $n < \sqrt{2}$

GO0081

Prism :

28. A ray of light is incident at 60° on a prism of refracting angle 30°. The emerging ray is at an angle 30° with the incident ray. The value of refractive index of the prism is :-

(A) $\frac{\sqrt{3}}{4}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\sqrt{3}$ (D) $\frac{2}{\sqrt{3}}$

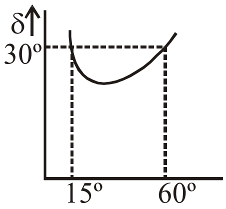
GO0082

29. The refracting angle of the prism is 60° and minimum deviation of 30°, then the angle of incidence is:-

(A) 30° (B) 45° (C) 25° (D) 60°

GO0083

30. Figure shows graph of deviation δ versus angle of incidence for a light ray striking a prism. Angle of prism is :-



(A) 30° (B) 45° (C) 60° (D) 75°

GO0084

31. There is a prism with refractive index equal to $\sqrt{2}$ and the refracting angle equal to 30°. One of the refracting surface of the prism is polished. A beam of monochromatic light will retrace its path if its angle of incidence over the first refracting surface of the prism is :-

(A) 0° (B) 30° (C) 45° (D) 60°

GO0085

32. The refractive index for the material of a 60° prism is 1.50. Then the angle of incidence for minimum deviation is nearly. ($\sin 42° \approx \frac{2}{3}$ and $\sin 49° \approx \frac{3}{4}$)

(A) 30° (B) 49° (C) 38° (D) 28°

GO0086

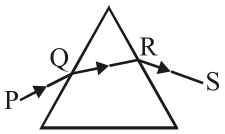
33. A ray of light is incident on an equilateral glass prism placed on a horizontal table. For minimum deviation which of the following is true ?

(A) PQ is horizontal

(B) QR is horizontal

(C) RS is horizontal

(D) Either PQ or RS is horizontal



GO0087

34. An isosceles glass prism having refractive index μ has one of its faces coated with silver. A ray of light is incident normally on the other face (which is equal to the silvered face). The ray of light is reflected twice on the same sized faces and then emerges through the base of the prism perpendicularly. The angles of prism are :-

(A) 40°, 70°, 70° (B) 50°, 65°, 65° (C) 36°, 72°, 72° (D) data insufficient

GO0088

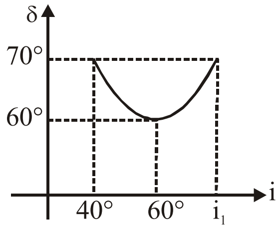
35. A given ray of light suffers minimum deviation in an equilateral prism P. If refractive index increases slightly then the ray will now suffer :-

(A) greater deviation (B) no deviation

(C) same deviation as before (D) Lesser deviation

GO0089

36. The curve of angle of incidence versus angle of deviation shown has been plotted for prism. The value of refractive index of the prism used is :



(A) $\sqrt{3}$ (B) $\sqrt{2}$ (C) $\frac{\sqrt{3}}{\sqrt{2}}$ (D) $\frac{2}{\sqrt{3}}$

GO0090

37. A beam of monochromatic light is incident at i = 50° on one face of an equilateral prism, the angle of emergence is 40°, then the angle of minimum deviation is :

(A) 30° (B) < 30° (C) ≤ 30° (D) ≥ 30°

GO0091

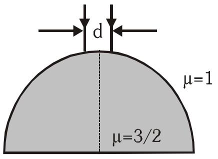
38. The angle of a prism is 6° and its refractive index for green light is 1.5. If a green ray passes through it, the deviation will be :-

(A) 30° (B) 15° (C) 9° (D) 3°

GO0092

Refraction at curved surface :

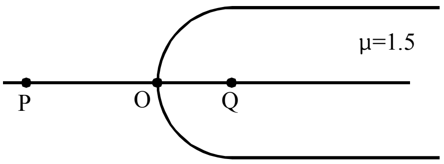
39. A beam of diameter 'd' is incident on a glass hemisphere as shown. If the radius of curvature of the hemisphere is very large in comparison to d, then the diameter of the beam at the base of the hemisphere will be :-



(A) $\frac{3}{4}$d (B) d (C) $\frac{d}{3}$ (D) $\frac{2}{3}d$

GO0093

40. One end of a glass rod of refractive index n = 1.5 is a spherical surface of radius of curvature R. The centre of the spherical surface lies inside the glass. A point object placed in air on the axis of the rod at the point P has its real image inside glass at the point Q (see fig.). A line joining the points P and Q cuts the surface at O such that OP = 2OQ. The distance PO is :-



(A) 8 R (B) 7 R (C) 2 R (D) None of these

GO0094

41. An air bubble is inside water. The refractive index of water is 4/3. At what distance from the air bubble should a point object be placed so as to form a real image at the same distance from the bubble:

(A) 2R (B) 3R

(C) 4R (D) The air bubble cannot form a real image

GO0095

42. A point object is placed at the centre of a glass sphere of radius 6 cm and refractive index 1.5. The distance of the virtual image from the surface of the sphere is :- **[IIT-JEE 2004 (Scr)]**

(A) 2 cm (B) 4 cm (C) 6 cm (D) 12 cm

GO0096

Lens :

43. A concave lens of glass, refractive index 1.5, has both surfaces of same radius of curvature R. On immersion in a medium of refractive index 1.75, it will behave as a :-

(A) convergent lens of focal length 3.5R (B) convergent lens of focal length 3.0 R.

(C) divergent lens of focal length 3.5 R (D) divergent lens of focal length 3.0 R

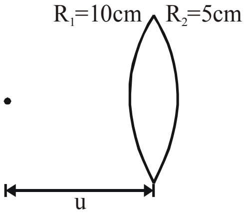
GO0097

44. A convergent (biconvex) lens is placed inside a jar filled with a liquid. The lens has focal length 20cm, when in air and its material has a refractive index of 1.5. If the liquid has a refractive index of 1.6, the focal length of the lens while in the jar, is :-

(A) –110 cm (B) –130 cm (C) –160 cm (D) –180 cm

GO0098

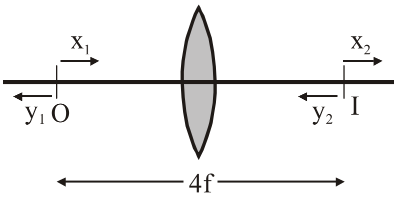
45. A thin convex lens is made of a material of refractive index 1.6. An object is kept at a distance of u from the lens on the principal axis as shown in the figure. The radius of curvature of the surfaces are 10 cm and 5cm. Now, the lens is reversed such that the face having radius of curvature 5cm lies close to the object. The difference in image position as obtained for both the cases is equal to :-



(A) 0.4 u (B) 0.6 u (C) 0.8 u (D) 0

GO0099

46. In a converging lens of focal length f and the distance between real object and its real image is $4f$. If the object moves x_1 distance towards lens its image moves x_2 distance away from the lens and when object moves y_1 distance away from the lens its image moves y_2 distance towards the lens, then choose the correct option:-



(A) $x_1 > x_2$ and $y_1 > y_2$ (B) $x_1 < x_2$ and $y_1 < y_2$

(C) $x_1 < x_2$ and $y_1 > y_2$ (D) $x_1 > x_2$ and $y_2 > y_1$

GO0100

47. A point object is placed on the principal axis of a converging lens and its image (I_1) is formed on its principal axis. If the lens is rotated by an small angle θ about its optical centre such that its principal axis also rotates by the same amount then the image (I_2) of the same object is formed at point P. Choose the correct option.

(A) Point P lies on the new principal axis.

(B) Point P lies on the old principal axis.

(C) Point P is anywhere between the two principal axes

(D) None of these

GO0101

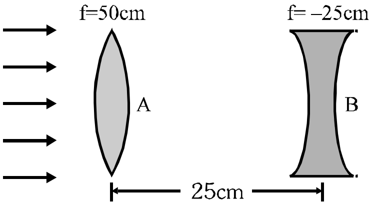
48. The two lenses shown are illuminated by a beam of parallel light from the left. Lens B is then moved slowly toward lens A. The beam emerging from lens B is :

(A) Initially parallel and then diverging

(B) Always diverging

(C) Initially converging and finally parallel

(D) Always parallel

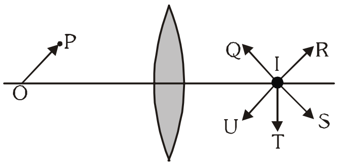


GO0102

49. A point object O moves from the principal axis of a converging lens in a direction OP. I the image of O, will move initially in the direction :

(A) IQ (B) IR

(C) IS (D) IU



GO0103

50. When the object is at distances u_1 and u_2 the images formed by the same lens are real and virtual respectively and of the same size. Then focal length of the lens is :

(A) $\frac{1}{2}\sqrt{u_1u_2}$ (B) $\frac{u_1+u_2}{2}$ (C) $\sqrt{u_1u_2}$ (D) $\sqrt{(u_1+u_2)}$

GO0104

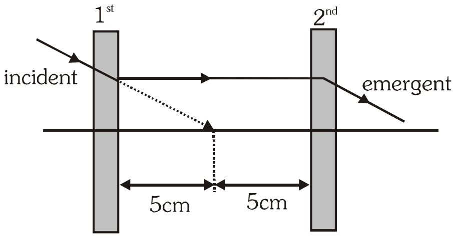
51. Look at the ray diagram shown, what will be the focal length of the 1st and the 2nd lens, if the incident light ray passes without any deviation ?

(A) –5 cm and +10 cm

(B) +5cm and +10cm

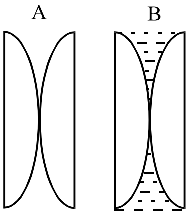
(C) –5cm and +5cm

(D) +5cm and – 5cm



GO0105

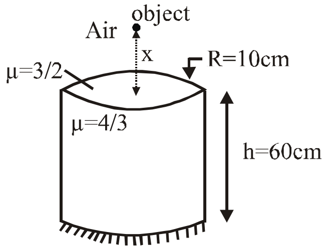
52. Figure A shows two identical plano-convex lenses in contact as shown. The combination has focal length 24 cm. Figure B shows the same with a liquid introduced between them. If refractive index of glass of the lenses is 1.50 and that of the liquid is 1.60, the focal length of the system in figure B will be :-



(A) –120 cm (B) 120 cm (C) –24 cm (D) 24 cm

GO0106

53. In the arrangement shown given that a biconvex lens of radius of curvature equal to 10 cm and concave mirror has focal length equal to 20cm. Then find the distance 'x' such that the final image formed by the system coincides with the object.



(A) 15 cm (B) 22.5 cm (C) 20 cm (D) 17.5 cm

GO0107

54. A convex lens of focal length 30 cm forms an image of height 2 cm for an object situated at infinity. If a convcave lens of focal length 20 cm is placed coaxially at a distance of 26 cm in front of convex lens then size image would be :- **[IIT-JEE 2003 (Scr)]**

(A) 2.5 cm (B) 5.0 (C) 1.25 (D) None

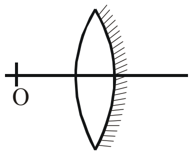
GO0108

55. A plano-convex lens of refractive index 1.5 and radius of curvature 30 cm is silvered at the curved surface. Now, this lens has been used to form the image of an object. At what distance from this lens, an object be placed in order to have a real image of the size of the object? **[AIEEE-2004]**

(A) 20 cm (B) 30 cm (C) 60 cm (D) 80 cm

GO0109

56. An equiconvex lens of refractive index μ and radius of curvature R has its one surface silvered. A point source O is placed before the silvered lens so that its image is coincident with it, the distance of the object from the lens is :-

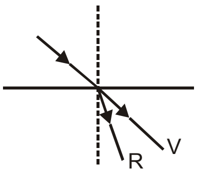


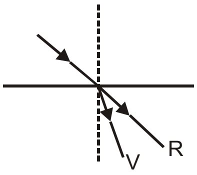
(A) $\frac{R}{(\mu-1)}$ (B) $\frac{2R}{(\mu-1)}$ (C) $\frac{R}{(2\mu-1)}$ (D) $\frac{2R}{(2\mu-1)}$

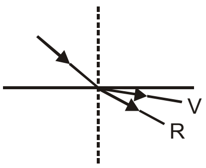
GO0110

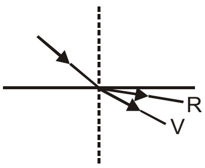
Dispersion :

57. A ray of light is coming from air to water. Which of the following figure shows correct dispersion of light ?

(A)


(B)


(C)


(D)


GO0111

58. When white light passes through a glass prism, one gets spectrum on the other side of the prism. In the emergent beam, the ray which is deviating least is or deviation by a prism is lowest for

(A) Violet ray (B) Green ray (C) Red ray (D) Yellow ray

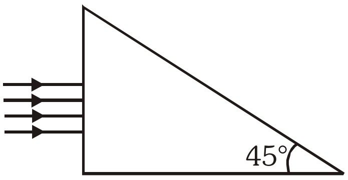
GO0112

59. The angle of prism is 5° and its refractive indices for red and violet colours are 1.5 and 1.6 respectively. The angular dispersion produced by the prism is :–

(A) 7.75° (B) 5° (C) 0.5° (D) 0.17°

GO0113

60. A beam of light consisting of red, green and blue and is incident on a right angled prism. The refractive index of the material of the prism for the above red, green and blue wavelengths are 1.39, 1.44 and 1.47 respectively. The prism will



(A) separate part of the red color from the green and blue colors

(B) separate part of the blue color from the red and green colors

(C) separate all the three colors from the other two colors

(D) not separate even partially, any colors from the other two colors

GO0114

61. Two thin prisms of flint glass, with refracting angles of 6º and 8º respectively, possess dispersive powers in the ratio :

(A) 4 : 3 (B) 3 : 4 (C) 1 : 1 (D) 9 : 16

GO0115

62. Chromatic aberration will be absent if for two thin lenses in contact :-

(A) $(\omega_1/F_1) + (\omega_2/F_2) = 0$ (B) $(\omega_1/F_2) + (\omega_2/F_1) = 0$

(C) $(F_1/\omega_2) + (F_2/\omega_1) = 0$ (D) $(\omega_1/\omega_2) + (F_1 + F_2) = 0$

GO0116

63. The dispersive powers of the materials of the two lenses are in the ratio 4 : 3. If the achromatic combination of these two lenses in contact is a convex lens of focal length 60 cm then the focal lengths of the component lenses are

(A) - 20 cm and 25 cm (B) 20 cm and - 25 cm

(C) - 15 cm and 40 cm (D) 15 cm and - 20 cm

GO0117

64. An achromatic convergent doublet of two lenses in contact has a power of + 2D. The convex lens has a power + 5D. What is the ratio of the dispersive powers of the convergent and divergent lenses

(A) 2 : 5 (B) 3 : 5 (C) 5 : 2 (D) 5 : 3

GO0118

65. It is desired to make an achromatic combination of two lenses (L_1 & L_2) made of materials having dispersive powers ω_1 and ω_2 ($<\omega_1$). If the combination of lenses is converging then

(A) L_1 is converging

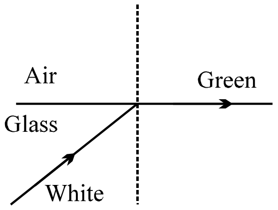
(B) L_2 is converging

(C) Power of L_1 is greater than the power of L_2

(D) None of these

GO0119

66. White light is incident on the interface of glass and air as shown in the figure. If green light is just totally internally reflected then the emerging ray in air contains **[IIT-JEE 2004 (Scr)]**



(A) yellow, orange, red

(B) violet, indigo, blue

(C) all colours

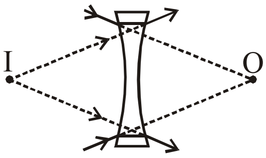
(D) all colours except green

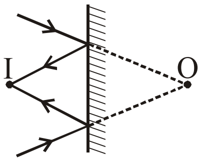
GO0120

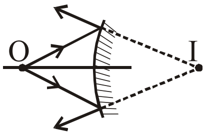
MULTIPLE CORRECT TYPE QUESTIONS

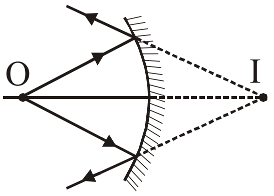
Spherical mirror :

67. The nature of object and image given with each of the optical condition is shown. Choose the **correct** option(s) :-

(A) Virtual image  Virtual object

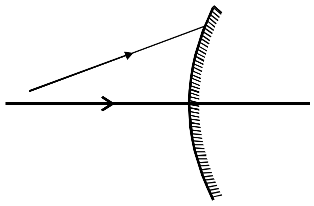
(B) Virtual image  Real object

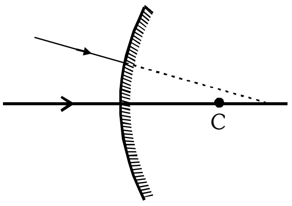
(C) Real object  Virtual image

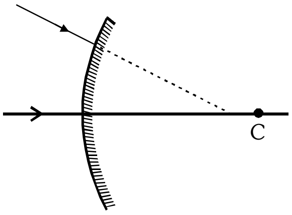
(D) Real object  Virtual image

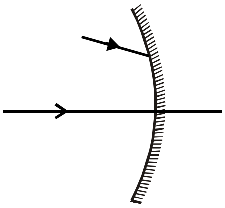
GO0121

68. In which of the following diagrams the image formed is virtual and inverted ?

(A) 

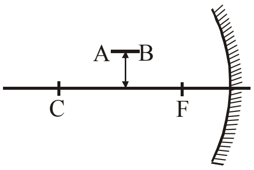
(B) 

(C) 

(D) 

GO0122

69. An object AB is placed parallel and close to the optical axis between focus F and centre of curvature C of a converging mirror of focal length f as shown in figure.



(A) Image of A will be closer than that of B from the mirror.

(B) Image of AB will be parallel to the optical axis.

(C) Length of image is equal to AB.

(D) Length of image is more than AB.

GO0123

70. Mark the **CORRECT** statement :-

(A) Virtual image of virtual object can be formed by a single optical element

(B) Virtual image of virtual object formed by a single optical element can be inverted

(C) Virtual image of real object can be formed by a single optical element

(D) Virtual image of real object formed by a single optical element can be inverted

GO0124

71. A concave mirror forms an image of the sun at a distance of 12 cm from it.

(A) the radius of curvature of this mirror is 6 cm.

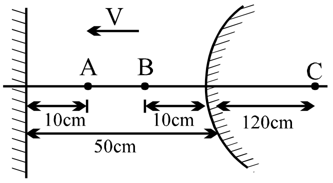
(B) to use it as a shaving mirror, it can be held at a distance of 8-10 cm from the face

(C) if an object is kept at a distance of 24 cm from it, the image formed will be of the same size as the object

(D) all the above alternatives are correct.

GO0125

72. In the figure shown consider the first reflection at the plane mirror and second at the convex mirror. AB is object.



(A) the second image is real , inverted of 1/5 th magnification w.r.t AB

(B) the second image is virtual and erect with magnification1/5 w.r.t AB

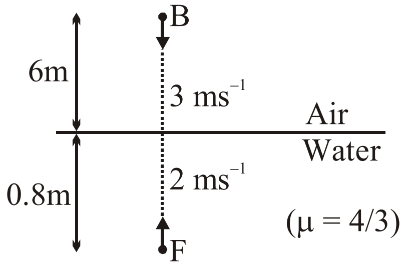
(C) the second image moves towards the convex mirror

(D) the second image moves away from the convex mirror.

GO0126

Refraction at plane surface :

73. A fish, F in the pond, is at a depth of 0.8 m from water surface and is moving vertically upwards with velocity 2 ms^{-1}. At the same instant, a bird B is at a height of 6 m from water surface and is moving downwards with velocity 3 ms^{-1}. At this instant both are on the same vertical lines as shown in the figure. Which of the following statement(s) is(are) correct?



(A) Height of B, observed by F (from itself) is equal to 8.00 m.

(B) Depth of F, observed by B (from itself) is equal to 6.60 m.

(C) Velocity of B, observed by F (relative to itself) is equal to 5.00 ms^{-1}.

(D) Velocity of F, observed by B (relative to itself) if equal to 4.50 ms^{-1}.

GO0127

Total internal reflection :

74. In the figure (a) light is incident at an angle θ which is slightly greater than the critical angle. Now, keeping the incident fixed a slab of refractive index n_3 is placed on surface AB in fig. (b). Which of the following statements are correct:

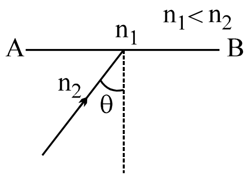


Fig. (a)

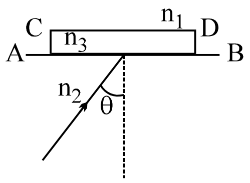


Fig. (b)

(A) total internal reflection occurs at AB for $n_3 < n_1$

(B) total internal reflection occurs at AB for $n_3 > n_1$

(C) the ray will return back to the same medium for all values of n_3

(D) total internal reflection occurs at CD for $n_3 < n_1$

GO0128

75. For the refraction of light through a prism

(A) For every angle of deviation there are two angles of incidence.

(B) The light travelling inside an equilateral prism is necessarily parallel to the base when prism is set for minimum deviation.

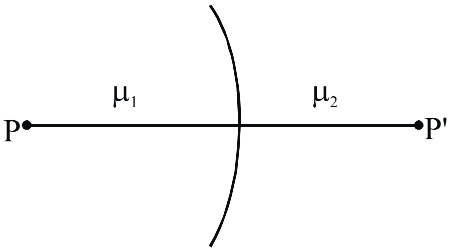
(C) There are two angles of incidence for maximum deviation.

(D) Angle of minimum deviation will increase if refractive index of prism is increased keeping the outside medium unchanged if $\mu_P > \mu_S$.

GO0129

Refraction at curved surface :

76. Two refracting media are separated by a spherical interface as shown in the figure. *PP*' is the principal axis, μ_1 and μ_2 are the refractive indices of medium of incidence and medium of refraction respectively, then



(A) if $\mu_2 > \mu_1$, then there cannot be a real image of real object.

(B) if $\mu_2 > \mu_1$, then there cannot be a real image of virtual object.

(C) if $\mu_1 > \mu_2$, then there cannot be a virtual image of virtual object.

(D) if $\mu_1 > \mu_2$, then there cannot be a real image of real object.

GO0130

Lens :

77. Optical axis of a thin equi-convex lens is the X-axis. The co-ordinate of a point object and its image are (–20 cm, 1 cm) and (25 cm, –2 cm) respectively

(A) the lens is located at x = 5 cm

(B) the lens is located at x = –5 cm

(C) the focal length of the lens is 10 cm

(D) the focal length of the lens is 15 cm

GO0131

78. The principal axis of an optical device is along y = –1, image of a small body placed at (–30,3) is formed at a point (60, –3). Then the optical device is

(A) A convex lens of focal length 20 cm

(B) A concave mirror of focal length 60 cm

(C) A concave lens of focal length 20 cm

(D) A convex mirror of focal length 60 cm

GO0132

79. In displacement method, the distance between object and screen is 96 cm. The ratio of length of two images formed by a convex lens placed between them is 4.84 :-

(A) Ratio of the length of object to the length of shorter image is 11/5.

(B) Distance between the two positions of the lens is 36 cm.

(C) Focal length of the lens is 20.625 cm.

(D) Distance of the lens from the shorter image is 30 cm.

GO0133

80. A lens is formed by a material having refecting index $\frac{3}{2}$ & radii of curvature 20cm & 10 cm. Then choose the correct option(s) :-

(A) If lens is concavo-convex & light is falling on the surface having radius of curvature 20 cm, then focal length is 40 cm

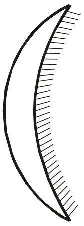
(B) If lens is concavo-convex & light is falling on the surface having radius of curvature 10 cm, then focal length is –40 cm

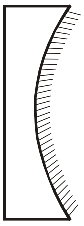
(C) If lens is convexo-concave & light is falling on the surface having radius of curvature 20 cm, then focal length is 40 cm

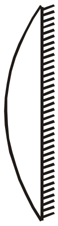
(D) If lens is convexo-concave & light is falling on the surface having radius of curvature 10 cm, then focal length is –40 cm

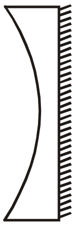
GO0134

81. Which of the following silvered lenses kept in air may form real image of a real object.

(A) 

(B) 

(C) 

(D) 

GO0135

82. A man wanted to get a picture of a Zebra. He photographed a white donkey after fitting a glass with black streaks onto the objective of his camera.

(A) the image will look like a white donkey on the photograph.

(B) the image will look like a Zebra on the photograph.

(C) the image will be more intense compared to the case in which no such glass is used.

(D) the image will be less intense compared to the case in which no such glass is used.

GO0136

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 83 to 85

A real object is placed perpendicular to the principal axis of a concave mirror at a position (1) such that the image formed is real with a magnification 2. The object is now shifted to another position (2) at a distance 15 cm from the position (1) and a real image is obtained with a magnification 8. Shifting the position of object from (2) to a third position (3) gives an image with magnification 1/2.

83. Position of the image when the object is at position (3) is :-

(A) between centre of curvature and ∞ (B) between focus and centre of curvature

(C) at focus (D) between pole and focus

GO0137

84. Focal length of the mirror is :-

(A) – 60 cm (B) –40 cm (C) –30 cm (D) –25 cm

GO0137

85. Distance of position (1) of the object from pole and that of position (2) of object from pole are , respectively :-

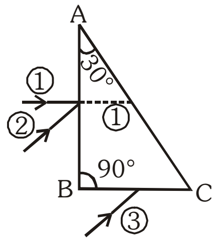
(A) 60 cm, 75 cm (B) 75 cm, 60 cm (C) 60 cm, 45 cm (D) 45 cm, 30 cm

GO0137

Paragraph for Question No. 86 to 88

ABC is a right-angled prism kept in air. A ray (1) is incident on the face AB along the normal. Refractive index of the material of prism is the minimum value that will be required so that ray (1) undergoes total internal reflection at the face AC.

Another ray (2) is incident on the face AB such that it emerges from face AC along the normal to AC. A third ray (3) falls on the face BC and emerges from face AC such that its angle of emergence is the same as that of incidence. Assuming light (1), (2) and (3) to have the same frequency, answer the following questions.



86. Refractive index of the material of prism is :-

(A) $\sqrt{2}$ (B) 2 (C) 1.5 (D) 2.2

GO0138

87. Angle of incidence of ray (2) is :-

(A) 0° (B) 45° (C) 60° (D) 90°

GO0138

88. Deviation suffered by ray (3) is :-

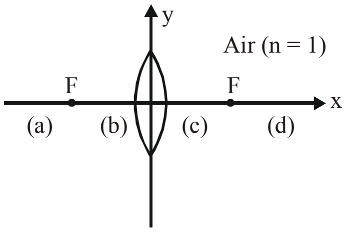
(A) 60° (B) 30° (C) 90° (D) 120°

GO0138

Paragraph for Question no. 89 and 90

This question concerns a symmetrical lens shown, along with its two focal points. It is made of plastic with (n = 1.2), and has a focal length f. Four different regions are shown :

Here (a) $-\infty < x < -f$; (b) $-f < x < 0$; (c) $0 < x < f$; (d) $f < x < \infty$



89. If an object is placed somewhere in region (a), in which region does the image appear ?

(A) (a) (B) (b) (C) (c) (D) (d)

GO0139

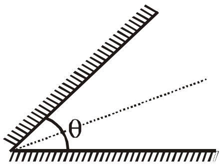
90. If incident rays are converging then in which region does the image appear ?

(A) (a) (B) (b) (C) (c) (D) (d)

GO0139

MATRIX MATCH TYPE QUESTIONS

91. Angle between two mirrors 'θ' and location of object is given in column I and some possible number of images are given in column II.



	Column–I		**Column–II**
(A)	$\theta = 30°$ and object is on bisector	(P)	9
(B)	$\theta = 36°$ and object is on bisector	(Q)	11
(C)	$\theta = 22.5°$ and object is on bisector	(R)	8
(D)	$\theta = 50°$ and object is on bisector	(S)	15

GO0140

92. An object O is kept perpendicular to the principal axis of a spherical mirror. In column I, coordinate of, object, u with sign in cm, type of mirror and its focal length in cm is given

	Column-I		**Column-II**
(A)	u = –18, concave mirror, 12	(P)	Image is real
(B)	u = –12, concave mirror, 18	(Q)	Image is virtual
(C)	u = –8, convex mirror, 10	(R)	Image diminished
(D)	u = –10, convex mirror, 8	(S)	Image is enlarged
		(T)	Image is inverted

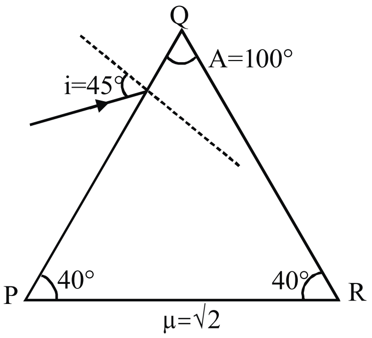
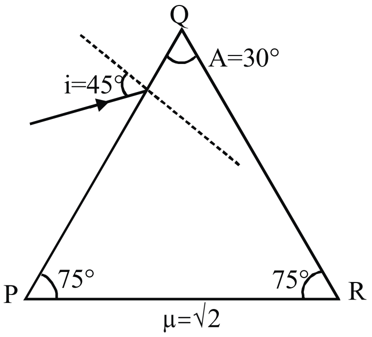
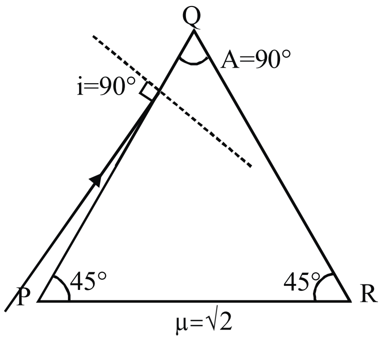
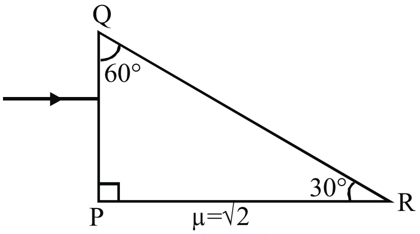
GO0141

93.

	Column-I		**Column-II**
(A)	An object is placed at a distance equal to focal length from pole before convex mirror	(P)	Maginification is (∞)
(B)	An object is placed at focus before a concave mirror	(Q)	Magnification is (0.5)
(C)	An object is placed at the centre of curvature before a concave mirror.	(R)	Magnification is (1/3)
(D)	An object is placed at a distance equal to radius of curvature before a convex mirror.	(S)	Magnification is (–1)

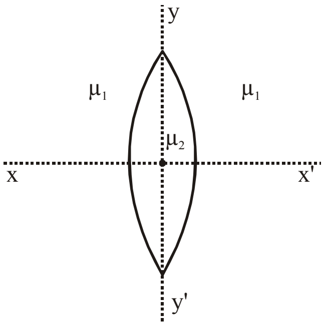
GO0142

94. Light is incident at surface PQ of prism as shown in column-I then match the column-I with column-II (surrounding medium is air in all cases)

Column-I	**Column-II**
(A) 	(P) Total internal reflection takes place at surface QR.
(B) 	(Q) Light emerges normally from the surface QR
(C)  Grazing incidence	(R) Light emerges parallel to surface QR
(D)  Normal incidence	(S) The light ray emerges from face PR perpendicularly
	(T) When light ray passes through the prism it is parallel to the base PR.

GO0143

95. An equi-convex lens of refractive index μ_2 and focal length f(in air) is kept in medium of refractive index μ_1.



	Column A		Column B
(P)	If lens is cut in two equal parts by a plane yy'. ($\mu_1 = 1$)	(1)	Lens will act as a glass slab
(Q)	If lens is cut in two equal parts by a plane xx' . ($\mu_1 = 1$)	(2)	Lens will be converging and focal length will change.
(R)	If $\mu_1 = \mu_2$.	(3)	Lens will be converging and focal length will remain same
(S)	If $\mu_1 > \mu_2$.	(4)	Lens will be diverging and focal length will change.

Codes :

	P	Q	R	S
(A)	2	4	1	3
(B)	2	3	1	4
(C)	3	4	2	1
(D)	1	2	3	4

GO0144

96. Column-II shows the optical phenomenon that can be associated with optical components given in column-I. Note that column-I may have more than one matching options in column-II.

	Column-I		Column-II
(A)	Convex mirror	(P)	Dispersion
(B)	Converging lens	(Q)	Deviation
(C)	Thin prism	(R)	Real image of real object
(D)	Glass slab	(S)	Virtual images of real object

GO0145

SUPPLEMENT FOR MAINS

Optical Instruments :

97. A long sighted person has a minimum distance of distinct vision of 50 cm. He wants to reduce it to 25cm. He should use a

(A) Concave lens of focal length 50 cm
(B) Convex lens of focal length 25 cm
(C) Convex lens of focal length 50 cm
(D) Concave lens of focal length 25 cm

GO0148

98. A person's near point is 50 cm and his far point is 3m. Powers of the lenses he required for (i) for reading (ii) for viewing distinct stars

(A) - 2D and 0.33 D
(B) 2D and - 0.33 D
(C) - 2D and 3D
(D) 2D and - 3 D

GO0149

99. When length of a compound microscope tube increase, its magnifying power

(A) decreases
(B) increases
(C) does not change
(D) may increase or decrease

GO0153

100. When the length of an astronomical telescope tube increases its magnifying power

(A) Decreases
(B) Increases
(C) Does not change
(D) May increase or decrease

GO0155

Paragraph for Question 110 to 113

101. If an astronomical telescope has objective and eye-pieces of focal lengths 200 cm and 4 cm respectively, then the magnifying power of the telescope for the normal vision is

(A) 42
(B) 50
(C) 58
(D) 204

GO0159

102. In the above question the length of the telescope for normal vision, is

(A) 204 cm
(B) 200 cm
(C) 196 cm
(D) 203.45 cm

GO0159

103. In the above question magnifying power of the telescope for distinct vision is

(A) 42
(B) 50
(C) 58
(D) 204

GO0159

104. In the above question the length of the telescope for distinct vision is

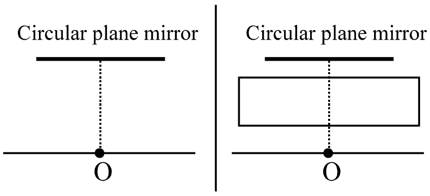
(A) 204 cm
(B) 200 cm
(C) 196 cm
(D) 203.45 cm

GO0159

EXERCISE O-2

SINGLE CORRECT TYPE QUESTIONS

1. In the diagram shown below, a point source O is placed vertically below the center of a circular plane mirror. The light rays starting from the source are reflected from the mirror such that a circular area A on the ground receives light. Now, a glass slab is placed between the mirror and the source O. What will the magnitude of the new area on the ground receiving light?



(A) A

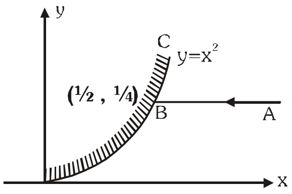
(B) Greater than A

(C) Less than A

(D) Cannot tell, as the information given is insufficient

GO0160

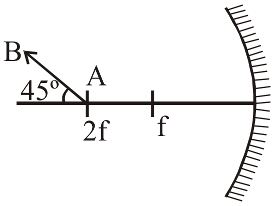
2. In the given figure, incident ray AB is parallel to x-axis. The angle of reflection is :-

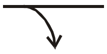
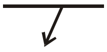


(A) 30° (B) 60° (C) 45° (D) None of these

GO0161

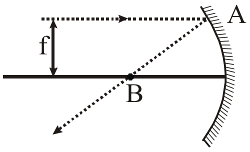
3. The shape of image formed of an object AB due to the concave mirror shown in the figure is best represented by (assume point A is at the centre of curvature of the mirror) :-



(A)  (B)  (C)  (D) 

GO0162

4. A ray of light is incident on a concave mirror. It is parallel to the principal axis and its height from principal axis is equal to the focal length of the mirror. The ratio of the distance of point B to the distance of the focus from the centre of curvature is (AB is the reflected ray) :-



(A) $\frac{2}{\sqrt{3}}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{2}{3}$ (D) $\frac{1}{2}$

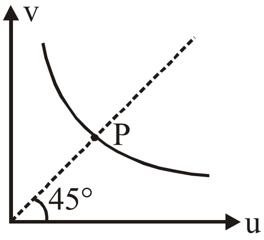
GO0163

5. An infinitely long rod lies along the axis of a concave mirror of focal length f. The near end of the rod is at distance u > f from the mirror. Its image will have a length–

(A) $\frac{uf}{u-f}$ (B) $\frac{uf}{u+f}$ (C) $\frac{f^2}{u+f}$ (D) $\frac{f^2}{u-f}$

GO0164

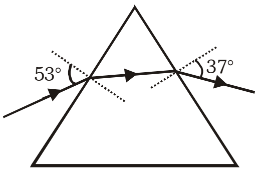
6. The graph shows the variation of v with change in u for a mirror. Points plotted above the point P on the curve are for values of v



(A) smaller than f (B) smaller than 2f (C) larger than 2f (D) larger than f

GO0165

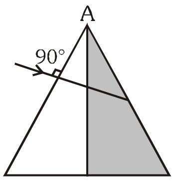
7. A ray incident at an angle 53° on a prism emerges at an angle 37° as shown. If the angle of incidence is made 50°, which of the following is a possible value of the angle of emergence?



(A) 35° (B) 42° (C) 40° (D) 38°

GO0166

8. A ray of light is incident normally on the first refracting face of the prism of refracting angle A. The ray of light comes out at grazing emergence. If one half of the prism (shaded position) is knocked off, the same ray will :–



(A) Emerge at an angle of emergence $\sin^{-1}\left(\frac{1}{2}\sec A/2\right)$

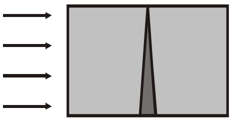
(B) Not emerge out of the prism

(C) Emerge at an angle of emergence $\sin^{-1}\left(\frac{1}{2}\sec A/4\right)$

(D) None of these

GO0167

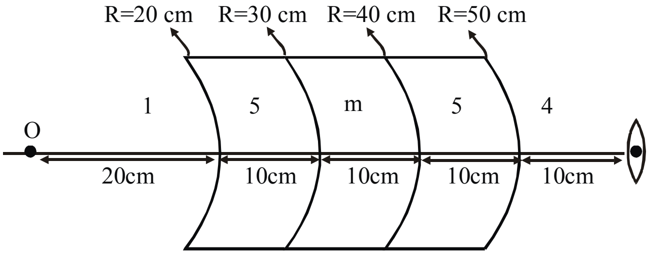
9. A thin isosceles prism with angle 4° and refractive index 5/4 is placed inside a transparent tube with liquid (refractive index = 1.5) as shown. The deviation of light due to prism will be :-



(A) 0.8° upward (B) 0.8° downward (C) 0.67° upward (D) 0.67° downward

GO0168

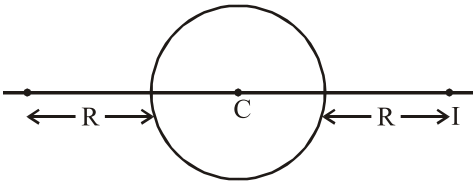
10. In the arrangement shown, separation between observer and object as seen by observer is



(A) 60 cm (B) more than 60 cm (C) less than 60 cm (D) depends on μ

GO0169

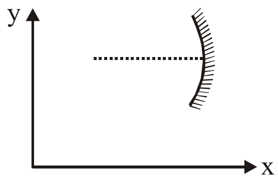
11. In the given situation, a spherical transparent surface of radius R is placed. The refractive index of sphere is such that image for an object kept at distance R is formed at the same distance of the opposite side. What is refractive index of sphere?



(A) 1.5 (B) 1.75 (C) 1.25 (D) 2

GO0170

12. Figure shows a concave mirror with its principal axis parallel to x–axis and focus on the y–axis. The centre of curvature is at (–108 cm, 54 cm). The coordinates of the image of a point source A (81 cm, 48 cm) will be :-



(A) (144 cm, 46 cm) (B) (144 cm, 62 cm) (C) (36 cm, 8 cm) (D) None of these

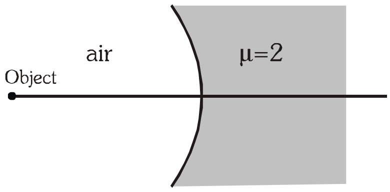
GO0171

13. A light ray is incident on a transparent sphere of index $=\sqrt{2}$, at an angle of incidence = 45°. What is the deviation of a tiny fraction of the ray, which enters the sphere, undergoes two internal reflections and then refracts out into air ?

(A) 270° (B) 240° (C) 120° (D) 180°

GO0172

14. A concave spherical surface with radius R separates a medium with index of refraction 2 from air. If an object is moved toward the surface from far away along the central axis, its image :



(A) Changes from virtual to real when it is R/2 from the surface

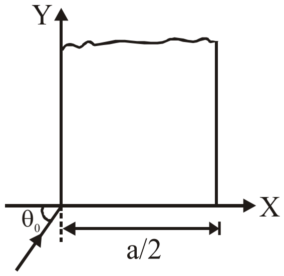
(B) Changes from virtual to real when it is 2R from the surface

(C) Changes from real to virtual when it is R/2 from the surface

(D) Remains virtual

GO0173

MULTIPLE CORRECT TYPE QUESTIONS



15. A glass slab is of thickness d = a/2 is shown in figure. The refractive index of the slab varies according to relation $\mu = \frac{a}{a-x}$ where a is positive constant. A ray of light enters the slab at origin at an angle $\theta_0 \to \pi/2$, then choose the **CORRECT** statement (s):-

(A) The path of the light ray inside medium is a circle given by $(x-a)^2 + y^2 = a^2$

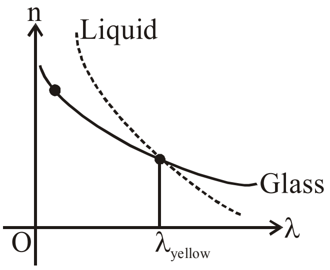
(B) The path of the light ray inside medium is a parabola given by $(x-a)^2 = y$

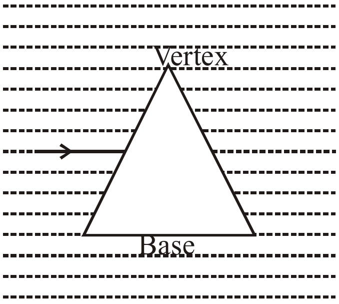
(C) The deviation of ray from its initial direction is $\delta = \pi/3$

(D) Ray emerges grazing at the surface.

GO0174

16. A glass prism is immersed in a hypothetical liquid. The curves showing the refractive index n as a function of wavelength λ for glass and liquid are as shown in the figure. When a ray of white light is incident on the prism parallel to the base :

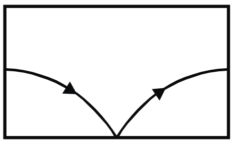


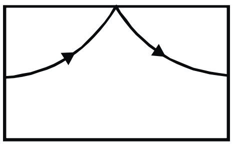


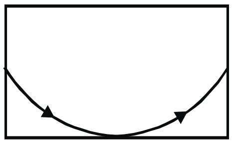
(A) yellow ray travels without deviation
(B) blue ray is deviated towards the vertex
(C) red ray is deviated towards the base
(D) there is no dispersion

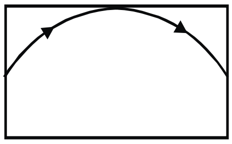
GO0175

17. A cubic container is filled with a liquid whose refractive index increases linearly from top to bottom. Which of the following figures may represent the path of a ray of light inside the liquid?

(A) 

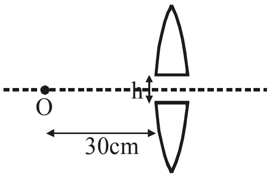
(B) 

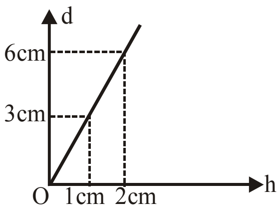
(C) 

(D) 

GO0176

18. Figure shows a convex lens cut symmetrically into two equal halves and separated laterally by a distance h. A point object O placed symmetrically at a distance 30 cm, from the lens halves, forms two images separated by a distance d. A plot of d versus h is shown in figure. The focal lengthof the lens is :-

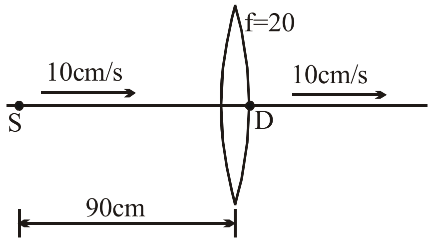




(A) 22.5 cm
(B) 40 cm
(C) 45 cm
(D) 20 cm

GO0177

19. The figure shows initial positions of a source of light S and a light detector D. Both S & D move with velocity 10 cm/s, as shown. The converging lens of focal length 20 cm is fixed. Time instant(s) when the detector detects maximum intensity of light is /are :-



(A) 3 sec

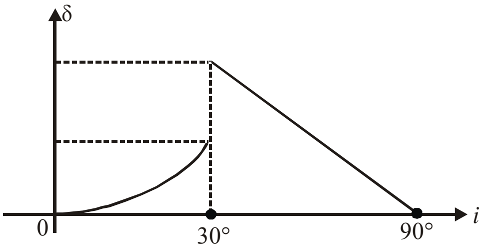
(B) 4.5 sec

(C) 6 sec

(D) equal intensity at all time

GO0178

20. Figure shows graph of angle of deviation v/s angle of incidence for a light ray. Incident ray goes from medium 1 (μ_1) to medium 2 (μ_2). Mark the **correct** option(s).



(A) $\frac{\mu_1}{\mu_2} = \frac{1}{2}$

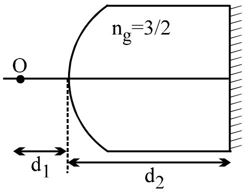
(B) Critical angle is 30°

(C) $\mu_1 > \mu_2$

(D) Maximum deviation is 120°

GO0179

21. In the figure shown a point object O is placed in air on the principal axis. The radius of curvature of the spherical surface is 60 cm. I_f is the final image formed after all the refractions and reflections.

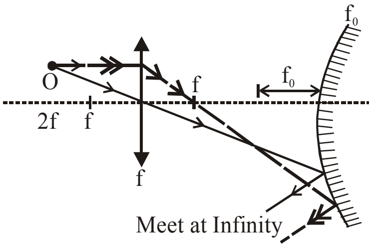


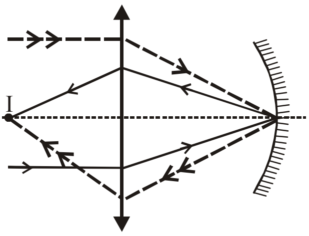
(A) If d_1 = 120 cm, then the 'I_f' is formed on 'O' for any value of d_2.
(B) If d_1 = 240 cm, then the 'I_f' is formed on 'O' only if d_2 = 360 cm.
(C) If d_1 = 240 cm, then the 'I_f' is formed on 'O' for all values of d_2.
(D) If d_1 = 240 cm, then the 'I_f' cannot be formed on 'O'.

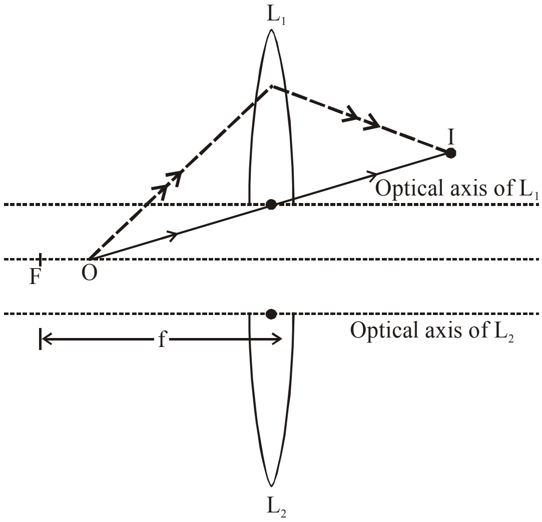
GO0180

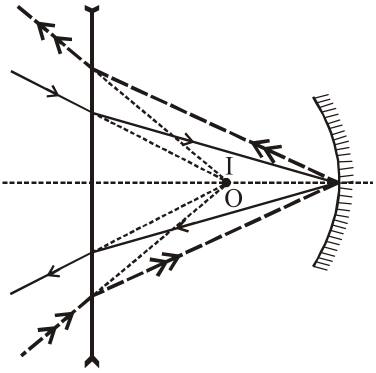
22. Choose the incorrect ray diagram(s). All the rays shown are paraxial.

[↕ denotes converging lens and ⌶ denotes diverging lens]

(A)


(B)


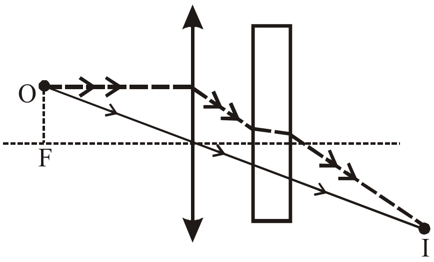
(C)


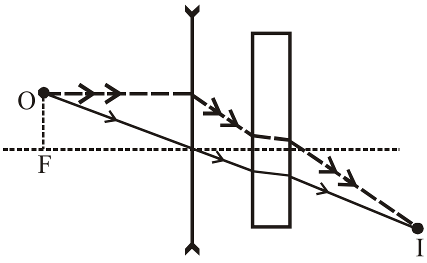
(D)


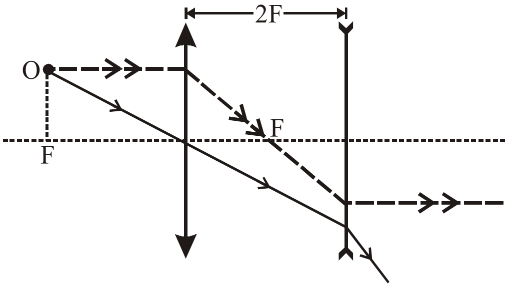
GO0181

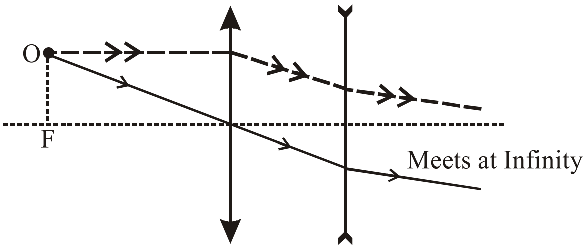
23. Choose incorrect ray diagram [↕ denotes converging lens and ⌶ denotes diverging lens]

All symbols have their usual meaning and all the rays shown are paraxial. (focal length of each lens is F)

(A)


(B)


(C)


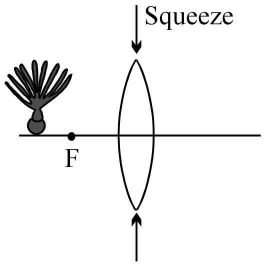
(D)


GO0182

COMPREHENSION TYPE QUESTIONS

Paragraph for Question 24 to 26

A turnip sits before a thin converging lens, outside the focal point of the lens. The lens is filled with a transparent gel so that it is flexible; by squeezing its ends toward its center [as indicated in figure(a)], you can change the curvature of its front and rear sides.



Figure(a)

24. When you squeeze the lens, the image.

(A) moves towards the lens (B) moves away from the lens

(C) shifts up (D) remains as it is

GO0183

25. When you squeeze the lens, the lateral height of image.

(A) increases (B) decreases (C) remains same (D) data insufficient

GO0183

26. Suppose that a sharp image must be formed on a card which is at a certain distance behind the lens [figure(b)], while you move the turnip away from the lens, then you should

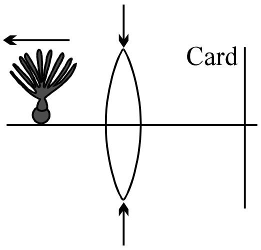


Figure (b)

(A) decrease the squeeze of the lens (B) increase the squeeze of the lens

(C) keep the card and lens as it is. (D) move the card away from the lens

GO0183

EXERCISE - (JM)

1. A car is fitted with a convex side-view mirror of focal length 20 cm. A second car 2.8 m behind the first car is overtaking the first car at a relative speed of 15 m/s. The speed of the image of the second car as seen in the mirror of the first one is:- **[AIEEE- 2011]**

(1) 10 m/s (2) 15 m/s (3) $\frac{1}{10}$ m/s (4) $\frac{1}{15}$ m/s

GO0184

2. When monochromatic red light is used instead of blue light in a convex lens, its focal length will :- **[AIEEE- 2011]**

(1) Does not depend on colour of light (2) Increase
(3) Decrease (4) Remain same

GO0185

3. A beaker contains water up to a height h_1 and kerosene of height h_2 above watger so that the total height of (water + kerosene) is $(h_1 + h_2)$. Refractive index of water is μ_1 and that of kerosene is μ_2. The apparent shift in the position of the bottom of the beaker when viewed from above is :- **[AIEEE- 2011]**

(1) $\left(1-\frac{1}{\mu_1}\right)h_2+\left(1-\frac{1}{\mu_2}\right)h_1$ (2) $\left(1+\frac{1}{\mu_1}\right)h_1-\left(1+\frac{1}{\mu_2}\right)h_2$

(3) $\left(1-\frac{1}{\mu_1}\right)h_1+\left(1-\frac{1}{\mu_2}\right)h_2$ (4) $\left(1+\frac{1}{\mu_1}\right)h_2-\left(1+\frac{1}{\mu_2}\right)h_1$

GO0186

4. An object 2.4 m in front of a lens forms a sharp image on a film 12 cm behind the lens. A glass plate 1 cm thick, of refractive index 1.50 is interposed between lens and film with its plane faces parallel to film. At what distance (from lens) should object be shifted to be in sharp focus on film ? **[AIEEE- 2012]**

(1) 5.6 m (2) 7.2 m (3) 2.4 m (4) 3.2 m

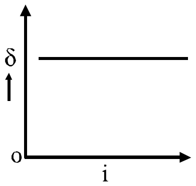
GO0187

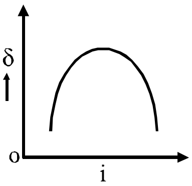
5. Diameter of a plano-convex lens is 6cm and thickness at the centre is 3 mm. If speed of light in material of lens is 2×10^8 m/s, the focal length of the lens is : **[JEE-Main- 2013]**

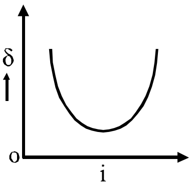
(1) 15 cm (2) 20 cm (3) 30 cm (4) 10 cm

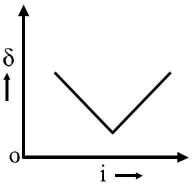
GO0188

6. The graph between angle of deviation (δ) and angle of incidence (i) for a triangular prism is represented by :- **[JEE-Main- 2013]**

(1)


(2)


(3)


(4)


GO0189

7. A thin convex lens made from crown glass $\left(\mu = \frac{3}{2}\right)$ has focal length f. When it is measured in two different liquids having refractive indices $\frac{4}{3}$ and $\frac{5}{3}$, it has the focal length f_1 and f_2 respectively. The correct relation between the focal lengths is : **[JEE-Main- 2014]**

(1) $f_2 > f$ and f_1 becomes negative
(2) f_1 and f_2 both become negative
(3) $f_1 = f_2 < f$
(4) $f_1 > f$ and f_2 become negative

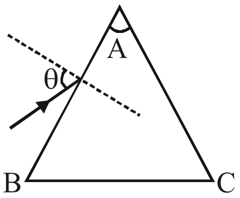
GO0190

8. A green light is incident from the water to the air - water interface at the critical angle (θ). Select the **correct** statement. **[JEE-Main- 2014]**

(1) The spectrum of visible light whose frequency is more than that of green light will come out to the air medium.
(2) The entire spectrum of visible light will come out of the water at various angles to the normal
(3) The entire spectrum of visible light will come out of the water at an angle of 90° to the normal.
(4) The spectrum of visible light whose frequency is less than that of green light will come out to the air medium.

GO0191

9. Monochromatic light is incident on a glass prism of angle A. If the refractive index of the material of the prism is μ, a ray, incident at an angle θ, on the face AB would get transmitted through the face AC of the prism provided : **[JEE-Main- 2015]**



(1) $\theta > \cos^{-1}\left[\mu \sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$
(2) $\theta < \cos^{-1}\left[\mu \sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$
(3) $\theta > \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$
(4) $\theta < \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

GO0192

10. An observer looks at a distant tree of height 10 m with a telescope of magnifying power of 20. To the observer the tree appears : **[JEE-Main- 2016]**

(1) 20 times nearer
(2) 10 times taller
(3) 10 times nearer
(4) 20 times taller

GO0193

11. In an experiment for determination of refractive index of glass of a prism by $i - \delta$, plot, it was found that a ray incident at angle 35°, suffers a deviation of 40° and that it emerges at angle 79°. In that case which of the following is closest to the maximum possible value of the refractive index? **[JEE-Main- 2016]**

(1) 1.8
(2) 1.5
(3) 1.6
(4) 1.7

GO0194

12. A diverging lens with magnitude of focal length 25 cm is placed at a distance of 15 cm from a converging lens of magnitude of focal length 20 cm. A beam of parallel light falls on the diverging lens. The final image formed is : **[JEE-Main- 2017]**

(1) real and at a distance of 40 cm from the divergent lens

(2) real and at a distance of 6 cm from the convergent lens

(3) real and at a distance of 40 cm from convergent lens

(4) virtual and at a distance of 40 cm from convergent lens.

GO0195

SELECTIVE PROBLEMS FROM JEE-MAINS ONLINE PAPERS

13. The eye can be regarded as a single refracting surface . The radius of curvature of this surface is equal to that of cornea (7.8 mm). This surface separates two media of refractive indices 1 and 1.34. Calculate the distance from the refracting surface at which a parallel beam of light will come to focus.

[JEE-Main-2019_Jan]

(1) 2 cm (2) 1 cm (3) 3.1 cm (4) 4.0 cm

GO0250

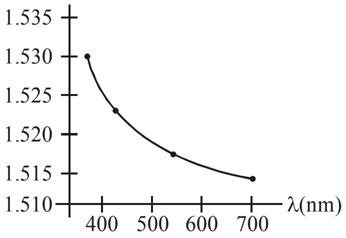
14. A plano convex lens of refractive index μ_1 and focal length f_1 is kept in contact with another plano concave lens of refractive index μ_2 and focal length f_2. If the radius of curvature of their spherical faces is R each and $f_1 = 2f_2$, then μ_1 and μ_2 are related as : **[JEE-Main-2019_Jan]**

(1) $\mu_1 + \mu_2 = 3$ (2) $2\mu_1 - \mu_2 = 1$ (3) $2\mu_2 - \mu_1 = 1$ (4) $3\mu_2 - 2\mu_1 = 1$

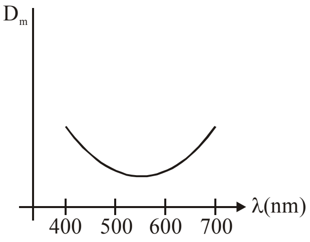
GO0251

15. The variation of refractive index of a crown glass thin prism with wavelength of the incident light is shown. Which of the following graphs is the correct one, if D_m is the angle of minimum deviation?

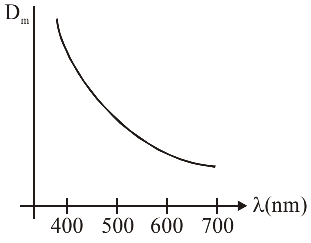
[JEE-Main-2019_Jan]



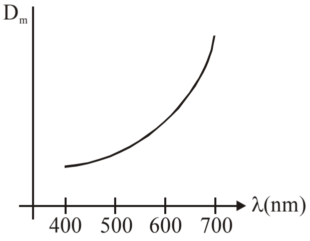
(1)



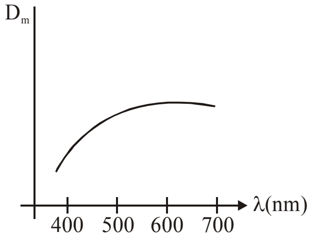
(2)



(3)



(4)



GO0252

16. An object is at a distacen of 20 m from a convex lens of focal length 0.3 m. The lens forms an image of the object. If the object moves away from the lens at a speed of 5 m/s, the speed and direction of the image will be : **[JEE-Main-2019_Jan]**

(1) 0.92×10^{-3} m/s away from the lens

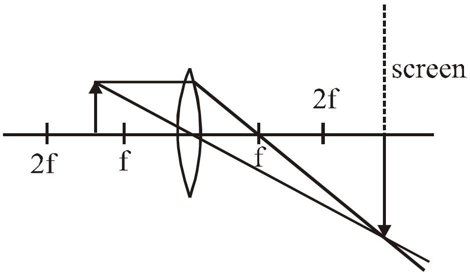
(2) 2.26×10^{-3} m/s away from the lens

(3) 1.16×10^{-3} m/s towards the lens

(4) 3.22×10^{-3} m/s towards the lens

GO0253

17. Formation of real image using a biconvex lens is shown below :



If the whole set up is immersed in water without disturbing the object and the screen position, what will one observe on the screen ? **[JEE-Main-2019_Jan]**

(1) Image disappears (2) No change (3) Erect real image (4) Magnified image

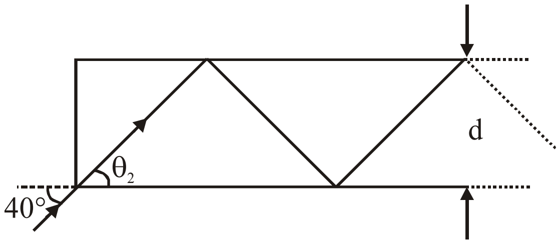
GO0254

18. A convex lens (of focal length 20 cm) and a concave mirror, having their principal axes along the same lines, are kept 80 cm apart from each other. The concave mirror is to the right of the convex lens. When an object is kept at a distance of 30 cm to the left of the convex lens, its image remains at the same position even if the concave mirror is removed. The maximum distance of the object for which this concave mirror, by itself would produce a virtual image would be :- **[JEE-Main-2019_April]**

(1) 20 cm (2) 10 cm (3) 25 cm (4) 30 cm

GO0255

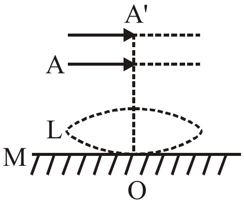
19. In figure, the optical fiber is $\ell = 2$m long and has a diameter of d = 20 μm. If a ray of light is incident on one end of the fiber at angle $\theta_1 = 40°$, the number of reflection it makes before emerging from the other end is close to: (refractive index of fibre is 1.31 and sin 40° = 0.64) **[JEE-Main-2019_April]**



(1) 55000 (2) 57000 (3) 66000 (4) 45000

GO0256

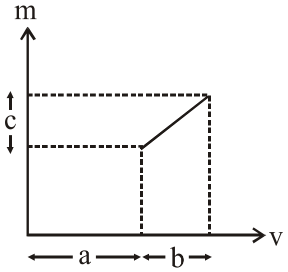
20. A thin convex lens L (refractive index = 1.5) is placed on a plane mirror M. When a pin is placed at A, such that OA = 18 cm, its real inverted image is formed at A itself, as shown in figure. When a liquid of refractive index μ_1 is put between the lens and the mirror, The pin has to be moved to A', such that OA' = 27 cm, to get its inverted real image at A' itself. The value of μ_1 will be :- **[JEE-Main-2019_April]**



(1) $\sqrt{2}$ (2) $\frac{4}{3}$ (3) $\sqrt{3}$ (4) $\frac{3}{2}$

GO0257

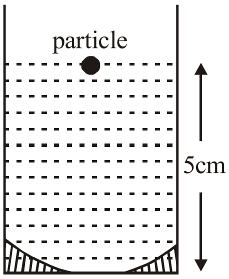
21. The graph shows how the magnification m produced by a thin lens varies with image distance v. What is the focal length of the lens used ? **[JEE-Main-2019_April]**



(1) $\frac{b^2c}{a}$ (2) $\frac{b^2}{ac}$ (3) $\frac{a}{c}$ (4) $\frac{b}{c}$

GO0258

22. A concave mirror has radius of curvature of 40 cm. It is at the bottom of a glass that has water filled up to 5 cm (see figure). If a small particle is floating on the surface of water, its image as seen, from directly above the glass, is at a distance d from the surface of water. The value of d is close to :
(Refractive index of water = 1.33) **[JEE-Main-2019_April]**



(1) 8.8 cm (2) 11.7 cm (3) 6.7 cm (4) 13.4 cm

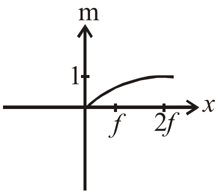
GO0259

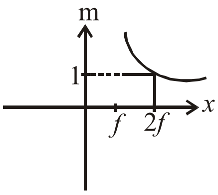
23. If we need a magnification of 375 from a compound microscope of tube length 150 mm and an objective of focal length 5 mm, the focal length of the eye-piece, should be close to : **[JEE-Main-2020_Jan]**
(1) 22 mm (2) 12 mm (3) 33 mm (4) 2 mm

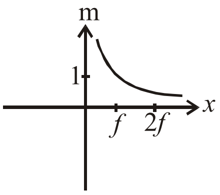
GO0260

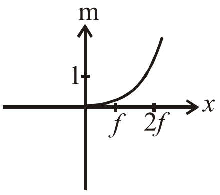
24. An object is gradually moving away from the focal point of a concave mirror along the axis of the mirror. The graphical representation of the magnitude of linear magnification (m) versus distance of the object from the mirror (x) is correctly given by : (Graphs are drawn schematically and are not to scale)

[JEE-Main-2020_Jan]

(1) 

(2) 

(3) 

(4) 

GO0261

25. There is a small source of light at some depth below the surface of water (refractive index = $\frac{4}{3}$) in a tank of large cross sectional surface area. Neglecting any reflection from the bottom and absorption by water, percentage of light that emerges out of surface is (nearly) :

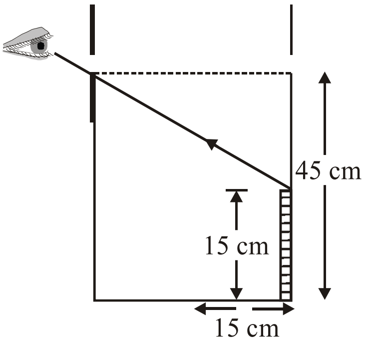
[Use the fact that surface area of a spherical cap of height h and radius of curvature r is $2\pi rh$]:

[JEE-Main-2020_Jan]

(1) 17% (2) 21% (3) 34% (4) 50%

GO0262

26. An observer can see through a small hole on the side of a jar (radius 15 cm) at a point at height of 15cm from the bottom (see figure). The hole is at a height of 45 cm. When the jar is filled with a liquid up to a height of 30 cm the same observer can see the edge at the bottom of the jar. If the refractive index of the liquid N/100, where N is an integer, the value of N is____. **[JEE-Main-2020_Sep]**



GO0263

27. The distance between an object and a screen is 100 cm. A lens can produce real image of the object on the screen for two different positions between the screen and the object. The distance between these two positions is 40 cm. If the power of the lens is close to $\left(\frac{N}{100}\right)$D where N is an integer, the value of N is ______.

[JEE-Main-2020_Sep]

GO0264

28. In a compound microscope, the magnified virtual image is formed at a distance of 25 cm from the eye-piece. The focal length of its objective lens is 1 cm. If the magnification is 100 and the tube length of the microscope is 20 cm, then the focal length of the eye-piece lens (in cm) is ________.

[JEE-Main-2020_Sep]

GO0265

EXERCISE-(JA)

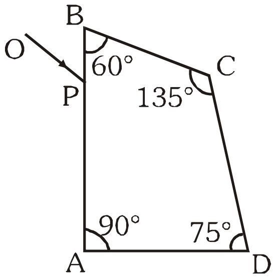
1. A biconvex lens of focal length 15 cm is in front of a plane mirror. The distance between the lens and the mirror is 10 cm. A small object is kept at a distance of 30 cm from the lens. The final image is **[IIT-JEE 2010]**

(A) virtual and at a distance of 16 cm from the mirror
(B) real and at a distance of 16 cm from the mirror
(C) virtual and at a distance of 20 cm from the mirror
(D) real and at a distance of 20 cm from the mirror

GO0196

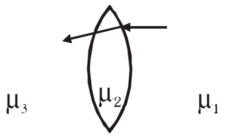
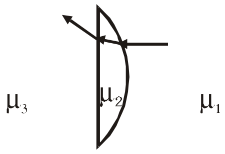
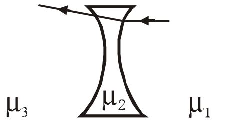
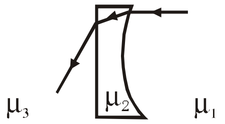
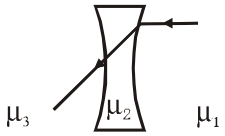
2. A ray OP of monochromatic light is incident on the face AB of prism ABCD near vertex B at an incident angle of 60° (see figure). If the refractive index of the material of the prism is $\sqrt{3}$, which of the following is (are) correct? **[IIT-JEE 2010]**

(A) The ray gets totally internally reflected at face CD
(B) The ray comes out through face AD
(C) The angle between the incident ray and the emergent ray is 90°
(D) The angle between the incident ray and the emergent ray is 120°



GO0197

3. Two transparent media of refractive indices μ_1 and μ_3 have a solid lens shaped transparent material of refractive index μ_2 between them as shown in figures in **Column II**. A ray traversing these media is also shown in the figures. In **Column I** different relationships between μ_1, μ_2 and μ_3 are given. Match them to the ray diagrams shown in **Column II**. **[IIT-JEE 2010]**

Column I	**Column II**
(A) $\mu_1 < \mu_2$	(p) 
(B) $\mu_1 > \mu_2$	(q) 
(C) $\mu_2 = \mu_3$	(r) 
(D) $\mu_2 > \mu_3$	(s) 
	(t) 

GO0198

4. The focal length of a thin biconvex lens is 20 cm. When an object is moved from a distance of 25 cm in front of it to 50 cm, the magnification of its image changes from m_{25} to m_{50}. The ratio $\frac{m_{25}}{m_{50}}$ is

[IIT-JEE 2010]

GO0199

5. Image of an object approaching a convex mirror of radius of curvature 20 m along its optical axis is observed to move from $\frac{25}{3}$ m to $\frac{50}{7}$ m in 30 seconds. What is the speed of the object in km per hour?

[IIT-JEE 2010]

GO0200

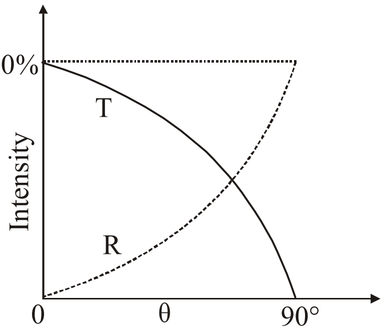
6. A large glass slab $\left(\mu = \frac{5}{3}\right)$ of thickness 8 cm is placed over a point source of light on a plane surface. It is seen that light emerges out of the top surface of the slab from a circular area of radius R cm. What is the value of R? **[IIT-JEE 2010]**

GO0201

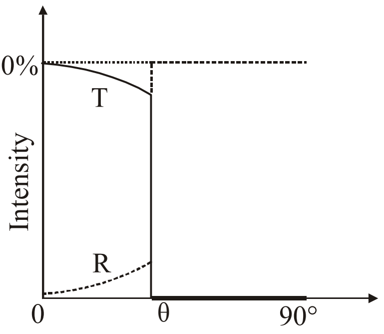
7. A light ray traveling in glass medium is incident on glass-air interface at an angle of incidence θ. The reflected (R) and transmitted (T) intensities, both as function of θ, are plotted. The correct sketch is

[IIT-JEE 2011]

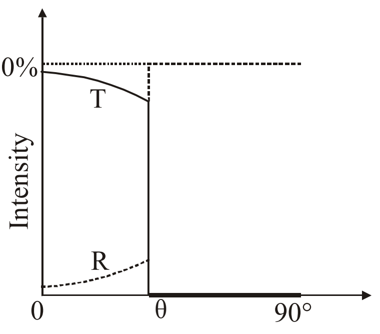
(A)



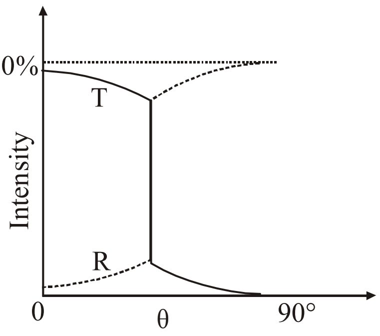
(B)



(C)

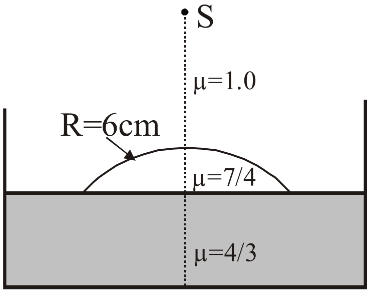


(D)



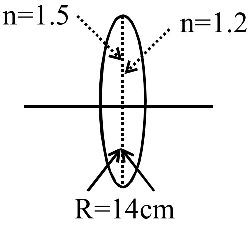
GO0202

8. Water (with refractive index = $\frac{4}{3}$) in a tank is 18 cm deep. Oil of refractive index $\frac{7}{4}$ lies on water making a convex surface of radius of curvature 'R = 6 cm' as shown. Consider oil to act as a thin lens. An object 'S' is placed 24 cm above water surface. The location of its image is at 'x' cm above the bottom of the tank. Then 'x' is **[IIT-JEE 2011]**



GO0203

9. A biconvex lens is formed with two thin plano-convex lenses as shown in the figure, Refractive index n of the first lens is 1.5 and that of the second lens is 1.2. Both the curved surfaces are of the same radius of curvature R = 14 cm. For this biconvex lens, for an object distance of 40 cm, the image distance will be :- **[IIT-JEE 2012]**



(A) –280.0 cm (B) 40.0 cm (C) 21.5 cm (D) 13.3 cm

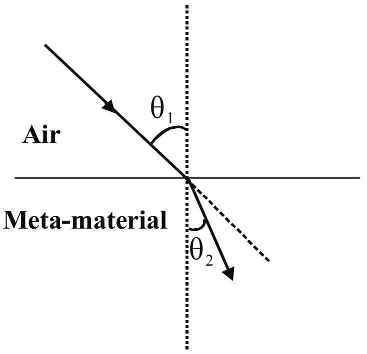
GO0204

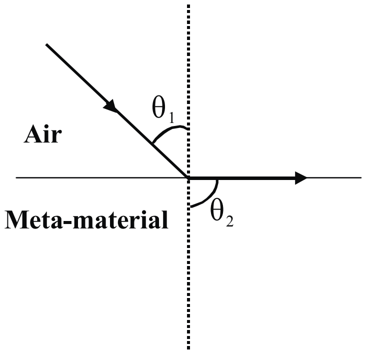
Paragraph for Questions 10 and 11

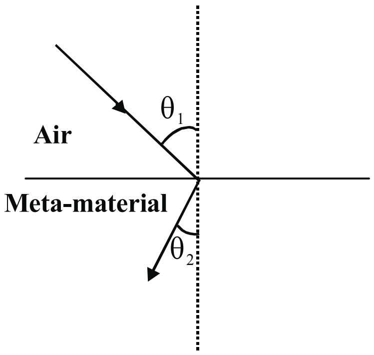
Most materials have the refractive index, n>1. So, when a light ray from air enters a naturally occurring material, then by Snell's law, $\frac{\sin\theta_1}{\sin\theta_2}=\frac{n_2}{n_1}$, it is understood that the refracted ray bends towards the normal. But it never emerges on the same side of the normal as the incident ray. According to electromagnetism, the refractive index of the medium is given by the relation, $n=\left(\frac{c}{v}\right)=\pm\sqrt{\varepsilon_r\mu_r}$, where c is the speed of electromagnetic waves in vacuum, v its speed in the medium, ε_r and μ_r are the relative permittivity and permeability of the medium respectively.

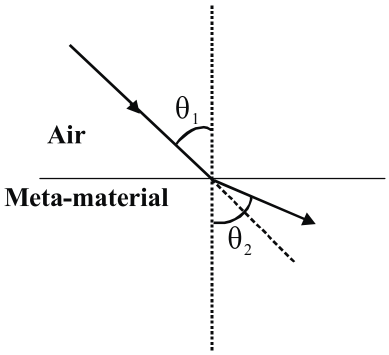
In normal materials, both ε_r and μ_r are positive, implying positive n for the medium. When both ε_r and μ_r are negative, one must choose the negative root of n. Such negative refractive index materials can now be artificially prepared and are called meta-materials. They exhibit significantly different optical behaviour, without violating any physical laws. Since n is negative, it results in a change in the direction of propagation of the refracted light. However, similar to normal materials, the frequency of light remains unchanged upon refraction even in meta-materials. **[IIT-JEE-2012]**

10. For light incident from air on a meta-material, the appropriate ray diagram is

(A)


(B)


(C)


(D)


GO0205

11. Choose the correct statement.

(A) The speed of light in the meta-material is $v = c|n|$

(B) The speed of light in the meta-material is $v = \frac{c}{|n|}$

(C) The speed of light in the meta-material is $v = c$.

(D) The wavelength of the light in the meta-material (λ_m) is given by $\lambda_m = \lambda_{air}|n|$, where λ_{air} is the wavelength of the light in air. **[IIT-JEE 2012]**

GO0205

12. A ray of light travelling in the direction $\frac{1}{2}(\hat{i}+\sqrt{3}\hat{j})$ is incident on a plane mirror. After reflection, it travels along the direction $\frac{1}{2}(\hat{i}-\sqrt{3}\hat{j})$. The angle of incidence is :- **[JEE-Advance-2013]**

(A) 30° (B) 45° (C) 60° (D) 75°

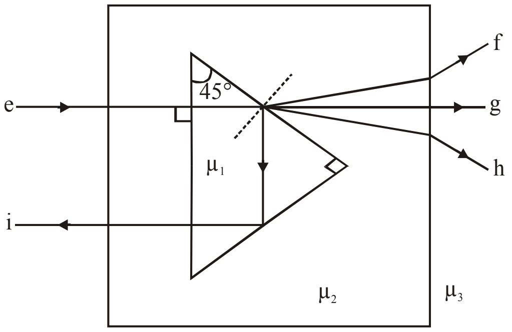
GO0206

13. The image of an object, formed by a plano-convex lens at a distance of 8 m behind the lens, is real and is one-third the size of the object. The wavelength of light inside the lens is $\frac{2}{3}$ times the wavelength in free space. The radius of the curved surface of the lens is :- **[JEE-Advance-2013]**

(A) 1 m (B) 2 m (C) 3 m (D) 6 m

GO0207

14. A right angled prism of refractive index μ_1 is placed in a rectangular block of refractive index μ_2, which is surrounded by a medium of refractive index μ_3, as shown in the figure. A ray of light 'e' enters the rectangular block at normal incidence. Depending upon the relationships between μ_1, μ_2, and μ_3, it takes one of the four possible paths 'ef', 'eg', 'eh' or 'ei'. **[JEE-Advance-2013]**



Match the paths in List I with conditions of refractive indices in List II and select the correct answer using the codes given below the lists :

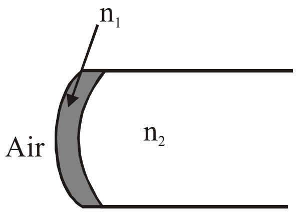
	List I		List II
P.	$e \rightarrow f$	1.	$\mu_1 > \sqrt{2}\mu_2$
Q.	$e \rightarrow g$	2.	$\mu_2 > \mu_1$ and $\mu_2 > \mu_3$
R.	$e \rightarrow h$	3.	$\mu_1 = \mu_2$
S.	$e \rightarrow i$	4.	$\mu_2 < \mu_1 < \sqrt{2}\mu_2$ and $\mu_2 > \mu_3$

Codes :

	P	Q	R	S
(A)	2	3	1	4
(B)	1	2	4	3
(C)	4	1	2	3
(D)	2	3	4	1

GO0208

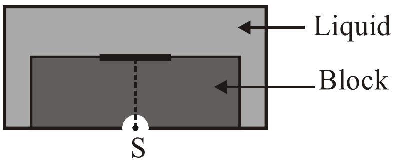
15. A transparent thin film of uniform thickness and refractive index $n_1 = 1.4$ is coated on the convex spherical surface of radius R at one end of a long solid glass cylinder of refractive index $n_2 = 1.5$, as shown in the figure. Rays of light parallel to the axis of the cylinder traversing through the film from air to glass get focused at distance f_1 from the film, while rays of light traversing from glass to air get focused at distance f_2 from the film. Then **[JEE-Advance-2014]**



(A) $|f_1| = 3R$ (B) $|f_1| = 2.8\ R$ (C) $|f_2| = 2R$ (D) $|f_2| = 1.4\ R$

GO0209

16. A point source S is placed at the bottom of a transparent block of height 10 mm and refractive index 2.72. It is immersed in a lower refractive index liquid as shown in the figure. It is found that the light emerging from the block to the liquid forms a circular bright spot of diameter 11.54 mm on the top of the block. The refractive index of the liquid is :- **[JEE-Advance-2014]**

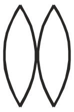
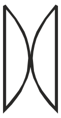
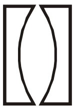


(A) 1.21 (B) 1.30 (C) 1.36 (D) 1.42

GO0210

17. Four combinations of two thin lenses are given in List-I. The radius of curvature of all curved surfaces is r and the refractive index of all the lenses is 1.5. Match lens combinations in List-I with their focal length in List-II and select the correct answer using the code given below the lists.

[JEE-Advance-2014]

List–I	**List–II**
(P) 	(1) 2r
(Q) 	(2) $\frac{r}{2}$
(R) 	(3) –r
(S) 	(4) r

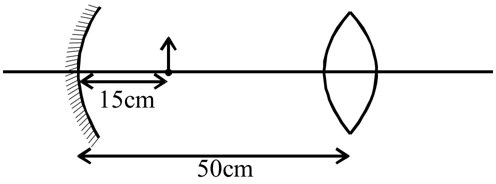
Code :

(A) P-1, Q-2, R-3, S-4 (B) P-2, Q-4, R-3, S-1
(C) P-4, Q-1, R-2, S-3 (D) P-2, Q-1, R-3, S-4

GO0211

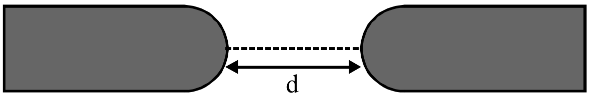
18. Consider a concave mirror and a convex lens (refractive index = 1.5) of focal length 10 cm each, separated by a distance of 50 cm in air (refractive index = 1) as shown in the figure. An object is placed at a distance of 15 cm from the mirror. Its erect image formed by this combination has magnification M_1. When the set-up is kept in a medium of refractive index 7/6 the magnification becomes M_2. The magnitude $\left|\frac{M_2}{M_1}\right|$ is.

[JEE-Advance-2015]



GO0212

19. Two identical glass rods S_1 and S_2 (refractive index = 1.5) have one convex end of radius of curvature 10 cm. They are placed with the curved surfaces at a distance d as shown in the figure, with their axes (shown by the dashded line) aligned. When a point source of light P is placed inside rod S_1 on its axis at a distance of 50 cm from the curved face, the light rays emanating from it are found to be parallel to the axis inside S_2. The distance d is : **[JEE-Advance-2015]**

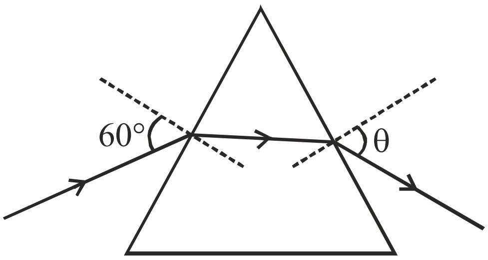


(A) 60 cm (B) 70 cm (C) 80 cm (D) 90 cm

GO0213

20. A monochromatic beam of light is incident at 60° on one face of an equilateral prism of refractive index n and emerges from the opposite face making an angle θ(n) with the normal (see the figure).

For n = $\sqrt{3}$ the value of θ is 60° and $\frac{d\theta}{dn} = m$. The value of m is **[JEE-Advance-2015]**

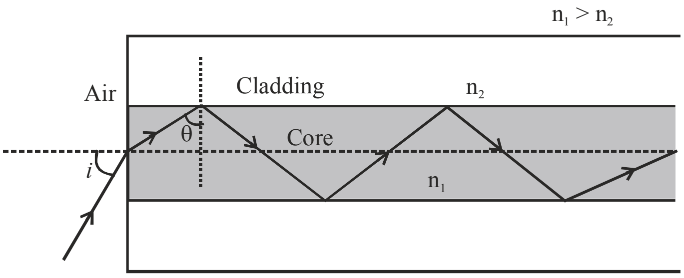


GO0214

Paragraph for Question No. 21 and 22

Light guidance in an optical fiber can be understood by considering a structure comprising of thin solid glass cylinder of refractive index n_1 surrounded by a medium of lower refractive index n_2. The light guidance in the structure takes place due to successive total internal reflections at the interface of the media n_1 and n_2 as shown in the figure. All rays with the angle of incidence i less than a particular value of i_m are confined in the medium of refractive index n_1. The numerical aperture (NA) of the structure is defined as $\sin i_m$.

[JEE-Advance-2015]



21. For two structures namely S_1 with $n_1 = \sqrt{45}/4$ and $n_2 = 3/2$, and S_2 with $n_1 = 8/5$ and $n_2 = 7/5$ and taking the refractive index of water to be 4/3 and that of air to be 1, the correct option(s) is (are)

(A) NA of S_1 immersed in water is the same as that of S_2 immersed in liquid of refractive index $\frac{16}{3\sqrt{15}}$.

(B) NA of S_1 immersed in liquid of refractive index $\frac{6}{\sqrt{15}}$ is the same as that of S_2 immersed in water.

(C) NA of S_1 placed in air is the same as that of S_2 immersed in liquid of refractive index $\frac{4}{\sqrt{15}}$.

(D) NA of S_1 placed in air is the same as that of S_2 placed in water.

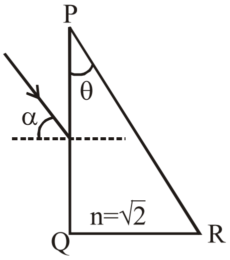
GO0215

22. If two structures of same cross-sectional area, but different numerical apertures NA_1 and NA_2 ($NA_2 < NA_1$) are joined longitudinally, the numerical aperture of the combined structure is

(A) $\frac{NA_1 NA_2}{NA_1 + NA_2}$ (B) $NA_1 + NA_2$ (C) NA_1 (D) NA_2

GO0215

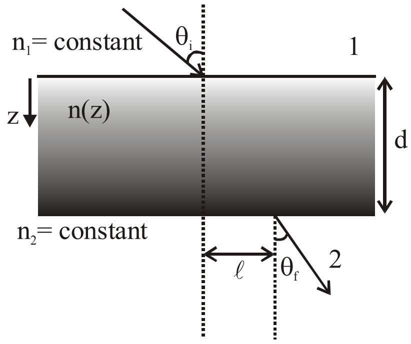
23. A parallel beam of light is incident from air at an angle α on the side PQ of a right angled triangular prism of refractive index $n = \sqrt{2}$. Light undergoes total internal reflection in the prism at the face PR when α has a minimum value of 45°. The angle θ of the prism is : **[JEE-Advance-2016]**



(A) 15° (B) 22.5° (C) 30° (D) 45°

GO0216

24. A transparent slab of thickness d has a refractive index n(z) that increases with z. Here z is the vertical distance inside the slab, measured from the top. The slab is placed between two media with uniform refractive indices n_1 and n_2 ($> n_1$), as shown in the figure. A ray of light is incident with angle θ_i from medium 1 and emerges in medium 2 with refraction angle θ_f with a lateral displacement ℓ. Which of the following statement(s) is(are) true ? **[JEE-Advance-2016]**



(A) ℓ is independent of n_2
(B) ℓ is dependent on n(z)
(C) $n_1\sin\theta_i = (n_2 - n_1)\sin\theta_f$
(D) $n_1\sin\theta_i = n_2\sin\theta_f$

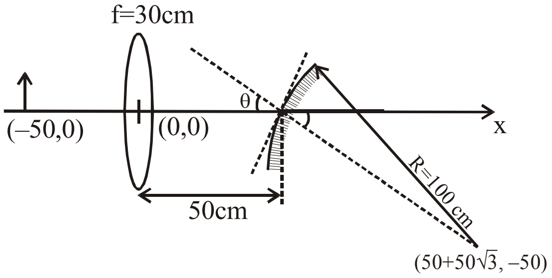
GO0217

25. A plano-convex lens is made of a material of refractive index n. When a small object is placed 30 cm away in front of the curved surface of the lens, an image of double the size of the object is produced. Due to reflection from the convex surface of the lens, another faint image is observed at a distance of 10 cm away from the lens. Which of the following statement(s) is(are) true? **[JEE-Advance-2016]**

(A) The refractive index of the lens is 2.5
(B) The radius of curvature of the convex surface is 45 cm
(C) The faint image is erect and real
(D) The focal length of the lens is 20 cm.

GO0218

26. A small object is placed 50 cm to the left of thin convex lens of focal length 30 cm. A convex spherical mirror of radius of curvature 100 cm is placed to the right of the lens at a distance of 50 cm. The mirror is tilted such that the axis of the mirror is at an angle θ = 30° to the axis of the lens, as shown in the figure. If the origin of the coordinate system is taken to be at the centre of the lens, the coordinates (in cm) of the point (x, y) at which the image is formed are : **[JEE-Advance-2016]**



(A) $(25, 25\sqrt{3})$
(B) $\left(\frac{125}{3}, \frac{25}{\sqrt{3}}\right)$
(C) $(50 - 25\sqrt{3}, 25)$
(D) (0, 0)

GO0219

27. For an isosceles prism of angle A and refractive index μ, it is found that the angle of minimum deviation δ_m = A. Which of the following options is/are correct ? **[JEE-Advance-2017]**

(A) At minimum deviation, the incident angle i_1 and the refracting angle r_1 at the first refracting surface are related by $r_1 = (i_1/2)$

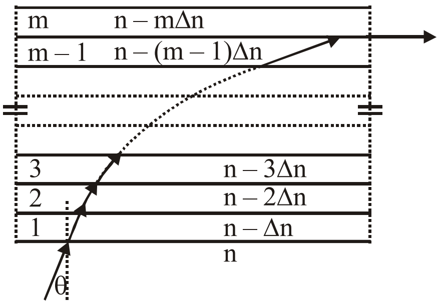
(B) For this prism, the refractive index μ and the angle of prism A are related as $A = \frac{1}{2}\cos^{-1}\left(\frac{\mu}{2}\right)$

(C) For this prism, the emergent ray at the second surface will be tangential to the surface when the angle of incidence at the first surface is $i_1 = \sin^{-1}\left[\sin A\sqrt{4\cos^2\frac{A}{2}-1}-\cos A\right]$

(D) For the angle of incidence i_1 = A, the ray inside the prism is parallel to the base of the prism.

GO0220

28. A monochromatic light is travelling in a medium of refractive index n = 1.6. It enters a stack of glass layers from the bottom side at an angle $\theta = 30°$. The interfaces of the glass layers are parallel to each other. The refractive indices of different glass layers are monotonically decreasing as $n_m = n - m\Delta n$, where n_m is the refractive index of the mth slab and $\Delta n = 0.1$ (see the figure). The ray is refracted out parallel to the interface between the (m – 1)th and mth slabs from the right side of the stack. What is the value of m ? **[JEE-Advance-2017]**

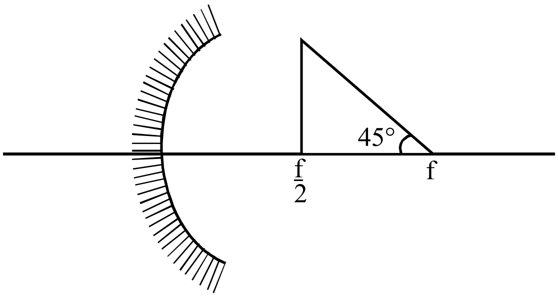


GO0221

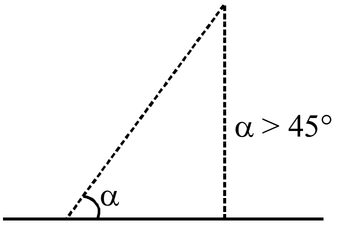
29. Sunlight of intensity 1.3 kW m^{-2} is incident normally on a thin convex lens of focal length 20 cm. Ignore the energy loss of light due to the lens and assume that the lens aperture size is much smaller than its focal length. The average intensity of light, in kW m^{-2}, at a distance 22 cm from the lens on the other side is _________. **[JEE-Advance-2018]**

GO0222

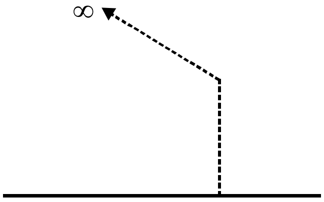
30. A wire is bent in the shape of a right angled triangle and is placed in front of a concave mirror of focal length f, as shown in the figure. Which of the figures shown in the four options qualitatively represent(s) the shape of the image of the bent wire ? (These figures are not to scale.) ? **[JEE-Advance-2018]**



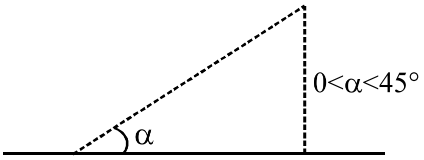
(A)



(B)



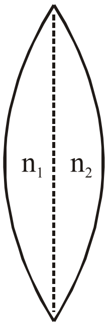
(C)



(D)

GO0223

31. A thin convex lens is made of two materials with refractive indices n_1 and n_2, as shown in figure. The radius of curvature of the left and right spherical surfaces are equal. f is the focal length of the lens when $n_1 = n_2 = n$. The focal length is $f + \Delta f$ when $n_1 = n$ and $n_2 = n + \Delta n$. Assuming $\Delta n << (n-1)$ and $1 < n < 2$, the correct statement(s) is/are : **[JEE-Advance-2019]**



(1) The relation between $\frac{\Delta f}{f}$ and $\frac{\Delta n}{n}$ remains unchanged if both the convex surfaces are replaced by concave surfaces of the same radius of curvature.

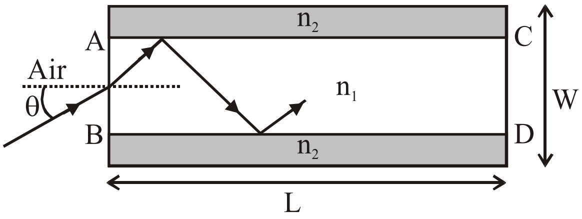
(2) $\left|\frac{\Delta f}{f}\right| < \left|\frac{\Delta n}{n}\right|$

(3) For $n = 1.5$, $\Delta n = 10^{-3}$ and $f = 20$ cm, the value of $|\Delta f|$ will be 0.02 cm (round off to 2nd decimal place)

(4) If $\frac{\Delta n}{n} < 0$ then $\frac{\Delta f}{f} > 0$

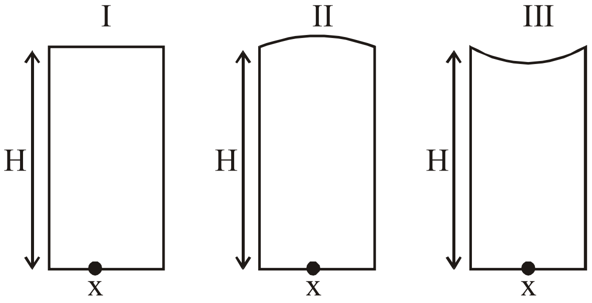
GO0224

32. A planar structure of length L and width W is made of two different optical media of refractive indices $n_1 = 1.5$ and $n_2 = 1.44$ as shown in figure. If L >> W, a ray entering from end AB will emerge from end CD only if the total internal reflection condition is met inside the structure. For L = 9.6 m, if the incident angle θ is varied, the maximum time taken by a ray to exit the plane CD is $t \times 10^{-9}$ s, where t is ____. [Speed of light $c = 3 \times 10^8$ m/s] **[JEE-Advance-2019]**



GO0225

33. Three glass cylinders of equal height H = 30 cm and same refractive index n = 1.5 are placed on a horizontal surfaces shown in figure. Cylinder I has a flat top, cylinder II has a convex top and cylinder III has a concave top. The radii of curvature of the two curved tops are same (R = 3m). If H_1, H_2 and H_3 are the apparent depths of a point X on the bottom of the three cylinders, respectively, the correct statement(s) is/are **[JEE-Advance-2019]**

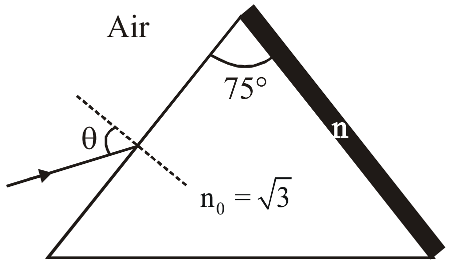


(1) $H_3 > H_1$ (2) $0.8 \text{ cm} < (H_2 - H_1) < 0.9 \text{ cm}$

(3) $H_2 > H_3$ (4) $H_2 > H_1$

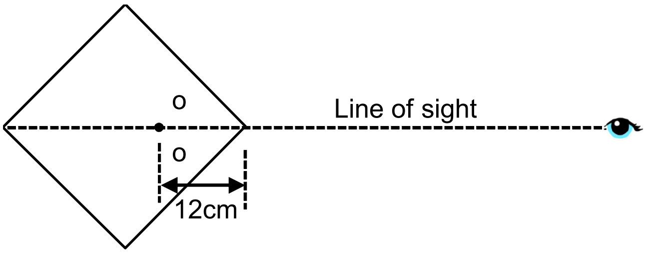
GO0226

34. A monochromatic light is incident from air on a refracting surface of a prism of angle 75° and refractive index $n_0 = \sqrt{3}$. The other refracting surface of a prism is coated by a thin film of material of refractive index n as shown in figure. The light suffers total internal reflection at the coated prism surface for an incidence angle of $\theta \le 60°$. The value of n^2 is_______. **[JEE-Advance-2019]**



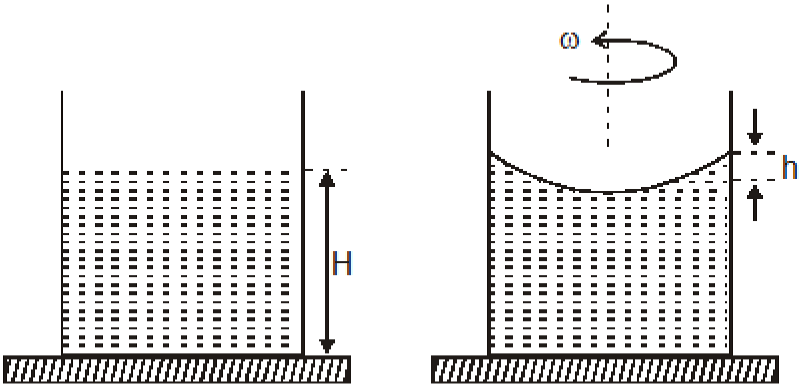
GO0227

35. A large square container with thin transparent vertical walls and filled with water (refractive index $\frac{4}{3}$) is kept on a horizontal table. A student holds a thin straight wire vertically inside the water 12 cm from one of its corners, as shown schematically in the figure. Looking at the wire from this corner, another student sees two images of the wire, located symmetrically on each side of the line of sight as shown. The separation (in cm) between these images is ____________. **[JEE-Advance-2020]**



GO0266

36. A beaker of radius r is filled with water (refractive index $\frac{4}{3}$) up to a height H as shown in the figure on the left. The beaker is kept on a horizontal table rotating with angular speed ω. This makes the water surface curved so that the difference in the height of water level at the center and at the circumference of the beaker is h (h << H, h << r), as shown in the figure on the right. Take this surface to be approximately spherical with a radius of curvature R. Which of the following is/are correct?
(g is the acceleration due to gravity) **[JEE-Advance-2020]**



(A) $R = \frac{h^2 + r^2}{2h}$

(B) $R = \frac{3r^2}{2h}$

(C) Apparent depth of the bottom of the beaker is close to $\frac{3H}{2}\left(1 + \frac{\omega^2 H}{2g}\right)^{-1}$

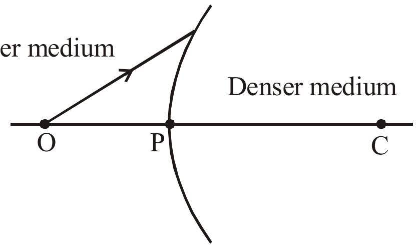
(D) Apparent depth of the bottom of the beaker is close to $\frac{3H}{4}\left(1 + \frac{\omega^2 H}{4g}\right)^{-1}$

GO0267

CBSE PREVIOUS YEAR'S QUESTIONS

1. Draw a ray diagram of an astronomical telescope in the normal adjustment position. Write down the expression for its magnifying power. **[2; CBSE-2004]**

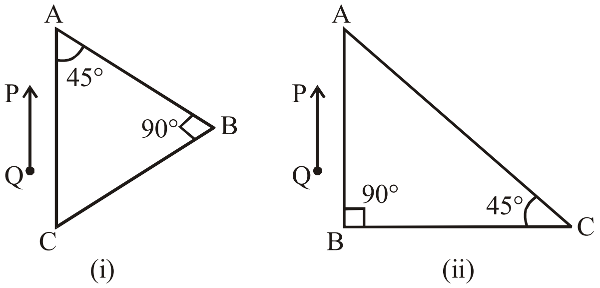
2. A spherical surface of radius of curvature R, separates a rarer and a denser medium as shown in the figure.



Complete the path of the incident ray of light, showing the formation of a real image. Hence derive the relation connecting object distance 'u', image distance 'v', radius of curvature R and the refractive indices n_1 and n_2 of the two media.

Briefly explain, how the focal length of a convex lens changes, with increase in wavelength of incident light. **[5; CBSE-2004]**

3. A right-angled crown glass prism with critical angle 41° is placed before an object PQ, in two positions as shown in the figures (i) and (ii). Trace the paths of the rays from P and Q the prisms in the two cases. **[2; CBSE-2005]**



4. (a) Draw a labelled ray diagram to show the formation of an image by a compound microscope. Write the expression for its magnifying power.

(b) How does the resolving power of a compound microscope change, when (i) refractive index of the medium between the object and the objective lens increases; and (ii) wavelength of the radiation used is increased? **[3; CBSE-2005]**

5. What is the function of cladding in a typical optical fiber? **[1; CBSE-2005]**

6. Draw a labelled ray diagram to show the image formation in a refracting type astronomical telescope. Why should the diameter of the objective of a telescope be large? **[2; CBSE-2006]**

7. A beam of light converges to a point P. A lens is placed in the path of the convergent beam 12 cm from P. At what point does the beam converge if the lens is

(i) a convex lens of focal length 20 cm,

(ii) a concave lens of focal length 16 cm ?

Do the required calculations. **[3; CBSE-2006]**

8. Define resolving power of a compound microscope. How does the resolving power of a compound microscope change when

(i) refractive index of the medium between the object and objective lens increases ?

(ii) wavelength of the radiation used is increased **|2; CBSE-2007]**

9. A double convex lens of glass of refractive index 1.6 has its both surfaces of equal radii of curvature of 30 cm each. An object of height 5 cm is placed at a distance of 12.5 cm from the lens. Calculate the size of the image formed. **[2; CBSE-2007]**

10. A glass lens of refractive index 1.5 is placed in a trough of liquid. What must be the refractive index of the liquid in order to make the lens disappear? **[1; CBSE-2008]**

11. A ray of light passing through an equilateral triangular glass prism from air undergoes minimum deviation when angle of incidence is 3/4th of the angle of prism. Calculate the speed of light in the prism. **[2; CBSE-2008]**

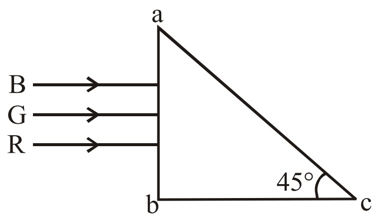
12. (a) For a ray of tight traveling from a denser medium of refractive Index n_1 to a rarer medium of refractive index n_2, prove that $\frac{n_2}{n_1} = \sin i_c$ where i_c is the critical angle of incidence for the media.

(b) Explain with the help of a diagram, how the above the principle is used for transmission of video signals using optical fibres. **[5; CBSE-2008]**

13. You are given following three lenses. Which two lenses will you use as an eyepiece and as an objective to construct an astronomical telescope? **[1; CBSE-2009]**

Lenses	**Power (P)**	**Aperture (A)**
LI	3D	8 cm
L2	6D	1 cm
L3	10D	1 cm

14. Three light rays red (R), green (G) and blue (B) are incident on a right angled prism 'abc' at face 'ab' The refractive indices of the material of the prism for red, green and blue wavelength are 1.39,1.44 and 1.47 respectively. Out of the three which colour ray will emerge out of face 'ac'? Justify your answer. Trace the path of these rays after passing through face 'ab'. **[3; CBSE-2009]**



15. Trace the rays of light showing the formation of an image due to a point object placed on the axis of a spherical surface separating the two media of refractive indices n_1 and n_2. Establish the relation between the distances of the object, the image and the radius of curvature from the central point the spherical surface. Hence derive the expression of lens maker's formula. **[5; CBSE-2009]**

16. Draw the labeled ray diagram for the formation of image by a compound microscope. Derive the expression for the total magnification of a compound microscope. Explain why both the objective and the eyepieces of a compound microscope must have short focal lengths. **[5; CBSE-2009]**

17. A converging lens is kept coaxially in contact with a diverging lens- both the lenses being of equal focal lengths. What is the focal length of the combination? **[1; CBSE-2010]**

18. (i) Draw a net labelled ray diagram of an astronomical telescope in normal adjustment. Explain briefly its working.

(ii) An astronomical telescope uses two lenses of powers 10 D and 1 D. What is its magnifying power in normal adjustment? **[3; CBSE-2010]**

19. (i) Draw a neat labelled ray diagram of a compound microscope. Explain briefly its working,

(ii) Why must both the objective and the eye piece of a compound microscope have short focal lengths? **[3; CBSE-2010]**

20. An illuminated object and a screen are placed 90 cm apart. Determine the focal length and nature of the lens required to produce a clear image on the screen, twice the size of the object,

[3; CBSE-2010]

21. A convex lens made up of glass of refractive index 1.5 is dipped, in turn, in (i) a medium of refractive index 1.65, (ii) a medium of refractive index 1.33.

(a) Will it behave as a converging or a diverging lens in the two cases ?

(b) How will its focal length change in the two media ? **|3; CBSE-2011]**

22. Use the mirror equation to show that (a) an object placed between f and 2f of a concave mirror producesa real image beyond 2f. (b) a convex mirror always produces a virtual image independent of the location of the object, (c) an object placed between the pole and focus of a concave mirror produces a virtual and enlarged image. **[3; CBSE-2011]**

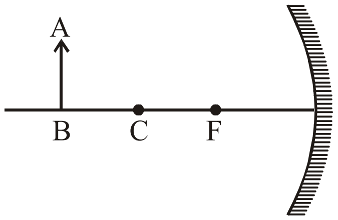
23. A compound microscope uses an objective lens of focal length 4 cm and eyepiece lens of focal length 10cm. An object is placed at 6 cm from the objective lens. Calculate the magnifying power of the compound microscope. Also calculate the length of the microscope. **[3; CBSE-2011]**

24. A giant refracting telescope at an observatory has an objective lens of focal length 1 m. If an eyepiece lens of focal length 1.0 cm is used, find the angular magnification of the telescope. If this telescope is used to view the moon, what is the diameter of the image of the moon formed by the objective lens ? The diameter of the moon is 3.42×10^6 m and the radius of the lunar orbit is 3.8×10^8 m. **[3; CBSE-2011]**

25. For the same value of angle of incidence, the angles of refraction in three media A, B and C are 15°, 25° and 35° respectively. In which medium would the velocity of light be minimum?

[1; CBSE-2012]

26. An object AB is kept in front of a concave mirror as shown in the figure.

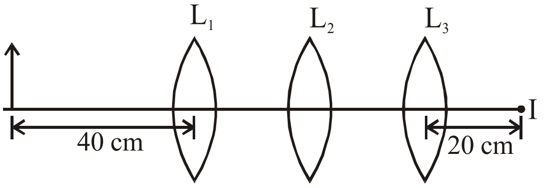


(i) Complete the ray diagram showing the image formation of the object.

(ii) How will the position and intensity of the image be affected if the lower half of the mirror's reflecting surface is painted black? **[2; CBSE-2012]**

27. Draw a labeled ray diagram of a reflecting telescope. Mention its two advantages over the refracting telescope. **[2; CBSE-2012]**

28. You are given three lenses L_1, L_2 and L_3 each of focal length 20 cm. A object is kept at 40 cm in front of L_1, as shown. The final real image is formed at the focus "I" of L_3. Find the separation between L_1, L_2 and L_3. **[3; CBSE-2012]**



29. A convex lens of focal length f_1 is kept in contact with a concave lens of focal length f_2. Find the focal length of the combination. **[CBSE-2013]**

30. Draw a labelled ray diagram of a refracting telescope. Define its magnifying power and write the expression for it. Write two important limitations of a refracting telescope over a reflecting type telescope. **[CBSE-2013]**

31. One day Chetan's mother developed a severe stomach ache all of a sudden. She was rushed to the doctor who suggested for an immediate endoscopy test and gave an estimate of expenditure for the same. Chetan immediately contacted his class teacher and shared the information with her. The class teacher arranged for the money and rushed to the hospital. On releasing that Chetan belonged to a below average income group family, even the doctor offered concession for the test fee. The test was conducted successfully. **[CBSE-2013]**

Answer the following questions based on the above information:

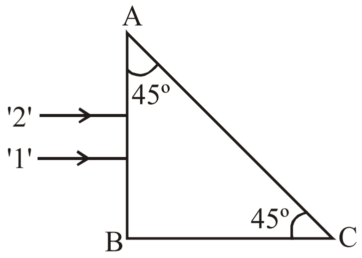
(a) Which principle in optics is made use of in endoscopy?

(b) Briefly explain the values reflected in the action taken by the teacher.

(c) In what way do you appreciate the response of the doctor on the given situation?

32. A biconvex lens made of a transparent material of refractive index 1.5 is immersed in water of refractive index 1.33. Will the lens behave as a converging or a diverging lens? Give reason. **[CBSE-2014]**

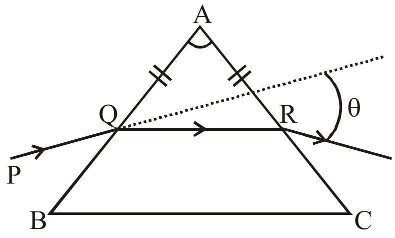
33. Two monochromatic rays of light are incident normally on the face AB of an isosceles right-angled prism ABC. The refractive indices of the glass prism for the two rays' 1' and '2' are respectively 1.3 and 1.5. Trace the path of these rays after entering through the prism. **[CBSE-2014]**



34. A Convex lens of focal length 20 cm is placed coaxially with a concave mirror of focal length 10 cm at a distance of 50 cm apart from each other. A beam of light coming parallel to the principal axis is incident on the convex lens. Find the position of the final image formed by this combination. Draw the ray diagram showing the formation of the image. **[CBSE-2014]**

35. When light travels from an optically denser medium to a rarer medium, why does the critical angle of incidence depend on the colour of light ? **[1; CBSE-2015]**

36. A ray PQ incident on the refracting face BA is refracted in the prism BAC as shown in the figure and emerges from the other refracting face AC as RS such that AQ = AR. If the angle of prism A = 60° and refractive index of material of prism is $\sqrt{3}$, calculate angle θ. **[CBSE-2016]**



37. (i) A screen is placed at a distance of 100 cm from an object. The image of the object is formed on the screen by a convex lens for two different locations of the lens separated by 20cm. Calculate the focal length of the lens used.

(ii) A converging lens is kept coaxially in contact with a diverging lens-both the lenses being of equal focal length. What is the focal length of the combination ? **[CBSE-2016]**

38. How does the angle of minimum deviation of a glass prism vary, if the incident violet light is replaced by red light ? Give reason. **[CBSE-2017]**

39. (a) Monochromatic light of wavelength 589 nm is incident from air on a water surface. If μ for water is 1·33, find the wavelength, frequency and speed of the refracted light. **[CBSE-2017]**

(b) A double convex lens is made of a glass of refractive index 1·55, with both faces of the same radius of curvature. Find the radius of curvature required, if the focal length is 20 cm.

40. (a) Draw a ray diagram depicting the formation of the image by an astronomical telescope in normal adjustment. **[CBSE-2017]**

(b) You are given the following three lenses. Which two lenses will you use as an eyepiece and as an objective to construct an astronomical telescope ? Give reason.

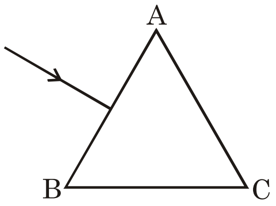
Lenses	Power (D)	Aperture (cm)
L_1	3	8
L_2	6	1
L_3	10	1

41. (a) Draw a ray diagram to show the image formation by a combination of two thin convex lenses in contact. Obtain the expression for the power of this combination in terms of the focal lengths of the lenses.

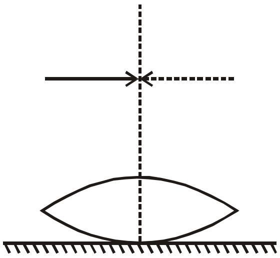
(b) A ray of light passing from air through an equilateral glass prism undergoes minimum deviation when the angle of incidence is $\frac{3}{4}$ th of the angle of prism. Calculate the speed of light in the prism.

[CBSE-2017]

42. (b) The figure shows a ray of light falling normally on the face AB of an equilateral glass prism having refractive index $\frac{3}{2}$, placed in water of refractive index $\frac{4}{3}$. Will this ray suffer total internal reflection on striking the face AC? Justify your answer. **[CBSE-2018]**



43. A symmetric biconvex lens of radius of curvature R and made of glass of refractive index 1.5, is placed on a layer of liquid placed on top of a plane mirror as shown in the figure. An optical needle with its tip on the principal axis of the lens is moved along the axis unti its real, inverted image coincides with the needle itselt. The distance of the needle from the lens is measured to be x. On removing the liquid layer and repeating the experiement, the distance is found to be y. Obtain the expression for the refractive index of the liquid in terms of x and y. **[CBSE-2018]**



44. (a) Draw a ray diagram to show image formation when the concave mirror produces a real, inverted and magnified image of the object.

(b) Obtain the mirror formula and write the expression for the linear magnification.

(c) Explain two advantages of a reflecting telescope over a refracting telescope. **[CBSE-2018]**

ANSWER KEY

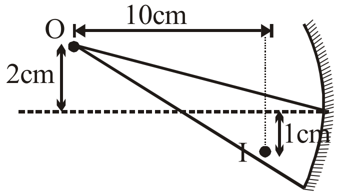
EXERCISE (S-1)

1. Ans. 3m

2. Ans. 2

3. Ans. 2

4. Ans. 20 cm from object, 20/3 cm from object



5. Ans. 25

6. Ans. d/2

7. Ans. 16 mm^3

8. Ans. 4

9. Ans. 4

10. Ans. 1

11. Ans. 5

12. Ans. 5

13. Ans. 42 cm

14. Ans. 4

15. Ans. 8

16. Ans. (a) $\sin^{-1}\left(\frac{1}{5}\right)$ (b) air

17. Ans. $\theta < \sin^{-1}(2\sin 15°)$

18. Ans. (i) $\lambda_0 = 600$ nm, n = 1.5 (ii) i = $\sin^{-1}(0.75) = 48.59°$

19. Ans. $\left(\frac{8}{3}\text{mm}\right)$

20. Ans. 2

21. Ans. 5cm

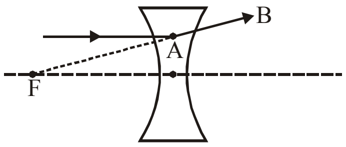
22. Ans. 8

23. Ans. $f = v = \frac{\mu_3 R}{\mu_3 - \mu_1}$

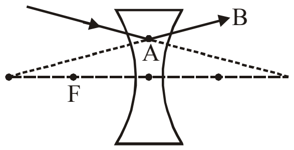
24. Ans. 30

25. Ans. Real, below principal axis, anywhere b/w P & ∞

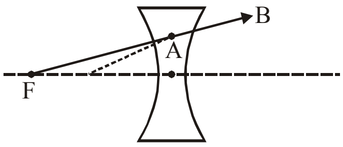
26. Ans. (1)



(2)



(3)



27. Ans. (i) 0.2 m, (ii) 0.4 m

28. Ans. (π/4) cm^2

29. Ans. 3

30. Ans. 5

31. Ans. (–20 cm)

32. Ans. 17.5 cm

33. Ans. 3

34. Ans. $y = -\frac{x}{1200} + 0.1$

35. Ans. 1

36. Ans. 4

37. Ans. (a) f = –20 cm, (b) + 80 cm, convergent achromatic lens]

EXERCISE (S-2)

1. Ans. 2 mm

2. Ans. (30 cm, – 14 mm)

3. Ans. $\frac{32}{9}$

4. Ans. 0180

5. Ans. 2

6. Ans. $\vec{r} = \frac{3}{5\sqrt{2}}\hat{i} + \frac{2\sqrt{2}}{5}\hat{j} - \frac{1}{\sqrt{2}}\hat{k}$(angle of incidence=$60^0$;r=$45^0$)

7. Ans. $\frac{1.514 \times 0.4}{0.1} = 6.06$ m correct upto two places of decimal.

8. Ans. On the object itself

9. Ans. (i) $\tan\theta = \frac{dy}{dx} = \cot i$; (ii) 1 ; (iii) $y = k^2(x/4)^4$; (iv) 4.0, 1; (v) It will become parallel to x-axis

10. Ans. $\frac{R}{\sqrt{3}}$

11. Ans. (a) $\frac{2}{\sqrt{3}}$ (b) Normal to surface (c) Retrace the path, $\sqrt{3}z + x = 10$

12. Ans. 0.09 m/s ; Magnitude of the rate of change of lateral magnification is 0.3 s^{-1}.

13. Ans. 15 cm right of the mirror, magnification =1.5; (–15 cm, –1.5 cm; – 15 cm, 0.3 cm)

14. Ans. $\frac{64}{3}$ cm

15. Ans. (i) –10 cm (ii) (10, 2)

16. Ans. (i) 2°, (ii) $\frac{4\pi}{9}$ mm

17. Ans. $\left(\frac{10}{3}\text{m}\right)$

EXERCISE (O-1)

1. Ans. (A) **2. Ans. (A)** **3. Ans. (B)** **4. Ans. (C)** **5. Ans. (A)** **6. Ans. (D)**
7. Ans. (D) **8. Ans. (D)** **9. Ans. (B)** **10. Ans. (B)** **11. Ans. (C)** **12. Ans. (D)**
13. Ans. (C) **14. Ans. (C)** **15. Ans. (C)** **16. Ans. (D)** **17. Ans. (B)** **18. Ans. (B)**
19. Ans. (A) **20. Ans. (B)** **21. Ans. (A)** **22. Ans. (B)** **23. Ans. (D)** **24. Ans. (C)**
25. Ans. (B) **26. Ans. (D)** **27. Ans. (B)** **28. Ans. (C)** **29. Ans. (B)** **30. Ans. (B)**
31. Ans. (C) **32. Ans. (B)** **33. Ans. (B)** **34. Ans. (C)** **35. Ans. (A)** **36. Ans. (A)**
37. Ans. (B) **38. Ans. (D)** **39. Ans. (D)** **40. Ans. (A)** **41. Ans. (D)** **42. Ans. (C)**
43. Ans. (A) **44. Ans. (C)** **45. Ans. (D)** **46. Ans. (C)** **47. Ans. (B)** **48. Ans. (A)**
49. Ans. (C) **50. Ans. (B)** **51. Ans. (C)** **52. Ans. (A)** **53. Ans. (B)** **54. Ans. (A)**
55. Ans. (A) **56. Ans. (C)** **57. Ans. (B)** **58. Ans. (C)** **59. Ans. (C)** **60. Ans. (A)**
61. Ans. (C) **62. Ans. (A)** **63. Ans. (D)** **64. Ans. (B)** **65. Ans. (B)** **66. Ans. (A)**
67. Ans. (A,C,D) **68. Ans. (B, C)** **69. Ans. (A,D)** **70. Ans. (A,B,C)**
71. Ans. (B,C) **72. Ans. (B,C)** **73. Ans. (B,D)** **74. Ans. (A,C)**
75. Ans. (B,C,D) **76. Ans. (A,C)** **77. Ans. (B,C)** **78. Ans. (A,B)**
79. Ans. (A,B,C,D) **80. Ans. (A,D)** **81. Ans. (A, C)** **82. Ans. (A, D)**
83. Ans. (B) **84. Ans. (B)** **85. Ans. (C)** **86. Ans. (B)**
87. Ans. (D) **88. Ans. (D)** **89. Ans. (D)** **90. Ans. (C)**
91. Ans. (A) → (Q), (B) → (P), (C) → (S), (D) → (R)
92. Ans. (A)→(P,S,T);(B)→(Q,S);(C)→(Q, R) (D)→(Q, R)
93. Ans. (A)-Q; (B)-P; (C)-S; (D)-R **94. Ans. (A)-P; (B)-Q; (C)-RT; (D)-PT**
95. Ans. (B) **96. Ans. (A)-Q,S; (B)-P,Q,R, S; (C)-P,Q,S (D)-S**
97. Ans. (C) **98. Ans. (B)** **99. Ans. (A)** **100. Ans. (A)**
101. Ans. (B) **102. Ans. (A)** **103. Ans. (C)** **104. Ans. (D)**

EXERCISE O-2

1. Ans. (A) **2. Ans. (C)** **3. Ans. (D)** **4. Ans. (A)** **5. Ans. (D)** **6. Ans. (C)**
7. Ans. (D) **8. Ans. (A)** **9. Ans. (C)** **10. Ans. (A)** **11. Ans. (D)** **12. Ans. (A)**
13. Ans. (A) **14. Ans. (D)** **15. Ans. (A,D)** **16. Ans. (A,B,C)** **17. Ans. (A,D)**
18. Ans. (B,D) **19. Ans. (A,C)** **20. Ans. (B,C,D)** **21. Ans. (A, B)**
22. Ans. (A,B,C) **23. Ans. (A,B,C,D)** **24. Ans. (A)** **25. Ans. (B)**
26. Ans. (A)

EXERCISE - (JM)

1. Ans. (4) **2. Ans. (2)** **3. Ans. (3)** **4. Ans. (1)** **5. Ans. (3)** **6. Ans. (3)**
7. Ans. (4) **8. Ans. (4)** **9. Ans. (3)** **10. Ans. (1)** **11. Ans. (2)** **12. Ans. (3)**
13. Ans. (3) **14. Ans. (2)** **15. Ans. (2)** **16. Ans. (3)** **17. Ans. (1)** **18. Ans. (2)**
19. Ans. (2) **20. Ans. (2)** **21. Ans. (4)** **22. Ans. (1)** **23. Ans. (1)** **24. Ans. (2)**
25. Ans. (1) **26. Ans. 158** **27. Ans. (5) or (476)** **28. Ans. (5) or (4.48)**

EXERCISE-(JA)

1. Ans. (B) **2. Ans. (A,B,C)** **3. Ans. (A)–pr, (B) –qst, (C) –prt, (D) –qs** **4. Ans. 6**
5. Ans. 3 **6. Ans. 6** **7. Ans. (C)** **8. Ans. 2** **9. Ans. (B)** **10. Ans. (C)**
11. Ans. (B) **12. Ans. (A)** **13. Ans. (C)** **14. Ans. (D)** **15. Ans. (A,C)** **16. Ans. (C)**
17. Ans. (B) **18. Ans. 7** **19. Ans. (B)** **20. Ans. 2** **21. Ans. (A,C)** **22. Ans. (D)**
23. Ans. (A) **24. Ans. (A,B,D)** **25. Ans. (A,D)** **26. Ans. (A)** **27. Ans. (A,C,D)**
28. Ans. 8 **29. Ans. 130 [129.95, 130.05]** **30. Ans. (D)** **31. Ans. (1,3,4)** **32. Ans. (50.00)**
33. Ans. (3,4) **34. Ans. (1.50)** **35. Ans. Bonus** **36. Ans. (A,D)**

Important Notes

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PHYSICS

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path to success

Chapter 01 Contents

WAVE OPTICS

Important Notes

WAVE OPTICS

KEY CONCEPTS

WAVE THEORY OF LIGHT

This theory was enunciated by Hygen in a hypothetical medium known as luminiferrous ether.

Ether is that imaginary medium which prevails in all space, in isotropic, perfectly elastic and massless.

The different colours of light are due to different wave lengths of these waves.

The velocity of light in a medium is constant but changes with change of medium.

This theory is valid for all types of waves.

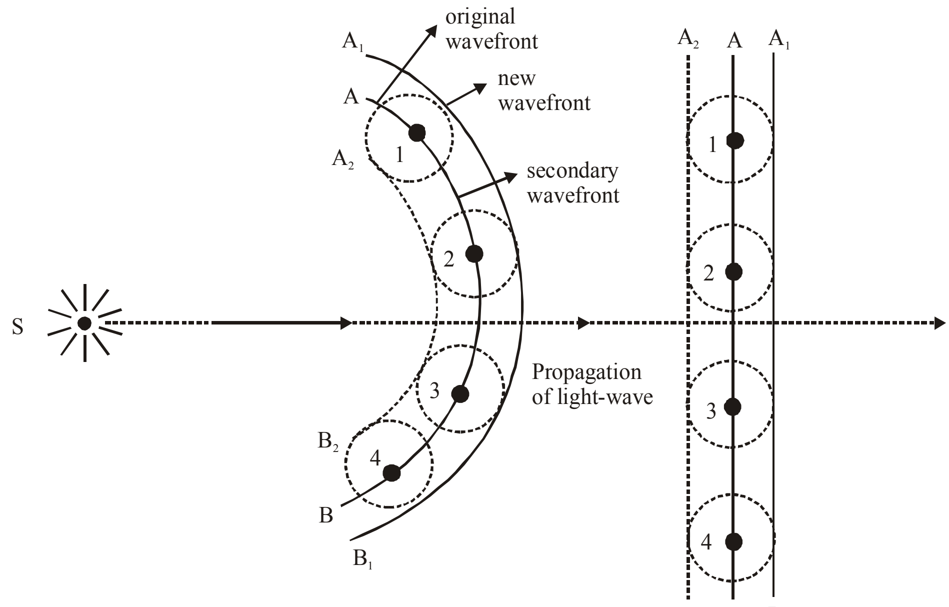
(i) The locus of all ether particles vibrating in same phase is known as wavefront.

(ii) Light travels in the medium in the form of wavefront.

(iii) When light travels in a medium then the particles of medium start vibrating and consequently a disturbance is created in the medium.

(iv) Every point on the wave front becomes the source of secondary wavelets. It emits secondary wavelets in all directions which travel with the speed of light (v),

The tangent plane to these secondary wavelets represents the new position of wave front.



(a) (b)

The phenomena explained by this theory

(i) Reflection, refraction, interference, diffraction, polarisation and double refraction.

(ii) Rectilinear propagation of light.

(iii) Velocity of light in rarer medium being grater than that in denser medium.

Phenomena not explained by this theory

(i) Photoelectric effect, Compton effect and Raman effect.

(ii) Backward propagation of light.

WAVE FRONT, VARIOUS TYPES OF WAVE FRONT AND RAYS

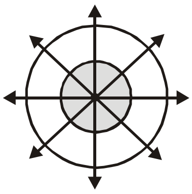
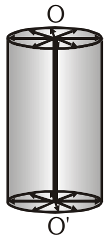
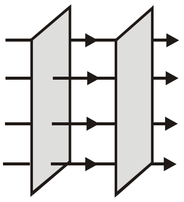
- **Wavefront**

  The locus of all the particles vibrating in the same phase is known as wavefront.

- **Types of wavefront**

  The shape of wavefront depends upon the shape of the light source originating that wavefront. On the basis of there are three types of wavefront.

Comparative study of three types of wavefront

S.No.	Wavefront	Shape of light source	Diagram of shape of wavefront	Variation of amplitude with distance	Variation of intensity with distance
1.	Spherical	Point source		$A \propto \frac{1}{d}$ or $A \propto \frac{1}{r}$	$I \propto \frac{1}{r^2}$
2.	cylindrical	Linear or slit		$A \propto \frac{1}{\sqrt{d}}$ or $A \propto \frac{1}{\sqrt{r}}$	$I \propto \frac{1}{r}$
3.	Plane	Extended large source situated at very large distance		A = constant	I = constant

CHARACTERISTIC OF WAVEFRONT

(a) The phase difference between various particles on the wavefront is zero.

(b) These wavefronts travel with the speed of light in all directions in an isotropic medium.

(c) A point source of light always gives rise to a spherical wavefront in an isotropic medium.

(d) In an anisotropic medium it travels with different velocities in different directions.

(e) Normal to the wavefront represents a ray of light.

(f) It always travels in the forward direction of the medium.

RAY OF LIGHT

The path of the light energy from one point to another is known as a ray of light.

(a) A line drawn at right angles to the wavefront is defined as a ray of light, which is shown by arrows in previous diagram of shape of wavefront.

(b) It represents the direction of propagation of light.

1. INTERFERENCE OF LIGHT

When two light waves of same frequency with zero initial phase difference or constant phase difference superimpose over each other, then the resultant intensity in the region of superposition is different from the sum of intensity of individual waves.

This modification in intensity in the region of superposition is called interference.

(a) Constructive interference

When resultant intensity is greater than the sum of two individual wave intensities $[I > (I_1 + I_2)]$, then the interference is said to be constructive.

(b) Destructive interference

When the resultant intensity is less than the sum of two individual wave intensities $[I < (I_1 + I_2)]$, then the interference is said to destructive. There is no violation of the law of conservation of energy in interference. Here, the energy from the points of minimum energy is shifted to the points of maximum energy.

2. TYPES OF SOURCES

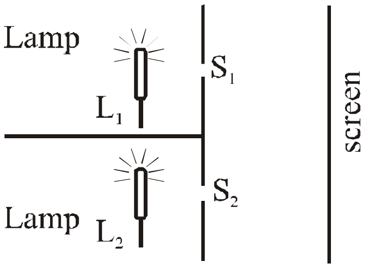
(a) Coherent sources

Two sources are said to be coherent if they emit light waves of the same wave length and start with same phase or have a constant phase difference.

Note : Laser is a source of monochromatic light waves of high degree of coherence.

(b) Incoherent sources

Two independent monochromatic sources, emit waves of same wavelength. But the waves are not in phase. So they are incoherent. This is because, atoms cannot emit light waves in same phase and these sources are said to be incoherent sources.



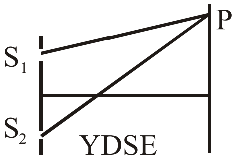
independent source
(noncoherent source)

3. METHOD FOR OBTAINING COHERENT SOURCES

(a) Division of wavefront

In this method, the wavefront is divided into two or more parts by use of mirrors, lenses or prisms.

Example : Young's double slit experiment. Fresnel's Biprism and Lloyd's single mirror method.



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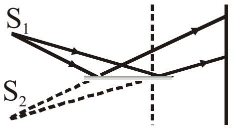
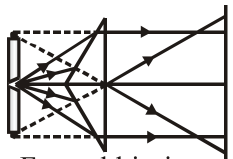


Lloyd's mirror

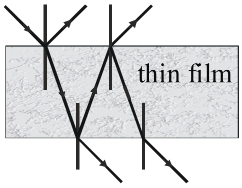


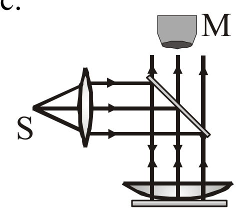
Fresnel biprism

(b) Division of amplitude

The amplitude of incoming beam is divided into two or more parts by partial reflection or refraction. These divided parts travel different paths and are finally brought together to produce interference.

Example : The brilliant colour seen in a thin film of transparent material like soap film, oil film, Michelson's Interferro Meter, Newtons' ring etc.





Newton's rings

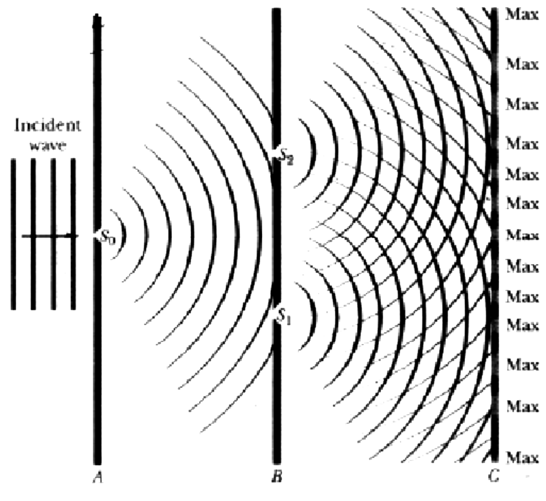
4. If two coherent waves with intensity I_1 and I_2 are superimposed with a phase difference of ϕ, the resulting wave intensity is

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\ \cos\phi$$

5. The phase difference between two waves at a point will depend upon

- **(i)** the difference in path lengths of two waves from their respective sources.(geometrical path difference)
- **(ii)** the refractive index of the medium (media)
- **(iii)** phase difference at source (if any).
- **(iv)** In case, the waves suffer reflection, the reflected wave differs in phase by π with respect to the incident wave if the incidence occurs in rarer medium. There would be no phase difference if incidence occurs in denser medium.

6. YOUNG'S DOUBLE SLIT EXPERIMENT

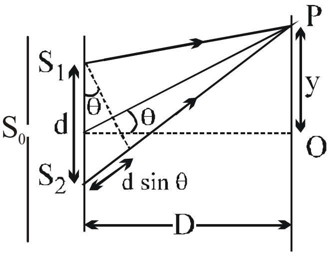


(i) If $d << D$

$\Delta x = S_2P - S_1P = d\sin\theta$

If $\lambda << d$ then $\sin\theta \approx \theta \approx \tan\theta$ as when P is close to D so θ is small.

$$\Delta x = d\left(\frac{y}{D}\right) = \frac{yd}{D}$$



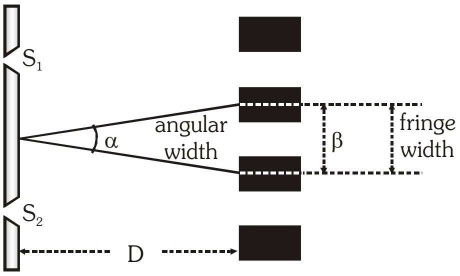
(ii) For maxima $\frac{yd}{D} = n\lambda$

or $y = 0, \pm\frac{D\lambda}{d}, \pm\frac{2D\lambda}{d}$

(iii) For minima $\frac{yd}{D} = [n + (1/2)]\lambda$ or $y = \pm\frac{D\lambda}{2d}, \pm\frac{3D\lambda}{2d}, \pm\frac{5D\lambda}{2d}$, so on

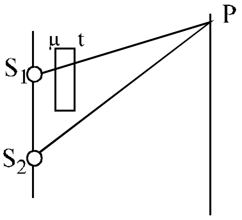
(iv) Fringe width = the distance between two successive maximas or minima $\beta = \frac{\lambda D}{d}$

(v) **Angular Fringe width**



Angular Fringe Width $\alpha = \frac{\beta}{D}$, $\alpha = \frac{\lambda}{d}$ $\left[\because \frac{\beta}{D} = \frac{\lambda}{d}\right]$

7. DISPLACEMENT OF FRINGE PATTERN



When a film of thickness t and refractive index μ is introduced in the path of one of the source's of light, then fringe shift occurs as the optical path difference changes.

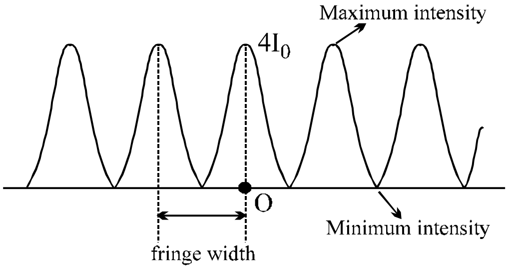
Optical path difference at P is

$\Delta x = S_2P - [S_1P + \mu t - t] = S_2P - S_1P - (\mu - 1)\,t = y.\,(d/D) - (\mu - 1)t$

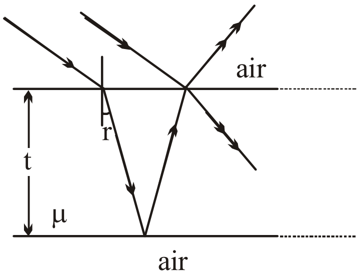
$\Rightarrow$ The fringe shift is given by $\Delta y = \frac{D(\mu-1)t}{d}$

8. INTENSITY VARIATION ON SCREEN

If I_0 is the intensity of light beam coming from each slit, the resultant intensity at a point where they have a phase difference of ϕ is $I = 4I_0\cos^2\frac{\phi}{2}$, where $\phi = \frac{2\pi(d\sin\theta)}{\lambda}$



9. INTERFERENCE AT THIN FILM



optical path difference $= 2\mu t \cos r$

$= 2\mu t$ (in case of near normal incidence)

(a) For interference in reflected light

(i) Condition of minima $2\mu t \cos r = n\lambda$

(ii) Condition of maxima $2\mu t \cos r = \left(n + \frac{1}{2}\right)\lambda$

(b) For interference in transmitted light

(i) Condition of maxima $2\mu t \cos r = n\lambda$

(ii) Condition of minima $2\mu t \cos r = \left(n + \frac{1}{2}\right)\lambda$

Ampere Law :

The general form of Ampere's law (sometimes called the **Ampere-Maxwell law**) as

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0\left(I + I_0\right) = \mu_0\left(I + \epsilon_0 \frac{d\Phi_E}{dt}\right)$$

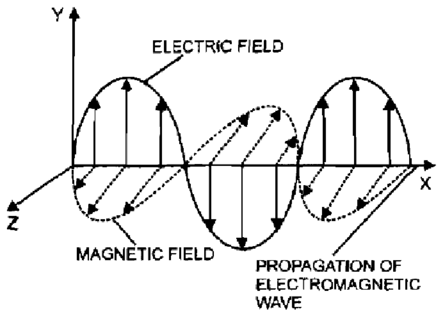
Hence, the displacement current through any surface is given by

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt}$$

By considering surface S_2, we can identify the displacement current as the source ofthe magnetic field

ELECTROMAGNETIC WAVES:

In electromagnetic waves, both the field vectors ($\vec{E}$ and $\vec{B}$) vary with time and space and have the same frequency and same phase. In figure, the electric field vector $\left(\vec{E}\right)$ and magnetic field vector $\left(\vec{B}\right)$ are vibrating along Y and Z directions and propagation of electromagnetic wave is shown in X-direction.



According to Maxwell the electromagnetic waves are of transverse in nature and they can pass through vacuum with the speed of light (= 3 × 10^8 ms^{-1}).

The velocity of eletromagnetic wave in a medium is given by

$$v = \frac{1}{\sqrt{\mu_0 \mu_r \in_0 \in_r}}$$

where, μ_0, μ_r = absolute permeability of space and relative permeability of medium,
$\in_0$, $\in_r$ = absolute permittivity of space and relative permittivity of medium
The velocity of electromagnetic waves ofdifferent frequency in vacuum is same but in a medium is different. It is more for red light and less for violet light.
The energy is shared equally between electric field vector and magnetic field vector.

$$U_{av} = \frac{1}{2} \in_0 E_0^2 = \frac{1}{2} \frac{B_0^2}{\mu_0}$$

It has been found that the velocity (c) of electromagnetic wave in free space is equal to the ratio of amplitude of electric field vector (E_0) and magnetic field vector (B_0) i.e $c = E_0/B_0$.
It was found that the accelerated charge or oscillating charge is a source of electromagnetic waves.

If the plane of electric field is oriented horizontally in respect to the earth, the electromagnetic wave is said to be horizontally polarised. On the other hand, if the plane of electric field vector is oriented vertically the electromagnetic wave is said to be vertically polarised.

The polarisation of electromagnetic wave is mainly the function of the antenna orientation.

Light may be polarized by passing it through a sheet of commercial material called Polaroid

Malus' Law

Suppose we have a second piece of Polaroid whose transmission axis makes an angle θ with that of the first one.

If $I_0 \cong E^2$ is the intensity between the two Polaroids, the intensity transmitted by both of them would be: $I(\theta) = I_0 \cos^2 \theta$.

Brewster's law

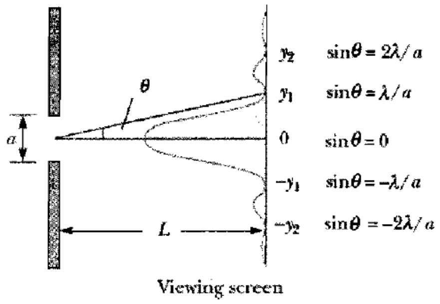
$\tan \theta_P = \frac{n_2}{n_1}$: **Brewster's law**

θ_P is the angle of incidence of unpolarized light which makes the reflected light completely polarized in the perpendicular direction to the plane of incidence (Sir David Brewster, 1812). When the angle of incidence of the initially unpolarized light is θ_P, the reflected and refracted rays are perpendicular to each other.

DIFFRACTION

When light passes through a narrow slit of width comparable to the wavelength of light, the light flares out of the slit; this bending or spreading of waves is called diffraction.

Properties of dark & bright fringes:



(a) It results from superposition of secondary wavelets originating from various parts of single coherent source.

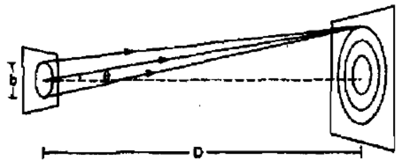
(b) Diffraction fringes are never of equal width.

(c) Intensity of all the bright fringes is not the same.

(d) Dark fringes are not perfectly dark.

FRAUNHOFER DIFFRACTION BY A CIRCULAR APERTURE

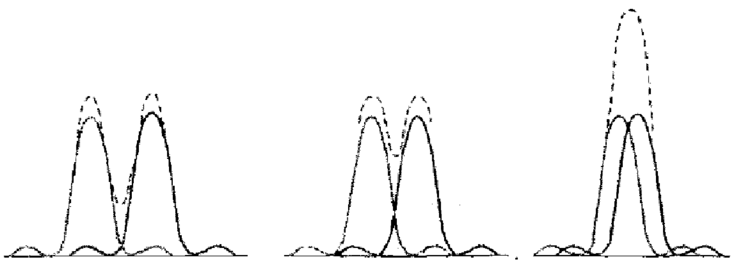
The mathematical analysis shows that the first dark ring is formed by the light diffracted from the hole at an angle θ with the axis $\sin\theta = \frac{1.22\lambda}{b}$.



The radius of the diffraction disc is given by $R = \frac{1.22\lambda D}{b}$,

LIMIT OF RESOLUTION

The fact that a lens forms a disc image of a point source, puts a limit on resolving two neighboring points imaged by a lens.



well resolved just resolved unresolved.

For two objects to be barely resolved, the angular separation between them should be at least:

$$\theta_R = \sin^{-1}\left(\frac{1.22\lambda}{b}\right)$$

EXERCISE (S-1)

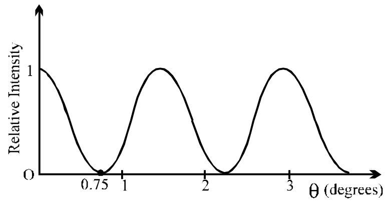
1. In a Young's double slit experiment for interference of light, the slits are 0.2 cm apart and are illuminated by yellow light ($\lambda = 600$ nm). What would be the fringe width on a screen placed 1 m from the plane of slits if the whole system is immersed in water of index 4/3?

WO0001

2. In Young's double slit experiment the slits are 0.5 mm apart and the interference is observed on a screen at a distance of 100 cm from the slit. It is found that the 9th bright fringe is at a distance of 7.5mm from the second dark fringe from the centre of the fringe pattern on same side. Find the wavelength of the light used.

WO0002

3. Light of wavelength 520 nm passing through a double slit, produces interference pattern of relative intensity versus angular position θ as shown in the figure. Find the separation d between the slits.



WO0003

4. In a Young's double slit experiment, two wavelengths of 500 nm and 700 nm were used. What is the minimum distance from the central maximum where their maximas coincide again? (Take $D/d = 10^3$. Symbols have their usual meanings.) **[IIT-JEE 2004]**

WO0004

5. In a YDSE apparatus, d = 1mm, λ = 600 nm and D = 1m. The slits individually produce same intensity on the screen. Find the minimum distance between two points on the screen having 75% intensity of the maximum intensity.

WO0005

6. The distance between two slits in a YDSE apparatus is 3mm. The distance of the screen from the slits is 1m. Microwaves of wavelength 1 mm are incident on the plane of the slits normally. Find the distance of the first maxima on the screen from the central maxima. Also find the total number of maxima on the screen.

WO0006

7. One slit of a double slit experiment is covered by a thin glass plate of refractive index 1.4 and the other by a thin glass plate of refractive index 1.7. The point on the screen, where central bright fringe was formed before the introduction of the glass sheets, is now occupied by the 5th bright fringe. Assuming that both the glass plates have same thickness and wavelength of light used is 4800 Å, find their thickness.

WO0007

8. A monochromatic light of $\lambda = 5000$ Å is incident on two slits separated by a distance of 5×10^{-4} m. The interference pattern is seen on a screen placed at a distance of 1 m from the slits. A thin glass plate of thickness 1.5×10^{-6} m & refractive index $\mu = 1.5$ is placed between one of the slits & the screen. Find the intensity at the centre of the screen, if the intensity there is I_0 in the absence of the plate. Also find the lateral shift of the central maximum.

WO0008

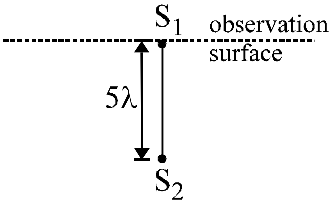
9. In a biprism experiment with sodium light, bands of width 0.0195 cm are observed on screen at 100cm from slit. On introducing a convex lens 30 cm away from the slit between biprism and screen, two images of the slit are seen 0.7 cm apart on screen. Calculate the wavelength of sodium light.

WO0009

10. A long narrow horizontal slit lies 1mm above a plane mirror as in Lloyd's mirror. The interference pattern produced by the slit and its image is viewed on a screen distant 1m from the slit. The wavelength of light is 600nm. Find the distance of first maximum above the mirror.

WO0010

11. Two microwave coherent point sources emitting waves of wavelength λ are placed at 5λ distance apart. The interference is being observed on a flat non-reflecting surface along a line passing through one source, in a direction perpendicular to the line joining the two sources (refer figure). Considering λ as 4 mm, calculate the positions of maxima and draw shape of interference pattern. Take initial phase difference between the two sources to be zero.



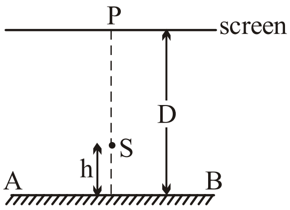
WO0011

12. A point source S emitting light of wavelength 600 nm is placed at a very small height h above the flat reflecting surface AB (see figure). The intensity of the reflected light is 36% of the incident intensity. Interference fringes are observed on a screen placed parallel to the reflecting surface at a very large distance D from it. **[IIT-JEE 2002]**

(i) What is the shape of the interference fringes on the screen?

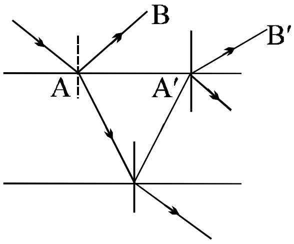
(ii) Calculate the ratio of the minimum to the maximum intensities in the interference fringes formed near the point P (shown in the figure).

(iii) If the intensities at point P corresponds to a maximum, calculate the minimum distance through which the reflecting surface AB should be shifted so that the intensity at P again becomes max.



WO0012

13. A ray of light of intensity I is incident on a parallel glass-slab at a point A as shown in figure. It undergoes partial reflection and refraction. At each reflection 20% of incident energy is reflected. The rays AB and A′ B′undergo interference. Find the ratio I_{max}/I_{min}. [Neglect the absorption of light]



WO0013

EXERCISE (S-2)

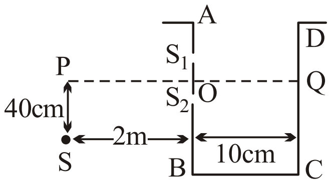
1. A thin glass plate of thickness t and refractive index μ is inserted between screen & one of the slits in a Young's experiment. If the intensity at the centre of the screen is I, what was the intensity at the same point prior to the introduction of the sheet.

WO0014

2. In Young's experiment, the source is red light of wavelength 7×10^{-7} m . When a thin glass plate of refractive index 1.5 at this wavelength is put in the path of one of the interfering beams, the central bright fringe shifts by 10^{-3} m to the position which was previously occupied by the 5th bright fringe. Find the thickness of the plate. When the source is now changed to green light of wavelength 5×10^{-7} m, the central fringe shifts to a position initially occupied by the 6th bright fringe due to red light without the plate. Find the refractive index of glass for the green light. Also estimate the change in fringe width due to the change in wavelength. **[IIT-JEE 1997 C]**

WO0015

3. A vessel $ABCD$ of 10 cm width has two small slits S_1 and S_2 sealed with identical glass plates of equal thickness. The distance between the slits is 0.8 mm. POQ is the line perpendicular to the plane AB and passing through O, the middle point of S_1 and S_2. A monochromatic light source is kept at S, 40cm below P and 2 m from the vessel, to illuminate the slits as shown in the figure below. Calculate the position of the central bright fringe on the other wall CD with respect to the line OQ. Now, a liquid is poured into the vessel and filled up to OQ. The central bright fringe is found to be at Q. Calculate the refractive index of the liquid. **[IIT-JEE 2001]**



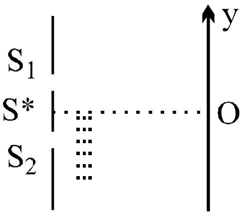
WO0016

4. The Young's double slit experiment is done in a medium of refractive index 4/3. A light of 600 nm wavelength is falling on the slits having 0.45 mm separation. The lower slit S_2 is covered by a thin glass sheet of thickness 10.4 μm and refractive index 1.5. The interference pattern is observed on a screen placed 1.5 m from the slits as shown **[IIT-JEE'99]**

(i) Find the location of the central maximum (bright fringe with zero path difference) on the y-axis.

(ii) Find the light intensity at point O relative to the maximum fringe intensity.

(iii) Now, if 600 nm light is replaced by white light of range 400 to 700 nm, find the wavelengths of the light that form maxima exactly at point O. [All wavelengths in this problem are for the given medium of refractive index 4/3. Ignore dispersion]

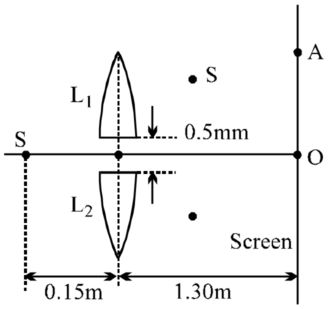


WO0017

5. In a Young's experiment, the upper slit is covered by a thin glass plate of refractive index 1.4 while the lower slit is covered by another glass plate having the same thickness as the first one but having refractive index 1.7. Interference pattern is observed using light of wavelength 5400 Å. It is found that the point P on the screen where the central maximum ($n = 0$) fell before the glass plates were inserted now has 3/4 the original intensity. It is further observed that what used to be the 5th maximum earlier, lies below the point P while the 6th minimum lies above P. Calculate the thickness of the glass plate. (Absorption of light by glass plate may be neglected). **[IIT-JEE 1997]**

WO0018

6. In the figure shown S is a monochromatic point source emitting light of wavelength $\lambda = 500$ nm. A thin lens of circular shape and focal length 0.10 m is cut into two identical halves L_1 and L_2 by a plane passing through a diameter. The two halves are placed symmetrically about the central axis SO with a gap of 0.5 mm. The distance along the axis from S to L_1 and L_2 is 0.15 m, while that from L_1 & L_2 to O is 1.30 m. The screen at O is normal to SO. **[IIT-JEE 1993]**

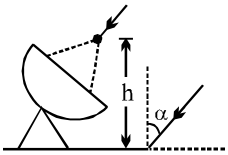


(i) If the third intensity maximum occurs at the point A on the screen, find the distance OA.

(ii) If the gap between L_1 & L_2 is reduced from its original value of 0.5 mm, will the distance OA increase, decrease or remain the same ?

WO0019

7. Radio waves coming at angle α to vertical are recieved by a radar after reflection from a nearby water surface & directly. What should be height of antenna from water surface so that it records a maximum intensity. (wavelength = λ).

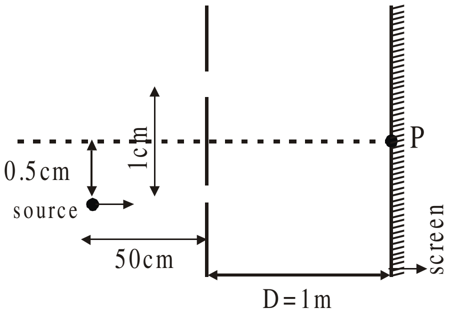


WO0020

8. One radio transmitter *A* operating at 60.0 MHz is 10.0 m from another similar transmitter *B* that is 180° out of phase with transmitter *A*. How far must an observer move from transmitter *A* toward transmitter *B* along the line connecting *A* and *B* to reach the nearest point where the two beams are in phase?

WO0021

9. In a typical Young's double slit experiment a point source of monochromatic light is kept as shown in the figure. If the source is given an instantaneous velocity v = 1 mm per second towards the screen, then the instantaneous velocity of central maxima is given as $\alpha \times 10^{-\beta}$ m/s upward. Find the value of $\alpha+\beta$.

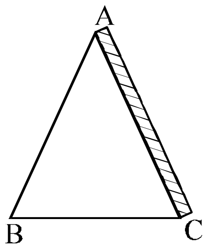


WO0022

10. A prism ($\mu_P = \sqrt{3}$) has an angle of prism $A = 30°$. A thin film ($\mu_f = 2.2$) is coated on face AC as shown in the figure. Light of wavelength 550 nm is incident on the face *AB* at 60° angle of incidence. Find **[IIT-JEE 2003]**

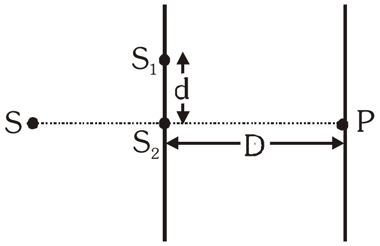
(i) the angle of its emergence from the face *AC* and

(ii) the minimum thickness (in nm) of the film for which the emerging light is of maximum possible intensity.



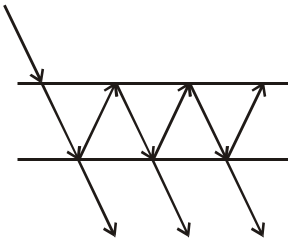
WO0023

11. In a YDSE experiment two slits S_1 and S_2 have separation of d = 2 mm. The distance of the screen is D = 8/5 m. Source S starts moving from a very large distance towards S_2 perpendicular to S_1S_2 as shown in figure. The wavelength of monochromatic light is 500 nm. The number of maximas observed on the screen at point P as the source moves towards S_2 is 3995 + n. Find the value of n.



WO0024

12. A narrow beam of light has entered a large thin glass plate. Each refraction is accompanied by reflection of one third of the beam's energy. What percentage of the light energy is transmitted through the plate?

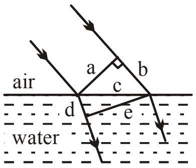


WO0025

EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

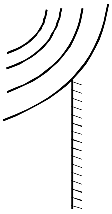
1. Figure shows plane waves refracted from air to water using Huygen's principle (where a, b, c, d, e are lengths on the diagram). The refractive index of water wrt air is the ratio :-

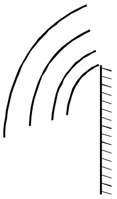
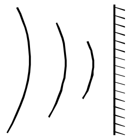
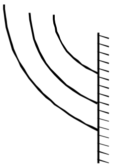
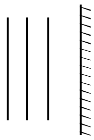


(A) a/e (B) b/e (C) b/d (D) d/b

WO0026

2. Spherical wavefronts shown in figure, strike a plane mirror. Reflected wavefronts will be as shown in



(A)

(B)

(C)

(D)


WO0027

3. Two beams of light having intensities I and $4I$ interfere to produce a fringe pattern on a screen. The phase difference between the beams is $\pi/2$ at point A and π at point B. Then the difference between the resultant intensities at A and B is : **[IIT-JEE (Scr.) 2001]**

(A) 2I (B) 4I (C) 5I (D) 7I

WO0028

4. In a young double slit experiment, 12 fringes are observed to be formed in a certain segment of the screen when light of wavelength 600 nm is used. If the wavelength of light is changed to 400 nm, number of fringes observed in the same segment of the screen is given by **[IIT-JEE (Scr.) 2001]**

(A) 12 (B) 18 (C) 24 (D) 30

WO0029

5. Two coherent monochromatic light beams of intensities *I* and 4*I* are superposed. The maximum and minimum possible intensities in the resulting beam are :

(A) 5*I* and *I* (B) 5I and 3I (C) 9*I* and *I* (D) 9*I* and 3*I*

WO0030

6. When light is refracted into a denser medium,

(A) its wavelength and frequency both increase

(B) its wavelength increases but frequency remains unchanged

(C) its wavelength decreases but frequency remains unchanged

(D) its wavelength and frequency both decrease.

WO0031

7. In YDSE how many maxima can be obtained on the screen if wavelength of light used is 200nm and $d = 700$ nm:

(A) 12 (B) 7 (C) 18 (D) none of these

WO0032

8. In a YDSE, the central bright fringe can be identified :

(A) as it has greater intensity than the other bright fringes.

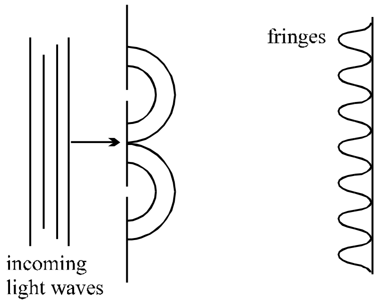
(B) as it is wider than the other bright fringes.

(C) as it is narrower than the other bright fringes.

(D) by using white light instead of single wavelength light.

WO0033

9. In a Young's double slit experiment, green light is incident on the two slits. The interference pattern is observed on a screen. Which of the following changes would cause the observed fringes to be more closely spaced?



(A) Reducing the separation between the slits

(B) Using blue light instead of green light

(C) Used red light instead of green light

(D) Moving the light source further away from the slits.

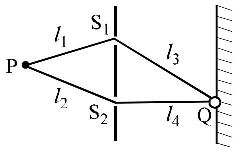
WO0034

10. In Young's double slit experiment, the wavelength of red light is 7800 Å and that of blue light is 5200 Å. The value of *n* for which n^{th} bright band due to red light coincides with $(n+1)^{th}$ bright band due to blue light, is :

(A) 1 (B) 2 (C) 3 (D) 4

WO0035

11. Two identical narrow slits S_1 and S_2 are illuminated by light of wavelength λ from a point source P. If, as shown in the diagram, the light is then allowed to fall on a screen, and if n is a positive integer, the condition for destructive interference at Q is :-



(A) $(\ell_1 - \ell_2) = (2n + 1)\lambda/2$ (B) $(\ell_3 - \ell_4) = (2n + 1)\lambda/2$
(C) $(\ell_1 + \ell_2) - (\ell_3 + \ell_4) = n\lambda$ (D) $(\ell_1 + \ell_3) - (\ell_2 + \ell_4) = (2n + 1)\lambda/2$

WO0036

12. In Young's double slit experiment, the two slits act as coherent sources of equal amplitude A and wavelength λ. In another experiment with the same setup the two slits are sources of equal amplitude A and wavelength λ but are incoherent. The ratio of the average intensity of light at the midpoint of the screen in the first case to that in the second case is :-

(A) 1 : 1 (B) 2 : 1 (C) 4 : 1 (D) none of these

WO0037

13. In a Young's double slit experiment, a small detector measures an intensity of illumination of I units at the centre of the fringe pattern. If one of the two (identical) slits is now covered, the measured intensity will be :-

(A) 2I (B) I (C) I/4 (D) I/2

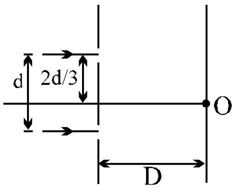
WO0038

14. In a Young's double slit experiment D equals the distance of screen and d is the separation between the slit. The distance of the nearest point to the central maximum where the intensity is same as that due to a single slit, is equal to :-

(A) $\frac{D\lambda}{d}$ (B) $\frac{D\lambda}{2d}$ (C) $\frac{D\lambda}{3d}$ (D) $\frac{2D\lambda}{d}$

WO0039

15. In the figure shown if a parallel beam of white light is incident on the plane of the slits then the distance of the white spot on the screen from O is [Assume $d << D$, $\lambda << d$]



(A) 0 (B) $d/2$ (C) $d/3$ (D) $d/6$

WO0040

16. In the above question if the light incident is monochromatic and point O is a maxima, then the wavelength of the light incident cannot be :-

(A) $d^2/3D$ (B) $d^2/6D$ (C) $d^2/12D$ (D) $\mathrm{d}^2/18\mathrm{D}$

WO0041

17. In a YDSE bi-chromatic light of wavelengths 400 nm and 560 nm are used. The distance between the slits is 0.1 mm and the distance between the plane of the slits and the screen is 1 m. The minimum distance between two successive regions of complete darkness is :- **[IIT-JEE' 2004 (Scr)]**

(A) 4 mm (B) 5.6 mm (C) 14 mm (D) 28 mm

WO0042

18. In Young's double slit arrangement, water is filled in the space between screen and slits. Then :

(A) fringe pattern shifts upwards but fringe width remains unchanged.

(B) fringe width decreases and central bright fringe shifts upwards.

(C) fringe width increases and central bright fringe does not shift.

(D) fringe width decreases and central bright fringe does not shift.

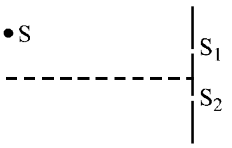
WO0043

19. Light of wavelength λ in air enters a medium of refractive index μ. Two points in this medium, lying along the path of this light, are at a distance x apart. The phase difference between these points is :

(A) $\dfrac{2\pi\mu x}{\lambda}$ (B) $\dfrac{2\pi x}{\mu\lambda}$ (C) $\dfrac{2\pi(\mu-1)x}{\lambda}$ (D) $\dfrac{2\pi x}{(\mu-1)\lambda}$

WO0044

20. In YDSE, the source placed symmetrically with respect to the slit is now moved parallel to the plane of the slits so that it is closer to the upper slit, as shown. Then,



(A) the fringe width will increase and fringe pattern will shift down.

(B) the fringe width will remain same but fringe pattern will shift up.

(C) the fringe width will decrease and fringe pattern will shift down.

(D) the fringe width will remain same but fringe pattern will shift down.

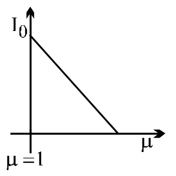
WO0045

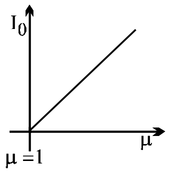
21. In the ideal double–slit experiment, when a glass–plate (refractive index 1.5) of thickness t is introduced in the path of one of the interfering beams (wavelength λ), the intensity at the position where the central maximum occurred previously remains unchanged. The minimum thickness of the glass–plate is :– **[IIT-JEE 2002]**

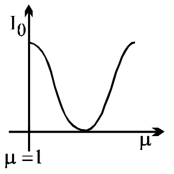
(A) 2λ (B) $\dfrac{2\lambda}{3}$ (C) $\dfrac{\lambda}{3}$ (D) λ

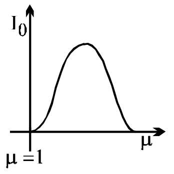
WO0046

22. In a YDSE experiment if a slab whose refractive index can be varied is placed in front of one of the slits then the variation of resultant intensity at mid-point of screen with μ ($\mu \geq 1$)will be best represented by [Assume slits of equal width and there is no absorption by slab]

(A)


(B)


(C)


(D)


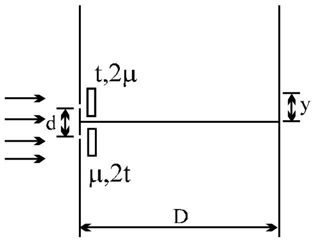
WO0047

23. In a Young's double-slit experiment, let A and B be the two slits. Films of thicknesses t_A and t_B and refractive indices μ_A and μ_B, are placed in front of A and B respectively. If $\mu_A t_A = \mu_B t_B$, the central maximum will :

(A) not shift
(B) shift towards A
(C) shift towards B
(D) option (B), if $t_B > t_A$; option (C) if $t_B < t_A$

WO0048

24. In the YDSE shown the two slits are covered with thin sheets having thickness t & $2t$ and refractive index 2μ and μ. Find the position (y) of central maxima



(A) zero
(B) $\frac{tD}{d}$
(C) $-\frac{tD}{d}$
(D) None

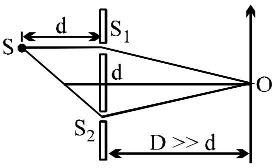
WO0049

25. In a double slit experiment, when the width of one slit is made twice as wide as the other in compared to normal YDSE having slits of equal width. Then, in the interference pattern **[IIT-JEE(Scr.) 2000]**

(A) the intensities of both the maxima and the minima increase.
(B) the intensity of the maxima increases and the minima has zero intensity.
(C) the intensity of the maxima decreases and that of the minima increases.
(D) the intensity of the maxima decreases and the minima has zero intensity.

WO0050

26. To make the central fringe at the centre O, a mica sheet of refractive index 1.5 is introduced. Choose the correct statements (s).



(A) The thickness of sheet is $2(\sqrt{2}-1)d$ in front of S_1.

(B) The thickness of sheet is $(\sqrt{2}-1)d$ in front of S_2.

(C) The thickness of sheet is $2\sqrt{2}\,d$ in front of S_1.

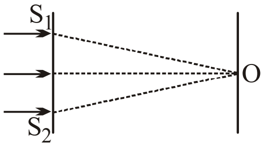
(D) The thickness of sheet is $(2\sqrt{2}-1)d$ in front of S_1.

WO0051

27. **Statement-1 :** In YDSE, as shown in figure, central bright fringe is formed at O. If a liquid is filled between plane of slits and screen, the central bright fringe is shifted in upward direction.

and

Statement-2 : If path difference at O increases, y-coordinate of central bright fringe will change.



(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

WO0052

28. **Statement-1 :** In glass, red light travels faster than blue light.

and

Statement-2 : Red light has a wavelength longer than blue.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

WO0053

29. **Statement-1 :**In standard YDSE set up with visible light, the position on screen where phase difference is zero appears bright.

and

Statement-2 : In YDSE set up magnitude of electromagnetic field at central bright fringe is not varying with time.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

WO0054

30. **Statement-1 :**In YDSE, the spacing between any two successive points having intensity half of the maximum intensity is same.

and

Statement-2 : The intensity on the screen in YDSE varies uniformly with distance from central maximum.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

WO0055

31. In a double slit experiment, the separation between the slits is $d = 0.25$ cm and the distance of the screen D = 100 cm from the slits. If the wavelength of light used is $\lambda = 6000$Å and I_0 is the intensity of the central bright fringe, the intensity at a distance $x = 4 \times 10^{-5}$ m from the central maxima is :-

(A) I_0 (B) $I_0/2$ (C) $3I_0/4$ (D) $I_0/3$

WO0056

32. Imagine a Young's double slit interference experiment performed with waves associated with fast moving electrons produced from an electron gun. The distance between successive maxima will decrease the most if :-

(A) the accelerating voltage in the electron gun is decreased

(B) the accelerating voltage is increased and the distance of the screen from the slits is decreased

(C) the distance of the screen from the slits is increased.

(D) the distance between the slits is decreased.

WO0057

33. Two monochromatic and coherent point sources of light are placed at a certain distance from each other in the horizontal plane. The locus of all those points in the horizontal plane which have constructive interference will be

(A) a hyperbola
(B) family of hyperbolas
(C) family of straight lines
(D) family of parabolas

WO0058

MULTIPLE CORRECT TYPE QUESTIONS

34. To observe a sustained interference pattern formed by two light waves, it is not necessary that they must have :

(A) the same frequency
(B) same amplitude
(C) a constant phase difference
(D) the same intensity

WO0059

35. As a wave propagates, **[IIT-JEE 1999]**

(A) the wave intensity remains constant for a plane wave
(B) the wave intensity decreases with distance from source and is proportional to inverse of the distance, for a spherical wave.
(C) the wave intensity decreases with distance from source and proportional to inverse square of the distance, for a spherical wave.
(D) total average power of the spherical wave over any spherical surface centered at the source remains same.

WO0060

36. In a YDSE apparatus, if we use white light then :

(A) the fringe next to the central will be red
(B) the central fringe will be white.
(C) the fringe next to the central will be violet
(D) there will not be a completely dark fringe.

WO0061

37. If the source of light used in a Young's Double Slit Experiment is changed from red to blue, then

(A) the fringes will become brighter
(B) consecutive fringes will come closer
(C) the number of maxima formed on the screen increases
(D) the central bright fringe will become a dark fringe.

WO0062

38. In a Young's double-slit experiment, let A and B be the two slits. A thin film of thickness t and refractive index μ is placed in front of A. Let $\beta =$ fringe width. The central maximum will shift :

(A) towards A
(B) towards B
(C) by $t(\mu - 1)\frac{\beta}{\lambda}$
(D) by $\mu t\frac{\beta}{\lambda}$

WO0063

39. If one of the slits of a standard YDSE apparatus is covered by a thin parallel sided glass slab so that it transmit only one half of the light intensity of the other, then :

(A) the fringe pattern will get shifted towards the covered slit.

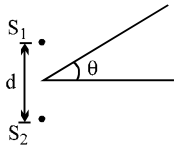
(B) the fringe pattern will get shifted away from the covered slit.

(C) the bright fringes will be less bright and the dark ones will be more bright.

(D) the fringe width will remain unchanged.

WO0064

40. In an interference arrangement similar to Young's double- slit experiment, the slits S_1 & S_2 are illuminated with coherent microwave sources, each of frequency 10^6 Hz. The sources are synchronized to have zero phase difference. The slits are separated by a distance $d = 150.0$ m. The intensity $I(\theta)$ is measured as a function of θ at a large distance from S_1 & S_2, where θ is defined as shown. If I_0 is the maximum intensity then $I(\theta)$ for $0 \le \theta \le 90°$ is given by :



(A) $I(\theta) = \frac{I_0}{2}$ for $\theta = 30°$

(B) $I(\theta) = \frac{I_0}{4}$ for $\theta = 90°$

(C) $I(\theta) = I_0$ for $\theta = 0°$

(D) $I(\theta)$ is constant for all values of θ .

WO0065

41. In a standard YDSE apparatus, a thin film ($\mu = 1.5$, $t = 2.1$ μm) is placed in front of upper slit. How far above or below the centre point of the screen are two nearest maxima located? Take $D = 1$ m, $d = 1$mm, $\lambda = 4500$Å. (Symbols have usual meaning)

(A) 1.5 mm (B) 0.6 mm (C) 0.15 mm (D) 0.3 mm

WO0066

42. In a YDSE with two identical slits, when the upper slit is covered with a thin, perfectly transparent sheet of mica, the intensity at the centre of screen reduces to 75% of the initial value. Second minima is observed to be above this point and third maxima below it. Which of the following can be a possible value of phase difference caused by the mica sheet

(A) $\frac{\pi}{3}$ (B) $\frac{13\pi}{3}$ (C) $\frac{17\pi}{3}$ (D) $\frac{11\pi}{3}$

WO0067

43. A beam of 2000 eV electrons are incident normally on the surface of crystal whose inter atomic separation is 0.11 nm. The mass of the electron can be taken as 9×10^{-31} kg. At what angle to the normal can we observe a diffraction maxima.

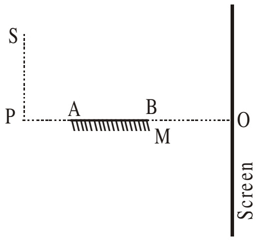
(A) $\sin^{-1}\left(\frac{1}{4}\right)$ (B) $2\cos^{-1}\left(\frac{1}{4}\right)$ (C) $\sin^{-1}\left(\frac{1}{2}\right)$ (D) $2\cos^{-1}\left(\frac{1}{2}\right)$

WO0068

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 44 and 45

In an experiment on interference due to single mirror, a light wave emitted directly by the source S (narrow slit) interferes with the wave reflected from the mirror M of length 2 mm. Source and screen are separated by distance 90 cm. Source *S* is at the height of 3 mm, from the point P and the middle point of mirror is at distance of 2mm from point *P*. Point *P* and mirror are in the same plane of screen is perpendicular to this plane.



44. If fringe width is 0.1 mm then what is the wavelength of light used?

(A) 3.3×10^{-7} m (B) 6.7×10^{-7} m (C) 1.0×10^{-7} m (D) 4×10^{-7} m

WO0069

45. If the mirror is shifted towards left then how does the fringe pattern on screen change?

(A) Fringe width decreases and the region in which interference is formed shifts downward.

(B) Fringe width decreases and region in which inteference is formed shifts upwards

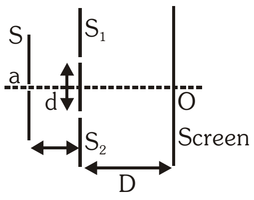
(C) Fringe width does not change and region in which interference is formed shifts upwards

(D) Fringe width does not change and region in which inteference is formed shifts downwards

WO0069

Paragraph for Question No. 46 to 48

The figure shows a schematic diagram showing the arrangement of Young's Double Slit Experiment



46. Choose the correct statement(s) related to the wavelength of light used

(A) Larger the wavelength of light larger the fringe width

(B) The position of central maxima depends on the wavelength of light used

(C) If white light is used in YDSE, then the violet colour forms its first maxima closest to the central maxima

(D) The central maxima of all the wavelengths coincide

WO0070

47. If the distance D is varied, then choose the correct statement(s)

(A) The angular fringe width does not change

(B) The fringe width changes in direct proportion

(C) The change in fringe width is same for all wavelengths

(D) The position of central maxima remains unchanged

WO0070

48. If the distance d is varied, then identify the correct statement(s)

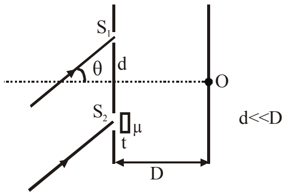
(A) The angular width does not change (B) The fringe width changes in inverse proportion

(C) The positions of all maxima change (D) The positions of all minima change

WO0070

Paragraph for Question No. 49 to 51

A monochromatic beam of light falls on Young's double slit experiment apparatus as shown in figure. A thin sheet of glass is inserted in front of lower slit S_2.



49. The central bright fringe can be obtained

(A) at O only (B) at O or below O only

(C) at O or above O only (D) Anywhere on the screen

WO0071

50. If central bright fringe is obtained on screen at O :-

(A) $(\mu-1)\,t = d\sin\theta$ (B) $(\mu-1)\,t = d\cos\theta$ (C) $(\mu-1)t + d\sin\theta = 0$ (D) $\dfrac{t}{\mu-1} = \dfrac{d}{\sin\theta}$

WO0071

51. The phase difference between the waves interfering at fifth minima is

(A) 5π (B) 7 π (C) 9π (D) 11π

WO0071

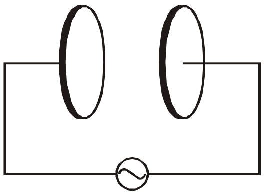
SUPPLEMENT FOR JEE-MAINS

52. A parallel plate capacitor (fig.) made of circular plates each of radius R = 6.0 cm has a capacitance C = 100 μF. The capacitor is connected to a 230 V ac supply with a (angular) frequency of 300 rad s^{-1}.

(a) What is the rms value of the conduction current?

(b) Is the conduction current equal to the displacement current?

(c) Determine the amplitude of B at a point 3.0 cm from the axis between the plates.



WO0072

53. The amplitude of the magnetic field part of a harmonic electromagnetic wave in vacuum is B_0 = 510nT. What is the amplitude of the electric field part of the wave?

WO0073

54. In a plane electromagnetic wave, the electric field oscillates sinusoidally at a frequency of 2.0×10^{10}Hz and amplitude 48 V m^{-1}.

(a) What is the wavelength of the wave?

(b) What is the amplitude of the oscillating magnetic field?

(c) Show that the average energy density of the E field equals the average energy density of the B field. [$c = 3 \times 10^8$ m s^{-1}]

WO0074

55. Suppose that the electric field part of an electromagnetic wave in vaccum is

$E = \{(3.1\text{ N/C}) \cos [(1.8 \text{ rad/m}) y + (5.4 \times 10^8 \text{ rad/s})t] \hat{i}$.

(a) What is the direction of propagation?

(b) What is the wavelength λ?

(c) What is the frequency v?

(d) What is the amplitude of the magnetic field part of the wave?

(e) Write an expression for the magnetic field part of the wave.

WO0075

56. Two coherent waves are described by the following expressions.

$$E_1 = E_0 \sin\left(\frac{2\pi x_1}{\lambda} - 2\pi ft + \frac{\pi}{6}\right);\ E_2 = E_0 \sin\left(\frac{2\pi x_2}{\lambda} - 2\pi ft + \frac{\pi}{8}\right)$$

Determine the relationship between x_1 and x_2 that produces constructive interference when the two waves are superposed.

WO0078

57. A nickel crystal is used as a diffraction grating for x-rays. Then the same crystal is used to diffract electrons. If the two diffraction patterns are identical, and the energy of each x-ray photon is E = 20.0 keY. What is the kinetic energy of each electron?

WO0079

58. Which of these statements correctly describes the orientation of the electric field $\left(\vec{E}\right)$, the magnetic field $\left(\vec{B}\right)$, and and velocity of propagation $\left(\vec{v}\right)$ of an electromagnetic wave?

(A) $\vec{E}$ is perpendicular to $\vec{B}$; $\vec{v}$ may have any orientation relative to $\vec{E}$.

(B) $\vec{E}$ is perpendicular to $\vec{B}$; $\vec{v}$ may have any orientation perpcndicular to $\vec{E}$

(C) $\vec{E}$ is parallel to $\vec{B}$: $\vec{v}$ is perpendicular to both $\vec{B}$ and $\vec{E}$.

(D) Each of the three vectors is perpendicular to the other two.

WO0081

59. The amplitude of electric field in a parallel light beam of intensity $4Wm^{-2}$ is :

(A) 35.5 NC^{-1} (B) 45.5 NC^{-1} (C) 49.5 NC^{-1} (D) 54.8 NC^{-1}

WO0082

60. Instantaneous displacement current of 1.0A in the space between the parallel plates of 1μF capacitor can be established by changing potential difference of:

(A) 10^{-6} V/s (B) 10^{6} V/s (C) 10^{-8} V/s (D) 10^{8} V/s

WO0083

61. A plane electromagnetic wave,

$E_z = 100 \cos (6 \times 10^8 t + 4x)$ V/m

propagates in a medium of dielectric constant:

(A) 1.5 (B) 2.0 (C) 2.4 (D) 4.0

WO0084

62. The rms value of the electric field of the light coming from the sun is 720 N/c. The average total energy density of the electromagnetic wave is :

(A) 4.58×10^{-6} J/m^3 (B) 6.37×10^{-9} J/m^3 (C) 81.35×10^{-12} J/m^3 (D) 3.3×10^{-3} J/m^3

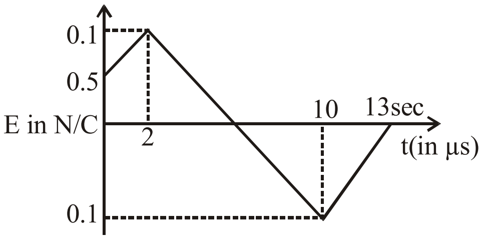
WO0086

63. A plane electromagnertic wave travels in free space along x-axis. At a particular point in space, the electric field along y-axis is 9.3 Vm^{-1}. The magnetic induction is:

(A) 3.1×10^{-8} T (B) 3×10^{-5} T (C) 3.1×10^{-6} T (D) 9.3×10^{-6} T

WO0088

64. The electric field through an area of 2m^2 varies with time as shown in the graph. The greatest displacement current through the area is at :-



(A) t = 1 sec. (B) t = 4 sec. (C) t = 8 sec. (D) t = 12 sec.

WO0089

65. A plane electromagnetic wave travelling along the X-direction has a wavelength of 3mm. The variation in the electric field occurs in the Y-direction with an amplitude 66Vm^{-1}.The equation for the electric and magnetic fields as a function of x and t are respectively.

(A) $E_y = 33\cos\pi\times10^{11}\left(t-\frac{x}{c}\right)$; $B_z = 1.1\times10^{-7}\cos\pi\times10^{11}\left(t-\frac{x}{c}\right)$

(B) $E_y = 11\cos2\pi\times10^{11}\left(t-\frac{x}{c}\right)$; $B_y = 1.1\times10^{-7}\cos2\pi\times10^{11}\left(t-\frac{x}{c}\right)$

(C) $E_x = 33\cos\pi\times10^{11}\left(t-\frac{x}{c}\right)$; $B_x = 1.1\times10^{-7}\cos\pi\times10^{11}\left(t-\frac{x}{c}\right)$

(D) $E_y = 66\cos2\pi\times10^{11}\left(t-\frac{x}{c}\right)$; $B_z = 2.2\times10^{-7}\cos2\pi\times10^{11}\left(t-\frac{x}{c}\right)$

WO0090

66. In a stack of three polarizing sheets the first and third are crossed while the middle one has its axis at 45° to the axes of the other two. The fraction of the intensity of an incident unpolarized beam of light that is transmitted by the stack is:

(A) 1/2 (B) 1/3 (C) 1/4 (D) 1/8

WO0091

67. A beam of light strikes a piece of glass at an angle of incidence of 60° and the reflected beam is completely plane polarised. The refractive index of the glass is :-

(A) 1.5 (B) $\sqrt{3}$ (C) $\sqrt{2}$ (D)(3/2)

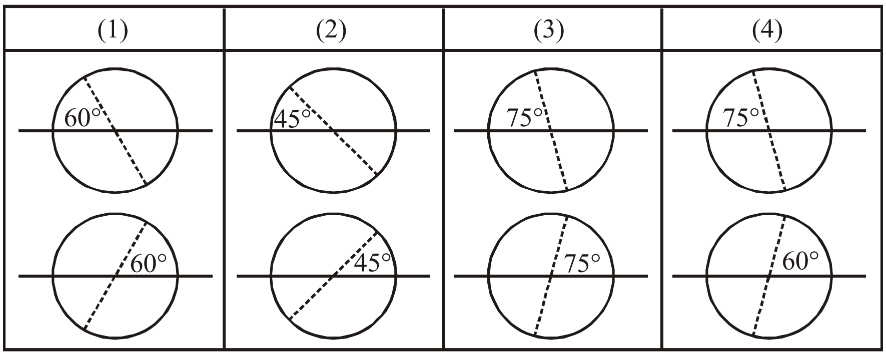
WO0092

68. The Sun is directly overhead and you are facing toward the north. Light coming to your eyes from the sky just above the horizon is :-

(A) Partially polarized north-south (B) Partially polarized east,west
(C) Partially polarized up-down (D) Randomly polarized

WO0093

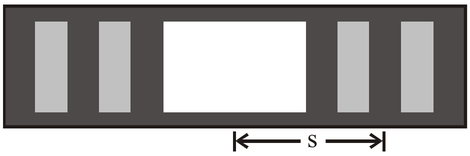
69. The diagrams show four pairs of polarizing sheets, with the polarizing directions indicated by dashed lines. The two 'sheets of each pair are placed one behind the other and the front sheet is illuminated by unpolarized light. The incident intensity is the same for all pairs of sheets. Rank the pairs according to the intensity of the transmitted light, least to greatest.



(A) 1,2, 3, 4 (B) 4, 2, 1, 3 (C) 2, 4, 3, 1 (D) 2, 1,4, 3

WO0094

70. Light of wavelength 600 nm is incident upon a single slit with width 4×10^{-4} m. The figure shows the pattern observed on a screen positioned 2 m from the slits. Determine the distance s.



(A) 0.002 m (B) 0.003 m (C) 0.004 m (D) 0.006 m

WO0095

71. The image of a star (effectively a point source) is made by a convergent lens of focal length 1m and diameter of aperture 5.0 cm. If the lens is ideal and the effective wavelength in image formation is taken as 5×10^{-5} cm, the diameter of the image formed will be nearest to:

(A) zero (B) 10^{-6} cm (C) 10^{-5} cm (D) 10^{-3} cm

WO0096

72. In a single slit diffraction pattern, as the width of the slit is increased,

(A) the peak intensity of central maxima increases and its width also increases

(B) the peak intensity of central maxima increases and its width decreases.

(C) the peak intensity of central maxima decreases and its width increases

(D) the peak intensity of central maxima decreases and its width also decreases.

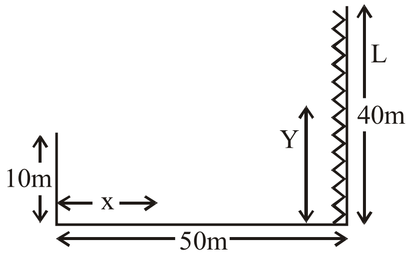
WO0097

73. A beam of electrons with de-broglie wavelength of 10^{-4} m pass through a slit 10^{-3} m wide. Calculate the angular spread introduced because of diffraction by slit.

(A) $\frac{9^\circ}{\pi}$ (B) $\frac{18^\circ}{\pi}$ (C) $\frac{36^\circ}{\pi}$ (D) $\frac{4.5^\circ}{\pi}$

WO0098

74. A person lives in a high-rise building on the bank of a river 50m wide. Across the river is a well lit tower of height 40 m. When the person, who is at a height of 10m, looks through a polarizer at an appropriate angle at light of the tower reflecting from the river surface, he notes that intensity of light coming from distance X from his building is the least and this corresponds to the light coming from light bulbs at height 'Y' on the tower. The values of X and Y are respectively close to (refractive index of water $\simeq$ 4/3)



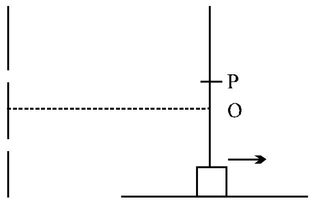
(A) 13 m, 27 m (B) 22 m, 13 m (C) 25 m, 10 m (D) 17 m, 29 m

WO0099

EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

1. In a Young's Double slit experiment, first maxima is observed at a fixed point *P* on the screen. Now the screen is continuously moved away from the plane of slits. The ratio of intensity at point P to the intensity at point *O* (centre of the screen)



(A) remains constant

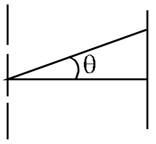
(B) keeps on decreasing

(C) first decreases and then increases

(D) First decreases and then becomes constant

WO0100

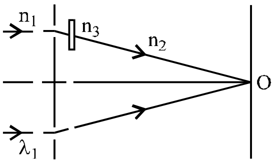
2. Two slits are separated by 0.3 mm. A beam of 500 nm light strikes the slits producing an interference pattern. The number of maxima observed in the angular range $-30° < \theta < 30°$.



(A) 300 (B) 150 (C) 599 (D) 601

WO0101

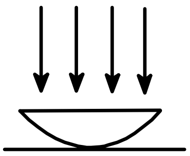
3. In the figure shown in YDSE, a parallel beam of light is incident on the slit from a medium of refractive index n_1. The wavelength of light in this medium is λ_1. A transparent slab of thickness t and refractive index n_3 is put in front of one slit. The medium between the screen and the plane of the slits is n_2. The phase difference between the light waves reaching point '*O*' (symmetrical, relative to the slits) is :



(A) $\frac{2\pi}{n_1\lambda_1}(n_3-n_2)\,t$ (B) $\frac{2\pi}{\lambda_1}(n_3-n_2)\,t$ (C) $\frac{2\pi n_1}{n_2\lambda_1}\left(\frac{n_3}{n_2}-1\right)t$ (D) $\frac{2\pi n_1}{\lambda_1}(n_3-n_1)t$

WO0102

4. A thin slice is cut out of a glass cylinder along a plane parallel to its axis. The slice is placed on a flat glass plate as shown. The observed interference fringes from this combination shall be **[IIT-JEE '99]**



(A) straight

(B) circular

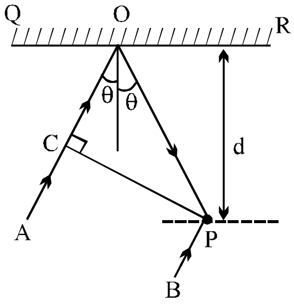
(C) equally spaced

(D) having fringe spacing which increases as we go outwards.

WO0103

5. In the adjacent diagram, *CP* represents a wavefront and *AO* and *BP*, the corresponding two rays. Find the condition on θ for constructive interference at P between the ray BP and reflected ray *OP*.

[JEE (Scr.) 2003]



(A) $\cos\theta = \frac{3\lambda}{2d}$ (B) $\cos\theta = \frac{\lambda}{4d}$ (C) $\sec\theta - \cos\theta = \frac{\lambda}{d}$ (D) $\sec\theta - \cos\theta = \frac{4\lambda}{d}$

WO0104

6. Two point monochromatic and coherent sources of light of wavelength λ are placed on the dotted line in front of a large screen. The sources emit waves in phase with each other. The distance between S_1 and S_2 is d while their distance from the screen is much larger.

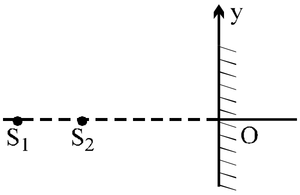
(1) If $d = 7\lambda/2$, *O* will be a minima

(2) If $d = 4.3\lambda$, there will be a total of 8 minima on y-axis.

(3) If $d = 7\lambda$, *O* will be a maxima.

(4) If $d = \lambda$, there will be only one maxima on the screen.

Which is the set of correct statement :



(A) 1, 2 & 3 (B) 2, 3 & 4 (C) 1, 2, 3 & 4 (D) 1, 3 & 4

WO0105

7. A parallel coherent beam of light falls on Fresnel biprism of refractive index μ and angle α. The fringe width on a screen at a distance D from biprism will be (wavelength = λ)

(A) $\frac{\lambda}{2(\mu-1)\alpha}$ (B) $\frac{\lambda D}{2(\mu-1)\alpha}$ (C) $\frac{D}{2(\mu-1)\alpha}$ (D) none

WO0106

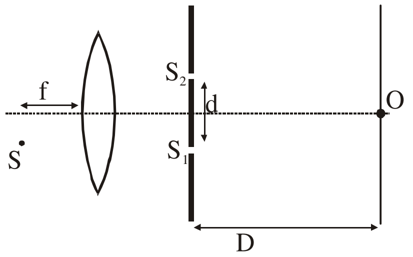
MATRIX MATCH TYPE QUESTIONS

8. Column-I shows some modifications in a standard YDSE setup. Column-II shows the associated characteristics.

Column-I

Column-II

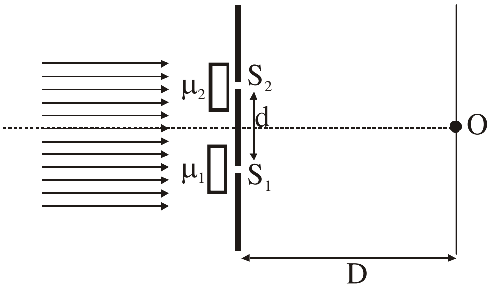
(A)



monochromatic point source
S placed in focal plane.

(P) Zero order maxima lies above O.

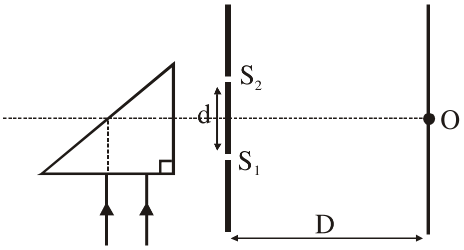
(B)



monochromatic parallel beam incident on S_1S_2 through transparent slabs of same thickness but $\mu_1 > \mu_2$

(Q) If a transparent mica sheet is introduced infront of S_2 central bright fringe can be obtained at O.

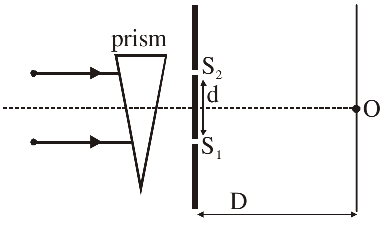
(C)



monochromatic parallel beam incident on a right angled isosceles prism of refractive index 1.50

(R) Fringe width $\beta = \frac{\lambda D}{d}$

(D)



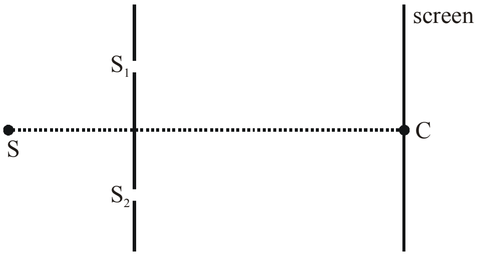
monochromatic parallel beam incident on thin prism

(S) Point O can be a minima

(T) Point O can be a least order minima.

WO0107

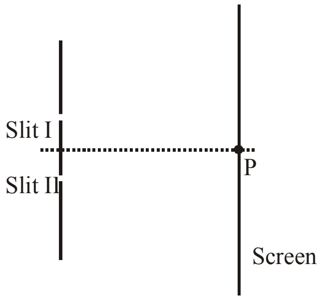
9. In a YDSE setup, light of wavelength 4000 Å is used. Distance of screen from the slits is 2m and distance between slits is 1mm. There are three slabs slab 1(thickness 2mm, $\mu = 2$), slab 2 (thickness 1mm, $\mu = 3$), slab 3(thickness 4mm $\mu = \frac{3}{2}$)



(A)	If slab 1 is placed in front of slit S_1	(P)	Central maxima at C
(B)	If the slab 2 is placed in front of slit S_2 along with condition (A)	(Q)	Central maxima above C
(C)	If slab 3 is placed in front of slab 1 along with condition (B)	(R)	Fringe width is equal to 0.8 mm
		(S)	No. of fringes crossing centre as a result of slab placing is 5000

WO0108

10. A double slit interference pattern is produced on a screen, as shown in the figure, using monochromatic light of wavelength 500nm. Point P is the location of the central bright fringe, that is produced when light waves arrive in phase without any path difference. A choice of three strips *A*, *B* and *C* of transparent materials with different thicknesses and refractive indices is available, as shown in the table. These are placed over one or both of the slits, singularly or in conjunction, causing the interference pattern to be shifted across the screen from the original pattern. In the column–I, how the strips have been placed, is mentioned whereas in the column–II, order of the fringe at point *P* on the screen that will be produced due to the placement of the strips(s), is shown. Correctly match both the column.



Film	A	B	C
Thickness (in µm)	5	1.5	0.25
Refractive index	1.5	2.5	2

	Column I		**Column II**
(A)	Only strip B is placed over slit–I	(P)	First Bright
(B)	Strip A is placed over slit–I and strip C is placed over slit–II	(Q)	Fourth Dark
(C)	Strip A is placed over the slit–I and strip B and strip C are placed over the slit–II in conjunction	(R)	Fifth Dark
(D)	Strip A and strip C are placed over slit–I (in conjuction) and strip B is placed over Slit–II	(S)	Central Bright

WO0109

EXERCISE-(J-M)

1. A mixture of light, consisting of wavelength 590 nm and an unknown wavelength, illuminates Young's double slit and gives rise to two overlapping interference patterns on the screen. The central maximum of both lights coincide. Further, it is observed that the third bright fringe of known light coincides with the 4th bright fringe of the unknown light. From this data, the wavelength of the unknown light is :- **[AIEEE-2009]**

(1) 442.5 nm (2) 776.8 nm (3) 393.4 nm (4) 885.0 nm

WO0110

Direction : Questions number 2 to 4 are based on the following paragraph.

An initially parallel cylindrical beam travels in a medium of refractive index $\mu(I) = \mu_0 + \mu_2 I$, where μ_0 and μ_2 are positive constants and I is the intensity of the light beam. The intensity of the beam is decreasing with increasing radius.

2. The initial shape of the wavefront of the beam is :- **[AIEEE-2010]**

(1) planar

(2) convex

(3) concave

(4) convex near the axis and concave near the periphery

WO0112

3. The speed of the light in the medium is :- **[AIEEE-2010]**

(1) maximum on the axis of the beam (2) minimum on the axis of the beam

(3) the same everywhere in the beam (4) directly proportional to the intensity I

WO0112

4. As the beam enters the medium, it will :- **[AIEEE-2010]**

(1) travel as a cylindrical beam

(2) diverge

(3) converge

(4) diverge near the axis and converge near the periphery

WO0112

5. At two points P and Q on screen in Young's double slit experiment, waves from slits S_1 and S_2 have a path difference of 0 and $\frac{\lambda}{4}$ respectively. the ratio of intensities at P and Q will be : **[AIEEE-2011]**

(1) 3 : 2 (2) 2 : 1 (3) $\sqrt{2}$: 1 (4) 4 : 1

WO0113

6. **Statement-1:** On viewing the clear blue portion of the sky through a Calcite Crystal, the intensity of transmitted light varies as the crystal is rotated.

Statement-1: The light coming from the sky is polarized due to scattering of sun light by particles in the atmosphere. The scattering is largest for blue light. **[AIEEE-2011]**

(1) Statement-1 is false, statement-2 is true

(2) Statement-1 is true, statement-2 is false

(3) Statement-1 is true, statement-2 true; statement-2 is the correct explanation of statement-1

(4) Statement-1 is true, statement-2 is true; statement -2 is not correct explanation of statement-1.

WO0114

7. In a Young's double slit experiment, the two slits act as coherent sources of waves of equal amplitude A and wavelength λ. In another experiment with the same arrangement the two slits are made to act as incoherent sources of waves of same amplitude and wavelength. If the intensity at the middle point of the screen in the first case is I_1 and in the second case I_2, then the ratio $\frac{I_1}{I_2}$ is :- **[AIEEE-2011]**

(1) 4 (2) 2 (3) 1 (4) 0.5

WO0115

8. Direction :

The question has a paragraph followed by two statement, Statement-1 and statement-2. Of the given four alternatives after the statements, choose the one that describes the statements.

A thin air film is formed by putting the convex surface of a plane-convex lens over a plane glass plate. With monochromatic light, this film gives an interference pattern due to light reflected from the top (convex) surface and the bottom (glass plate) surface of the film **[AIEEE-2011]**

Statement-1:

When light reflects from the air-glass plate interface, the reflected wave suffers a phase change of π.

Statement-2: The centre of the interference pattern is dark :-

(1) Statement-1 is true, Statement-2 is true and Statement-2 is not the correct explanation of Statement-1.

(2) Statement-1 is false, Statement-2 is true

(3) Statement-1 is true, Statement-2 is false

(4) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation of statement-1.

WO0116

9. In Young's double slit experiment, one of the slit is wider than other, so that the amplitude of the light from one slit is double of that from other slit. If I_m be the maximum intensity, the resultant intensity I when they interfere at phase difference ϕ is given by : **[AIEEE-2012]**

(1) $\frac{I_m}{9}(1+8\cos^2\frac{\phi}{2})$ (2) $\frac{I_m}{9}(4+5\cos\phi)$ (3) $\frac{I_m}{3}(1+2\cos^2\frac{\phi}{2})$ (4) $\frac{I_m}{5}(1+4\cos^2\frac{\phi}{2})$

WO0117

10. An electromagnetic wave in vacuum has the electric and magnetic fields $\vec{E}$ and $\vec{B}$, which are always perpendicular to each other. The direction of polarization is given by $\vec{X}$ and that ofwave propagation by $\vec{k}$. Then **[AIEEE-2012]**

(1) $\vec{X} \parallel \vec{B}$ and $\vec{k} \parallel \vec{E}\times\vec{B}$ (2) $\vec{X} \parallel \vec{E}$ and $\vec{k} \parallel \vec{B}\times\vec{E}$

(3) $\vec{X} \parallel \vec{B}$ and $\vec{k} \parallel \vec{B}\times\vec{E}$ (4) $\vec{X} \parallel \vec{E}$ and $\vec{k} \parallel \vec{E}\times\vec{B}$

WO0118

11. This question has Statement-l and statement-2. Of the four choices given after the Statements, choose the one that best describes the two statements. **[AIEEE - 2012]**

Statement-1 : Davisson - Germer experiment established the wave nature of electrons.

Statement-2: If electrons have wave nature, they can interfere and show diffraction.

(1) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation for Statement-1.

(2) Statement-1 is true, Statement-2 is true and Statement-2 is not the correct explanation of Statement-I

(3) Statement-1 is false, Statement-2 is true.

(4) Statement-1 is true, Statement-2 is false

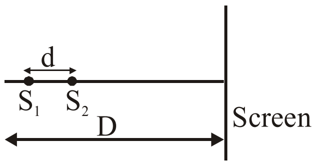
WO0119

12. A beam of unpolarised light of intensity I_0 is passed through a polaroid A and then through another polaroid B which is oriented so that its principal plane makes an angle of 45° relative to that of A. The intensity of the emergent light is :- **[JEE-Mains 2013]**

(1) I_0 (2) $I_0/2$ (3) $I_0/4$ (4) $I_0/8$

WO0120

13. Two coherent point sources S_1 and S_2 are separated by a small distance 'd' as shown. The fringes obtained on the screen will be : **[JEE-Mains 2013]**



(1) points (2) straight lines (3) semicircles (4) concentric circles

WO0121

14. During the propagation of electromagnetic waves in a medium : **[JEE-Mains 2014]**

(1) Electric energy density is equal to the magnetic energy density

(2) Both electric magnetic energy densities are zero

(3) Electric energy density is double of the magnetic energy density

(4) Electric energy density is half of the magnetic energy density.

WO0123

15. Two beams, A and B, of plane polarized light with mutually perpendicular planes of polarization are seen through a polaroid. From the position when the beam A has maximum intensity (and beam B has zero intensity), a rotation of polaroid through 30° makes the two beams appear equally bright. If the initial intensitites of the two beams are I_A and I_B respectively, then $\frac{I_A}{I_B}$ equals : **[JEE-Mains 2014]**

(1) 1 (2) $\frac{1}{3}$ (3) 3 (4) $\frac{3}{2}$

WO0124

16. A red LED emits light at 0.1 watt uniformly around it. The amplitude of the electric field of the light at a distance of 1 m from the diode is:- **[JEE-Mains 2015]**

(1) 5.48 V/m (2) 7.75 V/m (3) 1.73 V/m (4) 2.45 V/m

WO0125

17. Assuming human pupil to have a radius of 0.25cm and a comfortable viewing distance of 25cm, the minimum separation between two objects that human eye can resolve at 500 nm wavelength is :- **[JEE-Mains 2015]**

(1) 100 μm (2) 300 μm (3) 1 μm (4) 30 μm

WO0126

18. On a hot summer night, the refractive index of air is smallest near the ground and increases with height from the ground. When a light beam is directed horizontally, the Huygens' principle leads us to conclude that as it travels, the light beam : **[JEE-Mains 2015]**

(1) bends downwards (2) bends upwards

(3) becomes narrower (4) goes horizontally without any deflection

WO0127

19. The box of a pin hole camera, of length L, has a hole of radius a. It is assumed that when the hole is illuminated by a parallel beam of light of wavelength λ the spread of the spot (obtained on the opposite wall of the camera) is the sum of its geometrical spread and the spread due to diffraction. The spot would then have its minimum size (say b_{min}) when :- **[JEE-Mains 2016]**

(1) $a=\frac{\lambda^2}{L}$ and $b_{min}=\sqrt{4\lambda L}$ (2) $a=\frac{\lambda^2}{L}$ and $b_{min}=\left(\frac{2\lambda^2}{L}\right)$

(3) $a=\sqrt{\lambda L}$ and $b_{min}=\left(\frac{2\lambda^2}{L}\right)$ (4) $a=\sqrt{\lambda L}$ and $b_{min}=\sqrt{4\lambda L}$

WO0128

20. In a Young's double slit experiment, slits are separated by 0.5 mm, and the screen is placed 150 cm away. A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes on the screen. The least distance from the common central maximum to the point where the bright fringes due to both the wavelengths coincide is :- **[JEE-Main 2017]**

(1) 9.75 mm (2) 15.6 mm (3) 1.56 mm (4) 7.8 mm

WO0130

21. The angular width of the central maximum in a single slit diffraction pattern is 60°. The width of the slit is 1 μm. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it, Young's fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringe width is 1 cm, what is slit separation distance? (i.e. distance between the centres of each slit.) **[JEE-Main 2018]**

(1) 50 μm (2) 75 μm (3) 100 μm (4) 25 μm

WO0131

22. An EM wave from air enters a medium. The electric fields are $\vec{E}_1 = E_{01}\hat{x}\cos\left[2\pi\nu\left(\frac{z}{c}-t\right)\right]$ in air and $\vec{E}_2 = E_{02}\hat{x}\cos\left[k\left(2z-ct\right)\right]$ in medium, where the wave number k and frequency v refer to their values in air. The medium is non-magnetic. If ϵ_{r_1} and ϵ_{r_2} refer to relative permittivity of air and medium respectively, which of the following options is correct ? **[JEE-Main 2018]**

(1) $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = 2$ (2) $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = \frac{1}{4}$ (3) $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = \frac{1}{2}$ (4) $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = 4$

WO0132

SELECTED PROBLEMS FROM JEE-MAINS ONLINE PAPERS

23. In a Young's double slit experiment, the slits are placed 0.320 mm apart. Light of wavelength $\lambda = 500$ nm is incident on the slits. The total number of bright fringes that are observed in the angular range $-30° \le \theta \le 30°$ is: **[JEE Main-2019_Jan]**

(1) 320 (2) 641 (3) 321 (4) 640

24. In a double-slit experiment, green light (5303 Å) falls on a double slit having a separation of 19.44 μm and a width of 4.05 μm. The number of bright fringes between the first and the second diffraction minima is :- **[JEE Main-2019_Jan]**

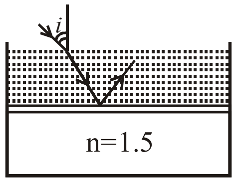
(1) 09 (2) 10 (3) 04 (4) 05

25. A light wave is incident normally on a glass slab of refractive index 1.5. If 4% of light gets reflected and the amplitude of the electric field of the incident light is 30V/m, then the amplitude of the electric field for the wave propogating in the glass medium will be: **[JEE Main-2019_Jan]**

(1) 10 V/m (2) 24 V/m (3) 30 V/m (4) 6 V/m

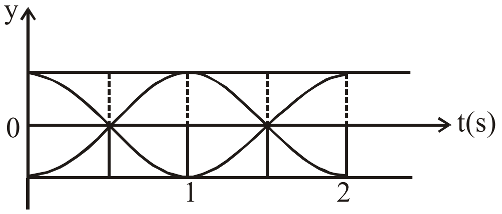
26. Consider a tank made of glass(reiractive index 1.5) with a thick bottom. It is filled with a liquid of refractive index μ,. A student finds that, irrespective of what the incident angle *i* (see figure) is for a beam of light entering the liquid, the light reflected from the liquid glass interface is never completely polarized. For this to happen, the minimum value of μ is :

[JEE Main-2019_Jan]



(1) $\frac{3}{\sqrt{5}}$ (2) $\frac{5}{\sqrt{3}}$ (3) $\sqrt{\frac{5}{3}}$ (4) $\frac{4}{3}$

27. The correct figure that shows, schematically, the wave pattern produced by superposition of two waves of frequencies 9 Hz and 11 Hz is : **[JEE Main-2019_April]**

(1)


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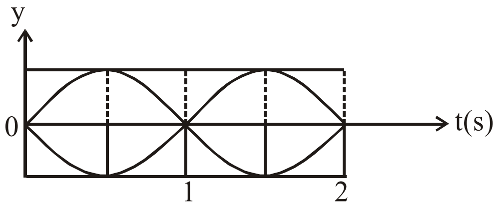

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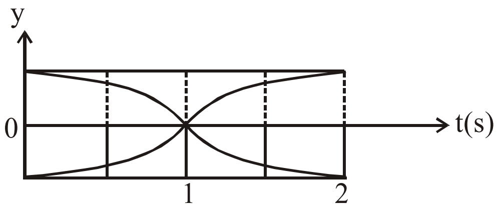

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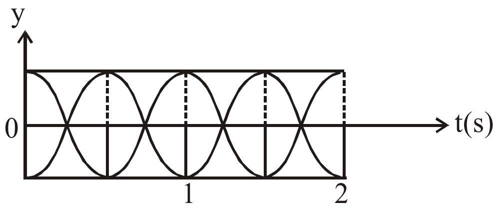


28. Calculate the limit of resolution of a telescope objective having a diameter of 200 cm, if it has to detect light of wavelength 500 nm coming from a star :- **[JEE Main-2019_April]**

(1) 305×10^{-9} radian (2) 152.5×10^{-9} radian (3) 610×10^{-9} radian (4) 457.5×10^{-9} radian

29. Diameter of the objective lens of a telescope is 250 cm. For light of wavelength 600nm. coming from a distant object, the limit of resolution of the telescope is close to :- **[JEE Main-2019_April]**

(1) 1.5×10^{-7} rad (2) 2.0×10^{-7} rad (3) 3.0×10^{-7} rad (4) 4.5×10^{-7} rad

30. The value of numerical aperature of the objective lens of a microscope is 1.25. If light of wavelength 5000 Å is used, the minimum separation between two points, to be seen as distinct, will be :

[JEE Main-2019_April]

(1) 0.24 μm (2) 0.48 μm (3) 0.12 μm (4) 0.38 μm

31. The aperture diameter of a telescope is 5m. The separation between the moon and the earth is 4×10^5 km. With light of wavelength of 5500 Å, the minimum separation between objects on the surface of moon, so that they are just resolved, is close to : **[JEE Main-2020_Jan]**

(1) 20 m (2) 600 m (3) 60 m (4) 200 m

32. A plane electromagnetic wave is propagating along the direction $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$, with its polarization along the direction $\hat{k}$. The correct form of the magnetic field of the wave would be (here B_0 is an appropriate constant) : **[JEE Main-2020_Jan]**

(1) $B_0\frac{\hat{i}-\hat{j}}{\sqrt{2}}\cos\left(\omega t-k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

(2) $B_0\frac{\hat{i}+\hat{j}}{\sqrt{2}}\cos\left(\omega t-k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

(3) $B_0\hat{k}\cos\left(\omega t-k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

(4) $B_0\frac{\hat{j}-\hat{i}}{\sqrt{2}}\cos\left(\omega t+k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

33. In a Young's double slit experiment, 16 fringes are observed in a certain segment of the screen when light of wavelength 700 nm is used. If the wavelength of light is changed to 400 nm, the number of fringes observed in the same segment of the screen would be : **[JEE Main-2020_Sep]**

(1) 28 (2) 24 (3) 18 (4) 30

34. A beam of plane polarised light of large cross sectional area and uniform intensity of 3.3 Wm^{-2} falls normally on a polariser (cross sectional area 3×10^{-4} m^2) which rotates about its axis with an angular speed of 31.4 rad/s. The energy of light passing through the polariser per revolution, is close to : **[JEE Main-2020_Sep]**

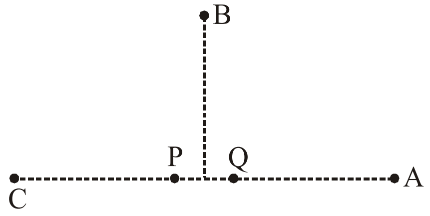
(1) 1.0×10^{-5} J (2) 5.0×10^{-4} J (3) 1.0×10^{-4} J (4) 1.5×10^{-4} J

35. A beam of electrons of energy E scatters from a target having atomic spacing of 1Å. The first maximum intensity occurs at $\theta = 60°$. Then E (in eV) is ______.

(Planck constant $h = 6.64 \times 10^{-34}$ Js, 1eV = 1.6×10^{-19} J, electron mass $m = 9.1 \times 10^{-31}$ kg)

[JEE Main-2020_Sep]

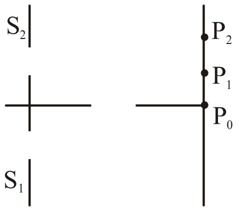
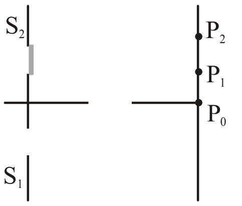
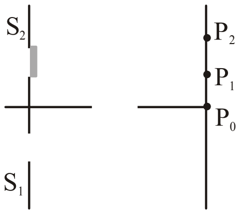
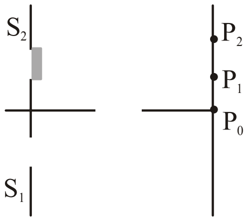
36. In the figure below, P and Q are two equally intense coherent sources emitting radiation of wavelength 20 m. The separation between P and Q is 5 m and the phase of P is ahead of that of Q by 90°. A, B and C are three distinct points of observation, each equidistant from the midpoint of PQ. The intensities of radiation at A, B, C will be in the ratio: **[JEE Main-2020_Sep]**



(1) 0 : 1 : 2 (2) 4 : 1 : 0 (3) 0 : 1 : 4 (4) 2 : 1 : 0

EXERCISE-(J-A)

1. **Column I** shows four situations of standard Young's double slit arrangement with the screen placed far away from the slits S_1 and S_2. In each of these cases $S_1P_0 = S_2P_0$, $S_1P_1 - S_2P_1 = \lambda/4$ and $S_1P_2 - S_2P_2 = \lambda/3$,, where λ is the wavelength of the light used. In the cases B, C and D, a transparent sheet of refractive index μ and thickness t is pasted on slit S_2. The thicknesses of the sheets are different in different cases. The phase difference between the light waves reaching a point P on the screen from the two slits is denoted by $\delta(P)$ and the intensity by $I(P)$. Match each situation given in **Column I** with the statement(s) in **Column II** valid for that situation. **[IIT-JEE-2009]**

Column-I	**Column-II**
(A) 	(p) $\delta(P_0) = 0$
(B) $(\mu - 1)\,t = \lambda/4$ 	(q) $\delta(P_1) = 0$
(C) $(\mu - 1)\,t = \lambda/2$ 	(r) $I(P_1) = 0$
(D) $(\mu - 1)\,t = 3\lambda/4$ 	(s) $I(P_0) > I(P_1)$
	(t) $I(P_2) > I(P_1)$

WO0133

2. Young's double slit experiment is carried out by using green, red and blue light, one color at a time. The fringe widths recorded are β_G, β_R and β_B, respectively. Then **[IIT-JEE-2012]**

(A) $\beta_G > \beta_B > \beta_R$
(B) $\beta_B > \beta_G > \beta_R$
(C) $\beta_R > \beta_B > \beta_G$
(D) $\beta_R > \beta_G > \beta_B$

WO0134

3. In the Young's double slit experiment using a monochromatic light of wavelength λ, the path difference (in terms of an integer n) corresponding to any point having half the peak intensity is :- **[JEE Advanced 2013]**

(A) $(2n+1)\frac{\lambda}{2}$
(B) $(2n+1)\frac{\lambda}{4}$
(C) $(2n+1)\frac{\lambda}{8}$
(D) $(2n+1)\frac{\lambda}{16}$

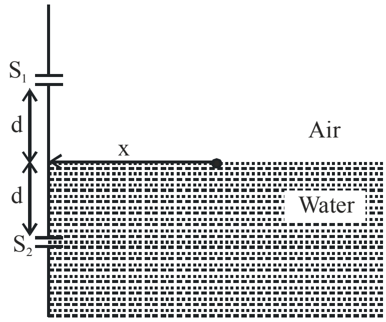
WO0135

4. A light source, which emits two wavelengths $\lambda_1 = 400$ nm and $\lambda_2 = 600$ nm, is used in a Young's double slit experiment. If recorded fringe widths for λ_1 and λ_2 are β_1 and β_2 and the number of fringes for them within a distance y on one side of the central maximum are m_1 and m_2, respectively, then :- **[JEE Advanced 2014]**

(A) $\beta_2 > \beta_1$
(B) $m_1 > m_2$
(C) From the central maximum, 3rd maximum of λ_2 overlaps with 5th minimum of λ_1
(D) The angular separation of fringes of λ_1 is greater than λ_2

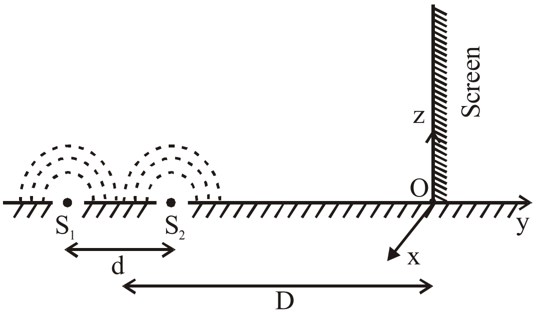
WO0136

5. A Young's double slit interference arrangement with slits S_1 and S_2 is immersed in water (refractive index = 4/3) as shown in the figure. The positions of maxima on the surface of water are given by $x^2 = p^2m^2\lambda^2 - d^2$, where λ is the wavelength of light in air (refractive index = 1), 2d is the separation between the slits and m is an integer. The value of p is. **[JEE Advanced 2015]**



WO0137

6. While conducting the Young's double slit experiment, a student replaced the two slits with a large opaque plate in the x-y plane containing two small holes that act as two coherent point sources (S_1, S_2) emitting light of wavelength 600 nm. The student mistakenly placed the screen parallel to the x-z plane (for z > 0) at a distance D = 3m from the mid-point of S_1S_2, as shown schematically in the figure. The distance between the sources d = 0.6003 mm. The origin O is at the intersection of the screen and the line joining S_1S_2. Which of the following is (are) true of the intensity pattern on the screen ? **[JEE Advanced 2016]**



(A) Hyperbolic bright and dark bands with foci symmetrically placed about O in the x-direction

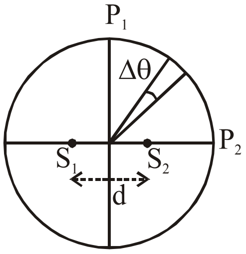
(B) Semi circular bright and dark bands centered at point O

(C) The region very close to the point O will be dark

(D) Straight bright and dark bands parallel to the x-axis

WO0138

7. Two coherent monochromatic point sources S_1 and S_2 of wavelength $\lambda = 600$ nm are placed symmetrically on either side of the center of the circle as shown. The sources are separated by a distance d = 1.8mm. This arrangement produces interference fringes visible as alternate bright and dark spots on the circumference of the circle. The angular separation between two consecutive bright spots is $\Delta\theta$. Which of the following options is/are correct ? **[JEE Advanced 2017]**



(A) A dark spot will be formed at the point P_2

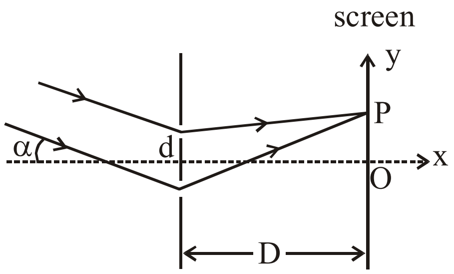
(B) The angular separation between two consecutive bright spots decreases as we move from P_1 to P_2 along the first quadrant

(C) At P_2 the order of the fringe will be maximum

(D) The total number of fringes produced between P_1 and P_2 in the first quadrant is close to 3000

WO0139

8. In a Young's double slit experiment, the slit separation d is 0.3 mm and the screen distance D is 1m. A parallel beam of light of wavelength 600nm is incident on the slits at angle α as shown in figure. On the screen, the point O is equidistant from the slits and distance PO is 11.0 mm. Which of the following statement(s) is/are correct ? **[JEE Advanced 2019]**



(1) For $\alpha = \dfrac{0.36}{\pi}$ degree, there will be destructive interference at point O.

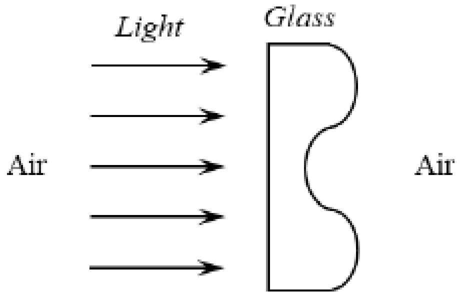
(2) Fringe spacing depends on α

(3) For $\alpha = \dfrac{0.36}{\pi}$ degree, there will be destructive interference at point P

(4) For α = 0, there will be constructive interference at point P.

WO0140

9. A parallel beam of light strikes a piece of transparent glass having cross section as shown in the figure below. Correct shape of the emergent wavefront will be (figures are schematic and not drawn to scale)- **[JEE Advanced 2020]**



(A) 

(B) 

(C) 

(D) 

WAVE OPTICS

CBSE Previous Year's Questions

1. Two narrow slits are illuminated by a single monochromatic source. Name the pattern obtained on the screen. One of the slits is now completely covered. What is the name of the pattern now obtained on the screen? Draw Intensity pattern obtained in the two cases. Also write two differences between the patterns obtained in the above two cases. **[3; CBSE-2004]**

2. Using Huygen's principle, draw a diagram to show propagation of a wave-front originating from a monochromatic point source.

Describe diffraction of light due to a single slit. Explain formation of a pattern of fringes obtained on the screen and plot showing variation of intensity with angle θ in single slit diffraction.

[5; CBSE-2005]

3. What is meant by a linearly polarized light? Which type of waves can be polarised? Briefly explain a method for producing polarised light. " Two Polaroids are placed at 90° to each other and the intensity of transmitted light is zero. What will be the intensity of transmitted light when one more Polaroid is placed between these two bisecting the angle between them? Take intensity of unpolarised light as λ.

[5; CBSE-2005]

4. Name the constituent radiation of electromagnetic spectrum which **[3; CBSE-2005]**

(a) Is used in satellite communication.
(b) Is used for studying crystal structure.
(c) Is similar to the radiations emitted during decay of radioactive nuclei?
(d) Has its wavelength range between 390 nm and 770 nm.
(e) Is absorbed from sunlight by ozone layer.
(f) Produces intense heating effect.

5. Mention the significance of Davisson-Germer experiment. An α particle and a proton are accelerated from rest through the same potential difference V. Find the ratio of de-Broglie wavelengths associated with them. **[3; CBSE-2005]**

6. What are coherent sources of light? State two conditions for two light sources to be coherent.

Derive a mathematical expression for the width of interference fringes obtained in Young's double slit experiment with the help of a suitable diagram. **[5; CBSE-2006]**

7. State Huygens' principle. Using the geometrical construction of secondary wave- lets, explain the refraction of a plane wave front incident at a plane surface. Hence verify Snell's law of refraction. Illustrate with the help of diagrams the action of (i) convex lens and (ii) concave mirror on a plane wave front incident on it. **[5; CBSE-2006]**

8. State the essential condition for diffraction of light to take place. Use Huygen's principle to explain diffraction of light due to a narrow single slit and the formation of a pattern of fringes obtained on the screen. Sketch the pattern of fringes formed due to diffraction at a single slit showing variation of intensity with angle θ. **[3; CBSE-2007]**

9. What are coherent sources of light ? Why are coherent sources required to obtain sustained interference pattern ? State three characteristic features which distinguish the interference pattern due to two coherently illuminated sources as compared to that observed in a diffraction pattern due to a single slit

[3; CBSE-2007]

10. Name the following constituent radiations of electromagnetic spectrum which
(i) produce intense heating effect,
(ii) is absorbed by the ozone layer in the atmosphere.
(iii) is used for studying crystal structure.
Write one more application for each of these radiations. **[2; CBSE-2007]**

11. Draw a schematic diagram of the experimental arrangement used by Davisson and Germer to establish the wave nature of electrons. Explain briefly how the de-Broglie relation was experimentally verified in case of electrons. **[3; CBSE-2007]**

12. How is a wavefront defined? Using Huygen's construction draw a figure showing the propagation of a plane wave refraction at a plane surface separating two media. Hence verify Snell s law of refraction. **[2; CBSE-2008]**

13. (a) What is plane polarised light? Two polaroids are placed at 90° to each other and the transmitted intensity is zero. What happens when one more Polaroid is placed between these two, bisecting the angle between them? How will the intensity of transmitted light vary on further rotating the third Polaroid?
(b) If a light beam shows no intensity variation when transmitted through a polaroid which is rotated, does it mean that the light is unpolarised? Explain briefly, **[5; CBSE-2008]**

14. Name the part of the electromagnetic spectrum of wavelength 10^{-2} m and mention its one application. **[1; CBSE-2008]**

15. The oscillating magnetic field in a plane electromagnetic wave is given by
$$B_y = (8 \times 10^{-6}) \sin [2 \times 10^{11}t + 300\pi x] T$$
(i) Calculate the wavelength of the electromagnetic wave.
(ii) Write down the expression for the oscillating electric field. **[2; CBSE-2008]**

16. Unpolarized light is incident on a plane surface of glass of refractive index μ at angle i. If the reflected light gets totally polarized, write the relation between the angle i and refractive index μ. **[1; CBSE-2009]**

17. Draw a diagram to show refraction of a plane wavefront incident in a convex lens and hence draw the refracted wave front. **[1; CBSE-2009]**

18. In a single slit diffraction experiment, when a tiny circular obstacle is placed in the path of light from distant source, a bright spot is seen at the centre of the shadow of the obstacle. Explain why? **[3; CBSE-2009]**

19. State two points of difference between the interference patterns obtained in Young's double slit experiment and the diffraction pattern due to a single slit. **[3; CBSE-2009]**

20. Name the electromagnetic radiation to which waves of wavelength in the range of 10^{-2} m belong. Give on use of this part of EM spectrum. **[l;CBSE-2009]**

21. How does a charge q oscillating at certain frequency produce electromagnetic waves? Sketch a schematic diagram depicting electric and magnetic field for an electromagnetic wave propagating along the Z-direction. **[2; CBSE-2009]**

22. In Young's double slit experiment, the two slits 0.15 mm apart are illuminated by monochromatic light of wavelength 450 nm. the screen is 1.0 m away from the slits. **[3; CBSE-2010]**
(a) Find the distance of the second (i) bright fringe, (ii) dark fringe from the central maximum.
(b) How will the fringe pattern change if the screen is moved away from the slits ?

23. How does an unpolarised light get polarised when passed through a polaroid ?

Two polaroids are set in crossed positions. A third polaroid is placed between the two making an angle θ with the pass axis of the first polaroid. Write the expression for the intensity of light transmitted from the second polaroid. In what orientations will the transmitted intensity be (i) minimum and (ii) maximum ? **[3; CBSE-2010]**

24. Name the part of electromagnetic spectrum whose wavelength lies in the rage of 10^{-10} m. Give its one use. **[1; CBSE-2010]**

25. Draw a sketch of a plane electromagnetic wave propagating along the z-direction. Depict clearly the directions of electric and magnetic field varying sinusoidally with z. **[2; CBSE-2010]**

26. State the importance of coherent sources in the phenomenon of interference.

In Young's double slit experiment to produce interference pattern, obtain the conditions for constructive and destructive interference. Hence deduce the expression for the fringe width. How does the fringes width get affected, if the entire experimental apparatus of Young is immersed in water ?

[5; CBSE-2011]

27. (a) State Huygens' principle. Using this principle explain how a diffraction pattern is obtained on a screen due to a narrow slit on which a narrow beam coming from a monochromatic source of light is incident normally.

(b) show that the angular width ofthe first diffraction fringe is half of that ofthe central fringe.

(c) If a monochromatic source of light is replaced by white light, what change would you observe in the diffraction pattern ? **[5; CBSE-2011]**

28. How does the angular separation between fringes in single-slit diffraction experiment change when the distance of separation between the slit screens is doubled? **[1; CBSE-2012]**

29. (a) In Young's double slit experiment, derive the condition for

(i) constructive interference and

(ii) destructive interference at a point on the screen

(b) A beam of light consisting of two wavelengths, 800 nm and 600 nm is used to obtain the interference fringes in a Young's double slit experiment on a screen placed 1.4m away. If the two slits are separated by 0.2 8 mm, calculate the least distance from the central bright maximum where the bright fringes of the two wavelength coincide.

OR

(a) How does an unpolarized light incident on a polaroid get polarized? Describe briefly, with the help of a necessary diagram, the polarization of light by reflection from a transparent medium.

(b) Two polaroids 'A' and 'B' are kept in crossed position. How should a third polaroid 'C' be placed between them so that the intensity of polarized light transmitted by polaroid B reduces to 1/8th of the intensity of unpolarized light incident on A? **(5; CBSE-2012]**

30. What are the directions of electric and magnetic field vectors relative to each other and relative to the direction of propagation of electromagnetic waves? **[1; CBSE-2012]**

31. A parallel beam of light of 500 nm falls on a narrow slit and the resulting diffraction pattern is observed on a screen 1 m away. It is observed that the first minimum is at a distance of 2-5 mm from the centre of the screen. Calculate the width ofthe slit. **[CBSE-2013]**

32. (a) What is linearly polarized light? Describe briefly using a diagram how sunlight is polarised.

(b) Unpolarised light is incident on a polaroid. How would the intensity of transmitted light change when the polaroid is rotated? **[CBSE-2013]**

33. Welders wear special goggles or face masks with glass windows to protect their eyes from electromagnetic radiations. Name the radiations and write the range of their frequency.

[1; CBSE-2013]

34. A capacitor, made of two parallel plates each of plate area A and separation d, is being charged by an external ac source. Show that the displacement current inside the capacitor is the same as the current charging the capacitor. **[3; CBSE-2013]**

35. Name the type of waves which are used for line of sight (LOS) communication. What is the range of their frequencies?

A transmitting antenna at the top of a tower has a height of 20 m and the height of the receiving antenna is 45 m. Calculate the maximum distance between them for satisfactory communication in LOS mode. (Radius of the Earth = $6{\cdot}4 \times 10^6$ m) **[3; CBSE-2013]**

36. To which part of the electromagnetic spectrum does a wave of frequency 3×10^{13} Hz belong ?

[CBSE-2014]

37. Considering the case of a parallel plate capacitor being charged, show how one i s required to generalize Ampere's circuital law to include the term due to displacement current. **[CBSE-2014]**

38. (1) Show, with the help of a diagram, how unpolarised sunlight gets polarised due to scattering,

(2) Two polaroids P_1 and P_2 are placed with their pass axes perpendicular to each other. Unpolarised light of intensity I_0 is incident on P_1. A third polaroid P_3 is kept in between P_1 and P_2 such that its pass axis makes an angle of 45° with that of P_1. Determine the intensity of light transmitted through P_1, P_2 and P_3. **[CBSE-2014]**

39. (a) In Young's double slit experiment, describe briefly how bright and dark fringes are obtained on the screen kept in front of a double slit. Hence obtain the expression for the fringe width. (b) The ratio of the intensities at minima to the maxima in the Young's double slit experiment is 9:25. Find the ratio of the widths of the two slits. **[CBSE-2014]**

OR

(a) Describe briefly how a diffraction pattern is obtained on a screen due to a single narrow slit illuminated by a monochromatic source of light. Hence obtain the conditions for the angular width of secondary maxima and secondary minima.

(b) Two wavelengths of sodium light of 590nm and 596 nm are used in turn to study the diffraction taking place at a single slit of aperture 2×10^6 m. The distance between the slit and the screen is 1.5m. Calculate the separation between the positions of first maxima of the diffraction pattern obtained in the two cases.

40. An electron microscope uses electrons accelerated by a voltage of 50 kV. Determine the de-Broglie wavelength associated with the electrons. Taking other factors, such as numerical aperture etc. to be same, how does the resolving power of an electron microscope compare with that of an optical microscope which uses yellow light? **[CBSE-2014]**

41. Define a wavefront. Using Huygens' principle, draw the shape of a refracted wavefront, when a plane wave is incident on a convex lens. **[CBSE-2015]**

OR

(a) When a wave is propagating from a rarer to a denser medium, which characteristic of the wave does not change and why ?

(b) What is the ratio of the velocity of the wave in the two media of refractive indices μ_1 and μ_2 ?

42. In Young's double slit experiment, the two slits are separated by a distance of 1.5 mm and the screen is placed 1 m away from the plane of the slits. A beam of light consisting of two wavelengths 650nm and 520 nm is used to obtain interference fringes. Find **[CBSE-2015]**

(a) the distance of the third bright fringe for $\lambda = 520$ nm on the screen from the central maximum.

(b) the least distance from the central maximum where the bright fringes due to both the wavelengths coincide.

43. How are electromagnetic waves produced ? What is the source of the energy carried by a propagating electromagnetic wave ? **[CBSE-2015]**

Identify the electromagnetic radiations used

(i) in remote switches of household electronic devices; and

(ii) as diagnostic tool in medicine.

44. Why can't we see clearly through fog ? Name the phenomenon responsible for it. **[1 ; CBSE-2016]**

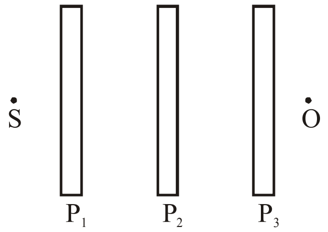
45. (i) Derive Snell's law on the basis of Huygen's wave theory when light is travelling from a denser to a rarer medium. **[3 ; CBSE-2016]**

(ii) Draw the sketches to differentiate between plane wavefront and spherical wavefront.

46. How are electromagnetic waves produced ? What is the source of energy of these waves ? Write mathematical expressions for electric and magnetic fields of an electromagnetic wave propagating along the z-axis. Write any two important properties of electromagnetic waves. **[3 ; CBSE-2016]**

47. (a) Why does unpolarised light from a source show a variation in intensity when viewed through a polaroid which is rotated ? Show with the help of a diagram, how unpolarised light from sun gets linearly polarised by scattering. **[5 ; CBSE-2016]**

(b) Three identical polaroid sheets P_1, P_2 and P_3 are oriented so that the pass axis of P_2 and P_3 are inclined at angles of 60° and 90° respectively with the pass axis of P_1. A monochromatic source S of unpolarized light of intensity I_0 is kept in front of the polaroid sheet P_1 as shown in the figure. Determine the intensities of light as observed by the observer at O, when polaroid P_3 is rotated with respect to P_2 at angles $\theta = 30°$ and 60°.



OR

(a) Derive an expression for path difference in Young's double slit experiment and obtain the conditions for constructive and destructive interference at a point on the screen.

(b) The intensity at the central maxima in Young's double slit experiment is I_0. Find out the intensity at a point where the path difference is $\frac{\lambda}{6}, \frac{\lambda}{4}$ and $\frac{\lambda}{3}$.

48. Do electromagnetic waves carry energy and momentum ? **[CBSE-2017]**

49. Draw the intensity pattern for single slit diffraction and double slit interference. Hence, state two differences between interference and diffraction patterns. **[CBSE-2017]**

OR

Unpolarised light is passed through a polaroid P_1. When this polarised beam passes through another polaroid P_2 and if the pass axis of P_2 makes angle θ with the pass axis of P_1, then write the expression for the polarised beam passing through P_2. Draw a plot showing the variation of intensity when θ varies from 0 to 2π.

50. Identify the electromagnetic waves whose wavelengths vary as **[CBSE-2017]**

(a) 10^{-12} m $< \lambda < 10^{-8}$ m

(b) 10^{-3} m $< \lambda < 10^{-1}$ m

Write one use for each.

51. (a) Define wavefront. Use Huygens' principle to verify the laws of refraction.

(b) How is linearly polarised light obtained by the process of scattering of light? Find the Brewster angle for air – glass interface, when the refractive index of glass = 1.5. **[CBSE-2017]**

52. Name the electromagnetic radiations used for (a) water purification, and (b) eye surgery. **[CBSE-2018]**

53. (a) Why are infra-red waves often called heat waves ? Explain. **[CBSE-2018]**

(b) What do you understand by the statement, "Electromagnetic waves transport momentum" ?

54. (a) If one of two identical slits producing interference in Young's experiment is covered with glass, so that the light intensity passing through it is reduced to 50%, find the ratio of the maximum and minimum intensity of the fringe in the interference pattern. **[CBSE-2018]**

(b) What kind of fringes do you expect to observe if white light is used instead of monochromatic light ?

55. (a) Define a wavefront. Using Huygen's principle, verify the laws of reflection at a plane surface.

(b) In a single slit diffraction experiment, the width of the slit is made double the original width. How does this affect the size and intensity of the central diffraction band ? Explain.

(c) When a tiny circular obstacle is placed in the path of light from a distant source, a bright spot is seen at the centre of the obstacle. Explain why. **[CBSE-2018]**

56. Show using a proper diagram how unpolarised light can be linearly polarised by reflection from a transparent glass surface. **[CBSE-2018]**

ANSWER KEY

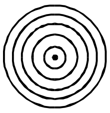
EXERCISE (S-1)

1. Ans. 0.225 mm **2. Ans.** 5000Å **3. Ans.** 1.99×10^{-2} mm

4. Ans. 3.5 mm **5. Ans.** 0.2 mm **6. Ans.** 35.35 cm approximate, 5

7. Ans. 8 μm **8. Ans.** 0, 1.5 mm **9. Ans.** $\lambda = 5850$ Å

10. Ans. 0.15 mm **11. Ans.** 48, 21, $\frac{32}{3}$, $\frac{9}{2}$, 0 m.m.;



12. Ans. (i) circular, (ii) $\frac{1}{16}$, (C) 3000Å **13. Ans.** 81 : 1

EXERCISE (S-2)

1. Ans. $I_0 = I\sec^2\left[\frac{\pi(\mu-1)t}{\lambda}\right]$ **2. Ans.** 7 μm, 1.6, $\frac{400}{7}$ μm (decrease)

3. Ans. (i) y = 2 cm, (ii) m = 1.0016

4. Ans. (i) y = – 13/3 mm, (ii) intensity at O = $0.75I_{max}$ (iii) 650 nm, 433.33 nm

5. Ans. 9.3 μm **6. Ans.** (i) 1 mm (ii) increase **7. Ans.** $\frac{\lambda}{4\cos\alpha}$

8. Ans. 1.25 m **9. Ans.** 7 **10. Ans.** 0, 125 nm **11. Ans.** 5

12. Ans. 50

EXERCISE (O-1)

1. Ans. (C) 2. Ans. (C) 3. Ans. (B) 4. Ans. (B) 5. Ans. (C) 6. Ans. (C)

7. Ans. (B) 8. Ans. (D) 9. Ans. (B) 10. Ans. (B) 11. Ans. (D) 12. Ans. (B)

13. Ans. (C) 14. Ans. (C) 15. Ans. (D) 16. Ans. (A) 17. Ans. (D) 18. Ans. (D)

19. Ans. (A) 20. Ans. (D) 21. Ans. (A) 22. Ans. (C) 23. Ans. (D) 24. Ans. (B)

25. Ans. (A) 26. Ans. (A) 27. Ans. (D) 28. Ans. (A) 29. Ans. (C) 30. Ans. (C)

31. Ans. (C) 32. Ans. (B) 33. Ans. (B) 34. Ans. (B,D) 35. Ans. (A,C,D)

36. Ans. (A,B,D) 37. Ans. (B,C) 38. Ans. (A,C) 39. Ans. (A,C,D)

40. Ans. (A,C) 41. Ans. (C,D) 42. Ans. (B,C,D) 43. Ans. (B,D)

44. Ans. (B) 45. Ans. (C) 46. Ans. (A,C,D) 47. Ans. (A,B,D)

48. Ans. (B,D) 49. Ans. (D) 50. Ans. (A) 51. Ans. (C)

52. Ans. (a) $I_{rms} = V_{rms}\,\omega C = 6.9$ A (b) Yes (c) $B_0 = \frac{\mu_0}{2\pi}\frac{r}{R^2}i_0$, $B_0 = 1.63 \times 10^{-5}$ T

53. Ans. 153 N/C

54. Ans. (a) $\lambda = (c/v) = 1.5 \times 10^{-2}$ m

(b) $B_0 = (E_0/c) = 1.6 \times 10^{-7}$ T

(c) Energy density in E field: $u_E = (1/2)\varepsilon_0 E^2$

Energy density in B field: $uB = (1/2\ \mu)\ B^2$

Using $E = cB$, and $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$, $u_E = u_B$

55. Ans.(a) $-\hat{j}$ (b) 3.5 m (b) 86 MHz (c) 10.3 nT (e) $\{(10.3\text{ nT})\cos[(1.8\text{ rad/m})\,y + (5.4 \times 10^8\text{ rad/s})t]\}\ \hat{k}$

56. Ans. $\left(n - \frac{1}{48}\right)\lambda = x_1 - x_2$ **57. Ans.** 391 eV **58. Ans. (D)** **59. Ans. (D)** **60. Ans. (B)**

61. Ans. (D) **62. Ans. (A)** **63. Ans. (A)** **64. Ans. (D)** **65. Ans. (D)** **66. Ans. (D)**

67. Ans. (B) **68. Ans. (B)** **69. Ans. (D)** **70. Ans. (D)** **71. Ans. (D)** **72. Ans. (B)**

73. Ans. (C) **74. Ans. (A)**

EXERCISE (O-2)

1. Ans. (C) **2. Ans. (C)** **3. Ans. (A)** **4. Ans. (A)** **5. Ans. (B)** **6. Ans. (C)**

7. Ans. (A) **8. Ans. (A) (PRST); (B) (QRST); (C) (R); (D) (PRST)**

9.Ans. (A)-R,S ; (B)-P,R ; (C)-R,S **10. Ans. (A) (R) ; (B) (R) ; (C) (S) ; (D) (P)**

EXERCISE-(J-M)

1. Ans. (1) **2. Ans. (1)** **3. Ans. (2)** **4. Ans. (3)** **5. Ans. (2)**
6. Ans. (3) **7. Ans. (2)** **8. Ans. (1)** **9. Ans. (1)** **10. Ans. (4)**
11. Ans. (1) **12. Ans. (3)** **13. Ans. (4)** **14. Ans. (1)** **15. Ans. (2)**
16. Ans. (4) **17. Ans. (4)** **18. Ans. (2)** **19. Ans. (4)** **20. Ans. (4)**
21. Ans. (4) **22. Ans. (2)**

SELECTED PROBLEMS FROM JEE-MAINS ONLINE PAPERS

23. Ans. (2) **24. Ans. (4) or (3)** **25. Ans. (2)** **26. Ans. (1)** **27. Ans. (4)**
28. Ans. (1) **29. Ans. (3)** **30. Ans. (1)** **31. Ans. (3)** **32. Ans. (1)**
33. Ans. (1) **34. Ans. (3)** **35. Ans. (50.00)** **36. Ans. (4)**

EXERCISE-(J-A)

1. Ans. (A) p, s ; (B) q ; (C) t ; (D) r,s,t **2. Ans. (D)** **3. Ans. (B)** **4. Ans. (A,B,C)**

5. Ans. 3 **6. Ans. (B, C)** **7. Ans. (C), (D)** **8. Ans. (3)** **9. Ans. (A)**

Important Notes

Chapter 02 Contents

WAVES ON STRING

Important Notes

WAVES ON STRING

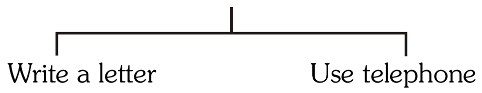
KEY CONCEPTS

INTRODUCTION OF WAVES

What is wave motion ?

- When a particle moves through space, it carries KE with itself. Wherever the particle goes, the energy goes with it. (One way of transport energy from one place to another place)
- There is another way (wave motion) to transport energy from one part of space to other without any bulk motion of material together with it. Sound is transmitted in air in this manner.

Ex. You (Kota) want to communicate your friend (Delhi)



1st option involves the concept of particle & the second choice involves the concept of wave.

Ex. When you say "Namaste" to your friend no material particle is ejected from your lips to fall on your friends ear. Basically you create some disturbance in the part of the air close to your lips. Energy is transferred to these air particles either by pushing them ahead or pulling them back. The density of the air in this part temporarily increases or decreases. These disturbed particles exert force on the next layer of air, transferring the disturbance to that layer. In this way, the disturbance proceeds in air and finally the air near the ear of the listener gets disturbed.

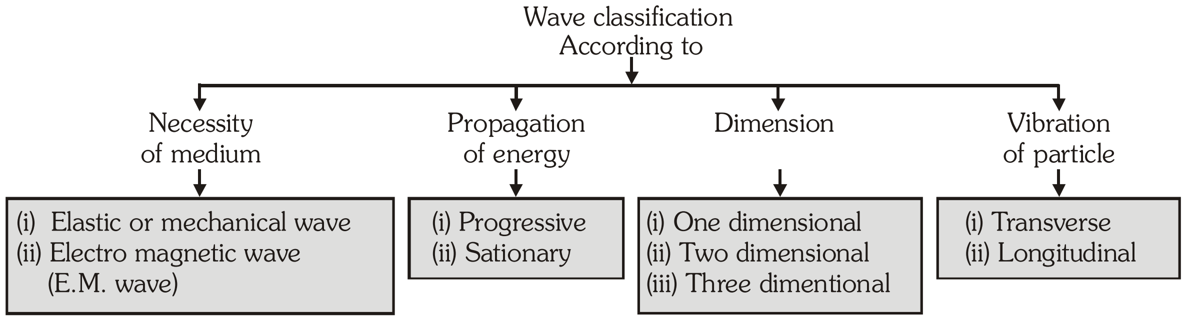
Note :- In the above example air itself does not move.

> A **wave** is a disturbance that propagates in space, transports energy and momentum from one point to another without the transport of matter.

Few examples of waves :

The ripples on a pond (water waves), the sound we hear, visible light, radio and TV signals etc.

CLASSIFICATION OF WAVES



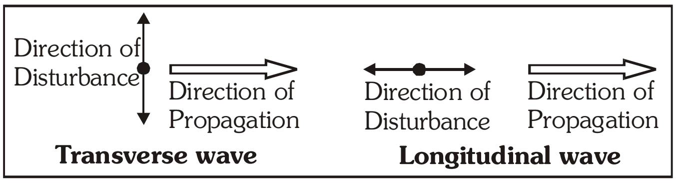
1. ***Based on medium necessity*** :- A wave may or may not require a medium for its propagation. The waves which do not require medium for their propagation are called non-mechanical, e.g. light, heat (infrared), radio waves etc. On the other hand the waves which require medium for their propagation are called mechanical waves. In the propagation of mechanical waves elasticity and density of the medium play an important role therefore mechanical waves are also known as **elastic waves**.

   Example : Sound waves in water, seismic waves in earth's crust.

2. **Based on energy propagation :-** Waves can be divided into two parts on the basis of energy propagation (i) Progressive wave (ii) Stationary waves. The progressive wave propagates with fixed velocity in a medium. In stationary waves particles of the medium vibrate with different amplitude but energy does not propagate.

3. ***Based on direction of propagation*** :- Waves can be one, two or three dimensional according to the number of dimensions in which they propagate energy. Waves moving along strings are one-dimensional. Surface waves or ripples on water are two dimensional , while sound or light waves from a point source are three dimensional.

4. ***Based on the motion of particles of medium*** :



Waves are of two types on the basis of motion of particles of the medium.

(i) Longitudinal waves

(ii) Transverse waves

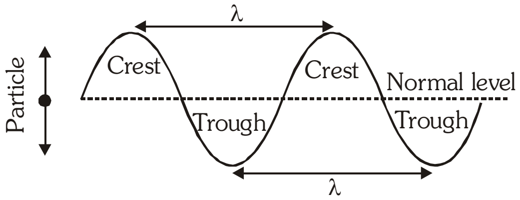
In the transverse wave the direction associated with the disturbance (i.e. motion of particles of the medium) is at right angle to the direction of propagation of wave while in the longitudinal wave the direction of disturbance is along the direction of propagation.

TRANSVERSE WAVE MOTION

Mechanical transverse waves produce in such type of medium which have shearing property, so they are known as shear wave or S-wave

Note :- Shearing is the property of a body by which it changes its shape on application of force.

⇒ Mechanical transverse waves are generated only in solids & surface of liquid.

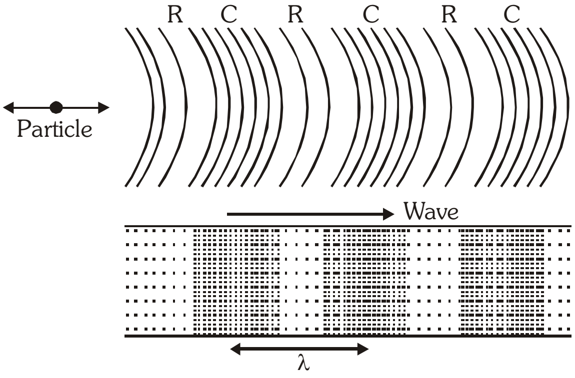


In this individual particles of the medium execute SHM about their mean position in direction $\perp^r$ to the direction of propagation of wave motion.

A **crest** is a portion of the medium, which is raised temporarily above the normal position of rest of particles of the medium , when a transverse wave passes.

A **trough** is a portion of the medium, which is depressed temporarily below the normal position of rest of particles of the medium , when a transverse wave passes.

LONGITUDINAL WAVE MOTION



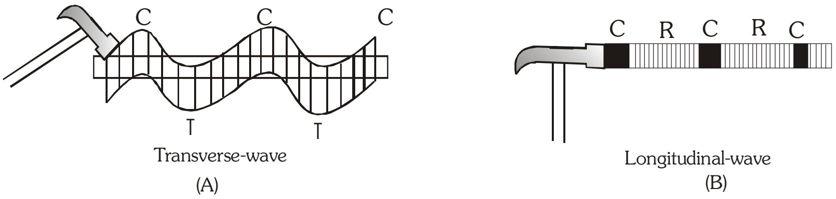
In this type of waves, oscillatory motion of the medium particles produces regions of compression (high pressure) and rarefaction (low pressure) which propagated in space with time (see figure).

Note : The regions of high particle density are called compressions and regions of low particle density are called rarefactions.

- The propagation of sound waves in air is visualized as the propagation of pressure or density fluctuations. The pressure fluctuations are of the order of 1 Pa, whereas atmospheric pressure is 10^5 Pa.

Mechanical Waves in Different Media

- A mechanical wave will be transverse or longitudinal depends on the nature of medium and mode of excitation.
- In strings mechanical waves are always transverse when string is under a tension. In gases and liquids mechanical waves are always longitudinal e.g. sound waves in air or water. This is because fluids cannot sustain shear.
- In solids, mechanical waves (may be sound) can be either transverse or longitudinal depending on the mode of excitation. The speed of the two waves in the same solid are different. (Longitudinal waves travels faster than transverse waves). e.g., if we struck a rod at an angle as shown in fig. (A) the waves in the rod will be transverse while if the rod is struck at the side as shown in fig. (B) or is rubbed with a cloth the waves in the rod will be longitudinal. In case of vibrating tuning fork waves in the prongs are transverse while in the stem are longitudinal.

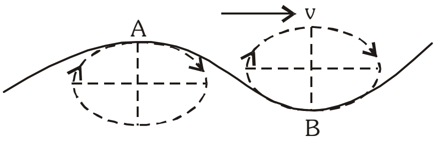


Transverse-wave (A)

Longitudinal-wave (B)

Further more in case of seismic waves produced by Earthquakes both S (shear) and P (pressure) waves are produced simultaneously which travel through the rock in the crust at different speeds [$v_S \cong 5$ km/s while $v_P \cong 9$ km/s] S–waves are transverse while P–waves longitudinal.

Some waves in nature are neither transverse nor longitudinal but a combination of the two. These waves are called 'ripple' and waves on the surface of a liquid are of this type. In these waves particles of the medium vibrate up and down and back and forth simultaneously describing ellipses in a vertical plane [Fig.]



Ripple

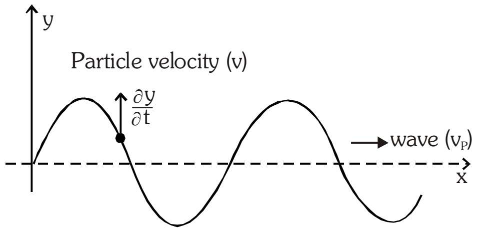
CHARACTERISTICS OF WAVE MOTION

Some of the important characteristics of wave motion are as follows :

- In a wave motion, the disturbance travels through the medium due to repeated periodic oscillations of the particles of the medium about their mean positions.
- The energy is transferred from place to another without any actual transfer of the particles of the medium.
- Each particle receives disturbance a little later than its preceding particle i.e., there is a regular phase difference between one particle and the next.
- The velocity with which a wave travels is different from the velocity of the particles with which they vibrate about their mean positions.
- The wave velocity remains constant in a given medium while the particle velocity changes continuously during its vibration about the mean position. It is maximum at the mean position and zero at the extreme position.
- For the propagation of a mechanical wave, the medium must possess the properties of inertia, elasticity and minimum friction amongst its particles.

SOME IMPORTANT TERMS CONNECTED WITH WAVE MOTION

- **Wavelength (λ) [length of one wave]**
  Distance travelled by the wave during the time, any one particle of the medium completes one vibration about its mean position. We may also define wavelength as the distance between any two nearest particles of the medium, vibrating in the same phase.
- **Frequency (n)** :Number of vibrations (Number of complete wavelengths) complete by a particle in one second.
- **Time period (T)** : Time taken by wave to travel a distance equal to one wavelength.
- **Amplitude (A)** : Maximum displacement of vibrating particle from its equilibrium position.
- **Angular frequency (ω) :** It is defined as $\omega = \frac{2\pi}{T} = 2\pi n$
- **Phase :** Phase is a quantity which contains all information related to any vibrating particle in a wave. For equation $y = A \sin(\omega t - kx)$; $(\omega t - kx)$ = phase.
- **Angular wave number (k) :** It is defined as $k = \frac{2\pi}{\lambda}$
- **Wave number ($\vec{v}$) :** It is defined as $\vec{v} = \frac{1}{\lambda} = \frac{k}{2\pi}$ = number of waves in a unit length of the wave pattern.
- **Particle velocity, wave velocity and particle's acceleration :** In plane progressive harmonic wave particles of the medium oscillate simple harmonically about their mean position. Therefore, all the formulae what we have read in SHM apply to the particles here also. For example, maximum particle velocity is ± $A\omega$ at mean position and it is zero at extreme positions etc. Similarly maximum particle acceleration is $\pm\omega^2A$ at extreme positions and zero at mean position. However the wave velocity is different from the particle velocity. This depends on certain characteristics of the medium. Unlike the particle velocity which oscillates simple harmonically (between $+ A\omega$ and $- A\omega$) the wave velocity is constant for given characteristics of the medium.
- **Particle velocity in wave motion :**
  The individual particles which make up the medium do not travel through the medium with the waves. They simply oscillate about their equilibrium positions. The instantaneous velocity of an oscillating particle of the medium, through which a wave is travelling, is known as "Particle velocity".



- **Wave velocity :** The velocity with which the disturbance, or planes of equal phase (wave front), travel through the medium is called wave (or phase) velocity.
- **Relation between particle velocity and wave velocity :**

Wave equation :- $y = A \sin(\omega t - kx)$, Particle velocity $v = \frac{\partial y}{\partial t} = A\omega \cos(\omega t - kx)$.

Wave velocity $= v_P = \frac{\lambda}{T} = \lambda\frac{\omega}{2\pi} = \frac{\omega}{k}$, $\frac{\partial y}{\partial x} = -Ak\cos(\omega t - kx) = -\frac{A}{\omega}\omega k\cos(\omega k - kx) = -\frac{1}{v_p}\frac{\partial y}{\partial t}$

$$\Rightarrow \frac{\partial y}{\partial x} = -\frac{1}{v_P}\frac{\partial y}{\partial t}$$

Note : $\frac{\partial y}{\partial x}$ represent the slope of the string (wave) at the point x.

Particle velocity at a given position and time is equal to negative of the product of wave velocity with slope of the wave at that point at that instant.

- **Differential equation of harmonic progressive waves :**

$$\frac{\partial^2 y}{\partial t^2} = -A\omega^2 \sin(\omega t - kx) \Rightarrow \frac{\partial^2 y}{\partial x^2} = -Ak^2 \sin(\omega t - kx) \Rightarrow \frac{\partial^2 y}{\partial x^2} = \frac{1}{v_P^2}\frac{\partial^2 y}{\partial t^2}$$

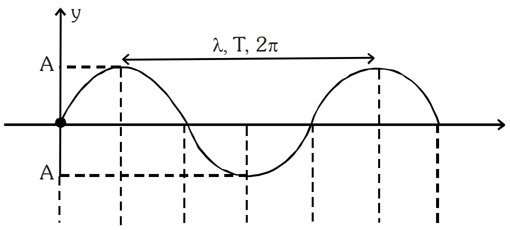
- **Particle velocity (v_p) and acceleration (a_p) in a sinusoidal wave :**

The acceleration of the particle is the second particle is the second partial derivative of y (x, t) with respect to t,

$$\therefore\ a_P = \frac{\partial^2 y(x,t)}{\partial t^2} = \omega^2 A \sin(kx - \omega t) = -\omega^2 y(x,t)$$

i.e., the acceleration of the particle equals $-\omega^2$ times its displacement, which is the result we obtained for SHM. Thus, $a_p = -w2$ (displacement)

- **Relation between Phase difference, Path difference & Time difference**



Phase (ϕ)	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$	3π
Wave length (λ)	0	$\frac{\lambda}{4}$	$\frac{\lambda}{2}$	$\frac{3\lambda}{4}$	λ	$\frac{5\lambda}{4}$	$\frac{3}{2}\lambda$
Time-period (T)	0	$\frac{T}{4}$	$\frac{T}{2}$	$\frac{3T}{4}$	T	$\frac{5T}{4}$	$\frac{3T}{2}$

$$\Rightarrow \frac{\Delta\phi}{2\pi} = \frac{\Delta\lambda}{\lambda} = \frac{\Delta T}{T} \Rightarrow \text{Path difference} = \left(\frac{\lambda}{2\pi}\right) \text{Phase difference}$$

Ex. A progressive wave of frequency 500 Hz is travelling with a velocity of 360 m/s. How far apart are two points 60° out of phase.

Sol. We know that for a wave $v = f\lambda$ So $\lambda = \frac{v}{f} = \frac{360}{500} = 0.72$ m

Phase difference $\Delta\phi = 60° = (\pi/180) \times 60 = (\pi/3)$ rad,

so path difference $\Delta x = \frac{\lambda}{2\pi}(\Delta\phi) = \frac{0.72}{2\pi} \times \frac{\pi}{3} = 0.12$ m

THE GENERAL EQUATION OF WAVE MOTION

Some physical quantity (say y) is made to oscillate at one place and these oscillations of y propagate to other places. The y may be,

(i) displacement of particles from their mean position in case of transverse wave in a rope or longitudinal sound wave in a gas.

(ii) pressure difference (dP) or density difference (dρ) in case of sound wave or

(iii) electric and magnetic fields in case of electromagnetic waves.

The oscillations of y may or may not be simple harmonic in nature. Consider one-dimensional wave travelling along x-axis. In this case y is a function of x and t. i.e. $y = f(x, t)$ But only those function of x & t, represent a wave motion which satisfy the differential equation. $\dfrac{\partial^2 y}{\partial t^2} = v^2 \dfrac{\partial^2 y}{\partial x^2}$ ...(i)

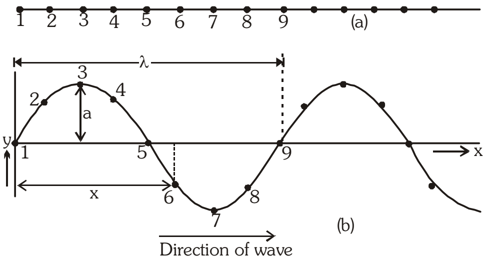
The general solution of this equation is of the form $y(x, t) = f(ax \pm bt)$ ...(ii)
Thus, any function of x and t and which satisfies equation (i) or which can be written as equation (ii) represents a wave. The only condition is that it should be finite everywhere and at all times, Further, if these conditions are satisfied, then speed of wave (v) is given by $v = \dfrac{\text{coefficient of t}}{\text{coefficient of x}} = \dfrac{b}{a}$

Ex. Which of the following functions represent a travelling wave ?

(a) $(x - vt)^2$ (b) $\ell n(x + vt)$ (c) $e^{-(x-vt)^2}$ (d) $\dfrac{1}{x + vt}$

Sol. Although all the four functions are written in the form $f(ax \pm bt)$, only (c) among the four functions is finite everywhere at all times. Hence only (c) represents a travelling wave.

Equation of a Plane Progressive Wave



If, on the propagation of wave in a medium, the particles of the medium perform simple harmonic motion then the wave is called a 'simple harmonic progressive wave'. Suppose, a simple harmonic progressive wave is propagating in a medium along the positive direction of the x-axis (from left to right). In fig. (a) are shown the equilibrium positions of the particles 1, 2, 3 ....

When the wave propagates, these particles oscillate about their equilibrium positions. In Fig. (b) are shown the instantaneous positions of these particles at a particular instant. The curve joining these positions represents the wave. Let the time be counted from the instant when the particle 1 situated at the origin starts oscillating. If y be the displacement of this particle after t seconds, then $y = a \sin \omega t$...(i)
where a is the amplitude of oscillation and $\omega = 2\pi n$, where n is the frequency. As the wave reaches the particles beyond the particle 1, the particles start oscillating. If the speed of the wave be v, then it will reach particle 6, distant x from the particle 1, in x/v sec. Therefore, the particle 6 will start oscillating x/v sec after the particle 1. It means that the displacement of the particle 6 at a time t will be the same as that of the particle 1 at a time x/v sec earlier i.e. at time $t - (x/v)$. The displacement of particle 1 at time $t - (x/v)$ can be the particle 6, distant x from the origin (particle 1), at time t is given by

$$y = a \sin \omega \left(t - \frac{x}{v}\right) \qquad \text{But } \omega = 2\pi n,\ y = a \sin(\omega t - kx) \left(k = \frac{\omega}{v}\right) \quad \text{...(ii)}$$

$$y = a \sin \left[\frac{2\pi}{T} t - \frac{2\pi}{\lambda} x\right] \qquad \text{Also } k = \frac{2\pi}{\lambda} \quad \text{...(iii)} \qquad y = a \sin 2\pi \left[\frac{t}{T} - \frac{x}{\lambda}\right] \text{...(iv)}$$

This is the equation of a simple harmonic wave travelling along +x direction. If the wave is travelling along the –x direction then inside the brackets in the above equations, instead of minus sign there will be plus sign. For example, equation (iv) will be of the following form : y =a sin 2π $\left(\frac{t}{T}+\frac{x}{\lambda}\right)$. If ϕ be the phase difference between the above wave travelling along the +x direction and an other wave, then the equation of that wave will be

$$y = a\sin\left\{2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right)\pm\phi\right\}$$

Ex. The equation of a wave is, $y(x,t) = 0.05\sin\left[\frac{\pi}{2}(10x-40t)-\frac{\pi}{4}\right]\text{m}$

Find: (a) The wavelength, the frequency and the wave velocity

(b) The particle velocity and acceleration at x=0.5 m and t=0.05 s.

Sol. : (a) The equation may be rewritten as, $y(x,t) = 0.05\sin\left(5\pi x - 20\pi t - \frac{\pi}{4}\right)\text{m}$

Comparing this with equation of plane progressive harmonic wave,

$y(x,t) = A\sin(kx-\omega t+\phi)$ we have, wave number $k = \frac{2\pi}{\lambda} = 5\pi\,\text{rad/m}$ $\therefore \lambda = 0.4\text{m}$

The angular frequency is, $\omega = 2\pi f = 20\pi\,\text{rad/s}$ $\therefore$ f = 10Hz

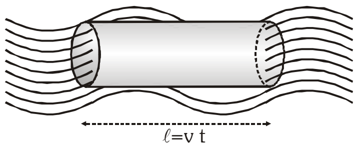
The wave velocity is, $v = f\lambda = \frac{\omega}{k} = 4\text{ms}^{-1}$ in + x direction

(b) The particle velocity and acceleration are, $v_p = \frac{\partial y}{\partial t} = -(20\pi)(0.05)\cos\left(\frac{5\pi}{2}-\pi-\frac{\pi}{4}\right)$ =2.22m/s

$a_p = \frac{\partial^2 y}{\partial t^2} = -(20\pi)^2(0.05)\sin\left(\frac{5\pi}{2}-\pi-\frac{\pi}{4}\right)$ =140 m/s^2

INTENSITY OF WAVE

The amount of energy flowing per unit area and per unit time is called the intensity of wave. It is represented by I. Its units are J/m^{2}s or watt/metre^2. $I = 2\pi^2 f^2 A^2 \rho v$ i.e. $I \propto A^2$ and $I \propto A^2$. If P is the power of an isotropic point source, then intensity at a distance r is given by,



$$I = \frac{P}{4\pi r^2} \text{ or } I \propto \frac{1}{r^2} \text{ (for a point source)}$$

If P is the power of a line source, then intensity at a distance r is given by,

$$I = \frac{P}{2\pi r\ell} \text{ or } I \propto \frac{1}{r} \text{ (for a line source) As, } I \propto A^2$$

Therefore, $A \propto \frac{1}{r}$ (for a point source) and $A \propto \frac{1}{\sqrt{r}}$ (for a line source)

SUPERPOSITION PRINCIPLE

Two or more waves can propagate in the same medium without affecting the motion of one another. If several waves propagate in a medium simultaneously, then the resultant displacement of any particle of the medium at any instant is equal to the vector sum of the displacements produced by individual wave. The phenomenon of intermixing of two or more waves to produce a new wave is called Superposition of waves. Therefore according to superposition principle.

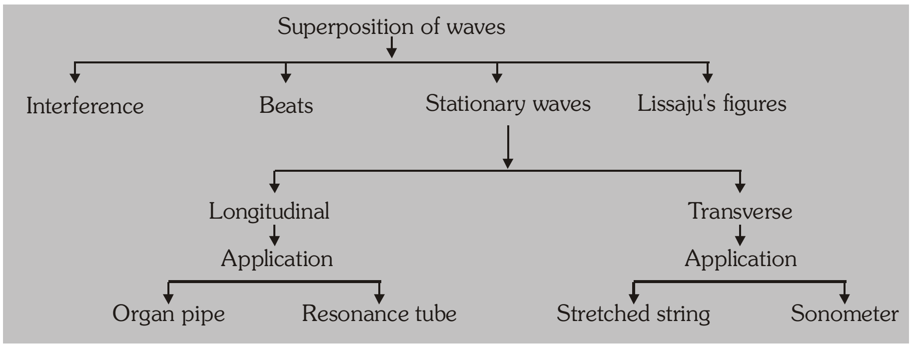
> The resultant displacement of a particle at any point of the medium, at any instant of time is the vector sum of the displacements caused to the particle by the individual waves.

If $\vec{y}_1, \vec{y}_2, \vec{y}_3, ...$ are the displacement of particle at a particular time due to individual waves, then the resultant displacement is given by $\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + ...$

Principle of superposition holds for all types of waves, i.e., mechanical as well as electromagnetic waves. But this principle is not applicable to the waves of very large amplitude.

Due to superposition of waves the following phenomenon can be seen

- **Interference :** Superposition of two waves having equal frequency and nearly equal amplitude.
- **Beats :** Superposition of two waves of nearly equal frequency in same direction.
- **Stationary waves :** Superposition of equal wave from opposite direction.
- **Lissajous' figure :** Superposition of perpendicular waves.



INTERFERENCE OF WAVES :

When two waves of equal frequency and nearly equal amplitude travelling in same direction having same state of polarisation in medium superimpose, then intensity is different at different points. At some points intensity is large, whereas at other points it is nearly zero.

Consider two waves $y_1 = A_1 \sin(\omega t - kx)$ and $y_2 = A_2 \sin(\omega t - kx + \phi)$

By principle of superposition $y = y_1 + y_2 = A \sin(\omega t - kx + \delta)$

where $$A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi \text{ and } \tan\delta = \frac{A_2 \sin\phi}{A_1 + A_2\cos\phi}$$

As intensity $I \propto A^2$ so $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

- **Constructive interference (maximum intensity) :**

  Phase difference $\phi = 2n\pi$ or path difference $= n\lambda$ where $n = 0, 1, 2, 3, ...$

  $$\Rightarrow A_{max} = A_1 + A_2 \quad \text{and} \quad I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

- **Destructive interference (minimum intensity) :**

  Phase difference $\phi = (2n+1)\pi$, or path difference $= (2n-1)\dfrac{\lambda}{2}$ where n = 0, 1, 2, 3, ...

  $\Rightarrow A_{min} = A_1 - A_2$ and $I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$

KEY POINTS

- Maximum and minimum intensities in any interference wave form. $\dfrac{I_{Max}}{I_{Min}} = \left(\dfrac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2 = \left(\dfrac{a_1+a_2}{a_1-a_2}\right)^2$

- Average intensity of interference wave form :– < I >or $I_{av} = \dfrac{I_{max}+I_{min}}{2} = I_1 + I_2$

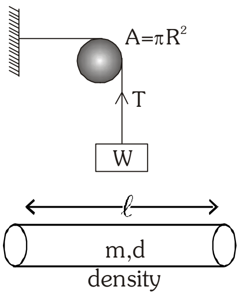
  if $a = a_1 = a_2$ and $I_1 = I_2 = I$ then $I_{max} = 4I$, $I_{min} = 0$ and $I_{AV} = 2I$
- Degree of interference Pattern (f) : Degree of hearing (Sound Wave) or

  Degree of visibility (Light Wave) $f = \dfrac{I_{max}-I_{min}}{I_{max}+I_{min}} \times 100$

  In condition of perfect interference degree of interference pattern is maximum $f_{max} = 1$ or 100%
- Condition of maximum contrast in interference wave form $a_1 = a_2$ and $I_1 = I_2$ then $I_{max} = 4I$ and $I_{min} = 0$

  For perfect destructive interference we have a maximum contrast in interference wave form.

VELOCITY OF TRANSVERSE WAVE



Mass of per unit length $m = \dfrac{\pi r^2 \ell \times d}{\ell}$, $m = \pi r^2 d$, where d = Density of matter

Velocity of transverse wave in any wire $v = \sqrt{\dfrac{T}{m}}$ or $\sqrt{\dfrac{T}{\pi r^2 d}} = \sqrt{\dfrac{T}{Ad}}$ $\because \pi r^2 = A$

- If m is constant then, $v \propto \sqrt{T}$ it is called tension law.

- If tension is constant then $v \propto \sqrt{\dfrac{1}{m}}$ ← it is called law of mass

- If T is constant & take wire of different radius for same material then $v \propto \dfrac{1}{r}$ ← it is called law of radius

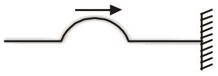
- If T is constant & take wire of same radius for different material. Then $v \propto \sqrt{\dfrac{1}{d}}$ ← law of density

REFLECTION FROM RIGID END

When the pulse reaches the right end which is clamped at the wall, the element at the right end exerts a force on the clamp and the clamp exerts equal and opposite force on the element. The element at the right end is thus acted upon by the force from the clamp. As this end remains fixed, the two forces are opposite to each other. The force from the left part of the string transmits the forward wave pulse and hence, the force exerted by the clamp sends a return pulse on the string whose shape is similar to a return pulse but is inverted. The original pulse tries to pull the element at the fixed end up and the return pulse sent by the clamp tries to pull it down, so the resultant displacement is zero. Thus, the wave is reflected from the fixed end and the reflected wave is inverted with respect to the original wave. The shape of the string at any time, while the pulse is being reflected, can be found by adding an inverted image pulse to the incident pulse.

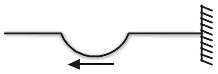
Equation of wave propagating in +ve x-axis

Incident wave $y_1 = a \sin(\omega t - kx)$



Reflected wave $y_2 = a \sin(\omega t + kx + \pi)$

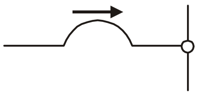
or $y_2 = -a \sin(\omega t + kx)$



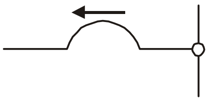
REFLECTION FROM FREE END

The right end of the string is attached to a light frictionless ring which can freely move on a vertical rod. A wave pulse is sent on the string from left. When the wave reaches the right end, the element at this end is acted on by the force from the left to go up. However, there is no corresponding restoring force from the right as the rod does not exert a vertical force on the ring. As a result, the right end is displaced in upward direction more than the height of the pulse i.e., it overshoots the normal maximum displacement. The lack of restoring force from right can be equivalent described in the following way. An extra force acts from right which sends a wave from right to left with its shape identical to the original one. The element at the end is acted upon by both the incident and the reflected wave and the displacements add. Thus, a wave is reflected by the free end without inversion.

Incident wave $y_1 = a \sin(\omega t - kx)$



Reflected wave $y_2 = a \sin(\omega t + kx)$



STATIONARY WAVES

* **Definition :** The wave propagating in such a medium will be reflected at the boundary and produce a wave of the same kind travelling in the opposite direction. The superposition of the two waves will give rise to a stationary wave. Formation of stationary wave is possible only and only in bounded medium.

ANALYTICAL METHOD FOR STATIONARY WAVES

- **From rigid end** : We know equation for progressive wave in positive x-direction $y_1 = a \sin(\omega t - kx)$
  After reflection from rigid end $y_2 = a \sin(\omega t + kx + \pi) = -a \sin(\omega t + kx)$
  By principle of super position. $y = y_1 + y_2 = a \sin(\omega t - kx) - a \sin(\omega t + kx) = -2a \sin kx \cos \omega t$
  This is equation of stationary wave reflected from rigid end

Amplitude $= 2a \sin kx$ **Velocity of particle** $v_{pa} = \frac{dy}{dt} = 2a\,\omega \sin kx \sin \omega t$

Strain $\frac{dy}{dx} = -2ak \cos kx \cos \omega t$ **Elasticity** $E = \frac{\text{stress}}{\text{strain}} = \frac{dp}{\frac{dy}{dx}}$ **Change in pressure** $dp = E\frac{dy}{dx}$

- **Node** $x = 0, \frac{\lambda}{2}, \lambda$ .... $A = 0$, $V_{pa} = 0$, strain → max **Change in pressure** → max
- **Antinode** $x = \frac{\lambda}{4}, \frac{3\lambda}{4}$ .... $A \to$ max, $-V_{pa} \to$ max. strain $= 0$ **Change in pressure** $= 0$

- **From free end** : we know equation for progressive wave in positive x-direction $y_1 = a \sin(\omega t - kx)$
  After reflection from free end $y_2 = a \sin(\omega t + kx)$

By Principle of superposition $y = y_1 + y_2 = a\sin(\omega t - kx) + a\sin(\omega t + kx) = 2a\sin\omega t\cos kx$

Amplitude = $2a\cos kx$, **Velocity of particle** = $v_{Pa} = \frac{dy}{dt} = 2a\,\omega\cos\omega t\cos kx$

Strain $\frac{dy}{dx} = -2ak\sin\omega t\sin kx$ **Change in pressure** $dp = E\frac{dy}{dx}$

- **Antinode :** $x = 0, \frac{\lambda}{2}, \lambda$ .... $A \to$ Max, $V_{pa} = \frac{dy}{dt} \to$ max. Strain = 0, dp = 0
- **Node :** $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$ .... $A = 0$, $V_{pa} = \frac{dy}{dt} = 0$, strain → max, $dp \to$ max

PROPERTIES OF STATIONARY WAVES

The stationary waves are formed due to the superposition of two identical simple harmonic waves travelling in opposite direction with the same speed.

Important characteristics of stationary waves are:–

(i) Stationary waves are produced in the bounded medium and the boundaries of bounded medium may be rigid or free.

(ii) In stationary waves nodes and antinodes are formed alternately. Nodes are the points which are always in rest having maximum strain. Antinodes are the points where the particles vibrate with maximum amplitude having minimum strain.

(iii) All the particles except at the nodes vibrate simple harmonically with the same period.

(iv) The distance between any two successive nodes or antinodes is λ/2.

(v) The amplitude of vibration gradually increases from zero to maximum value from node to antinode.

(vi) All the particles in one particular segment vibrate in the same phase, but the particle of two adjacent segments differ in phase by 180°

(vii) All points of the medium pass through their mean position simultaneously twice in each period.

(viii) Velocity of the particles while crossing mean position varies from maximum at antinodes to zero at nodes.

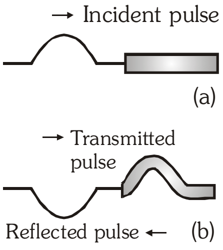
(ix) In a stationary wave the medium is splited into segments and each segment is vibrating up and down as a whole.

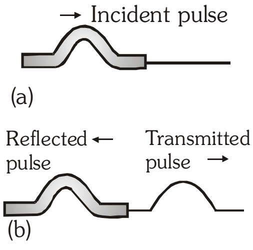
(x) In longitudinal stationary waves, condensation (compression) and refraction do not travel forward as in progressive waves but they appear and disappear alternately at the same place.

(xi) These waves do not transfer energy in the medium. Transmission of energy is not possible in a stationary wave.

TRANSMISSION OF WAVES

We may have a situation in which the boundary is intermediate between these two extreme cases, that is, one in which the boundary is neither rigid nor free. In this case, part of the incident energy is transmitted and part is reflected. For instance, suppose a light string is attached to a heavier string as in (figure). When a pulse travelling on the light reaches the knot, same part of it is reflected and inverted and same part of it is transmitted to the heavier string.





As one would expect, the reflected pulse has a smaller amplitude than the incident pulse, since part of the incident energy is transferred to the pulse in the heavier string. The inversion in the reflected wave is similar to the behaviour of a pulse meeting a rigid boundary, when it is totally reflected. When a pulse travelling on a heavy string strikes the boundary of a lighter string, as in (figure), again part is reflected and part is transmitted. However, in this case the reflected pulse is not inverted. In either case, the relative height of the reflected and transmitted pulses depend on the relative densities of the two string. In the previous section, we found that the speed of a wave on a string increases as the density of the string decreases. That is, a pulse travels more slowly on a heavy string than on a light string, if both are under the same tension. The following general rules apply to reflected waves. When a wave pulse travels from medium A to medium B and $v_A > v_B$ (that is, when B is denser than A), the pulse will be inverted upon reflection. When a wave pulse travels from medium A to medium B and $v_A < v_B$ (A is denser than B), it will not be inverted upon reflection.

KEY POINTS

Phenomenon of reflection and transmission of waves obeys the laws of reflection and refraction.

The frequency of these wave remains constant i.e. does not change. $\omega_i = \omega_r = \omega_t = \omega$

From rarer to denser medium $y_i = a_i \sin(\omega t - k_1x)$ $y_r = -a_i \sin(\omega t + k_1x)$ $y_t = a_t \sin(\omega t - k_2x)$

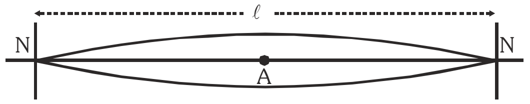
From denser to rarer medium $y_i = a_i \sin(\omega t - k_1x)$ $y_r = a_i \sin(\omega t + k_1x)$ $y_t = a_t \sin(\omega t - k_2x)$

STATIONARY WAVE ARE OF TWO TYPES :

(i) Transverse st. wave (stretched string) **(ii)** Longitudinal st. wave (organ pipes)

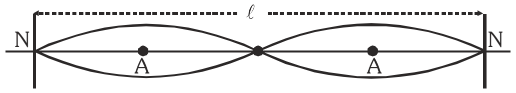
(i) Transverse Stationary wave

First Harmonic



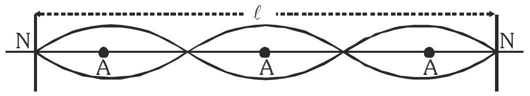
$$\frac{\lambda}{2} = \ell \Rightarrow \lambda = 2\ell$$

Second Harmonic



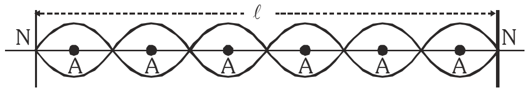
$$\frac{2\lambda}{2} = \ell \Rightarrow \lambda = \ell$$

Third Harmonic



$$\frac{3\lambda}{2} = \ell \Rightarrow \lambda = \frac{2\ell}{3}$$

p th harmonic

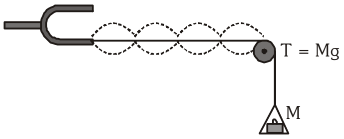


$$\frac{p\lambda}{2} = \ell \Rightarrow \lambda = \frac{2\ell}{p}$$

- **Law of length :** For a given string, under a given tension, the fundamental frequency of vibration is inversely proportional to the length of the string, i.e, $n \propto \frac{1}{\ell}$ (T and m are constant)
- **Law of tension :** The fundamental frequency of vibration of stretched string is directly proportional to the square root of the tension in the string, provided that length and mass per unit length of the string are kept constant. $n \propto \sqrt{T}$ (ℓ and m are constant)

- **Law of mass :** The fundamental frequency of vibration of a stretched string is inversely proportional to the square root of its mass per unit length provided that length of the string and tension in the string are kept constant, i.e., $n \propto \frac{1}{\sqrt{m}}$ (ℓ and T are constant)
- **Melde's experiment :** In Melde's experiment, one end of a flexible piece of thread is tied to the end of a tuning fork. The other end passed over a smooth pulley carries a pan which can be loaded. There are two arrangements to vibrate the tied fork with thread.

Transverse arrangement :

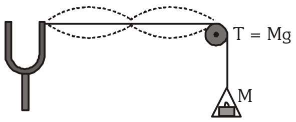


Case 1. In a vibrating string of fixed length, the product of number of loops and square root of tension are constant or $p\sqrt{T}$ = constant.

Case 2. When the tuning fork is set vibrating as shown in fig. then the prong vibrates at right angles to the thread. As a result the thread is set into motion. The frequency of vibration of the thread (string) is equal to the frequency of the tuning fork. If length and tension are properly adjusted then, standing waves are formed in the string. (This happens when frequency of one of the normal modes of the string matched with the frequency of the tuning fork). Then, if p loops are formed in the thread, then the frequency of the tuning fork is given by $n = \frac{p}{2\ell}\sqrt{\frac{T}{m}}$

Case 3. If the tuning fork is turned through a right angle, so that the prong vibrates along the length of the thread, then the string performs only a half oscillation for each complete vibrations of the prong. This is because the thread only makes node at the midpoint when the prong moves towards the pulley i.e. only once in a vibration.

Longitudinal arrangement :

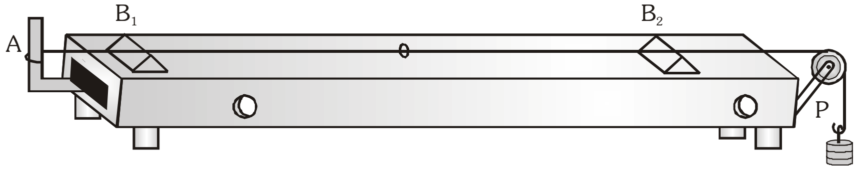


The thread performs sustained oscillations when the natural frequency of the given length of the thread under tension is half that of the fork.

Thus if p loops are formed in the thread, then the frequency of the tuning fork is $n = \frac{2p}{2\ell}\sqrt{\frac{T}{m}}$

SONOMETER :

Sonometer consists of a hollow rectangular box of light wood. One end of the experimental wire is fastened to one end of the box. The wire passes over a frictionless pulley P at the other end of the box. The wire is stretched by a tension T.



The box serves the purpose of increasing the loudness of the sound produced by the vibrating wire. If the length the wire between the two bridges is ℓ, then the frequency of vibration is $n = \frac{1}{2\ell}\sqrt{\frac{T}{m}}$

To test the tension of a tuning fork and string, a small paper rider is placed on the string. When a vibrating tuning fork is placed on the box, and if the length between the bridges is properly adjusted, then when the two frequencies are exactly equal, the string quickly picks up the vibrations of the fork and the rider is thrown off the wire. There are three laws of vibration of a wire.

COMPARISON OF PROGRESSIVE AND STATIONARY WAVES

	Progressive waves	**Stationary waves**
1.	These waves travels in a medium with definite velocity.	These waves do not travel and remain confined between two boundaries in the medium.
2.	These waves transmit energy in the medium.	These waves do not transmit energy in the medium.
3.	The phase of vibration varies continuously from particle to particle.	The phase of all the particles in between two nodes is always same. But particles of two Adjacent nodes differ in phase by 180°
4.	No particle of medium is Permanently at rest.	Particles at nodes are permanently at rest.
5.	All particles of the medium vibrate and amplitude of vibration is same.	The amplitude of vibration changes from particle to particle. The amplitude is zero for all at nodes and maximum at antinodes.
6.	All the particles do not attain the maximum displacement position simultaneously.	All the particles attain the maximum displacement

WORKED OUT EXAMPLES

Ex.1 A wave is propagating along x-axis. The displacement of particles of the medium in z-direction at t = 0 is given by: $z = \exp[-(x+2)^2]$, where 'x' is in meters. At t = 1s, the same wave disturbance is given by: $z = \exp[-(2-x)^2]$. Then, the wave propagation velocity is

(A) 4 m/s in + x direction
(B) 4 m/s in –x direction
(C) 2 m/s in + x direction
(D) 2 m/s in – x direction

Ans. **(D)**

Ex.2 A transverse wave is propagating along +x direction. At t = 2 sec, the particle at x = 4m is at y = 2 mm. With the passage of time its y coordinate increases and reaches to a maximum of 4 mm. The wave equation may be (using ω and k with their usual meanings)

(A) $y = 4\sin(\omega(t+2)+k(x-2)+\frac{\pi}{6})$
(B) $y = 4\sin(\omega(t+2)+k(x)+\frac{\pi}{6})$
(C) $y = 4\sin(\omega(t-2)-k(x-4)+\frac{5\pi}{6})$
(D) $y = 4\sin(\omega(t-2)-k(x-4)+\frac{\pi}{6})$

Ans. **(D)**

Ex.3 $Y(x,t) = 0.05 / [(4x + 2t)^2 + 5]$ represents a moving wave pulse, where x and y are in meters and t is in seconds. Then which statement(s) are **CORRECT**:

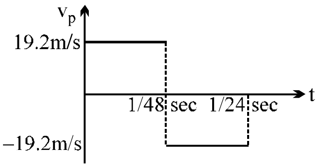
(A) Pulse is moving in – x direction (B) Wave speed is 0.5 m/s

(C) Maximum particle displacement is 1 cm (D) It is a symmetric pulse

Ans. **(A,B,C,D)**

Ex.4 A symmetrical triangular pulse of maximum height 0.4 m and total length 1 m is moving in the positive x-direction on a string on which the wave speed is 24 m/s. At t = 0 the pulse is entirely located between x = 0 and x = 1 m. Draw a graph of the transverse velocity of particle of string versus time at x = +1m.

Ans.



Ex.5 A transverse harmonic disturbance is produced in a string. The maximum transverse velocity is 3 m/s and maximum transverse acceleration is 90 m/s^2. If the wave velocity is 20 m/s then find the waveform. **[IIT-JEE 2005]**

Ans. $y = (10\text{ cm}) \sin\left(30t \pm \frac{3}{2}x + f\right)$

Ex.6 A non-uniform rope of mass M and length L has a variable linear mass density given by $\mu = kx$ where x is the distance from one end of the wire and k is a constant.

(a) Show that $M = kL^2/2$

(b) Show that the time required for a pulse generated at one end of the wire to travel to the other end is given by $t = \sqrt{8ML/9F}$ where F (constant)is the tension in the wire.

Ex.7 One end of a long string of linear mass density 10^{-2} kg m^{-1} is connected to an electrically driven tuning fork of frequency 150 Hz. The other end passes over a pulley and is tied to a pan containing a mass of 90 kg. The pulley end absorbs all the incoming energy so that reflected waves from this end have negligible amplitude. At t = 0, the left end (fork end) of the string is at x = 0 has a transverse displacement of 2.5 cm and is moving along positive y-direction. The amplitude of the wave is 5 cm. Write down the transverse displacement y (in cm) as function of x (in m) and t (in sec) that describes the wave on the string.

Ans. $y = 5\sin\left\{\pi\left(300t - x\right) + \frac{\pi}{6}\right\}$

Ex.8 If the tension in a stretched string fixed at both ends is changed by 20%, the fundamental frequency is found to increase by 15Hz. then the

(A) original frequency is 157 Hz

(B) velocity of propagation of the transverse wave along the string changes by 5%

(C) velocity of propagation of the transverse wave along the string changes by 10%.

(D) fundamental wave length on the string does not change.

Ans. **(A,C,D)**

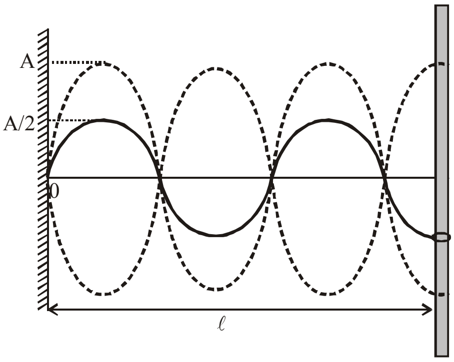
Ex.9 Here given snap shot at $t = \frac{T}{12}$ of a standing wave. Then the equations of the wave will be when particles are moving towards their extreme and when particles are moving towards the mean position respectively $\left(\text{Here, } T = \frac{2\pi}{\omega}\right)$.

(A) $y = A \sin kx \sin \omega t$, $y = A \sin\left(\omega t + \frac{2\pi}{3}\right) \sin kx$

(B) $y = A \sin kx \cos \omega t$, $y = A \sin\left(\omega t + \frac{2\pi}{3}\right) \sin kx$

(C) $y = A \cos kx \cos \omega t$, $y = A \cos\left(\omega t + \frac{2\pi}{3}\right) \sin kx$

(D) $y = A \cos kx \sin \omega t$, $y = A \cos\left(\omega t + \frac{2\pi}{3}\right) \cos kx$



Ans. (A)

Ex.10 A standing wave of time period T is set up in a string clamped between two rigid supports. At t = 0 antinode is at its maximum displacement 2A.

(A) The energy density of a node is equal to energy density of an antinode for the first time at t = T/4.

(B) The energy density of node and antinode becomes equal after T/2 second.

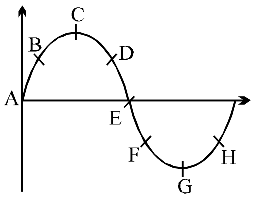
(C) The displacement of the particle at antinode at $t = \frac{T}{8}$ is $\sqrt{2}A$

(D) The displacement of the particle at node is zero

Ans. (C,D)

EXERCISE (S-1)

Wave equation



1. A transverse wave is travelling along a string from left to right. The figure represents the shape of the string (snap-shot) at a given instant. At this instant (a) which points have an upward velocity (b) which points will have downward velocity (c) which points have zero velocity (d) which points have maximum magnitude of velocity.

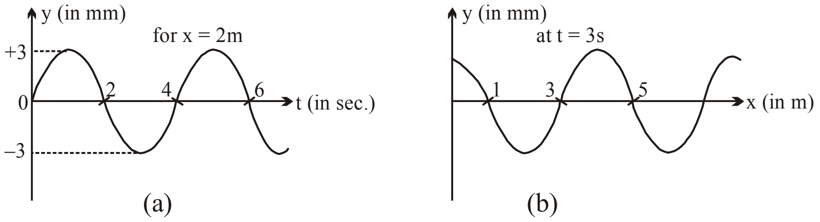
WA0001

2. A sinusoidal wave propagates along a string. In figure (a) and (b) 'y' represents displacement of particle from the mean position. 'x' & 't' have usual meanings. Find :

(a) wavelength, frequency and speed of the wave.

(b) maximum velocity and maximum acceleration of the particles

(c) the magnitude of slope of the string for x = 2 m at t = 4 sec.



(a) (b)

WA0002

3. A light pointer fixed to one prong of a tuning fork touches a vertical plate. The fork is set vibrating and plate is allowed to fall freely. 8 complete oscillations are counted when the plate falls through 10 cm. What is the frequency of the tuning fork ? **WA0003**

4. A long uniform string of mass density 0.1 kg/m is stretched with a force of 40 N. One end of the string (x = 0) is oscillated transversely (sinusoidally) with an amplitude of 0.02 m and a period of 0.1 sec, so that travelling waves in the +x direction are set up.

(a) What is the velocity of the waves?

(b) What is their wavelength?

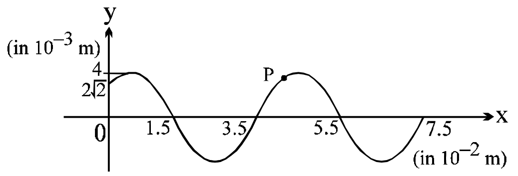
(c) If at the driving end (x = 0) the displacement (y) at t = 0 is 0.01 m with dy/dt negative, what is the equation of the travelling waves? **WA0004**

5. The figure shows a snap photograph of a vibrating string at t = 0. The particle P is observed moving up with velocity 20π cm/s. The angle made by string with x-axis at P is 6°.

(a) Find the direction in which the wave is moving

(b) the equation of the wave

(c) the total energy carried by the wave per cycle of the string, assuming that μ, the mass per unit length of the string = 50 gm/m.



WA0005

Velocity of wave

6. The extension in a string, obeying Hooke's law is x. The speed of wave in the stretched string is v. If the extension in the string is increased to 1.5 x find the new speed of wave.

WA0006

7. A uniform rope of length L and mass m is held at one end and whirled in a horizontal circle with angular velocity ω. Ignore gravity. Find the time required for a transverse wave to travel from one end of the rope to the other.

WA0007

8. A copper wire is held at the two ends by rigid supports. At 30°C, the wire is just taut, with negligible. Find the speed of transverse waves in this wire at 10°C.
Given : Young modulus of copper = 1.3×10^{11} N/m^2.
Coefficient of linear expansion of copper = 1.7×10^{-5} °C^{-1}.
Density of copper = 9×10^3 kg/m^3. **[IIT-1979]**

WA0008

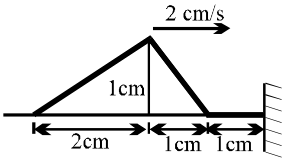
Energy of wave

9. A steel wire has a mass of 5g/m and is under tension 450 N. Find the maximum average power that can be carried by the transverse wave in the wire if the amplitude is not to exceed 20% of the wavelength.

WA0009

Superposition of waves

10. The figure shown a triangular pulse on a rope at t = 0. It is approaching a fixed end at 2 cm/s
(a) Draw the pulse at t = 2 sec.
(b) The particle speed on the leading edge at the instant depicted is _____.



WA0010

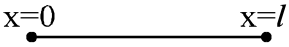
11. A 40 cm long wire having a mass 3.2 gm and area of cross-section 1 mm^2 is stretched between the supports 40.05 cm apart. In its fundamental mode it vibrate with a frequency 1000/64 Hz. Find the young's modulus of the wire.

WA0011

12. A plane wave given by equation $y = 0.04 \sin(0.5\pi x - 100\pi t)$, where x and y are in meter and t in sec is incident normally on a boundary between two media beyond which wave speed becomes doubled. State boundary condition and find the equation of the reflected and transmitted waves. Take x = 0 as the boundary between two media.

WA0012

13. A string between x = 0 and x = l vibrates in fundamental mode. The amplitude A, tension T and mass per unit length μ is given. Find the total energy of the string. **[IIT-JEE 2003(Scr)]**



EXERCISE (S-2)

1. A long string under tension of 100 N has one end at x = 0. A sinusoidal wave is generated at x = 0 whose equation is given by $y = (0.01\text{ cm})\sin\left[\left(\frac{\pi x}{10}\text{ m}\right) - 50\pi t\,(\text{sec})\right]$

(i) Sketch the shape of the string at $t = \frac{1}{50}$ sec.

(ii) Find the average power transmitted by the wave.

(iii) Draw velocity time graph of particle at x = 5 m.

WA0014

2. A string fixed at both ends is vibrating in the lowest mode of vibration for which a point at quarter of its length from one end is a point of maximum displacement. The frequency of vibration in this mode is 100 Hz. What will be the frequency emitted when it vibrates in the next mode such that this point is again a point of maximum displacement?

WA0015

3. A uniform string of length L and total mass M is suspended vertically and a transverse pulse is given at the top end of it. At the same moment a body is released from rest and falls freely from the top of the string. How far from the bottom does the body pass the pulse.

WA0016

4. A sinusoidal wave is moving along the positive x-direction as shown in figure (i) and (ii).

(i) Write the complete expression for the wave y (x, t)

(ii) Find the possible values of x_0 for which figure (ii) refers.

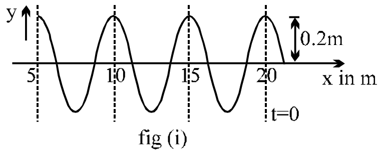


fig (i)

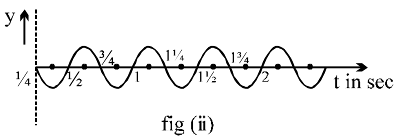


fig (ii)

WA0017

5. A plane progressive wave of frequency 25 Hz, amplitude 2.5×10^{-5} m and initial phase zero propagates along the negative x-direction with a velocity of 300 m/s. At any instant, the phase difference between the oscillations at two points 6m apart along the line of propagation is ...., and the corresponding amplitude difference is ...... m. **[IIT-1997C]**

WA0018

6. A steel wire 8×10^{-4} m in diameter is fixed to a support at one end and is wrapped round a cylindrical tuning peg 5 mm in diameter at the other end. The length of the wire between the peg and the support is 0.06 m. The wire is initially kept taut but without any tension. What will be the fundamental frequency of vibration of the wire if it is tightened by giving the peg a quarter of a turn?
Density of steel = 7800 kg/m^3, Y of steel = 20×10^{10} N/m^2.

WA0020

7. A steel of wire of length 25 cm is fixed at its ends to rigid walls. Young's modulus of steel = 200 GPa, coefficient of linear thermal expansion = 10^{-5} / °C. Initially, the wire is just taut at 20°C temperature. The density of steel = 8.0 g/cc. A tuning fork of frequency 200 Hz is touched to the wire, to execute oscillations. Simultaneously, the temperature is slowly lowered. At what temperature will resonance occur corresponding to the third overtone?

WA0021

8. A non-uniform rope of mass M and length L has a variable linear mass density given by $\mu = kx$ where x is the distance from one end of the wire and k is a constant.

(a) Show that $M = kL^2/2$

(b) Show that the time required for a pulse generated at one end of the wire to travel to the other end is given by $t = \sqrt{8ML/9F}$ where F (constant)is the tension in the wire.

WA0022

EXERCISE (O-1)

Wave equation

1. The function of x and t that does not represent a progressive wave is :-

(A) $y = 2\sin(4t - 3x)$ (B) $y = e^{(4+(4t-3x)]}$ (C) $y = [4t - 3x]^{-1}$ (D) $y = [4t-3x]$

WA0023

2. At x = 0 particle oscillate by law $y = \frac{3}{2t^2+1}$. If wave is propagating along –ve x axis with velocity 2m/s. Find equation of wave

(A) $y = \frac{3}{2\left(t-\frac{x}{2}\right)^2+1}$ (B) $y = \frac{3}{2\left(t+\frac{x}{2}\right)^2+1}$ (C) $y = \frac{3}{2\left(t-\frac{z}{2}\right)^2+1}$ (D) $y = \frac{3}{2\left(t+\frac{z}{2}\right)^2+1}$

WA0024

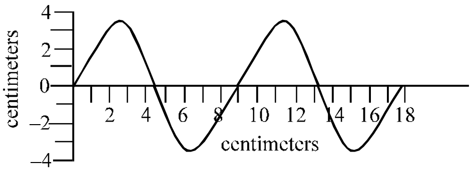
3. The shape of a wave propagating in the positive x or negative x- direction is given y = $\frac{1}{\sqrt{1+x^2}}$ at t =0 and $y = \frac{1}{\sqrt{2-2x+x^2}}$ at t = 1s where x and y are in meters. The shape the wave disturbance does not change during propagation. Find the velocity of the wave.

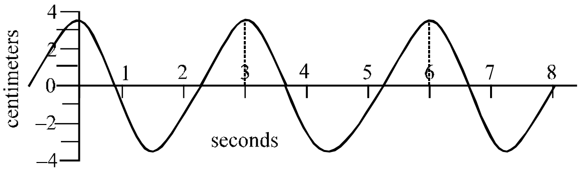
(A) 1 m/s in positive x direction (B) 1 m/s in negative x direction

(C) $\frac{1}{2}$ m/s in positive x direction (D) $\frac{1}{2}$ m/s in negative x direction

WA0025

4. A transverse wave is travelling along a horizontal string. The first picture shows the shape of the string at an instant of time. This picture is superimposed on a coordinate system to help you make any necessary measurements. The second picture is a graph of the vertical displacement of *one* point along the string as a function of time. How far does this wave travel along the string in one second?

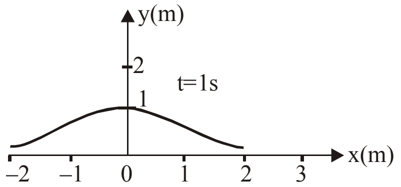


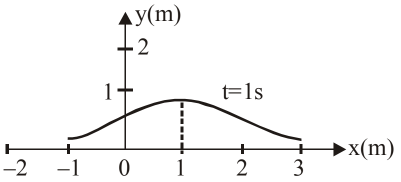


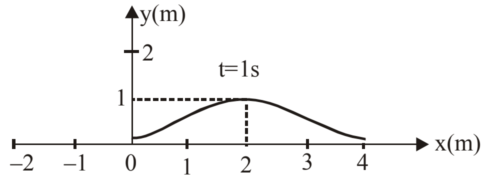
(A) 0.3 cm (B) 3.0 cm (C) 9.0 cm (D) 27 cm

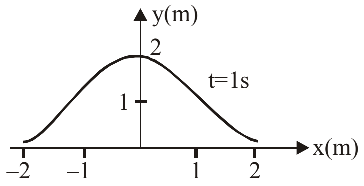
WA0026

5. A wave pulse is given by the equation $y = f(x, t) = A \exp(-B(x-vt)^2)$. Given $A = 1.0\text{m}$, $B = 1.0\ \text{m}^{-2}$ and $v = +2.0$ m/s, which of the following graph shows the correct wave profile at the instant t = 1 s?

(A)


(B)


(C)


(D)


WA0027

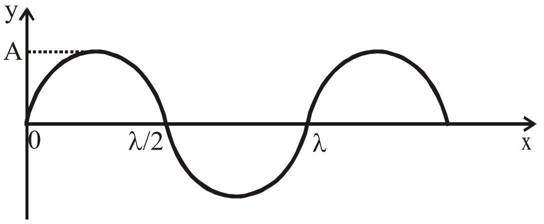
6. The displacement from the position of equilibrium of a point 4 cm from a source of sinusoidal oscillations is half the amplitude at the moment t = T/6 (T is the time period). Assume that the source was at mean position at t = 0. The wavelength of the running wave is

(A) 0.96 m (B) 0.48 m (C) 0.24 m (D) 0.12 m

WA0028

7. Here given snap shot of a progressive wave at t = 0 with time period = T. Then the equation of the wave if wave is going in +ve x-direction and if wave is going in –ve x-direction will be respectively.

$$\left(\text{Here, } T = \frac{2\pi}{\omega}\right)$$



(A) $y = A \sin(kx + \omega t)$, $y = A \sin(kx - \omega t)$ (B) $y = A \cos(kx + \omega t)$, $y = A \cos(kx - \omega t)$

(C) $y = A \sin(\omega t - kx)$, $y = A \sin(\omega t + kx)$ (D) $y = A \sin(kx - \omega t)$, $y = A \sin(kx + \omega t)$

WA0029

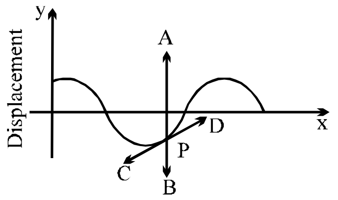
8. A sinusoidal wave travelling in the positive direction of x on a stretched string has amplitude 2.0 cm, wavelength 1 m and wave velocity 5.0 m/s. At x = 0 and t = 0, it is given that displacement y = 0 and $\frac{\partial y}{\partial t} < 0$. Express the wave function correctly in the form y = f(x, t) :-

(A) $y = (0.02\ \text{m}) \sin 2\pi (x-5t)$ (B) $y = (0.02\ \text{cm}) \cos 2\pi(x-5t)$

(C) $y = (0.02\ \text{m}) \sin 2\pi \left(x - 5t + \frac{1}{4}\right)$ (D) $y = (0.02\ \text{cm}) \cos 2\pi \left(x - 5t + \frac{1}{4}\right)$

WA0030

9. The figure below shows a snap photograph of a simple harmonic progressive wave, progressing in the negative X-axis, at a given instant. The direction of the velocity of the particle at the stage P on the figure is best represented by the arrow.

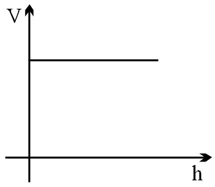


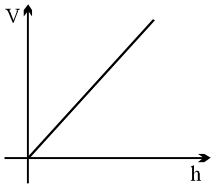
(A) $\vec{PA}$ (B) $\vec{PB}$ (C) $\vec{PC}$ (D) $\vec{PD}$

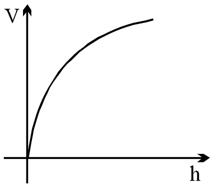
WA0031

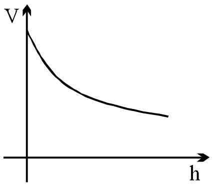
Velocity of wave

10. A uniform rope having some mass hanges vertically from a rigid support. A transverse wave pulse is produced at the lower end. The speed (v) of the wave pulse varies with height (h) from the lower end as:

(A)


(B)


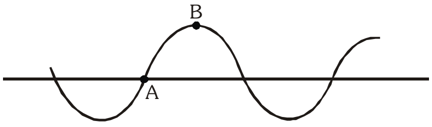
(C)


(D)


WA0032

Energy of wave

11. A progressive wave is travelling in a string as shown. Then which of the following statement about KE and potential energy of the elements A and B is true?



(A) For point A : kinetic energy is maximum and potential energy is min.

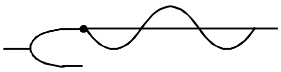
(B) For point B : kinetic energy is minimum and potential energy is min.

(C) For point A : kinetic energy is minimum and potential energy is max.

(D) For point B : kinetic energy is minimum and potential energy is max.

WA0033

12. The prong of a electrically operated tuning fork is connected to a long string of $\mu = 1$ kg/m and tension 25N. The maximum velocity of the prong is 1 cm/s, then the average power needed to drive the prong is:

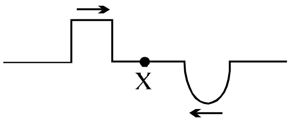


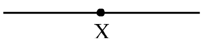
(A) 5×10^{-4} W (B) 2.5×10^{-4} W (C) 1×10^{-4} W (D) 10^{-3} W

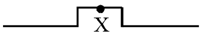
WA0034

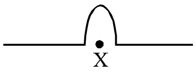
Superposition of waves

13. The diagram below shows two pulses traveling towards each other in a uniform medium with same speed. Pulses in the figure are at the same distance from X and has same height & width. Which diagram best represents the medium when the pulses meet at point X ?



(A) 

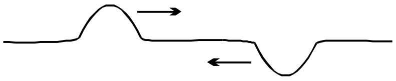
(B) 

(C) 

(D) 

WA0035

14. Two symmetric, identical pulses of opposite amplitude travel along a stretched string in opposite directions as shown in the figure below. Which one of the following statements most fully describes the situation ?



(A) There is an instant when the string is straight

(B) When the two pulses interfere completely, the energy of the wave is zero

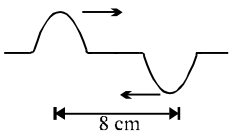
(C) There is a point on the string that does not move up or down

(D) Both A and C

(E) Both A and B

WA0036

15. Two pulses in a stretched string whose centres are initially 8 cm apart are moving towards each other as shown in figure. The speed of each pulse is 2 cm/s. After 2 seconds, the total energy of the pulses will be :-

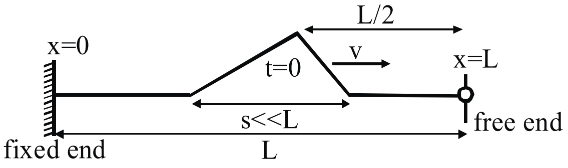


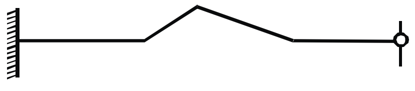
(A) zero (B) purely kinetic

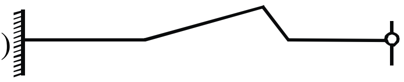
(C) purely potential (D) partly kinetic and partly potential

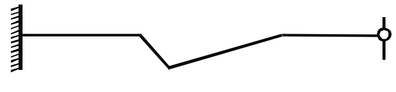
WA0037

16. A small pulse travelling with speed v in a string is shown at t=0, moving towards free end. Which of these is not **CORRECTLY** matched.



(i) $t = \frac{L}{v}$ (P) 

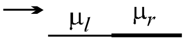
(ii) $t = \frac{2L}{v}$ (Q) 

(iii) $t = \frac{3L}{v}$ (R) 

(A) (i) (B) (ii) (C) (iii) (D) None of these

WA0038

17. A string consists of two parts attached at x = 0. The right part of the string (x > 0) has mass μ_r per unit length and the left part of the string (x < 0) has mass μ_l per unit length. The string tension is T. If a wave of unit amplitude travels along the left part of the string, as shown in the figure, what is the amplitude of the wave that is transmitted to the right part of the string ?



(A) 1 (B) $\frac{2}{1+\sqrt{\mu_l/\mu_r}}$ (C) $\frac{2\sqrt{\mu_l/\mu_r}}{1+\sqrt{\mu_l/\mu_r}}$ (D) $\frac{\sqrt{\mu_l/\mu_r}-1}{\sqrt{\mu_l/\mu_r}+1}$

WA0039

18. A wave travels on a light string .The equation of the wave is Y= Asin(kx−wt+30°) .It is reflected from a heavy string tied to an end of the light string at x = 0.If 64% of the incident energy is reflected the equation of the reflected wave is

(A)$Y = 0.8A\sin(kx-wt+30^0+180^0)$ (B)$Y = 0.8A\sin(kx+wt+30^0+180^0)$

(C)$Y = 0.8A\sin(kx+wt-30^0)$ (D)$Y = 0.8A\sin(kx+wt+30^0)$

WA0040

19. A wave is represented by the equation y = 10 sin 2π(100t–0.02x) + 10 sin 2π(100t+0.02x). The maximum amplitude and loop length are respectively

(A) 20 units and 30 units (B) 20 units and 25 units

(C) 30 units and 20 units (D) 25 units and 20 units

WA0041

20. A wave represented by the equation y = A cos (kx – ωt) is superimposed with another wave to form a statioary wave such that the point x = 0 is a node. The equation of the other wave is:

(A) –A sin (kx + ωt) (B) – A cos (kx + ωt) (C) A sin (kx + ωt) (D) A cos (kx + ωt)

WA0042

21. Five waveforms moving with equal speeds on the x-axis

$y_1 = 8\sin(\omega t + kx)$; $y_2 = 6\sin(\omega t + \frac{\pi}{2} + kx)$; $y_3 = 4\sin(\omega t + \pi + kx)$; $y_4 = 2\sin(\omega t + \frac{3\pi}{2} + kx)$;

$y_5 = 4\sqrt{2}\sin(\omega t - kx + \frac{\pi}{4})$ are superimposed on each other. The resulting wave is :

(A) $8\sqrt{2}\cos kx \sin(\omega t + \frac{\pi}{4})$ (B) $8\sqrt{2}\sin(\omega t - kx + \frac{\pi}{4})$

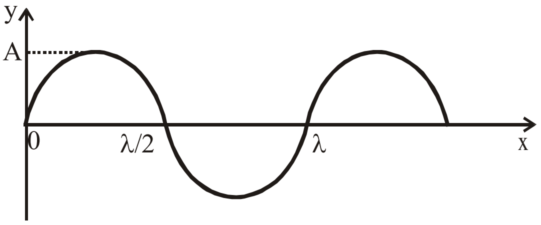
(C) $8\sqrt{2}\sin kx \cos(\omega t + \frac{\pi}{4})$ (D) $8\sin(\omega t + kx)$

WA0043

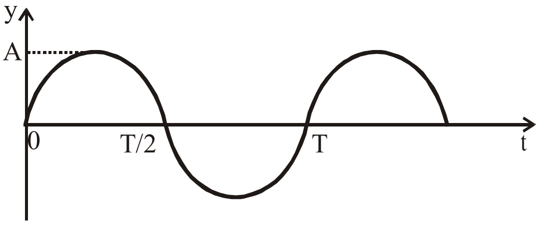
22. Here given figure (i) shows snap shot at t = T/4 and figure (ii) shows motion of particle at x = λ/4.

Then the possible equations of the wave will be $\left(\text{Here, } T = \dfrac{2\pi}{\omega}\right)$:-

(i)



(ii)



(A) $y = A \sin\left(\omega t + kx - \dfrac{\pi}{2}\right)$

(B) $y = A \sin\left(\omega t - kx + \dfrac{\pi}{2}\right)$

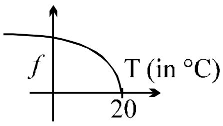
(C) Both $y = A \cos\left(\omega t + kx - \dfrac{\pi}{2}\right)$ and $y = A \cos\left(\omega t - kx + \dfrac{\pi}{2}\right)$

(D) Both $y = A \sin\left(\omega t + kx - \dfrac{\pi}{2}\right)$ and $y = A \sin\left(\omega t - kx + \dfrac{\pi}{2}\right)$

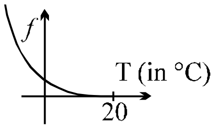
WA0044

23. A metal wire is clamped between two vertical walls. At 20 °C the unstrained length of the wire is exactly equal to the separation between walls. If the temperature of the wire is decreased the graph between fundamental frequency (*f*) and temperature (T) of the wire is

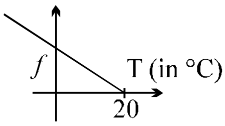
(A)



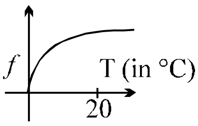
(B)



(C)



(D)



WA0045

24. What is the fractional change in the tension necessary in a sonometer of fixed length to produce a note one octave lower (half of original frequency) than before

(A) 1/4 (B) 1/2 (C) 2/3 (D) 3/4 (E) 2/1

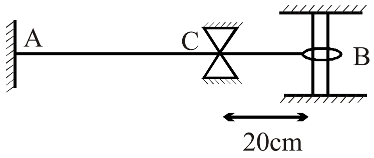
WA0046

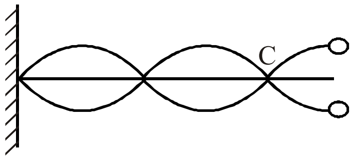
25. A string clamped at both ends is vibrating. At the moment the string looks flat, the instantaneous transverse velocity of points along the string, excluding its end-points, must be

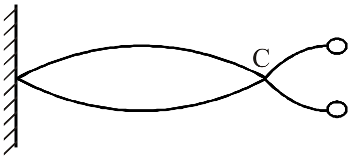
(A) zero everywhere
(B) dependent on the location along the string
(C) non zero everywhere
(D) non-zero and in the same direction everywhere

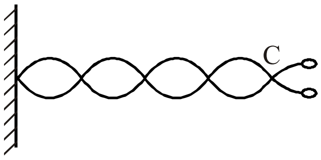
WA0047

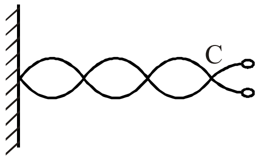
26. A 1m long wire having tension T is fixed at A and free at B. The point C, 20 cm from B is constrained to be stationary. What is shape of string for fundamental mode?



(A)


(B)


(C)


(D)


WA0048

27. The ends of a stretched wire of length L are fixed at x = 0 and x = L. In one experiment, the displacement of the wire is $y_1 = A \sin(\pi x/L) \sin \omega t$ and energy is E_1 and in another experiment its displacement is $y_2 = A\sin(2\pi x/L) \sin 2\omega t$ and energy is E_2. Then **[JEE 2001 (Scr)]**

(A) $E_2 = E_1$ (B) $E_2 = 2E_1$ (C) $E_2 = 4E_1$ (D) $E_2 = 16E_1$

WA0049

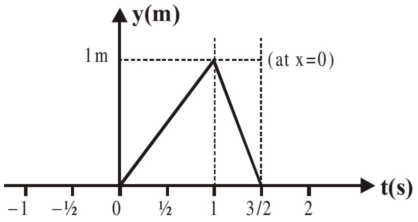
EXERCISE (O-2)

1. Consider a hypothetical wave pulse (at time t = 0) given by the following (y, x in meter)

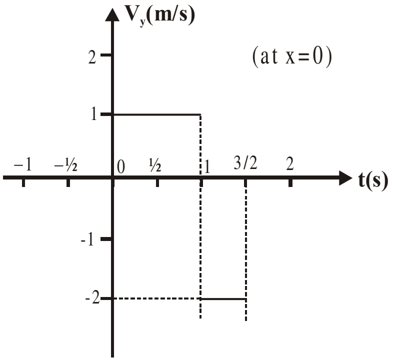
$y = 0, x < 0$
$y = x/2, 2 > x \geq 0$
$y = 3 - x, 3 \geq x \geq 2$
$y = 0, x > 3$

The pulse travels leftwards (negative x direction) at speed v = 2 m/s. Which of the following plots are correct? [V_y is the velocity of particle]

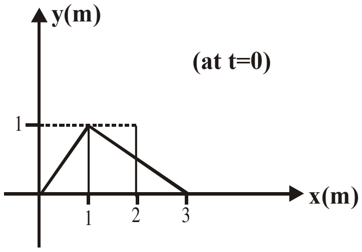
(A)



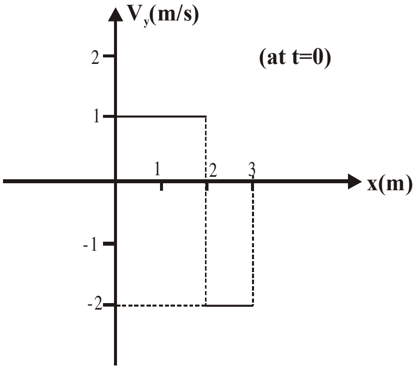
(B)



(C)

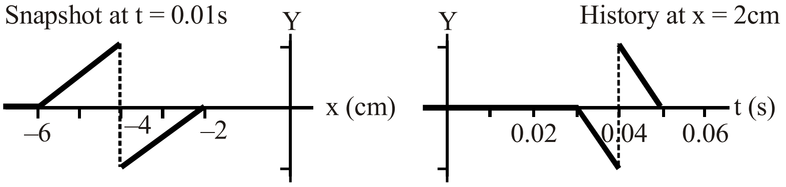


(D)



WA0050

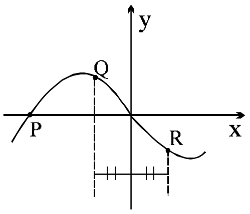
2. Figure shows a snapshot graph and a history graph for a wave pulse on a stretched string. They describe the same wave from two perspectives.



(A) the wave is travelling in positive x-direction (B) the wave is travelling in negative x-direction
(C) the speed of the wave is 2 m/s. (D) the peak is located at x = –6 cm at t = 0.

WA0051

3. At a certain moment, the photograph of a string on which a harmonic wave is travelling to the right is shown. Then, which of the following is true regarding the velocities of the points P, Q and R on the string.



(A) v_P is upwards (B) $v_Q = -v_R$ (C) $|v_P| > |v_Q| = |v_R|$ (D) $v_Q = v_R$

WA0052

4. An string has resonant frequencies given by 1001 Hz and 2639 Hz.

(A) If the string is fixed at one end only, 910 Hz can be a resonance frequency.

(B) If the string is fixed at one end only, 1911 Hz can be a resonance frequency.

(C) If the string is fixed at both the ends, 364 Hz can be one of the resonant frequency.

(D) 1001 Hz is definitely not the fundamental frequency of the string.

WA0053

5. In a travelling one dimensional mechanical sinusoidal, wave

(A) potential energy and kinetic energy of an element become maximum simultaneously.

(B) all particles oscillate with the same frequency and the same amplitude

(C) all particles may come to rest simultaneously

(D) we can find two particles, in a length equal to half of a wavelength, which have the same non zero acceleration simultaneously.

WA0054

Paragraph for Question Nos. 6 to 8

A wave represented by equation $y = 2(\text{mm}) \sin [4\pi(\text{sec}^{-1})t - 2\pi(\text{m}^{-1})x]$ is superimposed with another wave $y = 2\ (\text{mm}) \sin [4\pi(\text{sec}^{-1})t + 2\pi(\text{m}^{-1})x + \pi/3]$ on a tight string.

6. Phase difference between two particles which are located at $x_1 = 1/7$ and $x_2 = 5/12$ is :-

(A) 0 (B) $\frac{5\pi}{6}$ (C) π (D) $\frac{5\pi}{3}$

WA0055

7. Which of the following is not a location of antinode?

(A) $\frac{2}{3}$ (B) $\frac{11}{12}$ (C) $\frac{5}{12}$ (D) $\frac{17}{12}$

WA0055

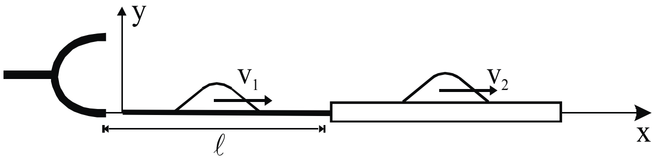
8. The location having maximum potential energy is

(A) 1/7 (B) 1/6 (C) 5/12 (D) 23/12

WA0055

Paragraph for Question Nos. 9 to 11

A harmonic oscillator at x = 0, oscillates with a frequency $\frac{\omega}{2\pi}$ and amplitude a. It is generating waves at end of a thin string in which velocity of wave is v_1 and which is connected to another heavier string in which velocity of wave is v_2 as shown, length of first string is ℓ.



9. If harmonic oscillator oscillates by an equation y = a sinωt. The equation of incident wave in first string is

(A) $y = a\sin\omega\left(t - \frac{x}{v_1}\right)$ (B) $y = a\sin\omega\left(t + \frac{x}{v_1}\right)$

(C) $y = a\sin\left[\omega\left(t - \frac{x}{v_1}\right) + \pi\right]$ (D) $y = a\sin\left[\omega\left(t + \frac{x}{v_1}\right) + \pi\right]$

WA0056

10. Equation of transmitted wave in second string if its amplitude is a_t is

(A) $y = a_t\sin\omega\left(t - \frac{x}{v_2}\right)$ (B) $y = a_t\sin\omega\left(t - \frac{\ell}{v_1}\right)$

(C) $y = a_t\sin\omega\left(t - \frac{\ell}{v_1} - \frac{x-\ell}{v_2}\right)$ (D) $y = a_t\sin\omega\left(t - \frac{x}{v_2}\right)$

WA0056

11. Equation of reflected wave, if it is reflecting at the joint and amplitude of reflected wave is a_R

(A) $y = a_R\sin\omega\left(t - \frac{x}{v_2}\right)$ (B) $y = a_R\sin\left[\omega\left(t - \frac{\ell}{v_1} - \frac{\ell-x}{v_1}\right) + \pi\right]$

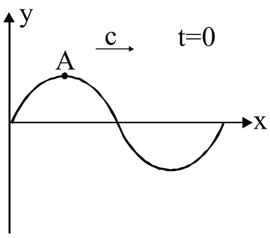
(C) $y = a_R\sin\left[\omega\left(t + \frac{x}{v_1}\right) + \pi\right]$ (D) $y = a_R\sin\left[\omega\left(t + \frac{2\ell+x}{v_1}\right) + \pi\right]$

WA0056

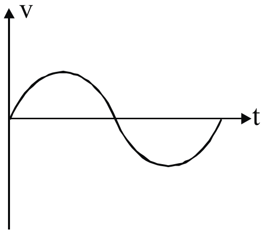
12. In column-I transverse waves travelling on a string at t = 0 is shown. Wave velocity is indicated by '*c*'. Column-II describes variation of different parameters for particle *A* or for all the particles.

Column-I **Column-II**

(A)

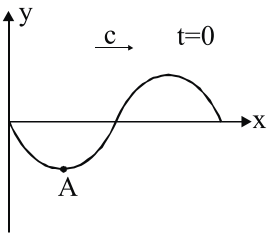


(P)

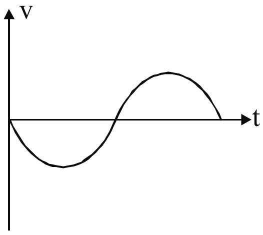


For particle A

(B)

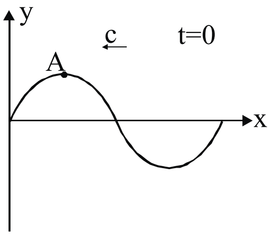


(Q)

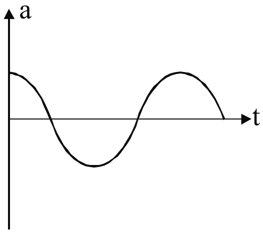


For particle A

(C)

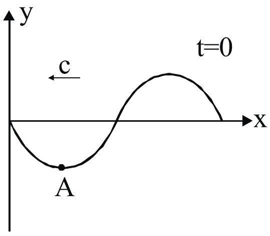


(R)

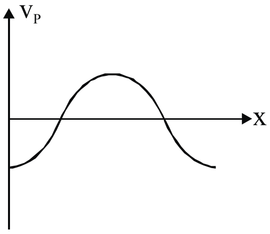


For particle A

(D)

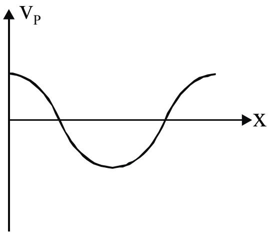


(S)



At t = 0 for all the particles

(T)



At t = 0 for all the particles

WA0057

13. In a string a standing wave is set up whose equation is given as $y = 2A \sin kx \cos \omega t$. The mass per unit length of the string is μ.

	Column-I		**Column-II**
(A)	at $t = 0$	(P)	Total energy per unit length at $x = 0$ is $2\mu A^2\omega^2$.
(B)	at $t = \frac{T}{8}$	(Q)	Total energy per unit length at $x = \lambda/4$ is $2\mu A^2\omega^2$.
(C)	at $t = \frac{T}{4}$	(R)	Total energy per unit length at $x = \lambda$ is $2\mu A^2\omega^2$.
(D)	at $t = \frac{T}{2}$	(S)	power transmitted through a point at $x = \lambda$ is 0.
		(T)	power transmitted through a point at $x = \lambda/4$ is 0.

WA0058

EXERCISE-JM

1. The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by

$y = 0.02(\text{m}) \sin\left[2\pi\left(\frac{t}{0.04(\text{s})} - \frac{x}{0.50(\text{m})}\right)\right]$. The tension in the string is : **[AIEEE - 2010]**

(1) 6.25 N (2) 4.0 N (3) 12.5 N (4) 0.5 N

WA0059

2. The transverse displacement y(x, t) of a wave on a string is given by $y(x,t) = e^{-\left(ax^2+bt^2+2\sqrt{ab}xt\right)}$. This represents a :- **[AIEEE - 2011]**

(1) standing wave of frequency $\sqrt{b}$ (2) standing wave of frequency $\frac{1}{\sqrt{b}}$

(3) wave moving in +x directionwith speed $\sqrt{\frac{a}{b}}$ (4) wave moving in –x direction with speed $\sqrt{\frac{b}{a}}$

WA0060

3. A travelling wave represented by y = A sin(ωt–kx) is superimposed on another wave represented by y = A sin (ωt + kx). The resultant is :- **[AIEEE-2011]**

(1) A standing wave having nodes at $x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}, n = 0,1,2$

(2) A wave travelling along + x direction

(3) A wave travelling along –x direction

(4) A standing wave having nodes at $x = \frac{n\lambda}{2}$; n = 0, 1, 2

WA0061

4. A sonometer wire of length 1.5m is made of steel. The tension in it produces an elastic strain of 1%. What is the fundamental frequency of steel if density and elasticity of steel are 7.7×10^3 kg/m^3 and 2.2×10^{11} N/m^2 respectively ? **[JEE-Main-2013]**

(1) 188.5 Hz (2) 178.2 Hz (3) 200.5 Hz (4) 770 Hz

WA0062

5. A uniform string of length 20m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is :-
(take g = 10 ms^{-2}) **[JEE-Main-2016]**

(1) $\sqrt{2}$ s (2) $2\pi\sqrt{2}$ s (3) 2s (4) $2\sqrt{2}$ s

WA0063

SELECTED PROBLEMS FROM JEE-MAINS ONLINE PAPERS

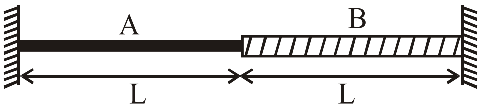
6. A heavy ball of mass M is suspended from the ceiling of a car by a light string of mass m (m<<M). When the car is at rest, the speed of transverse waves in the string is 60 ms^{-1}. When the car has acceleration a, the wave-speed increases to 60.5 ms^{-1}. The value of a, in terms of gravitational acceleration g, is closest to :

[JEE-Main-2019_Jan]

(1) $\frac{g}{5}$ (2) $\frac{g}{20}$ (3) $\frac{g}{10}$ (4) $\frac{g}{30}$

7. A wire of length 2L, is made by joining two wires A and B of same length but different radii r and 2r and made of the same material. It is vibrating at a frequency such that the joint of the two wires forms a node. If the number of antinodes in wire A is p and that in B is q then the ratio p : q is :

[JEE-Main-2019_April]



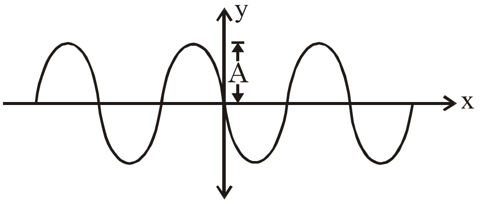
(1) 4 : 9 (2) 3 : 5 (3) 1 : 4 (4) 1 : 2

8. A string is clamped at both the ends and it is vibrating in its 4th harmonic. The equation of the stationary wave is Y = 0.3 sin(0.157x) cos(200πt). The length of the string is : (All quantities are in SI units.)

[JEE-Main-2019_April]

(1) 20 m (2) 80 m (3) 60 m (4) 40 m

9. A progressive wave travelling along the positive x-direction is represented by $y(x, t) = A \sin(kx - \omega t + \phi)$. Its snapshot at t = 0 is given in the figure:



For this wave, the phase ϕ is : **[JEE-Main-2019_April]**

(1) 0 (2) $-\frac{\pi}{2}$ (3) π (4) $\frac{\pi}{2}$

10. A transverse wave travels on a taut steel wire with a velocity of v when tension in it is 2.06×10^4 N. When the tension is changed to T, the velocity changed to v/2. The value of T is close to :

[JEE-Main-2020_Jan]

(1) 10.2×10^2 N (2) 5.15×10^3 N (3) 2.50×10^4 N (4) 30.5×10^4 N

EXERCISE (JA)

1. A horizontal stretched string, fixed at two ends, is vibrating in its fifth harmonic according to the equation, $y(x, t) = (0.01\text{m}) \sin [(62.8 \text{ m}^{-1})x] \cos [(628 \text{ s}^{-1})t]$. Assuming $\pi = 3.14$, the correct statement(s) is (are) **[JEE-Advance-2013]**

(A) The number of nodes is 5.

(B) The length of the string is 0.25 m.

(C) The maximum displacement of the midpoint of the string, from its equilibrium position is 0.01m.

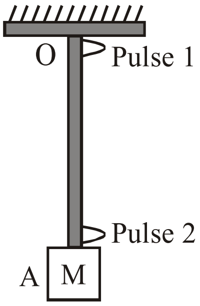
(D) The fundamental frequency is 100 Hz. **WA0065**

2. One end of a taut string of length 3m along the x–axis is fixed at x = 0. The speed of the waves in the string is 100 ms^{-1}. The other end of the string is vibrating in the y direction so that stationary waves are set up in the string. The possible waveform (s) of these stationary waves is(are) :- **[JEE-Advance-2014]**

(A) $y(t) = A\sin\frac{\pi x}{6}\cos\frac{50\pi t}{3}$ (B) $y(t) = A\sin\frac{\pi x}{3}\cos\frac{100\pi t}{3}$

(C) $y(t) = A\sin\frac{5\pi x}{6}\cos\frac{250\pi t}{3}$ (D) $y(t) = A\sin\frac{5\pi x}{2}\cos 250\pi t$ **WA0066**

3. A block M hangs vertically at the bottom end of a uniform rope of constant mass per unit length. The top end of the rope is attached to a fixed rigid support at O. A transverse wave pulse (Pulse 1) of wavelength λ_0 is produced at point O on the rope. The pulse takes time T_{OA} to reach point A. If the wave pulse of wavelength λ_0 is produced at point A (Pulse 2) without disturbing the position of M it takes time T_{AO} to reach point O. Which of the following options is/are **correct** ? **[JEE-Advance-2017]**



(A) The time $T_{AO} = T_{OA}$

(B) The velocities of the two pulses (Pulse 1 and Pulse 2) are the same at the midpoint of rope

(C) The wavelength of Pulse 1 becomes longer when it reaches point A

(D) The velocity of any pulse along the rope is independent of its frequency and wavelength. **WA0067**

4. **Answer the following by appropriately matching the lists based on the information given in the paragraph.**

A musical instrument is made using four different metal strings, 1,2,3 and 4 with mass per unit length μ, 2μ, 3μ and 4μ respectively. The instrument is played by vibrating the strings by varying the free length in between the range L_0 and $2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0.

List-I gives the above four strings while list-II lists the magnitude of some quantity.

If the tension in each string is T_0, the correct match for the highest fundamental frequency in f_0 units will be, **[JEE-Advance-2019]**

List-I	**List-II**
(I) String-1(μ)	(P) 1
(II) String-2 (2μ)	(Q) 1/2
(III) String-3 (3μ)	(R) $1/\sqrt{2}$
(IV) String-4 (4μ)	(S) $1/\sqrt{3}$
	(T) 3/16
	(U) 1/16

(A) I→P, II→R, III→S, IV→Q
(B) I→P, II→Q, III→T, IV→S
(C) I→Q, II→S, III→R, IV→P
(D) I→Q, II→P, III→R, IV→T **WA0068**

5. **Answer the following by appropriately matching the lists based on the information given in the paragraph.**

A musical instrument is made using four different metal strings, 1,2,3 and 4 with mass per unit length μ, 2μ, 3μ and 4μ respectively. The instrument is played by vibrating the strings by varying the free length in between the range L_0 and $2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0.

List-I gives the above four strings while list-II lists the magnitude of some quantity.

[JEE-Advance-2019]

List-I	**List-II**
(I) String-1(μ)	(P) 1
(II) String-2 (2μ)	(Q) 1/2
(III) String-3 (3μ)	(R) $1/\sqrt{2}$
(IV) String-4 (4μ)	(S) $1/\sqrt{3}$
	(T) 3/16
	(U) 1/16

(A) I→P, II→Q, III→T, IV→U
(B) I→T, II→Q, III→R, IV→U
(C) I→P, II→Q, III→R, IV→T
(D) I→P, II→R, III→T, IV→U **WA0069**

ANSWER KEY

EXERCISE (S-1)

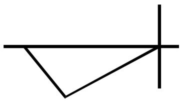
1. Ans. (a) DEF, (b) ABH, (c) CG, (D) AE

2. Ans. (a) $\lambda = 4\text{m}$, $f = \frac{1}{4}$ Hz, 1 m/s, (b) $\frac{3\pi}{2}$ mm/s, $\frac{3\pi^2}{4}$ mm/s^2, (c) $\frac{3\pi}{2}\times10^{-3}$ **3. Ans.** $40\sqrt{2}$

4. Ans. (a) 20 m/s; (b) 2 m; (c) $y(x,t) = 0.02\sin(\pi x - 20\pi t + \pi/6)]$

5. Ans. (a) negative x ; (b) $y = 4\times10^{-3}\sin 100\pi\left(3t + 0.5x + \frac{1}{400}\right)$ (x , y in meter) ; (c) $144\pi^2\times10^{-5}$ J

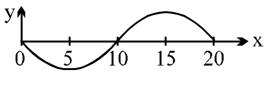
6. Ans. 1.22 v **7. Ans.** $\frac{\pi}{\sqrt{2}\omega}$ **8. Ans. 70 m/s** **9. Ans.** 10.8×10^4W

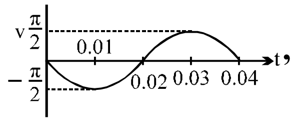
10. Ans. (a)  , (b) 2 cm/s **11. Ans.** 1×10^9 Nm^2

12. Ans. $A_t = \frac{4}{3}A_i$, $A_r = \frac{1}{3}A_i$, $y_r = -\frac{0.04}{3}\sin(0.5\,\pi x + 100\,\pi t)$; $y_t = +\frac{0.16}{3}\sin(0.25\pi x - 100\,\pi t)$

13. Ans. $E = \frac{A^2\pi^2 T}{4l}$

EXERCISE (S-2)

1. Ans. (i) 

(ii) 25 × 10^{-6} W (iii) 

, solved

2. Ans. 300 Hz **3. Ans. L/9** **4. Ans. 0.2 cos[$2\pi/5(x-10t)$], (ii) 5n – (15/4)]**

5. Ans. π, 0 **6. Ans. 10800 Hz** **7. Ans. 17.5°**

EXERCISE (O-1)

1. Ans. (C) **2. Ans. (B)** **3. Ans. (A)** **4. Ans. (B)** **5. Ans. (C)** **6. Ans. (B)**
7. Ans. (D) **8. Ans. (A)** **9. Ans. (A)** **10. Ans. (C)** **11. Ans. (B)** **12. Ans. (B)**
13. Ans. (D) **14. Ans. (D)** **15. Ans. (B)** **16. Ans. (B)** **17. Ans. (C)** **18. Ans. (C)**
19. Ans. (B) **20. Ans. (B)** **21. Ans. (A)** **22. Ans. (D)** **23. Ans. (A)** **24. Ans. (D)**
25. Ans. (B) **26. Ans. (A)** **27. Ans. (C)**

EXERCISE (O-2)

1. Ans. (A,B,D) **2. Ans. (A,C,D)** **3. Ans. (C,D)** **4. Ans. (B,C,D)** **5. Ans. (A,B,D)**

6. Ans. (C) **7. Ans. (A)** **8. Ans. (B)** **9. Ans. (A)** **10. Ans. (C)**

11. Ans. (B) **12. Ans. (A)–(Q,S) ; (B)–(P,R,T) ; (C)–(Q,T) ; (D)–(P,R,S)**

13. Ans. (A)–(P,R,S,T) ; (B)–(S,T) ; (C)–(Q,S,T) ; (D)–(P,R,S,T)

13. Ans. (A) – (P,R,S,T) ; (B) – (S,T) ; (C) – (Q,S,T) ; (D) – (P,R,S,T)

EXERCISE-JM

1. Ans. (1) **2. Ans. (4)** **3. Ans. (1)** **4. Ans. (2)** **5. Ans. (4)**

6. Ans. (1) **7. Ans. (4)** **8. Ans. (2)** **9. Ans. (3)** **10. Ans. (2)**

EXERCISE (JA)

1. Ans. (B,C) **2. Ans. (A,C,D)** **3. Ans. (A,D)** **4. Ans. (A)** **5. Ans. (A)**

ALLEN CAREER INSTITUTE
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Chapter 03 Contents

SOUND WAVES

Important Notes

SOUND WAVE

KEY CONCEPTS

Sound is a mechanical three dimensional and longitudinal wave that is created by a vibrating source such as a guitar string, the human vocal cords, the prongs of a tuning fork or the diaphragm of a loudspeaker.

Sound waves are the most common example of longitudinal waves. They travel through any material waves. They travel through any material medium with a speed that depends on the properties of the medium. As the waves travel through air, the elements of air vibrate to produce changes in density and pressure along the direction of motion of the wave. If the source of the sound waves vibrates sinusoidally, the pressure variations are also sinusoidal. The mathematical description of sinusoidally, the pressure variations are also sinusoidal sound waves is very similar to that of sinusoidal string waves.

EQUATION OF SOUND WAVES

As the piston oscillates sinusoidally, regions of compression and rarefaction are continuously set up. The distance between two successive compressions (or two successive rarefactions) equals the wavelength λ. As these regions travel through the tube, any small element of the medium moves with simple harmonic motion parallel to the direction of the wave. If s(x, t) is the position of a small element relative to its equilibrium position, We can express this harmonic position function as

$$s(x, t) = s_{max} \cos(kx - \omega t)$$

Where s_{max} is the maximum position of the element relative to equilibrium. This is often called the displacement amplitude of the wave. The parameter k is the wave number and ω is the angular frequency of the piston . Note that the displacement of the element is along s, in the direction of propagation of the sound wave, which means we are describing a longitudinal wave.

Consider a thin disk-shaped element of gas whose circular cross section is parallel to the piston in figure. This element will undergo changes in position, pressure, and density as a sound wave propagates through the gas. From the definition of bulk modulus , the pressure variation in the gas is

$$\Delta P = -B\frac{\Delta V}{V_i}$$

The element has a thickness Δx in the horizontal direction and a cross-sectional area A, so its volume is $V_i = A\Delta x$. The change in volume ΔV accompanying the pressure change is equal to AΔs, where Δs is the difference between the value of s at x + Δx and the value of s at x. Hence, we can express ΔP as

$$\Delta P = -B\frac{\Delta V}{V_i} = -B\frac{A\Delta s}{A\Delta x} = -B\frac{\Delta s}{\Delta x}$$

As Δx approaches zero , the ratio Δs/Δx becomes $\partial s/\partial x$ (The partial derivative indicates that we are interested in the variation of s with position at a fixed time.) Therefore,

$$\Delta P = -B\frac{\partial s}{\partial x}$$

If the position function is the simple sinusoidal function given by Equation , we find that

$$\Delta P = -B\,\frac{\partial}{\partial x}[s_{\max}\cos(kx-\omega t)] = Bs_{\max}\,k\sin(kx-\omega t)$$

$$\Delta P = \Delta P_{\max}\sin(kx-\omega t)$$

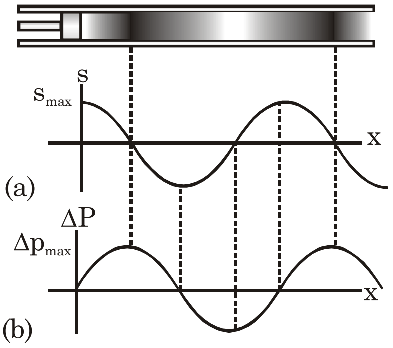


Figure : (a) Displacement amplitude and (b) pressure amplitude versus position for a sinusoidal longitudinal wave

Thus we can describe sound waves either in terms of excess pressure (equation 1.1) or in terms of the longitudinal displacement suffered by the particles of the medium.

If $s = s_0 \sin \omega(t - x/v)$ represents a sound wave where

s = displacement of medium particle from its mean position at x, then it can be proved that

$P = P_0 \sin \{w(t - x/v) + \pi/2\}$ ... (3.2)

represents that same sound wave where, P is excess pressure at position x, over and above the average atmospheric pressure and the pressure amplitude P_0 is given by

$P_0 = \dfrac{B\omega s_0}{v} = BKs_0$ ... (3.3)

(B = Bulk modulus of the medium, K = angular wave number)

Note from equation (3.1) and (3.2) that the displacement of a medium particle and excess pressure at any position are out of phase by $\frac{\pi}{2}$. Hence a displacement maxima corresponds to a pressure minima and vice-versa.

Ex. The equation of a sound wave in air is given by $P = 0.2 \sin [3000\,t - 9x]$, where all variables are in S.I. units.

(a) Find the frequency, wavelength and the speed of sound wave in air.

(b) If the equilibrium pressure of air is 1.0×10^5 N/m^2, what are the maximum and minimum pressures at a point as the wave passes through that point?

Sol. (a) Comparing with the standard form of a travelling wave

$P = P_0 \sin [\omega (t - x/v)]$

we see that ω 3000 s^{-1}. The frequency is

$$t = \frac{\omega}{2\pi} = \frac{3000}{2\pi}\text{ Hz}$$

Also from the same comparison, $\omega/v = 9.0$ m^{-1}.

$$\text{or,}\quad v = \frac{\omega}{9.0\,\text{m}^{-1}} = \frac{3000\,\text{s}^{-1}}{9.0\,\text{m}^{-1}} \qquad \frac{1000}{3}\ \text{m/s}^{-1}$$

$$\text{The wavelength is } \lambda = \frac{v}{f} = \frac{1000/3\text{m/s}}{3000/2\pi\,\text{Hz}} = \frac{2\pi}{9}\ \text{m}$$

(b) The pressure amplitude is $P_0 = 0.02$ N/m^2. Hence, the maximum and minimum pressures at a point in the wave motion will be $(1.01 \times 10^5 \pm 0.02)$ N/m^2.

Ex. A sound wave of wavelength 40cm travels in air. If the difference between the maximum and minimum pressure at a given point is 4.0×10^{-3} N/m^2, find the amplitude of vibration of the particles of the medium. The bulk modulus of air is 1.4×10^5 N/m^2.

Sol. The pressure amplitude is

$$p_0 = \frac{4.0 \times 10^{-3}\ \text{N/m}^2}{2} = 2 \times 10^{-3}\ \text{N/m}^2$$

The displacement amplitude s_0 is given by

$$p_0 = Bks_0$$

or $$s_0 = \frac{p_0}{Bk} = \frac{p_0\lambda}{2\pi B}$$

$$= \frac{2\times10^{-3}\ \text{N/m}^2 \times (40 \times 10^{-2}\ \text{m})}{2 \times \pi \times 14 \times 10^4\ \text{N/m}^2} = \frac{200}{7\pi}\ \text{Å}$$

$$= 13.2\ \text{Å}$$

SPEED OF SOUND WAVES :

Velocity of sound waves in a linear solid medium is given by

$$v = \sqrt{\frac{Y}{\rho}} \qquad (4.1)$$

where Y = young’s modulus of elasticity and ρ = density

Velocity of sound wave in a fluid medium (liquid or gas) is given by

$$v = \sqrt{\frac{B}{\rho}} \qquad (4.2)$$

where, ρ = density of the medium and B = Bulk modulus of the medium given by,

$$B = -V\frac{dP}{dV} \qquad (4.3)$$

Newton’s formula : Newton assumed propagation of sound through a gaseous medium to be an isothermal process.

$$PV = \text{constant}$$

$$\Rightarrow \qquad \frac{dP}{dV} = \frac{-P}{V}$$

and hence B = P

and thus he obtained for velocity of sound in a gas.

$$v = \sqrt{\frac{P}{\rho}} = \sqrt{\frac{RT}{M}}$$ where M = molar mass

Laplace's correction : Later Laplace established that propagation of sound in a gas is not an isothermal but an adiabatic process and hence PV' = constant

$$\Rightarrow \frac{dP}{dV} = -\gamma \frac{P}{V}$$

where, $B = -V\frac{dP}{dV} = \gamma P$

and hence speed of sound in a gas,

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}} \qquad (4.4)$$

FACTORS AFFECTING SPEED OF SOUND IN ATMOSPHERE

(a) Effect of temperature : as temperature (T) increases velocity (v) increases. For small change in temperature above room temperature v increases linearly by 0.6 m/s for every 1°C rise in temp.

(b) Effect of humidity : With increase in humidity density decreases. This is because the molar mass of water vapour is less than the molar mass of air.

(c) Effect of pressure :

The speed of sound in a gas is given by $v = \sqrt{\frac{\gamma P}{P}} = \sqrt{\frac{\gamma RT}{M}}$

So at constant temperature, if P changes then ρ also changes in such a way that P/ρ remains constant. Hence pressure does not have any effect on velocity of sound as long as temperature is constant.

Ex. The constant γ for oxygen as well as for hydrogen is 1.40. If the speed of sound in oxygen is 450 m/s, what will be the speed of hydrogen at the same temperature and pressure?

Sol. $v = \sqrt{\frac{\gamma RT}{M}}$

since temperature, T is constant

$$\therefore \frac{v_{(H_2)}}{v_{(O_2)}} = \sqrt{\frac{M_{O_2}}{M_{H_2}}} = \sqrt{\frac{32}{2}} = 4$$

$\Rightarrow v(H_2) = 4 \times 450 = 1800$ m/s

Aliter : The speed of sound in a gas is given by $u = \sqrt{\frac{\gamma P}{\rho}}$. At STP, 22.4 litres of oxygen has a mass of 32g whereas the same volume of hydrogen has a mass of 2g. Thus, the density of oxygen is 16times the density of hydrogen at the same temperature and pressure. As γ is same for both the gases.

$$\frac{f_{(\text{hydrogen})}}{f_{(\text{oxygen})}} = \sqrt{\frac{\rho_{(\text{oxygen})}}{\rho_{(\text{hydrogen})}}}$$

or $\quad f_{(\text{hydrogen})} = 4f_{(\text{oxygen})} = 4 \times 450 \text{ m/s} = 1800 \text{ m/s}$

INTENSITY OF PERIODIC SOUND WAVES

Like any other progressive wave, sound waves also carry energy from one point of space to the other. This energy can be used to work, for example, forcing the eardrums to vibrate or in the extreme case of a sonic boom created by supersonic jet, can even cause glass panes of windows to crack.

The amount of energy carried per unit by a wave is called its power and power per unit area held perpendicular a sound wave travelling along positive x-axis described by the equation.

We define the intensity I of a wave, or the power per unit area , to be the rate at which the energy being transported by the wave transfers through a unit area A perpendicular to the direction of travel of the wave :

$$I = \frac{P}{A}$$

In the present case, therefore, the intensity is

$$I = \frac{P}{A} = \frac{1}{2}\rho A v(\omega s_{max})^2$$

Thus, we see that the intensity of a periodic sound wave is proportional to the square of the displacement amplitude and to the square of the angular frequency (as in the case of a periodic string wave). this can also be written in terms of the pressure amplitude ΔP_{max} ; in this case, we use Equation to obtain

$$I = \frac{\Delta P_{max}^2}{2\rho v}$$

Ex. The pressure amplitude in a sound wave from a radio receiver is 4.0×10^{-3} N/m^2 and the intensity at a point is 10^{-6} W/m^2. If by fuming the "Volume" knob the pressure amplitude is increased to 6×10^{-3} N/m^2, evaluate the intensity.

Sol. The intensity is proportional to the square of the pressure amplitude.

Thus, $$\frac{I'}{I} = \left(\frac{p_0'}{p_0}\right)^2$$

or $$I' = \left(\frac{p_0'}{p_0}\right)^2 I = \left(\frac{p_0'}{p_0}\right)^2 \times 10^{-6}\ \text{W/m}^2 = 2.25 \times 10^{-16}\ \text{W/m}^2.$$

APPEARANCE OF SOUND TO HUMAN EAR

Pitch and Frequency

Frequency as we have discussed till now is an objective property measured its units of Hz and which can be assigned a unique value. However a person's perception of frequency is subjective. The brain interprets frequency primarily in terms of a subjective quality called Pitch. A pure note of high frequency is interpreted as high-pitched sound and a pure note of low frequency as low-pitched sound.

Pitch of a sound is that sensation by which we differentiate a buffalo voice, a male voice is of low pitch, a male voice has higher pitch and a female voice has (generally) still higher pitch. This sensation primarily depends on the dominant frequency present in the sound. Higher the frequency, higher will be the pitch and vice versa. The dominant frequency of a buffalo voice is smaller than that of a male voice which in turn is smaller than that of a female voice.

Loudness and Intensity

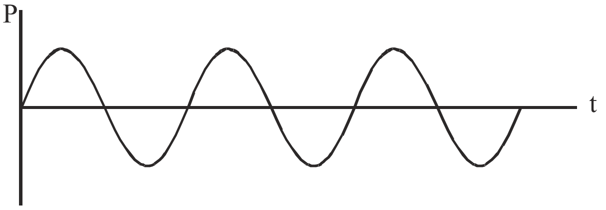
The loudness that we sense is related to the intensity of sound through it is not directly proportional to it. Our perception of loudness is better correlated with the sound level measured in decibels (abbreviated and dB) and defined as follows.

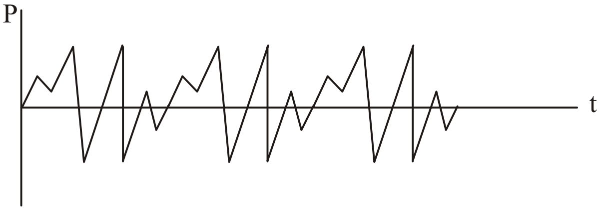
$$\beta = 10 \log_{10}\left(\frac{I}{I_0}\right),$$

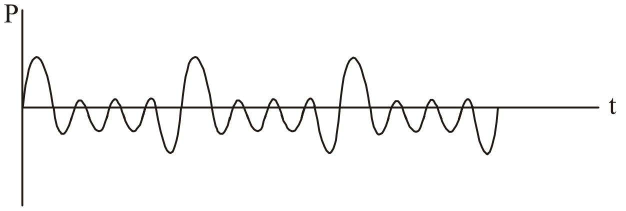
where I is the intensity of the sound and I_0 is a constant reference intensity 10^{-12} W/m^2. The reference intensity represents roughly the minimum intensity that is just audible at intermediate frequencies. For $I = I_0$, the sound level $\beta = 0$. Table shows the approximate sound levels of some of the sounds commonly encountered.

Quality and Waveform

A sound generated by a source may contain a number of frequency components in it. Different frequency components have different amplitudes and superposition of them results in the actual waveform. The appearance of sound depends on this waveform apart from the dominant frequency and intensity. Figure shows waveforms for a tuning fork, a clarinet and a cornet playing the same note (fundamental frequency = 440 Hz) with equal loudness.







We differentiate between the sound from a table and that from a mridang by saying that they have different quality. A musical sound has certain well-defined frequencies which have considerable amplitude. These frequencies are generally harmonics of a fundamental frequency. Such a sound is particularly pleasant to the ear. On the other hand, a noise has frequencies that do not bear well-defined relationship among themselves.

Ex. A bird is singing on a tree. A person approaches the tree and perceives that the intensity has increased by 10 dB. Find the ratio of initial and final separation between the man and the bird.

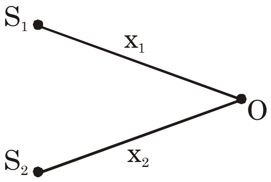
Sol. $\beta_1 = 10 \log \frac{I_1}{I_0}$

$\beta_2 = 10 \log \frac{I_2}{I_0} \Rightarrow \beta_2 - \beta_1 = 10 \log \frac{I_2}{I_1}$ or $10 = 10 \log_{10}\left(\frac{I_2}{I_1}\right)$

$\Rightarrow \frac{I_2}{I_1} = 10^1 = 10$

for point source $I \propto \frac{1}{r^2} \Rightarrow \frac{r_1}{r_2} = \sqrt{\frac{I_2}{I_1}} = \sqrt{10}$

INTERFERENCE OF SOUND WAVES :



If $p_1 = p_{m1} \sin(\omega t - kx_1 + \theta_1)$
and $p_2 = p_{m2} \sin(\omega t - kx_2 + \theta_1)$
resultant excess pressure at point O is

$p = p_1 + p_2$

$\Rightarrow p = p_0 \sin(\omega t - kx + \theta)$

where, $p_0 = \sqrt{p_{m_1}^2 + p_{m_2}^2 + 2p_{m_1}p_{m_2}\cos\phi}$, $\phi = |\,k(x_1 - x_2) + (\theta_1 - \theta_2)\,|$ ... (6.1)

(i) for constructive interference

$\phi = 2n\pi \qquad \Rightarrow p_0 = p_{m1} + p_{m2}$

(ii) for destructive interference

$\phi = (2n + 1)\pi \qquad \Rightarrow p_0 = |\,p_{m1} - p_{m2}\,|$

If ϕ is only due to path difference, then $\phi = \frac{2\pi}{\lambda}\Delta x$, and

Condition for constructive interference : $\Delta x = n\lambda$, $n = 0, \pm 1, \pm 2$

Condition for destructive interference : $\Delta x = (2n + 1)\frac{\lambda}{2}$, $n = 0 \pm 1, \pm 2$

from equation (6.1)

$$P_0^2 = P_{m_1}^2 + P_{m_2} + 2P_{m_1} P_{m_2} \cos\phi$$

Since intensity, $I \propto (\text{Pressure amplitude})^2$,

we have, for resultant intensity, $I = I_1 + I_2 + 2\sqrt{I_2 I_2}\cos\phi$ ... (6.2)

If $I_1 = I_2 = I_0$

$I = 2I_0(1 + \cos\phi)$

$\Rightarrow I = 4I_0 \cos^2\frac{\phi}{2}$ ... (6.3)

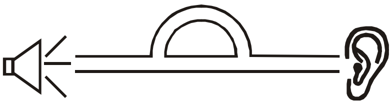
Hence in this case,

for constructive interference : $\phi = 0\ 2\pi, 4\pi$ ..... and $I_{max} = 4I_0$

and for destructive interference : $\phi = \pi, 3\pi$ .... and $I_{min} = 0$

Coherence : Two sources are said to be coherent if the phase difference between them does not change with time. In this case their resultant intensity at any point in space remains constant with time. Two independent sources of sound are generally incoherent in nature, i.e. phase difference between them changes with time and hence the resultant intensity due to them at any point in space changes with time.

Ex. Figure shows a tube structure in which a sound signal is sent from one end and is received at the other end. The semicircular part has a radius of 10.0 cm. The frequency of the sound source can be varied from 1 to 10 kHz. Find the frequencies at which the ear perceives maximum intensity. The speed of sound in air = 342 m/s.



Sol. The sound wave bifurcates at the junction of the straight and the semicircular parts. The wave through the straight part travels a distance $s_1 = 2 \times 10$ cm and the wave through the curved part travels a distance $s_2 = \pi\ 10$ cm = 31.4 cm before they meet again and travel to the receiver. The path difference between the two waves received is, therefore.

$$\Delta s = s_2 - s_1 = 31.4\text{ cm} - 20\text{ cm} = 11.4\text{ cm}$$

The wavelength of either wave is $\frac{v}{\upsilon} = \frac{330\text{ m/s}}{\upsilon}$. For constructive interference, $\Delta p = n\lambda$, where n is an integer.

$$\text{or , } \Delta p = n \cdot \frac{v}{v} \qquad \Rightarrow\ v = \frac{n \cdot v}{\Delta p} \Rightarrow \frac{n \cdot 342}{(0.114)} = 3000\ n$$

Thus, the frequencies within the specified range which cause maximum of intensity are

3000 × 1 3000 × 2 and 3000 × 3 Hz

LONGITUDINAL STANDING WAVES :

Two longitudinal waves of same frequency and amplitude travelling in opposite directions interfere to produce a standing wave.

If the two interfering wave are given by

$p_1 = p_0 \sin(\omega t - kx)$

and $p_2 = p_0 \sin(\omega t + kx + \phi)$

then the equation of the resultant standing wave would be given by

$$p + p_1 + p_2 = 2p_0 \cos\left(kx + \frac{\phi}{2}\right) \sin\left(\omega t + \frac{\phi}{2}\right)$$

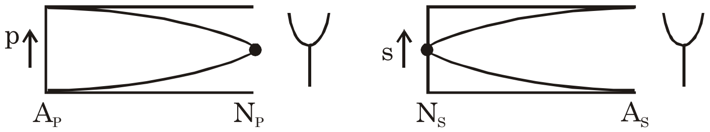
$$\Rightarrow\ p = p_0' \sin\left(\omega t + \frac{\phi}{2}\right) \qquad \text{... (8.1)}$$

VIBRATION OF AIR COLUMNS

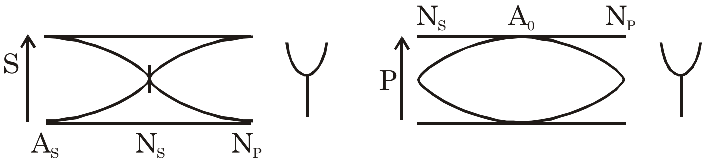
Standing waves can be set up in air-columns trapped inside cylindrical tubes if frequency of the tuning fork sounding the air column matches one of the natural frequency of air columns. In such a case the sound of the tuning fork becomes markedly louder, and we say there is resonance between the tuning fork and air column. To determine the natural frequency of the air column, notice that there is a displacement node (pressure antinode) at each closed end of the tube as air molecules there are not free to move, and a displacement antinode (pressure-node) at each open end of the air-column. In reality antinodes do not occurs exactly at the open end but a little distance outside. However if diameter of tube is small compared to its length, this end correction can be neglected.

Closed organ pipe

(In the diagram, A_P = Pressure antinode, A_S = displacement antinode, N_P = pressure node, N_s = displacement node.)

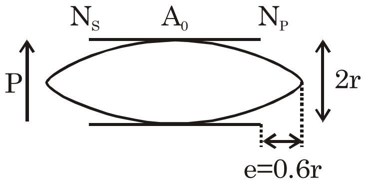


Open organ pipe :



End correction : As mentioned earlier the displacement antinode at an open end of an organ pipe lies slightly outside the open lend. The distance of the antinode from the open end is called end correction and its value is given by

$$e = 0.6\, r$$



where r = radius of the organ pipe.

with end correction, the fundamental frequency of a closed pipe (f_c) and an open argon pipe (f_0) will be given by

$$f_c = \frac{v}{4(\ell + 0.6r)} \text{ and}$$

$$f_0 = \frac{v}{2(\ell + 1.2r)} \qquad \text{.........} (9.5)$$

Ex. A tuning fork is vibrating at frequency 200 Hz. When another tuning fork is sounded simultaneously, 6 beats per second are heard. When some mass is added to the tuning fork of 200 Hz, beat frequency decreases. Find the frequency of the other tuning fork.

Sol. $|f - 200| = 6$

$\Rightarrow$ f = 194 or 206

when 1st tuning fork is loaded its frequency decreases and so does beat frequency

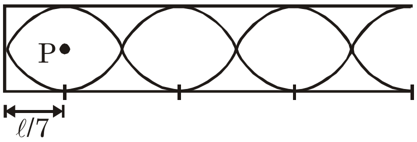
$\Rightarrow$ $200 > f$

$\Rightarrow$ f = 194 Hz

Ex. A closed organ pipe has length 'ℓ'. The air in it is vibrating in 3rd overtone with maximum amplitude 'a'. Find the amplitude at a distance of $\ell/7$ from closed end of the pipe.

Sol. The figure shows variation of displacement of particles in a closed organ pipe for 3rd overtone.

For third overtone $\ell = \frac{7\lambda}{4}$ or $\lambda = \frac{4\ell}{7}$ or $\frac{\lambda}{4} = \frac{\ell}{7}$



Hence the amplitude at P at a distance $\frac{\ell}{7}$ from closed end is 'a' because there is an antinode at that point.

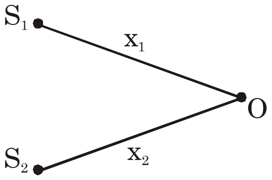
BEATS

When two sound waves of same amplitude and different frequency superimpose, then intensity at any point in space varies periodically with time. This effect is called beats.

If the equation of the two interfering sound waves emitted by s_1 and s_2 at point O are,

$p_1 = p_0 \sin(2\pi f_1 t - kx_1 + \theta_1)$

$p_2 = p_0 \sin(2\pi f_2 t - kx_2 + \theta_2)$



By principle of superposition

$p = p_1 + p_2$

$= 2\pi_0 \cos\{\pi(f_1 - f_2)t + \frac{\theta_1 + \theta_2}{2}\} \sin\{\pi(f_1 + f_2)t + \frac{\theta_1 + \theta_2}{2}\}$

i.e., the resultant sound at point O has frequency $\left(\frac{f_1 + f_2}{2}\right)$ while pressure amplitude p'_0 (t) variex with time as

$$p_0(t) = 2p_0 \cos\left\{\pi(f_1 - f_2)t + \frac{\phi_1 + \phi_2}{2}\right\}$$

Hence pressure amplitude at point O varies with time with a frequency of $\left(\frac{f_1 - f_2}{2}\right)$.

Hence sound intensity will vary with a frequency $f_1 - f_2$.

This frequency is called beat frequency (f_B) and the time interval between two successive intensity maxima (or minima) is called beat time period (T_B)

$$f_B = f_1 - f_2$$

$$T_B = \frac{1}{f_1 - f_2}$$

IMPORTANT POINTS :

(i) The frequency $|f_1 - f_2|$ should be less than 16 Hz, for it to be audible.

(ii) Beat phenomenon can be used for determining an unknown frequency by sounding it together with a sound of known frequency.

Ex. Two strings X and Y of a sitar produces a beat of frequency 4Hz. When the tension of string Y is slightly increased, the beat frequency is found to be 2Hz. If the frequency of X is 300 Hz, then the original frequency of Y was.

Ans. 296 Hz

DOPPLER EFFECT

We can express the general relationship for the observed frequency when a source is moving and an observer is at rest as equation , with the same sign convention applied to vs as was applied to v0 : a positive value is substituted for vs when the source moves toward the observer and a negative value is substituted when the source moves away from the observer.

Finally, we find the following general relationship for the observed frequency :

$$f' = \left(\frac{v + v0}{v - vs}\right)f$$

In this expression, the signs for the values substituted for v0 and vs depend on the direction of the velocity. A positive value is used for motion of the observer or the source toward the other, and a negative sign for motion of one away from the other.

A convenient rule concerning signs for you to remember when working with all Doppler effect problems is as follows :

The word toward is associated with an increase in observed frequency. The words away from are associated with a decrease in observed frequency.

Ex. A submarine (sub A) travels through water at a speed of 8.00 m/s, emitting a sonar wave at a frequency of 1400 Hz. The speed of sound in the water is 1533 m/s. A second submarine (sub B) is located such that both submarines are traveling directly toward one another. The second submarine is moving at 9.00 m/s.

(A) What frequency is detected by an observer riding on sub B as the subs approach each other?

(B) The subs barely miss each other and pass. What frequency is detected by an observer riding on sub B as the subs recede from each other?

Sol. (A) We use equation to find the Doppler - shifted frequency. As the two submarines approach each other, the observer in sub B hears the frequency

$$f' = \left(\frac{v + v0}{v - vs}\right)f = \left(\frac{1533\text{m/s} + (+9.00\text{m/s})}{1533\text{m/s} - (+8.00\text{m/s})}\right)(1400\text{ Hz}) = 1416\text{ Hz}$$

(B) As the two submarines recede from each other, the observer in sub B hears the frequency

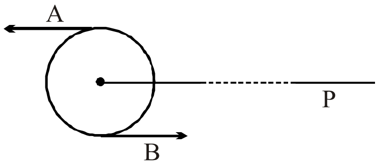
$$f' = \left(\frac{v+v0}{v-vs}\right)f$$

$$= \left(\frac{1533m/s+(-9.00m/s)}{1533m/s-(-8.00m/s)}\right)(1400\text{ Hz}) = 1385\text{ Hz}$$

Ex. A whistle of frequency 540 Hz is moving in a circle of radius 2m at a constant angular speed of 15rad/s. What are the lowest and height frequencies heard by a listener standing at rest, a long distance away from the centre of the circle ? (velocity of sound in air is 330 m/s ft/sec.)

Sol. The whistle is moving along a circular path with constant angular velocity ω. The linear velocity of the whistle is given by

$$v_s = \omega R$$



where, R is radius of the circle

At points A and B, the velocity v_s of whistle is parallel to line OP, i.e. with respect to observer at P, whistle has maximum velocity v_s away from P at point A, and towards P at point B. (Since distance OP is large compared to radius R, whistle may be assumed to be moving along line OP)

Observer, therefore, listens maximum frequency when source is at B moving towards observer.

$$f_{max} = f\frac{v}{v-v_s} = 540 \times \frac{330}{330-30} = 540 \times \frac{330}{300} = 594\text{ Hz}$$

where, v is speed of sound in air. Similarly, observer listens minimum frequency when source is at A, moving away from observer.

$$f_{min} = \frac{f\ v}{v+v_s} = 540 \times \frac{330}{360} = 495\text{ Hz}$$

1. Vibrating air columns :

(i) In a pipe of length L closed at one end, the fundamental note has a frequency $f_1 = \frac{v}{4L}$, where v is the velocity of sound in air.

(ii) The first overtone $f_2 = \frac{v}{L} = 2f_1$

2. Propagation of sound in solids :

(i) The velocity of propagation of a longitudinal wave in a rod of Young's modulus Y and density ρ is given by $v = \sqrt{\frac{Y}{\rho}}$

(ii) In a sonometer wire of length L and mass per unit length m under tension T vibrating in n loops

$$f_n = \frac{n}{2L}\sqrt{\frac{T}{m}}$$

(iii) Propagation of sound in gases

Laplace formula $v=\sqrt{\frac{\gamma P}{\rho}}$ where γ is the ratio of specific heats, P is the pressure and ρ is the density.

$$\frac{v_t}{v_0}=\sqrt{\frac{T}{T_0}}=\sqrt{\frac{273+t}{273}}$$

3. Doppler Effect :

(i) When a source of sound moves with a velocity v_s in a certain direction, the wavelength decreases in front of the source and increases behind the source.

$$\lambda'\text{ (in front)}=\frac{v-v_s}{f_s}\ ;\ f'=\frac{v}{\lambda'}=\frac{v}{v-v_s}f_s$$

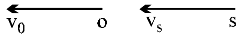
$$\lambda''\text{(behind)}=\frac{v+v_s}{f_s}\ ;\ f''=\frac{v}{\lambda''}=\frac{v}{v+v_s}f_s$$

Here v is the velocity of sound in air.

(ii) The apparent frequency $=\frac{v-v_0}{v}f_s$

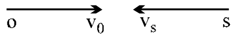
(a) When the source is moving towards the observer and the observer is moving away from the source, the apparent frequency

$$f_a=\frac{v-v_0}{v-v_s}f_s$$



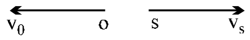
(b) When the source and the observer are moving towards each other.

$$f_a=\frac{v+v_0}{v-v_s}f_s$$



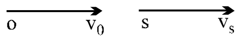
(c) When the source and observer are moving away from each other,

$$f_a=\frac{v-v_0}{v+v_s}f_s$$



(d) When the source is moving away from the observer and the observer is moving towards the source

$$f_a=\frac{v+v_0}{v+v_s}f_s$$



Here all velocities are relation to the medium.

4. Loudness of sound :

The loudness level B of sound is expressed in decibels, $B = 10 \log \frac{I}{I_0}$ where I is the intensity, I_0 is a reference intensity.

5. Beats :

When two tuning forks of close but different frequencies f_1 and f_2 are vibrating simultaneously at nearby places, a listener observes a fluctuation in the intensity of sound, called beats. The number of beats heard per second is $f_1 - f_2$.

WORKED OUT EXAMPLES

Ex.1 It is found that an increase in pressure of 100 kPa causes a certain volume of water to decrease by 5×10^{-3} percent of its original volume. Then the speed of sound in the water is about (density of water 10^3 kg/m^3)

(A) 330 m/s (B) 1414 m/s (C) 1732 m/s (D) 2500 m/s

Ans. **(B)**

Sol. Bulk modulus $\beta = -\dfrac{\Delta P}{\frac{\Delta v}{v}} = \dfrac{100 \times 10^3}{\frac{5 \times 10^{-3}}{100}} = 2 \times 10^9$

$$\text{speed } v = \sqrt{\frac{\beta}{\rho}} = \sqrt{\frac{2 \times 10^9}{10^3}} \approx 1414 \text{ m/s}$$

Ex.2 A sound wave is travelling in a uniform pipe with gas of adiabatic exponent γ. If u is the particle velocity at any point in medium and c is the wave velocity, then relative change in pressure $\frac{dP}{P}$ while wave passes through this point is :-

(A) $\dfrac{u}{\gamma c}$ (B) $\gamma\sqrt{\dfrac{u}{c}}$ (C) $\gamma\dfrac{u}{c}$ (D) $\dfrac{u^2}{\gamma c^2}$

Ans. **(C)**

Sol.

$$\Delta P = -B\left(\frac{dv}{v}\right)$$

$$\Delta P = -B\left(\frac{\delta y}{\delta x}\right)$$

$$\Delta P = -\gamma P \frac{\delta y}{\delta x}$$

$$\frac{\Delta P}{P} = \gamma\left(\frac{u}{c}\right)$$

Ex.3 A point source emits sound equally in all directions in a non-absorbing medium. Two points P and Q are at a distance of 9 meters and 25 meters respectively from the source. The ratio of the amplitudes of the waves at P and Q is :-

(A) 5 : 3 (B) 3 : 5 (C) 25 : 9 (D) 625 : 81

Ans. **(C)**

Sol. $I \propto A^2, I \propto \frac{1}{r^2}, \frac{A_1}{A_2} = \frac{r_2}{r_1}$

Ex.4 In a resonance tube experiment, an 80 cm air column is in resonance with a turning fork in first overtone. Which equation can represent correct pressure variation in the air column (x = 0 is the top point of the tube, neglect end correction, speed of sound = 320 m/sec) :-

(A) $A \sin \frac{15\pi}{8} x \cos 600\pi t$ (B) $A \cos \frac{15\pi}{8} x \sin 600\,\pi t$

(C) $A \cos \frac{15\pi}{8} x \sin 300\,\pi t$ (D) $A \sin \frac{15\pi}{8} x \sin 300\,\pi t$

Ans. **(A)**

Sol. $\frac{\omega}{k} = 320$

Ex.5 The displacement of the medium in a sound wave is given by the equation ; $y_1 = A\cos(ax + bt)$ where A, a & b are positive constants. The wave is reflected by an obstacle situated at x = 0. The intensity of the reflected wave is 0.64 times that of the incident wave.

(a) what are the wavelength & frequency of the incident wave.

(b) write the equation for the reflected wave.

(c) in the resultant wave formed after reflection, find the maximum & minimum values of the particle speeds in the medium.

Ans. (a) $2\pi/a$, $b/2\pi$, (b) $y_2 = \pm 0.8\,A\cos(ax - bt)$, (c) max.=1.8 bA, min. = 0,

Sol. (a) $\omega = b$ & $k = a$

$$f = \frac{2\pi}{\omega} \;\&\; \lambda = \frac{2\pi}{k}$$

(b) $I \propto A^2$

So $A_r = 0.8$ A

(c) $(A_{net})_{max} = A + 0.8\,A = 1.8\,A$

Ex.6 An observer moves towards a stationary source of sound, with a velocity one-fifth of the velocity of sound. what is the percentage increase in the apparent frequency ? **[AIEEE - 2005]**

(1) zero (2) 0.5% (3) 5% (4) 20%

Ans. **(4)**

Sol. $\frac{f_{app}}{f} = \left(\frac{v + \frac{v}{5}}{v}\right) = \frac{6}{5}$

Ex.7 The equation of a longitudinal standing wave due to superposition of the progressive waves produced by two sources of sound is $s = -20 \sin 10 \pi x \sin 100 \pi t$ where s is the displacement from mean position measured in mm ; x is in meters and t in seconds. The specific gravity of the medium is 10^{-3}. Find

(a) wavelength, frequency and velocity of the progressive waves.

(b) Bulk modulus of the medium and the pressure amplitude of the progressive waves.

(c) minimum distance between pressure antinode and the displacement antinode.

Ans. (a) 1/5 m, 50 Hz, 10 m/s; (b) 100 Pa, 10π Pa, (c) 1/20 m

Sol. $k = 10\pi$

$\omega = 100\,\pi$

$\because\ \lambda = \frac{1}{5}$ & $f = 50$ Hz

$v = \lambda f = 10$ m/s

$B = \rho v^2$

$P_0 = BKs$

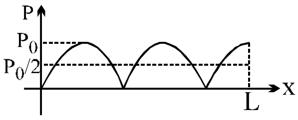
Minimum distance between pressure antinode and displacment antinode = $\frac{\lambda}{4}$

Ex.8 The air column in a pipe closed at one end and open to atmosphere at the other end is made to vibrate in its fifth harmonic by a tuning fork of frequency 470 Hz. The length of air column is $\frac{15}{16}$ m. Neglect end correction. Let p_0 denote the maximum gauge pressure at the closed end

(a) Find the speed of sound in air.

(b) Draw the graph of pressure amplitude vs distance from the open end of the tube.

(c) Find the points where the maximum gauge pressure is $\frac{p_0}{2}$.

Ans. (a) 352.5 m/s, (b) 

, (c) $\frac{l}{15}, \frac{5l}{15}, \frac{7l}{15}, \frac{11l}{15}, \frac{13l}{15}$

Sol. $\frac{5\lambda}{4} = \frac{15}{16}$

$\lambda = 0.75$ m

$f = 470$

$v = \lambda f = 352.5$ m/s

$$P = P_0 \sin\left(\frac{2\pi}{\lambda} x\right)$$

where $\lambda = \frac{4\ell}{5}$

$$P = P_0 \sin\left(\frac{5\pi}{2\ell}x\right)$$

Ex.9 A metal rod of length $l = 100$ cm is clamped at two points. Distance of each clamp from nearer end is a = 30cm. If density and Young's modulus of elasticity of rod material are $\rho = 9000$ kg m^{-3} and Y = 144 GPa respectively, calculate minimum and next higher frequency of natural longitudinal oscillations of the rod.

Ans. 10kHz, 30kHz

Sol. $v = \sqrt{\frac{Y}{\rho}} = 4 \times 10^3$ m/s

$\lambda = 0.4$ m

$f_0 = \frac{v}{\lambda} = 10\text{kHz}$

Ex.10 When two tuning forks (fork 1 and fork 2) are sounded simultaneously, 4 beats per second are heard. Now, some tape is attached on the prong of the fork 2. When the tuning forks are sounded again, 6beats per second are heard. If the frequency of fork 1 is 200 Hz, then what was the original frequency of fork 2 ? **[AIEEE - 2005]**

(1) 200 Hz (2) 202 Hz (3) 196 Hz (4) 204 Hz

Ans. **(3)**

Sol. $f_1 - f_2 = 4$ or $f_2 - f_1 = 4$

But according to question

$f_1 - f_2 = 4$

So $f_2 = 196$

Ex.11 A whistling train approaches a junction. An observer standing at junction observers the frequency to be 2.2 KHz and 1.8 KHz of the approaching and the receding train. Find the speed of the train (speed of sound = 300 m/s). **[JEE 2005]**

Ans. $V_s = 30$ m/s

Sol. $\left(\frac{v}{v - v_S}\right)f_0 = 2.2 \times 10^3$

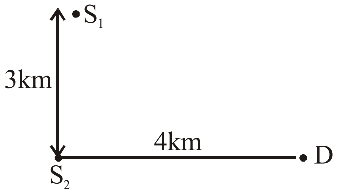
$\left(\frac{v}{v + v_S}\right)f_0 = 1.8 \times 10^3$

Dividing both we get

$\frac{v + v_S}{v - v_S} = \frac{11}{9}$

$v_S = 30$

Ex.12 Two point sound source S_1 and S_2 are both have the same power and send out sound waves in the same phase. The wavelength of both the waves is $\frac{48}{5}$ m. The intensity due to S_2 alone at D is 25 W/m^2. The resultant intensity at D is :



(A) 59 W/m^2 (B) 61 W/m^2 (C) 65 W/m^2 (D) None of these

Ans. **(B)**

Sol. $I \propto \frac{1}{r^2}$

Intensity due to S_1 alone at D = 16 W/m^2

Phase difference = $\frac{2\pi}{\left(\frac{48}{5}\right)} \times 1000 = \frac{625}{3}\pi$

$I_{res} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$

$= 25 + 16 + 2\,(5)\,(4)\left(\frac{1}{2}\right)$

Ex.13 Spherical sound waves are emitted uniformly in all directions from a point source. The variation in sound level SL as a function of distance r(> 0) from the source can be written as :- (where a and b are positive constant)

(A) SL = – b log r^a (B) SL = a – b log r (C) SL = a – b(log r)^2 (D) SL = a – $\frac{b}{r^2}$

Ans. **(B)**

Sol. $I \propto \frac{1}{r^2}$

$SL = 20\log_{10} = \frac{I}{I_0}$

EXERCISE (S-1)

Sound basics

1. Find the intensity of sound wave whose frequency is 250 Hz. The displacement amplitude of particles of the medium at this position is 1×10^{-8} m. The density of the medium is 1 kg/m^3, bulk modulus of elasticity of the medium is 400 N/m^2.

SW0001

2. In a mixture of gases, the average number of degrees of freedom per molecule is 6. The rms speed of the molecules of the gas is *c*. Find the velocity of sound in the gas.

SW0002

3. The loudness level at a distance R from a long linear source of sound is found to be 40dB. At this point, the amplitude of oscillations of air molecules is 0.01 cm. Then find the loudness level & amplitude at a point located at a distance '10R' from the source.

SW0003

Superposition of sound

4. The first overtone of a pipe closed at one end resonates with the third harmonic of a string fixed at its ends. The ratio of the speed of sound to the speed of transverse wave travelling on the string is 2 :1. Find the ratio of the length of pipe to the length of string.

SW0004

5. In a resonance-column experiment, a long tube, open at the top, is clamped vertically. By a separate device, water level inside the tube can be moved up or down. The section of the tube from the open end to the water level act as a closed organ pipe. A vibrating tuning fork is held above the open end, first and the second resonances occur when the water level is 24.1 cm and 74.1 cm respectively below the open end. Find the diameter of the tube.

SW0005

6. A tuning fork of frequency 480 Hz resonates with a tube closed at one end of length, 16 cm and diameter 5 cm in fundamental mode. Calculate velocity of sound in air. **[JEE 2003]**

SW0006

7. An open organ pipe filled with air has a fundamental frequency 500Hz. The first harmonic of another organ pipe closed at one end and filled with carbon dioxide has the same frequency as that of the first harmonic of the open organ pipe. Calculate the length of each pipe. Assume that the velocity of sound in air and in carbondioxide to be 330 and 264 m/s respectively.

SW0007

8. A steel rod having a length of 1 m is fastened at its middle. Assuming young's modulus to be 2×10^{11} Pa, and density to be 8 gm/cm^3 find the fundamental frequency of the longitudinal vibration and frequency of first overtone. **SW0008**

9. Two narrow cylindrical pipes A and B have the same length. Pipe A is open at both ends and is filled with a monoatomic gas of molar mass M_A. Pipe B is open at one end and closed at the other end, and is filled with a diatomic gas of molar mass M_B. Both gases are at the same temperature. **[JEE 2002]**

(a) If the frequency of the second harmonic of the fundamental mode in pipe A is equal to the frequency of the third harmonic of the fundamental mode in pipe B, determine the value of M_A/M_B.

(b) Now the open end of pipe B is also closed (so that the pipe is closed at both ends). Find the ratio of the fundamental frequency in pipe A to that in pipe B. **SW0009**

10. A tube of a certain diameter and of length 48 cm is open at both ends. Its fundamental frequency of resonance is found to be 320 Hz. The velocity of sound in air is 320m/sec. Estimate the diameter of the tube. **[IIT-1980]**

SW0010

Beats

11. A stretched uniform wire of a sonometer between two fixed knife edges, when vibrates in its second harmonic gives 1 beat per second with a vibrating tuning fork of frequency 200 Hz. Find the percentage change in the tension of the wire to be in unison with the tuning fork.

SW0011

12. A, B and C are three tuning forks. Frequency of A is 350Hz. Beats produced by A and B are 5 per second and by B and C are 4 per second. When a wax is put on A beat frequency between A and B is 2Hz and between A and C is 6Hz. Then, find the frequency of B and C respectively.

SW0012

13. A source of sound of frequency 256 Hz is moving rapidly towards wall with a velocity of 5 m/sec. How many beats per second will be heard if sound travels at a speed of 330 m/sec? **[IIT-1981]**

SW0013

14. Two tuning forks with natural frequencies of 340 Hz each move relative to a stationary observer. One fork moves away from the observer, while the other moves towards him at the same speed. The observer hears beats of frequency 3 Hz. Find the speed of the tuning fork (assume $v_{sound} = 340$ m/s) **[IIT-1986]**

SW0014

Doppler effect

15. Two tuning forks A and B lying on opposite sides of observer 'O' and of natural frequency 85 Hz move with velocity 10 m/s relative to stationary observer O. Fork A moves away from the observer while the fork B moves towards him. A wind with a speed 10 m/s is blowing in the direction of motion of fork A. Find the beat frequency measured by the observer in Hz. [Take speed of sound in air as 340 m/s]

SW0015

16. A car is moving towards a huge wall with a speed = c/10 , where c = speed of sound in still air. A wind is also blowing parallel to the velocity of the car in the same direction and with the same speed. If the car sounds a horn of frequency f, then what is the frequency of the reflected sound of the horn heard by driver of the car? **SW0016**

17. A plane sound wave of frequency f_0 and wavelength λ_0 travels horizontally toward the right. It strikes and is reflected from a large, rigid, vertical plane surface, perpendicular to the direction of propagation of the wave and moving towards the left with a speed v.

(a) How many positive wave crests strike the surface in a time interval t ?

(b) At the end of this time interval, how far to the left of the surface is the wave that was reflected at the beginning of the time interval ?

(c) What is the wavelength of the reflected waves, in terms of λ_0 ?

(d) What is the frequency, in terms of f_0 ?

(e) A listener is at rest at the left of the moving surface. Describe the sensation of sound that he hears as a result of the combined effect of the incident and reflected wave trains.

SW0017

18. A bus is moving towards a huge wall with a velocity of 5 ms^{-1}. The driver sounds a horn of frequency 200 Hz. The frequency of the beats heard by a passenger of the bus will be..... Hz (speed of sound in air = 342 ms^{-1}) **[IIT-1994]**

SW0018

EXERCISE (S-2)

1. A boat is travelling in a river with a speed of 10 m/s along the stream flowing with a speed 2 m/s. From this boat, a sound transmitter is lowered into the river through a rigid support. The wavelength of the sound emitted from the transmitter inside the water is 14.45 mm. Assume that attenuation of sound in water and air is negligible. **[JEE 2001]**

(a) What will be the frequency detected by a receiver kept inside the river downstream ?

(b) The transmitter and the receiver are now pulled up into air. The air is blowing with a speed 5 m/sec in the direction opposite the river stream. Determine the frequency of the sound detected by the receiver.

(Temperature of the air and water = 20°C ; Density of river water = 10^3 Kg/m^3; Bulk modulus of the water = 2.088×10^9 Pa; Gas constant R = 8.31 J/mol-K; Mean molecular mass of air = 28.8×10^{-3} kg/mol; C_P/C_V for air = 1.4)

Note: Boat velocity is with respect to ground & receiver is stationary w.r.t. ground

SW0019

2. The air column in a pipe closed at one end is made to vibrate in its second overtone by a tuning fork of frequency 440 Hz . The speed of sound in air is 330 ms^{-1}. End corrections may be neglected. Let P_0 denote the mean pressure at any point in the pipe & ΔP_0 the maximum amplitude of pressure variation. **[JEE '98]**

(i) Find the length L of the air column.

(ii) What is the amplitude of pressure variation at the middle of the column ?

(iii) What are the maximum & minimum pressures at the open end of the pipe .

(iv) What are the maximum & minimum pressures at the closed end of the pipe? **SW0020**

3. A train of length *l* is moving with a constant speed v along a circular track of radius R, The engine of the train emits a whistle of frequency f. Find the frequency heard by a guard at the rear end of the train. Make suitable assumption.

SW0021

4. A string 25 cm long and having a mass of 2.5 gm is under tension. A pipe closed at one end is 40 cm long. When the string is set vibrating in its first overtone and the air in the pipe in its fundamental frequency, 8 beats per second are heard. It is observed that decreasing the tension in the string decreases beat frequency. If the speed of sound in air is 320 m/s, find the tension in the string. **[IIT-1982]**

SW0022

5. A train approaching a hill at speed of 40 km/hr sound a whistle of frequency 580 Hz when it is at a distance of 1 km from a hill. A wind a speed of 40 km/hr blowing the direction of motion of the train. Find

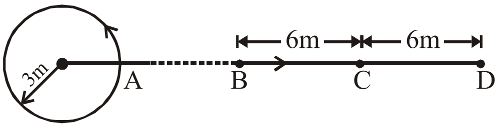
(i) the frequency of the whistle as heard by an observer on the hill.

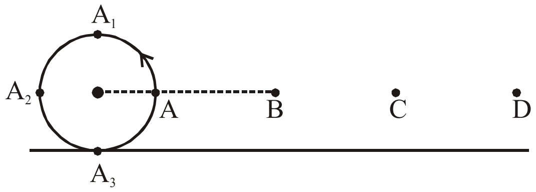
(ii) the distance from the hill at which the echo from the hill is heard by the driver and its frequency.

(Velocity of sound in air = 1,200 km/hr) **[IIT-1988]**

SW0023

6. A source of sound is moving along a circular orbit of radius 3 meteres with an angular velocity of 10 rad/s. A sound detector located far away from the source is executing linear simple harmonic motion along the line BD with an amplitude BC = CD = 6 metres. The frequency of oscillation of the detector is $\frac{5}{\pi}$ per second. The source is at the point A when the detector is at the point B. If the source is at the point A when the detector is at the point B. If the source emits a continuous sound wave of frequency 340 Hz, find the maximum and the minimum frequencies recorded by the detector. **[IIT-1990]**





SW0024

7. Two radio stations broadcast their programmes at the same amplitude A and at slight different frequencies ω_1 and ω_2 respectively, where $\omega_1 - \omega_2 = 10^3$ Hz A detector receives the signals from the two stations simultaneously. It can only detect signals of intensity $\geq 2\ A^2$.

(i) Find the time interval between successive maxima of the intensity of the signal received by the detector.

(ii) Find the time for which the detector remains idle in each cycle of the intensity of the signal.

[IIT-1993]

SW0025

8. A sonometer wire under tension of 64 Newtons vibrating in its fundamental mode is in resonance with a vibrating tuning fork. The vibrating portion of the sonometer wire has a length of 10 cm and a mass of 1 gm. The vibrating tuning fork is now moved away of 1 gm. The vibrating wire with a constant speed and an observer standing near the sonometer hears oen beat per second. Calculate the speed with which the tuning fork is moved if the speed of sound in air is 300 m/s. **[IIT-1983]**

SW0026

9. The displacement of the medium in a sound wave is given by the equation $y_1 = A\cos(ax + bt)$ where A, a and b are positive constants. The wave is reflected by an obstacle situated at x = 0. The intensity of the reflected wave is 0.64 times that of the incident wave. **[IIT-1991]**

(a) What are the wavelength and frequency of incident wave?

(b) Write the equation for the reflected wave.

(c) In the resultant wave formed after reflection, find the maximum and minimum values of the particle speeds in the medium.

(d) Express the resultant wave as a superpositions of standing wave and a travelling wave. What are the positions of the antinodes of the standing wave? What is the directions of propagation of travelling wave?

SW0027

10. The air colomn in a pipe closed at one end is made to vibrate in its second overtone by a tuning fork of frequency 440 Hz. The speed of sound in air is 330 ms^{-1}. End corrections may be neglected. Let P_0 denote the mean pressure at any point in the pipe, and ΔP_0 the maximum amplitude of pressure variation.

(a) Find the length L of the air column.

(b) What is the amplitude of pressure variation at the middle of the column?

(c) What are the maximum and minimum pressure at the open end of the pipe?

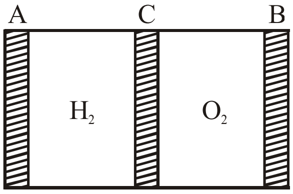
(d) What are the maximum and minimum pressure at the closed end of the pipe? **[IIT-1998]**

SW0028

11. A 3.6m long vertical pipe resonates with a source of frequency 212.5 Hz when water level is at certain height in the pipe. Find the height of water level (from the bottom of the pipe) at which resonance occurs. Neglect end correction. Now, the pipe is filled to a height H ($\approx$ 3.6 m). A small hole is drilled very close to its bottom and water is allowed to leak. Obtain and expression for the rate of fall of water level in the pipe as a function of H. If the radii of the pipe and the hole are 2×10^{-2} m and 1×10^{-3} m respectively calculate the time interval between the occurance of first two resonances. Speed of sound in air is 340 m/s and g = 10 m/s^2. **[IIT-2000]**

SW0029

12. AB is a cylinder of length 1m fitted a thin flexible diaphragm C at the middle and other thin flexible diaphragms A and B at the ends. The portions AC and BC contain hydrogen and oxygen gases respectively. The diaphragms A and B are set into vibrations of same frequency. What is the minimum frequency of these vibration for which diaphragm C is a not? (Under the conditions of experiment $v_{H_2} = 1100$ m/s, $v_{O_2} = 300$ m/s). **[IIT-1978]**



EXERCISE (O-1)

Sound basics

1. A sound wave has a wavelength of 3.0 m. The distance from a compression center to the adjacent rarefaction center is :-

(A) 0.75 m (B) 1.5 m

(C) 3.0 m (D) need to know wave speed **SW0030**

2. You are listening to an "A" note played on a violin string. Let the subscript "s" refer to the violin string and "a" refer to the air. Then :-

(A) $f_s = f_a$ but $\lambda_s \neq \lambda_a$ (B) $f_s = f_a$ and $\lambda_s = \lambda_a$

(C) $\lambda_s = \lambda_a$ but $f_s \neq f_a$ (D) $\lambda_s \neq \lambda_a$ and $f_s \neq f_a$ **SW0031**

3. The fig.(i) shows the graphical representation of the air molecules in a tube of air (length = L) at atmospheric pressure on the absolute pressure P(x) graph. Which one of the following pictures corresponds to the absolute pressure P(x) graph of fig. (ii).

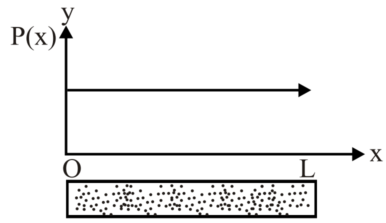


Figure (i)

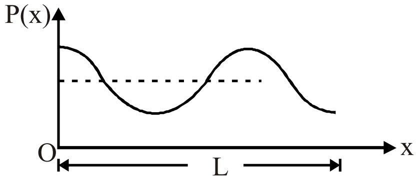
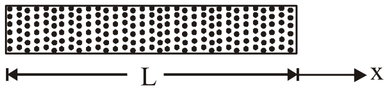
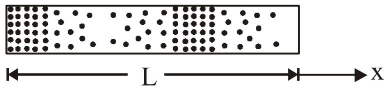
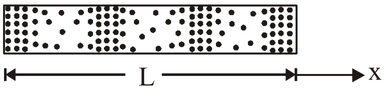
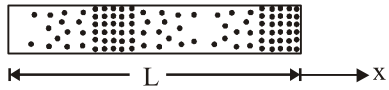


Figure (ii)

(A)


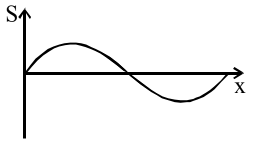
(B)


(C)


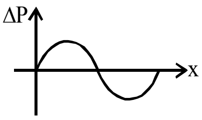
(D)


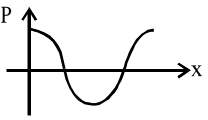
SW0032

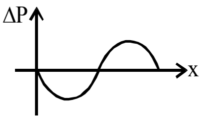
4. If a sound wave is travelling and snap shot at t = 0 is as shown in figure.

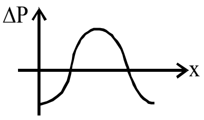


Choose snapshot of pressure variation.

(i)


(ii)


(iii)


(iv)


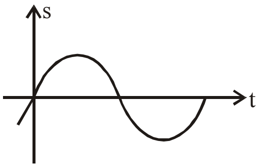
(A) For wave travelling towards right or left (i) is correct.

(B) For wave travelling towards right graph (iv) and for wave travelling towards left graph (iv) is correct.

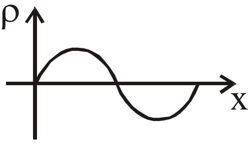
(C) For wave travelling towards right graph (i) and for wave travelling towards left graph (iii) is correct

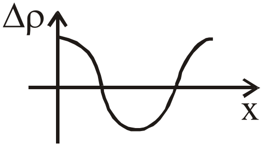
(D) For wave travelling towards right or left (ii) is correct. **SW0033**

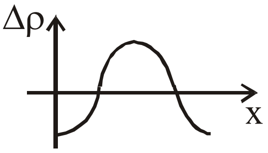
5. A sound waves is travelling towards right and its s-t graph is as shown for x = 0.

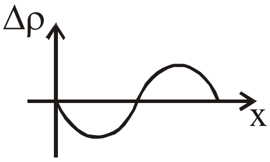


What will be the variation in density vs x graph at t = T/4 :-

(A)


(B)


(C)


(D)


SW0034

6. A point source of sound is located somewhere along the x-axis. Experiments show that the same wave front simultaneously reaches listeners at x = –8 m and x = +2.0 m.
A third listener is positioned along the positive y-axis. What is her y-coordinate (in m) if the same wave front reaches her at the same instant as it does the first two listeners ?

(A) 4 (B) 3 (C) 2 (D) 5

SW0035

7. Two monatomic ideal gases 1 and 2 of molecular masses m_1 and m_2 respectively are enclosed in separate container kept at the same temperature. The ratio of the speed of sound in gas 1 to that in gas 2 is given by **[JEE 2000 (Scr)]**

(A) $\sqrt{\frac{m_1}{m_2}}$ (B) $\sqrt{\frac{m_2}{m_1}}$ (C) $\frac{m_1}{m_2}$ (D) $\frac{m_2}{m_1}$

SW0036

8. A firecracker exploding on the surface of a lake is heard as two sounds a time interval *t* apart by a man on a boat close to water surface. Sound travels with a speed *u* in water and a speed *v* in air. The distance from the exploding firecracker to the boat is

(A) $\frac{uvt}{u+v}$ (B) $\frac{t(u+v)}{uv}$ (C) $\frac{t(u-v)}{uv}$ (D) $\frac{uvt}{u-v}$ **SW0037**

9. The speed of longitudinal wave is 100 times the speed of transverse wave in a taut brass wire. If the Young's modulus of brass is 1.0×10^{11} N/m^2, the stress in wire is :-

(A) 1.0×10^7 N/m^2 (B) 1.0×10^6 N/m^2 (C) 1.0×10^5 N/m^2 (D) 1.0×10^8 N/m^2

SW0038

10. The equations of S.H.M. of medium particle due to sound waves propagating in a medium are given by $s_1 = 2 \sin (200\pi t)$ and $s_2 = 5 \sin (150\pi t)$. The ratio of average intensities of sound at these points is:

(A) 4 : 25 (B) 9 : 100 (C) 8 : 15 (D) 64 : 225

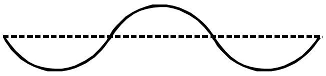
SW0039

11. A is singing a note and at the same time B is also singing a note with 1/8th the frequency of A. The energies of the two sounds are equal. The displacement amplitude of the note of B is:

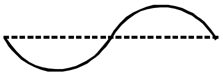
(A) same as that of A
(B) twice that of A
(C) four times that of A
(D) eight times that of A

SW0040

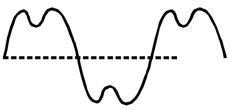
12. A microphone is connected to an oscilloscope. The diagram shows the trace on the screen when the microphone receives a pure note. Which trace can be obtained when a musical instrument produces a note of the same pitch but of a different quality ?



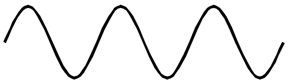
(A)



(B)



(C)



(D)

SW0041

13. Choose **correct** statement?

(A) Two different acoustic musical instruments can not have same loudness
(B) Two different acoustic musical instruments can not have same pitch
(C) Two different acoustic musical instruments can not have same quality
(D) Two different acoustic musical instruments can have more than two characteristics same

SW0042

14. A plane transverse wave is propagating in a direction making an angle of 30° with positive x-axis in the x-y plane. Find phase difference between points (0, 0, 0) and (1, 1, 1). Wavelength of the wave is 1m :-

(A) 2π rad
(B) $\left(\sqrt{3}+1\right)\pi$ rad
(C) $\left(\sqrt{2}+1\right)\pi$ rad
(D) None

SW0043

15. Which of the following is the equation of a spherical wave :-

(A) $S = S_0 \sin(Kx - \omega t)$
(B) $S = S_0 \cos(Kx - \omega t)$
(C) $S = (S_0/x) \sin(\omega t - Kx)$
(D) $S = (S_0/x^2) \sin(\omega t - Kx)$

SW0044

16. A note is produced when you blow air across the top of a test tube. Two students were asked about the effect of blowing harder.

Student-A : The pitch of sound would increase.

Student-B : The intensity of sound would increase

(A) A is correct, B is wrong
(B) B is correct, A is wrong
(C) both are correct
(D) both are wrong

SW0045

17. A sound absorber attenuates the sound level by 20 dB. The intensity decreases by a factor of-

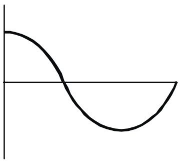
[AIEEE - 2007]

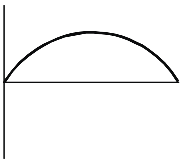
(A) 1000
(B) 10000
(C) 10
(D) 100

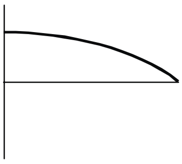
SW0046

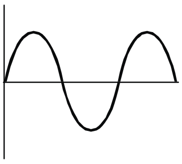
Standing waves

18. Which of the figures, shows the pressure difference from regular atmospheric pressure for an organ pipe of length L closed at one end, corresponds to the 1st overtone for the pipe?

(A) 

(B) 

(C) 

(D) 

SW0047

19. In an organ pipe whose one end is at x = 0, the pressure is expressed by

$p = p_0 \cos\frac{3\pi x}{2} \sin 300\pi t$ where x is in meter and t in sec. The organ pipe can be

(A) closed at one end, open at another with length = 0.5m

(B) open at both ends, length = 1m

(C) closed at both ends, length = 2m

(D) closed at one end, open at another with length = $\frac{2}{3}$m **SW0048**

20. If l_1 and l_2 are the lengths of air column for the first and second resonance when a tuning fork of frequency n is sounded on a resonance tube, then the distance of the displacement antinode from the top end of the resonance tube is:

(A) $2(l_2 - l_1)$ (B) $\frac{1}{2}(2l_1 - l_2)$ (C) $\frac{l_2 - 3l_1}{2}$ (D) $\frac{l_2 - l_1}{2}$

SW0049

21. A student is experimenting with resonance tube apparatus in Physics lab to find the speed of sound at room temperature. He got resonating lengths of air column as 17 cm and 51 cm, using tuning fork of frequency 512 Hz. Find speed of sound at room temperature and specify, whether the side water reservoir was moved upward or downward to obtain the second resonance (51 cm)?

(A) 348 m/s, downwards (B) 348 m/s, upwards

(C) 332 m/s, downwards (D) 332 m/s, upwards **SW0050**

Interference

22. The ratio of maximum to minimum intensity due to superposition of two waves is $\frac{49}{9}$. Then the ratio of the intensity of component waves is :-

(A) $\frac{25}{4}$ (B) $\frac{16}{25}$ (C) $\frac{4}{49}$ (D) $\frac{9}{49}$

SW0051

23. Two waves of sound having intensities I and 4I interfere to produce interference pattern. The phase difference between the waves is $\pi/2$ at point A and π at point B. Then the difference between the resultant intensities at A and B is

(A) 2I (B) 4I (C) 5I (D) 7I

SW0052

24. Three coherent waves of equal frequencies having amplitude 10 μm, 4μm and 7 μm respectively, arrive at a given point with successive phase difference of π/2. The amplitude of the resulting wave in μm is given by

(A) 5 (B) 6 (C) 3 (D) 4

SW0053

25. The ratio of intensities between two coherent soud sources is 4 : 1. The differenmce of loudness in dB between maximum and minimum intensities when they interfere in space is:

(A) 10 log 2 (B) 20 log 3 (C) 10 log 3 (D) 20 log 2

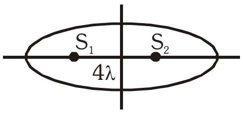
SW0054

26. In Quincke's tube a detector detects minimum intensity. Now one of the tube is displaced by 5 cm. During displacement detector detects maximum intensity 10 times, then finally a minimum intensity (when displacement is complete). The wavelength of sound is:

(A) 10/9 cm (B) 1 cm (C) 1/2 cm (D) 5/9 cm

SW0055

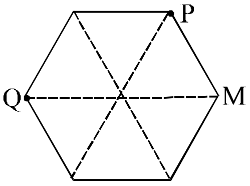
27. S_1, S_2 are two coherent sources of sound located along x – axis separated by 4λ where λ is wavelength of sound emitted by them. Number of maxima located on the elliptical boundary around it will be :



(A) 16 (B) 12 (C) 8 (D) 4

SW0056

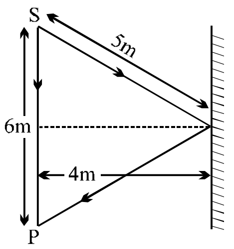
28. Two sound source emitting sound of wavelength 1 m are located at points P and Q as shown in figure. All sides of the polygon are equal and of length 1m. The intensity of sound at M due to both the individual sources is I_0. What will be the intensity of sound at point M when both the sources are on.



(A) $4I_0$ (B) $2I_0$ (C) I_0 (D) $(1/2)I_0$

SW0057

29. A person standing at a distance of 6 m from a source of sound receives sound wave in two ways, one directly from the source and other after reflection from a rigid boundary as shown in the figure. The maximum wavelength for which, the person will receive maximum sound intensity, is



(A) 4 m (B) $\frac{16}{3}$ m (C) 2 m (D) $\frac{8}{3}$ m

SW0058

Beats

30. Beats are heard when the A strings of two violins are played. The beat frequency decreases as the tension in the A string of violin 1 is slowly increased. Which of the following statement is correct?

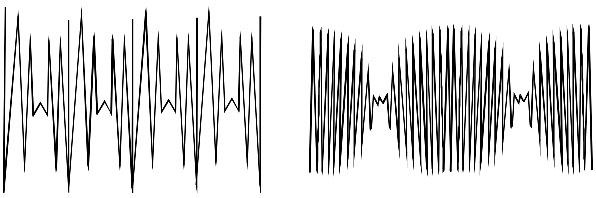
(A) the fundamental frequency of the A string in violin 1 is less than that for violin 2

(B) the fundamental frequency of the A string in violin 1 is greater than that for violin 2

(C) the fundamental frequency of the A string in violin 1 may be greater or less than that for violin 2 depending on the linear mass densities of the two strings.

(D) None of these **SW0059**

31. Two waves with similar frequencies are added. The resulting waveform oscillates with the average frequency and with an oscillating amplitude that changes with a frequency equal to the difference between the original frequencies. These oscillations in the amplitude are known as beats. The traces show the resulting waveforms that occur when two different pairs of waves are added. Graph is for the same time interval in both cases, which of the following statements is TRUE?



(A) On average, the waves on the left had higher frequencies, but the difference between frequencies less

(B) On average, the waves on the right had higher frequencies, but the difference between frequencies less

(C) On average, the waves on the left had higher frequencies, but the difference between frequencies more

(D) On average, the waves on the right had higher frequencies, but the difference between frequencies more

SW0060

Doppler effect

32. A source when at rest in a medium produces waves with a velocity v and a wavelength of λ. If the source is set in motion with a velocity v_s what would be the wavelengths produced directly in front of the source?

(A) $\lambda\left(1-\frac{v_s}{v}\right)$ (B) $\lambda\left(1+\frac{v_s}{v}\right)$ (C) $\lambda\left(1+\frac{v}{v_s}\right)$ (D) $\frac{\lambda v}{v+v_s}$

SW0061

33. A source of sound S having frequency f. Wind is blowing from source to observer O with velocity u. If speed of sound with respect to air is C, the wavelength of sound detected by O is:

(A) $\frac{C+u}{f}$ (B) $\frac{C-u}{f}$ (C) $\frac{C(C+u)}{(C-u)f}$ (D) $\frac{C}{f}$

SW0062

34. A train moving towards a hill at a speed of 72 km/hr sounds a whistle of frequency 500 Hz. A wind is blowing from the hill at a speed of 36 km/ hr. If the speed of sound in air is 340 m / s, the frequency heard by a man on the hill is

(A) 532.25 Hz. (B) 565.0 Hz. (C) 516.1 Hz. (D) none of the above.

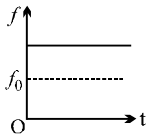
SW0063

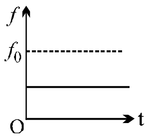
35. A source is moving with constant speed $v_s = 20$ m/sec towards a stationary observer due east of source. Wind is blowing at the speed of 20 m/sec due to 60° north of east. The source is generating of frequency 500 Hz. Then frequency registered by observer is: [Speed of sound in still air = 330 m/sec.]

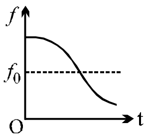
(A) 500 Hz (B) 532 Hz (C) 531 Hz (D) 530 Hz

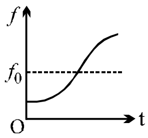
SW0064

36. Source and observer both start moving simultaneously from origin, one along x-axis and the other along y-axis with speed of source = twice the speed of observer. The graph between the apparent frequency observed by observer f and time t would approximately be :

(A) 

(B) 

(C) 

(D) 

SW0065

37. A siren placed at a railway platform is emitting sound of frequency 5 kHz. A passenger sitting in a moving train A records a frequency of 5.5 kHz while the train approaches the siren. During his return journey in a different train B he records a frequency of 6.0 kHz while approaching the same siren. The ratio of the velocity of train B to that of train A is :-

(A) 242/252 (B) 2 (C) 5/6 (D) 11/6

SW0066

38. A police van moving with velocity 22 m/s and emitting sound of frequency 176 Hz, follows a motor cycle in turn is moving towards a stationary car and away from the police van. The stationary car is emitting frequency 165 Hz. If motorcyclist does not hear any beats then his velocity is ($v_s = 330$ m/s)

(A) 22 m/s (B) 24 m/s (C) 20 m/s (D) 18 m/s

SW0067

EXERCISE (O-2)

1. Two tuning forks of frequency 250 Hz and 256 Hz produce beats. If a maximum of intensity is observed just now, after how much time the minimum is observed at the same place ?

(A) $\frac{1}{18}$ sec (B) $\frac{1}{4}$ sec. (C) $\frac{1}{3}$ sec. (D) $\frac{1}{12}$ sec.

SW0068

2. The particle displacement of a travelling longitudinal wave is represented by S = S (x, t). The midpoints of a compression zone and an adjacent rarefaction zone are represented by the letter 'C' and 'R'. Which of the following is true?

(A) $|\partial S / \partial x|_C = |\partial S / \partial x|_R$

(B) $|\partial S / \partial t|_C = |\partial S / \partial t|_R = 0$

(C) $(\text{pressure})_C - (\text{pressure})_R = 2\,|\partial S / \partial x|_C$ x Bulk modulus of air.

(D) Particles of air are stationary mid-way between 'C' and 'R'.

SW0069

3. Which of the following statements are wrong about the velocity of sound in air:

(A) decreases with increases in temperature (B) increases with decrease in temperature

(C) decreases as humidity increases (D) independent of density of air.

SW0070

4. A car moves towards a hill with speed v_c. It blows a horn of frequency f which is heard by an observer following the car with speed v_0. The speed of sound in air is v.

(A) the wavelength of sound reaching the hill is $\frac{v}{f}$

(B) the wavelength of sound reaching the hill is $\frac{v - v_c}{f}$

(C) the beat frequency observed by the observer is $\left(\frac{v + v_o}{v - v_c}\right)f$

(D) the beat frequency observed by the observer is $\frac{2v_c(v + v_o)f}{v^2 - v_c^2}$

SW0071

5. Three coherent source kept along the same line produce intensity I_0 each at point P on this line. When S_1 & S_2 are switched on simultaneously, intensity at point P is $2I_0$. When S_2 and S_3 are switched on simultaneously, intensity at point P is $2I_0$. Then

(A) When S_1 and S_3 are switched on simultaneously, intensity at point P can be $2I_0$

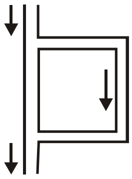
(B) When S_1 and S_3 are switched on simultaneously, intensity at point P can be 0

(C) When all 3 sources are switched on simultaneously, intensity at point P can be I_0

(D) When all 3 sources are switched on simultaneously, intensity at point P can be $3I_0$

SW0072

6. A sound consists of four frequencies → 300 Hz, 900 Hz, 2400 Hz and 4500 Hz. A sound 'filter' is made by passing this sound through a bifurcated pipe as shown. The sound waves have to travel a distance of 50 cm more in the right branch-pipe than in the straight pipe. The speed of sound in air is 300 m/s. Then, which of the following frequencies will be almost completely muffled or "silenced" at the outlet?



(A) 300 Hz (B) 900 Hz (C) 2400 Hz (D) 4500 Hz

SW0073

Paragraph for Question No. 7 to 9

A metallic rod of length 1m has one end free and other end rigidly clamped. Longitudinal stationary waves are set up in the rod in such a way that there are total six antinodes present along the rod. The amplitude of an antinode is 4×10^{-6} m. Young's modulus and density of the rod are 6.4×10^{10} N/m^2 and 4×10^3 Kg/m^3 respectively. Consider the free end to be at origin and at t=0 particles at free end are at positive extreme.

7. The equation describing displacements of particles about their mean positions is

(A) $s = 4\times10^{-6}\cos\left(\frac{11\pi}{2}x\right)\cos\left(22\pi\times10^{3}t\right)$ (B) $s = 4\times10^{-6}\cos\left(\frac{11\pi}{2}x\right)\sin\left(22\pi\times10^{3}t\right)$

(C) $s = 4\times10^{-6}\cos(5\pi x)\cos\left(20\pi\times10^{3}t\right)$ (D) $s = 4\times10^{-6}\cos(5\pi x)\sin\left(20\pi\times10^{3}t\right)$

SW0074

8. The equation describing stress developed in the rod is

(A) $140.8\pi\times10^{4}\cos\left(\frac{11}{2}\pi x+\pi\right)\cos\left(22\pi\times10^{3}t\right)$

(B) $140.8\pi\times10^{4}\sin\left(\frac{11}{2}\pi x+\pi\right)\cos\left(22\pi\times10^{3}t\right)$

(C) $128\pi\times10^{4}\cos(5\pi x+\pi)\cos\left(20\pi\times10^{3}t\right)$

(D) $128\pi\times10^{4}\sin(5\pi x+\pi)\cos\left(20\pi\times10^{3}t\right)$

SW0074

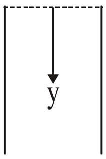
9. The magnitude of strain at midpoint of the rod at t= 1 sec is

(A) $11\sqrt{3}\pi\times10^{-6}$ (B) $11\sqrt{2}\pi\times10^{-6}$ (C) $10\sqrt{3}\pi\times10^{-6}$ (D) $10\sqrt{2}\pi\times10^{-6}$

SW0074

Paragraph for Question No. 10 to 12

In an organ pipe (may be closed or open) of length 1m standing wave is setup, whose equation for longitudinal displacement is given by $\xi = (0.1 \text{ mm}) \cos \frac{2\pi}{0.8}(y) \cos(400)t$ where y is measured from the top of the tube in meters and t in second.



10. The upper end and the lower ends of the tube are respectively:
(A) open – closed (B) closed – open (C) open – open (D) closed – closed
SW0075

11. The air column is vibrating in
(A) First overtone (B) Second overtone (C) Third harmonic (D) Fundamental mode
SW0075

12. Equation of the standing wave in terms of excess pressure is (Bulk modulus of air $B = 5 \times 10^5 \text{ N/m}^2$)

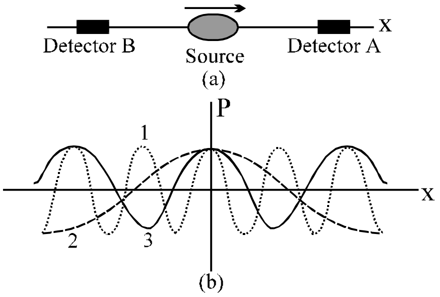
(A) $P_{ex} = (125\,\pi\text{N/m}^2) \sin \frac{2\pi}{0.8}(y) \cos(400t)$ (B) $P_{ex} = (125\,\pi\text{N/m}^2) \cos \frac{2\pi}{0.8}(y) \sin(400t)$

(C) $P_{ex} = (225\,\pi\text{N/m}^2) \sin \frac{2\pi}{0.8}(y) \cos(200t)$ (D) $P_{ex} = (225\,\pi\text{N/m}^2) \cos \frac{2\pi}{0.8}(y) \sin(200t)$

SW0075

Paragraph for Question No. 13 to 16

A source emitting a sound wave at a certain frequency moves with constant speed along an x-axis figure (a). The source moves directly towards a stationary detector A and directly away from another stationary detector B. The superimposed three plots of figure (b) indicate the pressure function P(x) of the sound wave as measured by detector A, by detector B, and by someone (c) in the rest frame of the source.



13. Which of the following plot corresponds to the measurement done by detector A?
(A) 1 (B) 2 (C) 3 (D) These plots are not possible
SW0076

14. The plot corresponding to the measurement done by detector B is
(A) 1 (B) 2 (C) 3 (D) These plots are not possible
SW0076

15. The plot corresponding to the measurement done by the detector C is

(A) 1 (B) 2 (C) 3 (D) These plots are not possible

SW0076

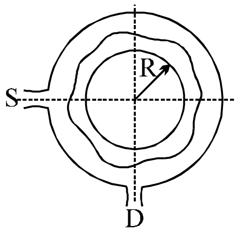
16. Now the source stops and begins to move along y-axis with same speed, the plot which corresponds to the measurement of B now is

(A) 1 (B) 2 (C) 3 (D) none of these

SW0076

Paragraph for Question No. 17 to 21

A narrow tube is bent in the form of a circle of radius R, as shown in the figure. Two small holes S and D are made in the tube at the positions right angle to each other. A source placed at S generated a wave of intensity I_0 which is equally divided into two parts : One part travels along the longer path, while the other travels along the shorter path. Both the part waves meet at the point D where a detector is placed



17. If a maxima is formed at the detector then, the magnitude of wavelength λ of the wave produced is given by :-

(A) πR (B) $\frac{\pi R}{2}$ (C) $\frac{\pi R}{4}$ (D) $\frac{2\pi R}{3}$

SW0077

18. If the minima is formed at the detector then, the magnitude of wavelength λ of the wave produced is given by :-

(A) $2\pi R$ (B) $\frac{3\pi R}{2}$ (C) $\frac{2\pi R}{3}$ (D) $\frac{2\pi R}{5}$

SW0077

19. The maximum intensity produced at D is given by :-

(A) $4I_0$ (B) $2I_0$ (C) I_0 (D) $3I_0$

SW0077

20. The maximum value of λ to produce a maxima at D is given by :-

(A) πR (B) $2\pi R$ (C) $\frac{\pi R}{2}$ (D) $\frac{3\pi R}{2}$

SW0077

21. The maximum value of λ to produce a minima at D is given by :-

(A) πR (B) $2\pi R$ (C) $\frac{\pi R}{2}$ (D) $\frac{3\pi R}{2}$

SW0077

Paragraph for Question Nos. 22 to 24

Two waves $y_1 = A\cos(0.5\pi x - 100\pi t)$ and $y_2 = A\cos(0.46\pi x - 92\pi t)$ are travelling in a pipe placed along x-axis. **[JEE 2006]**

22. Find the number of times intensity is maximum in time interval of 1 sec.

(A) 4 (B) 6 (C) 8 (D) 10

SW0078

23. Find wave velocity of louder sound

(A) 100 m/s (B) 192 m/s (C) 200 m/s (D) 96 m/s

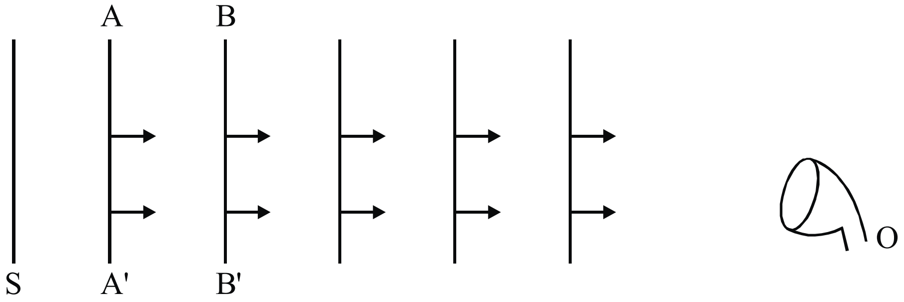
SW0078

24. At x = 0 how many times the value of $y_1 + y_2$ is zero in one second?

(A) 100 (B) 46 (C) 192 (D) 96

SW0078

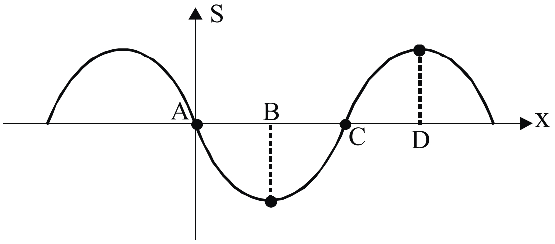
25. Consider a large plane diaphragm 'S' emitting sound and a detector 'O'. The diagram shows plane wavefronts for the sound wave travelling in air towards right when source, observer and medium are at rest. AA' and BB' are fixed imaginary planes. Column-I describes about the motion of source, observer or medium and column-II describes various effects. Match them correctly.



	Column-I		Column-II
(A)	Source starts moving towards right	(P)	Distance between any two wavefronts will increase.
(B)	Air starts moving towards right	(Q)	Distance between any two wavefronts will decrease.
(C)	Observer and source both move towards left with same speed.	(R)	The time needed by sound to move from plane AA' to BB' will increase.
(D)	Source and medium (air) both move towards right with same speed.	(S)	The time needed by sound to move from plane AA' to BB' will decrease.
		(T)	Frequency received by observer increases.

SW0079

26. Figure shows a graph of particle displacement function of x at t = 0 for a longitudinal wave travelling in positive x-direction in a gas. A,B,C,D denote position of particles in space.

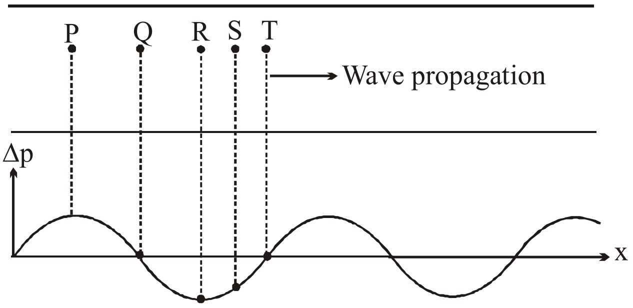


	Column-I		Column II
(A)	point A	(P)	Particle velocity is in direction of wave propagation
(B)	point B	(Q)	Maximum magnitude of strain
(C)	point C	(R)	Excess pressure is zero
(D)	point D	(S)	Maximum density
		(T)	Maximum magnitude of excess pressure

SW0080

27. Sound is travelling in a long tube towards right and the graph of excess pressure variation versus position (at some instant) is given below.

Match velocities in column-I with column-II. P,Q,R,S,T are medium particles inside the tube.



	Column-I		Column-II
(A)	velocity is towards right	(P)	P
(B)	velocity is towards left	(Q)	Q
(C)	velocity is zero	(R)	R
(D)	Speed is maximum	(S)	S
		(T)	T

SW0081

EXERCISE - JM

1. **Statement-1:** Two longitudinal waves given by equations : $y_1(x, t) = 2a \sin(\omega t - kx)$ and $y_2(x, t) = a \sin(2\omega t - 2kx)$ will have equal intensity.

Statement-1: Intensity of waves of given frequency in same medium is proportional to square of amplitude only. **[AIEEE - 2011]**

(1) Statement-1 is false, statement-2 is true

(2) Statement-1 is ture, statement-2 is false

(3) Statement-1 is ture, statement-2 true; statement-2 is the correct explanation of statement-1

(4) Statement-1 is true, statement-2 is true; statement -2 is not correct explanation of statement-1.

SW0082

2. A pipe of length 85 cm is closed from one end. Find the number of possible natural oscillations of air column in the pipe whose frequencies lie below 1250 Hz. The velocity of sound in air is 340 m/s.

[JEE Main - 2014]

(1) 6 (2) 4 (3) 12 (4) 8

SW0083

3. A train is moving on a straight track with speed 20 ms^{-1}. It is blowing its whistle at the frequency of 1000 Hz. The percentage change in the frequency heard by a person standing near the track as the train passes him is (speed of sound = 320 ms^{-1}) close to :-

[JEE Main - 2015]

(1) 18% (2) 24% (3) 6% (4) 12%

SW0084

4. A pipe open at both ends has a fundamental frequency f in air. The pipe is dipped vertically in water so that half of it is in water. The fundamental frequencty of the air column is now :-

[JEE-Main-2016]

(1) f (2) $\frac{f}{2}$ (3) $\frac{3f}{4}$ (4) 2f

SW0085

5. An observer is moving with half the speed of light towards a stationary microwave source emitting waves at frequency 10 GHz. What is the frequency of the microwave measured by the observer? (speed of light = 3×10^8 ms^{-1})

[JEE Main - 2017]

(1) 17.3 GHz (2) 15.3 GHz (3) 10.1 GHz (4) 12.1 GHz

SW0086

6. A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is 2.7×10^3 kg/m^3 and its Young's modulus is 9.27×10^{10} Pa. What will be the fundamental frequency of the longitudinal vibrations ?

[JEE-Main-2018]

(1) 2.5 kHz (2) 10 kHz (3) 7.5 kHz (4) 5 kHz

SELECTED PROBLEMS FROM JEE-MAINS ONLINE PAPERS

7. Two coherent sources produce waves of different intensities which interfere. After interference, the ratio of the maximum intensity to the minimum intensity is 16. The intensity of the waves are in the ratio: **[JEE-Main-2019_Jan]**

(1) 4 : 1 (2) 25 : 9 (3) 16 : 9 (4) 5 : 3

8. A resonance tube is old and has jagged end. It is still used in the laboratory to determine velocity of sound in air. A tuning fork of frequency 512 Hz produces first resonance when the tube is filled with water to a mark 11 cm below a reference mark, near the open end of the tube. The experiment is repeated with another fork of frequency 256 Hz which produces first resonance when water reaches a mark 27 cm below the reference mark. The velocity of sound in air, obtained in the experiment, is close to: **[JEE-Main-2019_Jan]**

(1) $328 ms^{-1}$ (2) $322 ms^{-1}$ (3) $341 ms^{-1}$ (4) $335 ms^{-1}$

9. A person standing on an open ground hears the sound of a jet aeroplane, coming from north at an angle 60° with ground level. But he finds the aeroplane right vertically above his position. If υ is the speed of sound, speed of the plane is : **[JEE-Main-2019_Jan]**

(1) $\frac{2\upsilon}{\sqrt{3}}$ (2) υ (3) $\frac{\upsilon}{2}$ (4) $\frac{\sqrt{3}}{2}\upsilon$

10. The pressure wave, $P = 0.01 \sin[1000t - 3x]\ Nm^{-2}$, corresponds to the sound produced by a vibrating blade on a day when atmospheric temperature is 0°C. On some other day, when temperature is T, the speed of sound produced by the same blade and at the same frequency is found to be $336\ ms^{-1}$. Approximate value of T is : **[JEE-Main-2019_April]**

(1) 15°C (2) 12°C (3) 4°C (4) 11°C

11. A tuning fork of frequency 480 Hz is used in an experiment for measuring speed of sound (ν) in air by resonance tube method. Resonance is observed to occur at two successive lengths of the air column, $l_1 = 30$ cm and $l_2 = 70$ cm. Then ν is equal to : **[JEE-Main-2019_April]**

(1) $332\ ms^{-1}$ (2) $379\ ms^{-1}$ (3) $384\ ms^{-1}$ (4) $338\ ms^{-1}$

12. A small speaker delivers 2 W of audio output. At what distance from the speaker will one detect 120dB intensity sound ? [Given reference intensity of sound as $10^{-12} W/m^2$]

[JEE-Main-2019_April]

(1) 10 cm (2) 30 cm (3) 40 cm (4) 20 cm

13. A submarine (A) travelling at 18 km/hr is being chased along the line of its velocity by another submarine (B) travelling at 27 km/hr. B sends a sonar signal of 500 Hz to detect A and receives a reflected sound of frequency ν. The value of ν is close to : (Speed of sound in water = $1500\ ms^{-1}$)

(1) 499 Hz (2) 502 Hz (3) 507 Hz (4) 504 Hz

14. A one metre long (both ends open) organ pipe is kept in a gas that has double the density of air at STP. Assuming the speed of sound in air at STP is 300 m/s, the frequency difference between the fundamental and second harmonic of this pipe is ________ Hz.

[JEE-Main-2020_Jan]

15. Three harmonic waves having equal frequency ν and same intensity I_0, have phase angles 0, $\frac{\pi}{4}$ and $-\frac{\pi}{4}$ respectively. When they are superimposed the intensity of the resultant wave is close to :

[JEE-Main-2020_Jan]

(1) $5.8\ I_0$ (2) $0.2\ I_0$ (3) I_0 (4) $3\ I_0$

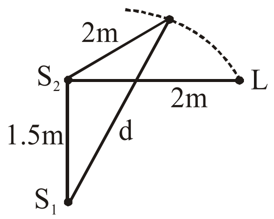
16. In a resonance tube experiment when the tube is filled with water up to height of 17.0 cm from bottom, it resonates with a given tuning fork. When the water level is raised the next resonance with the same tuning fork occurs at a height of 24.5 cm. If the velocity of sound in air is 330 m/s, the tuning fork frequency is: **[JEE-Main-2020_Sep]**

(1) 1100 Hz (2) 3300 Hz (3) 2200 Hz (4) 550 Hz

17. Assume that the displacement(s) of air is proportional to the pressure difference (Δp) created by a sound wave. Displacement(s) further depends on the speed of sound (v), density of air (ρ) and the frequency (f). If $\Delta p \sim 10$Pa, $v \sim 300$ m/s, $p \sim 1$ kg/m^3 and $f \sim 1000$Hz, then s will be the order of (take multiplicative constant to be 1) **[JEE-Main-2020_Sep]**

(1) 10 mm (2) $\frac{3}{100}$ mm (3) 1 mm (4) $\frac{1}{10}$ mm

18. Two coherent sources of sound, S_1 and S_2, produce sound waves of the same wavelength, $\lambda = 1$ m, in phase. S_1 and S_2 are placed 1.5 m apart (see fig.) A listener, located at L, directly in front of S_2 finds that the intensity is at a minimum when he is 2m away from S_2. The listener moves away from S_1, keeping his distance from S_2 fixed. The adjacent maximum of intensity is observed when the listener is at a distance d from S_1. Then, d is : **[JEE-Main-2020_Sep]**



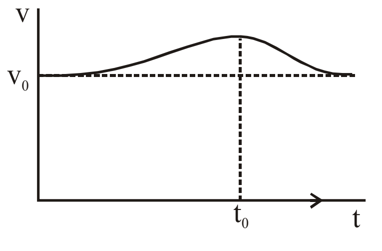
(1) 12m (2) 3m (3) 5m (4) 2m

19. A sound source S is moving along a straight track with speed v, and is emitting sound of frequency v_0 (see figure). An observer is standing at a finite distance, at the point O, from the track. The time variation of frequency heard by the observer is best represented by :

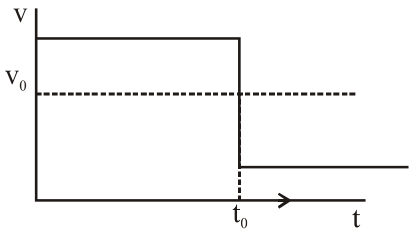
(t_0 represents the instant when the distance between the source and observer is minimum)

[JEE-Main-2020_Sep]

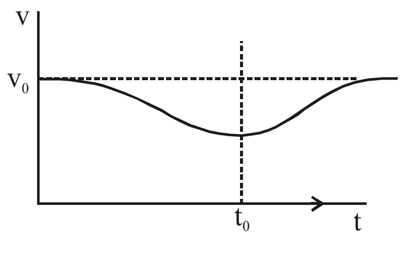
(1)



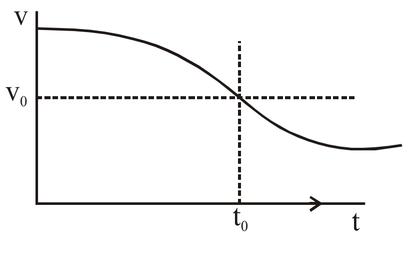
(2)



(3)



(4)



20. A wire of density 9×10^{-3} kg cm^{-3} is stretched between two clamps 1 m apart. The resulting strain in the wire is 4.9×10^{-4}. The lowest frequency of the transverse vibrations in the wire is (Young's modulus of wire $Y = 9 \times 10^{10}$ Nm^{-2}), (to the nearest integer),_________. **[JEE-Main-2020_Sep]**

EXERCISE (JA)

1. When two progressive waves $y_1 = 4\sin(2x - 6t)$ and $y_2 = 3\sin\left(2x - 6t - \frac{\pi}{2}\right)$ are superimposed, the amplitude of the resultant wave is **[IIT-JEE 2010]**

SW0087

2. **Column I** shows four systems, each of the same length L, for producing standing waves. The lowest possible natural frequency of a system is called its fundamental frequency, whose wavelength is denoted as λ_f. Match each system with statements given in **Column II** describing the nature and wavelength of the standing waves. **[IIT-JEE 2011]**

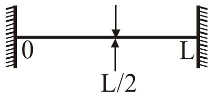
Column I

(A) Pipe closed at one end

(B) Pipe open at both ends

(C) Stretched wire clamped at both ends

(D) Stretched wire clamped at both ends and at mid-point



Column II

(p) Longitudinal waves

(q) Transverse Waves

(r) $\lambda_f = L$

(s) $\lambda_f = 2L$

(t) $\lambda_f = 4L$

SW0088

3. A police car with a siren of frequency 8 kHz is moving with uniform velocity 36 km/hr towards a tall building which reflects the sound waves. The speed of sound in air is 320 m/s. The frequency of the siren heard by the car driver is **[JEE 2011]**

(A) 8.50 kHz (B) 8.25 kHz (C) 7.75 kHz (D) 7.50 kHz

SW0089

4. A person blows into open-end of a long pipe. As a result, a high-pressure pulse of air travels down the pipe. When this pulse reaches the other end of the pipe. **[JEE 2012]**

(A) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is open

(B) a low -pressure pulse starts travelling up the pipe, if the other end of the pipe is open

(C) a low pressure pulse starts travelling up the pipe, if the other end of the pipe is closed

(D) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed

SW0090

5. A student is performing the experiment of Resonance Column. The diameter of the column tube is 4cm. The frequency of the tuning fork is 512 Hz. The air temperature is 38°C in which the speed of sound is 336 m/s. The zero of the meter scale coincides with the top end of the Resonance column tube. When the first resonance occurs, the reading of the water level in the column is :- **[JEE 2012]**

SW0091

(A) 14.0 cm (B) 15.2 cm (C) 16.4 cm (D) 17.6 cm

6. A student is performing an experiment using a resonance column and a tuning fork of frequency 244 s^{-1}. He is told that the air in the tube has been replaced by another gas (assume that the column remains filled with the gas). If the minimum height at which resonance occurs is (0.350 ± 0.005) m, the gas in the tube is

(**Useful information :** $\sqrt{167RT} = 640\ J^{1/2}\ mole^{-1/2}$; $\sqrt{140RT} = 590\ J^{1/2}\ mole^{-1/2}$. The molar masses M in grams are given in the options. Take the values of $\sqrt{\frac{10}{M}}$ for each gas as given there.)

[JEE Advanced-2014]

(A) Neon $\left(M = 20, \sqrt{\frac{10}{20}} = \frac{7}{10}\right)$ (B) Nitrogen $\left(M = 28, \sqrt{\frac{10}{28}} = \frac{3}{5}\right)$

(C) Oxygen $\left(M = 32, \sqrt{\frac{10}{32}} = \frac{9}{16}\right)$ (D) Argon $\left(M = 36, \sqrt{\frac{10}{36}} = \frac{17}{32}\right)$ **SW0092**

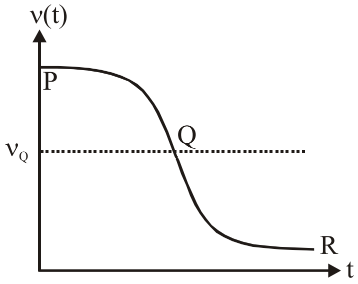
7. Four harmonic waves of equal frequencies and equal intensities I_0 have phase angles 0, π/3, 2π/3 and π. When they are superposed, the intensity of the resulting wave is nI_0. The value of n is.

[JEE-Advance-2015]

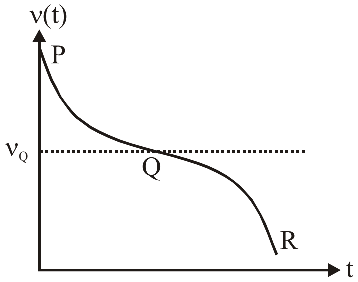
SW0093

8. Two loudspeakers M and N are located 20m apart and emit sound at frequencies 118 Hz and 121 Hz, respectively. A car is initially at a point P, 1800 m away from the midpoint Q of the line MN and moves towards Q constantly at 60 km/hr along the perpendicular bisector of MN. It crosses Q and eventually reaches a point R, 1800 m away from Q. Let $\nu(t)$ represent the beat frequency measured by a person sitting in the car at time t. Let ν_P, ν_Q and ν_R be the beat frequencies measured at locations P, Q and R, respectively. The speed of sound in air is 330 ms^{-1}. Which of the following statement(s) is(are) true regarding the sound heard by the person ? **[JEE Advanced 2016]**

(A) The plot below represents schematically the variation of beat frequency with time



(B) The plot below represents schematically the variations of beat frequency with time



(C) The rate of change in beat frequency is maximum when the car passes through Q

(D) $\nu_P + \nu_R = 2\nu_Q$

SW0094

9. A stationary source emits sound of frequency f_0 = 492 Hz. The sound is reflected by a large car approaching the source with a speed of 2 ms^{-1}. The reflected signal is received by the source and superposed with the original. What will be the beat frequency of the resulting signal in Hz ? (Given that the speed of sound in air is 330 ms^{-1} and the car reflects the sound at the frequency it has received).

[JEE Advanced 2017]

SW0095

10. Two men are walking along a horizontal straight line in the same direction. The man in front walks at a speed 1.0 ms^{-1} and the man behind walks at a speed 2.0 ms^{-1}. A third man is standing at a height 12m above the same horizontal line such that all three men are in a vertical plane. The two walking men are blowing identical whistles which emit a sound of frequency 1430 Hz. The speed of sound in air is 330 ms^{-1}. At the instant, when the moving men are 10 m apart, the stationary man is equidistant from them. The frequency of beats in Hz, heard by the stationary man at this instant, is _____

[JEE Advanced 2018]

SW0096

11. In an experiment to measure the speed of sound by a resonating air column, a tuning fork of frequency 500 Hz is used. The length of the air column is varied by changing the level of water in the resonance tube. Two successive resonances are heard at air columns of length 50.7 cm and 83.9 cm. Which of the following statements is (are) true ? **[JEE Advanced 2018]**

(A) The speed of sound determined from this experiment is 332 ms^{-1}

(B) The end correction in this experiment is 0.9 cm

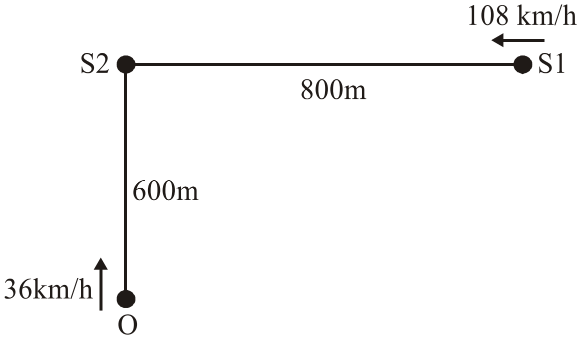
(C) The wavelength of the sound wave is 66.4 cm

(D) The resonance at 50.7 cm corresponds to the fundamental harmonic

SW0097

12. A train S1, moving with a uniform velocity of 108 km/h, approaches another train S2 standing on a platform. An observer O moves with a uniform velocity of 36 km/h towards S2, as shown in figure. Both the trains are blowing whistles of same frequency 120 Hz. When O is 600 m away from S2 and distance between S1 and S2 is 800 m, the number of beats heard by O is ____.

[Speed of the sound = 330 m/s] **[JEE Advanced 2019]**



SW0098

13. A stationary tuning fork is in resonance with an air column in a pipe. If the tuning fork is moved with a speed of 2 ms^{-1} in front of the open end of the pipe and parallel to it, the length of the pipe should be changed for the resonance to occur with the moving tuning fork. If the speed of sound in air is 320 ms^{-1}, the smallest value of the percentage change required in the length of the pipe is _________.-

[JEE Advanced 2020]

ANSWER KEY

EXERCISE (S-1)

1. Ans. $\frac{\pi^2 \times 10^{-9}}{4}$ W/m^2 **2. Ans.** 2c/3 **3. Ans.** 30 dB, $10\sqrt{10}$ μm **4. Ans.** 1 : 1

5. Ans. 3 cm **6. Ans.** 336 m/s **7. Ans.** 33 cm and 13.2 cm

8. Ans. 2.5 kHz, 7.5 kHz **9. Ans.** (a) 2.116, (b) $\frac{3}{4}$ **10. Ans.** 3.33 cm, 163 Hz

11. Ans. 1% **12. Ans.** 345, 341 or 349 Hz **13. Ans.** 8

14. Ans. 1.5 m/s **15. Ans. 5** **16. Ans.** 11f / 9

17. Ans. (a) $\left(\frac{v+\lambda_0 f_0}{\lambda_0}\right)t$ (b) $\left(\lambda_0 f_0 - v\right)t$ (c) $\lambda_0\left(\frac{\lambda_0 f_0 - v}{\lambda_0 f_0 + v}\right)$ (d) $f_0\left(\frac{\lambda_0 f_0 + v}{\lambda_0 f_0 - v}\right)$ (e) $\frac{2vf_0}{\lambda_0 f_0 - v}$ **18. Ans.** 6 Hz

EXERCISE (S-2)

1. Ans. (a) 100696 Hz (b) 103038 Hz

2. Ans. (i) $L = \frac{15}{16}$ m, (ii) $\frac{\Delta P_0}{\sqrt{2}}$, (iii) $P_{max} = P_{min} = P_0$ (iv) $P_{max} = P_0 + \Delta P_0$, $P_{min} = P_0 - \Delta P_0$

3. Ans. f **4. Ans.** 27.04 N **5. Ans.** (i) 599 Hz, (ii) 0.935 km, 620 Hz

6. Ans. 438.7 Hz, 257.3 Hz **7. Ans. (i) 10^{-3} sec, (ii) 2×10^{-3} sec**

8. Ans. 0.75 m/s

9. Ans. (a) $\frac{2\pi}{a}, \frac{b}{2\pi}$, (b) $y = -0.8\,A\cos(ax - bt)$ **OR** $y = 0.8\,A\cos(ax - bt)$, (c) 1.8 Ab, 0

(d) $y = -1.6\,A \sin ax \sin bt + 0.2\,A\cos(ax + bt)$, $\left[n + \frac{(-1)^2}{2}\right]\frac{\pi}{a}$, –X direction **OR**

$y = 0.2\,A\cos(ax + bt) + 1.6\,A\cos ax \cos bt$, $x = \frac{n\pi}{a}$, –X direction

10. Ans. (a) $\frac{15}{16}$m (b) $\frac{\Delta P_0}{\sqrt{2}}$ (c) equal to mean pressure (d) $P_0 + \Delta P_0$, $P_0 - \Delta P_0$

11. Ans. $\frac{-dH}{dt} = (1.11 \times 10^{-2})\sqrt{H}$, 43 sec. **12. Ans. 1650 Hz**

EXERCISE (O-1)

1. Ans. (B)
2. Ans. (A)
3. Ans. (B)
4. Ans. (B)
5. Ans. (A)
6. Ans. (A)
7. Ans. (B)
8. Ans. (D)
9. Ans. (A)
10. Ans. (D)
11. Ans. (D)
12. Ans. (B)
13. Ans. (C)
14. Ans. (B)
15. Ans. (C)
16. Ans. (B)
17. Ans. (D)
18. Ans. (A)
19. Ans. (C)
20. Ans. (C)
21. Ans. (A)
22. Ans. (A)
23. Ans. (B)
24. Ans. (A)
25. Ans. (B)
26. Ans. (B)
27. Ans. (A)
28. Ans. (A)
29. Ans. (A)
30. Ans. (A)
31. Ans. (B)
32. Ans. (A)
33. Ans. (A)
34. Ans. (A)
35. Ans. (C)
36. Ans. (B)
37. Ans. (B)
38. Ans. (A)

EXERCISE (O-2)

1. Ans. (B,D)
2. Ans. (A,C,D)
3. Ans. (A,B,C,D)
4. Ans. (B,D)
5. Ans. (B, C)
6. Ans. (A, B, D)
7. Ans. (A)
8. Ans. (B)
9. Ans. (B)
10. Ans. (A)
11. Ans. (B)
12. Ans. (A)
13. Ans. (A)
14. Ans. (B)
15. Ans. (C)
16. Ans. (D)
17. Ans. (A,B,C)
18. Ans. (A,C,D)
19. Ans. (B)
20. Ans. (A)
21. Ans. (B)
22. Ans. (A)
23. Ans. (C)
24. Ans. (A)
25. Ans. (A) (Q,T) ; (B) (P,S) ; (C) (P) (D) (S,T)
26. Ans. (A) (P,Q,S,T); (B) (R); (C) (Q,T); (D) (R)
27. Ans. (A) (P); (B) (R,S); (C) (Q,T); (D) (P,R)

EXERCISE (JM)

1. Ans. (4)
2. Ans. (1)
3. Ans. (4)
4. Ans. (1)
5. Ans. (1)
6. Ans. (4)

SELECTED PROBLEMS FROM JEE-MAINS ONLINE PAPERS

7. Ans. (2)
8. Ans. (1)
9. Ans. (3)
10. Ans. (3)
11. Ans. (3)
12. Ans. (3)
13. Ans. (2)
14. Ans. (106.00 to 107.20)
15. Ans. (1)
16. Ans. (3)
17. Ans. (2)
18. Ans. (2)
19. Ans. (4)
20. Ans. 35.00

EXERCISE (JA)

1. Ans. 5
2. Ans. (A) p,t (B) p,s (C) q,s (D) q,r
3. Ans. (A)
4. Ans. (B,D)
5. Ans. (B)
6. Ans. (D)
7. Ans. 3
8. Ans. (A, C, D)
9. Ans. 6
10. Ans. 5 [4.99, 5.01]
11. Ans. (A,B,C)
12. Ans. (8.12 to 8.13)
13. Ans. (0.62 to 0.64)

Important Notes

Important Notes

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E

PHYSICS

Chapter Contents 01

ELECTROSTATICS

Important Notes

ELECTROSTATICS

KEY CONCEPT

ELECTRIC CHARGE

Charge is the property associated with matter due to which it produces and experiences electrical and magnetic effects. The excess or deficiency of electrons in a body gives the concept of charge.

SI unit of charge : ampere × second i.e. Coulomb Dimension : [A T]

Practical units of charge are ampere × hour (=3600 C) and faraday (= 96500 C)

- Millikan calculated quanta of charge by 'Highest common factor' (H.C.F.) method and it is equal to charge of electron.
- 1 C = 3 × 10^9 state coulomb, 1 absolute - coulomb = 10 C, 1 Faraday = 96500 C.

SPECIFIC PROPERTIES OF CHARGE

- **Charge is a scalar quantity :** It represents excess or deficiency of electrons.
- **Charge is transferable :** If a charged body is put in contact with an another body, then charge can be transferred to another body.
- **Charge is always associated with mass**

  Charge cannot exist without mass though mass can exist without charge.
  - So the presence of charge itself is a convincing proof of existence of mass.
  - In charging, the mass of a body changes.
  - When body is given positive charge, its mass decreases.
  - When body is given negative charge, its mass increases.
- **Charge is quantized**

  The quantization of electric charge is the property by virtue of which all free charges are integral multiple of a basic unit of charge represented by e. Thus charge q of a body is always given by

  $q = ne$

  n = positive integer or negative integer

  The quantum of charge is the charge that an electron or proton carries.

  Note : Charge on a proton = (–) charge on an electron = 1.6 × 10^{-19} C
- **Charge is conserved**

  In an isolated system, total charge does not change with time, though individual charge may change i.e. charge can neither be created nor destroyed. Conservation of charge is also found to hold good in all types of reactions either chemical (atomic) or nuclear. No exceptions to the rule have ever been found.
- **Charge is invariant**

  Charge is independent of frame of reference. i.e. charge on a body does not change whatever be its speed.
- **Attraction – Repulsion**

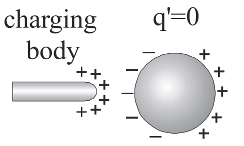
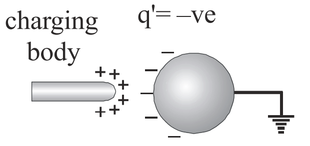
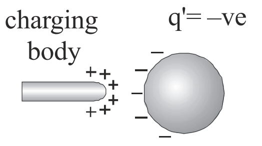
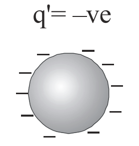
  Similar charges repel each other while dissimilar attract

METHODS OF CHARGING

- **Friction:** If we rub one body with other body, electrons are transferred from one body to the other.
- **Electrostatic induction**

  If a charged body is brought near a metallic neutral body, the charged body will attract opposite charge and repel similar charge present in the neutral body. As a result of this one side of the neutral body becomes negative while the other positive, this process is called 'electrostatic induction'.

  • Charging a body by induction (in four successive steps)

 charged body is brought near uncharged body step-1	 uncharged body is connected to earth step-2	 uncharged body is disconnected from the earth step-3	 charging body is removed step-4

Some important facts associated with induction-

(i) Inducing body neither gains nor loses charge

(ii) The nature of induced charge is always opposite to that of inducing charge

(iii) Induction takes place only in bodies (either conducting or non conducting) and not in particles.

- **Conduction**

  The process of transfer of charge by contact of two bodies is known as conduction. If a charged body is put in contact with uncharged body, the uncharged body becomes charged due to transfer of electrons from one body to the other.

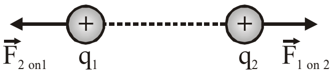
  - The charged body loses some of its charge (which is equal to the charge gained by the uncharged body)
  - The charge gained by the uncharged body is always lesser than initial charge present on the charged body.

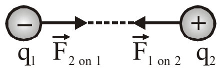
COULOMB'S LAW

The electrostatic force of interaction between two static point electric charges is directly proportional to the product of the charges, inversely proportional to the square of the distance between them and acts along the straight line joining the two charges.

Force of electrostatic interaction depends on the nature of medium between the charges. If two points charges q_1 and q_2 separated by a distance r. Let F be the electrostatic force between these two charges. According to Coulomb's law.

$$F \propto q_1 q_2 \text{ and } F \propto \frac{1}{r^2}$$

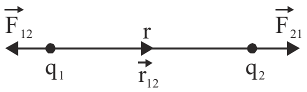




$$F_e = \frac{kq_1q_2}{r^2}$$

where $\left[k = \frac{1}{4\pi \in_0} = 9\times10^9 \frac{Nm^2}{C^2}\right]$ k = coulomb's constant or electrostatic force constant

Coulomb's law in vector form

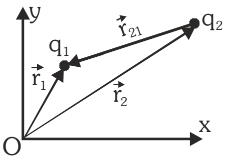


$\vec{F}_{12}$ = force on q_1 due to q_2 = $\frac{kq_1q_2}{r^2}\hat{r}_{21}$

$\vec{F}_{21} = \frac{kq_1q_2}{r^2}\hat{r}_{12}$ (here $\hat{r}_{12}$ is unit vector from q_1 to q_2)

Coulomb's law in terms of position vector

$$\vec{F}_{12} = \frac{kq_1q_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2)$$



Principle of superposition

When a number of charges are interacting, the total force on a given charge is vector sum of the forces exerted on it by all other charges individually

$$F = \frac{kq_0q_1}{r_1^2} + \frac{kq_0q_2}{r_2^2} + \cdots + \frac{kq_0q_i}{r_i^2} + \ldots \frac{kq_0q_n}{r_n^2}$$

in vector form $\vec{F} = kq_0 \sum_{i=1}^{n} \frac{q_i}{r_i^2}\hat{r}_i$

Some important points regarding Coulomb's law and electric force

- The law is based on physical observations and is not logically derivable from any other concept. Experiments till today reveal its universal nature.
- The force is a two body interaction, i.e., electrical force between two point charges is independent of presence or absence of other charges and so the principle of superposition is valid, i.e., force on a charged particle due to number of point charges is the resultant of forces due to individual point charges, i.e., $\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \ldots.$
- The net Coulomb's force on two charged particles in free space and in a medium filled upto infinity are

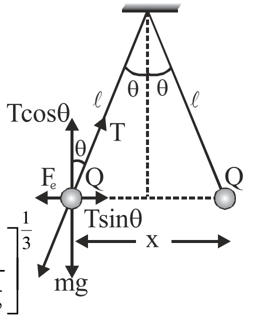
$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} \text{ and } F' = \frac{1}{4\pi\varepsilon}\frac{q_1q_2}{r^2}. \text{ So } \frac{F}{F'} = \frac{\varepsilon}{\varepsilon_0} = K,$$

- Dielectric constant (K) of a medium is numerically equal to the ratio of the force on two point charges in free space to that in the medium filled upto infinity.
- The law expresses the force between two point charges at rest. In applying it to the case of extended bodies of finite size care should be taken in assuming the whole charge of a body to be concentrated at its 'centre' as this is true only for spherical charged body, that too for external point.

Although net electric force on both particles change in the presence of dielectric but force due to one charge particle on another charge particle does not depend on the medium between them.

☐ Equilibrium of suspended point charge system

For equilibrium position

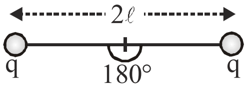


$$T\cos\theta = mg \text{ and } T\sin\theta = F_e = \frac{kQ^2}{x^2} \Rightarrow \tan\theta = \frac{F_e}{mg} = \frac{kQ^2}{x^2 mg}$$

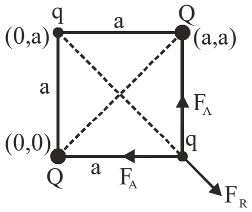
- If θ is small then

$$\tan\theta \approx \sin\theta = \frac{x}{2\ell} \Rightarrow \frac{x}{2\ell} = \frac{kQ^2}{x^2 mg} \Rightarrow x^3 = \frac{2kQ^2\ell}{mg} \Rightarrow x = \left[\frac{Q^2\ell}{2\pi\epsilon_0\, mg}\right]^{\frac{1}{3}}$$

- If whole set up is taken into an artificial satellite ($g_{eff} \simeq 0$) $T = F_e = \dfrac{kq^2}{4\ell^2}$



Ex. For the system shown in figure find Q for which resultant force on q is zero.



Sol. For force on q to be zero, charges q and Q must be of opposite of nature.

Net attraction force on q due to charges Q = Repulsion force on q due to q

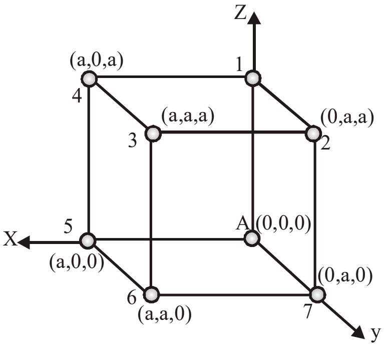
$$\sqrt{2}\,F_A = F_R \Rightarrow \sqrt{2}\,\frac{kQq}{a^2} = \frac{kq^2}{(\sqrt{2}a)^2} \Rightarrow q = 2\sqrt{2}\,Q \text{ Hence } q = -2\sqrt{2}\,Q$$

Ex. Given a cube with point charges q on each of its vertices. Calculate the force exerted on any of the charges due to rest of the 7 charges.

Sol. The net force on particle A can be given by vector sum of force experienced by this particle due to all the other charges on vertices of the cube. For this we use vector form of coulomb's law

$$\vec{F} = \frac{kq_1q_2}{\left|\vec{r}_1 - \vec{r}_2\right|^3}\left(\vec{r}_1 - \vec{r}_2\right)$$

From the figure the different forces acting on A are given as



$$\vec{F}_{A_1} = \frac{kq^2\left(-a\hat{k}\right)}{a^3}$$

$$\vec{F}_{A_2} = \frac{kq^2\left(-a\hat{j} - a\hat{k}\right)}{\left(\sqrt{2}a\right)^3},\ \vec{F}_{A_3} = \frac{kq^2\left(-a\hat{i} - a\hat{j} - a\hat{k}\right)}{\left(\sqrt{3}a\right)^3},$$

$$\vec{F}_{A_4} = \frac{kq^2\left(-a\hat{j} - a\hat{k}\right)}{\left(\sqrt{2}a\right)^3},\ \vec{F}_{A_5} = \frac{kq^2\left(-a\hat{j}\right)}{a^3},\ \vec{F}_{A_6} = \frac{kq^2\left(-a\hat{i} - a\hat{j}\right)}{\left(\sqrt{2}a\right)^3},\ \vec{F}_{A_7} = \frac{kq^2\left(-a\hat{j}\right)}{a^3}$$

The net force experienced by A can be given as

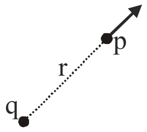
$$\vec{F}_{net} = \vec{F}_{A_1} + \vec{F}_{A_2} + \vec{F}_{A_3} + \vec{F}_{A_4} + \vec{F}_{A_5} + \vec{F}_{A_6} + \vec{F}_{A_7} = \frac{-kq^2}{a^2}\left[\left(\frac{1}{3\sqrt{3}} + \frac{1}{\sqrt{2}} + 1\right)\left(\hat{i} + \hat{j} + \hat{k}\right)\right]$$

ELECTRIC FIELD

In order to explain 'action at a distance', i.e., 'force without contact' between charges it is assumed that a charge or charge distribution produces a field in space surrounding it. So the region surrounding a charge (or charge distribution) in which its electrical effects are perceptible is called the electric field of the given charge. Electric field at a point is characterized either by a vector function of position $\vec{E}$ called 'electric intensity' or by a scalar function of position V called 'electric potential'. The electric field in a certain space is also visualized graphically in terms of 'lines of force.' So electric intensity, potential and lines of force are different ways of describing the same field.

Intensity of electric field due to point charge

Electric field intensity is defined as force on unit test charge.



$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0} = \frac{kq}{r^2}\hat{r} = \frac{kq}{r^3}\vec{r}$$

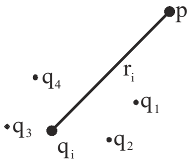
Note : Test charge (q_0) is a fictitious charge that exerts no force on nearby charges but experiences forces due to them.

Unit : N/C, V/m **Dimensions** : $[MLT^{-3}A^{-1}]$

Due to discrete distribution of charge

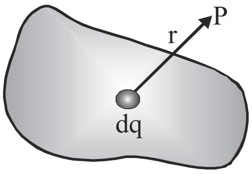
Field produced by a charge distribution for discrete distribution:–

By principle of superposition intensity of electric field due to i^th charge $\vec{E}_{ip} = \frac{kq}{r_i^3}\vec{r}_i$



$\therefore$ Net electric field due to whole distribution of charge $\vec{E}_p = \sum_{i=1} \vec{E}_i$

Continuous distribution of charge



Treating a small element as particle $\vec{E} = \frac{1}{4\pi \in_0} \int \frac{dq}{r^3}\vec{r}$

Due to linear charge distribution $E = k\int_{\ell} \frac{\lambda d\ell}{r^2}$ [λ = charge per unit length]

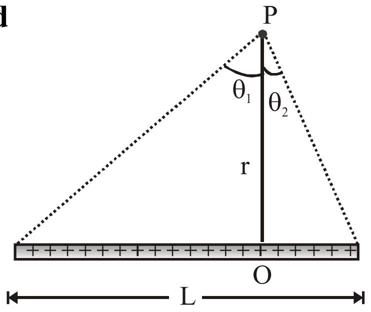
Due to surface charge distribution $E = k\int_{s} \frac{\sigma ds}{r^2}$ [σ = charge per unit area]

Due to volume charge distribution $E = k\int_{v} \frac{\rho dv}{r^2}$ [ρ = charge per unit volume]

Key points

- Charged particle in an electric field always experiences a force either it is at rest or in motion.
- In presence of a dielectric , electric field decreases and becomes $\frac{1}{\in_r}$ times of its value in free space.
- Test charge is always a unit (+ ve) charge. $\vec{E} = \frac{\vec{F}_{test}}{\text{test charge}}$
- If identical charges are placed on each vertices of a regular polygon, then $\vec{E}$ at centre = zero.

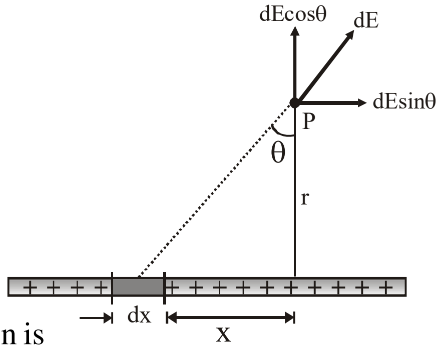
Electric field strength at a general point due to a uniformly charged rod



As shown in figure, if P is any general point in the surrounding of rod, to find electric field strength at P, we consider an element on rod of length dx at a distance x from point O as shown in figure. Now if dE be the electric field at P due to the element, then

$$dE = \frac{kdq}{\left(x^2 + r^2\right)} \quad \text{Here } dq = \frac{Q}{L}dx$$

Electric field strength in x–direction due to dq at P is



$$dE_x = dE\sin\theta = \left[\frac{kdq}{\left(x^2+r^2\right)}\right]\sin\theta = \frac{kQ\sin\theta}{L\left(x^2+r^2\right)}dx$$

Here we have x = r tanθ and dx = r sec²θ dθ

$$\text{Thus } dE_x = \frac{kQ}{L}\ \frac{r\sec^2\theta\ d\theta}{r^2\sec^2\theta}\ \sin\theta, \text{ Strength} = \frac{kQ}{Lr}\ \sin\theta\ d\theta$$

Net electric field strength due to dq at point P in x–direction is

$$E_x = \int dE_x = \frac{kQ}{Lr}\int_{-\theta_2}^{+\theta}\sin\theta\ d\theta = \frac{kQ}{Lr}\left[-\cos\theta\right]_{-\theta_2}^{+\theta_1} = \frac{kQ}{Lr}\left[\cos\theta_2 - \cos\theta_1\right]$$

Similarly, electric field strength at point P due to dq in y–direction is

$$dE_y = dE\cos\theta = \frac{kQdx}{L\left(r^2+x^2\right)}\times\cos\theta$$

Again we have x = r tanθ and dx = r sec² θdθ.

$$\text{Thus we have } dE_y = \frac{kQ}{L}\cos\theta \times \frac{r\sec^2\theta}{r^2\sec^2\theta}d\theta = \frac{kQ}{Lr}\ \cos\theta\ d\theta$$

Net electric field strength at P due to dq in y–direction is

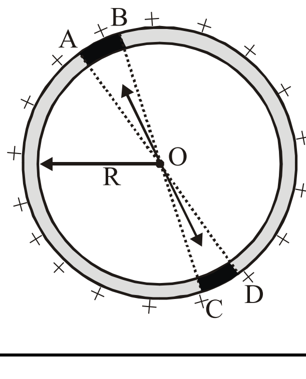
$$E_y = \int dE_y = \frac{kQ}{Lr}\int_{-\theta_2}^{+\theta_1}\cos\theta\ d\theta = \frac{kQ}{Lr}\left[+\sin\theta\right]_{-\theta_2}^{+\theta_1} = \frac{kQ}{Lr}\left[\sin\theta_1 + \sin\theta_2\right]$$

Thus electric field at a general point in the surrounding of a uniformly charged rod which subtend angles θ_1 and θ_2 at the two corners of rod can be given as

in x–direction : $E_x = \frac{kQ}{Lr}\ (\cos\theta_2 - \cos\theta_1)$ and in y–direction $E_y = \frac{kQ}{Lr}\ (\sin\theta_1 - \sin\theta_2)$

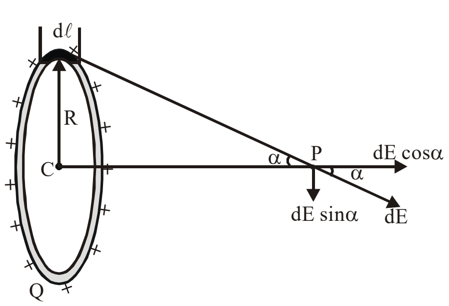
Electric field due to a uniformly charged ring

Case – I : At its centre



Here by symmetry we can say that electric field strength at centre due to every small segment on ring is cancelled by the electric field at centre due to the segment exactly opposite to it. The electric field strength at centre due to segment AB is cancelled by that due to segment CD. This net electric field strength at the centre of a uniformly charged ring is E_0=0

Case II : At a point on the axis of Ring



Here we'll find the electric field strength at point P due to the ring which is situated at a distance x from the ring centre. For this we consider a small section of length dℓ on ring as shown. The charge on this elemental section is

$$dq = \frac{Q}{2\pi R} d\ell \quad \text{[Q= total charge of ring]}$$

Due to the element dℓ, electric field strength dE at point P can be given as $dE = \dfrac{Kdq}{\left(R^2 + x^2\right)}$

The component of this field strength dE sin α which is normal to the axis of ring will be cancelled out due to the ring section opposite to dℓ. The component of electric field strength along the axis of ring $dE_{\cos\alpha}$ due to all the sections will be added up. Hence total electric field strength at point P due to the ring is

$$E_p = \int dE\cos\alpha = \int_O^{2\pi R} \frac{kdq}{\left(R^2+x^2\right)} \times \frac{x}{\sqrt{R^2+x^2}} = \int_O^{2\pi R} \frac{kQx}{2\pi R\left(R^2+x^2\right)} d\ell = \frac{kQx}{2\pi R\left(R^2+x^2\right)^{3/2}} \int_O^{2\pi R} d\ell$$

$$= \frac{kQ_x}{2\pi R\left(R^2+x^2\right)^{3/2}}[2\pi R] = \frac{kQx}{\left(R^2+x^2\right)^{3/2}}$$

Electric field strength due to a charged circular arc at its centre :

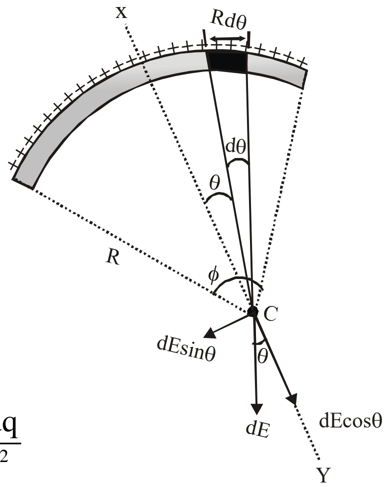


Figure shows a circular arc of radius R which subtend an angle φ at its centre. To find electric field strength at C, we consider a polar segment on arc of angular width dθ at an angle θ from the angular bisector XY as shown. The length of elemental segment is Rdθ, the charge on this element dℓ is $dq = \left(\dfrac{Q}{\phi}\right).d\theta$

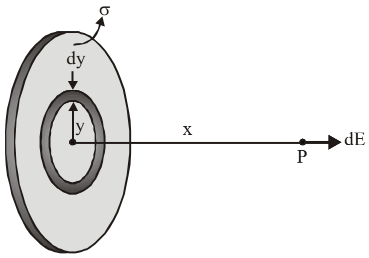
Due to this dq, electric field at centre of arc C is given as $dE = \dfrac{kdq}{R^2}$

Now electric field component due to this segment dEsinθ which is perpendicular to the angular bisector gets cancelled out in integration and net electric field at centre will be along angular bisector which can be calculated by integrating dEcosθ within limits from $-\dfrac{\phi}{2}$ to $\dfrac{\phi}{2}$.

Hence net electric field strength at centre C is $E_C = \int dE\cos\theta$

$$= \int_{-\phi/2}^{+\phi/2} \frac{kQ}{\phi R^2}\cos\theta d\theta = \frac{kQ}{\phi R^2}\int_{-\phi/2}^{+\phi/2}\cos\theta d\theta = \frac{kQ}{\phi R^2}[\sin\theta]_{-\phi/2}^{+\phi/2} = \frac{kQ}{\phi R^2}\left[\sin\frac{\phi}{2} + \sin\frac{\phi}{2}\right] = \frac{2kQ\sin\left(\frac{\phi}{2}\right)}{\phi R^2}$$

Electric field strength due to a uniformly surface charged disc :



If there is a disc of radius R, charged on its surface with surface charge density σ, we wish to find electric field strength due to this disc at a distance x from the centre of disc on its axis at point P shown in figure.

To find electric field at point P due to this disc, we consider an elemental ring of radius y and width dy in the disc as shown in figure. The charge on this elemental ring $dq = \sigma.2\pi y dy$ [Area of elemental ring $ds = 2\pi y\ dy$]

Now we know that electric field strength due to a ring of radius R, charge Q, at a distance x from its centre on its axis can be given as $E = \dfrac{kQx}{\left(x^2+R^2\right)^{3/2}}$

Here due to the elemental ring electric field strength dE at point P can be given as

$$dE = \frac{kdqx}{\left(x^2+y^2\right)^{3/2}} = \frac{k\sigma 2\pi y\ dy\ x}{\left(x^2+y^2\right)^{3/2}}$$

Net electric field at point P due to this disc is given by integrating above expression from 0 to R as

$$E = \int dE = \int_0^R \frac{k\sigma 2\pi xydy}{\left(x^2+y^2\right)^{3/2}} = k\sigma\pi x\int_0^R \frac{2y\ dy}{\left(x^2+y^2\right)^{3/2}} = 2k\sigma\pi x\left[-\frac{1}{\sqrt{x^2+y^2}}\right]_0^R = \frac{\sigma}{2\epsilon_0}\left[1-\frac{x}{\sqrt{x^2+R^2}}\right]$$

Ex. Calculate the electric field at origin due to infinite number of charges as shown in figures below.

fig (a)

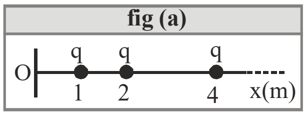
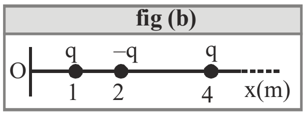


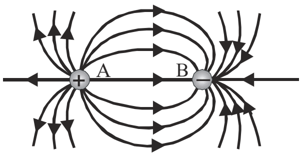
fig (b)



Sol. (a) $E_0 = kq\left[\frac{1}{1}+\frac{1}{4}+\frac{1}{16}+----\right] = \dfrac{kq.1}{(1-1/4)} = \dfrac{4kq}{3}$ $\left[\because\ S_\infty = \dfrac{a}{1-r}\right.$, a = 1 and r = $\left.\frac{1}{4}\right]$

(b) $E_0 = kq\left[\frac{1}{1}-\frac{1}{4}+\frac{1}{16}-----\right] = \dfrac{kq.1}{\left(1-(-1/4)\right)} = \dfrac{4kq}{5}$

ELECTRIC LINES OF FORCE



$q_A > q_B$

Electric lines of electrostatic field have following properties

(i) Imaginary

(ii) Can never cross each other

(iii) Can never be closed loops

(iv) The number of lines originating or terminating on a charge is proportional to the magnitude of charge. In rationalised MKS system $(1/\varepsilon_0)$ electric lines are associated with unit charge, so if a body encloses a charge q , total lines of force associated with it (called flux) will be q/ε_0.

(v) If there is no electric field there will be no lines of force.

(vi) Lines of force per unit area normal to the area at a point represents magnitude of intensity , crowded lines represent strong field while distant lines weak field.

(vii)Tangent to the line of force at a point in an electric field gives the direction of intensity. So a positive charge free to move follow the line of force.

- Lines of force starts from (+ve) charge and ends on (–ve) charge.

Electric flux (ϕ)

The word "flux" comes from a Latin word meaning "to flow" and you can consider the flux of a vector field to be a measure of the flow through an imaginary fixed element of surface in the field.

Electric flux is defined as $\phi_E = \int \vec{E}\cdot d\vec{A}$

This surface integral indicates that the surface in question is to be divided into infinitesimal elements of area $d\vec{A}$ and the scalar quantity $\vec{E}\cdot d\vec{A}$ is to be evaluated for each element and summed over the entire surface.

Electric flux :

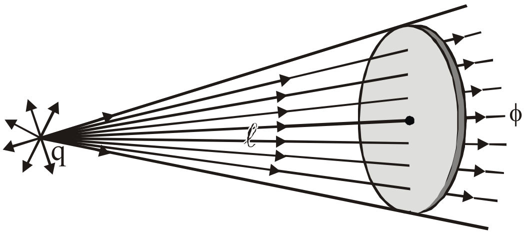
(i) It is a scalar quantity (ii) **Units** (V–m) and N – m^2/C

Dimensions : [$ML^3T^{-3}A^{-1}$]

(iii) The value of ϕ does not depend upon the distribution of charges and the distance between them inside the closed surface.

Electric Flux through a circular Disc :

Figure shows a point charge q placed at a distance ℓ from a disc of radius R. Here we wish to find the electric flux through the disc surface due to the point charge q. We know a point charge q originates electric flux in radially outward direction. The flux is originated in cone shown in figure passes through the disc surface.

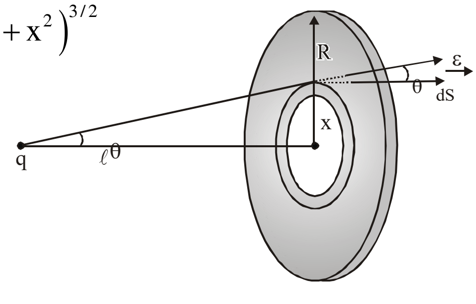


To calculate this flux, we consider on elemental ring an disc surface of radius x and width dx as shown. Area of this ring (strip) is $dS = 2\pi x\ dx$. The electric field due to q at this elemental ring is given as $E = \dfrac{kq}{\left(x^2+\ell^2\right)}$

If $d\phi$ is the flux passing through this elemental ring, then

$$d\phi = EdS\cos\theta = \frac{kq}{\left(x^2+\ell^2\right)} \times 2\pi x\ dx \times \frac{\ell}{\sqrt{x^2+\ell^2}} = \frac{2\pi kq\ell\ x\ dx}{\left(\ell^2+x^2\right)^{3/2}}$$

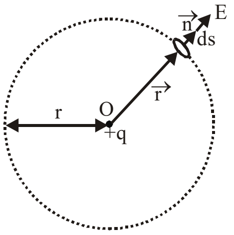
$$\phi = \int d\phi = \int_0^R \frac{q\ell}{2\epsilon_0} \frac{x\ dx}{\left(\ell^2+x^2\right)^{3/2}} = \frac{q\ell}{2\epsilon_0}\int_0^R \frac{x\ dx}{\left(\ell^2+x^2\right)^{3/2}}$$



$$= \frac{q\ell}{2\epsilon_0}\left[-\frac{1}{\sqrt{\ell^2+x^2}}\right]_0^R = \frac{q\ell}{2\epsilon_0}\left[\frac{1}{\ell} - \frac{1}{\sqrt{\ell^2+x^2}}\right]$$

The above result can be obtained in a much simpler way by using the concept of solid angle and Gauss's law.

GAUSS'S LAW



It relates with the total flux of an electric field through a closed surface to the net charge enclosed by that surface and according to it, the total flux linked with a closed surface is $(1/\varepsilon_0)$ times the charge enclosed by the closed surface i.e., $\int_S \vec{E}.\vec{ds} = \frac{q}{\epsilon_0}$

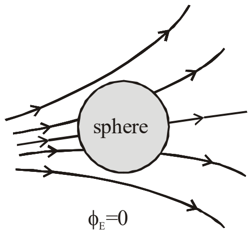
Regarding Gauss's law it is worth noting that :

(1) Gauss's law and Coulomb's law are equivalent, i.e., if we assume Coulomb's law we can prove Gauss's law and vice–versa. To prove Gauss's law from Coulomb's law consider a hypothetical spherical surface [called Gaussian–surface] of radius r with point charge q at its centre as shown in figure. By Coulomb's law intensity at a point P on the surface will be, $\vec{E} = \frac{1}{4\pi\varepsilon_0 r^3}\vec{r}$ And hence electric flux linked with area $\vec{ds} \Rightarrow \vec{E}.\vec{ds} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^3}\vec{r}.\vec{ds}$

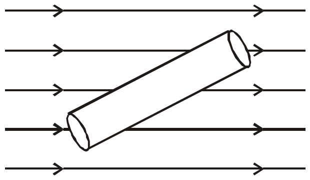
Here direction of $\vec{r}$ and $\vec{ds}$ are same, i.e., $\int_S \vec{E}.\vec{ds} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\oint_S ds = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}\left(4\pi r^2\right) \oint_S \vec{E}.\vec{ds} = \frac{q}{\epsilon_0}$

Which is the required result. Though here in proving it we have assumed the surface to be spherical, it is true for any arbitrary surface provided the surface is closed.

(2) (a) If a closed body (not enclosing any charge) is placed in an electric field (either uniform or non–uniform) total flux linked with it will be zero.

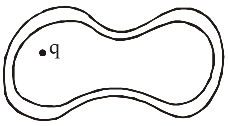


$\phi_E=0$
(A)

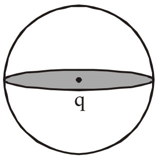


$\phi_E=0$
(B)

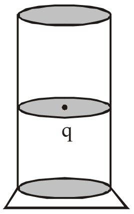
(b) If a closed body encloses a charge q, then total flux linked with the body will be $\oint_S \vec{E}.\vec{ds} = \frac{q}{\epsilon_0}$

(A)


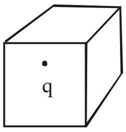
$\phi_E=(q/\varepsilon_0)$

(B)


$\phi_E=(q/\varepsilon_0)$

(C)


$\phi_E=(q/\varepsilon_0)$

(D)


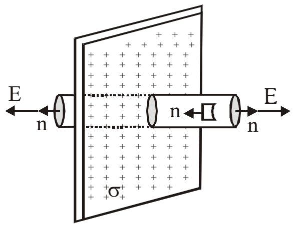
$\phi_E=(q/\varepsilon_0)$

From this expression it is clear that the flux linked with a closed body is independent of the shape and size of the body and position of charge inside it.[figure]

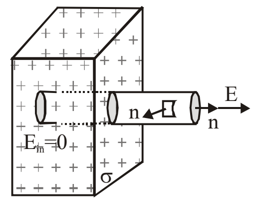
Note : So in case of closed symmetrical body with charge at its centre, flux linked with each half will be $\frac{1}{2}(\phi_E)=\left(\frac{q}{2\varepsilon_0}\right)$ and the symmetrical closed body has n identical faces with point charge at its centre,

flux linked with each face will be $\left(\frac{\phi_E}{n}\right)=\left(\frac{q}{n\varepsilon_0}\right)$

(3) Gauss's law is a powerful tool for calculating electric intensity in case of symmetrical charge distribution by choosing a Gaussian– surface in such a way that $\vec{E}$ is either parallel or perpendicular to its various faces. As an example, consider the case of a plane sheet of charge having charge density σ. To calculate E at a point P close to it consider a Gaussian surface in the form of a 'pill box' of cross–section S as shown in figure.



Sheet of charge
(A)



Conductor
(B)

The charge enclosed by the Gaussian–surface = σS and the flux linked with the pill box = ES + 0 + ES = 2ES (as E is parallel to curved surface and perpendicular to plane faces)

So from Gauss's law, $\phi_E=\frac{1}{\varepsilon_0}(q)$, $2ES=\frac{1}{\varepsilon_0}(\sigma S)\Rightarrow E=\frac{\sigma}{2\varepsilon_0}$

(4) If $\vec{E}=0$, $\phi=\oint\vec{E}.\vec{ds}=0$, so q = 0 but if q = 0, $\oint\vec{E}.\vec{ds}=0$ So $\vec{E}$ may or may not be zero.

If a dipole is enclosed by a closed surface then, q = 0, so $\oint\vec{E}.\vec{ds}=0$, but $\vec{E}\neq 0$

Note : If instead of plane sheet of charge, we have a charged conductor, then as shown in figure (B)

$E_{in}=0$. So $\phi_E=ES$ and hence in this case $E=\frac{\sigma}{\epsilon_0}$. This result can be verified from the fact that intensity at the surface of a charged spherical conductor of radius R is, $E=\frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$ with $q=4\pi R^2\sigma$

So for a point close to the surface of conductor, $E=\frac{1}{4\pi\varepsilon_0 R^2}\times\left(4\pi R^2\sigma\right)=\frac{\sigma}{\varepsilon_0}$

Ex. If a point charge q is placed at the centre of a cube.
What is the flux linked (a) with the cube? (b) with each face of the cube?

Sol. (a) According to Gauss's law flux linked with a closed body is $(1/\varepsilon_0)$ times the charge enclosed and here the closed body cube is enclosing a charge q so, $\phi_T=\frac{1}{\varepsilon_0}(q)$

(b) Now as cube is a symmetrical body with 6 –faces and the point charge is at its centre, so electric flux linked with each face will be $\phi_F=\frac{1}{6}(\phi_T)=\frac{q}{6\varepsilon_0}$

Note: (i) Here flux linked with cube or one of its faces is independent of the side of cube.
(ii) If charge is not at the centre of cube (but anywhere inside it), total flux will not change, but the flux linked with different faces will be different.

Ex. Consider $\vec{E} = 3 \times 10^3 \hat{i}$ (N/C) then what is the flux through the square of 10 cm side, if the normal of its plane makes 60° angle with the X axis.

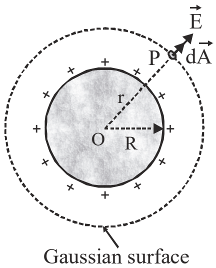
Sol. $\phi = ES\cos\theta = 3 \times 10^3 \times [10 \times 10^{-2}]^2 \times \cos 60° = 3 \times 10^3 \times 10^{-2} \times ½ = 15\ \text{Nm}^2/\text{C}$

Ex. Find the electric field due to an infinitely long cylindrical charge distribution of radius R and having linear charge density λ at a distance half of the radius from its axis.

Sol. $r = \dfrac{R}{2}$ point will be inside so $E = \dfrac{2k\lambda r}{R^2} = \dfrac{2k\lambda}{R^2}\left(\dfrac{R}{2}\right) = \dfrac{\lambda}{4\pi \in_0 R}$

ELECTRIC FIELD DUE TO SOLID CONDUCTING OR HOLLOW SPHERE

- **For outside point (r > R)**



Using Gauss's theorem $\oint \vec{E}.d\vec{s} = \dfrac{\Sigma q}{\in_0}$

∵ At every point on the Gaussian surface $\vec{E} \| d\vec{s}$;

⇒ $\vec{E}.d\vec{s} = E\, ds \cos 0° = E\, ds$

∴ $\oint E.ds = \dfrac{\Sigma q}{\in_0}$ [E is constant over the gaussian surface] ⇒ $E \times 4\pi r^2 = \dfrac{q}{\in_0}$ ⇒ $E_p = \dfrac{q}{4\pi \in_0 r^2}$

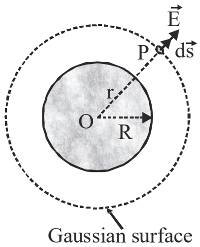
- **For surface point r = R** : $E_S = \dfrac{q}{4\pi \in_0 R^2}$
- **For Inside point (r < R)** : Because charge inside the conducting sphere or hollow is zero.

(i.e. Σq = 0) So $\oint \vec{E}.d\vec{s} = \dfrac{\Sigma q}{\in_0} = 0 \Rightarrow E_{in} = 0$

ELECTRIC FIELD DUE TO SOLID NON CONDUCTING SPHERE

- **Outside (r > R)**

From Gauss's theorem

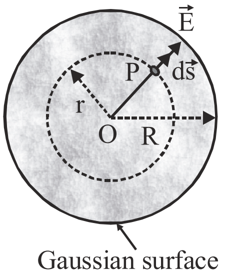


$$\oint_s \vec{E}.d\vec{s} = \frac{\Sigma q}{\in_0} \Rightarrow E \times 4\pi r^2 = \frac{q}{\in_0} \Rightarrow E_P = \frac{q}{4\pi \in_0 r^2}$$

- **At surface (r = R)**

$$E_S = \frac{q}{4\pi \in_0 R^2} \text{ Put } r = R$$

- **Inside (r < R) :**



From Gauss's theorem $\oint_s \vec{E}.d\vec{s} = \dfrac{\Sigma q}{\in_0}$

Where Σq charge contained within Gaussian surface of radius r

$$E(4\pi r^2) = \frac{\Sigma q}{\in_0} \Rightarrow E = \frac{\Sigma q}{4\pi r^2 \in_0} \text{ ...(i)}$$

As the sphere is uniformly charged, the volume charge density (charge/volume) ρ is constant throughout the sphere $\rho = \frac{q}{\frac{4}{3}\pi R^3}$ ⇒ charge enclosed in gaussian surface

$$= \rho\left(\frac{4}{3}\pi r^3\right) = \left(\frac{q}{(4/3)\pi R^3}\right)\left(\frac{4}{3}\pi r^3\right) \Rightarrow \sum q = \frac{qr^3}{R^3}$$

put this value in equation (i) $E_{in} = \frac{1}{4\pi \in_0}\frac{qr}{R^3}$

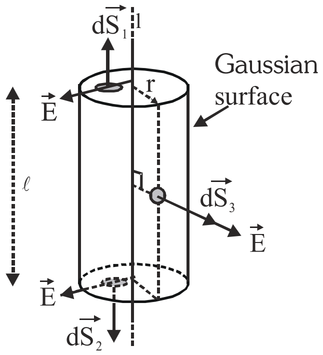
ELECTRIC FIELD DUE TO AN INFINITE LINE DISTRIBUTION OF CHARGE

Let a wire of infinite length is uniformly charged having a constant linear charge density λ.

P is the point where electric field is to be calculated.

Let us draw a coaxial Gaussian cylindrical surfaces of length ℓ.

From Gauss's theorem



$$q\int_{s_1}\vec{E}.d\vec{S}_1 + \int_{s_2}\vec{E}.d\vec{S}_2 + \int_{s_3}\vec{E}.d\vec{S}_3 = \frac{q}{\in_0}$$

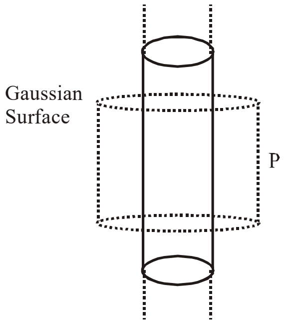
$$\Sigma\vec{E} \perp d\vec{S}_1 \text{ so } \vec{E}.d\vec{S}_1 = 0 \text{ and } \vec{E} \perp d\vec{S}_2 \text{ so } \vec{E}.d\vec{S}_2 = 0$$

$$E \times 2\pi r\ell = \frac{q}{\in_0} \quad [\because \vec{E} \,||\, d\vec{S}_3]$$

Charge enclosed in the Gaussian surface $q = \lambda\ell$.

So $E \times 2\pi r\ell = \frac{\lambda\ell}{\in_0} \Rightarrow E = \frac{\lambda}{2\pi \in_0 r}$ or $E = \frac{2k\lambda}{r}$ where $k = \frac{1}{4\pi \in_0}$

Charged cylindrical nonconductor of infinite length



Electric field at outside point $\vec{E}_A = \frac{2k\lambda}{r}\hat{r}$ $\quad r > R$

Electric field at inside point $\vec{E}_B = \frac{\lambda\vec{r}}{2\pi \in_0 R^2}$ $\quad r < R$

KEY POINTS

- Electric field inside a solid conductor is always zero.
- Electric field inside a hollow conductor may or may not be zero ($E \neq 0$ if non zero charge is inside the sphere).
- The electric field due to a circular loop of charge and a point charge are identical provided the distance of the observation point from the circular loop is quite large as compared to its radius i.e. x >>> R.

Ex. A charged particle is kept in equilibrium in the electric field between the plates of millikan oil drop experiment. If the direction of the electric field between the plates is reversed, then calculate acceleration of the charged particle.

Sol. Let mass of the particle = m,

charge on particle = q

Intensity of electric field in between plates = E initially $mg = qE$

After reversing the field $ma = mg + qE \Rightarrow ma = 2mg$

$\therefore$ Acceleration of particle $\Rightarrow a = 2g$

ELECTROSTATIC POTENTIAL ENERGY

Potential energy of a system of particles is defined only in conservative fields. As electric field is also conservative, we define potential energy in it. Potential energy of a system of particles we define as the work done in assembling the system in a given configuration against the interaction forces of particles. Electrostatic potential energy is defined in two ways.

(i) Interaction energy of charged particles of a system

(ii) Self energy of a charged object

INFINITE SHEET CHARGE

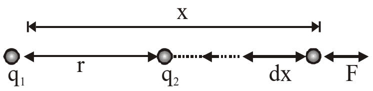
- **Electrostatic Interaction Energy**

Electrostatic interaction energy of a system of charged particles is defined as the external work required to assemble the particles from infinity to the given configuration. When some charged particles are at infinite separation, their potential energy is taken zero as no interaction is there between them. When these charges are brought close to a given configuration, external work is required if the force between these particles is repulsive and energy is supplied to the system, hence final potential energy of system will be positive. If the force between the particle is attractive, work will be done by the system and final potential energy of system will be negative.

- **Interaction Energy of a system of two charged particles**

Figure shows two + ve charges q_1 and q_2 separated by a distance r. The electrostatic interaction energy of this system can be given as work done in bringing q_2 from infinity to the given separation from q_1. It can be calculated as

$$W = \int_{\infty}^{r} \vec{F}.\overrightarrow{dx} = -\int_{\infty}^{r} \frac{kq_1q_2}{x^2}dx \quad [-\text{ve sign shows that x is decreasing}]$$

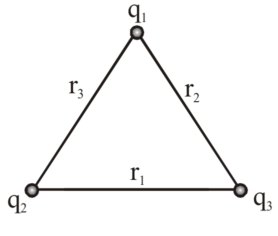


$$W = \frac{kq_1q_2}{r} = U \text{ [interaction energy]}$$

If the two charges here are of opposite sign, the potential energy will be negative as $U = -\frac{kq_1q_2}{r}$

- **Interaction Energy for a system of charged particles**

When more than two charged particles are there in a system, the interaction energy can be given by sum of interaction energies of all the pairs of particles. For example if a system of three particles having charges q_1, q_2 and q_3 is given as shown in figure. The total interaction energy of this system can be given as $U = \frac{kq_1q_2}{r_3} + \frac{kq_1q_3}{r_2} + \frac{kq_2q_3}{r_1}$



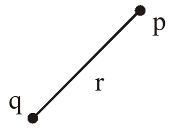
ELECTRIC POTENTIAL

Electric potential is a scalar property of every point in the region of electric field. At a point in electric field potential is defined as the interaction energy of a unit positive charge. If at a point in electric field a charge q_0 has potential energy U, then electric potential at that point can be given as

$$V = \frac{U}{q_0} \text{ joule/coulomb}$$

Potential energy of a charge in electric field is defined as work done in bringing the charge from infinity to the given point in electric field. Similarly we can define electric potential as "work done in bringing a unit positive charge from infinity to the given point against the electric forces.

- **Electric Potential due to a point charge in its surrounding :**



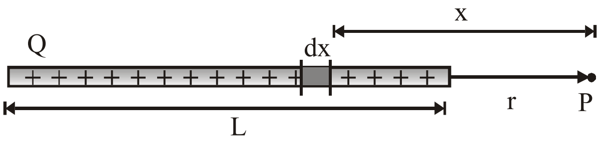
The potential at a point P at a distance r from the charge q , $V_P = \frac{U}{q_0}$

Where U is the potential energy of charge q_0 at point p, $U = \frac{kqq_0}{r}$

Thus potential at point P is $V_P = \frac{kq}{r}$

Electric Potential due to a charge Rod :

Figure shows a rod of length L, uniformly charged with a charge Q. Due to this we'll find electric potential at a point P at a distance r from one end of the rod as shown in figure.



For this we consider an element of width dx at a distance x from the point P.

Charge on this element is $dQ = \frac{Q}{L}\,dx$

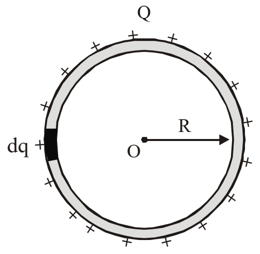
The potential dV due to this element at point P can be given by using the result of a point charge as

$$dV = \frac{kdq}{x} = \frac{kQ}{Lx}dx$$

Net electric potential at point P : $V = \int dV = \int_r^{r+L} \frac{kQ}{Lx}\,dx = \frac{kQ}{L}\ell n\left(\frac{r+L}{r}\right)$

Electric potential due to a charged ring

Case–I : At its centre

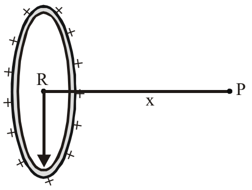


To find potential at the centre C of the ring, we first find potential dV at centre due to an elemental charge dq on ring which is given as $dV = \frac{kdq}{R}$

Total potential at C is $V = \int dV = \int \frac{kdq}{R} = \frac{kQ}{R}$.

As all dq's of the ring are situated at same distance R from the ring centre C, simply the potential due to all dq's is added as being a scalar quantity, we can directly say that the total electric potential at ring centre is $\frac{kQ}{R}$. Here we can also state that even if charge Q is non–uniformly distributed on ring, the electric potential C will remain same.

Case II : At a point on axis of ring



We find the electric potential at a point P on the axis of ring as shown, we can directly state the result as here also all points of ring are at same distance $\sqrt{x^2+R^2}$ from the point P, thus the potential at P can be given as $V_P = \frac{kQ}{\sqrt{R^2+x^2}}$

Electric potential due to a uniformly charged disc :

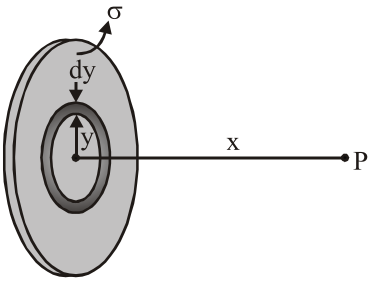


Figure shows a uniformly disc of radius R with surface charge density ρ coul/m^2. To find electric potential at point P we consider an elemental ring of radius y and width dy, charge on this elemental ring is dq=σ2πy dy. Due to this ring, the electric potential at point P can be given as

$$dV = \frac{kdq}{\sqrt{x^2+y^2}} = \frac{k.\sigma.2\pi y\ dy}{\sqrt{x^2+y^2}}$$

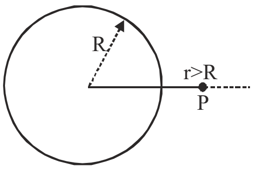
Net electric potential at Point P due to whole disc can be given as

$$V = \int dV = \int_0^R \frac{\sigma}{2\in_0} \cdot \frac{y\ dy}{\sqrt{x^2+y^2}} = \frac{\sigma}{2\in_0}\left[\sqrt{x^2+y^2}\right]_0^R = \frac{\sigma}{2\in_0} = \left[\sqrt{x^2+R^2}-x\right]$$

ELECTRIC POTENTIAL DUE TO HOLLOW OR CONDUCTING SPHERE

- **At outside sphere**

According to definition of electric potential, electric potential at point P



$$V = -\int_\infty^r \vec{E}.d\vec{r} = -\int_\infty^r \frac{q}{4\pi \in_0 r^2} dr \left[\because \ E_{out} = \frac{q}{4\pi \in_0 r^2}\right]; \ V = -\frac{q}{4\pi \in_0}\int_\infty^r \frac{1}{r^2} dr = \frac{q}{4\pi \in_0}\left[\frac{1}{r}\right]_\infty^r \ \frac{q}{4\pi \in_0 r}$$

- **At surface**

$$= V = -\int_\infty^R \vec{E}.d\vec{r} = -\int_\infty^R \frac{q}{4\pi \in_0 r^2} dr \left[\because \ E_{out} = \frac{q}{4\pi\varepsilon_0 r^2}\right]; V = -\frac{q}{4\pi \in_0}\int_\infty^R \left(\frac{1}{r^2}\right) dr = \frac{q}{4\pi \in_0}\left[\frac{1}{r}\right]_\infty^R \Rightarrow V = \frac{q}{4\pi \in_0 R}$$

- **Inside the surface** :

$$\because \text{ Inside the surface } E = 0 \left[E = -\frac{dV}{dr}\right] \frac{dV}{dr} = 0 \text{ or } V = \text{constant so } V = \frac{q}{4\pi \in_0 R}$$

ELECTRIC POTENTIAL DUE TO SOLID NON CONDUCTING SPHERE

- **At outside sphere** Same as conducting sphere.
- **At Surface** Same as conducting sphere.
- **Inside the sphere**

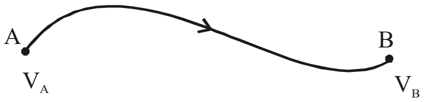
$$V = -\int_{\infty}^{r} \vec{E}.d\vec{r} \qquad \Rightarrow V = -\left[\int_{\infty}^{R} E_1 dr + \int_{R}^{r} E_2 dr\right]$$

$$V = -\left[\int_{\infty}^{R}\left(\frac{kq}{r^2}\right)dr + \int_{R}^{r}\left(\frac{kqr}{R^3}\right)dr\right] \qquad \Rightarrow V = -\left[kq\left(-\frac{1}{r}\right)_{\infty}^{R} + \frac{kq}{R^3}\left(\frac{r^2}{2}\right)_{R}^{r}\right]$$

$$V = -kq\left[-\frac{1}{R} + \frac{r^2}{2R^3} - \frac{R^2}{2R^3}\right] \qquad \Rightarrow V = \frac{kq}{2R^3}\left[3R^2 - r^2\right]$$

Potential Difference Between Two points in electric field

Potential difference between two points in electric field can be defined as work done in displacing a unit positive charge from one point to another against the electric forces.



If a unit +ve charge is displaced from a point A to B as shown work required can be given as

$$V_B - V_A = -\int_A^B \vec{E}.\vec{d}x$$

If a charge q is shifted from point A to B, work done against electric forces can be given as W = q $(V_B - V_A)$

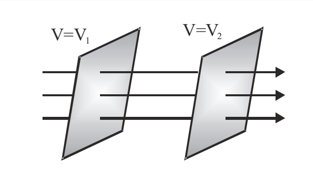
If in a situation work done by electric forces is asked, we use W = q $(V_A - V_B)$

If $V_B < V_A$, then charges must have tendency to move toward B (low potential point) it implies that electric forces carry the charge from high potential to low potential points. Hence we can say that in the direction of electric field always electric potential decreases.

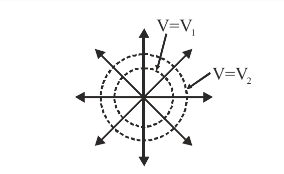
Equipotential surfaces

For a given charge distribution, locus of all points having same potential is called 'equipotential surface'.

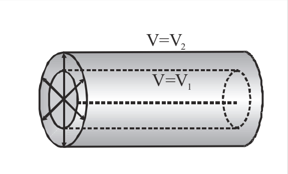
- Equipotential surfaces can never cross each other (otherwise potential at a point will have two values which is absurd)
- Equipotential surfaces are always perpendicular to direction of electric field.
- If a charge is moved from one point to the other over an equipotential surface then work done $W_{AB} = -U_{AB} = q(V_B - V_A) = 0 \quad [\because V_B = V_A]$
- Shapes of equipotential surfaces



for uniform electric field equipotential surfaces are parallel plane



for a point charge, spherical conductor equipotential surfaces are spherical



for a line distribution of charge equipotential surfaces are cylinderical

- The intensity of electric field along an equipotential surface is always zero.

Electric Potential Gradient

The maximum rate of change of potential at right angles to an equipotential surface in an electric field is defined as potential gradient. $\vec{E} = -\vec{\nabla}V = -\text{grad } V$

Note : Potential is a scalar quantity but the gradient of potential is a vector quantity

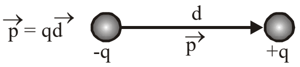
In cartesian co–ordinates $\vec{\nabla}V = \left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right]$

Ex. If $V = -5x + 3y + \sqrt{15}\,z$ then E(x, y, z) = ?

Sol. $\vec{E} = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right] = -\left(-5\hat{i} + 3\hat{j} + \sqrt{15}\hat{k}\right) \Rightarrow |\vec{E}| = \sqrt{25+9+15} = \sqrt{49} = 7$ unit

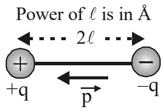
ELECTRIC DIPOLE

A system of two equal and opposite charges separated by a small distance is called electric dipole, shown in figure. Every dipole has a characteristic property called dipole moment. It is defined as the product of magnitude of either charge and the separation between the charges, given as



In some molecules, the centres of positive and negative charges do not coincide. This results in the formation of electric dipole. Atom is non – polar because in it the centres of positive and negative charges coincide. Polarity can be induced in an atom by the application of electric field. Hence it can be called as induced dipole.

- **Dipole Moment** : Dipole moment $\vec{p} = q\vec{d}$



(i) Vector quantity , directed from negative to positive charge

(ii) **Dimension** : [LTA], **Units** : coulomb × metre (or Cm)

(iii) Practical unit is "debye" ≡ Two equal and opposite point charges each having charge 10^{-10} frankline ($\simeq$ e) and separation of 1Å then the value of dipole moment $(\vec{p})$ is 1 debye.

$$1 \text{ Debye} = 10^{-10} \times 10^{-10} \text{ Fr} \times \text{m} = 10^{-20} \times \frac{C \times m}{3 \times 10^{9}} \simeq 3.3 \times 10^{-30} \text{ C} \times \text{m}$$

- **Dipole Placed in uniform Electric Field**

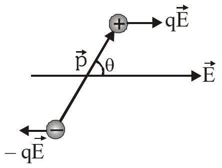


Figure shows a dipole of dipole moment $\vec{p}$ placed at an angle θ to the direction of electric field. Here the charges of dipole experience forces qE in opposite direction as shown. $\vec{F}_{net} = \left[q\vec{E} + (-q)\,\vec{E}\right] = \vec{0}$

Thus we can state that when a dipole is placed in a uniform electric field, net force on the dipole is zero. But as equal and opposite forces act with a separation in their line of action, they produce a couple which tend to align the dipole along the direction of electric field. The torque due to this couple can be given as

τ = Force × separation between lines of actions of forces = qE × d sin θ = pE sin θ

$$\vec{\tau} = \vec{r} \times \vec{F} = \vec{d} \times q\vec{E} = q\vec{d} \times \vec{E} = \vec{p} \times \vec{E}$$

Work done in Rotation of a Dipole in Electric field

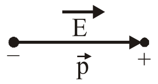
When a dipole is placed in an electric field at an angle θ, the torque on it due to electric field is $\tau = pE \sin\theta$

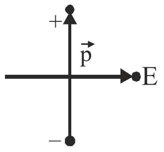
Work done in rotating an electric dipole from θ_1 to θ_2 [uniform field]

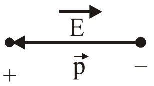
$$dW = \tau\, d\theta \text{ so } W = \int dW = \int \tau d\theta \text{ and } W_{\theta_1 \to \theta_2} = W = \int_{\theta_1}^{\theta_2} pE \sin\theta\, d\theta = pE(\cos\theta_1 - \cos\theta_2)$$

e.g. $W_{0 \to 180} = pE[1-(-1)] = 2pE$ $\qquad W_{0 \to 90} = pE(1-0) = pE$

If a dipole is rotated from field direction ($\theta = 0°$) to θ then $W = pE(1 - \cos\theta)$







$\theta = 0$	$\theta = 90°$	$\theta = 180°$
τ = minimum = 0	τ = maximum = pE	τ = minimum = 0
W = minimum = 0	W = pE	W = maximum = 2pE

Electrostatic potential energy :

Electrostatic potential energy of a dipole placed in a uniform field is defined as work done in rotating a dipole from a direction perpendicular to the field to the given direction i.e.,

$$W_{90° \to \theta} = \int_{90°}^{\theta} pE \sin\theta\, d\theta = -pE\cos\theta = -\vec{p}.\vec{E}$$

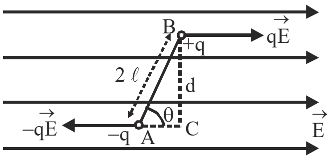
$\vec{E}$ is a conservative field so what ever work is done in rotating a dipole from θ_1 to θ_2 is just equal to change in electrostatic potential energy $W_{\theta_1 \to \theta_2} = U_{\theta_2} - U_{\theta_1} = pE(\cos\theta_1 - \cos\theta_2)$

Work done in rotating an electric dipole in an electric field

Suppose at any instant, the dipole makes an angle θ with the electric field.

The torque acting on dipole. $\tau = qEd = (q\, 2\ell \sin\theta)E = pE \sin\theta$

The work done in rotating dipole from θ_1 to θ_2



$$W = \int_{\theta_1}^{\theta_2} \tau d\theta = \int_{\theta_1}^{\theta_2} pE \sin\theta\, d\theta$$

$$W = pE(\cos\theta_1 - \cos\theta_2) = U_2 - U_1 \quad (\because U = -pE\cos\theta)$$

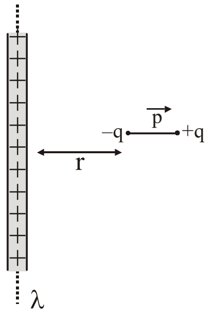
Force on an electric dipole in Non–uniform electric field :

If in a non–uniform electric field dipole is placed at a point where electric field is E, the interaction energy of dipole at this point $U = -\vec{p}.\vec{E}$. Now the force on dipole due to electric field $F = -\dfrac{\Delta U}{\Delta x}$

For unidirectional variation in electric field, $F = -\dfrac{d}{dx}\left(\vec{p}.\vec{E}\right)$

If dipole is placed in the direction of electric field then $F = -p\dfrac{dE}{dx}$

Ex. Calculate force on a dipole in the surrounding of a long charged wire as shown in the figure.



Sol. In the situation shown in figure, the electric field strength due to the wire, at the position of dipole as $E = \dfrac{2k\lambda}{r}$

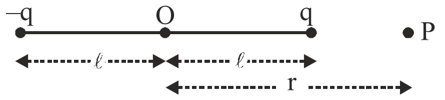
Thus force on dipole is $F = -p.\dfrac{dE}{dr} = -p\left[-\dfrac{2k\lambda}{r^2}\right] = \dfrac{2kp\lambda}{r^2}$

Here –ve charge of dipole is close to wire hence net force an dipole due to wire will be attractive.

ELECTRIC POTENTIAL DUE TO DIPOLE

- **At axial point**

Electric potential due to +q charge $V_1 = \dfrac{kq}{(r-\ell)}$



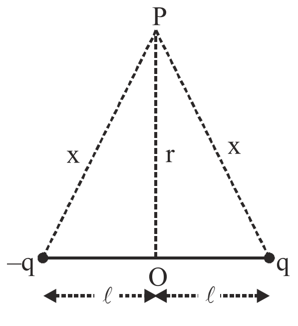
Electric potential due to –q charge $V_2 = \dfrac{-kq}{(r+\ell)}$

Net electric potential $V = V_1 + V_2 = \dfrac{kq}{(r-\ell)} + \dfrac{-kq}{(r+\ell)} = \dfrac{kq\times 2\ell}{(r^2-\ell^2)} = \dfrac{kp}{r^2-\ell^2}$

If $r >>> \ell \Rightarrow V = \dfrac{kp}{r^2}$

- **At equatorial point**

Electric potential of P due to +q charge $V_1 = \dfrac{kq}{x}$



Electric potential of P due to –q charge $V_2 = -\dfrac{kq}{x}$

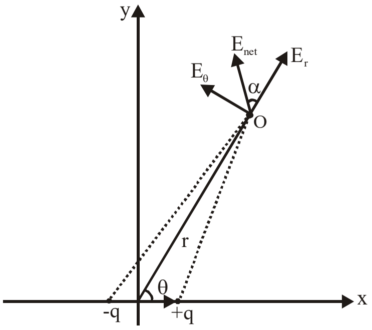
Net potential $V = V_1 + V_2 = \dfrac{kq}{x} - \dfrac{kq}{x} = 0 \therefore V = 0$

- **At general point**

$$V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{\vec{p}\,.\,\vec{r}}{4\pi\varepsilon_0 r^3} \quad \vec{p} = q\vec{a} \text{ electric dipole moment}$$

Electric field due to an electric dipole

Figure shows an electric dipole placed on x–axis at origin. Here we wish to find the electric field and potential at a point O having coordinates (r, θ). Due to the positive charge of dipole electric field at O is in radially outward direction and due to the negative charge it is radially inward as shown in figure.



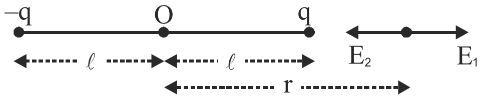
$$E_r = -\frac{\partial V}{\partial r} = \frac{2kp\cos\theta}{r^3} \text{ and } E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} = \frac{kp\sin\theta}{r^3}$$

Thus net electric field at point O, $E_{net} = \sqrt{E_r^2 + E_\theta^2} = \frac{kp}{r^3}\sqrt{1+3\cos^2\theta}$

If the direction of E_{net} is at an angle α from radial direction, then $\alpha = \tan^{-1}\frac{E_\theta}{E_r} = \left(\frac{1}{2}\tan\theta\right)$

Thus the inclination of net electric field at point O is $(\theta + \alpha)$

- **At a point on the axis of a dipole :**



Electric field due to +q charge $E_1 = \frac{kq}{(r-\ell)^2}$

Electric field due to –q charge $E_2 = \frac{kq}{(r+\ell)^2}$

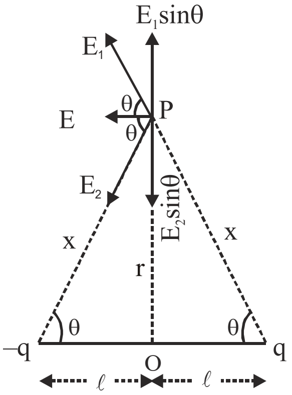
Net electric field $E = E_1 - E_2 = \frac{kq}{(r-\ell)^2} - \frac{kq}{(r+\ell)^2} = \frac{kq \times 4r\ell}{(r^2-\ell^2)^2}$ $[\because\ p = q \times\ 2\ell = \text{Dipole moment}]$

$$E = \frac{2kpr}{(r^2-\ell^2)^2} \text{ If } r >>> \ell \text{ then } E = \frac{2kp}{r^3}$$

- **At a point on equatorial line of dipole :**

Electric field due to +q charge $E_1 = \frac{kq}{x^2}$; Electric field due to –q charge $E_2 = \frac{kq}{x^2}$

Vertical component of E_1 and E_2 will cancel each other and horizontal components will be added



So net electric field at P

$$E = E_1\cos\theta + E_2\cos\theta \ [\because E_1 = E_2]$$

$$E = 2E_1\cos\theta = \frac{2kq}{x^2}\cos\theta \because \cos\theta = \frac{\ell}{x} \text{ and } x = \sqrt{r^2+\ell^2}$$

$$E = \frac{2kq\ell}{x^3} = \frac{2kq\ell}{(r^2+\ell^2)^{3/2}} = \frac{kp}{(r^2+\ell^2)^{3/2}} \quad \text{If } r >>> \ell$$

then $E = \frac{kp}{r^3}$ or $\vec{E} = \frac{-k\vec{p}}{r^3}$

Ex. A short electric dipole is situated at the origin of coordinate axis with its axis along x–axis and equator along y–axis. It is found that the magnitudes of the electric intensity and electric potential due to the dipole are equal at a point distant r = $\sqrt{5}$ m from origin. Find the position vector of the point.

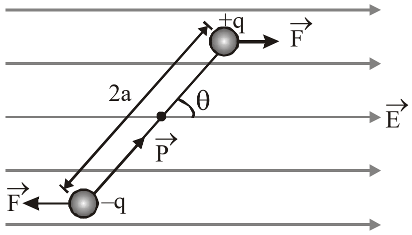
Sol. $\because |E_P| = |V_P| \ \therefore \ \frac{kp}{r^3}\sqrt{1+3\cos^2\theta} = \frac{kp\sin\theta}{r^2} \Rightarrow 1 + 3\cos^2\theta = 5\cos^2\theta \Rightarrow \cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^0$

Position vector $\vec{r}$ of point P is $\vec{r} = \frac{\sqrt{5}}{2}\left(\hat{i}+\hat{j}\right)$

Ex. Prove that the frequency of oscillation of an electric dipole of moment p and rotational inertia I for small amplitudes about its equilibrium position in a uniform electric field strength E is $\frac{1}{2\pi}\sqrt{\left(\frac{pE}{I}\right)}$

Sol. Let an electric dipole (charge q and –q at a distance 2a apart) placed in a uniform external electric field of strength E.

Restoring torque on dipole



$\tau = -pE\sin\theta = -pE\,\theta$ (as θ is small)

Here – ve sign shows the restoring tendency of torque.

$\because \tau = I\alpha \therefore$ angular acceleration = $\alpha = \frac{\tau}{I} = \frac{PE}{I}\theta$

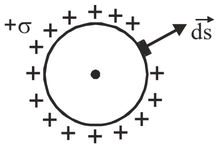
For SHM $\alpha = w^2\theta$ comparing we get $\omega = \sqrt{\frac{pE}{I}}$

Thus frequency of oscillations of dipole n = $\frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\left(\frac{pE}{I}\right)}$

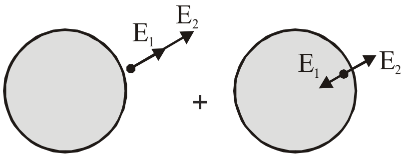
ELECTROSTATIC PRESSURE

Force due to electrostatic pressure is directed normally outwards to the surface. Force on small element ds of charged conductor

dF = (Charge on ds) x Electric field = (σ ds) $\frac{\sigma}{2\epsilon_0}$ $dF = \frac{\sigma^2}{2\epsilon_0}ds$



Inside $E_1 - E_2 = 0 \Rightarrow E_1 = E_2$



Just outside $E = E_1 + E_2 = 2E_2 \Rightarrow E_2 = \frac{\sigma}{2\epsilon_0}$

(E_1 is field due to point charge on the surface and E_2 is field due to rest of the sphere).

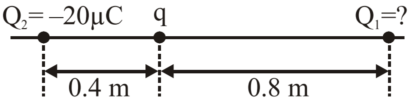
The electric force acting per unit area of charged surface is defined as electrostatic pressure.

$$P_{eleectrostatic} = \frac{dF}{dS} = \frac{\sigma^2}{2\epsilon_0}$$

EXERCISE (S-1)

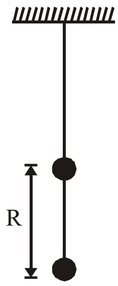
Properties of charge and Coulomb's law

1. Three charges are positioned in a straight line as depicted in the diagram. Using the information given in the diagram, determine the magnitude of charge 'Q_1' (in μC) if the charge q is to remain stationary.



ES0001

2. Two identical balls of mass m = 0.9 g each are charged by the same charges, joined by a thread and suspended from the ceiling (figure). Find the charge (in μC) that each ball should have so that the tension in both the threads are same? The distance between the centers of balls is R = 3m.

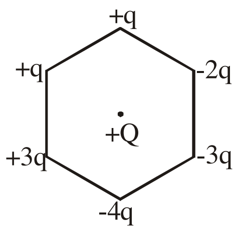


ES0002

3. Four identical charges q are placed at the corners of a square of side a, what charges Q must be placed at the centre of the square so that whole system of charges is in equilibrium?

ES0003

4. Six charges are kept at the vertices of a regular hexagon as shown in the figure. If magnitude of force applied by +Q on +q charge is F, then net electric force on the +Q is nF. Find the value of n.



ES0004

5. Two charged balls are connected by an inextensible thread of length 3 m. Masses of balls are 2kg and 1 kg, the charges are +20 μC and –100 μC. What minimum constant external force F (in N) must be applied to the ball of mass 1 kg so that the thread does not slack? Neglect gravity and friction.

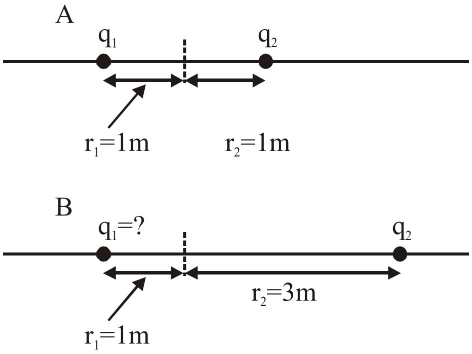
ES0005

Electric field

6. Two point charge –4Q and 9Q are placed at a distance 2m from each other. The position at which net electric field is zero from the charge –4Q is x (in m). What is the value of x.

ES0006

7. Two charges are separated as shown in the diagram A below. Each charge has a magnitude of 50 nC in diagram A. If the charge on the right is moved further to the right as depicted in diagram B, additional charge that must be removed from the left in order to maintain the initial electric field magnitude at the center (in nC) is given by $\frac{\alpha}{9}$ nC. Fill the value of α in OMR sheet.

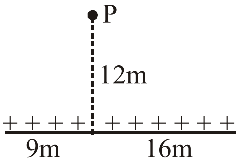


ES0007

8. A clock face has negative charges $-q, -2q, -3q, ..., -12q$ fixed at the positions of the corresponding numerals on the dial. Assume that the clock hands do not disturb the net field due to point charges. At what time does the hour hand point in the same direction is electric field at the centre of the dial.

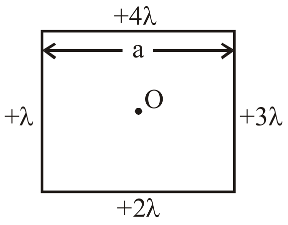
ES0008

9. Find the magnitude of electric field (in N/C) due to a line charge of $\lambda = \left(2\sqrt{2}\right)$ nC/m at a point P as shown.



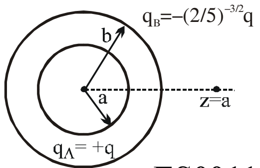
ES0009

10. Four uniformly charged wires of length a are arranged to form a square. Linear charge density of each wire is as shown. Electric field intensity at centre of square is $\frac{nk\lambda}{a}$ then value of n



ES0010

11. Two concentric rings, one of radius 'a' and the other of radius 'b' have the charges +q and $-(2/5)^{-3/2}$ q respectively as shown in the figure. Find the ratio b/a if a charge particle placed on the axis at z = a is in equilibrium.

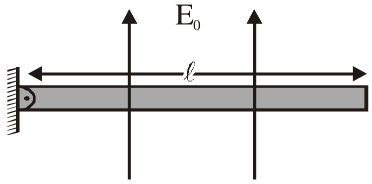


ES0011

12. The intensity of elecric field is required to exert a force on a proton equal to its weight at sea level is given by $\alpha \times 10^{-7}$ V/m. Fill the value of α (Given g = 10 m/s^2, mass of proton = 1.6×10^{-27} kg and charge on proton = 1.6×10^{-19} C).

ES0012

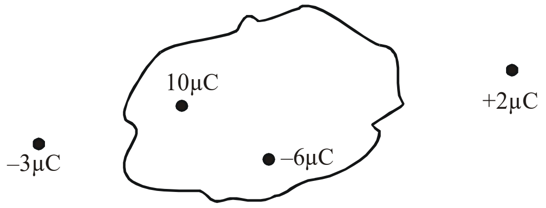
13. A thin insulating uniformly charged (linearly charged density λ) rod is hinged about one of its ends. It can rotate in vertical plane. If rod is in equilibrium by applying vertical electric field E as shown in figure. Find the value of E(in N/C). (Given that mass of rod 2 kg, $\lambda = 10\ C/m$, ℓ=1m, g=10 m/s^2)



ES0013

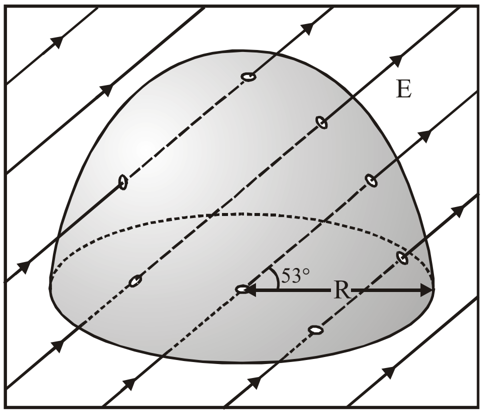
Gauss' law

14. In a particular system, if number of electric field lines associated by 1C charge is 10^9. Then net number of electric field lines passing through the given closed surface is $n \times 10^3$ find n.



ES0014

15. A uniform electric field E = 500 N/C passes through a hemispherical surface of radius R = 1.2 m as shown in figure. The net electric flux (in S.I. units) through the hemispherical surface only is Nπ. Then find the value of N.



ES0015

16. A non-uniform electric field in x-direction is increasing uniformly from 2N/C at x = 1m to 8 N/C at x = 7m. The center of cube is at x = 2m. If the charge enclosed inside a small cube of side length 10cm is 8.85×10^{-5n} C. The value of n will be.

ES0016

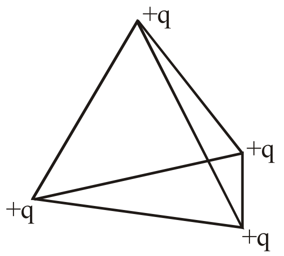
17. 20 μC charge is placed inside a closed surface then flux related to surface is ϕ. If 80 μC charge is added inside the surface then change in flux is given by $\alpha\phi$. Fill the value of α.

ES0017

18. A sphere of radius R has charge density given by $\rho = \rho_0\left(1 - \frac{nr}{3R}\right)$, where ρ_0 is a constant, r is distance from centre of sphere. For a spherical gaussian surface of radius R centered at the centre of sphere, the flux is zero. Find 'n'.

ES0018

19. Four point charges are kept at the vertices of a regular tetrahedron of side R. Total electrostatic energy of the configuration is $\frac{\alpha k q^2}{R}$. The value of α is



ES0019

Electric potential energy and electric potential

20. Four identical free point positive charges q (=1μC) each are located at the four corners of a square of side 1 m. A negative charge is placed at the centre of the square to obtain equilibrium of all the charges. What is the total potential energy (in milli Joules) of the system assuming the reference point at infinity?

ES0020

21. Consider the configuration of a system of four charges each of value +q. Find the work done by external agent in changing the configuration of the system from figure (i) to fig (ii).

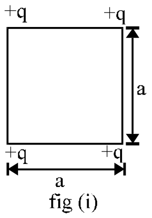


fig (i)

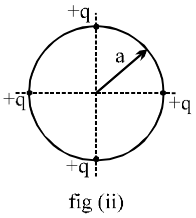
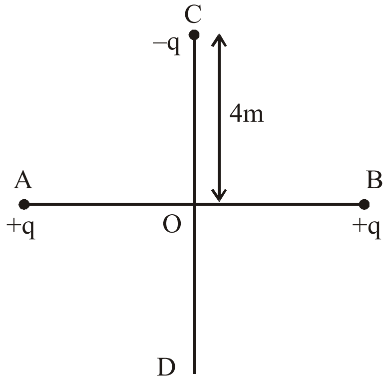


fig (ii)

ES0021

22. Two fixed, equal positive charges, each +q are located at point A & B separated by a distance of 6 m. A particle of mass m having equal and opposite charge –q moves towards them along the perpendicular bisector line COD where O is the centre of line joining A and B. If –q charge is released from rest from point C, then speed of charge –q at O is given by $v = q\sqrt{\dfrac{x}{15\pi \in_0 m}}$ find value of x. (Neglect gravity)



ES0022

23. An electric field $\left(-30\hat{i}+20\hat{j}\right)$ Vm^{-1} exists in the space. If potential at the origin is zero then find the potential at (5m,3m) in volts.

ES0023

24. A circular ring of radius a with uniform charge density λ is in the xy plane with centre at origin. A particle of mass m and charge q is projected from P (0, 0, a√3) on +z-axis towards origin with initial velocity u. The minimum value of the velocity so that the particle does not return to P is $\sqrt{\dfrac{\lambda q}{x\varepsilon_0 m}}$.

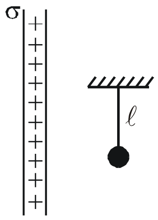
Find 'x' (neglect gravity).

ES0024

25. A particle of mass m carrying charge 'q' is projected with velocity (v) from point P towards an infinite line charge from a distance 'a'. Its speed reduces to zero momentarily at point Q which is at a distance a/2 from the line charge. If another particle with mass m and charge –q is projected with the same velocity v from point P towards the line charge. Its speed is found to be $\dfrac{Nv}{\sqrt{2}}$ at point 'Q'. Find the value of N.

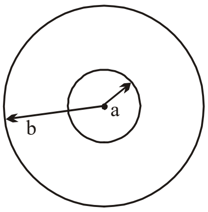
ES0025

26. A simple pendulum of length ℓ and bob mass *m* is hanging in front of a large nonconducting sheet of surface charge density σ. If suddenly a charge +*q* is given to the bob in the position shown in figure. Find the maximum angle through which the string is deflected from vertical.



ES0026

27. Outer cylinder of the coaxial nonconductor of radius 'b' is given a positive potential V relative to the inner cylinder of radius 'a' as shown in the figure (charge distribution is uniform). A charge q (mass = m) is set free with negligible velocity at the surface of the inner cylinder. Find the velocity (in m/s), when it hits the outer cylinder. [consider V = 10, q = –20, m = 1 all in S.I. Units]



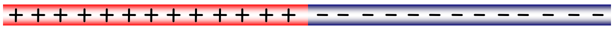
ES0027

28. A positive charge Q is uniformly distributed throughout the volume of a nonconducting sphere of radius R. A point mass having charge $+q$ and mass m is fired towards the centre of the sphere with velocity v from a point at distance $r(r > R)$ from the centre of the sphere. Find the minimum velocity v so that it can penetrate $R/2$ distance of the sphere. Neglect any resistance other than electric interaction. Charge on the small mass remains constant throughout the motion.

ES0028

Electric dipole

29. A plastic rod of length 1.0 m carries uniform positive charge +4.0 μC on half of its length and uniform negative charge –4.0 μ C on the remaining half of its length. Find magnitude of it's net dipole moment in μC–m.

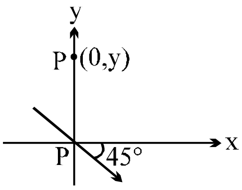


ES0029

30. A dipole of dipole moment $\vec{p} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ is placed at point A(2,–3,1). The electric potential due to this dipole at the point B(4,–1,0) is (ab) × 10^9 volt here 'a' represents sign (for negative answer select 0 for positive answer select 1. Write the value of $(a + b)^2$. The parameters specified here are in S.I. units.

ES0030

31. A dipole is placed at origin of coordinate system as shown in figure, find the electric field at point P (0, y).



ES0031

32. A small rigid object carries positive and negative charge of magnitude 4 coulomb each. It is oriented so that the positive charge has coordinates (–1.2mm, 1.1 mm) and negative charge has coordinates (1.4 mm, –1.3 mm). The object is kept in an electric field of $\left(2500\hat{i}-5000\hat{j}\right)$ N/C. Find the magnitude of torque (in N-m) acting on the dipole.

ES0032

33. A charge $+Q$ is fixed at the origin of the coordinate system while a small electric dipole of dipole-moment $\vec{p}$ pointing away from the charge along the x-axis is set free from a point far away from the origin.

(a) calculate the K.E. of the dipole when it reaches to a point $(d, 0)$ **[IIT-JEE 2003]**

(b) calculate the force on the charge $+Q$ due to the dipole at this moment.

ES0033

Conductors

34. A conducting sphere of radius R has two spherical cavities of radius a and b. The cavities have charges q_a and q_b respectively at their centres. Find:

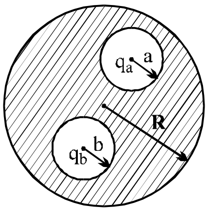
(a) The electric field and electric potential at a distance r

(i) r (distance from O, the centre of sphere > R)

(ii) r (distance from B, the centre of cavity b) < b

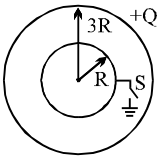
(b) Surface charge densities on the surface of radius R, radius a and radius b.

(c) What is the force on q_a and q_b?



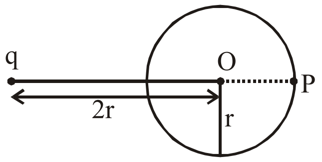
ES0034

35. Two thin conducting shells of radii R and $3R$ are shown in figure. The outer shell carries a charge $+Q$ and the inner shell is neutral. The inner shell is earthed with the help of switch S. Find the charge attained by the inner shell.



ES0035

36. A point charge q is placed at a distance 2r from the centre O of a conducting neutral sphere of radius r. Due to the induced charge on the sphere, the electric potential at point P on the surface of sphere is x volt. Then find the value of x. (If $\frac{kq}{r}$ = 18 volt)

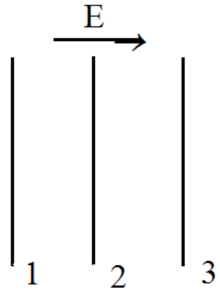


ES0036

37. A conducting liquid bubble of radius a and thickness t ($t << a$) is charged to potential V. If the bubble collapses to a droplet, find the potential on the droplet. **[IIT-JEE 2005]**

ES0037

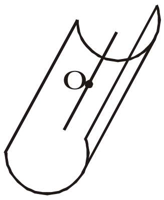
38. Three uncharge conducing large plates are placed parallel to each other in a uniform electric field. Find the induced charge density on each surface of each plate.



ES0038

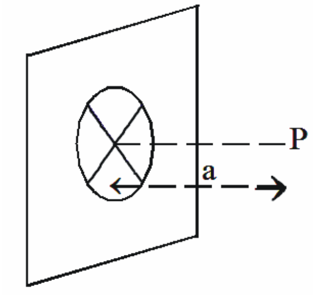
EXERCISE (S-2)

1. A thin long strip whose cross-section is a semicircle carries a uniform surface charge of density σ on its inner surface. Find the electric field at a point O located midway on its axis.



ES0039

2. An infinitely long non-conducting plane of charge density σ has circular aperture of certain radius carved out from it. The electric field at a point which is at a distance 'a' from the centre of the aperture is $\sigma/2\sqrt{2}\in_0$. Find the radius of aperture.

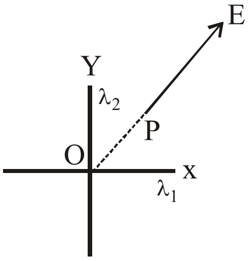


ES0040

3. A disc of radius R is kept such that its axis coincide with the x-axis and its centre is at (d,0,0). The thickness of disc is t and it carries a uniform volume charge density ρ. The external electric field in the space is given by $\vec{E} = K\vec{r}$ where K = Constant and $\vec{r}$ is position vector of any point in space with respect to the origin of the coordinate system. Find the electric force on the disc.

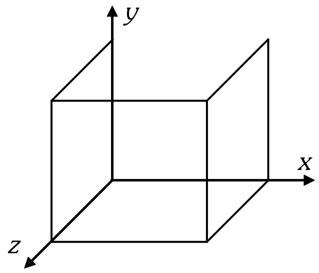
ES0041

4. Two mutually perpendicular infinite wires along x-axis and y-axis carry charge densities λ_1 and λ_2. The electric line of force at P is along the line $y = \frac{1}{\sqrt{3}}x$, where P is also a point lying on the same line then find λ_2/λ_1.



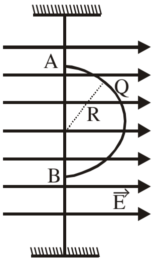
ES0042

5. An electric field given by $\vec{E} = 4\hat{i} - 3(y^2 + 2)\hat{j}$ pierces Gaussian's cube of side 1m placed at origin such that its three sides represents x, y and z axes. The net charge enclosed within the cube is given by $-n\varepsilon_0$. Find the value of n.



ES0043

6. The charge $Q = \pi C$ is distributed on a thin semicircular ring of radius R = 2m. There is a uniform electrostatic field $|\vec{E}| = 2N/C$ directed horizontally. The semicircular ring can rotate freely about a fixed vertical axis AB. Initially the ring is in static equilibrium as shown in figure. If we want to rotate it about the fixed axis by 90° then minimum work required on the ring is *x* J. Find the value of x.



ES0044

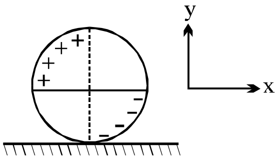
7. Four point charges +8μC, –1μC, –1μC and +8μC, are fixed at the points, $-\sqrt{\frac{27}{2}}$ m, $-\sqrt{\frac{3}{2}}$ m, $+\sqrt{\frac{3}{2}}$ m and $+\sqrt{\frac{27}{2}}$ m respectively on the y-axis. A particle of mass 6×10^{-4} kg and of charge + 0.1 μC moves along the $-x$ direction. Its speed at $x = +\infty$ is v_0. Find the least value of v_0 for which the particle will cross the origin . Find also the kinetic energy of the particle at the origin. Assume that space is gravity free. (Given : $1/(4\pi\varepsilon_0) = 9 \times 10^9$ Nm^2/C^2)

ES0045

8. The electric potential in a region is given by $V(x, y, z) = ax^2 + ay^2 + abz^2$. 'a' is a positive constant of appropriate dimensions and b, a positive constant such that V is volts when x, y, z are in m. Let b = 2. The work done by the electric field when a point charge +4μC moves from the point (0, 0, 0.1m) to the origin is 50 μJ. The radius of the circle of the equipotential curve corresponding to V = 6250 volts and $z = \sqrt{2}$ m is α m. Fill α^2 in OMR sheet.

ES0046

9. A nonconducting ring of mass m and radius R is charged as shown. The charged density i.e. charge per unit length is λ. It is then placed on a rough nonconducting horizontal surface plane. At time t = 0, a uniform electric field $\vec{E} = E_0 i$ is switched on and the ring start rolling without sliding. Determine the friction force (magnitude and direction) acting on the ring, when it starts moving.

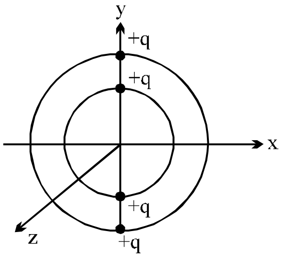


ES0047

10. Small identical balls with equal charges are fixed at vertices of regular 2008 - gon with side *a*. At a certain instant, one of the balls is released & a sufficiently long time interval later, the ball adjacent to the first released ball is freed. The kinetic energies of the released balls are found to differ by *K* at a sufficiently long distance from the polygon. Determine the charge *q* of each ball.

ES0048

11. Two concentric rings of radii r and 2r are placed with centre at origin. Two charges +q each are fixed at the diametrically opposite points of the rings as shown in figure. Smaller ring is now rotated by an angle 90° about Z-axis then it is again rotated by 90° about Y-axis. Find the work done by electrostatic forces in each step. If finally larger ring is rotated by 90° about X-axis, find the total work required to perform all three steps.



ES0049

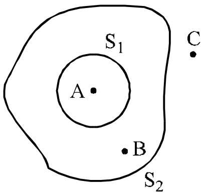
12. A non-conducting disc of radius a and uniform positive surface charge density σ is placed on the ground, with its axis vertical. A particle of mass m & positive charge q is dropped, along the axis of the disc, from a height H with zero initial velocity. The particle has $\frac{q}{m} = \frac{4\varepsilon_0 g}{\sigma}$.

(a) Find the value of H if the particle just reaches the disc.

(b) Sketch the potential energy of the particle as a function of its height and find its equilibrium position.

ES0050

13. S_1 and S_2 are two conducting surfaces. Between S_1 and S_2 and inside S_1 is air. S_1 is spherical with A its centre. S_1 has total charge Q. S_2 is uncharged. Find (if possible) :



(i) Charges induced on inner and outer surface of S_2.

(ii) Total electric field at A, B.

(iii) Electric field at B due to induced charges on S_2.

(iv) Electric field at C due to induced charges on inner surface of S_2.

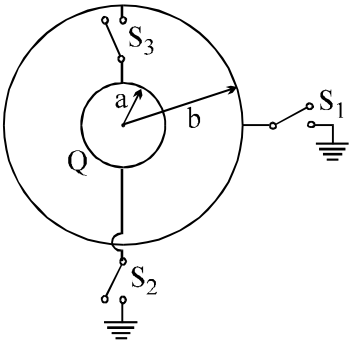
(v) Electric field produced by induced charges on outer surface of S_2 inside the body of S_2.

(vi) Can you find electric field at C easily ?

(take the required distances from A). Which charge will produce electric field here.

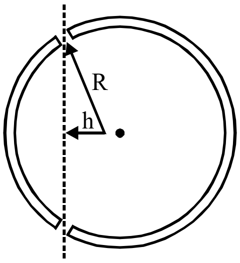
ES0051

14. The figure shows a conducting sphere 'A' of radius 'a' which is surrounded by a neutral conducting spherical shell B of radius 'b' (>a). Initially switches S_1, S_2 and S_3 are open and sphere 'A' carries a charge Q. First the switch 'S_1' is closed to connect the shell B with the ground and then opened. Now the switch 'S_2' is closed so that the sphere 'A' is grounded and then S_2 is opened. Finally, the switch 'S_3' is closed to connect the spheres together. Find the heat (in joule) which is produced after closing the switch S_3. [Consider b = 4 cm, a = 2 cm and Q = 80 μC]



ES0052

15. Consider a metal sphere, of radius R that is cut in two along a plane whose minimum distance from the sphere's centre is h. Sphere is uniformly charged by a total electric charge Q. What force is necessary to hold the two parts of the sphere together ?



ES0053

16. Two fixed charges –2Q and Q are located at the points with co-ordinates (–3a, 0) and (3a, 0) respectively in the x-y plane. (a) Show that all the points in the x-y plane where the electric potential due to the two charges is zero lie on a circle. Find its radius and the location of its centre. (b) Give the expression for the potential $V_{(x)}$ at a general point on the x-axis and sketch the function $V_{(x)}$ on the whole x-axis. (c) If a particle of charge +q starts from rest at the centre of the circle, show by a short qualitative argument that the particle eventually crosses the circle. Find its speed when it does so.

ES0054

17. The electric field strength depends only on the x, y and z coordinates according to the law $E = \dfrac{a(x\hat{i} + y\hat{j} + z\hat{k})}{\left(x^2 + y^2 + z^2\right)^{3/2}}$, where a = 122.5 SI unit and is a constant. Find the potential difference (in volt) between (3, 2, 6) and (0, 3, 4).

ES0055

EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

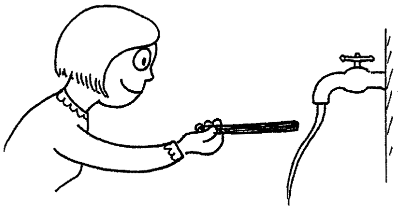
Properties of charges and coulomb's law

1. If an object made of substance A rubs an object made of substance B, then A becomes positively charged and B becomes negatively charged. If, however, an object made of substance A is rubbed against an object made of substance C, then A becomes negatively charged. What will happen if an object made of substance B is rubbed against an object made of substance C?

(A) B becomes positively charged and C becomes positively charged.
(B) B becomes positively charged and C becomes negatively charged.
(C) B becomes negatively charged and C becomes positively charged.
(D) B becomes negatively charged and C becomes negatively charged.

ES0056

2. In normal cases thin stream of water bends toward a negatively charged rod. When a positively charged rod is placed near the stream, it will bend in the



(A) Opposite direction. (B) Same direction.
(C) It won't bend at all. (D) Can't be predicted.

ES0057

3. Two charged bodies A and B exert repulsive forces on each other. If charge on A is more then that on B, which of the following statement is true.

(A) Body A experiences more Colombian force then B.
(B) Body A experiences less Colombian force then B.
(C) Both of them experience Colombian forces of equal magnitude.
(D) It depends whether the bodies can be treated as point like charges or not.

ES0058

4. Given are four arrangements of three fixed electric charges. In each arrangement, a point labeled P is also identified — test charge, +q, is placed at point P. All of the charges are of the same magnitude, Q, but they can be either positive or negative as indicated. The charges and point P all lie on a straight line. The distances between adjacent items, either between two charges or between a charge and point P, are all the same. Correct order of choices in a decreasing order of magnitude of force on P is

I. ⊕ ⊕ ⊕ ● P
II. ⊕ ⊕ P ● ⊖
III. ⊕ ⊕ ⊖ P ●
IV. ⊕ ⊖ ⊕ P ●

(A) II > I > III > IV (B) I > II > III > IV (C) II > I > IV > III (D) III > IV > I > II

ES0059

5. Two point charge of 100 μC and –4 μC are positioned at points $(-2\sqrt{3}, 3\sqrt{3}, -4)$ and $(4\sqrt{3}, -5\sqrt{3}, 6)$ respectively of a Cartesian coordinate system. Find the force vector on the –4 μC charge? All the coordinates are in meters.

(A) $9\times10^{-4}\left(3\sqrt{3}\hat{i}-4\sqrt{3}\hat{j}+5\hat{k}\right)$

(B) $9\times10^{-4}\left(-3\sqrt{3}\hat{i}+4\sqrt{3}\hat{j}-5\hat{k}\right)$

(C) $2.25\times10^{-4}\left(-3\sqrt{3}\hat{i}+4\sqrt{3}\hat{j}-5\hat{k}\right)$

(D) $2.25\times10^{-4}\left(3\sqrt{3}\hat{i}-4\sqrt{3}\hat{j}+5\hat{k}\right)$

ES0060

Electric field

6. Five positive equal charges are placed at vertices of a regular hexagon and net electric field at the centre is E_1. A negative charge having equal magnitude is placed sixth vertex and then net electric field is E_2. Find $\frac{E_2}{E_1}$.

(A) 2 (B) 1 (C) 3 (D) None of these

ES0061

7. There are two point charges q_1 and q_2 lying on a circle of unit radius. Electric field intensity at the center of circle due to these charges is $\vec{E}$. Find the position vector of the center with respect to q_2 if the position vector of the center with respect to q_1 is $\vec{r}_1$.

(A) $\frac{\vec{E}+kq_1\vec{r}_1}{kq_2}$

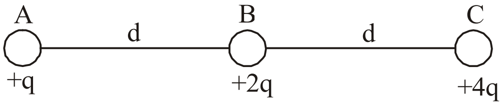
(B) $-\left(\frac{\vec{E}+kq_1\vec{r}_1}{kq_2}\right)$

(C) $\frac{kq_1\vec{r}_1-\vec{E}}{kq_2}$

(D) $\frac{\vec{E}-kq_1\vec{r}_1}{kq_2}$

ES0062

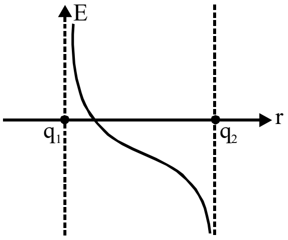
8. Three charges +q, +2q and +4q are connected by strings as shown in the figure. What is ratio of tensions in the strings AB and BC.



(A) 1 : 2 (B) 1 : 3 (C) 2 : 1 (D) 3 : 1

ES0063

9. The variation of electric field between the two charges q_1 and q_2 along the line joining the charges is plotted against distance from q_1 (taking rightward direction of electric field as positive) as shown in the figure. Then the correct statement is :-

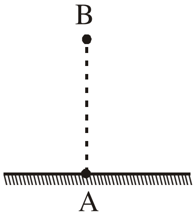


(A) q_1 and q_2 are positive and $q_1 < q_2$ (B) q_1 and q_2 are positive and $q_1 > q_2$

(C) q_1 is positive and q_2 is negative and $q_1 < q_2$ (D) q_1 and q_2 are negative and $q_1 < q_2$

ES0064

10. Particle *B* of charge *Q* and mass m is in equilibrium under weight and electrostatics force applied by a fixed charged *A*, which is directly beneath the particle *B* as shown in figure. When particle B is disturbed along vertical, the equilibrium is



(A) stable (B) unstable

(C) neutral (D) there can not be in equilibrium

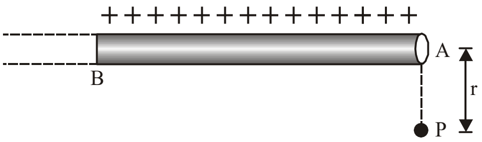
ES0065

11. A charge q is placed at the centroid of an equilateral triangle. Three equal charges Q are placed at the vertices of the triangle. The system of four charges will be in equilibrium if q is equal to :-

(A) $\frac{-Q}{\sqrt{3}}$ (B) $\frac{-Q}{3}$ (C) $-Q\sqrt{3}$ (D) $\frac{Q}{\sqrt{3}}$

ES0066

12. A semi-infinite insulating rod has linear charge density λ. The electric field at the point P shown in figure is :-

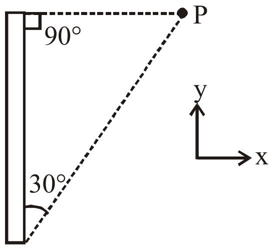


(A) $\frac{2\lambda^2}{(4\pi\varepsilon_0 r)^2}$ at 45° with AB (B) $\frac{\sqrt{2}\lambda}{4\pi\varepsilon_0 r^2}$ at 45° with AB

(C) $\frac{\sqrt{2}\lambda}{4\pi\varepsilon_0 r}$ at 45° with AB (D) $\frac{\sqrt{2}\lambda}{4\pi\varepsilon_0 r}$ at perpendicular to AB

ES0067

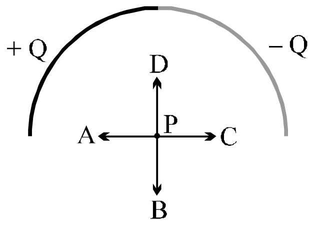
13. The direction (θ) of $\vec{E}$ at point P due to uniformly charged finite rod will be :-



(A) at angle 30^0 from x-axis
(B) 45^0 from x - axis
(C) 60^0 from x-axis
(D) none of these

ES0068

14. As shown in the figure, an insulating rod is set into the shape of a semicircle. The left half of the rod has a charge of + Q uniformly distributed along its length, and the right half of the rod has a charge of –Q uniformly distributed along its length. What vector shows the correct direction of the electric field at point P, the centre of the semicircle ?



(A) A
(B) B
(C) C
(D) D

ES0069

15. A nonconducting ring of radius *R* has uniformly distributed positive charge *Q*. A small part of the ring, of length d, is removed ($d << R$). The electric field at the centre of the ring will now be
(A) directed towards the gap, inversely proportional to R^3.
(B) directed towards the gap, inversely proportional to R^2.
(C) directed away from the gap, inversely proportional to R^3.
(D) directed away from the gap, inversely proportional to R^2.

ES0070

16. The maximum electric field at a point on the axis a uniformly charged ring is E_0. At how many points on the axis will the magnitude of electric field be $E_0/2$:-
(A) 1
(B) 2
(C) 3
(D) 4

ES0071

17. A particle of mass m, charge –Q is constrained to move along the axis of a ring of radius a. The ring carries a uniform charge density + λ along its circumference. Initially, the particle lies in the plane of the ring at a point where no net force acts on it. The period of oscillation of the particle when it is displaced slightly from its equilibrium position is

(A) $T = 4\pi\sqrt{\dfrac{\varepsilon_0 ma^2}{\lambda Q}}$
(B) $T = 2\pi\sqrt{\dfrac{2\varepsilon_0 ma^2}{\lambda Q}}$
(C) $T = 2\pi\sqrt{\dfrac{4\varepsilon_0 ma^2}{\lambda Q}}$
(D) $T = 2\pi\sqrt{\dfrac{\varepsilon_0 ma^2}{2\lambda Q}}$

ES0072

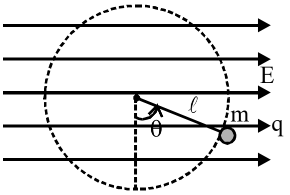
18. The surface charge density of a thin charged disc of radius R is σ. The value of the electric field at the centre of the disc is $\frac{\sigma}{2\epsilon_0}$. With respect to the field at the centre, the electric field along the axis at a distance $\sqrt{3}$ R from the centre of the disc :

(A) reduces by 70.7% (B) reduces by 29.3%

(C) reduces by 86.6% (D) reduces by 13.4%

ES0073

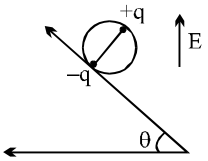
19. A small ball of mass m and charge +q tied with a string of length ℓ, is rotating in a vertical circle under gravity and a uniform horizontal electric field E as shown. The tension in the string will be minimum for:-



(A) $\theta = \tan^{-1}\left(\frac{qE}{mg}\right)$ (B) $\theta = \pi$ (C) $\theta = \pi - \tan^{-1}\left(\frac{qE}{mg}\right)$ (D) $\theta = \pi + \tan^{-1}\left(\frac{qE}{mg}\right)$

ES0074

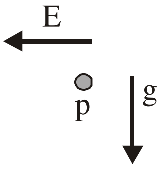
20. A wheel having mass m has charges +q and –q on diametrically opposite points. It remains in equilibrium on a rough inclined plane in the presence of uniform vertical electric field E =

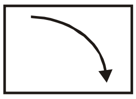


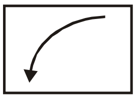
(A) $\frac{mg}{q}$ (B) $\frac{mg}{2q}$ (C) $\frac{mg\tan\theta}{2q}$ (D) none

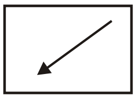
ES0075

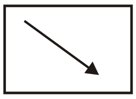
21. A negatively charged particle *p* is placed, initially at rest, in a constant, uniform gravitational field and a constant, uniform electric field as shown in the diagram. What qualitatively, is the shape of the trajectory of the electron ?



(A) 

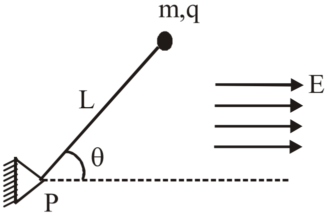
(B) 

(C) 

(D) 

ES0076

22. A particle of mass m and charge q is attached to a light rod of length L. The rod can rotate freely in the plane of paper about the other end, which is hinged at P. The entire assembly lies in a uniform electric field E also acting in the plane of paper as shown. The rod is released from rest when it makes an angle θ with the electric field direction. Determine the speed of the particle when the rod is parallel to the electric field.

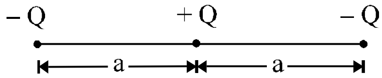


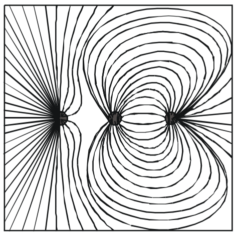
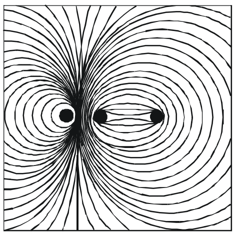
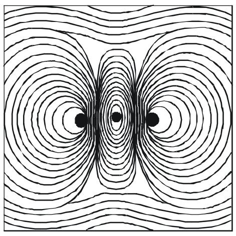
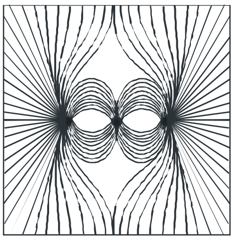
(A) $\left(\frac{2qEL(1-\cos\theta)}{m}\right)^{1/2}$ (B) $\left(\frac{2qEL(1-\sin\theta)}{m}\right)^{1/2}$

(C) $\left(\frac{qEL(1-\cos\theta)}{2m}\right)^{1/2}$ (D) $\left(\frac{2qEL\cos\theta}{m}\right)^{1/2}$

ES0077

23. The fig. shows the distribution of three charges –Q, +Q and –Q on the X-axis. Which of the following figures shows the possible electric field lines for the distribution?

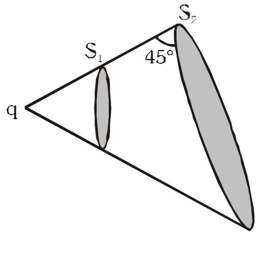


(A)  (B)  (C)  (D) 

ES0078

Gauss' law

24. In the given figure flux through surface S_1 is ϕ_1 & through S_2 is ϕ_2. Which is correct ?

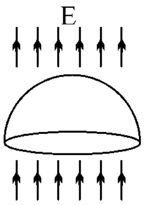


(A) $\phi_1 = \phi_2$ (B) $\phi_1 > \phi_2$

(C) $\phi_1 < \phi_2$ (D) None of these

ES0079

25. A hemispherical surface (half of a spherical surface) of radius R is located in a uniform electric field E that is parallel to the axis of the hemisphere. What is the magnitude of the electric flux through the hemisphere surface?

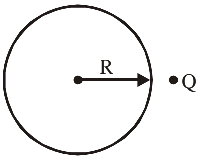


(A) 0 (B) $4\pi R^2E/3$ (C) $2\pi R^2E$ (D) πR^2E

ES0080

26. **Statement 1:** A charge is outside the Gaussian sphere of radius R. Then electric field on the surface of sphere is zero.

and



Statement 2 : As $\oint \vec{E}\cdot d\vec{s} = \frac{q_{in}}{\varepsilon_0}$, for the sphere q_{in} is zero, so $\oint \vec{E}\cdot d\vec{s} = 0$.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

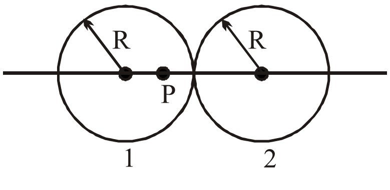
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

ES0081

27. Figure shows, in cross section, two solid spheres with uniformly distributed charge throughout their volumes. Each has radius R. Point P lies on a line connecting the centres of the spheres, at radial distance R/2 from the center of sphere 1. If the net electric field at point P is zero and Q_1 is 64 μC, what is Q_2(in μC).



(A) 64 (B) 36 (C) 32 (D) 72

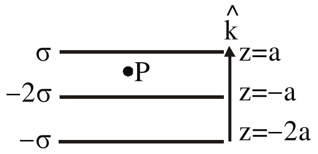
ES0082

28. A sphere of radius R carries charge density proportional to the square of the distance from the center: $\rho = Ar^2$, where A is a positive constant. At a distance of R/2 from the center, the magnitude of the electric field is :-

(A) $A/4\pi\varepsilon_0$ (B) $AR^3/40\varepsilon_0$ (C) $AR^3/24\varepsilon_0$ (D) $AR^3/5\varepsilon_0$

ES0083

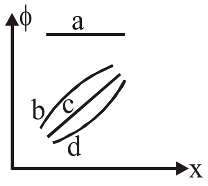
29. Three large parallel plates have uniform surface charge densities as shown in the figure. What is the electric field at *P*? **[IIT-JEE 2005 (Scr)]**



(A) $-\frac{4\sigma}{\in_0}\hat{k}$ (B) $\frac{4\sigma}{\in_0}\hat{k}$ (C) $-\frac{2\sigma}{\in_0}\hat{k}$ (D) $\frac{2\sigma}{\in_0}\hat{k}$

ES0084

30. A line of charge extends along a X-axis whose linear charge density varies directly as x. Imagine a spherical volume with its centre located on X-axis and is moving gradually along it. Which of the graphs shown in figure correspond to the flux ϕ with the x coordinate of the centre of the volume?



(A) a (B) b (C) c (D) d

ES0085

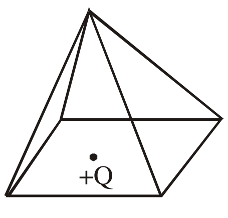
31. The electric field in a region is given by $\vec{E} = 200\hat{i}$ N/C for x > 0 and $-200\hat{i}$ N/C for x < 0. A closed cylinder of length 2m and cross-section area 10^2 m^2 is kept in such a way that the axis of cylinder is along X-axis and its centre coincides with origin. The total charge inside the cylinder is

[Take : $\varepsilon_0 = 8.85 \times 10^{-12}$ C^{2}m^2.N)

(A) 0 (B) 1.86×10^{-5}C (C) 1.77×10^{-11}C (D) 35.4×10^{-8} C

ES0086

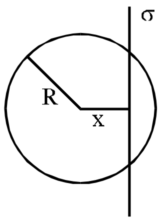
32. A point charge +Q is positioned at the center of the base of a square pyramid as shown. The flux through one of the four identical upper faces of the pyramid is :-



(A) $\frac{Q}{16\varepsilon_0}$ (B) $\frac{Q}{4\varepsilon_0}$ (C) $\frac{Q}{8\varepsilon_0}$ (D) None of these

ES0087

33. An infinite, uniformly charged sheet with surface charge density σ cuts through a spherical Gaussian surface of radius R at a distance x from its center, as shown in the figure. The electric flux Φ through the Gaussian surface is :-



(A) $\frac{\pi R^2 \sigma}{\varepsilon_0}$ (B) $\frac{2\pi\left(R^2 - x^2\right)\sigma}{\varepsilon_0}$ (C) $\frac{\pi(R-x)^2\sigma}{\varepsilon_0}$ (D) $\frac{\pi\left(R^2 - x^2\right)\sigma}{\varepsilon_0}$

ES0088

34. Consider a circle of radius R. A point charge lies at a distance 'a' from its center and on its axis such that $R = a\sqrt{3}$. If electric flux passing through the circle is ϕ then the magnitude of the point charge is:-

(A) $\sqrt{3}\varepsilon_0\phi$ (B) $2\varepsilon_0\phi$ (C) $4\varepsilon_0\phi/\sqrt{3}$ (D) $4\,\varepsilon_0\phi$

ES0089

Electric potential energy and electric potential

35. Two particles X and Y, of equal mass and with unequal positive charges, are free to move and are initially far away from each other. With Y at rest, X begins to move towards it with initial velocity u. After a long time, finally :-

(A) X will stop, Y will move with velocity u.

(B) X and Y will both move with velocities u/2 each.

(C) X will stop, Y will move with velocity < u.

(D) both will move with velocities < u/2.

ES0090

36. Two positively charged particles X and Y are initially far away from each other and at rest. X begins to move towards Y with some initial velocity. The total momentum and energy of the system are p and E.

(A) If Y is fixed, both p and E are conserved.

(B) If Y is fixed, E is conserved, but not p.

(C) If both are free to move, p is conserved but not E.

(D) If both are free, E is conserved, but not p.

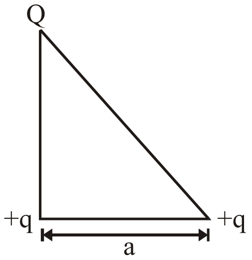
ES0091

37. Potential energy of a system comprising of point charges is U_1.When a charge q is added in the system without disturbing other charges, the potential energy becomes U_2.The potential of the point where the charge q is placed in the system is

(A) $\frac{U_2-U_1}{q}$ (B) $\frac{U_1-U_2}{q}$ (C) $\frac{U_1+U_2}{2q}$ (D) $\frac{U_2-U_1}{2q}$

ES0092

38. Three charges Q, +q and +q are placed at the vertices of a right-angled isosceles triangle as shown. The net electrostatic energy of the configuration is zero if Q is equal to : **[JEE 2000(Scr) 1 + 1]**



(A) $\frac{-q}{1+\sqrt{2}}$ (B) $\frac{-2q}{2+\sqrt{2}}$ (C) $-2q$ (D) $+q$

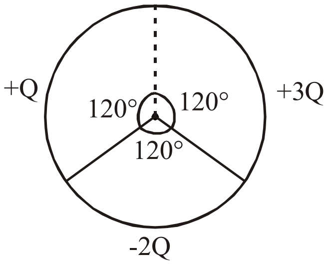
ES0093

39. Two fixed charges A and B of 5 μC each are separated by a distance of 6m. C is the mid point of the line joining A and B. A charge 'Q' of –5μC is shot perpendicular to the line joining A and B through C with a kinetic energy of 0.06J. The charge 'Q' comes to rest at a point D. The distance CD is:-

(A) 3 m (B) $\sqrt{3}$ m (C) $3\sqrt{3}$ m (D) 4 m

ES0094

40. Figure shows three circular arcs, each of radius R and total charge as indicated. The net electric potential at the centre of curvature is :-

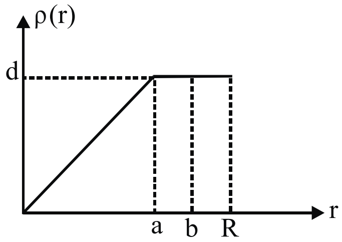


(A) $\frac{Q}{4\pi \epsilon_0 R}$ (B) $\frac{Q}{2\pi \epsilon_0 R}$

(C) $\frac{2Q}{\pi \epsilon_0 R}$ (D) $\frac{Q}{\pi \epsilon_0 R}$

ES0095

41. The nuclear charge (Ze) is non-uniformly distribute within a nucleus of radius R. The charge density ρ(r) [charge per unit volume] is dependent on the radial distance r from the centre of the nucleus as shown in figure. Select correct alternative/s.



(A) Electric field at r = R is independent of b

(B) Electric potential at r = R is proportional to b

(C) Electric field at r = R is proportional to a

(D) Electric potential at r = R is proportional to a

ES0096

42. A solid sphere of radius R is charged uniformly. At what distance from its surface is the electrostatic potential half of the potential at the centre?

(A) R (B) R/2 (C) R/3 (D) 2R

ES0097

43. When a negative charge is released and moves in electric field, it moves toward a position of

(A) lower electric potential and lower potential energy

(B) lower electric potential and higher potential energy

(C) higher electric potential and lower potential energy

(D) higher electric potential and higher potential energy

ES0098

44. A positively charged particle is released from rest in an uniform electric field. The electric potential energy of the charge

(A) remains a constant because the electric field is uniform.

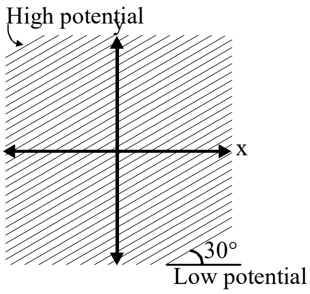
(B) increases because the charge moves along the electric field.

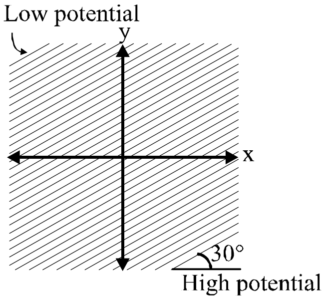
(C) decreases because the charge moves along the electric field.

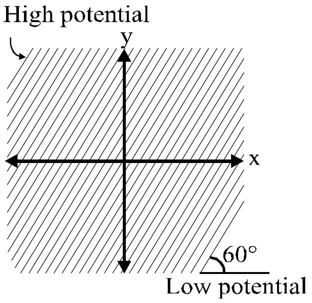
(D) decreases because the charge moves opposite to the electric field.

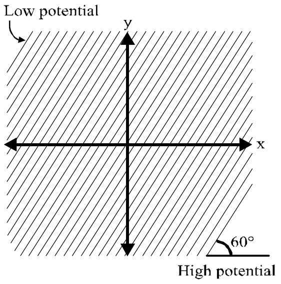
ES0099

45. The electric field intensity at all points in space is given by $\vec{E} = \sqrt{3}\hat{i} - \hat{j}$ volts/metre. The nature of equipotential lines in x-y plane is given by :-

(A) 

(B) 

(C) 

(D) 

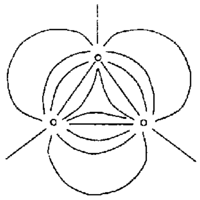
ES0100

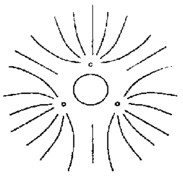
46. Equipotential at a great distance from a collection of charges whose total sum is not zero are approximately

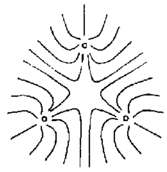
(A) spheres.
(B) planes.
(C) paraboloids
(D) ellipsoids.

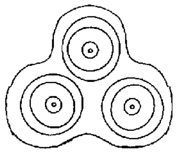
ES0101

47. Three positive charges of equal value q are placed at the vertices of an equilateral triangle. The resulting lines of force should be sketched as in **[JEE 2001 (Scr)]**

(A) 

(B) 

(C) 

(D) 

ES0102

48. In an electric field the potential at a point is given by the following relation V = $\frac{343}{r}$. The electric field at $\vec{r} = 3\hat{i} + 2\hat{j} + 6\hat{k}$ is :

(A) $21\hat{i} + 14\hat{j} + 42\hat{k}$
(B) $3\hat{i} + 2\hat{j} + 6\hat{k}$
(C) $\frac{1}{7}(3\hat{i} + 2\hat{j} + 6\hat{k})$
(D) $-(3\hat{i} + 2\hat{j} + 6\hat{k})$

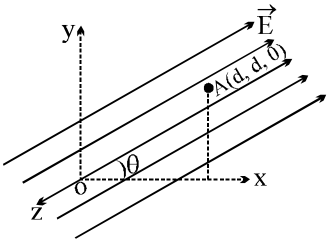
ES0103

49. From a point if we move in a direction making an angle θ measured from +ve x-axis , the potential gradient is given as $\frac{dv}{dr}$=2 cos θ. Find the direction and magnitude of electric field at that point.

(A) $2\hat{i}$ (B) $-2\hat{i}$ (C) $\hat{i}+\hat{j}$ (D) $-\hat{i}+\hat{j}$

ES0104

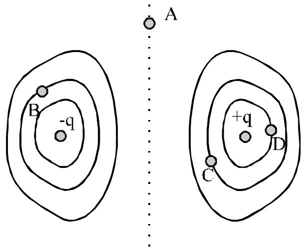
50. A uniform electric field having strength $\vec{E}$ is existing in x-y plane as shown in figure. Find the p.d. between origin O & A(d, d, 0)



(A) Ed (cosθ + sinθ) (B) –Ed (sinθ – cosθ) (C) $\sqrt{2}$ Ed (D) none of these

ES0105

51. Figure shows equi-potential surfaces for a two charges system. At which of the labeled points point will an electron have the highest potential energy?



(A) Point A (B) Point B (C) Point C (D) Point D

ES0106

52. Figure shows some equipotential lines distributed in space. A charged object is moved from point A to point B.

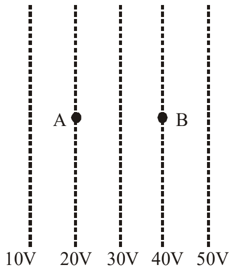


Figure-I
(i)

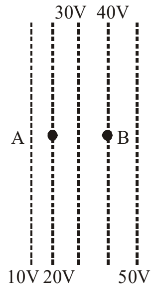


Figure-II
(ii)

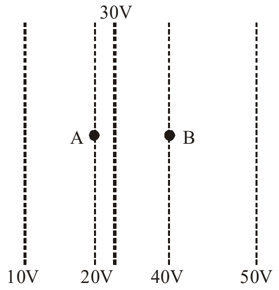


Figure-III
(iii)

(A) The work done in Fig. (i) is the greatest.
(B) The work done in Fig. (ii) is least.
(C) The work done is the same in Fig. (i), Fig. (ii) and Fig. (iii).
(D) The work done in Fig. (iii) is greater than Fig. (ii)but equal to that in Fig. (i).

ES0107

53. Consider the following conclusions regarding the components of an electric field at a certain point in space given by

$E_x = -Ky$ $\quad E_y = Kx$ $\quad E_z = 0$

(I) The field is conservative. (II) The field is non-conservative.

(III) The lines of force are staright lines (IV) The lines of force are circles.

Of these conclusions

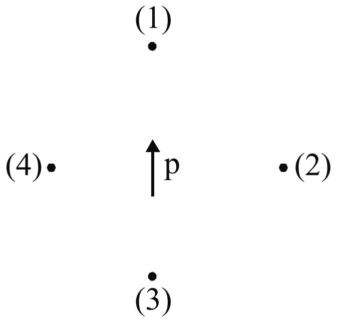
(A) II and IV are valid (B) I and III are valid

(C) I and IV are valid (D) II and III are valid

ES0108

Electric dipole

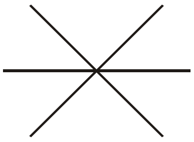
54. The drawing shows four points surrounding an electric dipole. Which one of the following expressions best ranks the electric potential at these four locations?

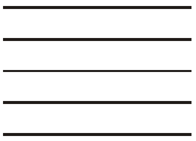


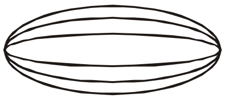
(A) $1 > 2 = 4 > 3$ (B) $3 > 2 > 4 > 1$ (C) $3 > 2 = 4 > 1$ (D) $2 = 4 > 1 = 3$

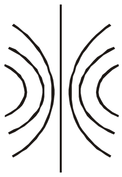
ES0109

55. Which of the following represents the equipotential lines of a dipole (two equal and opposite charges placed at small separation)?

(A) 

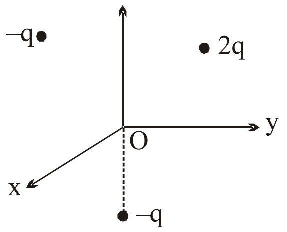
(B) 

(C) 

(D) 

ES0110

56. Three point charges 2q, –q and –q are located respectively at (0, a, a), (0, a, –a) and (0, 0, – a) as shown. The dipole moment of this distribution is :-



(A) 2qa in the y–z plane at $\tan^{-1}\left(\frac{1}{4}\right)$ with z-axis

(B) $\sqrt{17}$qa in the y–z plane at $\tan^{-1}\left(\frac{1}{4}\right)$ with z-axis

(C) $\sqrt{5}$qa in the x–y plane at $\tan^{-1}(4)$ with y-axis

(D) 4qa in the x–y plane at $\tan^{-1}(4)$ with y-axis

ES0111

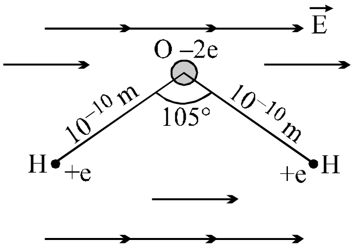
57. Point P lies on the axis of a dipole. If the dipole is rotated by 90° anticlock wise, the electric field vector $\vec{E}$ at P will rotate by

(A) 90° clock wise (B) 180° clock wise

(C) 90° anti clock wise (D) none

ES0112

58. A water molecule as shown is in a region of uniform electric field $\vec{E} = 1000\hat{i}$ V/m. This molecule experiences

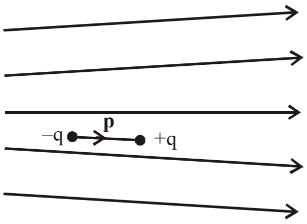


(A) A counterclockwise torque (B) A clockwise torque

(C) A net force to the right (D) A net force to the left

ES0113

59. Electric field lines in which an electric dipole **p** is placed as shown. Which of the following statements is correct ?



(A) The dipole will not experience any force.

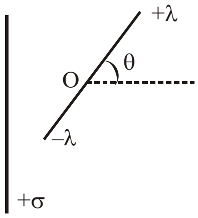
(B) The dipole will experience a force towards right.

(C) The dipole will experience a force towards left.

(D) The dipole will experience a force upwards.

ES0114

60. A large sheet carries uniform surface charge density σ. A rod of length 2ℓ has a linear charge density λ on one half and – λ on the other half. The rod is hinged at mid point O and makes angle θ with the normal to the sheet. The torque experienced by the rod is :-



(A) $\dfrac{\sigma\lambda\ell^2}{2\varepsilon_0}\cos\theta$ (B) $\dfrac{\sigma\lambda\ell}{\varepsilon_0}\cos^2\theta$

(C) $\dfrac{\sigma\lambda\ell^2\sin\theta}{2\varepsilon_0}$ (D) $\dfrac{\sigma\lambda\ell\sin^2\theta}{\varepsilon_0}$

ES0115

61. An electric dipole (dipole moment p) is placed at a radial distance r >> a (where a is dipole length) from a infinite line of charge having linear charge density +λ. Dipole moment vector is aligned along radial vector $\vec{r}$ force experienced by dipole is :-

(A) $\frac{\lambda p}{2\pi\varepsilon_0 r^2}$, attractive (B) $\frac{\lambda p}{2\pi\varepsilon_0 r^3}$, attractive

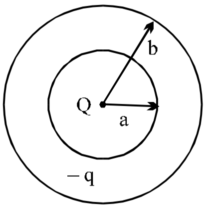
(C) $\frac{\lambda p}{2\pi\varepsilon_0 r^2}$, repulsive (D) $\frac{\lambda p}{2\pi\varepsilon_0 r^3}$, repulsive

ES0116

Conductors

62. Both question (a) and (b) refer to the system of charges as shown in the figure. A spherical shell with an inner radius 'a' and an outer radius 'b' is made of conducting material. A point charge +Q is placed at the centre of the spherical shell and a total charge – q is placed on the shell.

a. Charge – q is distributed on the surfaces as



(A) – Q on the inner surface, – q on outer surface

(B) – Q on the inner surface, – q + Q on the outer surface

(C) +Q on the inner surface, –q – Q on the outer surface

(D) The charge –q is spread uniformly between the inner and outer surface.

b. Assume that the electrostatic potential is zero at an infinite distance from the spherical shell. The electrostatic potential at a distance R (a < R < b) from the centre of the shell is (where $K = \frac{1}{4\pi\varepsilon_0}$)

(A) 0 (B) $\frac{KQ}{a}$ (C) $K\frac{Q-q}{R}$ (D) $K\frac{Q-q}{b}$

ES0117

63. The electrostatic potential on the surface of a charged conducting sphere is 100V. Two statements are made in this regard:

S_1 : At any point inside the sphere, electric intensity is zero.

S_2 : At any point inside the sphere, the electrostatic potential is 100V.

Which of the following is a correct statement?

(A) S_1 is true but S_2 is false.

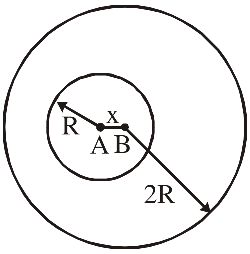
(B) Both S_1 & S_2 are false.

(C) S_1 is true, S_2 is also true and S_1 is the cause of S_2.

(D) S_1 is true, S_2 is also true but the statements are independent.

ES0118

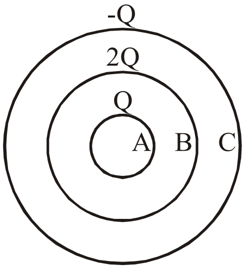
64. Figure shows two shells of radii R and 2R. The inner shell (centre at A) is nonconducting and uniformly charged wih charge Q while the outer shell (centre at B) is conducting and uncharged. the potential at the point B is :-



(A) zero (B) $\frac{KQ}{R}$ (C) $\frac{KQ}{x}$ (D) None of these

ES0119

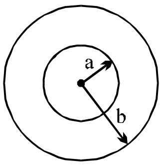
65. Charge Q, 2Q and –Q are given to three concentric conducting spherical shells A, B and C respectively. The ratio of charges on the inner and the outer surfaces of the shell 'C' will be :-



(A) $\frac{3}{4}$ (B) $-\frac{3}{4}$ (C) $\frac{3}{2}$ (D) $-\frac{3}{2}$

ES0120

66. If the electric potential of the inner metal sphere is 10 volt & that of the outer shell is 5 volt, then the potential at the centre will be :



(A) 10 volt (B) 5 volt (C) 15 volt (D) 0

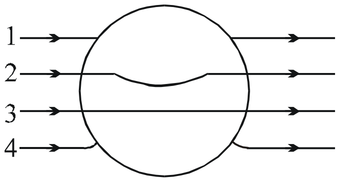
ES0121

67. n small drops of same size are charged to V volts each. If they coalesce to form a signal large drop, then its potential will be :-

(A) V/n (B) Vn (C) $Vn^{1/3}$ (D) $Vn^{2/3}$

ES0122

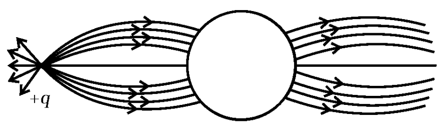
68. A metallic solid sphere is placed in a uniform electric field. The lines of force follow the path (s) shown in figure as :



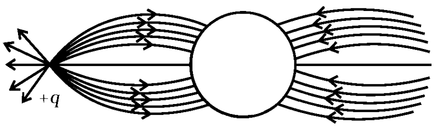
(A) 1 (B) 2 (C) 3 (D) 4

ES0123

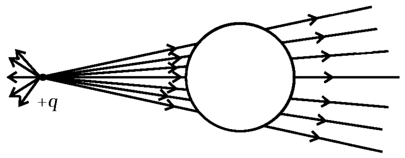
69. A point positive charge is brought near an isolated conducting sphere. The electric field is best given by



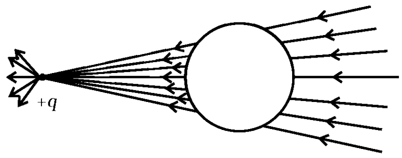
(i)



(ii)



(iii)



(iv)

(A) Fig (i) (B) Fig (iii) (C) Fig (ii) (D) Fig (iv)

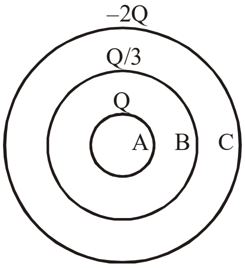
ES0124

70. There are two uncharged identical metallic spheres 1 and 2 of radius r separated by a distance d (d >> r). A charged metallic sphere of same radius having charge q is touched with one of the sphere. After some time it is moved away from the system. Now the uncharged sphere is earthed. Charge on earthed sphere is

(A) q (B) – q (C) – qr/2d (D) 0

ES0125

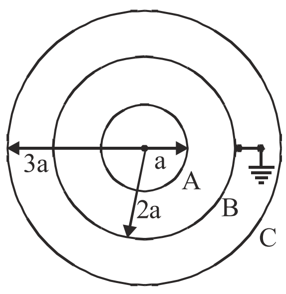
71. Three conducting concentric spherical shells of radius R, 2R and 3R have charges Q, $\frac{Q}{3}$ and – 2Q respectively. The intermediate shell is now grounded. Find the charge flow into the earth.



(A) $\frac{Q}{3}$ (B) $\frac{2Q}{3}$ (C) Q (D) 0

ES0126

72. Figure shows a system of three concentric metal shells A, B and C with radii a, 2a and 3a respectively. Shell B is earthed and shell C is given a charge Q. Now if shell C is connected to shell A, then the final charge on the shell B, is equal to :

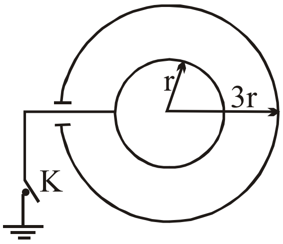


(A) $-\frac{4Q}{13}$ (B) $-\frac{8Q}{11}$

(C) $-\frac{5Q}{3}$ (D) $-\frac{3Q}{7}$

ES0127

73. Figure shows two conducting thin concentric shells of radii r and 3r. The outer shell carries charge q and inner shell is neutral. The amount of charge which flows from inner shell to the earth after the key K is closed, is equal to :-

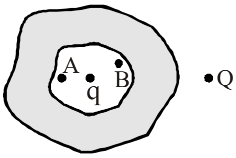


(A) –q/3 (B) q/3 (C) 3q (D) –3q

ES0128

74. **Statement-1 :** A point charge q is placed inside a cavity of conductor as shown. Another point charge Q is placed outside the conductor as shown. Now as the point charge Q is pushed away from conductor, the potential difference ($V_A - V_B$) between two points A and B within the cavity of sphere remains constant.

Statement-2 : The electric field due to charges on outer surface of conductor and outside the conductor is zero at all points inside the conductor.



(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

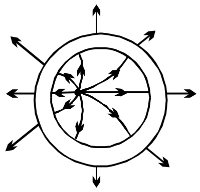
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

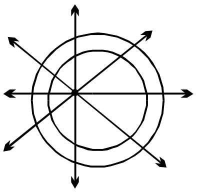
(C) Statement-1 is true, statement-2 is false.

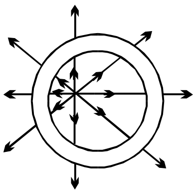
(D) Statement-1 is false, statement-2 is true.

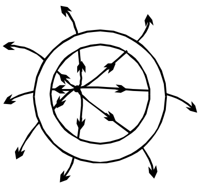
ES0129

75. A point charge *q* is placed at a point inside a hollow conducting sphere. Which of the following electric force pattern is correct ? **[IIT-JEE'2003 (scr)]**

(A)


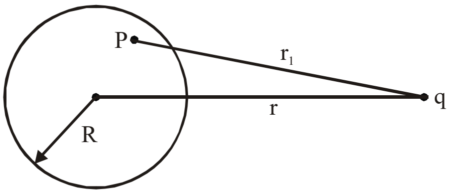
(B)


(C)


(D)


ES0130

76. A point charge q is placed at a distance r from center of a conducting neutral sphere of radius R (r>R). The potential at any point P inside the sphere at a distance r_1 from point charge due to induced charge of the sphere is given by :-



(A) $\frac{Kq}{r_1}-\frac{Kq}{R}$ (B) $\frac{Kq}{r_1}-\frac{Kq}{R}$ (C) $\frac{Kq}{r}-\frac{Kq}{r_1}$ (D) $-\frac{Kq}{r_1}+\frac{Kq}{R}$

ES0131

77. Consider a conductor with a spherical cavity in it. A point charge q_0 is placed at the centre of cavity and a point charge Q is placed outside conductor.

Statement-1 : Total charge induced on cavity wall is equal and opposite to the charge inside.

Statement-2 : If cavity is surrounded by a Gaussian surface, where all parts of Gaussian surface are located inside the conductor, $\oint \vec{E}.d\vec{A}=0$; hence $q_{\text{induced}}=-q_0$

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

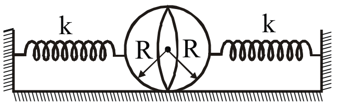
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

ES0132

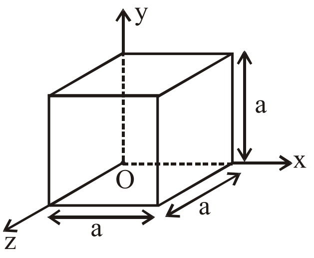
78. Two non-conducting hemispherical surfaces, which are having uniform charge density σ are placed on smooth horizontal surface as shown in figure. Assuming springs are ideal, calculate compression in each spring if both the hemispherical surface are just touching each other.



(A) $\frac{\sigma^2 R^2}{2\varepsilon_0 k}$ (B) R (C) $\frac{\sigma^2 \pi R^2}{2\varepsilon_0 k}$ (D) None of these

ES0133

79. Electric field in a region is found to be $E=3y\hat{j}$. The total energy stored in electric field inside the cube shown will be



(A) $9a^5 \in_0$ (B) $3 \in_0 a^5$

(C) $\frac{3}{2}\in_0 a^5$ (D) 0

ES0134

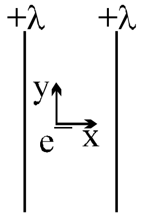
MULTIPLE CORRECT TYPE QUESTIONS

80. If a body is charged by rubbing it with another body then its weight :-

(A) may increase slightly
(B) may decrease slightly
(C) must increase slightly
(D) remains precisely constant

ES0135

81. An electron is placed just in the middle between two long fixed line charges of charge density $+\lambda$ each. The wires are in the xy plane (Do not consider gravity)



(A) The equilibrium of the electron will be unstable along x-direction
(B) The equilibrium of the electron will be neutral along y-direction
(C) The equilibrium of the electron will be stable along z-direction
(D) The equilibrium of the electron will be stable along y-direction

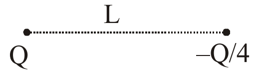
ES0136

82. Which statement(s) concerning electrostatic fields is/are **CORRECT**?

(A) Electric field lines never cross each other.
(B) Positive charge experiences force in the direction of electric field line.
(C) Electric field lines always start on negative charges and end on positive charges.
(D) Electric field lines that are closer indicate a stronger electric field while electric field lines that are far apart indicate a weaker electric field.

ES0137

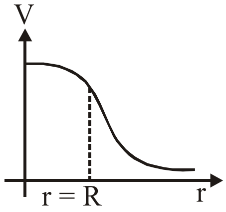
83. Two point charges Q and $-\dfrac{Q}{4}$ are separated by distance L then

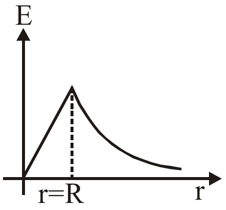


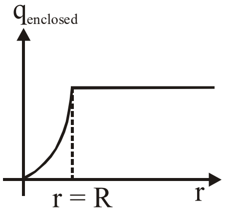
(A) Potential is zero at a point on the axis which is at a distance L/3 on the right side of charge $-\dfrac{Q}{4}$
(B) Potential is zero at a point on the axis which is at a distance L/5 on the left side of charge $-\dfrac{Q}{4}$
(C) Electric field is zero at a point on the axis which is at a distance L on the right side of charge $-\dfrac{Q}{4}$
(D) There exist two points on the axis, where electric field is zero

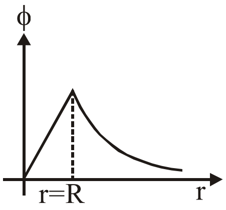
ES0138

84. Graph shows the variation of potential (V), magnitude of electric field (E), charge enclosed ($q_{enclosed}$) within the concentric sphere and net flux (ϕ) through a concentric spherical Gaussian surface as a function of distance (r) from centre of a uniformly positive charged solid sphere of radius R. Choose the **CORRECT** option(s):-

(A)


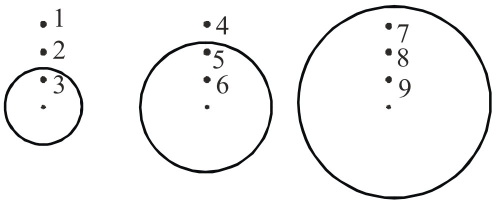
(B)


(C)


(D)


ES0139

85. Figure shows three spherical shells in separate situations, with each shell having the same uniform positive net charge. Points 1, 4 and 7 are at the same radial distances from the centre of the their respective shells; so are points 2, 5 and 8 ; and so are points 3, 6 and 9. With the electric potential taken equals to zero at an infinite distance, choose correct statement.



(A) Point 3 has highest potential
(B) point 1, 4 and 7 are at same potential
(C) Point 9 has lowest potential
(D) point 5 and 8 are at same potential

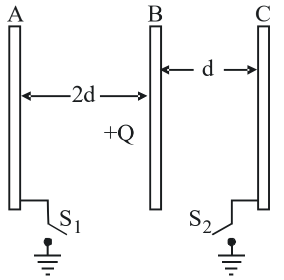
ES0140

86. Which of the following represent(s) an electrostatic field :-

(A) $\vec{E} = y\hat{i} + x\hat{j}$
(B) $\vec{E} = y\hat{i}$
(C) $\vec{E} = y\hat{i} + x\hat{j} + z\hat{k}$
(D) $\vec{E} = \alpha\vec{r}$ (α is constant)

ES0141

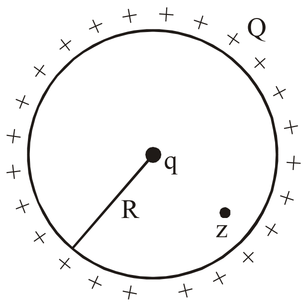
87. Three identical, parallel conducting plates A, B and C are placed as shown. Switches S_1 and S_2 are open, and can connect A and C to earth when closed. +Q charge is given to B.



(A) If S_1 is closed with S_2 open, a charge of amount Q will pass through S_1
(B) If S_2 is closed with S_1 open, a charge of amount Q will pass through S_2
(C) If S_1 and S_2 are closed together, a charge of amount Q/3 will pass through S_1, and a charge of amount 2Q/3 will pass through S_2.
(D) All the above statements are incorrect

ES0142

88. A positive charge Q is uniformly distributed along a circular ring of radius R. A small test charge q is placed at the centre of the ring. Then



(A) If $q > 0$ and is displaced away from the centre in the plane of the ring, it will be pushed back towards the centre.

(B) If $q < 0$ and is displaced away from the centre in the plane of the ring, it will never return to the centre and will continue moving till it hits the ring.

(C) If $q < 0$, it will perform SHM for small displacement along the axis.

(D) q at the centre of the ring is in an unstable equilibrium within the plane of the ring for $q > 0$.

ES0143

89. If there were only one type of charge in the universe, then

(A) $\oint_s E.dS \neq 0$ on any surface.

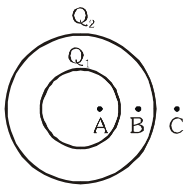
(B) $\oint_s E.dS = 0$ if the charge is outside the surface.

(C) $\oint_s E.dS$ could not be defined.

(D) $\oint_s E.dS = \frac{q}{\varepsilon_0}$ if charges of magnitude q were inside the surface.

ES0144

90. Figure shows two uniform charged concentric spherical shell. Both charges are positive, Select correct statement



(A) Electric field intensity at B may be greater than electric field intensity at C.

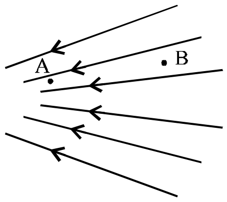
(B) Electric field intensity at B must be greater than electric field intensity at C.

(C) Potential at A greater than potential at B

(D) If a charge moves from B to C work done by electric force must be positive.

ES0145

91. Which of the following is true for the figure showing electric lines of force? (E is electrical field, V is potential)



(A) $E_A > E_B$ (B) $E_B > E_A$ (C) $V_A > V_B$ (D) $V_B > V_A$

ES0146

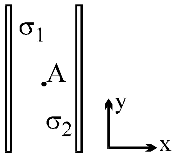
92. Consider a uniform electric field in the $\hat{z}$ direction. The potential is a constant

(A) in all space. (B) for any x for a given z.

(C) for any y for a given z. (D) on the x-y plane for a given z.

ES0147

93. Two large conducting sheets are kept parallel to each other as shown. In equilibrium, the charge density on facing surfaces is σ_1 and σ_2. What is the value of electric field at A.



(A) $\frac{\sigma_1}{\varepsilon_0}\hat{i}$ (B) $-\frac{\sigma_2}{\varepsilon_0}\hat{i}$ (C) $\frac{\sigma_1+\sigma_2}{2\varepsilon_0}\hat{i}$ (D) $\frac{\sigma_1-\sigma_2}{2\varepsilon_0}\hat{i}$

ES0148

94. A hollow closed conductor of irregular shape is given some charge. Which of the following statements are correct ?

(A) The entire charge will appear on its outer surface.

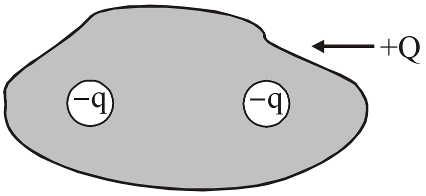
(B) All points on the conductor will have the same potential.

(C) All points on its surface will have the same charge density.

(D) All points just outside it will have the same electric intensity.

ES0149

95. A conducting body is given charge Q and charge –q has been placed in each of the cavity, which of the following statements is/are true?



(A)If Q = 2q, then conducting body will be at zero potential.

(B)If an external electric field is applied then the charge distribution on the outer surface of conductor would change.

(C)The potential of any point inside the cavity is less than that of conducting body.

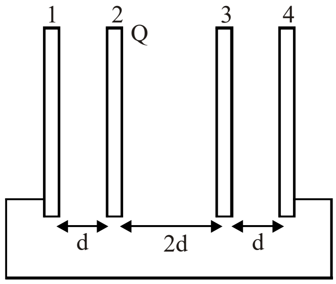
(D)None of these

ES0150

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 96 to 98

Four metallic plates are placed as shown in the figure. Plate 2 is given a charge Q whereas all other plates are uncharged. Plates 1 and 4 are joined together. The area of each plate is same.



96. The charge appearing on the right side of plate 3 is

(A) 0 (B) +Q/4 (C) –3Q/4 (D) Q/2

ES0151

97. The charge appearing on right side of plate 4 is

(A) 0 (B) –Q/4 (C) –3Q/4 (D) +Q/2

ES0151

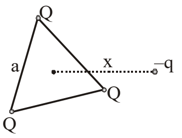
98. The potential difference between plates 1 and 2 is

(A) $\frac{3}{2}\frac{Qd}{\varepsilon_0 A}$ (B) $\frac{Qd}{\varepsilon_0 A}$ (C) $\frac{3}{4}\frac{Qd}{\varepsilon_0 A}$ (D) $\frac{3Qd}{\varepsilon_0 A}$

ES0151

Paragraph for Questions No. 99 to 101

Three charged particles each of +Q are fixed at the corners of an equilateral triangle of side 'a'. A fourth particle of charge –q and mass m is placed at a point on the line passing through centroid of triangle and perpendicular to the plane of triangle at a distance x from the centre of triangle.



99. Magnitude of resultant force on the fourth charged particle is

(A) $\frac{1}{4\pi\epsilon_0}\frac{9\sqrt{3}\,Qqx}{(3x^2+a^2)^{3/2}}$

(B) $\frac{1}{4\pi\epsilon_0}\frac{27\sqrt{3}\,Qqx}{(3x^2+a^2)^{3/2}}$

(C) $\frac{1}{4\pi\epsilon_0}\frac{2\sqrt{2}\,Qqx}{(2x^2+a^2)^{3/2}}$

(D) $\frac{1}{4\pi\epsilon_0}\frac{4\sqrt{2}\,Qqx}{(2x^2+a^2)^{3/2}}$

ES0152

100. Value of x for which the force is maximum is

(A) $\frac{a}{\sqrt{3}}$ (B) $\frac{a}{\sqrt{2}}$ (C) $\frac{a}{\sqrt{6}}$ (D) $\frac{a}{\sqrt{5}}$

ES0152

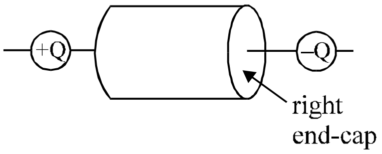
101. For small oscillation the period of oscillation of fourth particle is

(A) $2\pi\sqrt{\frac{4\pi \in_0 ma^3}{9\sqrt{3}Qq}}$ (B) $\pi\sqrt{\frac{4\pi \in_0 ma^3}{9\sqrt{3}\,Qq}}$ (C) $2\pi\sqrt{\frac{2\pi \in_0 ma^3}{27\sqrt{3}\,Qq}}$ (D) $2\pi\sqrt{\frac{\pi \in_0 ma^3}{27\sqrt{3}\,Qq}}$

ES0152

Paragraph for Question No. 102 and 103

The figure applies to the following two questions. Positive and negative charges of equal magnitude lie along the symmetry axis of a cylinder. The distance from the positive charge to the left end-cap of the cylinder is the same as the distance from the negative charge to the right end -cap.



102. What is the flux of the electric field through the closed cylinder ?

(A) 0 (B) + Q / ε_0 (C) + 2Q / ε_0 (D) – Q / ε_0

ES0153

103. What is the sign of the flux through the right end-cap of the cylinder ?

(A) Positive

(B) Negative

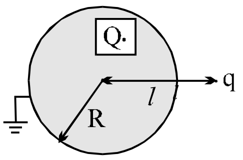
(C) There is no flux through the right end-cap.

(D) None of these

ES0153

Paragraph for Question No. 104 to 106

There is a cubical cavity inside a conducting sphere of radius R. A positive point charge Q is placed at the centre of the cube and another positive charge q is placed at a distance *l* (>R) from the centre of the sphere. The sphere is earthed



104. Charge induced on the inner surface of cavity is

(A) –Q, uniformly distributed (B) –Q, non-uniformly distributed

(C) –(Q+q) non-uniformly distributed (D) none

ES0154

105. Net charge on the outer surface of conducting sphere is

(A) +Q (B) Q – qR/l

(C) –qR/l (D) none

ES0154

106. Potential at a point inside the cavity is

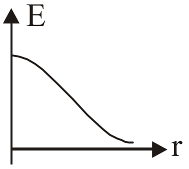
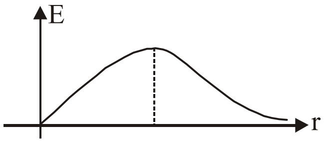
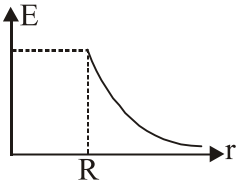
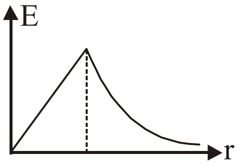
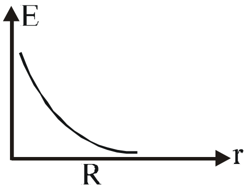
(A) zero (B) positive

(C) negative (D) can not be determined.

ES0154

MATRIX MATCH TYPE QUESTION

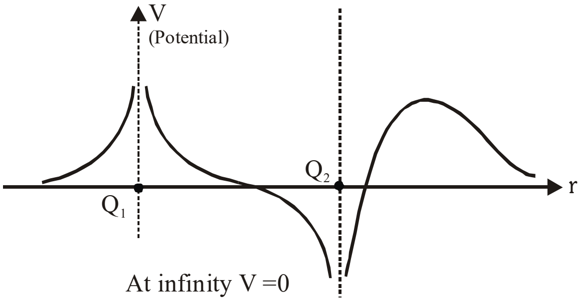
107. Column II corresponds to the graph of magnitude of electric field versus distance from centre of charge distribution in Column I.

	Column-I		**Column-II**
(A)	Ring along its axis	(P)	
(B)	Uniformly charged solid sphere	(Q)	
(C)	Uniformly charged spherical shell	(R)	
(D)	Combination of charge +Q and –Q at the perpendicular bisector	(S)	
		(T)	

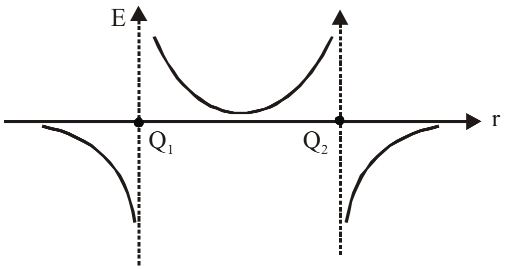
ES0155

108. As shown in column I their are graphs of electric field (E) and potential (V) along the line joining charges Q_1 and Q_2 are drawn against distance (r) on x–axis for charges Q_1 and Q_2. Take potential at infinity equal to zero. [Take direction of E in righward direction as positive]

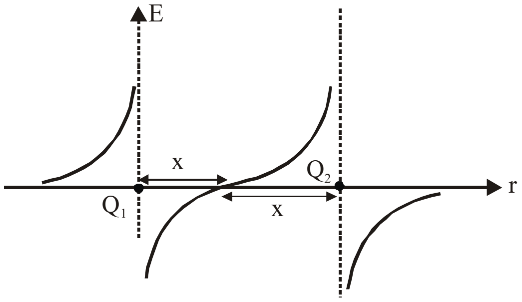
Column-I **Column-II**

(A)


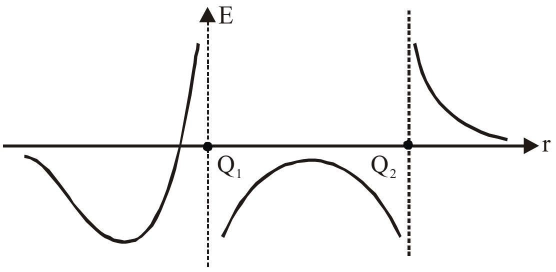
(P) $Q_1 < 0, Q_2 > 0$

(B)


(Q) $Q_1 > 0, Q_2 < 0$

(C)


(R) $|Q_1|>|Q_2|$

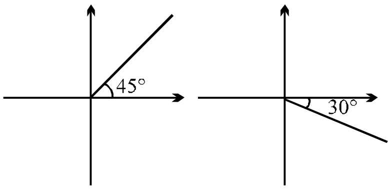
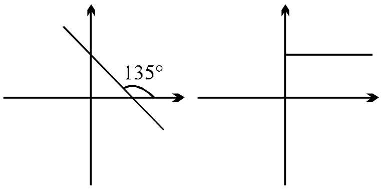
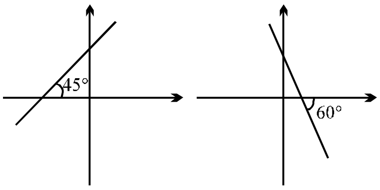
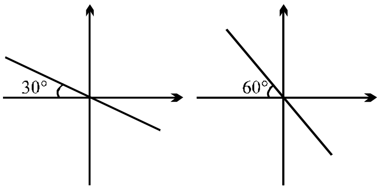
(D)


(S) $Q_1 < 0, Q_2 < 0$

(T) $|Q_1| = |Q_2|$

ES0156

109. Column I shows graphs of electric potential V versus x and y in a certain region for four situations. Column II shows the range of angle which the electric field vector makes with positive x-direction.

Column I V–x, V–y		Column II (Range of angle)	
(A)		(P)	$0 \le \theta < 45°$
(B)		(Q)	$45° \le \theta < 90°$
(C)		(R)	$90° \le \theta < 135°$
(D)		(S)	$135° \le \theta \le 180°$

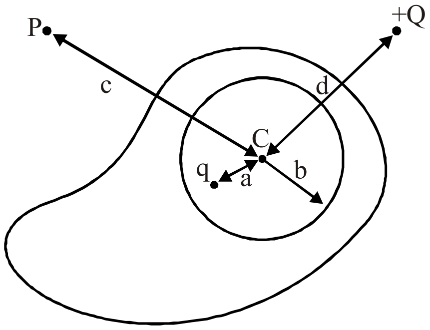
ES0157

110. A spherical metallic conductor has a spherical cavity. A positive charge is placed inside the cavity at its centre. Another positive charge is placed outside it. The conductor is initially electrically neutral.

	Column I (Cause)		Column II (Effect)
(A)	If outside charge is shifted to other position	(P)	distribution of charge on inner surface of cavity changes
(B)	If inside charge is shifted to other position within cavity	(Q)	distribution of charge on outer surface of of conductor changes
(C)	If magnitude of charge inside cavity is increased	(R)	electric potential at centre of conductor due to charges present on outer surface of conductor changes
(D)	If conductor is earthed	(S)	force on the charge placed inside cavity changes

ES0158

111. In the shown figure the conductor is uncharged and a charge q is placed inside a spherical cavity at a distance a from its centre (C). Point P and charge $+Q$ are as shown. a, b, c, d are known.



	Column-I		**Column-II**
(A)	Electric field due to induced charges on the inner surface of cavity at point P	(P)	zero
(B)	Electric potential due to charges on the inner surface of cavity and q at P	(Q)	non-zero
(C)	Electric field due to induced charges on the outer surface of conductor and Q at C	(R)	value can be stated with the given data.
(D)	Electric potential due to induced charges on the inner surface of cavity at C	(S)	value cannot be stated from the given data

ES0159

EXERCISE (O-2)

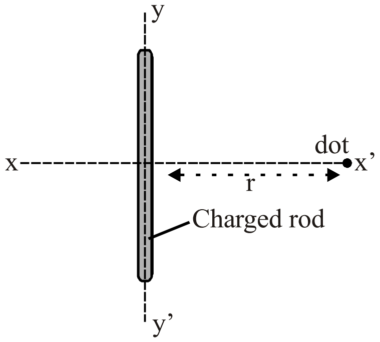
SINGLE CORRECT TYPE QUESTIONS

1. A square of side b centred at the origin with sides parallel to axes of x and y has surface charge density $\sigma(x, y) = \sigma_0 xy$ within its boundaries. Total charge on the square is

(A) 0 (B) $\sigma_0 b^2$ (C) $2\sigma_0 b^2$ (D) $4\sigma_0 b^2$

ES0160

2. A uniformly charged rod is kept on y-axis with centre at origin, as shown. Which of the following actions will increase the electric field strength at the position of the dot ?



(A) make the rod longer without changing the charge

(B) make the rod shorter without changing the charge

(C) make the rod shorter without changing the linear charge density

(D) rotate the rod about yy'

ES0161

3. A charged particle having some mass is resting in equilibrium at a height H above the centre of a uniformly charged non-conducting horizontal ring of radius R. The force of gravity acts downwards. The equilibrium of the particle will be stable

(A) for all values of H (B) only if $H > \frac{R}{\sqrt{2}}$ (C) only if $H < \frac{R}{\sqrt{2}}$ (D) only if $H = \frac{R}{\sqrt{2}}$

ES0162

4. **Statement-1 :** A positive point charge initially at rest in a uniform electric field starts moving along electric lines of forces. (Neglect all other forces except electric forces)

Statement-2 : Electric lines of force represents path of charged particle which is released from rest in it.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

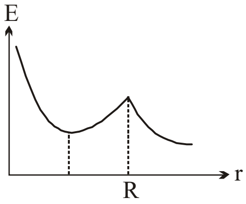
ES0163

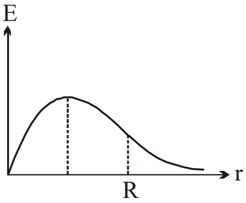
5. A system consists of uniformly charged sphere of radius R and a surrounding medium filled by a charge with the volume density $\rho = \frac{\alpha}{r}$, where α is a positive constant and r is the distance from the centre of the sphere. The charge of the sphere for which electric field intensity E outside the sphere is independent of r is

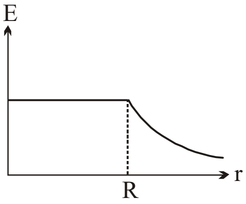
(A) $\frac{\alpha}{2 \in_0}$ (B) $\frac{2}{\alpha \in_0}$ (C) $2\pi\alpha R^2$ (D) αR^2

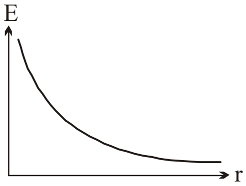
ES0164

6. A spherical insulator of radius R is charged uniformly with a charge Q throughout its volume and contains a point charge $\frac{Q}{16}$ located at its centre. Which of the following graphs best represent qualitatively, the variation of electric field intensity E with distance r from the centre.

(A)


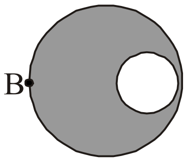
(B)


(C)


(D)


ES0165

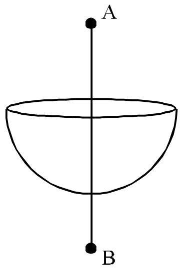
7. A positively charged sphere of radius r_0 carries a volume charge density ρ_E (Figure). A spherical cavity of radius $r_0/2$ is then scooped out and left empty, as shown. What is the direction and magnitude of the electric field at point B ?



(A) $\frac{17\rho r_0}{54 \in_0}$ left (B) $\frac{\rho r_0}{6 \in_0}$ left (C) $\frac{17\rho r_0}{54 \in_0}$ right (D) $\frac{\rho r_0}{6 \in_0}$ right

ES0166

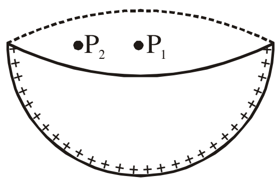
8. The diagram shows a uniformly charged hemisphere of radius R. It has volume charge density ρ. If the electric field at a point 2R distance above its center is E then what is the electric field at the point which is 2R below its center?



(A) $\rho R/6\varepsilon_0 + E$ (B) $\rho R/12\varepsilon_0 - E$ (C) $-\rho R/6\varepsilon_0 + E$ (D) $\rho R/24\varepsilon_0 + E$

ES0167

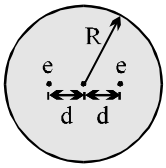
9. Consider a uniformly charged hemispherical shell shown below. Indicate the directions (not magnitude) of the electric field at the central point P_1 and an off-centre point P_2 on the drumhead of the shell.



(A) ↑ ; ↖ (B) ↑ ; ↗ (C) ↑ ; ↑ (D) ↑ ; ←

ES0168

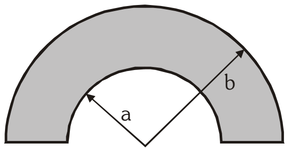
10. Using thomson's model of the atom, consider an atom consisting of two electrons, each of charge –e, embeded in a sphere of charge + 2e and radius R. In equilibrium each electron is at distance d from the centre of the atom. What is equilibrium separation between electrons?



(A) R (B) R/2 (C) R/3 (D) R/4

ES0169

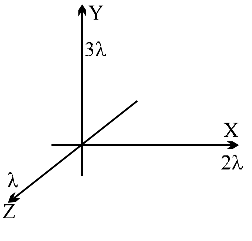
11. A non conducting semicircular disc (as shown in figure) has a uniform surface charge density σ. The electric potential at the centre of the disc :-



(A) $\frac{\sigma}{2\pi\epsilon_0}\frac{\ell n(b/a)}{(b-a)}$ (B) $\frac{\sigma(b-a)}{2\epsilon_0}$ (C) $\frac{\sigma(b-a)}{4\epsilon_0}$ (D) $\frac{\sigma(b-a)}{4\pi\epsilon_0}$

ES0170

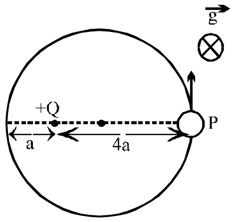
12. The diagram shows three infinitely long uniform line charges placed on the X, Y and Z axis. The work done in moving a unit positive charge from (1, 1, 1) to (0, 1, 1) is equal to



(A) (λ ln 2) / $2\pi\varepsilon_0$ (B) (λ ln 2) /$\pi\varepsilon_0$ (C) (3λ ln 2) / $2\pi\varepsilon_0$ (D) None

ES0171

13. The diagram shows a small bead of mass m carrying charge q. The bead can freely move on the smooth fixed ring placed on a smooth horizontal plane. In the same plane a charge +Q has also been fixed as shown. The potential at the point P due to +Q is V. The velocity with which the bead should projected from the point P so that it can complete a circle should be greater than



(A) $\sqrt{\frac{6qV}{m}}$ (B) $\sqrt{\frac{qV}{m}}$ (C) $\sqrt{\frac{3qV}{m}}$ (D) none

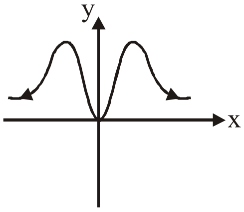
ES0172

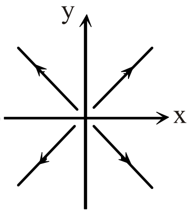
14. A charged particle of charge Q is held fixed and another charged particle of mass m and charge q (of the same sign) is released from a distance r. The impulse of the force exerted by the external agent on the fixed charge by the time distance between Q and q becomes 2r is

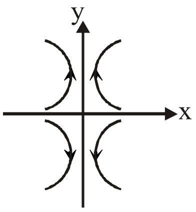
(A) $\sqrt{\frac{Qq}{4\pi \in_0 mr}}$ (B) $\sqrt{\frac{Qqm}{4\pi \in_0 r}}$ (C) $\sqrt{\frac{Qqm}{\pi \in_0 r}}$ (D) $\sqrt{\frac{Qqm}{2\pi \in_0 r}}$

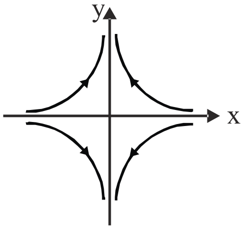
ES0173

15. In a certain region of space, the potential field depends on x and y coordinates as $V = (x^2 - y^2)$. The corresponding electric field lines in x-y plane are correctly represented by :

(A) 

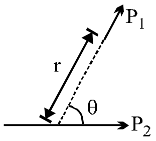
(B) 

(C) 

(D) 

ES0174

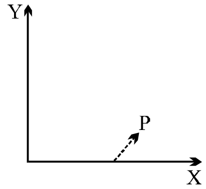
16. Two short electric dipoles are placed as shown. The energy of electric interaction between these dipoles will be :-



(A) $\frac{2kP_1P_2\cos\theta}{r^3}$ (B) $\frac{-2kP_1P_2\cos\theta}{r^3}$ (C) $\frac{-2kP_1P_2\sin\theta}{r^3}$ (D) $\frac{-4kP_1P_2\cos\theta}{r^3}$

ES0175

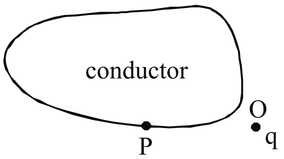
17. A small electric dipole $\vec{P}$ is placed on the X axis at the point (1, 0). The dipole vector forms an angle of 30° with the X axis. Consider a non uniform electric field to have been applied in the region given by the vector $\vec{E} = x^2\hat{i} + y^2\hat{j}$. What is the force acting on the dipole?



(A) 2Pcos 30° $(\hat{i} + 2\hat{j})$ (B) 2Pcos 30° $(\hat{i})$ (C) 2Pcos 30° $(2\hat{j})$ (D) None

ES0176

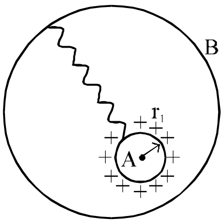
18. The density of charge at P on the conductor is σ. The resultant electric field near P will [$\hat{n}$ = unit normal vector at point P. $\vec{r}$ = vector along OP] where K = 1/4 πε



(A) $\left(\frac{\sigma}{\epsilon_0}\right)\hat{n} + \left(\frac{Kq\vec{r}}{r^3}\right)$ (B) $\left(\frac{\sigma}{2\,\epsilon_0}\right)\hat{n} + \left(\frac{Kq\vec{r}}{r^3}\right)$ (C) $\left(\frac{\sigma}{\epsilon_0}\right)\hat{n}$ (D) $\left(\frac{\sigma}{2\,\epsilon_0}\right)\hat{n}$

ES0177

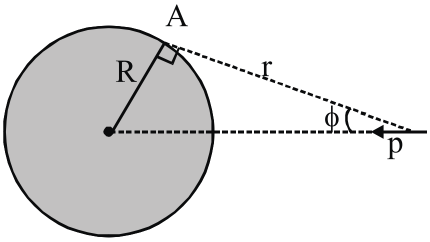
19. A metal sphere A of radius r_1 charged to a potential ϕ_1 is enveloped by a thin walled conducting spherical shell B of radius r_2. Then ϕ_2 of the sphere A after it is connected by a thin wire to the shell B will be :-



(A) $\phi_1 \frac{r_1}{r_2}$ (B) $\phi_1 \left(\frac{r_2}{r_1}\right)$ (C) $\phi_1 \left(1 - \frac{r_1}{r_2}\right)$ (D) $\phi_1 \left(\frac{r_1 r_2}{r_1 + r_2}\right)$

ES0178

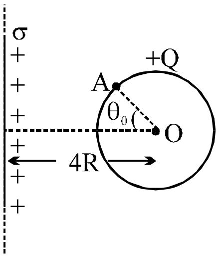
20. A dipole having dipole moment p is placed in front of a solid uncharged conducting sphere as shown in the diagram. The net potential at point A lying on the surface of the sphere is :-



(A) $\frac{kp\cos\phi}{r^2}$ (B) $\frac{kp\cos^2\phi}{r^2}$ (C) 0 (D) $\frac{2kp\cos^2\phi}{r^2}$

ES0179

21. A conducting sphere of radius R and charge Q is placed near a uniformly charged nonconducting infinitely large thin plate having surface charge density σ. Then find the potential at point A (on the surface of sphere) due to charge on sphere (here $K = \frac{1}{4\pi \in_0}$, $\theta_0 = \frac{\pi}{3}$)



(A) $K\frac{Q}{R}-\frac{\sigma}{4\in_0}R$ (B) $K\frac{Q}{R}-\frac{\sigma R}{\in_0}$ (C) $K\frac{Q}{R}$ (D) none of these

ES0180

22. The intensity of an electric field depends only on the coordinates x, y and z as follows :

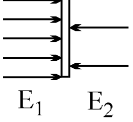
$$\vec{E} = a\frac{\left(x\hat{i}+y\hat{j}+z\hat{k}\right)}{\left(x^2+y^2+z^2\right)^{3/2}}\text{unit}$$

The electrostatic energy stored between two imaginary concentric spherical shells of radii R and 2R with centre at origin is :-

(A) $\frac{4\pi\varepsilon_0 a^2}{R}$ (B) $\frac{2\pi\varepsilon_0 a^2}{R}$ (C) $\frac{\pi\varepsilon_0 a^2}{R}$ (D) $\frac{\pi\varepsilon_0 a^2}{2R}$

ES0181

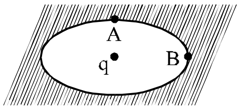
23. A charged large metal sheet is placed into uniform electric field, perpendicularly to the electric field lines. After placing the sheet into the field, the electric field on the left side of the sheet is $E_1 = 5\times10^5$ V/m and on the right it is $E_2 = 3\times10^5$ V/m. The sheet experiences a net electric force of 0.08 N. Find the area of one face of the sheet. Assume external field to remain constant after introducing the large sheet. Use $\left(\frac{1}{4\pi\varepsilon_0}\right) = 9\times10^9$ Nm²/C²



(A) 3.6 π × 10^{-2} m² (B) 0.9 π × 10^{-2} m² (C) 1.8 π × 10^{-2} m² (D) none

ES0182

24. An ellipsoidal cavity is carved within a perfect conductor. A positive charge q is placed at the center of the cavity. The points A & B are on the cavity surface as shown in the figure. Then :

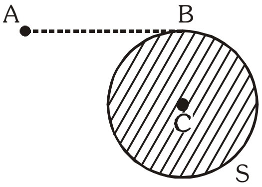


(A) electric field near A in the cavity = electric field near B in the cavity
(B) charge density at A = charge density at B
(C) potential at A = potential at B
(D) total electric field flux through the surface of the cavity is q/ε_0.

ES0183

MULTIPLE CORRECT TYPE QUESTIONS

25. S is a solid neutral conducting sphere. A point charge q of 1×10^{-6}C is placed at point A. C is the centre of sphere and AB is a tangent. BC = 3m and AB = 4m.



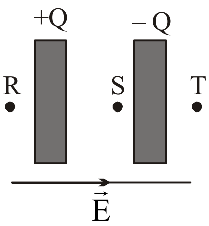
(A) The electric potential of the conductor is 1.8 kV.
(B) The electric potential of the conductor is 2.25 kV.
(C) The electric potential at B due to induced charges on the sphere is –0.45 kV.
(D) The electric potential at B due to induced charges on the sphere is 0.45 kV.

ES0184

26. Four identical particles each having mass m and charge q are placed at the vertices of a square of side ℓ. All the particles are free to move without any friction and released simultaneously from rest. Then
(A) At all instants, the particles remains at vertices of square whose edge length is changing
(B) The configuration is changing (not remaining square) as the time passes
(C) The speed of the particles when one of the particles get displaced by $\frac{\ell}{\sqrt{2}}$ is $\sqrt{\frac{q^2}{8\pi\varepsilon_0 m\ell}\left(2+\frac{1}{\sqrt{2}}\right)}$
(D) Speed of the particles can not be found

ES0185

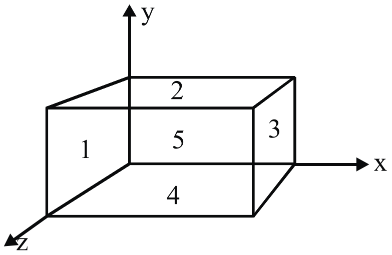
27. Two large thin conducting plates with small gap in between are placed in a uniform electric field E (perpendicular to the plates). Area of each plate is A and charges $+Q$ and $-Q$ are given to these plates as shown in the figure. If points R,S and T as shown in the figure are three points in space, then the



(A) field at point R is E
(B) field at point S is E
(C) field at point T is $\left(E+\frac{Q}{\varepsilon_o A}\right)$
(D) field at point S is $\left(E+\frac{Q}{A\varepsilon_o}\right)$

ES0186

28. In a region of space, the electric field $\vec{E} = E_0 x\hat{i} + E_0 y\hat{j}$. Consider an imaginary cubical volume of edge 'a' with its edges parallel to the axes of coordinates. Now,



(A) the total electric flux through the faces 1 and 3 is E_0a^3
(B) the charge inside the cubical volume is $2\varepsilon_0E_0a^3$
(C) the total electric flux through the faces 2 and 4 is $2E_0a^3$
(D) the charge inside the cubical volume is $\varepsilon_0E_0a^3$

ES0187

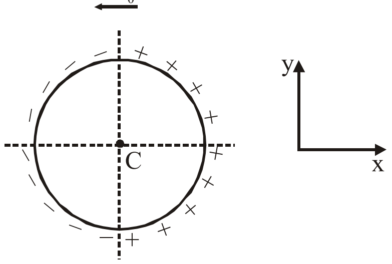
29. Equipotential surfaces :-
(A) are closer in regions of large electric fields compared to regions of lower electric fields.
(B) will be more crowded near sharp edges of a conductor.
(C) will be more crowded near regions of large charge densities.
(D) will always be equally spaced.

ES0188

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 30 and 31

A uniform ring of mass m and radius R can rotate freely about an axis passing through centre C and perpendicular to plane of paper. Half of ring is positively charge and other half is negatively charge. Uniform electric field E_0 is switched on along –ve x-axis (axis are shown in figure) [Magnitude of charge density λ]



30. The dipole moment of ring is :-
(A) $2\lambda R^2$ (B) $4\lambda R^2$ (C) $2\pi\lambda R^2$ (D) $4\pi\lambda R^2$

ES0189

31. If ring is slightly disturb from given position, find the angular speed of ring when it rotate by π/2.

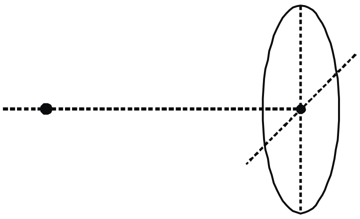
(A) $2\sqrt{\frac{\lambda E_0}{m}}$ (B) $\sqrt{\frac{\lambda E_0}{m}}$ (C) $\sqrt{\frac{8\lambda E_0}{m}}$ (D) None

ES0189

MATCHING LIST TYPE (4 × 4 × 4) SINGLE OPTION CORRECT (THREE COLUMNS AND FOUR ROWS)

Answer Q.32, 33 & 34 by appropriately matching the information given in the three columns of the following table.

Consider a non conducting ring of radius r and mass m and a particle of same mass, both at rest in free space. The particle is on the axis of the ring and far away from the ring. An amount Q of positive charge is uniformly distributed on the ring and the particle is given a positive charge q. The particle is imparted a velocity v towards the centre of the ring. Consider the consequences given in the columns and answer the following questions.



	Column 1		Column 2		Column 3
(I)	Maximum speed of the ring is v	(i)	Minimum speed of the particle is v/2	(P)	Final speed of the ring and particle is v and zero respectively.
(II)	Maximum speed of the ring is v/2	(ii)	Minimum speed of the particle is zero	(Q)	Final speed of the ring and particle is zero and v respectively.
(III)	Maximum speed of the ring is $\frac{v}{2}\left[1+\sqrt{1-\frac{Qq}{\pi\varepsilon_0 mrv^2}}\right]$	(iii)	Minimum speed of the particle is $\frac{v}{2}\left[1+\sqrt{1-\frac{Qq}{\pi\varepsilon_0 mrv^2}}\right]$	(R)	Final speed of the ring and particle is v/4 and 3v/4 respectively.
(IV)	Maximum speed of the ring is $\frac{v}{2}\left[1-\sqrt{1-\frac{Qq}{\pi\varepsilon_0 mrv^2}}\right]$	(iv)	Minimum speed of the particle is $\frac{v}{2}\left[1-\sqrt{1-\frac{Qq}{\pi\varepsilon_0 mrv^2}}\right]$	(S)	Final speed of the ring and particle is v/2 for both.

32. Which of following options is the correct representation if $v=\sqrt{\frac{Qq}{2\pi\varepsilon_0 mr}}$

(A) (II) (i) S (B) (I) (ii) P (C) (III) (iv) R (D) (IV) (iii) Q

ES0190

33. Which of following options is the correct representation if $v=\sqrt{\frac{Qq}{\pi\varepsilon_0 mr}}$

(A) (II) (i) S (B) (I) (ii) P (C) (III) (iv) R (D) (IV) (iii) Q

ES0190

34. Which of following options is the correct representation if $v = \sqrt{\dfrac{2Qq}{\pi\varepsilon_0 mr}}$

(A) (II) (i) S (B) (I) (ii) P (C) (III) (iv) Q (D) (IV) (iii) (Q)

ES0190

MATRIX MATCH TYPE QUESTION

35. Column-II shows some charge distributions and column-I has some statements about electric field at four points A, B, C, D. Match column-I with column-II.

Column-I

(A) $\vec{E}_A$ has x component only

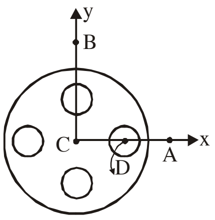
(B) $\vec{E}_B$ has y component only

(C) $\vec{E}_C$ has y component only

(D) $|\vec{E}_D|$ is zero

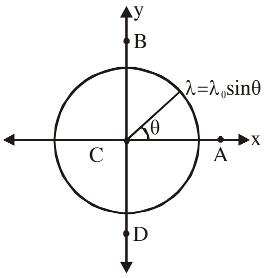
Column-II

(P)



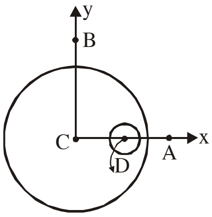
A solid non conducting sphere of radius R of volumetric charge density ρ with four symmetrical spherical cavities. All the five sphere's centre lie in same plane.

(Q)



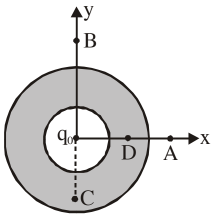
A very small circular filament lying in xy-plane. All points lie in same plane. A, B and D are at large distance compared to radius of circle.

(R)



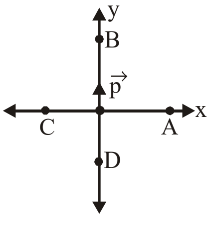
A charged spherical conductor with an empty cavity in it.

(S)



A hollow thick spherical neutral conductor with a concentric cavity. Charge q_0 is placed inside at centre of cavity.

(T)



A small electric dipole $\vec{p}$ is placed at origin. A, B, C and D are four points at large distance from origin.

ES0191

EXERCISE-JM

1. Two identical charged spheres suspended from a common point by two massless string of length ℓ are initially a distance $d(d << \ell)$ apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result the charges approach each other with a velocity v. Then as a function of distance x between them :- **[AIEEE - 2011]**

(1) $v \propto x^{1/2}$ (2) $v \propto x$ (3) $v \propto x^{-1/2}$ (4) $v \propto x^{-1}$

ES0192

2. The electrostatic potential inside a charged spherical ball is given by $\phi = ar^2 + b$ where r is the distance from the centre; a, b are constant. Then the charge density inside the ball is :- **[AIEEE - 2011]**

(1) $-24\pi a\epsilon_0$ (2) $-6 a\epsilon_0$ (3) $-24\pi a\epsilon_0 r$ (4) $-6 a\epsilon_0 r$

ES0193

3. Two positive charges of magnitude 'q' are placed at the ends of a side (side 1) of a square of side '2a'. Two negative charges of the same magnitude are kept at the other corners. Starting from rest, if a charge Q moves from the middle of side 1 to the centre of square, its kinetic energy at the centre of square is :- **[AIEEE - 2011]**

(1) $\frac{1}{4\pi\epsilon_0}\frac{2qQ}{a}\left(1-\frac{1}{\sqrt{5}}\right)$ (2) zero

(3) $\frac{1}{4\pi\epsilon_0}\frac{2qQ}{a}\left(1+\frac{1}{\sqrt{5}}\right)$ (4) $\frac{1}{4\pi\epsilon_0}\frac{2qQ}{a}\left(1-\frac{2}{\sqrt{5}}\right)$

ES0194

4. **This question has Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.** **[AIEEE - 2012]**

An insulating solid sphere of radius R has a uniformaly positive charge density ρ. As a result of this uniform charge distribution there is a finite value of electric potential at the centre of the sphere, at the surface of the sphre and also at a point out side the sphere. The electric potential at infinity is zero.

Statement-1: When a charge 'q' is taken from the centre to the surface of the sphere, its potential energy changes by $\frac{q\rho}{3\epsilon_0}$

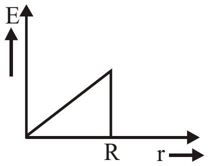
Statement-2 : The electric field at a distance r (r < R) from the centre of the sphere is $\frac{\rho r}{3\epsilon_0}$

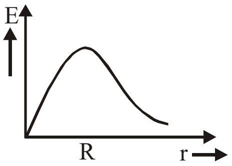
(1) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation of Statement-1.
(2) Statement-1 is true, Statement-2 is true and Statement-2 is not the correct explanation of statement-1.
(3) Statement-1 is true, Statement-2 is false
(4) Statement-1 is false, Statement-2 is true

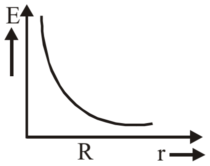
ES0195

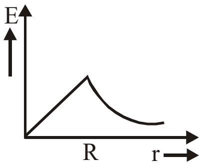
5. In a uniformly charged sphere of total charge Q and radius R, the electric field E is plotted as a function of distance from the centre. The graph which would correspond to the above will be :-

[AIEEE - 2012]

(1) 

(2) 

(3) 

(4) 

ES0196

6. Let $[\in_0]$ denote the dimensional formula of the permittivity of vacuum. If M = mass, L = Length, T = Time and A = electric current, then : **[JEE-Main-2013]**

(1) $[\in_0] = [M^{-1} L^{-3} T^2 A]$
(2) $[\in_0] = [M^{-1} L^{-3} T^4 A^2]$
(3) $[\in_0] = [M^{-1} L^2 T^{-1} A^{-2}]$
(4) $[\in_0] = [M^{-1} L^2 T^{-1} A]$

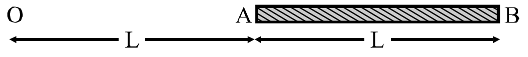
ES0197

7. Two charges, each equal to q, are kept at x = –a and x = a on the x-axis. A particle of mass m and charge $q_0 = \frac{q}{2}$ is placed at the origin. If charge q_0 is given a small displacement ($y << a$) along the y-axis, the net force acting on the particle is proportional to **[JEE-Main-2013]**

(1) y
(2) –y
(3) $\frac{1}{y}$
(4) $-\frac{1}{y}$

ES0198

8. A charge Q is uniformly distributed over a long rod AB of length L as shown in the figure. The electric potential at the point O lying at a distance L from the end A is :- **[JEE-Main-2013]**



(1) $\frac{Q}{8\pi \in_0 L}$
(2) $\frac{3Q}{4\pi \in_0 L}$
(3) $\frac{Q}{4\pi \in_0 L \ln 2}$
(4) $\frac{Q \ln 2}{4\pi \in_0 L}$

ES0199

9. Assume that an electric field $\vec{E} = 30x^2\hat{i}$ exists in space. Then the potential difference $V_A - V_O$, where V_O is the potential at the origin and V_A the potential at x = 2 m is :- **[JEE-Main-2014]**

(1) –80 J
(2) 80 J
(3) 120 J
(4) –120 J

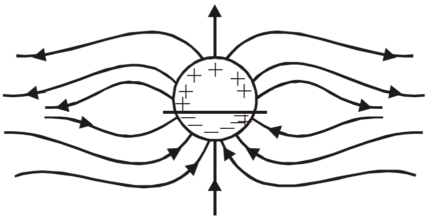
ES0200

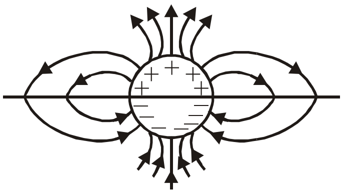
10. A uniformally charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere the equipotential surfaces with potentials $\frac{3V_0}{2}, \frac{5V_0}{4}, \frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R_1, R_2, R_3 and R_4 respectively. Then **[JEE-Main-2015]**

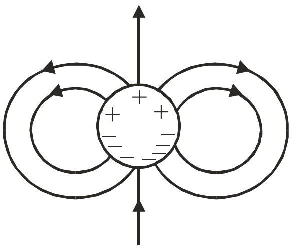
(1) $R_1 = 0$ and $R_2 < (R_4 - R_3)$
(2) $2R < R_4$
(3) $R_1 = 0$ and $R_2 > (R_4 - R_3)$
(4) $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$

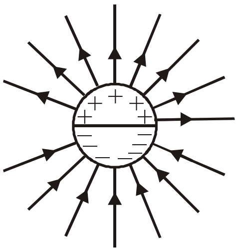
ES0201

11. A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge –σ in the lower half. The electric field lines around the cylinder will look like figure given in: (figures are schematic and not drawn to scale) **[JEE-Main-2015]**

(1)


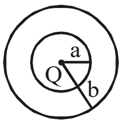
(2)


(3)


(4)


ES0202

12. The region between two concentric spheres of radii 'a' and 'b', respectively (see figure), has volume charge density $\rho = \frac{A}{r}$, where A is a constant and r is the distance from the centre. At the centre of the spheres is a point charge Q. The value of A such that the electric field in the region between the spheres will be constant, is :- **[JEE-Main-2016]**



(1) $\frac{2Q}{\pi a^2}$ (2) $\frac{Q}{2\pi a^2}$ (3) $\frac{Q}{2\pi(b^2 - a^2)}$ (4) $\frac{2Q}{\pi(a^2 - b^2)}$

ES0203

13. An electric dipole has a fixed dipole moment $\vec{p}$, which makes angle θ with respect to x-axis. When subjected to an electric field $\vec{E}_1 = E\hat{i}$, it experiences a torque $\vec{T}_1 = \tau\hat{k}$. When subjected to another electric field $\vec{E}_2 = \sqrt{3}E_1\hat{j}$ it experiences torque $\vec{T}_2 = -\vec{T}_1$. The angle θ is : **[JEE-Main-2017]**

(1) 60° (2) 90° (3) 30° (4) 45°

ES0204

14. Three concentric metal shells A, B and C of respective radii a, b and c (a < b < c) have surface charge densities +σ, –σ and +σ respectively. The potential of shell B is :- **[JEE-Main-2018]**

(1) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2 - b^2}{b} + c\right]$ (2) $\frac{\sigma}{\varepsilon_0}\left[\frac{b^2 - c^2}{b} + a\right]$ (3) $\frac{\sigma}{\varepsilon_0}\left[\frac{b^2 - c^2}{c} + a\right]$ (4) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2 - b^2}{a} + c\right]$

ES0205

SELECTED PROBLEMS FROM JEE-MAINS ONLINE PAPERS

15. Charge is distributed within a sphere of radius R with a volume charge density $\rho(r)=\frac{A}{r^2}e^{-2r/a}$, where A and a are constants. If Q is the total charge of this charge distribution, the radius R is :

[JEE-Main-2019_Jan]

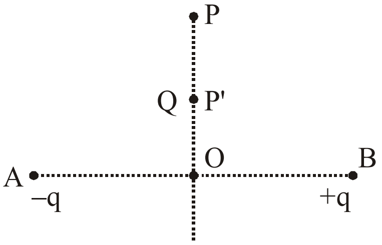
(1) $\frac{a}{2}\log\left(1-\frac{Q}{2\pi aA}\right)$ (2) $a\log\left(1-\frac{Q}{2\pi aA}\right)$

(3) $a\log\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$ (4) $\frac{a}{2}\log\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$

ES0247

16. Charges –q and +q located at A and B, respectively, constitute an electric dipole. Distance AB = 2a, O is the mid point of the dipole and OP is perpendicular to AB. A charge Q is placed at P where OP = y and y >> 2a. The charge Q experiences and electrostatic force F. If Q is now moved along the equatorial line to P' such that OP'=$\left(\frac{y}{3}\right)$, the force on Q will be close to : $\left(\frac{y}{3}>>2a\right)$

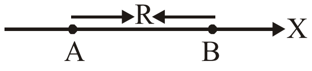
[JEE-Main-2019_Jan]



(1) $\frac{F}{3}$ (2) 3F (3) 9F (4) 27F

ES0248

17. Two electric dipoles, A, B with respective dipole moments $\vec{d}_A=-4qa\hat{i}$ and $\vec{d}_B=-2qa\hat{i}$ placed on the x-axis with a separation R, as shown in the figure



The distance from A at which both of them produce the same potential is : **[JEE-Main-2019_Jan]**

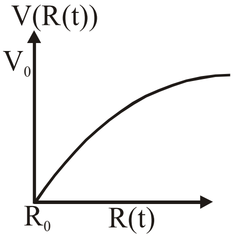
(1) $\frac{\sqrt{2}R}{\sqrt{2}+1}$ (2) $\frac{R}{\sqrt{2}+1}$ (3) $\frac{\sqrt{2}\,R}{\sqrt{2}-1}$ (4) $\frac{R}{\sqrt{2}-1}$

ES0249

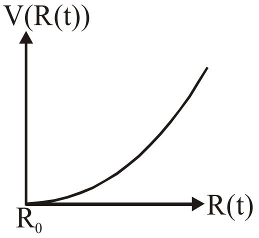
18. There is a uniform spherically symmetric surface charge density at a distance R_0 from the origin. The charge distribution is initially at rest and starts expanding because of mutual repulsion. The figure that represents best the speed V(R(t)) of the distribution as a function of its instantaneous radius R (t) is:

[JEE-Main-2019_Jan]

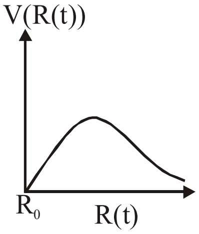
(1)



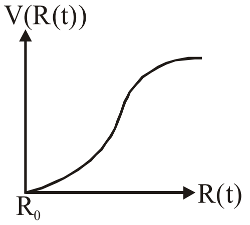
(2)



(3)

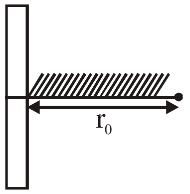


(4)



ES0250

19. A positive point charge is released from rest at a distance r_0 from a positive line charge with uniform density. The speed (v) of the point charge, as a function of instantaneous distance r from line charge, is proportional to :- **[JEE-Main-2019_April]**



(1) $v \propto e^{+r/r_0}$ (2) $v \propto \ell n\left(\frac{r}{r_0}\right)$ (3) $v \propto \left(\frac{r}{r_0}\right)$ (4) $v \propto \sqrt{\ell n\left(\frac{r}{r_0}\right)}$

ES0251

20. A solid conducting sphere, having a charge Q, is surrounded by an uncharged conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V. If the shell is now given a charge of –4 Q, the new potential difference between the same two surfaces is : **[JEE-Main-2019_April]**

(1) V (2) 2V (3) –2V (4) 4V

ES0252

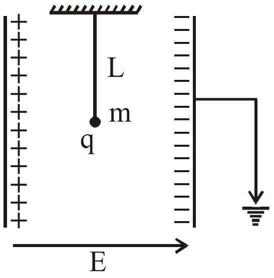
21. Four point charges –q, +q, +q and –q are placed on y-axis at y = –2d, y = –d, y = +d and y = +2d, respectively. The magnitude of the electric field E at a point on the x-axis at x = D, with D >> d, will behave as :- **[JEE-Main-2019_April]**

(1) $E \propto \frac{1}{D}$ (2) $E \propto \frac{1}{D^3}$ (3) $E \propto \frac{1}{D^2}$ (4) $E \propto \frac{1}{D^4}$

ES0253

22. A simple pendulum of length L is placed between the plates of a parallel plate capacitor having electric field E, as shown in figure. Its bob has mass m and charge q. The time period of the pendulum is given by :

[JEE-Main-2019_April]



(1) $2\pi\sqrt{\dfrac{L}{\sqrt{g^2+\left(\dfrac{qE}{m}\right)^2}}}$ (2) $2\pi\sqrt{\dfrac{L}{\left(g+\dfrac{qE}{m}\right)}}$ (3) $2\pi\sqrt{\dfrac{L}{\left(g-\dfrac{qE}{m}\right)}}$ (4) $2\pi\sqrt{\dfrac{L}{\sqrt{g^2-\dfrac{q^2E^2}{m^2}}}}$

ES0254

23. A uniformly charged ring of radius 3a and total charge q is placed in xy-plane centred at origin. A point charge q is moving towards the ring along the z-axis and has speed *u* at z = 4a. The minimum value of *u* such that it crosses the origin is : **[JEE-Main-2019_April]**

(1) $\sqrt{\dfrac{2}{m}}\left(\dfrac{1}{15}\dfrac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$ (2) $\sqrt{\dfrac{2}{m}}\left(\dfrac{2}{15}\dfrac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$ (3) $\sqrt{\dfrac{2}{m}}\left(\dfrac{4}{15}\dfrac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$ (4) $\sqrt{\dfrac{2}{m}}\left(\dfrac{1}{5}\dfrac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$

ES0255

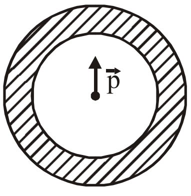
24. Let a total charge 2Q be distributed in a sphere of radius R, with the charge density given by $\rho(r) = kr$, where r is the distance from the centre. Two charges A and B, of –Q each, are placed on diametrically opposite points, at equal distance, a, from the centre. If A and B do not experience any force, then :

[JEE-Main-2019_April]

(1) $a=\dfrac{3R}{2^{1/4}}$ (2) $a = R/\sqrt{3}$ (3) $a = 8^{-1/4}R$ (4) $a = 2^{-1/4}\,R$

ES0256

25. Shown in the figure is a shell made of a conductor. It has inner radius a and outer radius b, and carries charge Q. At its centre is a dipole $\vec{p}$ as shown. In this case : **[JEE-Main-2019_April]**



(1) Electric field outside the shell is the same as that of a point charge at the centre of the shell.

(2) Surface charge density on the inner surface of the shell is zero everywhere.

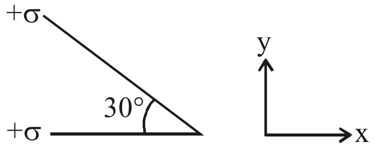
(3) Surface charge density on the inner surface is uniform and equal to $\dfrac{(Q/2)}{4\pi a^2}$.

(4) Surface charge density on the outer surface depends on $|\vec{p}|$

ES0257

26. Two infinite planes each with uniform surface charge density + σ are kept in such a way that the angle between them is 30°. The electric field in the region shown between them is given by:

[JEE-Main-2020_Jan]



(1) $\frac{\sigma}{\varepsilon_0}\left[\left(1+\frac{\sqrt{3}}{2}\right)\hat{y}+\frac{\hat{x}}{2}\right]$

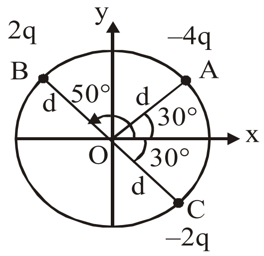
(2) $\frac{\sigma}{2\varepsilon_0}\left[\left(1-\frac{\sqrt{3}}{2}\right)\hat{y}-\frac{\hat{x}}{2}\right]$

(3) $\frac{\sigma}{2\varepsilon_0}\left[\left(1+\sqrt{3}\right)\hat{y}+\frac{\hat{x}}{2}\right]$

(4) $\frac{\sigma}{2\varepsilon_0}\left[\left(1+\sqrt{3}\right)\hat{y}-\frac{\hat{x}}{2}\right]$

ES0258

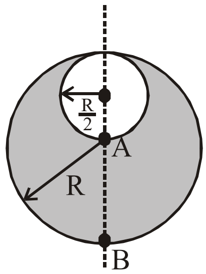
27. Three charged particle A, B and C with charges –4q, 2q and –2q are present on the circumference of a circle of radius d. the charged particles A, C and centre O of the circle formed an equilateral triangle as shown in figure. Electric field at O along x-direction is : **[JEE-Main-2020_Jan]**



(1) $\frac{2\sqrt{3}\,q}{\pi\varepsilon_0 d^2}$ (2) $\frac{\sqrt{3}\,q}{4\pi\varepsilon_0 d^2}$ (3) $\frac{3\sqrt{3}\,q}{4\pi\varepsilon_0 d^2}$ (4) $\frac{\sqrt{3}\,q}{\pi\varepsilon_0 d^2}$

ES0259

28. Consider a sphere of radius R which carries a uniform charge density ρ. If a sphere of radius $\frac{R}{2}$ is carved out of it, as shown, the ratio $\frac{|\vec{E}_A|}{|\vec{E}_B|}$ of magnitude of electric field $\vec{E}_A$ and $\vec{E}_B$, respectively, at points A and B due to the remaining portion is : **[JEE-Main-2020_Jan]**



(1) $\frac{18}{54}$ (2) $\frac{21}{34}$ (3) $\frac{17}{54}$ (4) $\frac{18}{34}$

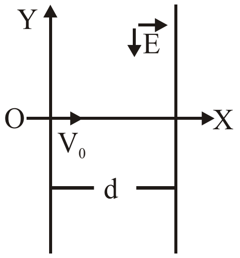
ES0260

29. An electric dipole of moment $\vec{p} = (-\hat{i} - 3\hat{j} + 2\hat{k}) \times 10^{-29}$ C.m is at the origin (0, 0, 0). The electric field due to this dipole at $\vec{r} = +\hat{i} + 3\hat{j} + 5\hat{k}$ (note that $\vec{r}.\vec{p} = 0$) is parallel to: **[JEE-Main-2020_Jan]**

(1) $(-\hat{i} + 3\hat{j} - 2\hat{k})$ (2) $(+\hat{i} - 3\hat{j} - 2\hat{k})$ (3) $(+\hat{i} + 3\hat{j} - 2\hat{k})$ (4) $(-\hat{i} - 3\hat{j} + 2\hat{k})$

ES0261

30. A charged particle (mass m and charge q) moves along X axis with velocity V_0. When it passes through the origin it enters a region having uniform electric field $\vec{E} = -E\hat{j}$ which extends upto x = d. Equation of path of electron in the region x > d is : **[JEE-Main-2020_Sep]**



(1) $y = \frac{qEd}{mV_0^2}\left(\frac{d}{2} - x\right)$ (2) $y = \frac{qEd}{mV_0^2}(x - d)$ (3) $y = \frac{qEd}{mV_0^2}x$ (4) $y = \frac{qEd^2}{mV_0^2}x$

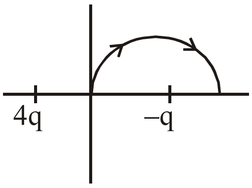
ES0262

31. Two isolated conducting spheres S_1 and S_2 of radius $\frac{2}{3}R$ and $\frac{1}{3}R$ have 12 μC and –3 μC charges, respectively, and are at a large distance from each other. They are now connected by a conducting wire. A long time after this is done the charges on S_1 and S_2 are respectively : **[JEE-Main-2020_Sep]**

(1) 6 μC and 3 μC (2) +4.5 μC and –4.5 μC
(3) 3 μC and 6 μC (4) 4.5 μC on both

ES0263

32. A two point charges 4q and –q are fixed on the x-axis at $x = -\frac{d}{2}$ and $x = \frac{d}{2}$, respectively. If a third point charge 'q' is taken from the origin to x = d along the semicircle as shown in the figure, the energy of the charge will : **[JEE-Main-2020_Sep]**



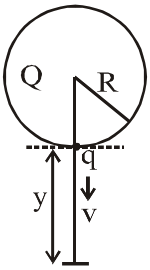
(1) increase by $\frac{2q^2}{3\pi\varepsilon_0 d}$ (2) increase by $\frac{3q^2}{4\pi\varepsilon_0 d}$

(3) decrease by $\frac{4q^2}{3\pi\varepsilon_0 d}$ (4) decrease by $\frac{q^2}{4\pi\varepsilon_0 d}$

ES0264

33. A solid sphere of radius R carries a charge (Q + q) distributed uniformly over its volume. A very small point like piece of it of mass m gets detached from the bottom of the sphere and falls down vertically under gravity. This piece carries charge q. If it acquires a speed v when it has fallen through a vertical height y (see figure), then : (assume the remaining portion to be spherical).

[JEE-Main-2020_Sep]



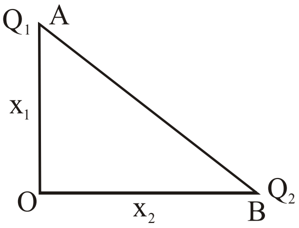
(1) $v^2 = 2y\left[\dfrac{qQ}{4\pi\epsilon_0 R(R+y)m} + g\right]$ (2) $v^2 = y\left[\dfrac{qQ}{4\pi\epsilon_0 R^2 ym} + g\right]$

(3) $v^2 = 2y\left[\dfrac{qQR}{4\pi\epsilon_0 (R+y)^3 m} + g\right]$ (4) $v^2 = y\left[\dfrac{qQ}{4\pi\epsilon_0 R(R+y)m} + g\right]$

ES0265

34. Charges Q_1 and Q_2 arc at points A and B of a right angle triangle OAB (see figure). The resultant electric field at point O is perpendicular to the hypotenuse, then Q_1/Q_2 is proportional to :

[JEE-Main-2020_Sep]

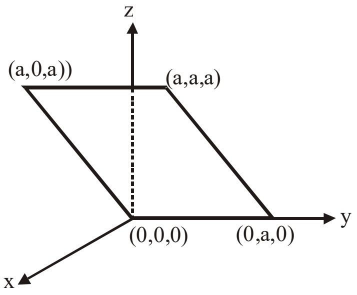


(1) $\dfrac{x_2^2}{x_1^2}$ (2) $\dfrac{x_1^3}{x_2^3}$ (3) $\dfrac{x_1}{x_2}$ (4) $\dfrac{x_2}{x_1}$

ES0266

EXERCISE-(JA)

1. Consider an electric field $\vec{E} = E_0\hat{x}$, where E_0 is a constant. The flux through the shaded area (as shown in the figure) due to this field is **[IIT-JEE 2011]**



(A) $2E_0a^2$ (B) $\sqrt{2}E_0a^2$ (C) E_0a^2 (D) $\frac{E_0a^2}{\sqrt{2}}$

ES0206

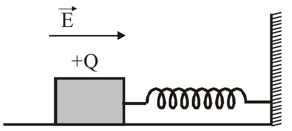
2. A spherical metal shell A of radius R_A and a solid metal sphere B of radius $R_B(<R_A)$ are kept far apart and each is given charge $+Q$. Now they are connected by a thin metal wire. Then

[IIT-JEE 2011]

(A) $E_A^{inside} = 0$ (B) $Q_A > Q_B$ (C) $\frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A}$ (D) $E_A^{on\ \text{surface}} < E_B^{on\ \text{surface}}$

ES0207

3. A wooden block performs SHM on a frictionless surface with frequency, ν_0. The block carries a charge $+Q$ on its surface. If now a uniform electric field $\vec{E}$ is switched-on as shown, then the SHM of the block will be :- **[IIT-JEE 2011]**



(A) of the same frequency and with shifted mean position

(B) of the same frequency and with the same mean position

(C) of changed frequency and with shifted mean position

(D) of changed frequency and with the same mean position

ES0208

4. Which of the following statement(s) is/are correct? **[IIT-JEE 2011]**

(A) If the electric field due to a point charge varies as $r^{-2.5}$ instead of r^{-2}, then the Gauss law will still be valid.

(B) The Gauss law can be used to calculate the field distribution around an electric dipole.

(C) If the electric field between two point charges is zero somewhere, then the sign of the two charges is the same.

(D) The work done by the external force in moving a unit positive charge from point A at potential V_A to point B at potential V_B is $(V_B - V_A)$

ES0209

5. Four point charges, each of +q, are rigidly fixed at the four corners of a square planar soap film of side 'α'. The surface tension of the soap film is γ. The system of charges and planar film are in equilibrium, and $a = k\left[\dfrac{q^2}{\gamma}\right]^{1/N}$, where 'k' is a constant. Then N is **[IIT-JEE 2011]**

ES0210

Paragraph for Question Nos. 6 and 7

A dense collection of equal number of electrons and positive ions is called neutral plasma. Certain solids containing fixed positive ions surrounded by free electrons can be treated as neutral plasma. Let 'N' be the number density of free electrons, each of mass 'm'. When the electrons are subjected to an electric field, they are displaced relatively away from the heavy positive ions. If the electric field becomes zero, the electrons begins to oscillate about the positive ions with a natural angular frequency 'ω_ρ', which is called the plasma frequency. To sustain the oscillations, a time varying electric field needs to be applied that has an angular frequency ω, where a part of the energy is absorbed and a part of it is reflected. As ω approaches ω_ρ, all the free electrons are set to resonance together and all the energy is reflected. This is the explanation of high reflectivity of metals.

[IIT-JEE 2011]

6. Taking the electronic charge as 'e' and the permittivity as 'ε_0', use dimensional analysis to determine the correct expression for ω_ρ.

(A) $\sqrt{\dfrac{Ne}{m\varepsilon_0}}$ (B) $\sqrt{\dfrac{m\varepsilon_0}{Ne}}$ (C) $\sqrt{\dfrac{Ne^2}{m\varepsilon_0}}$ (D) $\sqrt{\dfrac{m\varepsilon_0}{Ne^2}}$

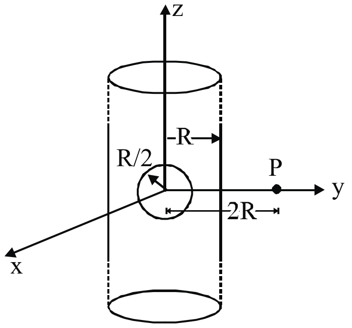
ES0211

7. Estimate the wavelength at which plasma reflection will occur for a metal having the density of electrons $N \approx 4 \times 10^{27} m^{-3}$. Take $\varepsilon_0 \approx 10^{-11}$ and $m \approx 10^{-30}$, where these quantities are in proper SI units

(A) 800 nm (B) 600 nm (C) 300 nm (D) 200 nm

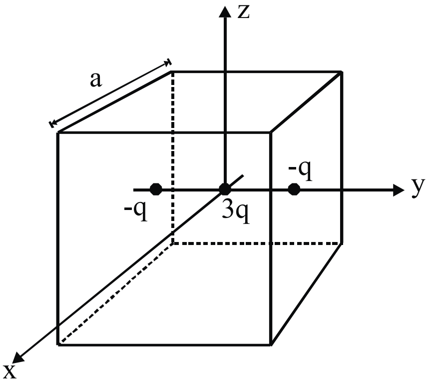
ES0211

8. An infinitely long solid cylinder of radius R has a uniform volume charge density ρ. It has a spherical cavity of radius R/2 with its centre on the axis of the cylinder, as shown in the figure. The magnitude of the electric field at the point P, which is at a distance 2R from the axis of the cylinder, is given by the expression $\frac{23\rho R}{16k\varepsilon_0}$. The value of k is **[IIT-JEE 2012]**



ES0212

9. A cubical region of side a has its centre at the origin. It encloses three fixed point charges, –q at (0, –a/4, 0), +3q at (0,0,0) and –q at (0, +a/4, 0). Choose the correct option(s). **[IIT-JEE 2012]**



(A) The net electric flux crossing the plane x = +a/2 is equal to the net electric flux crossing the plane x = – a/2

(B) The net electric flux crossing the plane y = +a/2 is more than the net electric flux crossing the plane y = –a/2.

(C) The net electric flux crossing the entire region is $\frac{q}{\varepsilon_0}$

(D) The net electric flux crossing the plane z = +a/2 is equal to the net electric flux crossing the plane x = +a/2.

ES0213

10. Two large vertical and parallel metal plates having a separation of 1 cm are connected to a DC voltage source of potential difference X. A proton is released at rest midway between the two plates. It is found to move at 45° to the vertical JUST after release. Then X is nearly

[IIT-JEE 2012]

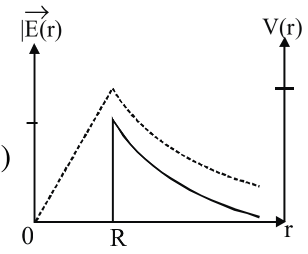
(A) 1×10^{-5} V
(B) 1×10^{-7} V
(C) 1×10^{-9} V
(D) 1×10^{-10} V

ES0214

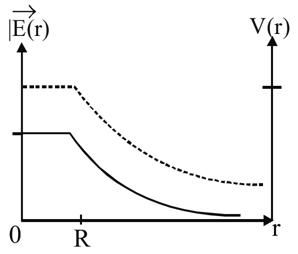
11. Consider a thin spherical shell of radius R with its centre at the origin, carrying uniform positive surface charge density. The variation of the magnitude of the electric field $|\vec{E}(r)|$ and the electric potential V(r) with the distance r from the centre, is best represented by which graph?

[IIT-JEE 2012]

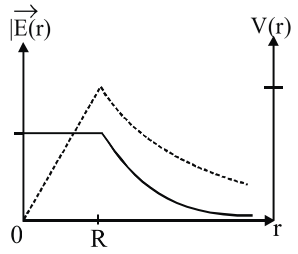
(A)



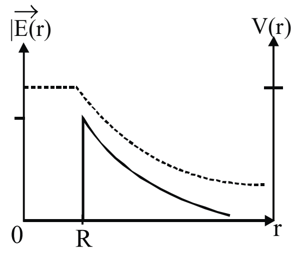
(B)



(C)

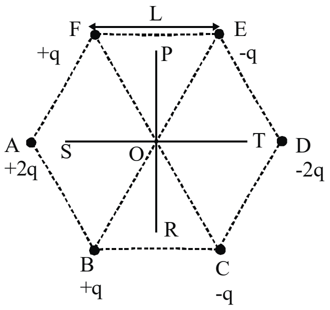


(D)



ES0215

12. Six point charges are kept at the vertices of a regular hexagon of side L and centre O, as shown in figure. Given that $K = \frac{1}{4\pi \in_0}\frac{q}{L^2}$, which of the following statement(s) is(are) correct? **[IIT-JEE 2012]**



(A) The electric field at O is 6 K along OD.
(B) The potential at O is zero.
(C) The potential at all points on the line PR is same.
(D) The potential at all points on the line ST is same.

ES0216

13. Two non-conducting solid spheres of radii R and 2R, having uniform volume charge densities ρ_1 and ρ_2 respectively, touch each other. The net electric field at a distance 2R from the centre of the smaller sphere, along the line joining the centres of the spheres, is zero. The ratio $\frac{\rho_1}{\rho_2}$ can be

[JEE-Advance-2013]

(A) –4 (B) $-\frac{32}{25}$ (C) $\frac{32}{25}$ (D) 4

ES0217

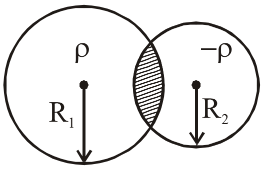
14. Two non-conducting spheres of radii R_1 and R_2 and carrying uniform volume charge densities $+\rho$ and $-\rho$, respectively, are placed such that they partially overlap, as shown in the figure. At all points in the overlapping region :- **[JEE-Advance-2013]**

(A) the electrostatic field is zero

(B) the electrostatic potential is constant

(C) the electrostatic field is constant in magnitude

(D) the electrostatic field has same direction



ES0218

15. Let $E_1(r)$, $E_2(r)$ and $E_3(r)$ be the respective electric fields at a distance r from a point charge Q, an infinitely long wire with constant linear charge density λ, and an infinite plane with uniform surface charge density σ. If $E_1(r_0) = E_2(r_0) = E_3(r_0)$ at a given distance r_0, then :-

[JEE-Advance-2014]

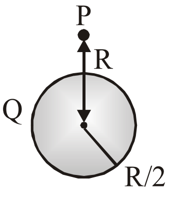
(A) $Q = 4\sigma\pi r_0^2$ (B) $r_0 = \frac{\lambda}{2\pi\sigma}$

(C) $E_1(r_0/2) = 2E_2(r_0/2)$ (D) $E_2(r_0/2) = 4E_3(r_0/2)$

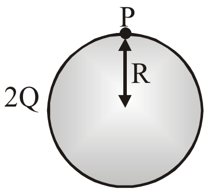
ES0219

16. Charges Q, 2Q and 4Q are uniformly distributed in three dielectric solid spheres 1, 2 and 3 of radii R/2, R and 2R respectively, as shown in figure. If magnitudes of the electric fields at point P at a distance R from the centre of spheres 1, 2 and 3 are E_1, E_2 and E_2 respectively, then

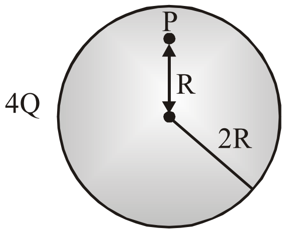
[JEE-Advance-2014]



Sphere 1



Sphere 2



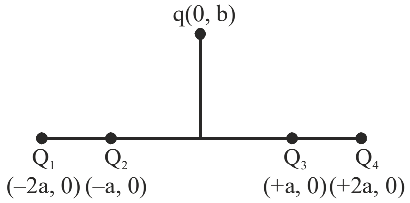
Sphere 3

(A) $E_1 > E_2 > E_3$ (B) $E_3 > E_1 > E_2$ (C) $E_2 > E_1 > E_3$ (D) $E_3 > E_2 > E_1$

ES0220

17. Four charges Q_1, Q_2, Q_3 and Q_4 of same magnitude are fixed along the x axis at x = –2a, –a, +a and +2a, respectively. A positive charge q is placed on the positive y axis at a distance b > 0. Four options of the signs of these charges are given in List I. The direction of the forces on the charge q is given in List II. Match List I with List II and select the correct answer using the code given below the lists. **[JEE-Advance-2014]**

	List-I		List-II
(P)	Q_1, Q_2, Q_3, Q_4 all positive	(1)	+x
(Q)	Q_1, Q_2 positive ; Q_3, Q_4 negative	(2)	–x
(R)	Q_1, Q_4 positive ; Q_2, Q_3 negative	(3)	+y
(S)	Q_1, Q_3 positive ; Q_2, Q_4 negative	(4)	–y

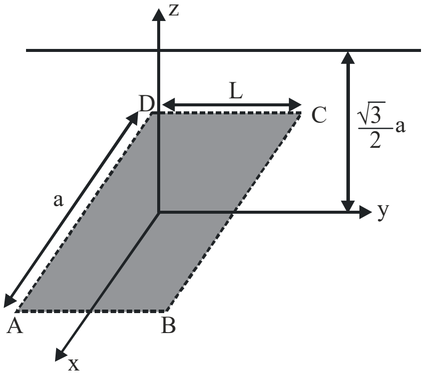


Code :

(A) P-3, Q-1, R-4, S-2
(B) P-4, Q-2, R-3, S-1
(C) P-3, Q-1, R-2, S-4
(D) P-4, Q-2, R-1, S-3

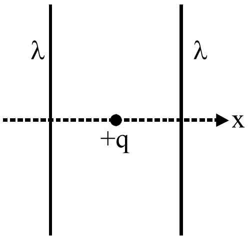
ES0221

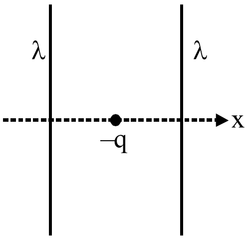
18. An infinitely long uniform line charge distribution of charge per unit length λ lies parallel to the y-axis in the y-z plane at $z = \frac{\sqrt{3}}{2}a$ (see figure). If the magnitude of the flux of the electric field through the rectangular surface ABCD lying in the x-y plane with its centre at the origin is $\frac{\lambda L}{n\varepsilon_0}$ (ε_0 = perimittivity of free space) then the value of n is. **[JEE-Advance-2015]**



ES0222

19. The figures below depict two situations in which two infinitely long static line charges of constant positive line charge density λ are kept parallel to each other. In their resulting electric field, point charges q and –q are kept in equilibrium between them .The point charges are confined to move in the x direction only. If they are given a small displacement about their equlibrium positions, then the correct statement(s) is (are) : **[JEE-Advance-2015]**





(A) Both charges execute simple harmonic motion

(B) Both charges will continue moving in the direction of their displacement

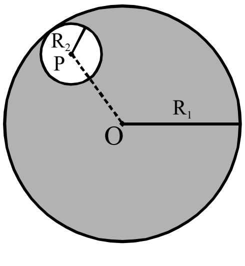
(C) Charge +q executes simple harmonic motion while charge –q continues moving in the direction of its displacement.

(D) Charge –q executes simple harmonic motion while charge +q continues moving in the direction of its displacement.

ES0223

20. Consider a uniform spherical distirbution of radius R_1 centred at the origin O. In this distibution, a spherical cavity of radius R_2, centred at P with distance OP = a = $R_1 - R_2$ (see figure) is made. If the electric field inside the cavity at position $\vec{r}$ is $\vec{E}(\vec{r})$, then the correct statement(s) is(are) :

[JEE-Advance-2015]



(A) $\vec{E}$ is uniform, its magnitude is independent of R_2 but its direction depends on $\vec{r}$

(B) $\vec{E}$ is uniform, its magnitude depends on R_2 and its direction depends on $\vec{r}$

(C) $\vec{E}$ is uniform, its magnitude is independent of a but its direction depends on $\vec{a}$

(D) $\vec{E}$ is uniform and both its magnitude and direction depend on $\vec{a}$

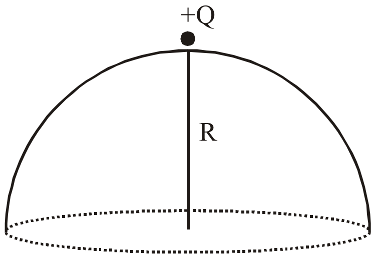
ES0224

21. A length-scale (ℓ) depends on the permittivity (ε) of a dielectric material, Boltzmann constant k_B, the absolute temperature T, the number per unit volume (n) of certain charged particles, and the charge (q) carried by each of the particles, Which of the following expressions(s) for ℓ is(are) dimensionally correct? **[JEE-Advance-2016]**

(A) $\ell = \sqrt{\left(\frac{nq^2}{\varepsilon k_B T}\right)}$ (B) $\ell = \sqrt{\left(\frac{\varepsilon k_B T}{nq^2}\right)}$ (C) $\ell = \sqrt{\left(\frac{q^2}{\varepsilon n^{2/3} k_B T}\right)}$ (D) $\ell = \sqrt{\left(\frac{q^2}{\varepsilon n^{1/3} k_B T}\right)}$

ES0225

22. A point charge +Q is placed just outside an imaginary hemispherical surface of radius R as shown in the figure. Which of the following statements is/are correct ? **[JEE-Advance-2017]**



(A) The circumference of the flat surface is an equipotential

(B) The electric flux passing through the curved surface of the hemisphere is $-\frac{Q}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right)$

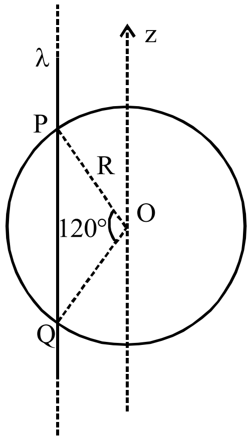
(C) Total flux through the curved and the flat surfaces is $\frac{Q}{\varepsilon_0}$

(D) The component of the electric field normal to the flat surface is constant over the surface.

ES0226

23. An infinitely long thin non-conducting wire is parallel to the z-axis and carries a uniform line charge density λ. It pierces a thin non-conducting spherical shell of radius R in such a way that the arc PQ subtends an angle 120° at the centre O of the spherical shell, as shown in the figure. The permittivity of free space is ε_0. Which of the following statements is (are) true ?

[JEE-Advance-2018]



(A) The electric flux through the shell is $\sqrt{3}\,R\lambda/\varepsilon_0$

(B) The z-component of the electric field is zero at all the points on the surface of the shell

(C) The electric flux through the shell is $\sqrt{2}\,R\lambda/\varepsilon_0$

(D) The electric field is normal to the surface of the shell at all points

ES0227

24. A particle, of mass 10^{-3} kg and charge 1.0 C, is initially at rest. At time t = 0, the particle comes under the influence of an electric field $\vec{E}(t) = E_0 \sin\omega t\,\hat{i}$ where E_0 = 1.0 N C^{-1} and ω=10^3 rad s^{-1}. Consider the effect of only the electrical force on the particle. Then the maximum speed, in ms^{-1}, attained by the particle at subsequent times is.......................... **[JEE-Advance-2018]**

ES0228

25. The electric field E is measured at a point P(0,0,d) generated due to various charge distributions and the dependence of E on d is found to be different for different charge distributions. List-I contains different relations between E and d. List-II describes different electric charge distributions, along with their locations. Match the functions in List-I with the related charge distributions in List-II.

[JEE-Advance-2018]

	List–I		**List–II**
P.	E is indpendent of d	1.	A point charge Q at the origin
Q.	$E \propto \frac{1}{d}$	2.	A small dipole with point charges Q at $(0,0,\ell)$ and – Q at $(0,0,-\ell)$. Take $2\ell << d$
R.	$E \propto \frac{1}{d^2}$	3.	An infinite line charge coincident with the x-axis, with uniform linear charge density λ.
S.	$E \propto \frac{1}{d^3}$	4.	Two infinite wires carrying uniform linear Charge density parallel to the x - axis. The one along $(y = 0, z = \ell)$ has a charge density $+\lambda$ and the one along $(y = 0, z = -\ell)$ has a charge density $-\lambda$. Take $2\ell << d$
		5.	Infinite plane charge coincident with the xy-plane with uniform surface charge density

(A) $P \rightarrow 5$; $Q \rightarrow 3, 4$; $R \rightarrow 1$; $S \rightarrow 2$
(B) $P \rightarrow 5$; $Q \rightarrow 3,$; $R \rightarrow 1,4$; $S \rightarrow 2$
(C) $P \rightarrow 5$; $Q \rightarrow 3,$; $R \rightarrow 1,2$; $S \rightarrow 4$
(D) $P \rightarrow 4$; $Q \rightarrow 2, 3$; $R \rightarrow 1$; $S \rightarrow 5$

ES0229

26. A thin spherical insulating shell of radius R carries a uniformly distributed charge such that the potential at its surface is V_0. A hole with a small area $\alpha 4\pi R^2$ ($\alpha<<1$) is made on the shell without affecting the rest of the shell. Which one of the following statements is correct ?

[JEE-Advance-2019]

(1) The ratio of the potential at the center of the shell to that of the point at $\frac{1}{2}R$ from center towards the hole will be $\frac{1-\alpha}{1-2\alpha}$

(2) The magnitude of electric field at the center of the shell is reduced by $\frac{\alpha V_0}{2R}$

(3) The magnitude of electric field at a point, located on a line passing through the hole and shell's center on a distance 2R from the center of the spherical shell will be reduced by $\frac{\alpha V_0}{2R}$

(4) The potential at the center of the shell is reduced by $2\alpha V_0$

ES0230

27. A charged shell of radius R carries a total charge Q. Given Φ as the flux of electric field through a closed cylindrical surface of height h, radius r and with its center same as that of the shell. Here, center of the cylinder is a point on the axis of the cylinder which is equidistant from its top and bottom surfaces. Which of the following option(s) is/are correct ?

[$\in_0$ is the permittivity of free space] **[JEE-Advance-2019]**

(1) If h > 2R and r > R then $\Phi = \frac{Q}{\in_0}$

(2) If $h < \frac{8R}{5}$ and $r = \frac{3R}{5}$ then $\Phi = 0$

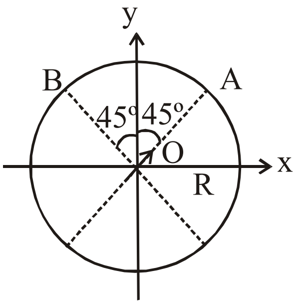
(3) If h > 2R and $r = \frac{4R}{5}$ then $\Phi = \frac{Q}{5\in_0}$

(4) If h > 2R and $r = \frac{3R}{5}$ then $\Phi = \frac{Q}{5\in_0}$

ES0231

28. An electric dipole with dipole moment $\frac{p_0}{\sqrt{2}}(\hat{i}+\hat{j})$ is held fixed at the origin O in the presence of an uniform electric field of magnitude E_0. If the potential is constant on a circle of radius R centered at the origin as shown in figure, then the correct statement(s) is/are:

(ε_0 is permittivity of free space, R >> dipole size) **[JEE-Advance-2019]**



(1) $R = \left(\frac{p_0}{4\pi\varepsilon_0 E_0}\right)^{1/3}$

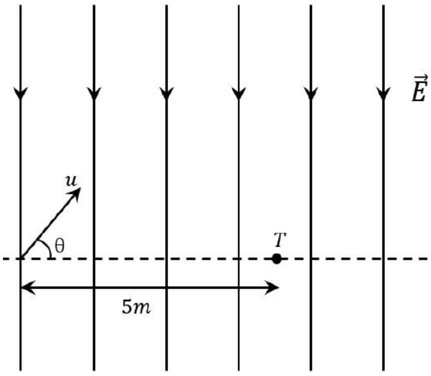
(2) The magnitude of total electric field on any two points of the circle will be same

(3) Total electric field at point A is $\vec{E}_A = \sqrt{2}E_0(\hat{i}+\hat{j})$

(4) Total electric field at point B is $\vec{E}_B = 0$

ES0232

29. A uniform electric field, $\vec{E} = -400\sqrt{3}\hat{y}$ NC^{-1} is applied in a region. A charged particle of mass m carrying positive charge q is projected in this region with an initial speed of $2\sqrt{10} \times 10^6$ ms^{-1}. This particle is aimed to hit a target T, which is 5 m away from its entry point into the field as shown schematically in the figure. Take $\frac{q}{m} = 10^{10}$ Ckg^{-1}. Then:- **[JEE-Advance-2020]**



(A) the particle will hit T if projected at an angle 45° from the horizontal

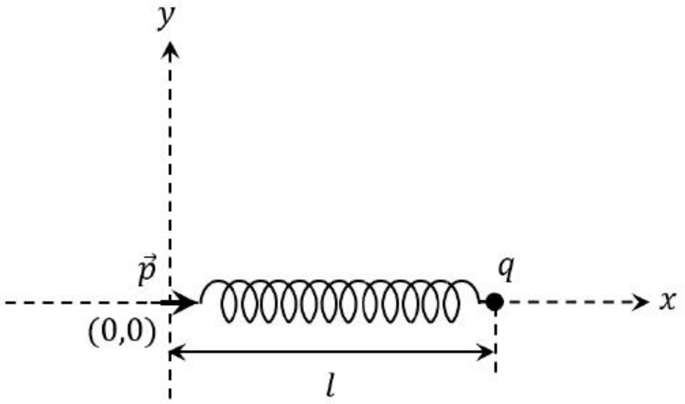
(B) the particle will hit T if projected either at an angle 30° or 60° from the horizontal

(C) time taken by the particle to hit T could be $\sqrt{\frac{5}{6}}$ μs as well as $\sqrt{\frac{5}{2}}$ μs

(D) time taken by the particle to hit T is $\sqrt{\frac{5}{3}}$ μs

ES0267

30. One end of a spring of negligible unstretched length and spring constant k is fixed at the origin (0,0). A point particle of mass m carrying a positive charge q is attached at its other end. The entire system is kept on a smooth horizontal surface. When a point dipole $\vec{p}$ pointing towards the charge q is fixed at the origin, the spring gets stretched to a length ℓ and attains a new equilibrium position (see figure below). If the point mass is now displaced slightly by $\Delta\ell << \ell$ from its equilibrium position and released, it is found to oscillate at frequency $\frac{1}{\delta}\sqrt{\frac{k}{m}}$. The value of δ is. **[JEE-Advanced-2020]**



ES0268

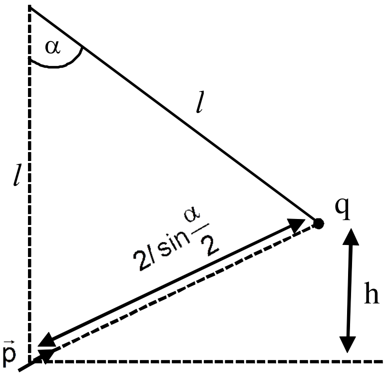
31. A circular disc of radius R carries surface charge density $\sigma(r) = \sigma_0\left(1-\frac{r}{R}\right)$, where σ_0 is a constant and r is the distance from the center of the disc. Electric flux through a large spherical surface that encloses the charged disc completely is ϕ_0. Electric flux through another spherical surface of radius $\frac{R}{4}$ and concentric with the disc is ϕ. Then the ratio $\frac{\phi_0}{\phi}$ is_________. -

[JEE-Advance-2020]

ES0269

32. A point charge q of mass m is suspended vertically by a string of length ℓ. A point dipole of dipole moment $\vec{p}$ is now brought towards q from infinity so that the charge moves away. The final equilibrium position of the system including the direction of the dipole, the angles and distances is shown in the figure below. If the work done in bringing the dipole to this position is N × (mgh), where g is the acceleration due to gravity, then the value of N is _________ . (Note that for three coplanar forces keeping a point mass in equilibrium, $\frac{F}{\sin\theta}$ is the same for all forces, where F is any one of the forces and θ is the angle between the other two forces)

[JEE-Advance-2020]



ES0270

33. Two identical non-conducting solid spheres of same mass and charge are suspended in air from a common point by two non-conducting, massless strings of same length. At equilibrium, the angle between the strings is α. The spheres are now immersed in a dielectric liquid of density 800 kg m^{-3} and dielectric constant 21. If the angle between the strings remains the same after the immersion, then

[JEE-Advance-2020]

(A) electric force between the spheres remains unchanged

(B) electric force between the spheres reduces

(C) mass density of the spheres is 840 kg m^{-3}

(D) the tension in the strings holding the spheres remains unchanged

ES0271

ELECTROSTATICS

(CBSE Previous Year's Questions)

1. An electrostatic field line cannot be discontinuous. Why ? **[1; CBSE-2005]**

2. Define electric field intensity. Write its S.1 unit. Write the magnitude and direction of electric field intensity due to an electric dipole of length 2a at the mid- point of the line joining the two charges. **[2; CBSE-2005]**

3. State Gauss' theorem. Apply this theorem to obtain the expression for the electric field intensity at a point due to an infinitely long, thin, uniformly charged straight wire. **[3; CBSE-2005]**

4. Define the term electric dipole moment. Is it a scalar or a vector quantity? **[1; CBSE-2006]**

5. A point charge 'q' is placed at O as shown in the figure. **[2; CBSE-2006]**

Is $V_P - V_Q$ positive or negative when (i) $q > 0$, (ii) $q < 0$? Justify your answer.

6. Using Gauss's theorem, show mathematically that for any point outside the shell, the field due to a uniformly charged thin spherical shell is the sai as if the entire charge of the shell is concentrated at the centre. Why do you expect the electric field inside the shell to be zero according to this theorem? **[3; CBSE-2006]**

7. Two point charges 4μC and –2μC are separated by a distance of 1 m in air. Calculate at what point on the line joining the two charges is the electric potential zero. **[1; CBSE-2007]**

8. State Gauss's theorem in electrostatics. Apply this theorem to derive an expression for electric field intensity at a point near an infinitely long straight charged wire. **[2; CBSE-2007]**

9. A 500 μC Charge is at the centre of a square of side 10cm. Find the work done in moving a charge of 10 μC between two diagonally opposite points on the square. **[1; CBSE-2008]**

10. Derive the expression for the electric potential at any point along the axial line of an electric dipole. **[1; CBSE-2008]**

11. (i) Can two equi- potential surfaces intersect each other? Give reasons.

(ii) Two charges –q and +q are located at points A (0,0, –a) and B (0,0, + a) respectively. How much work is done in moving a test charge from point P (7,0,0) to Q (–3,0,0)? **[2; CBSE-2009]**

12. State Gauss's law in electrostatics. Using this law derive an expression for the electric field due to a uniformly charged infinite plane sheet. **[3; CBSE-2009]**

13. Name the physical quantity whose S .1. unit is JC^{-1}. Is it a scalar or a vector quantity ? **[1; CBSE-2010]**

14. Define electric dipole moment. Write its S.I. unit. **[1; CBSE-201I]**

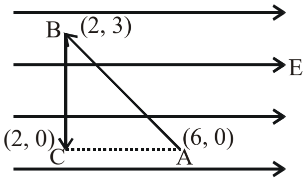
15. A hollow metal sphere of radius 5 cm is charged such that the potential on its surface is 10 V, What is the potential at the centre of the sphere ? **[1; CBSE-2011]**

16. A thin straight infinitely long conducting wire having charge density λ is enclosed by a cylindrical surface of radius r and length *l*, its axis coinciding with the length of the wire. Find the expression for the electric flux through the surface of the cylinder. **[2; CBSE-2011]**

17. Plot a graph showing the variation of coulomb force (F) versus $\left(\frac{1}{r^2}\right)$, where r is the distance between the two charges of each pair of charges: (1μC, 2μC) and (2μC, – 3μC), interpret the graphs obtained. **[2; CBSE-2011]**

18. Two wires of equal length, one of copper and the other of manganin have the same resistance. Which wire is thicker? **[1; CBSE-2012]**

19. Acharge 'q' is moved without acceleration from A to C along the path from Ato B and then from B to C in electric field E as shown in the figure, (i) Calculate the potential difference between A and C. (ii) At which point (of the two) is the electric potential more and why?



20. An electric dipole is held in a uniform electric field.

(i) Show that the net force acting on it is zero

(ii) The dipole is aligned parallel to the field. Find the work done in rotating it through the angle of 180°. **[2; CBSE-2012]**

21. Two charges of magnitudes – 2Q and + Q are located at points (a, 0) and (4a, 0) respectively. What is the electric flux due to these charges through a sphere of radius '3a' with its centre at the origin?

[CBSE-2013]

22. (a) Define electric dipole moment. Is it a scalar or a vector? Derive the expression for the electric field of a dipole at a point on the equatorial plane of the dipole.

(b) Draw the equipotential surfaces due to an electric dipole. Locate the points where the potential due to the dipole is zero.

OR

Using Gauss' law deduce the expression for the electric field due to a uniformly charged spherical conducting shell of radius R at a point (i) outside and (ii) inside the shell.

Plot a graph showing variation of electric field as a function of r > Rand r < R. (r being the distance from the centre of the shell) **[CBSE-2013]**

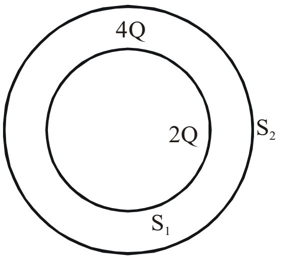
23. Why do the electrostatic field lines not form closed loops ? **[CBSE-2014]**

24. Draw a labelled diagram of Van de Graaft generator. State its working principle to show how by introducing a small charged sphere, into a larger sphere, a large amount of charge can be transferred to the outer sphere state the use of this machine and also point out its limitations.

OR

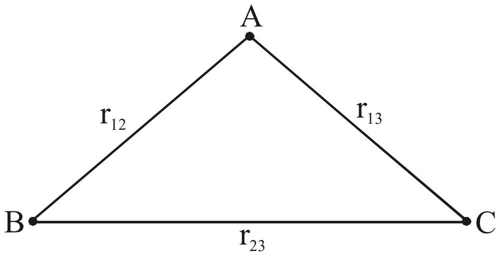
(a) Deduce the expression for the torque acting on a dipole of dipole moment $\vec{p}$ in the presence of a uniform electric field $\vec{E}$.

(b) Consider two hollow concentric spheres, S_1 and S_2, enclosing charges 2Q and 4Q respectively as shown in the figure, (i) Find out the ratio of the electric flux through them, (ii) How will the electric flux through the sphere S_1 change if a medium of dielectric constant 'ε_r' is introduced in the space inside S_1 in place of air ? Deduce the necessary expression. **[CBSE-2014]**



25. Determine the distance of closest approach when an alpha particle of kinetic energy 4.5 MeV strikes a nucleus of Z = 80, stops and reverses its direction. **[2; CBSE-2015]**

26. (a) State Gauss's law in electrostatics. Show, with the help of a suitable example along with the figure, that the outward flux due to a point charge 'q', in vacuum within a closed surface, is independent of its size or shape and is given by q/ε_0. **[5; CBSE-2015]**



(b) Two parallel uniformly charged infinite plane sheets, '1' and '2', have charge densities + σ and –2 σ respectively. Give the magnitude and direction of the net electric field at a point

(i) in between the two sheets and

(ii) outside near the sheet '1'.

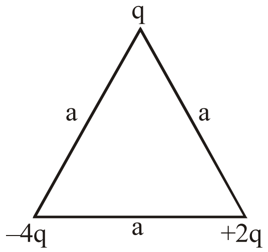
27. What is the amount of work done in moving a point charge Q around a circular arc of radius 'r' at the centre of which another point charge 'q' is located? **[1; CBSE-2016]**

28. Find the electric field intensity due to a uniformly charged spherical shell at a point (i) outside the shell and (ii) inside the shell. Plot the graph of electric field with distance from the centre of the shell. **[3; CBSE-2016]**

29. (a) Explain why, for any charge configuration, the equipotential surface through a point is normal to the electric field at the point. **[5; CBSE-2016]**

Draw a sketch of equipotential surfaces due to a single charge (–q), depicting the electric field lines due to the charge.

(b) Obtain an expression for the work done to dissociate the system of three charges placed at the vertices of an equilateral triangle of side 'a' as shown below.



30. (a) Derive an expression for the electric field E due to a dipole of length '2a' at a point distant r from the centre of the dipole on the axial line. **[CBSE-2017]**

(b) Draw a graph of E versus r for r >> a.

(c) If this dipole were kept in a uniform external electric field E_0, diagrammatically represent the position of the dipole in stable and unstable equilibrium and write the expressions for the torque acting on the dipole in both the cases.

OR

(a) Use Gauss's theorem to find the electric field due to a uniformly charged infinitely large plane thin sheet with surface charge density σ.

(b) An infinitely large thin plane sheet has a uniform surface charge density $+\sigma$. Obtain the expression for the amount of work done in bringing a point charge q from infinity to a point, distant r, in front of the charged plane sheet.

ANSWER KEY

EXERCISE (S-1)

1. Ans. 80

2. Ans. 003

3. Ans. $(1 + 2\sqrt{2})/4\,Q$

4. Ans. 9

5. Ans. 3

6. Ans. 4

7. Ans. 400

8. Ans. 9.30

9. Ans. 3

10. Ans. 8

11. Ans. 2

12. Ans. 1

13. Ans. 2

14. Ans. 4

15. Ans. 576

16. Ans. 3

17. Ans. 4

18. Ans. 4

19. Ans. 6

20. Ans. 0

21. Ans. $-\frac{kq^2}{a}\left(3-\sqrt{2}\right)$

22. Ans. 2

23. Ans. 90

24. Ans. 2

25. Ans. 2

26. Ans. $2\tan^{-1}\left(\frac{\sigma q}{2\varepsilon_0 mg}\right)$

27. Ans. 20

28. Ans. $\left[\frac{2KQq}{mR}\left(\frac{r-R}{r}+\frac{3}{8}\right)\right]^{1/2}$

29. Ans. 2

30. Ans. 4

31. Ans. $\frac{kP}{\sqrt{2}\,y^3}(-\hat{i}-2\hat{j})$

32. Ans. 028

33. Ans. (a) $K.E = \frac{P}{4\pi\varepsilon_0}\frac{Q}{d^2}$, (b) $\frac{QP}{2\pi\varepsilon_0 d^3}$ along positive x-axis

34. Ans. (a) (i) $V = \frac{k(q_a+q_b)}{r}$, $E = \frac{k(q_a+q_b)}{r^2}$; (ii) $\frac{k(q_a+q_b)}{R} + \frac{kq_b}{r} - \frac{kq_b}{b}$; $\frac{kq_b}{r^2}$

(b) $\sigma_R = \left(\frac{q_a+q_b}{4\pi R^2}\right)$, $\sigma_a = -\frac{q_a}{4\pi a^2}$; $\sigma_b = -\frac{q_b}{4\pi b^2}$; (c) $f = 0$]

35. Ans. $-Q/3$

36. Ans. 3

37. Ans. $V' = \left(\frac{a}{3t}\right)^{1/3} V$

38. Ans. -εE,εE and so on

EXERCISE (S-2)

1. Ans. $\frac{\sigma}{\pi \epsilon_0}$

2. Ans. a

3. Ans. $\pi K d t \rho R^2$

4. Ans. 3

5. Ans. 3

6. Ans. 8

7. Ans. $v_0 = 3$ m/s ; K.E. at the origin $= \left(27-10\sqrt{6}\right) \times 10^{-4}$ J approx.2.5×10^{-4} J

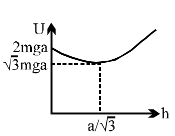
8. Ans. 6

9. Ans. $\lambda R E_0 \hat{i}$

10. Ans. $\sqrt{4\pi\varepsilon_o K a}$

11. Ans. $W_{\text{first step}} = \left(\frac{8}{3}-\frac{4}{\sqrt{5}}\right)\frac{Kq^2}{r}$, $W_{\text{second step}} = 0$, $W_{\text{total}} = 0$

12. Ans. (a) $H = \frac{4a}{3}$, (b) $U = mg\left[2\sqrt{h^2+a^2}-h\right]$ equilibrium at $h = \frac{a}{\sqrt{3}}$,



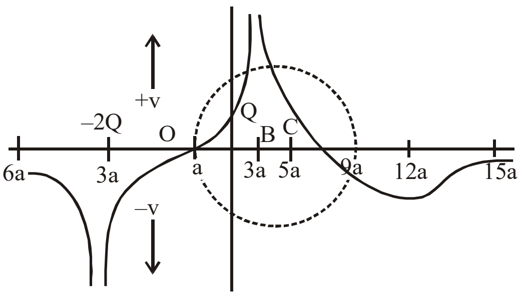
13. Ans. (i) -Q, + Q (ii) $E_A = 0$, can't be found (iii) can't be found (iv) can't be found (v) 0 (vi) No, induced charge on outer surface]

14. Ans. 180 **15. Ans.** $F = \frac{Q^2}{32\pi\varepsilon_0 R^4}\left(R^2 - h^2\right)$

16. Ans. (a) radius = 4a & center (5a, 0)

(b) $V = \frac{1}{4\pi\varepsilon_0}\left[\frac{-2Q}{|(x+3a)|}+\frac{Q}{|(x-3a)|}\right]$

(c) speed $v = \sqrt{\frac{1}{4\pi\varepsilon_0}\times\frac{qQ}{2ma}}$



17. Ans. 7

EXERCISE (O-1)

1. Ans. (C) 2. Ans. (B) 3. Ans. (C) 4. Ans. (C) 5. Ans. (B) 6. Ans. (A)
7. Ans. (D) 8. Ans. (B) 9. Ans. (A) 10. Ans. (A) 11. Ans. (A) 12. Ans. (C)
13. Ans. (A) 14. Ans. (C) 15. Ans. (A) 16. Ans. (D) 17. Ans. (B) 18. Ans. (C)
19. Ans. (D) 20. Ans. (B) 21. Ans. (D) 22. Ans. (A) 23. Ans. (D) 24. Ans. (A)
25. Ans. (D) 26. Ans. (D) 27. Ans. (D) 28. Ans. (B) 29. Ans. (C) 30. Ans. (C)
31. Ans. (D) 32. Ans. (C) 33. Ans. (D) 34. Ans. (D) 35. Ans. (A) 36. Ans. (B)
37. Ans. (A) 38. Ans. (B) 39. Ans. (D) 40. Ans. (B) 41. Ans. (A) 42. Ans. (C)
43. Ans. (C) 44. Ans. (C) 45. Ans. (C) 46. Ans. (A) 47. Ans. (C) 48. Ans. (B)
49. Ans. (B) 50. Ans. (A) 51. Ans. (B) 52. Ans. (C) 53. Ans. (A) 54. Ans. (A)
55. Ans. (D) 56. Ans. (B) 57. Ans. (A) 58. Ans. (A) 59. Ans. (C) 60. Ans. (C)
61. Ans. (A) 62. Ans. a. (B); b. (D) 63. Ans. (C) 64. Ans. (B) 65. Ans. (D)
66. Ans. (A) 67. Ans. (D) 68. Ans. (D) 69. Ans. (A) 70. Ans. (C) 71. Ans. (D)
72. Ans. (B) 73. Ans. (B) 74. Ans. (A) 75. Ans. (A) 76. Ans. (C) 77. Ans. (A)
78. Ans. (C) 79. Ans. (C) 80. Ans. (A,B) 81. Ans. (A,B,C) 82. Ans. (A,B,D)
83. Ans. (A,B,C) 84. Ans. (A,B,C) 85. Ans. (AC) 86. Ans. (A,C,D)
87. Ans. (A,B,C) 88. Ans. (A,B,C) 89. Ans. (B,D) 90. Ans. (A,C)
91. Ans. (A, D) 92. Ans. (B,C,D) 93. Ans. (A, B, D) 94. Ans. (A, B)
95. Ans. (A,B,C) 96. Ans. (B) 97. Ans. (D) 98. Ans. (C) 99. Ans. (A)
100. Ans. (C) 101. Ans. (A) 102. Ans. (A) 103. Ans. (A) 104. Ans. (B)
105. Ans. (C) 106. Ans. (B) 107. Ans. (A)→(Q); (B)→(S); (C)→(R); (D)→(P)
108. Ans. (A) → (Q, R) ; (B) → (Q,T) ; (C) → (S,T) (D) → (P)
109. Ans. (A) – S; (B) – P; (C) – R; (D) – Q
110. Ans. A → Q; B → P, S; C → P, Q, R; D → Q, R
111. Ans. (A)-Q,S, (B)-P,R (C)-P,R (D)-Q,R

EXERCISE (O-2)

1. Ans. (A) 2. Ans. (B) 3. Ans. (B) 4. Ans. (C) 5. Ans. (C) 6. Ans. (A)
7. Ans. (A) 8. Ans. (B) 9. Ans. (C) 10. Ans. (A) 11. Ans. (C) 12. Ans. (B)
13. Ans. (A) 14. Ans. (B) 15. Ans. (D) 16. Ans. (B) 17. Ans. (B) 18. Ans. (C)
19. Ans. (A) 20. Ans. (B) 21. Ans. (A) 22. Ans. (C) 23. Ans. (A) 24. Ans. (C)
25. Ans. (A, C) 26. Ans. (A,C) 27. Ans. (A,D) 28. Ans. (A,B) 29. Ans. (A,B,C)
30. Ans. (B) 31. Ans. (C) 32. Ans. (B) 33. Ans. (A) 34. Ans. (D)
35. Ans. (A) P,R,S (B) P,Q,R,S,T (C) Q,T (D) R,S

EXERCISE-JM

1. Ans. (3) 2. Ans. (2) 3. Ans. (1) 4. Ans. (4) 5. Ans. (4) 6. Ans. (2)
7. Ans. (1) 8. Ans. (4) 9. Ans. (1) 10. Ans. (1 or 2) 11. Ans. (3) 12. Ans. (2)
13. Ans. (1) 14. Ans. (1) 15. Ans. (4) 16. Ans. (4) 17. Ans. (3) 18. Ans. (1)
19. Ans. (4) 20. Ans. (1) 21. Ans. (4) 22. Ans. (1) 23. Ans. (2) 24. Ans. (3)
25. Ans. (1) 26. Ans. (2) 27. Ans. (4) 28. Ans. (4) 29. Ans. (3) 30. Ans. (1)
31. Ans. (1) 32. Ans. (3) 33. Ans. (1) 34. Ans. (3)

EXERCISE-(JA)

1. Ans. (C) 2. Ans. (A,B,C,D) 3. Ans. (A) 4. Ans. (C,D) 5. Ans. 3
6. Ans. (C) 7. Ans. (B) 8. Ans. 6 9. Ans. (A,C,D) 10. Ans. (C)
11. Ans. (D) 12. Ans. (A, B, C) 13. Ans. (B, D) 14. Ans. (C, D) 15. Ans. (C)
16. Ans. (C) 17. Ans. (A) 18. Ans. 6 19. Ans. (C) 20. Ans. (D)
21. Ans. (B,D) 22. Ans. (A) (B) 23. Ans. (A,B) 24. Ans. 2 [1.99, 2.01]
25. Ans. (B) 26. Ans. (1) 27. Ans. (1,2,4) 28.Ans. (1,4) 29. Ans. (B,C)
30. Ans. (3.14) 31. Ans. (6.40) 32. Ans. 2 33. Ans. (A,C)

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

path to success

Chapter 02

Contents

GRAVITATION

Important Notes

GRAVITATION

KEY CONCEPT

The discovery of the law of gravitation

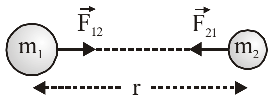
The way the law of universal gravitation was discovered is often considered the paradigm of modern scientific technique. The major steps involved were

- The hypothesis about planetary motion given by **Nicolaus Copernicus (1473–1543)**.
- The careful experimental measurements of the positions of the planets and the Sun by **Tycho Brahe (1546–1601)**.
- Analysis of the data and the formulation of empirical laws by **Johannes Kepler (1571–1630)**.
- The development of a general theory by **Isaac Newton (1642–1727)**.

Newton's law of Gravitation

It states that every body in the universe attracts every other body with a force which is directly proportional to the product of their masses and is inversely proportional to the square of the distance between them.

$F \propto m_1m_2$ and $F \propto \frac{1}{r^2}$ so $F \propto \frac{m_1m_2}{r^2}$



$\therefore\ F = -\frac{Gm_1m_2}{r^2}\hat{r}$ [G = Universal gravitational constant]

Note : This formula is only applicable for spherical symmetric masses or point masses.

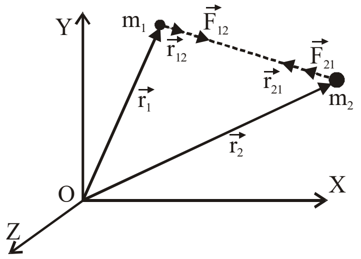
Vector form of Newton's law of Gravitation :

Let $\vec{r}_{12}$ = Displacement vector from m_1 to m_2

$\vec{r}_{21}$ = Displacement vector from m_2 to m_1

$\vec{F}_{21}$ = Gravitational force exerted on m_2 by m_1

$\vec{F}_{12}$ = Gravitational force exerted on m_1 by m_2



$$\vec{F}_{12} = -\frac{Gm_1m_2}{r_{21}^2}\hat{r}_{21} = -\frac{Gm_1m_2}{r_{21}^3}\vec{r}_{21}$$

Negative sign shows that :

(i) The direction of $\vec{F}_{12}$ is opposite to that $\hat{r}_{21}$

(ii) The gravitational force is attractive in nature

Similarly $\vec{F}_{21} = -\frac{Gm_1m_2}{r_{12}^2}\hat{r}_{12}$ or $\vec{F}_{21} = -\frac{Gm_1m_2}{r_{12}^3}\vec{r}_{12} \Rightarrow \vec{F}_{12} = -\vec{F}_{21}$

The gravitational force between two bodies are equal in magnitude and opposite in direction.

Gravitational constant "G"

- Gravitational constant is a scalar quantity.
- **Unit :** **S I :** G = 6.67 × 10^{-11} N–m^2/kg^2

  CGS : 6.67 × 10^{-8} dyne–cm^2/g^2

  Dimensions : $M^{-1}L^3T^{-2}$
- Its value is same throughout the universe, G does not depend on the nature and size of the bodies, it also does not depend upon nature of the medium between the bodies.
- Its value was first find out by the scientist **"Henry Cavendish"** with the help of "Torsion Balance" experiment.
- Value of G is small therefore gravitational force is weaker than electrostatic and nuclear forces.

Ex. Two particles of masses 1 kg and 2 kg are placed at a separation of 50 cm. Assuming that the only forces acting on the particles are their mutual gravitation, find the initial acceleration of heavier particle.

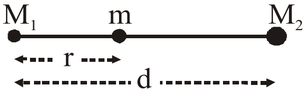
Sol. Force exerted by one particle on another $F=\frac{Gm_1m_2}{r^2}=\frac{6.67\times10^{-11}\times1\times2}{(0.5)^2}=5.3\times10^{-10}\,N$

Acceleration of heavier particle = $\frac{F}{m_2}=\frac{5.3\times10^{-10}}{2}=2.65\times10^{-10}\,ms^{-2}$

This example shows that gravitation is very weak but only this force keep bind our solar system and also this universe, all galaxies and other interstellar system.

Ex. Two stationary particles of masses M_1 and M_2 are at a distance 'd' apart. A third particle lying on the line joining the particles, experiences no resultant gravitational forces. What is the distance of this particle from M_1.

Sol. The force on m towards M_1 is $F_1=\frac{GM_1m}{r^2}$



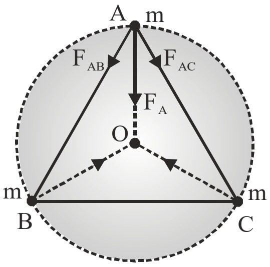
The force on m towards M_2 is $F_2=\frac{GM_2m}{(d-r)^2}$

According to question net force on m is zero i.e. $F_1=F_2$

$$\Rightarrow \frac{GM_1m}{r^2}=\frac{GM_2m}{(d-r)^2} \Rightarrow \left(\frac{d-r}{r}\right)^2=\frac{M_2}{M_1}$$

$$\Rightarrow \frac{d}{r}-1=\frac{\sqrt{M_2}}{\sqrt{M_1}} \Rightarrow r = d\left[\frac{\sqrt{M_1}}{\sqrt{M_1}+\sqrt{M_2}}\right]$$

Ex. Three particles, each of mass m, are situated at the vertices of an equilateral triangle of side 'a'. The only forces acting on the particles are their mutual gravitational forces. It is desired that each particle moves in a circle while maintaining their original separation 'a'. Determine the initial velocity that should be given to each particle and time period of circular motion.



Sol. The resultant force on particle at A due to other two particles is

$$F_A = \sqrt{F_{AB}^2 + F_{AC}^2 + 2F_{AB}F_{AC}\cos 60^\circ} = \sqrt{3}\frac{Gm^2}{a^2} \;...(i) \quad \left[\because F_{AB} = F_{AC} = \frac{Gm^2}{a^2}\right]$$

Radius of the circle $r = \frac{2}{3} \times a\sin 60^\circ = \frac{a}{\sqrt{3}}$

If each particle is given a tangential velocity v, so that F acts as the centripetal force,

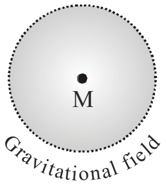
Now $\frac{mv^2}{r} = \sqrt{3}\frac{mv^2}{a}$ ...(ii)

From (i) and (ii) $\sqrt{3}\frac{mv^2}{a} = \frac{Gm^2\sqrt{3}}{a^2} \Rightarrow v = \sqrt{\frac{Gm}{a}}$

Time period $T = \frac{2\pi r}{v} = \frac{2\pi a}{\sqrt{3}}\sqrt{\frac{a}{Gm}} = 2\pi\sqrt{\frac{a^3}{3Gm}}$

- Gravitational forces are central forces as they act along the line joining the centres of two bodies.
- The gravitational forces are conservative forces so work done by gravitational force does not depends upon path and therefore if any particle moves along a closed path under the action of gravitational force then the work done by this force is always zero.
- The total gravitational force on one particle due to number of particles is the resultant of forces of attraction exerted on the given particle due to individual particles i.e. $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ...$ it means the principle of superposition is valid.

Gravitational Field

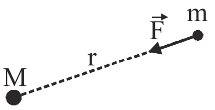


The gravitational field is the space around a mass or an assembly of masses over which it can exert gravitational forces on other masses.

Theoretically speaking, the gravitational field extends up to infinity. However, in actual practice, the gravitational field may become too weak to be measured beyond a particular distance.

Gravitational Field Intensity [g or E_g]

Gravitational force acting per unit mass at any point in the gravitational field is called Gravitational field intensity.

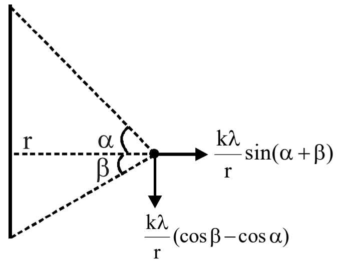
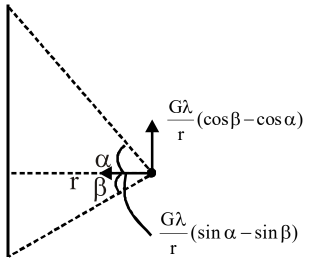
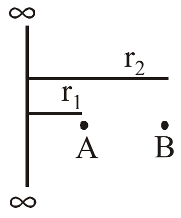
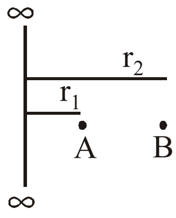


$$g = \frac{GMm}{r^2}\Big/m = \frac{GM}{r^2} \quad \text{Vector form : } \vec{g} = \frac{\vec{F}}{m} \text{ or } \vec{g} = -\frac{GM}{r^2}\hat{r}$$

Gravitational field intensity is a vector quantity having dimension [LT^{-2}] and unit N/kg.

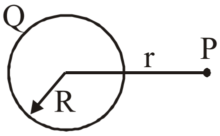
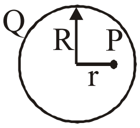
- Since the force between two point masses is having the similar expression as that of force between two point charges, we can write the gravitational field & gravitational potential in the same manner as the electric field & electric potential.

Analogy betwen Electrostatics & Gravitation

(1)	**Point Charge**	**Point Mass**
(a)	$E = \dfrac{kQ}{r^2}$	$g = \dfrac{GM}{r^2}$
(b)	$V = \dfrac{kQ}{r}$	$V_G = \dfrac{-GM}{r}$
(2)	**Uniform charged ring**	**Ring of uniform mass distribution**
(a)	$E = \dfrac{kQx}{(r^2+x^2)^{3/2}}$ on axis	$g = \dfrac{GMx}{(r^2+x^2)^{3/2}}$ on axis
	E is max. when $x = \dfrac{r}{\sqrt{2}}$	g is max. when $x = \dfrac{r}{\sqrt{2}}$
(b)	$V = \dfrac{kQ}{\sqrt{r^2+x^2}}$ on axis , $\dfrac{kQ}{r}$ at center	$V_G = \dfrac{-GM}{\sqrt{r^2+x^2}}$ on axis, $\dfrac{-GM}{r}$ at center
(3)	**Uniform linear charge**	**Uniform linear mass**
	 	 (λ=mass per unit length)
(4)	**Infinite Linear charge**	**Infinite linear mass**
	 	
(a)	$E = \dfrac{2K\lambda}{r}$	$g = \dfrac{2G\lambda}{r}$
(b)	$V_B - V_A = -2K\lambda \ln\dfrac{r_2}{r_1}$	$V_B - V_A = 2G\lambda \ln\left(\dfrac{r_2}{r_1}\right)$
(5)	**Infinite Sheet of charge**	**Infinite Sheet of mass**
	$E = \dfrac{\sigma}{2\varepsilon_0}$	$g = \dfrac{\sigma}{2}\times 4\pi G = 2\pi G\sigma$ (σ = mass per unit area)

* Notice gravitational force is always attractive and hence gravitational potential is always –ve. (for a repulsive force potential is postive). This can be explained from the sign of W_{ext} in moving the test charge from ∞ to the point under consideration.

** Since $\vec{g}$ points from B towards A potential increases as we move from A to B. Just like electric potencial gravitational potential also increases opposite to field direction.

(6)	**Uniformly charged hollow shpere** Charge Q, radius R distance of field point from center r	**Hollow sphere of uniform mass** Mass M , radius R distance of field point from center r
	Case I r > R	
		$g = \frac{GM}{r^2}$
	$E = \frac{kQ}{r^2}$	$V_G = -\frac{GM}{r}$
	$V = \frac{kQ}{r}$	
	Case II r < R	
		$g = 0$
	$E = 0$	$V_G = -\frac{GM}{R}$
	$V = \frac{kQ}{r}$	
(7)	**Electrostatics self energy of uniformly charged thin spherical shell.**	**Gravitational self energy of uniform thin spherical shell.**
	$U = \frac{KQ^2}{2R}$	$U = \frac{GM^2}{2R}$
(8)	**Uniformly charged solid sphere** mass M, radius R	**Uniformly solid sphere** mass M, radius R
	$E = \frac{kQ}{r^2}, r > R$	$g = \frac{GM}{r^2}, r > R$
	$\frac{kQr}{R^3}, r < R$	$\frac{GM}{R^3}r, r < R$
	$V = \frac{kQ}{r}, r > R$	$V_a = -\frac{GM}{r}, r > R$
	$\frac{kQ}{2R^3}\left(3R^3 - r^2\right), r > R$	$\frac{-GM}{2R^3}\left(3R^3 - r^2\right), r > R$

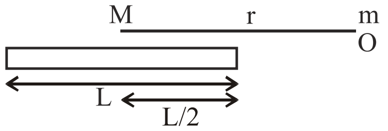
(9) Electrostatics self energy of uniformly charged solid sphere.

$$U = \frac{3}{5}\frac{KQ^2}{R}$$

Gravitational self energy of uniform solid sphere.

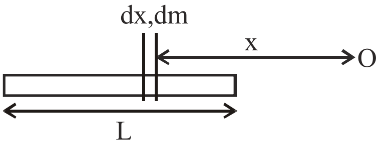
$$U = \frac{3}{5}\frac{GM^2}{R}$$

Ex. Find gravitational force between the point mass & the rod of uniform mass.



Ans. $\dfrac{GMm}{\left(r+\frac{1}{2}\right)\left(r-\frac{1}{2}\right)}$

Sol. $dm = \dfrac{M}{L}dx$



$$dF = \frac{G(dM)m}{x^2}$$

$$\int_0^F dF = \frac{GMm}{L}\int_{x_1=\left(r-\frac{L}{2}\right)}^{x_2=\left(r+\frac{L}{2}\right)} \frac{dx}{x^2}$$

$$F = \frac{GMm}{L}\left[\frac{1}{x}\right]_{x_2}^{x_1} = \frac{GMm}{L}\frac{L}{\left(r-\frac{L}{2}\right)\left(r+\frac{L}{2}\right)} = \frac{GMm}{\left(r-\frac{L}{2}\right)\left(r+\frac{L}{2}\right)}$$

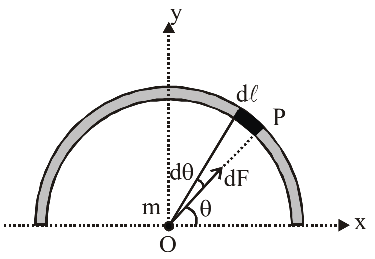
Ex. A thin rod of mass M and length L is bent in a semicircle as shown in figure.

(a) What is its gravitational force (both magnitude and direction) on a particle with mass m at O, the centre of curvature?

(b) What would be the force on m if the rod is, in the form of a complete circle?

Sol. (a) Considering an element of rod of length $d\ell$ as shown in figure and treating it as a point of mass $(M/L)\,d\ell$ situated at a distance R from P, the gravitational force due to this element on the particle will be

$$dF = \frac{Gm(M/L)(Rd\theta)}{R^2} \text{ along OP [as } d\ell = Rd\theta\text{]}$$



So the component of this force along x and y–axis will be

$$dF_x = dF\cos\theta = \frac{GmM\ \cos\theta\ d\theta}{LR}\ ;\ dF_y = dF\sin\theta = \frac{GmM\ \sin\theta\ d\theta}{LR}$$

$$\text{So that } F_x = \frac{GmM}{LR}\int_0^{\pi}\cos\theta\ d\theta = \frac{GmM}{LR}\left[\sin\theta\right]_0^{\pi} = 0$$

$$\text{and } F_y = \frac{GmM}{LR}\int_0^{\pi}\sin\theta\ d\theta = \frac{GmM}{LR}\left[-\cos\theta\right]_0^{\pi} = \frac{2\pi GmM}{L^2}\left[\text{as } R=\frac{L}{\pi}\right]$$

So $$F=\sqrt{F_x^2+F_y^2}=F_y=\frac{2\pi GmM}{L^2}\quad [\text{as } F_x \text{ is zero}]$$

i.e., the resultant force is along the y–axis and has magnitude ($2\pi GmM/L^2$)

(b) If the rod was bent into a complete circle,

$$F_x=\frac{GmM}{LR}\int_0^{2\pi}\cos\theta\, d\theta=0 \text{ and also } F_y=\frac{GmM}{LR}\int_0^{2\pi}\sin\theta\ d\theta=0$$

i.e, the resultant force on m at O due to the ring is zero.

Ex. Find ratio of gravitational field on the surface of two planets which are of uniform mass density & have radius R_1 & R_2 if

(a) They are of same mass

(b) They are of same density

Ans. (a) $\frac{g_1}{g_2}=\frac{R_2^2}{R_1^2}$ (b) $\frac{g_1}{g_2}=\frac{R_1}{R_2}$

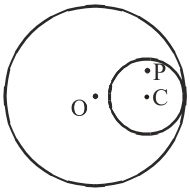
Sol. $$g=\frac{GM}{R^2}=\frac{G\frac{4}{3}\pi R^3\rho}{R^2}=\left(\frac{4\pi GR}{3}\right)\rho$$

Ex. A uniform solid sphere of density ρ and radius R has a spherical cavity of radius r inside it as shown. Find gravitation field at

(a) O

(b) C

(c) P (prove that field inside cavity is uniform)



Ans. (a) $\frac{k\left(\frac{4}{3}\pi r^3\rho\right)}{(OC)^2}\hat{CO}$ (b) $\frac{4\pi G\rho\overrightarrow{OC}}{3}$ (c) $\frac{4}{3}\pi G\rho\overrightarrow{O_1O_2}$

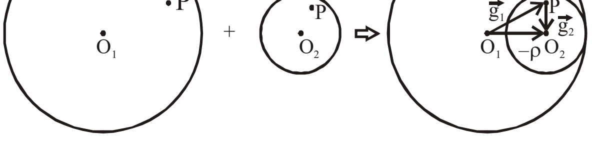
Sol. $\vec{g}$ = gravitational field at any point inside sphere

$$\vec{g}=\frac{GM}{R^3}\vec{r}$$

$$=\frac{G}{R^3}\frac{4}{3}\pi R^3\rho\vec{r}$$

$$\vec{g}=\frac{4}{3}\pi G\rho\,\vec{r}$$

Let the sphere with cavity is formed by superimposing it with a small sphere of density (–ρ) as shown



Resultant field $\vec{g} = \vec{g}_1 + \vec{g}_2$

$$= \left(\frac{4}{3}\pi G\rho\right)\overrightarrow{O_1P} + \left(\frac{4}{3}\pi G\rho\right)\overrightarrow{PO_2}$$

$$= \frac{4}{3}\pi G\rho\overrightarrow{O_1O_2} \qquad \left[\overrightarrow{O_1O_2} = \overrightarrow{OC}\right]$$

It is indepndent of position of point inside cavity

At O $\vec{g} = \vec{g}_1 + \vec{g}_2$

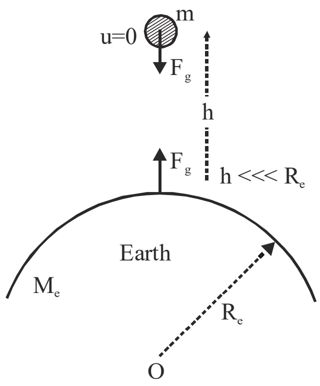
$$= 0 + \frac{GM}{\left(\overrightarrow{CO}\right)^2}\left(\widehat{CO}\right)$$

$$= \frac{G\frac{4}{3}\pi r^2\rho}{\left(\overrightarrow{CO}\right)^2}\widehat{CO}$$

$$= \frac{\left(\frac{4}{3}\pi G r^2\rho\right)}{\left(\overrightarrow{CO}\right)^2}\widehat{CO}$$

Acceleration Due to Gravity (g)

Gravitational Force F_g = ma if R_e = Radius of Earth, M_e = Mass of Earth.



then $\frac{GM_em}{R_e^{\ 2}} = ma_g \Rightarrow a_g = g = \frac{GM_e}{R_e^{\ 2}}$ $(GM_e = gR_e^{\ 2})$ ... (i)

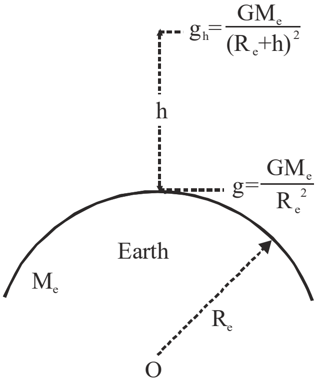
- In form of density $g = \frac{GM_e}{R_e^2} = \frac{G}{R_e^2} \times \frac{4}{3}\pi R_e^{\ 3} \times \rho$

$\therefore\ g = \frac{4}{3}\pi GR_e\rho$ .... (ii)

If ρ is constant then $g \propto R_e$

Variation in Acceleration due gravity

(a) Due to Altitude (height)



$$\frac{g_h}{g} = \frac{R_e^2}{(R_e+h)^2}$$

By binomial expansion $\left(1 + \frac{h}{R_e}\right)^{-2} \simeq \left(1 - \frac{2h}{R_e}\right)$

[If h << R_e, then higher power terms are negligible] $\therefore\ g_h = g\left[1 - \frac{2h}{R_e}\right]$

Ex. Two equal masses m and m are hung from a balance whose scale pans differ in vertical height by 'h'. Determine the error in weighing in terms of density of the Earth ρ.

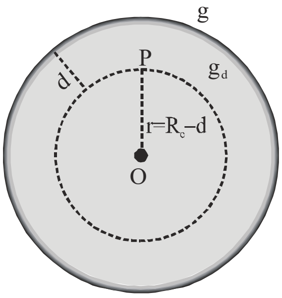
Sol. $g_h = g\left[1-\frac{2h}{R_e}\right]$, $W_2 - W_1 = mg_2 - mg_1 = 2mg\left[\frac{h_1}{R_e}-\frac{h_2}{R_e}\right] = 2m\frac{GM}{R_e^2}\times\frac{h}{R_e}$ $\left[\because g = \frac{GM}{R_e^2}\ \&\ h_1 - h_2 = h\right]$

Error in weighing = $W_2 - W_1 = 2mG\frac{4}{3}\pi R_e^3\rho\frac{h}{R_e^3} = \frac{8\pi}{3}Gm\rho h$

(b) Due to depth :

Assuming density of Earth remains same throughout. At depth d inside the Earth :

$g_d = g\left[1-\frac{d}{R_e}\right]$ valid for any depth



Decrement in g with depth $= \Delta g_d = g - g_d = g - g\left[1-\frac{d}{R_e}\right]$

$\therefore\ \frac{\Delta g_d}{g} = \frac{d}{R_e}$

Ex. At which depth from Earth surface, acceleration due to gravity is decreased by 1%

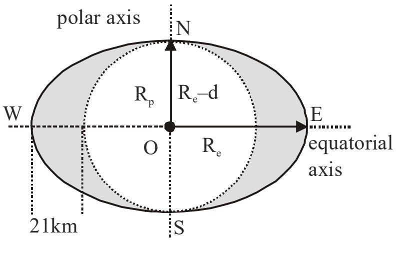
Sol. $\frac{\Delta g_d}{g} = \frac{d}{R_e} \Rightarrow \frac{1}{100} = \frac{d}{6400}\quad \therefore\quad d = 64$ km

(c) Due to shape of the Earth

$R_p < R_e$

$\therefore\ g_e < g_p$

$\Rightarrow$ by putting the values $g_p - g_e = 0.02\ m/s^2$



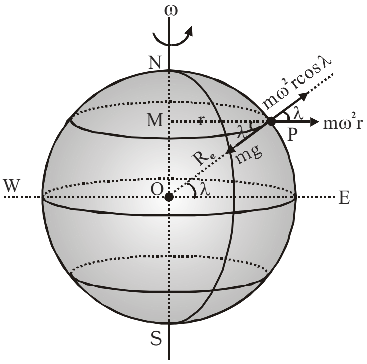
(d) Due to Rotation of the Earth

$g' = g - R_e\omega^2\cos^2\lambda$

If latitude angle $\lambda = 0$. It means at equator. $g'_{min} = g_e = g - R_e\omega^2$

If latitude angle $\lambda = 90°$. It means at poles. $g'_{max.} = g_p = g \Rightarrow g_p > g_e$

Change in "g" only due to rotation $\Delta g_{rot.} = g_p - g_e = 0.03\ m/s^2$



$$\boxed{\Delta g_{total} = g_p - g_e = (0.05\ m/s^2) \longrightarrow \begin{cases} 0.02\ m/s^2 & \text{(due to shape)} \\ 0.03\ m/s^2 & \text{(due to rotation)} \end{cases}}$$

Weightlessness

State of the free fall $\left(\vec{a} = -\dfrac{GM}{r^2}\vec{r}\right)$ is called state of weightlessness. If a body is in a satellite (which does not produce its own gravity) orbiting the Earth at a height h above its surface then

$$\text{True weight} = mg_h = \frac{mGM}{(R+h)^2} = \frac{mg}{\left(1+\dfrac{h}{R}\right)^2}$$

$$\text{Apparent weight} = m(g_h - a)$$

$$\text{but } a = \frac{v_0^2}{r} = \frac{GM}{r^2} = \frac{GM}{(R+h)^2} = g_h \Rightarrow \text{Apparent weight} = m(g_h - g_h) = 0$$

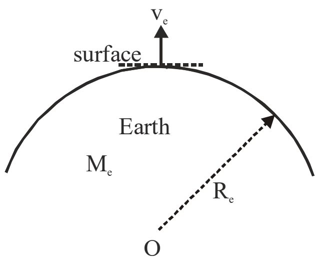
Note : The condition of weightlessness can be overcome by creating artificial gravity by rotating the satellite in addition to its revolution.

Escape speed (v_e)

Minimum speed required for an object at Earth's surface so that it just escapes the Earth's gravitational field.

Escape energy

Minimum energy given to a particle in form of kinetic energy so that it can just escape from Earth's gravitational field.



$$\text{Escape energy} = \frac{GM_e m}{R_e} \quad \text{(–ve of PE of Earth's surface)}$$

$$\text{Escape energy = Kinetic Energy} \Rightarrow \frac{GM_e m}{R_e} = \frac{1}{2}mv_e^2 \Rightarrow v_e = \sqrt{\frac{2GM_e}{R_e}}$$

- $v_e = \sqrt{\dfrac{2GM_e}{R_e}}$ (In form of mass) If M = constant $v_e \propto \dfrac{1}{\sqrt{R_e}}$
- $v_e = \sqrt{2gR_e}$ (In form of g) If g = constant $v_e \propto \sqrt{R_e}$
- $v_e = R_e\sqrt{\dfrac{8\pi G.\rho}{3}}$ (In form of density) If ρ = constant $v_e \propto R_e$
- Escape velocity does not depend on mass of body, angle of projection or direction of projection.

  $v_e \propto m^0$ and $v_e \propto \theta°$
- Escape velocity at : Earth surface v_e = 11.2 km/s Moon surface v_e = 2.31 km/s
- Atmosphere on Moon is absent because root mean square velocity of gas particle is greater then escape velocity. $v_{rms} > v_e$

Ex. A space–ship is launched into a circular orbit close to the Earth's surface. What additional speed should now be imparted to the spaceship so that orbit to overcome the gravitational pull of the Earth.

Sol. Let ΔK be the additional kinetic energy imparted to the spaceship to overcome the gravitation pull

then by energy conservation $-\frac{GMm}{2R} + \Delta K = 0 + 0 \Rightarrow \Delta K = \frac{GMm}{2R}$

Total kinetic energy $= \frac{GMm}{2R} + \Delta K = \frac{GMm}{2R} + \frac{GMm}{2R} = \frac{GMm}{R}$ then $\frac{1}{2}mv_2^2 = \frac{GMm}{R} \Rightarrow v_2 = \sqrt{\frac{2GM}{R}}$

But $v_1 = \sqrt{\frac{GM}{R}}$. So Additional velocity $= v_2 - v_1 = \sqrt{\frac{2GM}{R}} - \sqrt{\frac{GM}{R}} = \left(\sqrt{2}-1\right)\sqrt{\frac{GM}{R}}$

Ex. Find the minimum speed with which an object should be projected vertically upward from earth's surface to reach a height equal to radius of earth, R_e.

Ans. $\sqrt{\frac{GM}{R_e}}$

Sol. $-\frac{GMm}{R_e} + \frac{1}{2}mv^2 = -\frac{GMm}{2R_e}$

$\therefore \; v = \sqrt{\frac{GM}{R_e}}$

Ex. The distance between the centres of two stars is 10a. The masses of the stars are M and 16M and their radii a and 2a respectively. A body of mass m is fired straight from surface of the larger star towards smaller star.

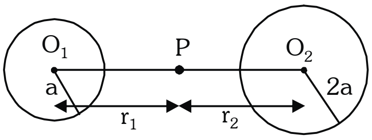
(a) Find the distance between centre of smaller star and the point of zero gravitational field strength:

Sol. P is the point where field strength is zero.

$$\frac{GM}{r_1^2} = \frac{G(16M)}{r_2^2}$$

and $r_1 + r_2 = 10a$

So, $r_1 = 2a$, $r_2 = 8a$



(b) The initial minimum speed of the body to reach smaller star is $K\sqrt{\frac{GM}{a}}$. Find the value of K:

Sol. From conservation of mechanical energy.

$\frac{1}{2}mv_{min}^2 =$ Potential energy of body at P– Potential energy of body at larger star

$$= \left[-\frac{GMm}{r_1} - \frac{16GMm}{r_2}\right] - \left[-\frac{GMm}{(10a-2a)} - \frac{16GMm}{2a}\right]$$

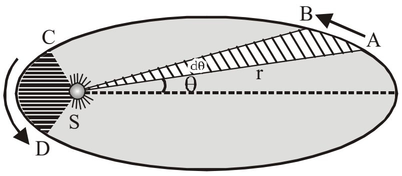
$$= \frac{3\sqrt{5}}{2}\left[\sqrt{\frac{GM}{a}}\right]$$

Kepler's Laws

Kepler found important regularities in the motion of the planets. These regularities are known as 'Kepler's three laws of planetary motion'.

(a) **First Law (Law of Orbits) :** All planets move around the Sun in elliptical orbits, having the Sun at one focus of the orbit.

(b) **Second Law (Law of Areas) :** A line joining any planet to the Sun sweeps out equal areas in equal times, that is, the areal speed of the planet remains constant.



According to the second law, when the planet is nearest the Sun, then its speed is maximum and when it is farthest from the Sun, then its speed is minimum. In figure if a planet moves from A to B in a given time–interval, and from C to D in the same time–interval, then the areas ASB and CSD will be equal.

$$\frac{dA}{dt}=\frac{J}{2m} \qquad \text{...(iii)}$$

Now, the areal speed dA/dt of the planet is constant, according to Kepler's second law. Therefore, according to eq. (iii), the angular momentum J of the planet is also constant, that is, the angular momentum of the planet is conserved. Thus, Kepler's second law is equivalent to conservation of angular momentum.

(c) **Third Law : (Law of Periods) :** The square of the period of revolution (time of one complete revolution) of any planet around the Sun is directly proportional to the cube of the semi–major axis of its elliptical orbit.

$$T^2 \propto a^3$$

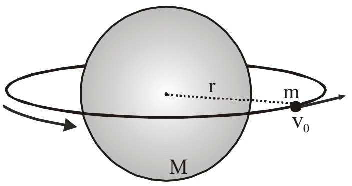
So it is clear through this rule that the farthest planet from the Sun has largest period of revolution. The period of revolution of the closest planet Mercury is 88 days, while that of the farthest dwarf planet Pluto is 248 years.

Satellite motion

A light body revolving round a heavier body due to gravitational attraction, is called satellite. Earth is a satellite of the Sun while Moon is satellite of Earth.

Orbital velocity (v_0) : A satellite of mass m moving in an orbit of radius r with speed v_0 then required centripetal force is provided by gravitation.

$$F_{cp}=F_g \Rightarrow \frac{mv_0^2}{r}=\frac{GMm}{r^2} \Rightarrow v_0=\sqrt{\frac{GM}{r}}=\sqrt{\frac{GM}{(R_e+h)}} \quad (r=R_e+h)$$



For a satellite very close to the Earth surface $h << R_e \therefore r = R_e$

$$v_0=\sqrt{\frac{GM}{R_e}}=\sqrt{gR_e}=8 \text{ km/s}$$

- If a body is taken at some height from Earth and given horizontal velocity of magnitude 8 km/sec then the body becomes satellite of Earth.
- v_o depends upon : Mass of planet, Radius of circular orbit of satellite, g (at planet), Density of planet
- If orbital velocity of a near by satellite becomes $\sqrt{2}\,v_o$ (or increased by 41.4%, or K.E. is doubled) then the satellite escapes from gravitational field of Earth.

Time Period of a Satellite $T = \dfrac{2\pi r}{v_0} = \dfrac{2\pi r^{3/2}}{\sqrt{GM}} = \dfrac{2\pi r^{3/2}}{R\sqrt{g}} \Rightarrow T^2 = \dfrac{4\pi^2}{GM} r^3 \Rightarrow T^2 \propto r^3 \, (r = R + h)$

For Geostationary Satellite $T = 24$ hr, $h = 36{,}000$ km $\simeq 6\,R_e$ $(r \simeq 7\,R_e)$, $v_0 = 3.1$ km/s

For Near by satellite $v_0 = \sqrt{\dfrac{GM_e}{R_e}} \simeq 8$ km/s

$$T_{Ns} = 2\pi\sqrt{\frac{R_e}{g}} = 84 \text{ minute} = 1 \text{ hour } 24 \text{ minute} = 1.4 \text{ hr} = 5063 \text{ s}$$

In terms of density $T_{Ns} = \dfrac{2\pi (R_e)^{1/2}}{(G \times 4/3\,\pi R_e \times \rho)^{1/2}} = \sqrt{\dfrac{3\pi}{G\rho}}$

Time period of near by satellite only depends upon density of planet.

For Moon $h_m = 380{,}000$ km and $T_m = 27$ days

$$v_{om} = \frac{2\pi(R_e + h)}{T_m} = \frac{2\pi(386400 \times 10^3)}{27 \times 24 \times 60 \times 60} \simeq 1.04 \text{ km/sec.}$$

Energies of a Satellite Kinetic energy $\quad$ $K.E. = \dfrac{1}{2} m v_0^2 = \dfrac{GMm}{2r} = \dfrac{L^2}{2mr^2}$

Potential energy $\quad$ $P.E. = -\dfrac{GMm}{r} = -m v_0^2 = -\dfrac{L^2}{mr^2}$

Total mechanical energy $T.E. = P.E. + K.E. = -\dfrac{m v_0^2}{2} = -\dfrac{GMm}{2r} = -\dfrac{L^2}{2mr^2}$

Essential Condition's for Satellite Motion

- Centre of satellite's orbit coincide with centre of Earth.
- Plane of orbit of satellite is passing through centre of Earth.

Special Points about Geo–Stationary Satellite

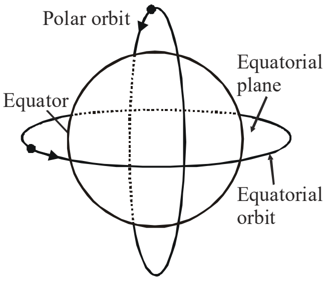
- All three essential conditions for satellite motion should be followed.
- It rotates in equatorial plane.
- Its height from Earth surface is 36000 km. ($\sim 6R_e$)
- Its angular velocity and time period should be same as that of Earth.
- Its rotating direction should be same as that of Earth (West to East).
- Its orbit is called parking orbit and its orbital velocity is 3.1 km./sec.
- Maximum latitude at which message can be recieved by geostationary satellite is

$$\theta = \cos^{-1}\left(\frac{R_e}{R_e + h}\right)$$

- The area of earth's surface covered by geostationary satellite is $S = \Omega R_e^2 = \dfrac{2\pi h R_e^2}{R_e + h}$

- **Polar Satellite (Sun – synchronous satellite)**

  It is that satellite which revolves in polar orbit around Earth. A polar orbit is that orbit whose angle of inclination with equatorial plane of Earth is 90° and a satellite in polar orbit will pass over both the north and south geographic poles once per orbit. Polar satellites are Sun–synchronous satellites. Every location on Earth lies within the observation of polar satellite twice each day. The polar satellites are used for getting the cloud images, atmospheric data, ozone layer in the atmosphere and to detect the ozone hole over Antarctica.



  Only the equatorial orbits are stable for a satellite. For any satellite to orbit around in a stable orbit, it must move in such an orbit so that the centre of Earth lies at the centre of the orbit.

Binding energy

Total mechanical energy (potential + kinetic) of a closed system is negative. The modulus of this total mechanical energy is known as the binding energy of the system. This is the energy due to which system is closed or different parts of the system are bound to each other.

Binding energy of satellite (system)

B.E. = – T.E. $\text{B.E.} = \frac{1}{2}mv_0^2 = \frac{GMm}{2r} = \frac{L^2}{2mr^2}$ Hence B.E. = K.E. = – T.E. = $\frac{-\text{P.E.}}{2}$

Work done in Changing the Orbit of Satellite

W = Change in mechanical energy of system but $E = \frac{-GMm}{2r}$ so $W = E_2 - E_1 = \frac{GMm}{2}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

Ex. A satellite moves eastwards very near the surface of the Earth in equatorial plane with speed (v_0). Another satellite moves at the same height with the same speed in the equatorial plane but westwards. If R = radius of the Earth and ω be its angular speed of the Earth about its own axis. Then find the approximate difference in the two time period as observed on the Earth.

Sol. $T_{west} = \frac{2\pi R}{v_0 + R\omega}$ and $T_{east} = \frac{2\pi R}{v_0 - R\omega} \Rightarrow \Delta T = T_{east} - T_{west} = 2\pi R\left[\frac{2R\omega}{v_0^2 - R^2\omega^2}\right] = \frac{4\pi\omega R^2}{v_0^2 - R^2\omega^2}$

Ex. An artificial satellite (mass m) of a planet (mass M) revolves in a circular orbit whose radius is n times the radius R of the planet. In the process of motion, the satellite experiences a slight resistance due to cosmic dust. Assuming the force of resistance on satellite to depend on velocity as $F = av^2$ where 'a' is a constant, calculate how long the satellite will stay in the space before it falls onto the planet's surface.

Sol. Air resistance $F = -av^2$, where orbital velocity $v = \sqrt{\frac{GM}{r}}$

r = the distance of the satellite from planet's centre $\Rightarrow F = -\frac{GMa}{r}$

The work done by the resistance force $dW = Fdx = Fvdt = \frac{GMa}{r}\sqrt{\frac{GM}{r}}dt = \frac{(GM)^{3/2}a}{r^{3/2}}dt$ ....(i)

The loss of energy of the satellite = dE $\therefore \frac{dE}{dr} = \frac{d}{dr}\left[-\frac{GM\,m}{2r}\right] = \frac{GMm}{2r^2} \Rightarrow dE = \frac{GMm}{2r^2}dr$ ...(ii)

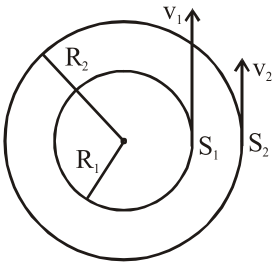
Since dE = – dW (work energy theorem) $-\frac{GMm}{2r^2}dr = \frac{(GM)^{3/2}}{r^{3/2}}dt$

$$\Rightarrow t = -\frac{m}{2a\sqrt{GM}}\int_{nR}^{R}\frac{dr}{\sqrt{r}} = \frac{m\sqrt{R}\left(\sqrt{n}-1\right)}{a\sqrt{GM}} = \left(\sqrt{n}-1\right)\frac{m}{a\sqrt{gR}}$$

Ex. Two satellites S_1 and S_2 revolve round a planet in coplanar circular orbits in the same sense. Their periods of revolution are 1 h and 8h respectively. The radius of the orbit of S_1 is 10^4 km. When S_2 is closest to S_1, find (a) the speed of S_2 relative to S_1 and (b) the angular speed of S_2 as observed by an astronaut in S_1.

Sol. Let the mass of the planet be M, that of S_1 be m_1 and S_2 be m_2. Let the radius of the orbit of S_1 be R_1(= 10^4 km) and of S_2 be R_2.

Let v_1 and v_2 be the linear speeds of S_1 and S_2 with respect to the planet. Figure shows the situation. As the square of the time period is proportional to the cube of the radius.



$$\left(\frac{R_2}{R_1}\right)^3 = \left(\frac{T_2}{T_1}\right)^2 = \left(\frac{8h}{1h}\right)^2 = 64$$

or, $\frac{R_2}{R_1} = 4$

or, $R_2 = 4R_1 = 4 \times 10^4$ m.

Now the time period of S_1 is 1 h. So,

$$\frac{2\pi R_1}{v_1} = 1h$$

or, $v_1 = \frac{2\pi R_i}{1h} = 2\pi \times 10^4 \text{ km/h}$

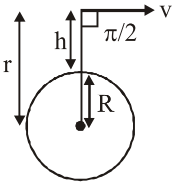
similarly, $v_2 = \frac{2\pi R_2}{8h} = \pi \times 10^4 \text{ km/h}$

(a) At the closest separation, the are moving in the same direction. Hence the speed of S_2 with respect to S_1 is $|v_2 - v_1| = \pi \times 10^4$ km/h.

(b) As seen from S_1, the satellite S_2 is a distance $R_2 - R_1 = 3 \times 10^4$ km at the closest separation. Also is moving at $\pi \times 10^4$ km/h in a direction perpendicular to the line joining them. Thus, the angular speed of S_2 as observed by S_1 is

$$\omega = \frac{\pi \times 10^4 \text{ km/h}}{3 \times 10^4 \text{ km/h}} = \frac{\pi}{3}\text{ rad/h}.$$

Conditions for different trajectroy



For a body being projected tangentially from above earth's surface, say at a distance r from earth's center, the trajectory would depend on the velocity of projection v.

Velocity	Orbit
1. velocity, $v<\sqrt{\frac{GM}{r}\left(\frac{2R}{r+R}\right)}$	Body returns to earth
2. $\sqrt{\frac{GM}{r}}>v>\sqrt{\frac{GM}{r}\left(\frac{2R}{r+R}\right)}$	Body acquires an elliptical orbit with earth as the far-focus w.r.t. the point of projection.
3. Velocity is equal to the critical velocity of the orbit, $v=\sqrt{\frac{GM_e}{r}}$	Circular orbit with radius r
4. Velocity is between the critical and escape velocity of the orbit $\sqrt{\frac{2GM_e}{r}}>v>\sqrt{\frac{GM_e}{r}}$	Body acquires an elliptical orbit with earth as the near focus w.r.t. the point of projection.
5. $v=v_{esc}=\sqrt{\frac{2GM_e}{r}}$	Body just escapes earth's gravity, along a parabolic path.
6. $v>v_{esc}=\sqrt{\frac{2GM_e}{r}}$	Body escape earth's gravity along a hyperbolic path.

Binary star system

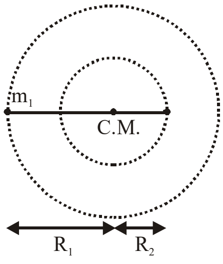
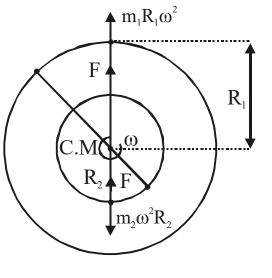


Figure shows two particles moving due to mutually attractive gravitational force about center of mass. Since there is no external force CM of system remains fixed and time period of revolution must be same.

Both bodies have comparable mass and both are moving in circular orbit centre of mass as shown in diagram

$$\omega=\sqrt{\frac{G\left(m_1+m_2\right)}{R^3}}$$



Angular momentum of the system about centre of mass.

$$L=\left(\frac{m_1m_2}{m_1+m_2}\right)R^2\omega$$

$$\text{Kinetic energy}=\frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)R^2\omega^2$$

The time period of a simple pendulum of infinite length.

In deriving the formula $T_0 = 2\pi\sqrt{\left(\frac{L}{g}\right)}$ we have assumed that length of the pendulum L is much less than the radius of the earth R so that 'g' always remains vertical. However, if length of pendulum is comparable of the radius of earth, 'g' will not remain vertical but will be directed towards the centre of the earth.

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{1}{g\left[\frac{1}{L}+\frac{1}{R}\right]}} \qquad (< T_0)$$

From this expression it is clear that :

(a) If L << R, (1/L) >> (1/R) so T = $2\pi\sqrt{\left(\frac{L}{g}\right)}$ which is expected.

(b) If L >> R (→ ∞) (1/L) << (1/R) so

$$T = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{6.4\times10^6}{10}}$$

= 800 × 2π sec ≈ 83.8 minute

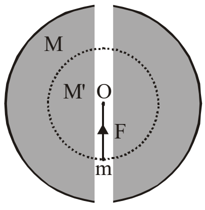
And it is also the maximum time period which an oscillating simple pendulum can have.

(c) If L is comparable to R (say L = R),

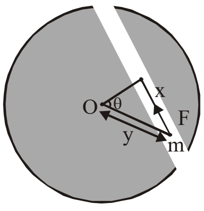
$$T = 2\pi\sqrt{\frac{R}{2g}} \simeq 1\text{hour}.$$

Motion of a ball in a tunnel through the earth :

Case I : If the tunnel is along a diameter and the ball is released from the suface. The ball executes SHM.



(A)



(B)

so that $T = 2\pi\sqrt{\frac{R^3}{GM}}$; $T = 2\pi\sqrt{\left(\frac{R}{g}\right)}$

Which is same as that of a simple pendulum of infinite length and is equal to 84.6 minute.

Case II : If the tunnel is along a chord and ball is released from the suface. The motion is SHM with he same time period.

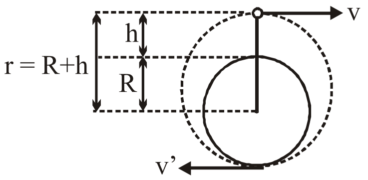
Gravitaional pressure :

A uniform sphere has a mass M and radius R. The pressure p inside the sphere, caused by gravitational compression, as a function of the distance r from its centre can be found to be $p = \frac{3}{8}\,(1 - r^2/R^2)\,\gamma M^2/\pi R^4$.

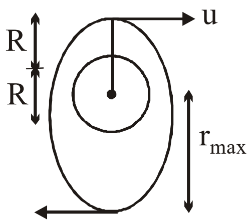
Launching of an artificial satellite around earth

Ex. A satellite is launched tangentially from a height h above earth's surface as shown.

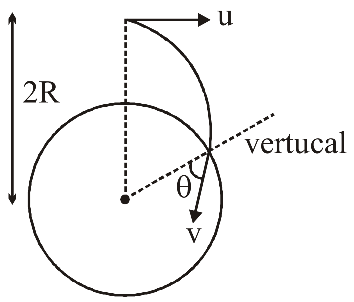
I. Find minimum launch speed 'v' so that it just touches the earth's surface



II. If h = R and satellite is launched tangentially with speed = $\sqrt{\frac{3GM}{5R}}$ find the maximum distance of satellite from earth's center



III. If h = R and satellite is launched tangentially with a speed $u = \sqrt{\frac{GM}{7R}}$. Find the angle w.r.t. vertical at which the satellite will crash on earth's surface.



Sol.

(I) Angular momentum conservation

$$mv(R+h) = mv'R$$

Energy conservation

$$\frac{-GMm}{R+h} + \frac{1}{2}mv^2 = -\frac{GMm}{R} + \frac{1}{2}mv'^2$$

Solving , $v = \sqrt{\frac{2GMR}{r(R+r)}}$

(II) Angular momentum conservation $mu \cdot 2R = mvr_{max}$.

Energy conservation :

$$\frac{-GMm}{2R} + \frac{1}{2}mu^2 = \frac{-GMm}{r_{max}} + \frac{1}{2}mv^2$$

$$\Rightarrow \; GMm\left(\frac{1}{2R} - \frac{1}{r_{max}}\right) = \frac{1}{2}mu^2\left(1 - \left(\frac{2R}{r_{max}}\right)^2\right)$$

$$\Rightarrow \; GMm\left(\frac{1}{2R} - \frac{1}{r_{max}}\right) = \frac{3GM}{10R}\left(1 - \frac{4R^2}{r_{max}^2}\right)$$

$$\Rightarrow \; 2r_{max}^2 - 10Rr_{max} + 12R^2 = 0$$

$$\Rightarrow \; (r_{max} - 2R)(r_{max} - 3R) = 0$$

$$\Rightarrow \; r_{max} = 3R$$

(III) Energy conservation

$$\frac{1}{2}mu^2 - \frac{GMm}{2R} = \frac{1}{2}mv^2 - \frac{GMm}{R}$$

$$\Rightarrow \; \frac{1}{2}m(v^2 - u^2) = \frac{GMm}{2R}$$

$$v^2 = u^2 + \frac{GM}{R} \Rightarrow v = \sqrt{\frac{8GM}{7R}}$$

Angular momentum conservation

$mu \cdot 2R = mvR\sin\theta$

$$\Rightarrow \sin\theta = \frac{2u}{v}$$

$$\Rightarrow \theta = 45°$$

EXERCISE (S-1)

1. A particle is fired vertically from the surface of the earth with a velocity $k\upsilon_e$, where υ_e is the escape velocity and $k < 1$. Neglecting air resistance and assuming earth's radius as R_e. Calculate the height to which it will rise from the surface of the earth.

GR0001

2. Calculate the distance from the surface of the earth at which above and below the surface acceleration due to gravity is the same.

GR0002

3. An object is projected vertically upward from the surface of the earth of mass M with a velocity such that the maximum height reached is eight times the radius R of the earth. Calculate:
(i) the initial speed of projection
(ii) the speed at half the maximum height.

GR0003

4. A satellite close to the earth is in orbit above the equator with a period of rotation of 1.5 hours. If it is above a point P on the equator at some time, it will be above P again after time________.

GR0004

5. A satellite is moving in a circular orbit around the earth. The total energy of the satellite is $E = -2\times10^5$J. The amount of energy to be imparted to the satellite to transfer it to a circular orbit where its potential energy is $U = -2\times10^5$J is equal to ________.

GR0005

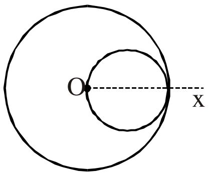
6. A satellite of mass m is orbiting the earth in a circular orbit of radius r. It starts losing energy due to small air resistance at the rate of C J/ s. Then the time taken for the satellite to reach the earth is _______.

GR0006

7. A pair of stars rotates about a common center of mass. One of the stars has a mass M which is twice as large as the mass m of the other. Their centres are at a distance d apart, d being large compared to the size of either star. (a) Derive an expression for the period of rotation of the stars about their common centre of mass in terms of d,m, G. (b) Compare the angular momentum of the two stars about their common centre of mass by calculating the ratio L_m/L_M. (c) Compare the kinetic energies of the two stars by calculating the ratio K_m/K_M.

GR0007

8. A sphere of radius R has its centre at the origin. It has a uniform mass density ρ_0 except that there is a spherical hole of radius $r = R/2$ whose centre is at $x = R/2$ as in fig. (a) Find gravitational field at points on the axis for $|x| > R$ (b) Show that the gravitational field inside the hole is uniform, find its magnitude and direction.



GR0008

EXERCISE (S-2)

1. A small body of mass m is projected with a velocity just sufficient to make it reach from the surface of a planet (of radius 2R and mass 3M) to the surface of another planet (of radius R and mass M). The distance between the centers of the two spherical planets is 6R. The distance of the body from the center of bigger planet is 'x' at any moment. During the journey, find the distance x where the speed of the body is (a) maximum (b) minimum. Assume motion of body along the line joining centres of planets.

GR0009

2. A body is launched from the earth's surface at an angle $\alpha=30^{o}$ to the horizontal at a speed $v_0=\sqrt{\frac{1.5GM}{R}}$. Neglecting air resistance and earth's rotation, find (a) the height to which the body will rise. (b) the radius of curvature of trajectory at its top point.

GR0010

3. A body moving radially away from a planet of mass M, when at distance r from planet, explodes in such a way that two of its many fragments move in mutually perpendicular circular orbits around the planet. What will be (a) then velocity in circular orbits. (b) maximum distance between the two fragments before collision and (c) magnitude of their relative velocity just before they collide.

GR0011

4. A cord of length 64 m is used to connect a 100 kg astronaut to spaceship whose mass is much larger than that of the astronaut. Estimate the value of the tension in the cord. Assume that the spaceship is orbiting near earth surface. Assume that the spaceship and the astronaut fall on a straight line from the earth centre. The radius of the earth is 6400 km.

GR0012

5. A hypothetical spherical planet of radius R and its density varies as $\rho = Kr$, where K is constant and r is the distance from the center. Determine the pressure caused by gravitational pull inside ($r < R$) the planet at a distance r measured from its center.

GR0013

6. The Earth may be regarded as a spherically shaped uniform core of density ρ_1 and radius R/2 surrounded by a uniform shell of thickness R/2 and density ρ_2. Find the ratio of $\frac{\rho_1}{\rho_2}$ if the value of acceleration due to gravity is the same at surface as at depth R/2 from the surface.

GR0014

7. A binary star has a period (T) of 2 earth years while distance L between its components having masses M_1 and M_2 is four astronomical units. If $M_1 = M_S$ where M_S is the mass of sun, find the ratio $M_2/5M_S$.

GR0015

8. Two uniform spherical stars made of same material have radii R and 2R. Mass of the smaller planet is m. They start moving from rest towards each other from a large distance under mutual force of gravity. The collision between the stars is inelastic with coefficient of restitution 1/2.
(a) Find the kinetic energy of the system just after the collision.
(b) Find the maximum separation between their centres after their first collision.

GR0016

9. A remote sensing satellite is revolving in an orbit of radius x over the equator of earth. Find the area on earth surface in which satellite can not send message.

GR0017

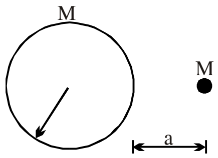
EXERCISE (O-1)

1. If the distance between the centres of Earth and Moon is D and mass of Earth is 81 times that of Moon. At what distance from the centre of Earth gravitational field will be zero?

(A) $\frac{D}{2}$ (B) $\frac{2D}{3}$ (C) $\frac{4D}{5}$ (D) $\frac{9D}{10}$

GR0018

2. A particle of mass M is at a distance a from surface of a thin spherical shell of equal mass and having radius a.



(A) Gravitational field and potential both are zero at centre of the shell.

(B) Gravitational field is zero not only inside the shell but at a point outside the shell also.

(C) Inside the shell, gravitational field alone is zero.

(D) Neither gravitational field nor gravitational potential is zero inside the shell.

GR0019

3. A hollow spherical shell is compressed to half its radius. The gravitational potential at the centre

(A) increases (B) decreases

(C) remains same (D) during the compression increases then returns at the previous value.

GR0020

4. Let ω be the angular velocity of the earth's rotation about its axis. Assume that the acceleration due to gravity on the earth's surface has the same value at the equator and the poles in absence of rotation of earth. An object weighed at the equator gives the same reading as a reading taken at a depth d below earth's surface at a pole (d<<R) The value of d is

(A) $\frac{\omega^2 R^2}{g}$ (B) $\frac{\omega^2 R^2}{2g}$ (C) $\frac{2\omega^2 R^2}{g}$ (D) $\frac{\sqrt{Rg}}{g}$

GR0021

5. If the radius of the earth be increased by a factor of 5, by what factor its density be changed to keep the value of g the same?

(A) 1/25 (B) 1/5 (C) $1/\sqrt{5}$ (D) 5

GR0022

6. The mass and diameter of a planet are twice those of earth. What will be the period of oscillation of a pendulum on this planet if it is a seconds pendulum on earth?

(A) $\sqrt{2}$ second (B) $2\sqrt{2}$ seconds (C) $\frac{1}{\sqrt{2}}$ second (D) $\frac{1}{2\sqrt{2}}$ second

GR0023

7. Two identical satellites are at the heights R and 7R from the Earth's surface. Then which of the following statement is incorrect. (R = radius of the Earth)

(A) Ratio of total energy of both is 5

(B) Ratio of kinetic energy of both is 4

(C) Ratio of potential energy of both 4

(D) Ratio of total energy of both is 4 and ratio of magnitude of potential to kinetic energy is 2

GR0024

8. A spherical uniform planet is rotating about its axis. The velocity of a point on its equator is V. Due to the rotation of planet about its axis the acceleration due to gravity g at equator is 1/2 of g at poles. The escape velocity of a particle on the pole of planet in terms of V is

(A) $V_e = 2V$ (B) $V_e = V$ (C) $V_e = V/2$ (D) $V_e = \sqrt{3}\,V$

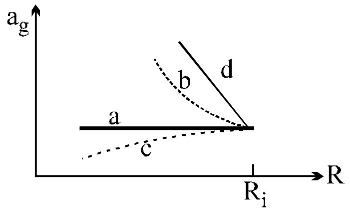
GR0025

9. The escape velocity for a planet is v_e. A tunnel is dug along a diameter of the planet and a small body is dropped into it at the surface. When the body reaches the centre of the planet, its speed will be

(A) v_e (B) $\frac{v_e}{\sqrt{2}}$ (C) $\frac{v_e}{2}$ (D) 0

GR0026

10. A (nonrotating) star collapses onto itself from an initial radius R_i with its mass remaining unchanged. Which curve in figure best gives the gravitational acceleration a_g on the surface of the star as a function of the radius of the star during the collapse?



(A) a (B) b (C) c (D) d

GR0027

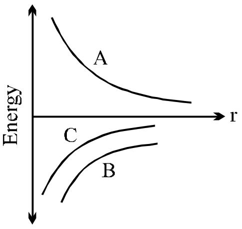
11. A satellite of mass m, initially at rest on the earth, is launched into a circular orbit at a height equal to the radius of the earth. The minimum energy required is

(A) $\frac{\sqrt{3}}{4}$ mgR (B) $\frac{1}{2}$ mgR

(C) $\frac{1}{4}$ mgR (D) $\frac{3}{4}$ mgR

GR0028

12. The figure shows the variation of energy with the orbit radius of a body in circular planetary motion. Find the correct statement about the curves A, B and C



(A) A shows the kinetic energy, B the total energy and C the potential energy of the system.

(B) C shows the total energy, B the kinetic energy and A the potential energy of the system.

(C) C and A are kinetic and potential energies respectively and B is the total energy of the system.

(D) A and B are kinetic and potential energies and C is the total energy of the system.

GR0029

13. A satellite of mass 5M orbits the earth in a circular orbit. At one point in its orbit, the satellite explodes into two pieces, one of mass M and the other of mass 4M. After the explosion the mass M ends up travelling in the same circular orbit, but in opposite direction. After explosion the mass 4M is :-

(A) In a circular orbit

(B) unbound

(C) elliptical orbit

(D) data is insufficient to determine the nature of the orbit.

GR0030

14. A satellite can be in a geostationary orbit around earth at a distance r from the centre. If the angular velocity of earth about its axis doubles, a satellite can now be in a geostationary orbit around earth if its distance from the centre is :-

(A) $\frac{r}{2}$ (B) $\frac{r}{2\sqrt{2}}$ (C) $\frac{r}{(4)^{1/3}}$ (D) $\frac{r}{(2)^{1/3}}$

GR0031

15. An earth satellite is moved from one stable circular orbit to another larger and stable circular orbit. The following quantities increase for the satellite as a result of this change:-

(A) gravitational potential energy (B) angular velocity

(C) linear orbital velocity (D) centripetal acceleration

GR0032

16. Satellites A and B are orbiting around the earth in orbits of ratio R and 4R respectively. The ratio of their areal velocities is :

(A) 1 : 2 (B) 1 : 4 (C) 1 : 8 (D) 1 : 16

GR0033

17. The fractional change in the value of free–fall acceleration g for a particle when it is lifted from the surface to an elevation h (h<<R) is

(A) $\frac{h}{R}$ (B) $\frac{2h}{R}$ (C) $-\frac{2h}{R}$ (D) $-\frac{h}{R}$

GR0034

18. If suddenly the gravitational force of attraction between earth and a satellite revolving around it becomes zero, then the satellite will- **[AIEEE-2002]**

(A) continue to move in its orbit with same velocity

(B) move tangentially to the original orbit with same velocity

(C) become stationary in its orbit

(D) move towards the earth

GR0035

19. The time period of a satellite of earth is 5 hours. If the separation between the centre of earth and the satellite is increased to 4 times the previous value, the new time period will become- **[AIEEE-2003]**

(A) 10 h (B) 80 h (C) 40 h (D) 20 h

GR0036

20. A communications Earth satellite

(A) goes round the earth from east to west

(B) can be in the equatorial plane only

(C) can be vertically above any place on the earth

(D) goes round the earth from west to east

GR0037

21. If a satellite orbits as close to the earth's surface as possible,

(A) its speed is maximum

(B) time period of its rotation is minimum

(C) the total energy of the 'earth plus satellite' system is minimum

(D) the total energy of the 'earth plus satellite'system is maximum

GR0038

22. For a satellite to orbit around the earth, which of the following must be true?

(A) It must be above the equator at some time

(B) It cannot pass over the poles at any time

(C) Its height above the surface cannot exceed 36,000 km

(D) Its period of rotation must be > $2\pi\sqrt{R/g}$ where R is radius of earth

GR0039

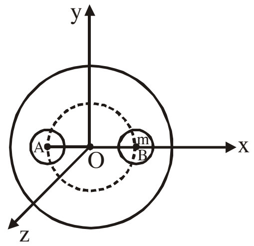
23. In elliptical orbit of a planet, as the planet moves from apogee position to perigee position,

	Column–I		**Column–II**
(A)	Speed of planet	(P)	Remains same
(B)	Distance of planet from centre of Sun	(Q)	Decreases
(C)	Potential energy	(R)	Increases
(D)	Angular momentum about centre of Sun	(S)	Can not say

GR0040

EXERCISE (O-2)

1. A solid sphere of uniform density and radius 4 units is located with its centre at the origin O of coordinates. Two spheres of equal radii 1 unit, with their centres at A (–2, 0, 0) and B (2, 0, 0) respectively, are taken out of the solid leaving behind spherical cavities as shown in figure. Then



(A) The gravitational field due to this object at the origin is zero

(B) The gravitational field at the point B (2, 0, 0) is zero

(C) The gravitational potential is the same at all points of circle $y^2 + z^2 = 36$

(D) The gravitational potential is the same at all points on the circle $y^2 + z^2 = 4$

GR0041

2. A planet of mass m is in an elliptical orbit about the sun ($m << M_{sun}$) with an orbital period T. If A be the area of orbit, then its angular momentum would be :

(A) $\frac{2mA}{T}$ (B) mAT (C) $\frac{mA}{2T}$ (D) 2mAT

GR0042

3. A planet revolves about the sun in elliptical orbit. The arial velocity $\left(\frac{dA}{dt}\right)$ of the planet is 4.0×10^{16} m²/s. The least distance between planet and the sun is 2×10^{12} m. Then the maximum speed of the planet in km/s is :

(A) 10 (B) 20 (C) 40 (D) None of these

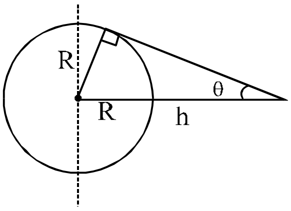
GR0043

4. The Sun travels in approximately circular orbit of radius R around the center of the galaxy and completes one revolution in time T. The Earth also revolves around the Sun in time t. Assume orbit of the Earth to be a circle of radius r (r << R) and whole mass of the galaxy centered on its center. By using only these given informations, find an expression for the ratio of the mass of the galaxy to that of the Sun.

(A) $\left(\frac{R}{r}\right)^3\left(\frac{t}{T}\right)^2$ (B) $\left(\frac{R}{r}\right)^3\left(\frac{T}{t}\right)^2$ (C) $\left(\frac{R}{r}\right)^2\left(\frac{t}{T}\right)^3$ (D) $\left(\frac{R}{r}\right)^2\left(\frac{T}{t}\right)^3$

GR0044

5. A geostationary satellite is at a height h above the surface of earth. If earth radius is R



(A) The minimum colatitude on earth upto which the satellite can be used for communication is $\sin^{-1}\left(R/R+h\right)$.

(B) The maximum latitudes on earth upto which the satellite can be used for communication is $\cos^{-1}\left(R/R+h\right)$.

(C) The area on earth escaped from this satellite is given as $2\pi R^2\,(1 + \sin\theta)$

(D) The area on earth escaped from this satellite is given as $2\pi R^2\,(1 + \cos\theta)$

GR0045

6. When a satellite in a circular orbit around the earth enters the atmospheric region, it encounters small air resistance to its motion. Then

(A) its kinetic energy increases

(B) its kinetic energy decreases

(C) its angular momentum about the earth decreases

(D) its period of revolution around the earth increases

GR0046

7. Two satellites s_1 & s_2 of equal masses revolve in the same sense around a heavy planet in coplanar circular orbit of radii R & 4R

(A) the ratio of period of revolution of s_1 & s_2 is 1 : 8.

(B) their velocities are in the ratio 2 : 1

(C) their angular momentum about the planet are in the ratio 2 : 1

(D) the ratio of angular velocities of s_2 w.r.t. s_1 when all three are in the same line is 9 : 5.

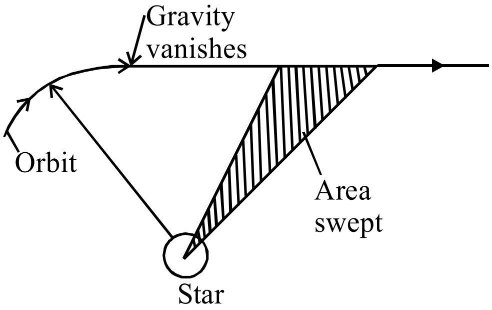
GR0047

8. A double star is a system of two stars of masses m and 2m, rotating about their centre of mass only under their mutual gravitational attraction. If r is the separation between these two stars then their time period of rotation about their centre of mass will be proportional to :

(A) $r^{3/2}$ (B) r (C) $m^{1/2}$ (D) $m^{-1/2}$

GR0048

9. A planet is orbiting a star when for no apparent reason the star's gravity suddenly vanishes. After which planet moves in a straight line. Mark the **CORRECT** statement(s) :



(A) Newton's first law is obeyed on planet after gravity vanishes

(B) Kepler's law of areas is obeyed only till the planet is in gravity of star

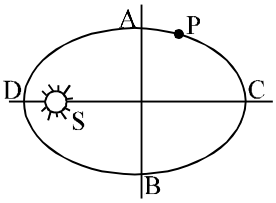
(C) Kepler's law of areas is obeyed even after gravity vanishes

(D) Angular momentum of planet about centre of star is conserved through out its motion

GR0049

Paragraph for Question No. 10 and 11

Figure shows the orbit of a planet P around the sun S. AB and CD are the minor and major axes of the ellipse.



10. If t_1 is the time taken by the planet to travel along ACB and t_2 the time along BDA, then

(A) $t_1 = t_2$ (B) $t_1 > t_2$

(C) $t_1 < t_2$ (D) nothing can be concluded

GR0050

11. If U is the potential energy and K kinetic energy then $|U| > |K|$ at

(A) Only D (B) Only C (C) both D & C (D) neither D nor C

GR0050

EXERCISE (JM)

1. Two bodies of masses m and 4m are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is :- **[AIEEE - 2011]**

(1) $-\frac{6Gm}{r}$ (2) $-\frac{9Gm}{r}$ (3) zero (4) $-\frac{4Gm}{r}$

GR0051

2. Two particles of equal mass 'm' go around a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle with respect to their centre of mass is:- **[AIEEE-2011]**

(1) $\sqrt{\frac{Gm}{R}}$ (2) $\sqrt{\frac{Gm}{4R}}$ (3) $\sqrt{\frac{Gm}{3R}}$ (4) $\sqrt{\frac{Gm}{2R}}$

GR0052

3. The mass of a spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of 'g' and 'R' (radius of earth) are 10 m/s^2 and 6400 km respectively. The required energy for this work will be :- **[AIEEE-2012]**

(1) 6.4×10^{10} Joules (2) 6.4×10^{11} Joules (3) 6.4×10^{8} Joules (4) 6.4×10^{9} Joules

GR0053

4. What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of 2R ? **[JEE-Main 2013]**

(1) $\frac{5GmM}{6R}$ (2) $\frac{2GmM}{3R}$ (3) $\frac{GmM}{2R}$ (4) $\frac{GmM}{3R}$

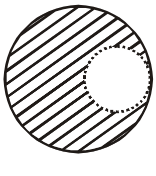
GR0054

5. Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is : **[JEE-Main 2014]**

(1) $\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$ (2) $\frac{1}{2}\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$ (3) $\sqrt{\frac{GM}{R}}$ (4) $\sqrt{2\sqrt{2}\frac{GM}{R}}$

GR0055

6. From a solid sphere of mass M and radius R, a spherical portion of radius $\frac{R}{2}$ is removed, as shown in the figure. Taking gravitational potential V = 0 at r = ∞, the potential at the centre of the cavity thus formed is : (G = *gravitational constant*)



(1) $\frac{-2GM}{3R}$ (2) $\frac{-2GM}{R}$ (3) $\frac{-GM}{2R}$ (4) $\frac{-GM}{R}$

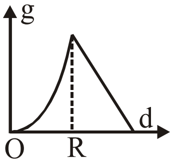
GR0056

7. A satellite is reolving in a circular orbit at a height 'h' from the earth's surface (radius of earth R ; h << R). The minimum increase in its orbital velocity required, so that the satellite could escape from the earth's gravitational field, is close to : (Neglect the effect of atmosphere). **[JEE-Main 2016]**

(1) $\sqrt{gR}\left(\sqrt{2}-1\right)$ (2) $\sqrt{2gR}$ (3) $\sqrt{gR}$ (4) $\sqrt{gR/2}$

GR0057

8. The variation of acceleration due to gravity g with distance d from centre of the earth is best represented by (R = Earth's radius) :- **[JEE-Main 2017]**

(1)


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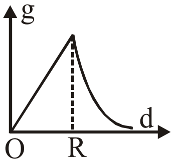

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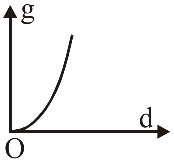

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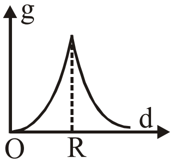


GR0058

SELECTED PROBLEMS FROM JEE-MAINS ONLINE PAPERS

9. Two stars of masses 3×10^{31} kg each, and at distance 2×10^{11}m rotate in a plane about their common centre of mass O. A meteorite passes through O moving perpendicular to the star's rotation plane. In order to escape from the gravitational field of this double star, the minimum speed that meteorite should have at O is : (Take Gravitational constant $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}) **[JEE-Main-2019_Jan]**

(1) 1.4×10^5 m/s (2) 24×10^4 m/s (3) 3.8×10^4 m/s (4) 2.8×10^5 m/s

GR0096

10. A satellite is moving with a constant speed v in circular orbit around the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of ejection, the kinetic energy of the object is : **[JEE-Main-2019_Jan]**

(1) $\frac{3}{2}mv^2$ (2) mv^2 (3) $2mv^2$ (4) $\frac{1}{2}mv^2$

GR0097

11. A straight rod of length L extends from x = a to x = L + a. The gravitational force is exerts on a point mass 'm' at x = 0, if the mass per unit length of the rod is $A + Bx^2$, is given by: **[JEE-Main-2019_Jan]**

(1) $Gm\left[A\left(\frac{1}{a+L}-\frac{1}{a}\right)-BL\right]$ (2) $Gm\left[A\left(\frac{1}{a}-\frac{1}{a+L}\right)+BL\right]$

(3) $Gm\left[A\left(\frac{1}{a+L}-\frac{1}{a}\right)+BL\right]$ (4) $Gm\left[A\left(\frac{1}{a}-\frac{1}{a+L}\right)-BL\right]$

GR0098

12. A satellite of mass M is in a circular orbit of radius R about the centre of the earth. A meteorite of the same mass, falling towards the earth, collides with the satellite completely inelastically. The speeds of the satellite and the meteorite are the same, just before the collision. The subsequent motion of the combined body will be : **[JEE-Main-2019_Jan]**

(1) in a circular orbit of a different radius
(2) in the same circular orbit of radius R
(3) in an elliptical orbit
(4) such that it escapes to infinity

GR0099

13. A rocket has to be launched from earth in such a way that it never returns. If E is the minimum energy delivered by the rocket launcher, what should be the minimum energy that the launcher should have if the same rocket is to be launched from the surface of the moon ? Assume that the density of the earth and the moon are equal and that the earth's volume is 64 times the volume of the moon :-

[JEE-Main-2019_April]

(1) $\frac{E}{4}$ (2) $\frac{E}{16}$ (3) $\frac{E}{32}$ (4) $\frac{E}{64}$

GR0100

14. A test particle is moving in a circular orbit in the gravitational field produced by a mass density $\rho(r) = \frac{K}{r^2}$. Identify the correct relation between the radius R of the particle's orbit and its period T :

[JEE-Main-2019_April]

(1) T/R^2 is a constant (2) TR is a constant (3) T^2/R^3 is a constant (4) T/R is a constant

GR0101

15. A spaceship orbits around a planet at a height of 20 km from its surface. Assuming that only gravitational field of the planet acts on the spaceship, what will be the number of complete revolutions made by the spaceship in 24 hours around the planet ? [Given : Mass of planet = 8×10^{22} kg ; Radius of planet = 2×10^6 m, Gravitational constant $G = 6.67 \times 10^{-11}$ Nm^2/kg^2] **[JEE-Main-2019_April]**

(1) 9 (2) 11 (3) 13 (4) 17

GR0102

16. A satellite of mass m is launched vertically upwards with an initial speed u from the surface of the earth. After it reaches height R (R = radius of the earth), it ejects a rocket of mass $\frac{m}{10}$ so that subsequently the satellite moves in a circular orbit. The kinetic energy of the rocket is (G is the gravitational constant; M is the mass of the earth): **[JEE-Main-2020_Jan]**

(1) $\frac{m}{20}\left(u-\sqrt{\frac{2GM}{3R}}\right)^2$
(2) $5m\left(u^2-\frac{119}{200}\frac{GM}{R}\right)$

(3) $\frac{3m}{8}\left(u+\sqrt{\frac{5GM}{6R}}\right)^2$
(4) $\frac{m}{20}\left(u^2+\frac{113}{200}\frac{GM}{R}\right)$

GR0103

17. A body A of mass m is moving in a circular orbit of radius R about a planet. Another body B of mass $\frac{m}{2}$ collides with A with a velocity which is half $\left(\frac{\vec{v}}{2}\right)$ the instantaneous velocity $\vec{v}$ of A. The collision is completely inelastic. Then, the combined body : **[JEE-Main-2020_Jan]**

(1) starts moving in an elliptical orbit around the planet.

(2) continues to move in a circular orbit

(3) Falls vertically downwards towards the planet

(4) Escapes from the Planet's Gravitational field.

GR0104

18. The mass density of a spherical galaxy varies as $\frac{K}{r}$ over a large distance 'r' from its centre. In that region, a small star is in a circular orbit of radius R. Then the period of revolution, T depends on R as: **[JEE-Main-2020_Sep]**

(1) $T \propto R$ (2) $T^2 \propto \frac{1}{R^3}$ (3) $T^2 \propto R$ (4) $T^2 \propto R^3$

GR0105

19. A satellite is moving in a low nearly circular orbit around the earth. Its radius is roughly equal to that of the earth's radius R_e. By firing rockets attached to it, its speed is instantaneously increased in the direction of its motion so that is become $\sqrt{\frac{3}{2}}$ times larger. Due to this the farthest distance from the centre of the earth that the satellite reaches is R, value of R is : **[JEE-Main-2020_Sep]**

(1) $4R_e$ (2) $3R_e$ (3) $2R_e$ (4) $2.5R_e$

GR0106

20. On the x-axis and a dsitance x from the origin, the gravitational field due to a mass distribution is given by $\frac{Ax}{(x^2+a^2)^{3/2}}$ in the x-direction. The magnitude of gravitational potential on the x-axis at a distance x, taking its value to be zero at infinity, is : **[JEE-Main-2020_Sep]**

(1) $\frac{A}{(x^2+a^2)^{1/2}}$ (2) $\frac{A}{(x^2+a^2)^{3/2}}$ (3) $A(x^2+a^2)^{3/2}$ (4) $A(x^2+a^2)^{1/2}$

GR0107

EXERCISE (JA)

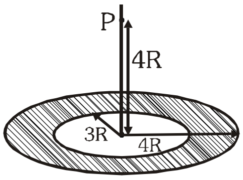
1. Gravitational acceleration on the surface of a planet is $\frac{\sqrt{6}}{11}g$, where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is $\frac{2}{3}$ times that of the Earth. If the escape speed on the surface of the earth is taken to be 11 kms^{-1}, the escape speed on the surface of the planet in kms^{-1} will be **[IIT-JEE 2010]**

GR0059

2. A binary star consists of two stars A (mass 2.2 M_S) and B (mass 11 M_S), where M_S is the mass of the sun. They are separated by distance d and are rotating about their centre of mass, which is stationary. The ratio of the total angular momentum of the binary star to the angular momentum of star B about the centre of mass is :- **[IIT-JEE 2010]**

GR0060

3. A thin uniform annular disc (see figure) of mass M has outer radius 4R and inner radius 3R. The work required to take a unit mass from point P on its axis to infinity is **[IIT-JEE 2010]**



(A) $\frac{2GM}{7R}\left(4\sqrt{2}-5\right)$ (B) $-\frac{2GM}{7R}\left(4\sqrt{2}-5\right)$ (C) $\frac{GM}{4R}$ (D) $\frac{2GM}{5R}\left(\sqrt{2}-1\right)$

GR0061

4. A satellite is moving with a constant speed V in a circular orbit about the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection, the kinetic energy of the object is :- **[IIT-JEE 2011]**

(A) $\frac{1}{2}mV^2$ (B) mV^2 (C) $\frac{3}{2}mV^2$ (D) $2mV^2$

GR0062

5. Two spherical planets P and Q have the same uniform density ρ, masses M_P and M_Q, and surface areas A and 4A, respectively. A spherical planet R also has uniform density ρ and its mass is $(M_P + M_Q)$. The escape velocities from the planets P, Q and R, are V_P, V_Q and V_R, respectively. Then **[IIT-JEE 2012]**

(A) $V_Q > V_R > V_P$ (B) $V_R > V_Q > V_P$ (C) $V_R/V_P = 3$ (D) $V_P/V_Q = 1/2$

GR0063

6. A planet of radius $R = \frac{1}{10} \times$ (radius of Earth) has the same mass density as Earth. Scientists dig a well of depth $\frac{R}{5}$ on it and lower a wire of the same length and of linear mass density 10^{-3} kgm^{-1} into it. If the wire is not touching anywhere, the force applied at the top of the wire by a person holding it in place is (take the radius of Earth = 6×10^6 m and the acceleration due to gravity on Earth is 10 ms^{-2}) **[JEE-Advance 2014]**

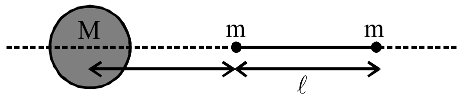
(A) 96 N (B) 108 N (C) 120 N (D) 150 N

GR0064

7. A bullet is fired vertically upwards with velocity v from the surface of a spherical planet. When it reaches its maximum height, its acceleration due to the planet's gravity is 1/4th of its value at the surface of the planet. If the escape velocity from the planet is $v_{esc} = v\sqrt{N}$, then the value of N is (ignore energy loss due to atmosphere) **[JEE-Advance 2015]**

GR0065

8. A large spherical mass M is fixed at one position and two identical point masses m are kept on a line passing through the centre of M (see figure). The point masses are connected by a rigid massless rod of length ℓ and this assembly is free to move along the line connecting them. All three masses interact only through their mutual gravitational interaction. When the point mass nearer to M is at a distance $r = 3\ell$ from M, the tension in the rod is zero for $m = k\left(\frac{M}{288}\right)$. The value of k is : **[JEE-Advance 2015]**



GR0066

9. A spherical body of radius R consists of a fluid of constant density and is in equilibrium under its own gravity. If P(r) is the pressure at r(r < R), then the correct option(s) is(are) :- **[JEE-Advance 2015]**

(A) $P(r = 0) = 0$ (B) $\frac{P(r = 3R/4)}{P(r = 2R/3)} = \frac{63}{80}$

(C) $\frac{P(r = 3R/5)}{P(r = 2R/5)} = \frac{16}{21}$ (D) $\frac{P(r = R/2)}{P(r = R/3)} = \frac{20}{27}$

GR0067

10. A rocket is launched normal to the surface of the Earth, away from the Sun, along the line joining the sun and the Earth. The Sun is 3×10^5 times heavier than the Earth and is at a distance 2.5×10^4 times larger than the radius of the Earth. The escape velocity from Earth's gravitational field is $v_e = 11.2$ km s^{-1}. The minimum initial velocity (v_s) required for the rocket to be able to leave the *Sun-Earth system* is closest to (*Ignore the rotation and revolution of the Earth and the presence of any other planet)* **[JEE-Advance 2017]**

(A) $v_s = 22$ km s^{-1} (B) $v_s = 72$ km s^{-1} (C) $v_s = 42$ km s^{-1} (D) $v_s = 62$ km s^{-1}

GR0068

11. A planet of mass M, has two natural satellites with masses m_1 and m_2. The radii of their circular orbits are R_1 and R_2 respectively. Ignore the gravitational force between the satellites. Define v_1, L_1, K_1 and T_1 to be, respectively, the orbital speed, angular momentum, kinetic energy and time period of revolution of satellite 1 ; and v_2, L_2, K_2 and T_2 to be the corresponding quantities of satellite 2. Given $m_1/m_2 = 2$ and $R_1/R_2 = 1/4$, match the ratios in List-I to the numbers in List-II. **[JEE-Advance 2018]**

	List–I		**List–II**
P.	$\frac{v_1}{v_2}$	1.	$\frac{1}{8}$
Q.	$\frac{L_1}{L_2}$	2.	1
R.	$\frac{K_1}{K_2}$	3.	2
S.	$\frac{T_1}{T_2}$	4.	8

(A) P $\rightarrow$ 4 ; Q $\rightarrow$ 2 ; R $\rightarrow$ 1 ; S $\rightarrow$ 3 (B) P $\rightarrow$ 3 ; Q $\rightarrow$ 2 ; R $\rightarrow$ 4 ; S $\rightarrow$ 1

(C) P $\rightarrow$ 2 ; Q $\rightarrow$ 3 ; R $\rightarrow$ 1 ; S $\rightarrow$ 4 (D) P $\rightarrow$ 2 ; Q $\rightarrow$ 3 ; R $\rightarrow$ 4 ; S $\rightarrow$ 1

GR0069

12. Consider a spherical gaseous cloud of mass density $\rho(r)$ in free space where r is the radial distance from its center. The gaseous cloud is made of particles of equal mass m moving in circular orbits about the common center with the same kinetic energy K. The force acting on the particles is their mutual gravitational force. If $\rho(r)$ is constant in time, the particle number density $n(r) = \rho(r)/m$ is : [G is universal gravitational constant] **[JEE-Advance 2019]**

(A) $\frac{K}{\pi r^2 m^2 G}$ (B) $\frac{K}{6\pi r^2 m^2 G}$ (C) $\frac{3K}{\pi r^2 m^2 G}$ (D) $\frac{K}{2\pi r^2 m^2 G}$

GR0070

ANSWER KEY

EXERCISE (S-1)

1. Ans. $\frac{R_e k^2}{1-k^2}$ **2. Ans.** $h = \frac{\sqrt{5}-1}{2}R$ **3. Ans.** *(i)* $\frac{4}{3}\sqrt{\frac{Gm}{R}}$, *(ii)* $\frac{2}{3}\sqrt{\frac{2Gm}{5R}}$

4. Ans. 1.6 hours if it is rotating from west to east, 24/17 hours if it is rotating from east to west

5. Ans. 1×10^5J **6. Ans.** $t = \frac{GMm}{2C}\left(\frac{1}{R_e}-\frac{1}{r}\right)$ **7. Ans.** (a) $T=\frac{2\pi d^{3/2}}{\sqrt{3Gm}}$, (b) 2, (c) 2

8. Ans. $\vec{g} = +\frac{\pi G\rho_0 R^3}{6}\left[\frac{1}{(x-(R/2))^2}-\frac{8}{x^2}\right]\hat{i}$, $\vec{g} = -\frac{2\pi G\rho_0 R}{3}\hat{i}$

EXERCISE (S-2)

1. Ans. 2R, $3R[3-\sqrt{3}$ **2. Ans.** (a) $h=\left(\frac{\sqrt{7}}{2}+1\right)R$, (b) 1.125 R **3. Ans.** (a) $\sqrt{\frac{GM}{r}}$; (b) $r\sqrt{2}$; (c) $\sqrt{\frac{2GM}{r}}$

4. Ans. $T = 3 \times 10^{-2}$ N **5. Ans.** $\frac{1}{4}K^2G\pi(R^4 - r^4)$ **6. Ans.** 7/3 **7. Ans. 3**

8. Ans. (a) $\frac{2Gm^2}{3R}$ (b) 4R **9. Ans.** $\left(1-\frac{\sqrt{x^2-R^2}}{x}\right)4\pi R^2$

EXERCISE (O-1)

1. Ans. (D) 2. Ans. (D) 3. Ans. (B) 4. Ans. (A) 5. Ans. (B) 6. Ans. (B)
7. Ans. (A) 8. Ans. (A) 9. Ans. (B) 10. Ans. (B) 11. Ans. (D) 12. Ans. (D)
13. Ans. (B) 14. Ans. (C) 15. Ans. (A) 16. Ans. (A) 17. Ans. (C) 18. Ans. (B)
19. Ans. (C) 20. Ans. (B,D) 21. Ans. (A,B,C) 22. Ans. (A, D)
23. Ans. (A)-R, (B)-Q, (C)-Q (D)-P

EXERCISE O-2

1. Ans. (A,C,D) 2. Ans. (A) 3. Ans. (C) 4. Ans. (A) 5. Ans. (A,B,C) 6. Ans. (A,C)
7. Ans. (A,B,D) 8. Ans. (A,D) 9. Ans. (A,C,D) 10. Ans. (B) 11. Ans. (C)

EXERCISE - (JM)

1. Ans. (2) 2. Ans. (2) 3. Ans. (1) 4. Ans. (1) 5. Ans. (2) 6. Ans. (4)
7. Ans. (1) 8. Ans. (2) 9. Ans. (4) 10. Ans. (2) 11. Ans. (2) 12. Ans. (3)
13. Ans. (2) 14. Ans. (4) 15. Ans. (2) 16. Ans. (2) 17. Ans. (1) 18. Ans. (3)
19. Ans. (2) 20. Ans. (1)

EXERCISE-(JA)

1. Ans. 3 2. Ans. 6 3. Ans. (A) 4. Ans. (B) 5. Ans. (B,D) 6. Ans. (B)
7. Ans. 2 8. Ans. 7 9. Ans. (B,C) 10. Ans. (C) 11. Ans. (B) 12. Ans. (D)

Chapter Contents 03

CURRENT ELECTRICITY

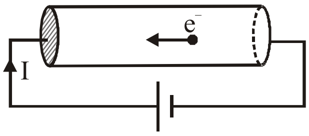
Important Notes

CURRENT ELECTRICITY

KEY CONCEPT

In previous chapters we deal largely with electrostatics that is, with charges at rest. With this chapter we begin to focus on electric currents, that is, charges in motion.

ELECTRIC CURRENT



Electric charges in motion constitute an electric current. Any medium having practically free electric charges, free to migrate is a conductor of electricity. The electric charge flows from higher potential energy state to lower potential energy state.

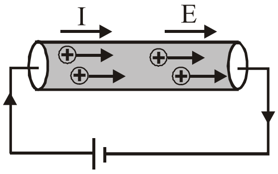
Positive charge flows from higher to lower potential and negative charge flows from lower to higher. Metals such as gold, silver, copper, aluminium etc. are good conductors.

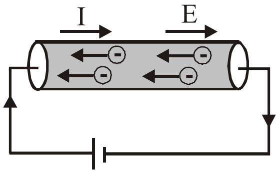
When charge flows in a conductor from one place to the other, then the rate of flow of charge is called electric current (I). When there is a transfer of charge from one point to other point in a conductor, we say that there is an electric current through the area. If the moving charges are positive, the current is in the direction of motion of charge. If they are negative the current is opposite to the direction of motion. If a charge ΔQ crosses an area in time Δt then the average electric current through the area, during this time as

- Average current $I_{av} = \dfrac{\Delta Q}{\Delta t}$ • Instantaneous current $I = \lim\limits_{\Delta t \to 0} \dfrac{\Delta Q}{\Delta t} = \dfrac{dQ}{dt}$

- Current is a fundamental quantity with dimension $[M^0L^0T^0A^1]$
- Current is a scalar quantity with its SI unit **ampere.**

  Ampere : The current through a conductor is said to be one ampere if one coulomb of charge is flowing per second through a cross–section of wire.
- The conventional direction of current is the direction of flow of positive charge or applied field. It is opposite to direction of flow of negatively charged electrons.





- The conductor remains uncharged when current flows through it because the charge entering at one end per second is equal to charge leaving the other end per second.
- For a given conductor current does not change with change in its cross–section because current is simply rate of flow of charge.
- If there are n particles per unit volume each having a charge q and moving with velocity v then current through cross–sectional area A is $I = \dfrac{\Delta q}{\Delta t} = nqvA$
- If a charge q is moving in a circle of radius r with speed v then its time period is $T = 2\pi r/v$. The equivalent current $I = \dfrac{q}{T} = \dfrac{qv}{2\pi r}$.

Behavior of conductor in absence of applied potential difference :

In absence of applied potential difference electrons have random motion. The average displacement and average velocity is zero. There is no flow of current due to thermal motion of free electrons in a conductor.

The free electrons present in a conductor gain energy from temperature of surrounding and move randomly in the conductor.

The speed gained by virtue of temperature is called as thermal speed of an electron $\frac{1}{2}mv_{rms}^2 = \frac{3}{2}kt$

So thermal speed $v_{rms} = \sqrt{\frac{3kT}{m}}$ where m is mass of electron

At room temperature T = 300 K, $v_{rms} = 10^5$ m/s

- **Mean free path λ :** $(\lambda \sim 10\text{Å}) = \lambda = \frac{\text{total distance travelled}}{\text{number of collisions}}$
- **Relaxation time :** The time taken by an electron between two successive collisions is called as relaxation time τ : $(\tau \sim 10^{-14}\text{s})$, Relaxation time : $\tau = \frac{\text{total time taken}}{\text{number of collisions}}$

Behavior of conductor in presence of applied potential difference :

When two ends of a conductors are joined to a battery then one end is at higher potential and another at lower potential. This produces an electric field inside the conductor from point of higher to lower potential

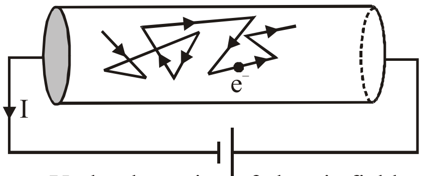
$E = \frac{V}{L}$ where V = emf of the battery, L = length of the conductor.

The field exerts an electric force on free electrons causing acceleration of each electron.

Acceleration of electron $\vec{a} = \frac{\vec{F}}{m} = \frac{-e\vec{E}}{m}$

DRIFT VELOCITY

Drift velocity is defined as the velocity with which the free electrons get drifted towards the positive terminal under the effect of the applied external electric field. In addition to its thermal velocity, due to acceleration given by applied electric field, the electron acquires a velocity component in a direction opposite to the direction of the electric field. The gain in velocity due to the applied field is very small and is lost in the next collision.



Under the action of electric field : Random motion of an electron with superimposed drift

At any given time, an electron has a velocity $\vec{v}_1 = \vec{u}_1 + \vec{a}\tau_1$, where $\vec{u}_1$ = the thermal velocity and $\vec{a}\tau_1$ = the velocity acquired by the electron under the influence of the applied electric field.

τ_1 = the time that has elapsed since the last collision. Similarly, the velocities of the other electrons are $\vec{v}_2 = \vec{u}_2 + \vec{a}\tau_2, \vec{v}_3 = \vec{u}_3 + \vec{a}\tau_3, ...\vec{v}_N = \vec{u}_N + \vec{a}\tau_N$.

The average velocity of all the free electrons in the conductor is equal to the drift velocity $\vec{v}_d$ of the free electrons

$$\vec{v}_d=\frac{\vec{v}_1+\vec{v}_2+\vec{v}_3+\ldots\vec{v}_N}{N}=\frac{(u_1+\vec{a}\tau_1)+(\vec{u}_2+\vec{a}\tau_2)+\ldots+(\vec{u}_N+\vec{a}\tau_N)}{N}=\frac{(\vec{u}_1+\vec{u}_2+\ldots+\vec{u}_N)}{N}+\vec{a}\left(\frac{\tau_1+\tau_2+\ldots+\tau_N}{N}\right)$$

$$\because\ \frac{\vec{u}_1+\vec{u}_2+\ldots+\vec{u}_N}{N}=0\ \therefore\ \vec{v}_d=\vec{a}\left(\frac{\tau_1+\tau_2+\ldots+\tau_N}{N}\right)\Rightarrow\vec{v}_d=\vec{a}\tau=-\frac{e\vec{E}}{m}\tau$$

Note : Order of drift velocity is 10^{-4} m/s.

Relation between current and drift velocity :

Let n= number density of free electrons and A= area of cross–section of conductor.

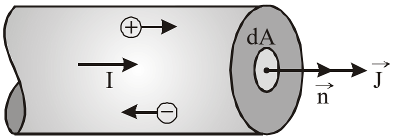
Number of free electrons in conductor of length L = nAL, Total charge on these free electrons $\Delta q = neAL$

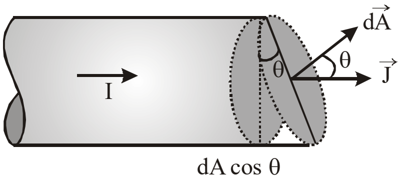
Time taken by drifting electrons to cross conductor $\Delta t=\frac{L}{v_d}$ $\therefore$ current $I=\frac{\Delta q}{\Delta t}=neAL\left(\frac{v_d}{L}\right)=neAv_d$

CURRENT DENSITY (J)

Current is a macroscopic quantity and deals with the overall rate of flow of charge through a section. To specify the current with direction in the microscopic level at a point, the term current density is introduced. Current density at any point inside a conductor is defined as a vector having magnitude equal to current per unit area surrounding that point. Remember area is normal to the direction of charge flow (or current passes) through that point.

- Current density at point P is given by $\vec{J}=\frac{dI}{dA}\vec{n}$





- If the cross–sectional area is not normal to the current, but makes an angle θ with the direction of current then $J=\frac{dI}{dA\cos\theta}\Rightarrow dI=JdA\cos\theta=\vec{J}.d\vec{A}\Rightarrow I=\int\vec{J}.\overrightarrow{dA}$
- Current density $\vec{J}$ is a vector quantity. It's direction is same as that of $\vec{E}$. It's S.I. unit is ampere/m^2 and dimension [$L^{-2}A$].

Ex. The current density at a point is $\vec{J}=\left(2\times10^4\hat{j}\right)Jm^{-2}$.

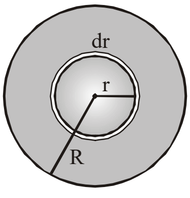
Find the rate of charge flow through a cross sectional area $\vec{S}=\left(2\hat{i}+3\hat{j}\right)cm^2$

Sol. The rate of flow of charge = current $=I=\int\vec{J}.d\vec{S}\Rightarrow I=\vec{J}.\vec{S}=(2\times10^4)\left[\hat{j}\cdot\left(2\hat{i}+3\hat{j}\right)\right]\times10^{-4}A=6A$

Ex. A potential difference applied to the ends of a wire made up of an alloy drives a current through it. The current density varies as J = 3 + 2r, where r is the distance of the point from the axis. If R be the radius of the wire, then the total current through any cross section of the wire.

Sol. Consider a circular strip of radius r and thickness dr

$$dI=\vec{J}.d\vec{S}=(3+2r)(2\pi rdr)\cos0^\circ=2\pi\left(3r+2r^2\right)dr$$



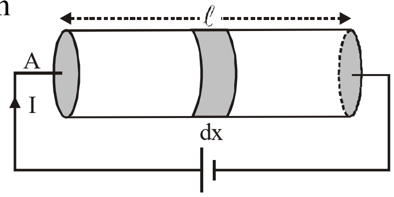
$$I=\int_0^R2\pi\left(3r+2r^2\right)dr=2\pi\left(\frac{3r^2}{2}+\frac{2}{3}r^3\right)_0^R=2\pi\left(\frac{3R^2}{2}+\frac{2R^3}{3}\right)\text{ units}$$

RELATION BETWEEN CURRENT DENSITY, CONDUCTIVITY AND ELECTRIC FIELD

Let the number of free electrons per unit volume in a conductor = n

Total number of electrons in dx distance = n (Adx)

Total charge dQ = n (Adx)e



Current $I=\frac{dQ}{dt}=nAe\frac{dx}{dt}=neAv_d$, Current density $J=\frac{I}{A}=nev_d$

$$=ne\left(\frac{eE}{m}\right)\tau \because v_d=\left(\frac{eE}{m}\right)\tau \Rightarrow J=\left(\frac{ne^2\tau}{m}\right)E \Rightarrow J=\sigma E, \text{ where conductivity } \sigma=\frac{ne^2\tau}{m}$$

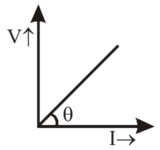
σ depends only on the material of the conductor and its temperature.

In vector form $\vec{J}=\sigma\vec{E}$ *Ohm's law (at microscopic level)*

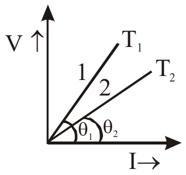
RELATION BETWEEN POTENTIAL DIFFERENCE AND CURRENT (Ohm's Law)

If the physical conditions of the conductor (length, temperature, mechanical strain etc.) remains same, then the current flowing through the conductor is directly proportional to the potential difference across it's two ends i.e. $I \propto V \Rightarrow V = IR$ where R is a proportionality constant, known as electric resistance. *Ohm's law (at macroscopic level)*

- Ohm's law is not a universal law. The substances, which obey ohm's law are known as ohmic.
- Graph between V and I for a metallic conductor is a straight line as shown.



Slope of the line $=\tan\theta=\frac{V}{I}=R$



At different temperatures V–I curves are different.

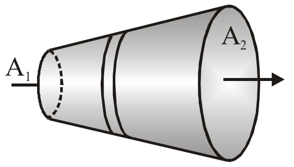
Here $\tan\theta_1 > \tan\theta_2$ So $R_1 > R_2$ i.e. $T_1 > T_2$

- 1 ampere of current means the flow of 6.25×10^{18} electrons per second through any cross section of conductor.
- Current is a scalar quantity but current density is a vector quantity.
- Order of free electron density in conductors = 10^{28} electrons/m^3
-

Terms	Thermal speed v_T	Mean free path λ	Relaxation time τ	Drift speed v_d
Order	10^5 m/s	10 Å	10^{-14} m/s	10^{-4} m/s

- If a steady current flows in a metallic conductor of non uniform cross section.

  Current density and drift velocity depends on area



$$I_1 = I_2,\ A_1 < A_2 \Rightarrow J_1 > J_2,\ E_1 > E_2,\ v_{d_1} > v_{d_2}$$

- If the temperature of the conductor increases, the amplitude of the vibrations of the positive ions in the conductor also increase. Due to this, the free electrons collide more frequently with the vibrating ions and as a result, the average relaxation time decreases.

Ex. What will be the number of electron passing through a heater wire in one minute, if it carries a current of 8 A.

Sol. $I=\frac{Ne}{t}\Rightarrow N=\frac{It}{e}=\frac{8\times60}{1.6\times10^{-19}}=3\times10^{21}$ electrons

Ex. A current of 1.34 A exists in a copper wire of cross–section 1.0 mm^2. Assuming each copper atom contributes one free electron. Calculate the drift speed of the free electrons in the wire. The density of copper is 8990 kg/m^3 and atomic mass = 63.50.

Sol. Mass of 1m^3 volume of the copper is = 8990 kg = 8990 × 10^3 g

Number of moles in 1m^3 $= \dfrac{8990\times10^3}{63.5} = 1.4\times10^5$

Since each mole contains 6 × 10^{23} atoms therefore number of atoms in 1m^3

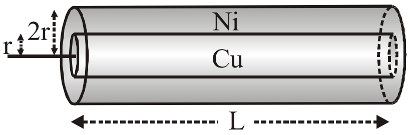
$n = (1.4\times10^5)\times(6\times10^{23}) = 8.4\times10^{28}$

$$\because I = neAv_d \therefore v_d = \frac{I}{neA} = \frac{1.34}{8.4\times10^{28}\times1.6\times10^{-19}\times10^{-6}} = 10^{-4}\text{ m/s} = 0.1\text{ mm/s } (\because 1\text{ mm}^2 = 10^{-6}\text{ m}^2)$$

Ex. A copper wire of length 'ℓ' and radius 'r' is nickel plated till its final radius is 2r. If the resistivity of the copper and nickel are ρ_{Cu} and ρ_{Ni}, then find the equivalent resistance of wire?

Sol. $R = \rho\dfrac{\ell}{A}$; Resistance of copper wire $R_{Cu} = \rho_{Cu}\dfrac{\ell}{\pi r^2}$

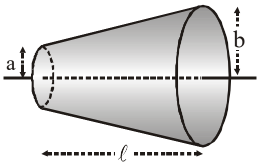
$(\because A = \pi r^2)$



$\because A_{Ni} = \pi(2r)^2 - \pi r^2 = 3\pi r^2 \Rightarrow$ Resistance of Nickel wire $R_{Ni} = \rho_{Ni}\dfrac{\ell}{3\pi r^2}$

Both wire are connected in parallel. So equivalent resistance $R = \dfrac{R_{Cu}R_{Ni}}{R_{Cu}+R_{Ni}} = \left(\dfrac{\rho_{Cu}\rho_{Ni}}{3\rho_{Cu}+\rho_{Ni}}\right)\dfrac{\ell}{\pi r^2}$

Ex. Figure shows a conductor of length ℓ carrying current I and having a circular cross – section. The radius of cross section varies linearly from a to b. Assuming that (b – a) << ℓ. Calculate current density at distance x from left end.



Sol. Since radius at left end is a and that of right end is b,
Therefore increase in radius over length ℓ is (b – a).

Hence rate of increase of radius per unit length $= \left(\dfrac{b-a}{\ell}\right)$ Increase in radius over length x $= \left(\dfrac{b-a}{\ell}\right)x$

Since radius at left end is a so radius at distance x, $r = a + \left(\dfrac{b-a}{\ell}\right)x$

Area at this particular section $A = \pi r^2 = \pi\left[a + \left(\dfrac{b-a}{\ell}\right)x\right]^2$

Hence current density $J = \dfrac{I}{A} = \dfrac{I}{\pi r^2} = \dfrac{I}{\pi\left[a + \dfrac{x(b-a)}{\ell}\right]^2}$

RESISTANCE

The resistance of a conductor is the opposition which the conductor offers to the flow of charge. When a potential difference is applied across a conductor, free electrons get accelerated and collide with positive ions and their motion is thus opposed. This opposition offered by the ions is called resistance of the conductor.

Resistance is the property of a conductor by virtue of which it opposes the flow of current in it.

Unit : ohm, volt/ampere,

Dimension = $M L^2 T^{-3} A^{-2}$

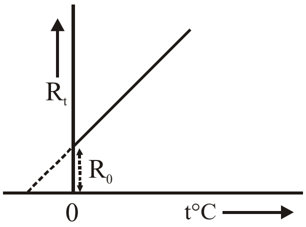
Resistance depends on :

- Length of the conductor ($R \propto \ell$)
- Area of cross-section of the conductor $R \propto \frac{1}{A}$
- Nature of material of the conductor $R = \frac{\rho\ell}{A}$
- Temperature $R_t = R_0 (1 + \alpha\Delta t)$

  Where R_t = Resistance at t° C, R_0 = Resistance at 0° C

  Δt = Change in temperature, α = Temperature coefficient of resistance

  [*For metals* : α positive *for semiconductors* : α negative]

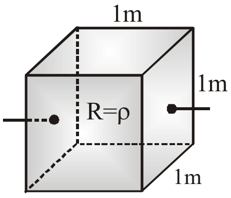


RESISTIVITY

Resistivity : $\rho = RA/\ell$ if ℓ = 1m, A = 1m^2 then $\rho = R$

The specific resistance of a material is equal to the resistance of the wire of that material with unit cross – section area and unit length.

Resistivity depends on (i) Nature of material (ii) Temperature of material ρ does not depend on the size and shape of the material because it is the characteristic property of the conductor material.



Specific use of conducting materials :

- The **heating element** of devices like heater, geyser, press etc are made of **michrome** because it has high resistivity and high melting point. It does not react with air and acquires steady state when red hot at 800°C.
- **Fuse wire** is made of **tin lead alloy** because it has low melting point and low resistivity. The fuse is used in series, and melts to produce open circuit when current exceeds the safety limit.
- **Resistances** of resistance box are made of **manganin** or **constantan** because they have moderate resistivity and very small temperature coefficient of resistance. The resistivity is nearly independent of temperature.
- The **filament of bulb** is made up of **tungsten** because it has low resistivity, high melting point of 3300 K and gives light at 2400 K. The bulb is filled with inert gas because at high temperature it reacts with air forming oxide.
- The **connection wires** are made of **copper** because it has low resistance and resistivity.

KIRCHHOFF'S LAW

There are two laws given by Kirchhoff for determination of potential difference and current in different branches of any complicated network.

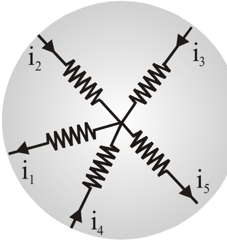
Law of conservation of charge is a consequence of continuity equation

- **First law (Junction Law or Current Law)**

  In an electric circuit, the algebraic sum of the current meeting at any junction in the circuit is zero or Sum of the currents entering the Junction is equal to sum of the current leaving the Junction. $\Sigma i=0$

  $i_1 - i_2 - i_3 - i_4 + i_5 = 0 \Rightarrow i_1 + i_5 = i_2 + i_3 + i_4$

  This is based on law of conservation of charge.



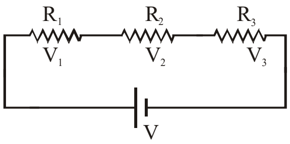
- **Second law (loop rule or potential law)**

  In any closed circuit the algebraic sum of all potential differences and e.m.f. is zero. $\Sigma E - \Sigma IR = 0$ while moving from negative to positive terminal inside the cell, e.m.f. is taken as positive while moving in the direction of current in a circuit the potential drop (i.e. IR) across resistance is taken as positive.

  This law is based on law of conservation of energy.

COMBINATION OF RESISTANCE

Series Combination



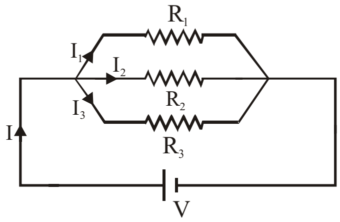
- Same current passes through each resistance
- Voltage across each resistance is directly proportional to it's value

$$V_1 = IR_1,\ V_2 = IR_2\ ,\ V_3 = IR_3$$

- Sum of the voltage across resistance is equal to the voltage applied across the circuit.

$V = V_1 + V_2 + V_3 \Rightarrow IR = IR_1 + IR_2 + IR_3 \Rightarrow\ R = R_1 + R_2 + R_3$ Where R = equivalent resistance

Parallel Combination



- There is same drop of potential across each resistance.
- Current in each resistance is inversely proportional to the value of resistance. $I_1 = \frac{V}{R_1}\ ,\ I_2 = \frac{V}{R_2}\ ,\ I_3 = \frac{V}{R_3}$
- Current flowing in the circuit is sum of the currents in individual resistance.

$$I = I_1 + I_2 + I_3\ \Rightarrow \frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \Rightarrow \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

- To get maximum resistance, resistance must be connected in series and in series the resultant is greater than largest individual.
- To get minimum resistance, resistance must be connected in parallel and the equivalent resistance of parallel combination is lower than the value of lowest resistance in the combination.
- **In general :**

(i) Resistivity of alloys is greater than their metals.

(ii) Temperature coefficient of alloys is lower than pure metals.

(iv) The resistivity of an insulator (e.g. amber) is greater then the metal by a factor of 10^{22}

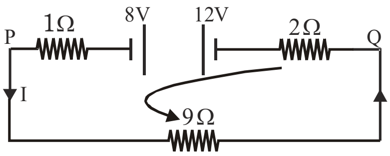
Ex. The resistance 4 R, 16 R, 64 R ... ∞ are connected in series. Find their equivalent resistance.

Sol. Resultant of the given combination $R_{eq} = 4R + 16R + 64R + \ldots \infty = \infty$

Ex. Resistance R, 2R, 4R, 8R...∞ are connected in parallel. What is their resultant resistance ?

Sol. $$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{2R} + \frac{1}{4R} + \frac{1}{8R}............\infty = \frac{1}{R}\left[1 + \frac{1}{2} + \frac{1}{4} + \ldots\infty\right] = \frac{1}{R}\left[\frac{1}{1-\frac{1}{2}}\right] = \frac{2}{R} \Rightarrow R_{eq} = \frac{R}{2}$$

Ex. In the given circuit calculate potential difference between the points P and Q.

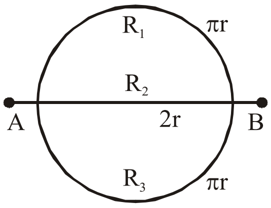


Sol. Applying Kirchhoff's voltage law (KVL)

$$12 - 8 = (1)\,I + (9)\,I + (2)\,I \Rightarrow I = \frac{1}{3}A$$

Potential difference between the points P and Q, $V_P - V_Q = 9 \times \frac{1}{3} = 3$ volt

Ex. A wire of $\rho_L = 10^{-6}\ \Omega/\text{m}$ is turned in the form of a circle of diameter 2 m. A piece of same material is connected in diameter AB. Then find resistance between A and B.



Sol. $\because\ R = \rho_L \times \text{length}$

$\therefore\ R_1 = \pi \times 10^{-6}\ \Omega,\ R_2 = 2 \times 10^{-6}\,\Omega,\ R_3 = \pi \times 10^{-6}\ \Omega$

$$\frac{1}{R_{AB}} = \frac{1}{\pi\times10^{-6}} + \frac{1}{2\times10^{-6}} + \frac{1}{\pi\times10^{-6}}\ ;\ R_{AB} = 0.88 \times 10^{-6}\ \text{ohm.}$$

CELL

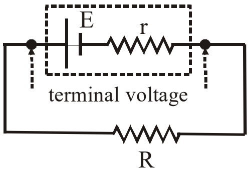
Cell convert chemical energy into electrical energy.

ELECTRO MOTIVE FORCE (E. M. F.)

The potential difference across the terminals of a cell when it is not giving any current is called emf of the cell. The energy given by the cell in the flow of unit charge in the whole circuit (including the cell) is called the emf of the cell.

- emf depends on : (i) nature of electrolyte (ii) metal of electrodes
- emf does not depend on : (i) area of plates (ii) distance between the electrodes (iii) quantity of electrolyte (iv) size of cell

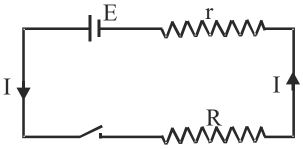
TERMINAL VOLTAGE (V)



- When current is drawn through the cell or current is supplied to cell then, the potential difference across its terminals called terminal voltage.
- When I current is drawn from cell, then terminal voltage is less than it's e.m.f. V = E – Ir

- **Terminal Potential Difference** *:* The potential difference between the two electrodes of a cell in a closed circuit i.e. when current is being drawn from the cell is called terminal potential difference.

(a) When cell is discharging :

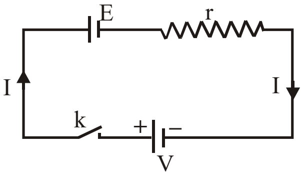


Current inside the cell is from cathode to anode.

Current $I = \dfrac{E}{r+R} \Rightarrow E = IR + Ir = V + Ir \Rightarrow V = E - Ir$

When current is drawn from the cell potential difference is less than emf of cell. Greater is the current drawn from the cell smaller is the terminal voltage. When a large current is drawn from a cell its terminal voltage is reduced.

(b) When cell is charging :



Current inside the cell is from anode to cathode.

Current $I = \dfrac{V-E}{r} \Rightarrow V = E + Ir$

During charging terminal potential difference is greater than emf of cell.

(c) When cell is in open circuit :

In open circuit $R = \infty\ \therefore\ I = \dfrac{E}{R+r} = 0 \Rightarrow V = E$

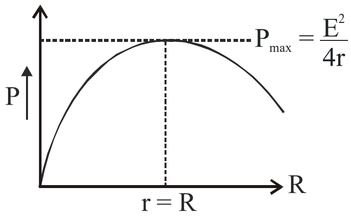
In open circuit terminal potential difference is equal to emf and is the maximum potential difference which a cell can provide.

(d) When cell is short circuited :

In short circuit $R = 0 \Rightarrow I = \frac{E}{R+r} = \frac{E}{r}$ and $V = IR = 0$

In short circuit current from cell is maximum and terminal potential difference is zero.

(e) Power transferred to load by cell :



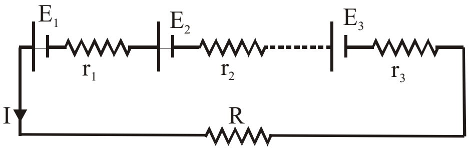
$$P = I^2R = \frac{E^2R}{(r+R)^2} \Rightarrow P = P_{max} \text{ if } \frac{dP}{dR} = 0 \Rightarrow r = R$$

Power transferred by cell to load is maximum when

$r = R$ and $P_{max} = \frac{E^2}{4r} = \frac{E^2}{4R}$

COMBINATION OF CELLS

- **Series combination**



When the cells are connected in series the total e.m.f. of the series combination is equal to the sum of the e.m.f.'s of the individual cells and internal resistance of the cells also come in series.

Equivalent internal resistance $r = r_1 + r_2 + r_3 + ...$

Equivalent emf = $E = E_1 + E_2 + E_3 + ...$

Current $I = \frac{E_{net}}{r_{net} + R}$, If all n cell are identical then $I = \frac{nE}{nr + R}$

• If nr >> R, $I = \frac{E}{r} \simeq$ current from one cell • If nr << R, $I = \frac{nE}{R} \simeq n \times$ current from one cell

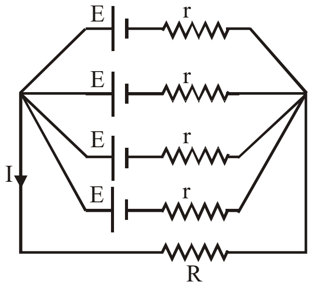
- **Parallel combination**

When the cells are connected in parallel, the total e.m.f. of the parallel combination remains equal to the e.m.f. of a single cell and internal resistance of the cell also come in parallel. If m identical cell connected in parallel then total internal resistance of this combination $r_{net} = \frac{r}{m}$. Total e.m.f. of this combination = E

Current in the circuit $I = \frac{E}{R + \frac{r}{m}} = \frac{mE}{mR + r}$

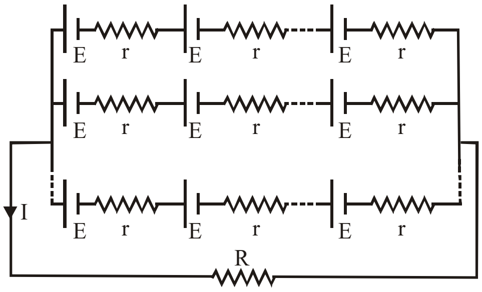
If r << mR I=E/R = Current from one cell

If r>> mR $I = \frac{mE}{r} = m \times$ current from one cell



- **Mixed combination**

If n cells connected in series and their are m such branches in the circuit then total number of identical cell in this circuit is nm. The internal resistance of the cells connected in a row = nr . Since there are such m rows,



Total internal resistance of the circuit $r_{net} = \frac{nr}{m}$

Total e.m.f. of the circuit = total e.m.f. of the cells connected in a row

$E_{net} = nE$

Current in the circuit $I = \frac{E_{net}}{R + r_{net}} = \frac{nE}{R + \frac{nr}{m}}$

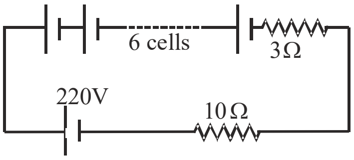
Current in the circuit is maximum when external resistance in the circuit is equal to the total internal resistance of the cells $R = \frac{nr}{m}$

- At the time of charging a cell when current is supplied to the cell, the terminal voltage is greater than the e.m.f. E, V = E + Ir
- Series combination is useful when internal resistance is less than external resistance of the cell.
- Parallel combination is useful when internal resistance is greater than external resistance of the cell.
- Power in R (given resistance) is maximum, if its value is equal to net resistance of remaining circuit.
- Internal resistance of ideal cell = 0
- if external resistance is zero than current given by circuit is maximum.

Ex. A battery of six cells each of e.m.f. 2 V and internal resistance 0.5 Ω is being charged by D. C. mains of e.m.f. 220 V by using an external resistance of 10 Ω. What will be the charging current.

Sol. Net e.m.f of the battery = 12V and total internal resistance = 3Ω

Total resistance of the circuit = 3 + 10 = 13 Ω

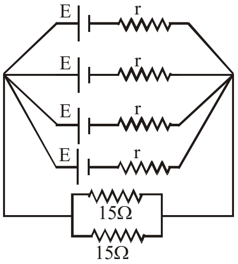


Charging current $I = \frac{\text{Net e.m.f.}}{\text{total resistance}} = \frac{220-12}{13} = 16\,\text{A}$

Ex. A battery of six cells each of e.m.f. 2 V and internal resistance 0.5 Ω is being charged by D. C. mains of e.m.f. 220 V by using an external resistance of 10 Ω. What is the potential difference across the battery ?

Sol In case of charging of battery, terminal potential V = E + Ir = 12 + 16 × 3 = 60 volt.

Ex. Four identical cells each of e.m.f. 2V are joined in parallel providing supply of current to external circuit consisting of two 15Ω resistors joined in parallel. The terminal voltage of the equivalent cell as read by an ideal voltmeter is 1.6V calculate the internal resistance of each cell.

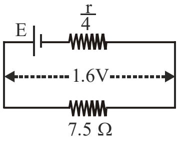


Sol. Total internal resistance of the combination $r_{eq} = \frac{r}{4}$

Total e.m.f. $E_{eq} = 2V$

Total external resistance $R = \frac{15\times15}{15+15} = \frac{15}{2} = 7.5\Omega$

Current drawn from equivalent cell $I = \frac{\text{terminal potential}}{\text{external resistance}} = \frac{1.6}{7.5}\text{A}$



$$\because\ E - I\left(\frac{r}{4}\right) = 1.6 \ \therefore\ E - I\left(\frac{r}{4}\right) = 1.6 \Rightarrow r = 7.5\ \Omega$$

Ex. The e.m.f. of a primary cell is 2 V, when it is shorted then it gives a current of 4 A. Calculate internal resistance of primary cell.

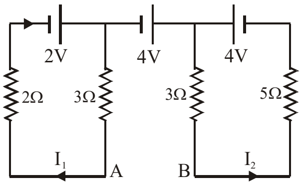
Sol. $I=\dfrac{E}{r+R}$, If cell is shorted then R = 0, $I=\dfrac{E}{r}$ $\therefore$ $r=\dfrac{E}{I}=\dfrac{2}{4}=0.5\ \Omega$

Ex. n rows each containing m cells in series, are joined in parallel. Maximum current is taken from this combination in a 3 Ω resistance. If the total number of cells used is 24 and internal resistance of each cell is 0.5 Ω , find the value of m and n.

Sol. Total number of cell mn = 24, For maximum current $\dfrac{mr}{n}=R \Rightarrow 0.5\ m = 3\ n,\ m=\dfrac{3n}{0.5}=6n$

$\therefore 6n \times n = 24 \Rightarrow n = 2$ and $m \times 2 = 24 \Rightarrow m = 12$

Ex. In the given circuit calculate potential difference between A and B.



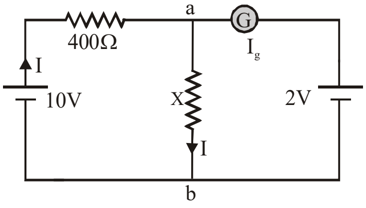
Sol. First applying KVL on left mesh $2 - 3\ I_1 - 2\ I_1 = 0 \Rightarrow I_1 = 0.4$ amp.

Now applying KVL on right mesh. $4 - 5\ I_2 - 3\ I_2 = 0 \Rightarrow I_2 = 0.5$ amp.

Potential difference between points A and B

$V_A - V_B = -3 \times 0.4 - 4 + 3 \times 0.5 = -3.7$ volt.

Ex. In the following circuit diagram, the galvanometer reading is zero. If the internal resistance of cells are negligible then what is the value of X ?

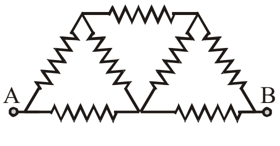


Sol. $\because\ I_g = 0$ $\therefore\ I=\dfrac{10}{400+X}$

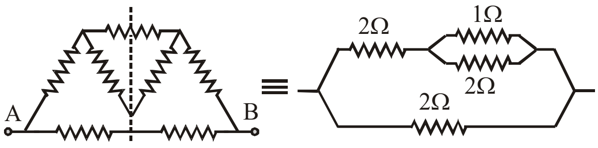
also potential difference across X is 2V $\Rightarrow$ I X = 2

$$\frac{10X}{400+X}=2\left(\because\ I=\frac{10}{400+X}\right)\Rightarrow\ X = 100\Omega$$

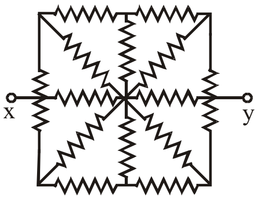
Ex. Each resistance is of 1 Ω in the circuit diagram shown in figure. Find out equivalent resistance between A and B



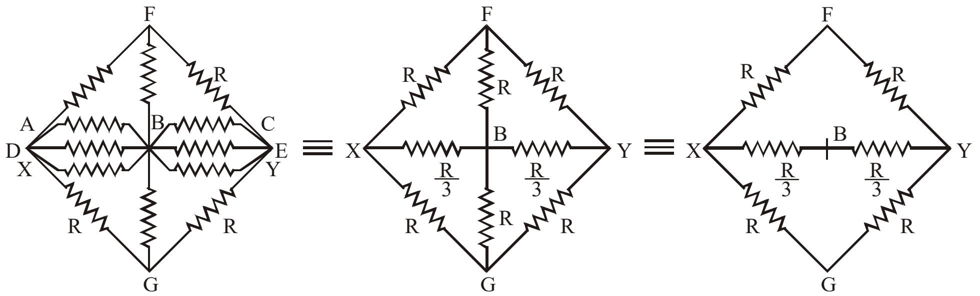
Sol. By symmetric line method $R_{AB} = (2 + 1 \,||\, 2) \,||\, 2 = \dfrac{8}{7}\ \Omega$



Ex. Identical resistance of resistance R are connected as in figure then find out net resistance between x and y.



Sol. Given circuit can be modified according to following figures



$$\frac{1}{R_{xy}}=\frac{1}{2R}+\frac{3}{2R}+\frac{1}{2R}=\frac{5}{2R}\Rightarrow R_{xy}=\frac{2R}{5}$$

HEATING EFFECT OF CURRENT

CAUSE OF HEATING

The potential difference applied across the two ends of conductor sets up electric field. Under the effect of electric field, electrons accelerate and as they move, they collide against the ions and atoms in the conductor, the energy of electrons transferred to the atoms and ions appears as heat.

- **Joules's Law of Heating**

When a current I is made to flow through a passive or ohmic resistance R for time t, heat Q is produced such that

$$Q = I^2 R t = P \times t = V I t = \frac{V^2}{R} t$$

Heat produced in conductor does not depend upon the direction of current.

• **SI unit** : joule ;

Practical Units : 1 kilowatt hour (kWh)

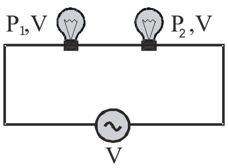
1kWh = 3.6×10^6 joule = 1 unit 1 BTU (British Thermal Unit) = 1055 J

• **Power :** $P = V I = \frac{V^2}{R} = I^2R$ • **SI unit :** Watt

The watt–hour meter placed on the premises of every consumer records the electrical energy consumed.

- **Series combination of resistors (bulbs)**

Total power consumed $P_{total}=\frac{P_1P_2}{P_1+P_2}$.



If n bulbs are identical $P_{total}=\frac{P}{n}$

In series combination of bulbs : Brightness $\propto$ Power consumed by bulb $\propto V \propto R \propto \frac{1}{P_{rated}}$

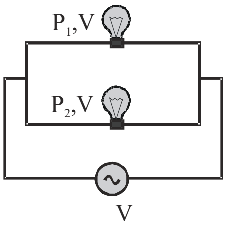
Bulb of lesser wattage will shine more. For same current $P = I^2R \quad P \propto R \quad R\uparrow \Rightarrow P\uparrow$

- **Parallel combination of resistors (bulbs)**

  Total power consumed $P_{total} = P_1 + P_2$

  If n bulbs are identical $P_{total} = nP$

  In parallel combination of bulbs



  Brightness $\propto$ Power consumed by bulb $\propto I \propto \frac{1}{R}$

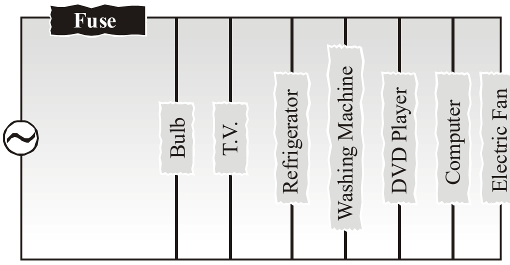
  Bulb of greater wattage will shine more.

  For same V more power will be consumed in smaller resistance $P \propto \frac{1}{R}$

- Two identical heater coils gives total heat H_S when connected in series and H_p when connected in parallel than $\frac{H_P}{H_S} = 4$ [In this, it is assumed that supply voltage is same]
- If a heater boils m kg water in time T_1 and another heater boils the same water in T_2, then both connected in series will boil the same water in time $T_s = T_1 + T_2$ and in parallel $T_P = \frac{T_1T_2}{T_1 + T_2}$ [Use time taken $\propto$ Resistance]
- Instruments based on heating effect of current, works on both A.C. and D.C. Equal value of A.C. (RMS) and D.C. produces, equal heating effect. That why brightness of bulb is same whether it is operated by A.C. or same value D.C.

FUSE WIRE

The fuse wire for an electric circuit is chosen keeping in view the value of safe current through the circuit.



- The fuse wire should have high resistance per unit length and low melting point.
- However the melting point of the material of fuse wire should be above the temperature that will be reached on the passage of the current through the circuit
- A fuse wire is made of alloys of lead (Pb) and tin (Sn).
- Length of fuse wire is immaterial.
- The material of the filament of a heater should have high resistivity and high melting point.
- The temperature of the wire increases to such a value at which, the heat produced per second equals heat lost per second due to radiation from the surface of wire $I^2\left(\frac{\rho\ell}{\pi r^2}\right) = H \times 2\pi r\ell \quad I^2 \propto r^3$

H = heat lost per second per unit area due to radiation.

Ex. An electric heater and an electric bulb are rated 500 W, 220 V and 100 W, 220 V respectively. Both are connected in series to a 220 V a.c. mains. Calculate power consumed by (i) heater (ii) bulb.

Sol. $P=\dfrac{V^2}{R}$ or $R=\dfrac{V^2}{P}$, For heater. Resistance $R_h=\dfrac{(220)^2}{500}=96.8\ \Omega$,

For bulb resistance $R_L=\dfrac{(220)^2}{100}=484\ \Omega$

Current in the circuit when both are connected in series $I=\dfrac{V}{R_L+R_h}=\dfrac{220}{484+96.8}=0.38$ A

(i) Power consumed by heater = $I^2R_h=(0.38)^2\times 96.8=13.98$ W

(ii) Power consumed by bulb = $I^2R_L=(0.38)^2\times 484=69.89$ W

Ex. A heater coil is rated 100 W, 200 V. It is cut into two identical parts. Both parts are connected together in parallel, to the same source of 200 V. Calculate the energy liberated per second in the new combination.

Sol. $\because P=\dfrac{V^2}{R}$ $\quad\therefore R=\dfrac{V^2}{P}=\dfrac{(200)^2}{100}=400\ \Omega$

Resistance of half piece = $\dfrac{400}{2}=200\ \Omega$

Resistance of pieces connected in parallel = $\dfrac{400}{2}=100\ \Omega$

Energy liberated/second $P=\dfrac{V^2}{R}=\dfrac{200\times 200}{100}=400$ W

Ex. The power of a heater is 500W at 800°C. What will be its power at 200°C. If $\alpha=4\times 10^{-4}$ per °C?

Sol. $P=\dfrac{V^2}{R}\ \therefore\ \dfrac{P_{200}}{P_{800}}=\dfrac{R_{800}}{R_{200}}=\dfrac{R_0(1+4\times10^{-4}\times800)}{R_0(1+4\times10^{-4}\times200)}\Rightarrow P_{200}=\dfrac{500\times1.32}{1.08}=611\text{W}$

Ex. When a battery sends current through a resistance R_1 for time t, the heat produced in the resistor is Q. When the same battery sends current through another resistance R_2 for time t, the heat produced in R_2 is again Q. Determine the internal resistance of battery.

Sol. $\left[\dfrac{E}{R_1+r}\right]^2R_1=\left[\dfrac{E}{R_2+r}\right]^2R_2\Rightarrow r=\sqrt{R_1R_2}$

Ex. How much time heater will take to increase the temperature of 100 g water by 50°C if resistance of heating coil is 484Ω and supply voltage is 220V a.c.

Sol. Heat given by heater = heat taken by water $\Rightarrow\dfrac{V^2}{R}t=msJ\Delta\theta\Rightarrow\dfrac{220\times220}{484}$

$t=(100\times10^{-3})(4.2\times10^{3})(50)\Rightarrow t=210$s

GALVANOMETER

The instrument used to measure strength of current, by measuring the deflection of the coil due to torque produced by a magnetic field, is known as galvanometer.

SHUNT

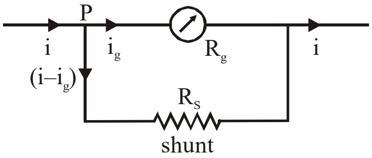
The small resistance connected in parallel to galvanometer coil, in order to control current flowing through the galvanometer, is known as shunt.

- **Merits of shunt**
  (i) To protect the galvanometer coil from burning.
  (ii) Any galvanometer can be converted into ammeter of desired range with the help of shunt.
  (iii) The range an ammeter can be changed by using shunt resistance of different values.
- **Demerits of shunt**
  Shunt resistance decreases the sensitivity of galvanometer.

CONVERSION OF GALVANOMETER INTO AMMETER

A galvanometer can be converted into an ammeter by connecting low resistance in parallel to its coil.

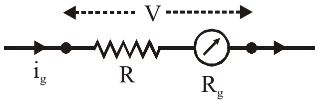
- The value of shunt resistance to be connected in parallel to galvanometer coil is given by : $R_S = \frac{R_g i_g}{i - i_g}$



Where i = Range of ammeter
i_g = Current required for full scale deflection of galvanometer.
R_g = Resistance of galvanometer coil.

CONVERSION OF GALVANOMETER INTO VOLTMETER

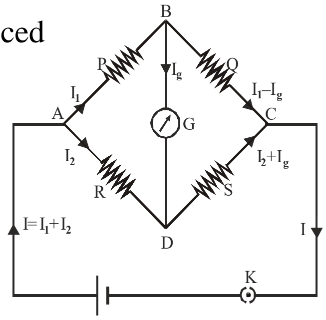
- The galvanometer can be converted into voltmeter by connecting high resistance in series with its coil.
- The high resistance to be connected in series with galvanometer coil is given by $R = \frac{V}{i_g} - R_g$



- The rate of variation of deflection depends upon the magnitude of deflection itself and so the accuracy of the instrument.
- A suspended coil galvanometer can measure currents of the order of 10^{-9} ampere.
- I_g is the current for full scale deflection. If the current for a deflection, of one division on the galvanometer scale is k and N is the total number of divisions on one side of the zero of galvanometer scale, then $I_g = k \times N$.
- A ballistic galvanometer is a specially designed moving coil galvanometer, used to measure charge flowing through the circuit for small time intervals.

WHEAT STONE BRIDGE

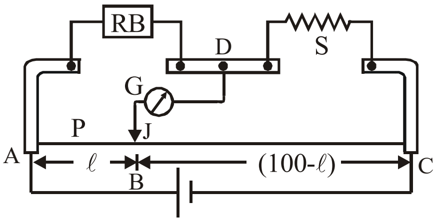
- The configuration in the adjacent figure is called Wheat Stone Bridge.
- If current in galvanometer is zero ($I_g = 0$) then bridge is said to be balanced



$$V_D = V_B \Rightarrow I_1P = I_2R \ \&\ I_1Q = I_2S \Rightarrow \frac{P}{Q} = \frac{R}{S}$$

- If $\frac{P}{Q} < \frac{R}{S}$ then $V_B > V_D$ and current will flow from B to D.
- If $\frac{P}{Q} > \frac{R}{S}$ then $V_B < V_D$ and current will flow from D to B.

METRE BRIDGE



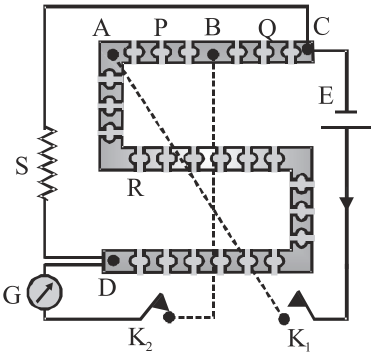
It is based on principle of whetstone bridge. It is used to find out unknown resistance of wire. AC is 1 m long uniform wire R.B. is known resistance and S is unknown resistance. A cell is connected across 1 m long wire and Galvanometer is connected between Jockey and midpoint D. To find out unknown resistance we touch jockey from A to C and find balance condition. Let balance is at B point on wire.

AB = ℓ cm	P = r ℓ
BC = $(100 - \ell)$ cm	Q = r$(100 - \ell)$
	where r = resistance per unit length on wire.

At balance condition : $\frac{P}{Q} = \frac{R}{S} \Rightarrow \frac{r\ell}{r(100-\ell)} = \frac{R}{S} \Rightarrow S = \frac{(100-\ell)}{\ell}R$

POST OFFICE BOX

It is also based on wheat stone bridge. The resistance of 10Ω, 100Ω, and 1000Ω are often connected between AB and BC. These are known as ratio arms. Resistance from 1Ω to 5000Ω are connected between A and D, this is known arm. Unknown resistance is connected between C and D.

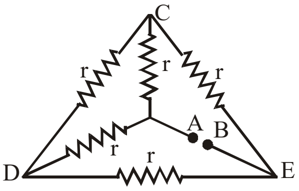


A cell is connected between A and C with key K_1 and Galvanometer is connected between B and D with key K_2.

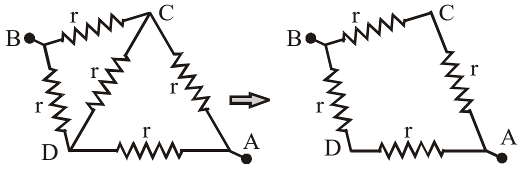
First we select ratio of resistance Q and P. For given value of S we will take value of resistance from known arm in such a way that Galvanometer show null deflection $S=\frac{Q}{P}R$. On decreasing the value of $\frac{Q}{P}$ the sensitivity of the box increases. It is used to find out the breakage in telegraph line in post and telegraph offices.

- To increase the range of an ammeter a shunt is connected in parallel with the galvanometer.
- To convert an ammeter of range I ampere and resistance R_g Ω into an ammeter of range nI ampere, the value of resistance to be connected in parallel will be $R_g(n-1)$
- To increase the range of a voltmeter a high resistance is connected in series with it.
- To convert a voltmeter of resistance R_g Ω and range V volt into a voltmeter of range nV volt, the value of resistance to be connected in series will be $(n-1)R_g$.
- Resistance of ideal ammeter is zero & resistance of ideal voltmeter is infinite.
- The bridge is most sensitive when the resistance in all the four branches of the bridge is of same order.

Ex. In the adjoining network of resistors each is of resistance r Ω. Find the equivalent resistance between point A and B



Sol. Given circuit is balanced Wheat stone Bridge



$$\therefore \frac{1}{R_{AB}}=\frac{1}{2r}+\frac{1}{2r}=\frac{1}{r}\ R_{AB}=r$$

Ex. A 100 volt voltmeter whose resistance is 20 kΩ is connected in series to a very high resistance R. When it is joined in a line of 110 volt, it reads 5 volt. What is the magnitude of resistance R ?

Sol. When voltmeter connected in 110 volt line, Current through the voltmeter $I=\frac{110}{(20\times10^3+R)}$

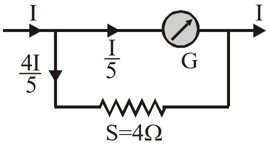
The potential difference across the voltmeter $V = IR_V$

$$\Rightarrow 5=\frac{110\times20\times10^3}{(20\times10^3+R)}$$

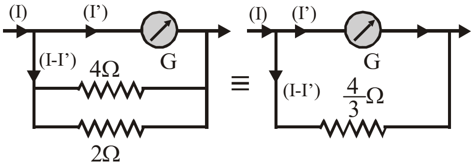
$\Rightarrow 20\times 10^3 + R = 440\times 10^3 \Rightarrow R = 420\times 10^3$ Ω

Ex. When a shunt of 4Ω is attached to a galvanometer, the deflection reduces to 1/5th. If an additional shunt of 2Ω is attached what will be the deflection ?

Sol. Initial condition : When shunt of 4Ω used $\frac{I}{5} \times G = \frac{4}{5} I \times 4 \Rightarrow G = 16\Omega$



When additional shunt of 2Ω used $I' \times 16 = (I - I')\frac{4}{3} \Rightarrow I' = \frac{I}{13}$



$\therefore$ it will reduce to $\frac{I}{13}$ of the initial deflection

Ex. A galvanometer having 30 divisions has current sensitivity of 20μA/division. It has a resistance of 25 Ω.

(i) How will you convert it into an ammeter measuring upto 1 ampere.

(ii) How will you convert this ammeter into a voltmeter upto 1 volt.

Sol The current required for full scale deflection $I_g = 20\ \mu A \times 30 = 600\ \mu A = 6 \times 10^{-4} A$

(i) To convert it into ammeter, a shunt is required in parallel with it

$$\text{shunt resistance } R'_S = \frac{I_g R_g}{(I - I_g)} = \left(\frac{6 \times 10^{-4}}{1 - 6 \times 10^{-4}}\right) 25 = 0.015\Omega$$

(ii) To convert galvanometer into voltmeter, a high resistance in series with it is required

$$\text{series resistance } R = \frac{V}{i_g} - R_g = \frac{1}{6 \times 10^{-4}} - 25 = 1666.67 - 25 = 1641.67\ \Omega$$

POTENTIOMETER

- **Necessity of potentiometer**

  Practically voltameter has a finite resistance. (ideally it should be ∞) in other words it draws some current from the circuit . To overcome this problem potentiometer is used because at the instant of measurement , it draws no current from the circuit. It means its effective resistance is infinite.

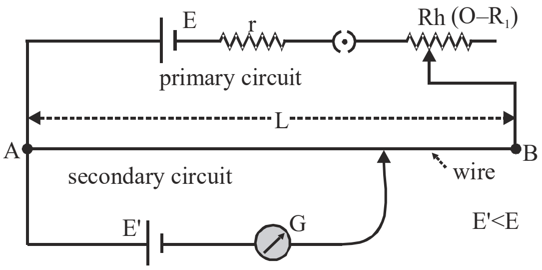
- **Working principle of potentiometer**

  Any unknown potential difference is balanced on a known potential difference which is uniformly distributed over entire length of potentiometer wire. This process is named as zero deflection or null deflection method.

- **Potentiometer wire**

  Made up of alloys of magnin, constantan, Eureka. Specific properties of these alloys are high specific resistance, negligible temperature co–efficient of resistance (α). Invariability of resistance of potentiometer wire over a long period.

CIRCUITS OF POTENTIOMETER



- **Primary circuit** contains constant source of voltage rheostat or Resistance Box
- **Secondary, Unknown or galvanometer circuit**

  Let ρ = Resistance per unit length of potentiometer wire
- **Potential gradient (x) (V/m)**

  The fall of potential per unit length of potentiometer wire is called potential gradient.

$$x=\frac{V}{L}=\frac{\text{current }\times\text{ resitance of potentiometer wire}}{\text{length of potentiometer wire}}=I\left(\frac{R}{L}\right)$$

  The potential gradient depends only on primary circuit and is independent of secondary circuit.
- **Applications of potentiometer**

• To measure potential difference across a resistance. • To find out emf of a cell • Comparison of two emfs E_1/E_2 • To find out internal resistance of a primary cell • Comparison of two resistance. • To find out an unknown resistance which is connected in series with the given resistance.	• To find out current in a given circuit • Calibration of an ammeter or to have a check on reading of (A) • Calibration of a voltmeter or to have a check on reading of (V) • To find out thermocouple emf (e_t) (mV or mV)

Different between potentiometer and voltmeter

Potentiomer	Voltmeter
• It measures the unknown emf very accurately	It measures the unknown emf approximately.
• While measuring emf it does not draw any current from the driving source of know emf.	While measuring emf it draws some current from the source of emf.
• While measuring unknown potential difference the resistance of potentiometer becomes infinite.	While measuring unknown potential difference the resistance of voltmeter is high but finite.
• It is based on zero deflection method.	It is based on deflection method.
• It has a high sensitivity.	Its sensitivity is low.
• it is used for various applications like measurement of internal restiance of cell, calibration of ammeter and voltmeter, measurement of thermo emf, comparison of emf's etc.	It is only used to measured emf or unknown potential difference.

Ex. There is a definite potential difference between the two ends of a potentiometer. Two cells are connected in such a way that first time help each other, and second time they oppose each other. They are balanced on the potentiometer wire at 120 cm and 60 cm length respectively. Compare the electromotive force of the cells.

Sol. Suppose the potential gradient along the potentiometer wire = x and the emf's of the two cells are E_1 and E_2.

When the cells help each other, the resultant emf = $(E_1 + E_2)$

$$E_1 + E_2 = x \times 120 \text{ cm} \quad \text{...(i)}$$

When the cells oppose each other, the resultant emf = $(E_1 - E_2)$

$$E_1 - E_2 = x \times 60 \text{ cm} \quad \text{...(ii)}$$

From equation (i) and (ii) $\dfrac{E_1+E_2}{E_1-E_2} = \dfrac{120\,\text{cm}}{60\text{cm}} = \dfrac{2}{1} \Rightarrow E_1 + E_2 = 2(E_1 - E_2) \Rightarrow 3E_2 = E_1 \Rightarrow \dfrac{E_1}{E_2} = \dfrac{3}{1}$

COLOUR CODE FOR CARBON RESISTORS

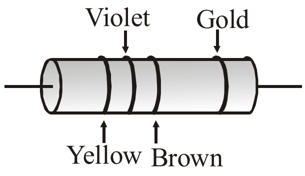
Colour	Strip A	Strip B	Strip C	Strip D (Tolerance)
Black	0	0	10^0	
Brown	1	1	10^1	
Red	2	2	10^2	
Orange	3	3	10^3	
Yellow	4	4	10^4	
Green	5	5	10^5	
Blue	6	6	10^6	
Violet	7	7	10^7	
Grey	8	8	10^8	
White	9	9	10^9	
Gold	-	-	10^{-1}	± 5 %
Silver	-	-	10^{-2}	± 10 %
No colour	-	-	-	± 20 %

Aid to memory BBROY
Great Britain
Very Good watch of gold & silver

Ex. Draw a colour code for 42 k Ω ± 10% carbon resistance.

Sol. According to colour code colour for digit 4 is yellow, for digit 2 it is red, for 3 colour is orange and 10% tolerance is represented by silver colour. So colour code should be yellow, red, orange and silver.

Ex. What is resistance of following resistor.



Sol. Number for yellow is 4, Number of violet is 7

Brown colour gives multiplier 10^1, Gold gives a tolerance of ± 5%

So resistance of resistor is $47 \times 10^1\ \Omega \pm 5\% = 470 \pm 5\%\ \Omega$.

EXERCISE # S-1

Microscophic analysis

1. A copper wire of length L, and cross section area A carries a current I. If the specific resistance of copper is ρ, the electric field in the wire is ______ .

CE0001

2. A copper wire carries a current density j (= current per unit area). Assuming that n = No. of free electrons per unit volume, e = electronic charge, <v> = average speed due to thermal agitation. The distance which will be covered by an electron during its displacement l along the wire _____.

CE0002

3. The total momentum of electrons in a straight wire of length ℓ carrying a current I is ______ (mass of electron = m_e, charge of electron = e)

CE0003

4. Two conductors are made of the same material and have the same length. Conductor A is a solid wire of diameter 1mm. Conductor B is a hollow tube of outer diameter 2mm and inner diameter 1mm. Find the ratio of resistance R_A to R_B.

CE0004

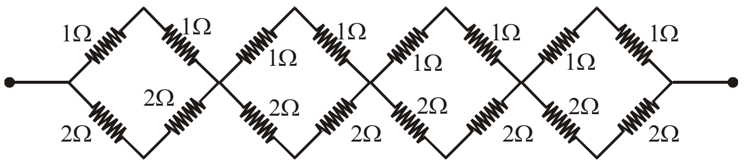
Ohm's law and circuit analysis

5. (a) Given n resistors each of resistance R, how will you combine them to get the (i) maximum (ii) minimum effective resistance? What Is the ratio of the maximum to minimum resistance?

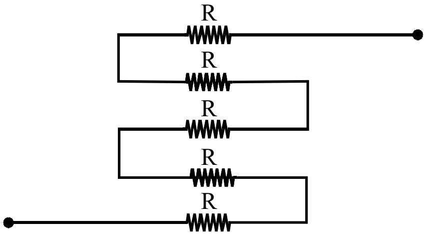
(b) Given the resistances of 1Ω, 2Ω, 3Ω, how will be combine them to get an equivalent resistance of

(i) (11/3) Ω (ii) (11/5) Ω (iii) 6Ω (iv) (6/11) Ω ?

(c) Determine the equivalent resistance of networks shown in figures



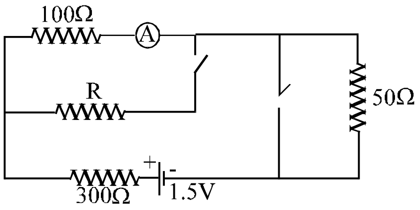
(a)



(b)

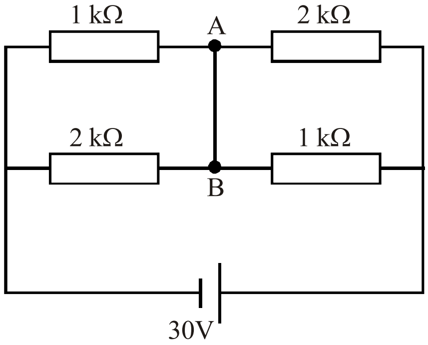
CE0005

6. In the circuit shown in figure the reading of ammeter is the same with both switches open as with both closed. Then find the resistance R. (ammeter is ideal)



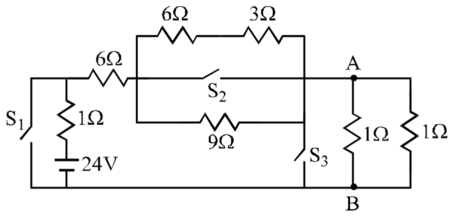
CE0006

7. Find the current (in mA) in the wire between points A and B.



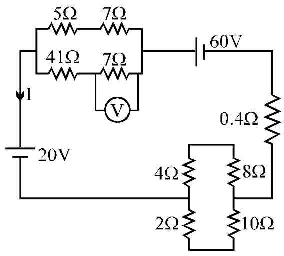
CE0007

8. If the switches S_1, S_2 and S_3 in the figure are arranged such that current through the battery is minimum, find the voltage across points A and B.



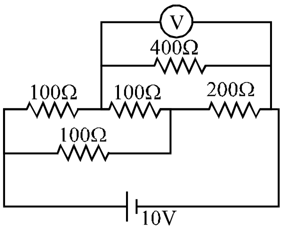
CE0008

9. Find the current I & voltage V in the circuit shown.



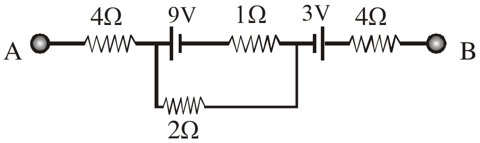
CE0009

10. An electrical circuit is shown in the figure. Calculate the potential difference across the resistance of 400 ohm, as will be measured by the voltmeter V of resistance 400 ohm, either by applying Kirchhoff's rules or otherwise.



CE0010

11. In the circuit shown in figure potential difference between point A and B is 16V. Find the current passing through 2Ω resistance.

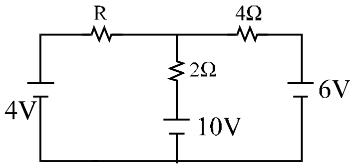


CE0011

12. An enquiring physics student connects a cell to a circuit and measures the current drawn from the cell to I_1. When he joins a second identical cell in series with the first, the current becomes I_2. When the cells are connected are in parallel, the current through the circuit is I_3. Show that relation between the current is $3 I_3 I_2 = 2 I_1 (I_2 + I_3)$

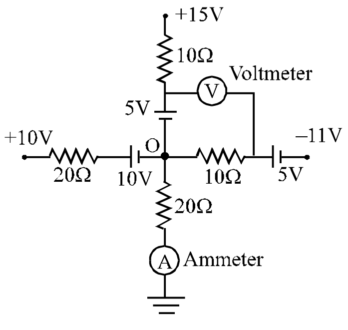
CE0012

13. For what value of R in circuit, current through 4Ω resistance is zero.



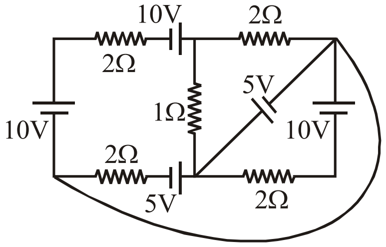
CE0013

14. The potential of certain points in the circuit are maintained at the values indicated. The Voltmeter and Ammeter are ideal. Find the potential of the cross junction point in the circuit (at center O) and the readings of Voltmeter and Ammeter. All cells are ideal.



CE0014

15. In the given circuit diagram, the current through the 1Ω resistor is given by I amp. Fill 2 I in OMR sheet.



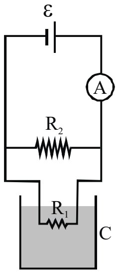
CE0015

Joule heating

16. If a cell of constant E.M.F. produces the same amount of the heat during the same time in two independent resistors R_1 and R_2, when they are separately connected across the terminals of the cell, one after the another, find the internal resistance of the cell.

CE0016

17. The coil of a calorimeter C has a resistance of $R_1 = 60\Omega$. The coil R_1 is connected to the circuit as shown in figure. What is the rise in temperature (°C) of 240 grams of water poured into the calorimeter when it is heated for 7 minutes during which a current flows through the coil and the ammeter shows 3A? The resistance $R_2 = 30\Omega$. [Disregard the resistances of the battery and the ammeter, and the heat losses and heat capacity of the calorimeter and the resistor and specific heat of water = 4200 J/kg°C]

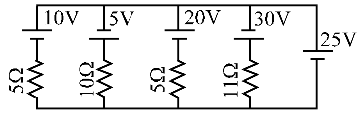


CE0017

18. An electric kettle has two windings. When one of them is switched on, the water in the kettle begins to boil in 15 minutes, and when the other is switched on it takes 30 minutes for water to boil. If the two windings are joined in series and switched on, water in the kettle begin to boil in $\frac{\alpha}{4}$ hr. Assuming no heat loss to the surrounding fill the value of α in OMR sheet.

CE0018

19. Find the current through 25V cell & power supplied by 20V cell in the figure shown.



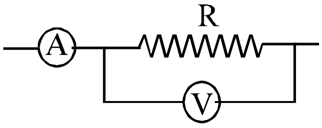
CE0019

20. A person decides to use his bath tub water to generate electric power to run a 40 watt bulb. The bath tub is located at a height of 10 m from the ground & it holds 200 litres of water. If we install a water driven wheel generator on the ground, at what rate should the water drain from the bath tub to light bulb? How long can we keep the bulb on, if the bath tub was full initially. The efficiency of generator is 90%.($g = 10 m/s^2$)

CE0020

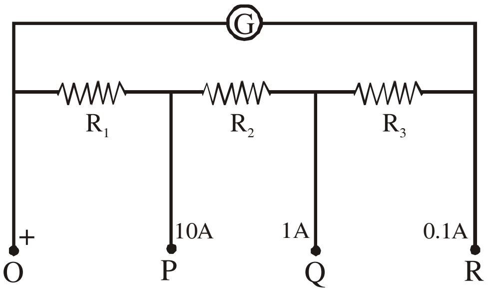
Instruments

21. A part of a circuit is shown in figure. Here reading of ammeter is 5 ampere and voltmeter is 96V & voltmeter resistance is 480 ohm. Then find the resistance R



CE0021

22. The resistance of the galvanometer G in the circuit is 25Ω. The meter deflects full scale for a current of 10 mA. The meter behaves as an ammeter of three different ranges. The range is 0–10A, if the terminals O and P are taken; range is 0–1 A between O and Q; range is 0–0.1 A between O and R. Calculate the resistance R_1, R_2 and R_3.



CE0022

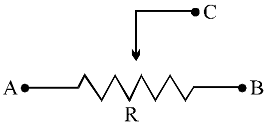
23. A galvanometer having 50 divisions provided with a variable shunt is used to measure the current as an ammeter when connected in series with a resistance of 90 Ω and a battery of internal resistance 10Ω. It is observed that when the shunt resistance are 10Ω, 50Ω, respectively the deflection are respectively 9 & 30 divisions. What is the resistance of the galvanometer? Further if the full scale deflection of the galvanometer movement is 300 mA, find the emf of the cell.

CE0023

24. Draw the circuit for experimental verification of Ohm's law using a source of variable D.C. voltage, a main resistance of 100 Ω, two galvanometers and two resistances of values 10^6 Ω and 10^{-3} Ω respectively. Clearly show the positions of the voltmeter and the ammeter. **[IIT-JEE' 2004]**

CE0024

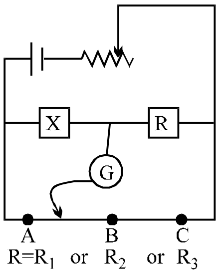
25. How a battery is to be connected so that shown rheostat will behave like a potential divider? Also indicate the points about which output can be taken. **[IIT-JEE' 2003]**



CE0025

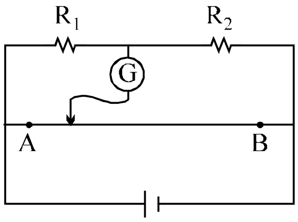
26. An unknown resistance X is to be determined using resistances R_1, R_2 or R_3. Their corresponding null points are A, B and C. Find which of the above will give the most accurate reading and why?

[IIT-JEE 2005]



CE0026

27. In the figure shown for which values of R_1 and R_2 the balance point for Jockey is at 40 cm from A. When R_2 is shunted by a resistance of 10 Ω, balance shifts to 50 cm. Find R_1 and R_2. (AB = 1 m):



CE0027

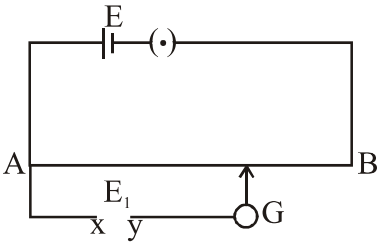
28. While doing an experiment with potentiometer it was found that the deflection is one sided and two casses are possible

(i) the deflection decreased while moving from one end A of the wire to the end B;

(ii) the deflection increased. while the jockey was moved towards the end B. Then

(a) Which terminal +or –ve of the cell E_1, is connected at X in case (i) and how is E_1 related to E?

(b) Which terminal of the cell E_1 is connected at X in case (ii)?

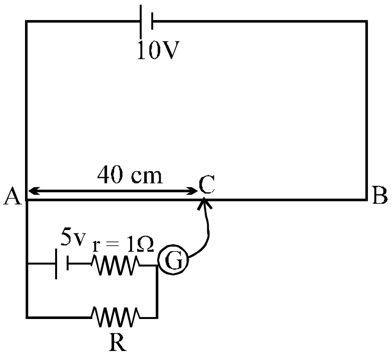


CE0028

29. In a potentiometer arrangement, a cell of emf 1.25 V gives a balance point at 35.0 cm length of the wire. If the cell is replaced by another cell and the balance point shifts to 63.0 cm, what is the emf of the second cell ?

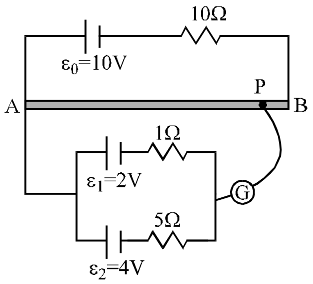
CE0029

30. A potentiometer wire AB is 100 cm long and has a total resistance of 10 ohm. If the galvanometer shows zero deflection at the position C, then find the value of unknown resistance R.



CE0030

31. A battery of emf $\varepsilon_0 = 10$ V is connected across a 1 m long uniform wire having resistance 10Ω/m. Two cells of emf $\varepsilon_1 = 2$V and $\varepsilon_2 = 4$V having internal resistances 1Ω and 5Ω respectively are connected as shown in the figure. If a galvanometer shows no deflection at the point P, find the distance of point P from the point A.



CE0031

EXERCISE # S-2

1. A long cylinder with uniformly charged surface and cross sectional radius a = 1.0 cm moves with a constant velocity v = 10 m/s, along its axis. An electric field strength at the surface of the cylinder is equal to E = 0.9 KV/cm. Find the resulting covection current, that is, the current caused by mechanical transfer of charge.

CE0032

2. A resistance R of thermal coefficient of resistivity = α is connected in parallel with a resistance = 3R, having thermal coefficient of resistivity = 2α. Find the value of α_{eff}.

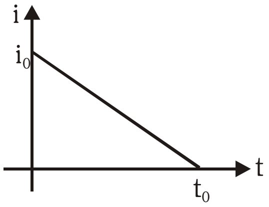
CE0033

3. Relation between current in conductor and time is shown in figure then determine.

(i) Total charge flow through the conductor

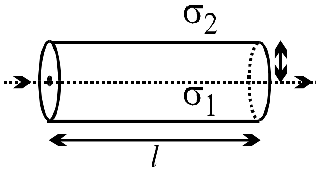
(ii) Write expression of current in terms of time

(iii) If resistance of conductor is R then total heat dissipated across resistance R is



CE0034

4. A long conductor of circular cross-section has radius r and lengh l as shown in the figure. The conductivity of the material near the axis is σ_1 and increases linearly with the distance from axis and becomes σ_2 near the surface. Find the resistance of the conductor if the current enters from the one end and leaves from the other end.



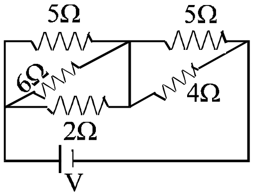
CE0035

5. (a) The current density across a cylindrical conductor of radius R varies according to the equation $J = J_0\left(1-\frac{r}{R}\right)$, where r is the distance from the axis. Thus the current density is a maximum J_0 at the axis r = 0 and decreases linearly to zero at the surface r = R. Calculate the current in terms of J_0 and the conductor's cross sectional area is $A = \pi R^2$.

(b) Suppose that instead the current density is a maximum J_0 at the surface and decreases linearly to zero at the axis so that $J = J_0\frac{r}{R}$. Calculate the current.

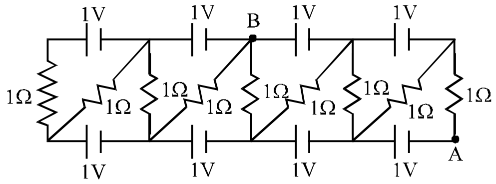
CE0036

6. Find the resistor in which maximum heat will be produced.



CE0037

7. Find the potential difference $V_A - V_B$ for the circuit shown in the figure.



CE0038

8. A galvanometer (coil resistance 99Ω) is converted into a ammeter using a shunt of 1 Ω and connected as shown in the figure (i). The ammeter reads 3A. The same galvanometer is converted into a voltmeter by connected a resistance of 101Ω in series. This voltmeter is connected as shown in figure (ii). Its reading is found to be 4/5 of the full scale reading. Find

(i) internal resistance r of the cell

(ii) range of the ammeter and voltmeter

(iii) full scale deflection current of the galvanometer

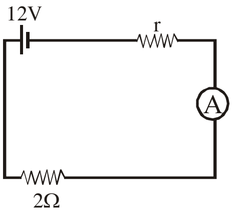


Fig. (i)

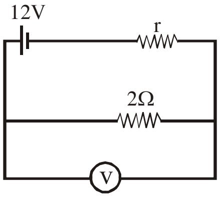


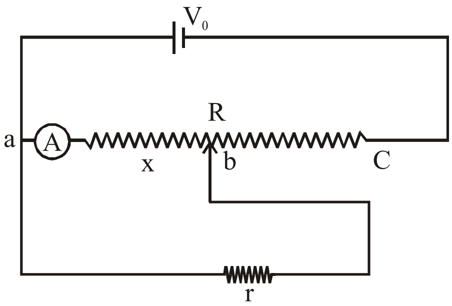
Fig. (ii)

CE0039

9. An accumulator of emf 2 Volt and negligible internal resistance is connected across a uniform wire of length 10m and resistance 30Ω. The appropriate terminals of a cell of emf 1.5 Volt and internal resistance 1Ω is connected to one end of the wire, and the other terminal of the cell is connected through a sensitive galvanometer to a slider on the wire. What length of the wire will be required to produce zero deflection of the galvanometer ? How will the balancing change (a) when a coil of resistance 5Ω is placed in series with the accumulator, (b) the cell of 1.5 volt is shunted with 5Ω resistor?

CE0040

10. A constant voltage V_0 (= 12V) is applied to a potential divider of resistance R (= 4Ω), connected to an ideal ammeter. A constant resistor r (= 2 Ω) is connected to the sliding contact of the potential divider (as shown). Find the minimum current (in A) measured by ammeter.



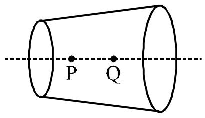
CE0041

EXERCISE # O-1

SINGLE CORRECT TYPE QUESTIONS

Microscopic analysis

1. A wire has a non-uniform cross-section as shown in figure. A steady current flows through it. The drift speed of electrons at points P and Q is v_P and v_Q.



(A) $v_P = v_Q$ (B) $v_P < v_Q$ (C) $v_P > v_Q$ (D) Data insufficient

CE0042

2. Two wires each of radius of cross section r but of different materials are connected together end to end (in series). If the densities of charge carriers in the two wires are in the ratio 1 : 4, the drift velocity of electrons in the two wires will be in the ratio:

(A) 1 : 2 (B) 2 : 1 (C) 4 : 1 (D) 1 : 4

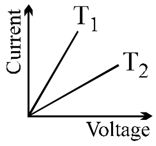
CE0043

3. An insulating pipe of cross-section area 'A' contains an electrolyte which has two types of ions→ their charges being –e and +2e. A potential difference applied between the ends of the pipe result in the drifting of the two types of ions, having drift speed = v (–ve ion) and v/4 (+ve ion). Both ions have the same number per unit volume = n. The current flowing through the pipe is

(A) nev A/2 (B) nev A/4 (C) 5nev A/2 (D) 3nev A/2

CE0044

4. The current in a metallic conductor is plotted against voltage at two different temperatures T_1 and T_2. Which is correct



(A) $T_1 > T_2$ (B) $T_1 < T_2$

(C) $T_1 = T_2$ (D) none

CE0045

5. A metal rod of length 10 cm and a rectangular cross-section of 1 cm $\times \frac{1}{2}$ cm is connected to a battery across opposite faces. The resistance will be

(A) maximum when the battery is connected across 1 cm $\times \frac{1}{2}$ cm faces.

(B) maximum when the battery is connected across 10 cm × 1 cm faces.

(C) maximum when the battery is connected across 10 cm $\times \frac{1}{2}$ cm faces.

(D) same irrespective of the three faces.

CE0046

6. Consider a current carrying wire (current I) in the shape of a circle. Note that as the current progresses along the wire, the direction of **j** (current density) changes in an exact manner, while the current I remain unaffected. The agent that is *essentially* responsible for is

(A) source of emf.

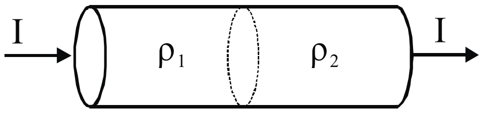
(B) electric field produced by charges accumulated on the surface of wire.

(C) the charges just behind a given segment of wire which push them just the right way by repulsion.

(D) the charges ahead.

CE0047

7. Two long straight cylindrical conductors with resistivities ρ_1 and ρ_2 respectively are joined together as shown in figure. If current I flows through the conductors, the magnitude of the total free charge at the interface of the two conductors is :-



(A) zero (B) $\dfrac{(\rho_1-\rho_2)I\varepsilon_0}{2}$ (C) $\varepsilon_0 I|\rho_1-\rho_2|$ (D) $\varepsilon_0 I|\rho_1+\rho_2|$

CE0048

8. **Statement 1 :** The drift speed of electrons in metals is small (in the order of a few mm/s) and the charge of an electron is also very small ($= 1.6 \times 10^{-19}$C), yet we can obtain a large current in a metal.

and

Statement 2: At room temperature, the thermal speed of electron is very high (about 10^7 times the drift speed).

(A) Statement–1 is True, Statement–2 is True ; Statement–2 is a correct explanation for Statement–1.

(B) Statement–1 is True, Statement–2 is True ; Statement–2 is not a correct explanation for Statement–1.

(C) Statement–1 is True, Statement–2 is False.

(D) Statement–1 is False, Statement–2 is True.

CE0049

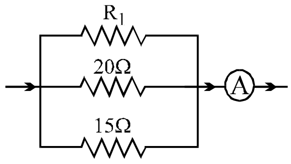
Ohm's law & circuit analysis

9. A storage battery is connected to a charger for charging with a voltage of 12.5Volts. The internal resistance of the storage battery is 1Ω. When the charging current is 0.5 A, the emf of the storage battery is:

(A) 13 Volts (B) 12.5 Volts (C) 12 Volts (D) 11.5 Volts

CE0050

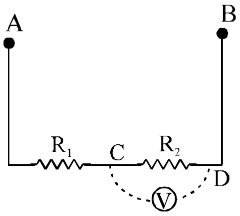
10. In the given circuit the current flowing through the resisitance 20 ohms is 0.3 ampere while the ammetre reads 0.8 ampere. What is the value of R_1?



(A) 30 ohms (B) 40 ohms (C) 50 ohms (D) 60 ohms

CE0051

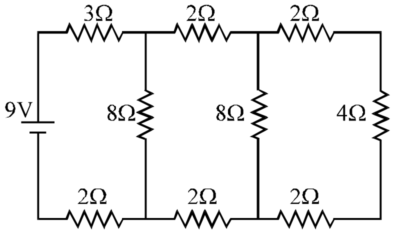
11. Resistances R_1 and R_2 each 60Ω are connected in series as shown in figure. The Potential difference between A and B is kept 120 volt. Then what will be the reading of voltmeter connected between the point C & D if resistance of voltmeter is 120Ω.



(A) 48 V (B) 24 V (C) 40V (D) None

CE0052

12. In the circuit shown in the figure, the current through :



(A) the 3Ω resistor is 0.50 A (B) the 3Ω resistor is 0.25 A
(C) $4\ \Omega$ resistor is 0.50 A (D) the 4Ω resistor is 0.25 A

CE0053

13. A simple circuit contains an ideal battery and a resistance R. If a second resistor is placed in parallel with the first,

(A) the potential across R will decrease
(B) the current through R will decreased
(C) the current delivered by the battery will increase
(D) the power dissipated by R will increased.

CE0054

14. The equivalent resistance of a group of resistances is R. If another resistance is connected in parallel to the group, its new equivalent becomes R_1 & if it is connected in series to the group, its new equivalent becomes R_2 we have :

(A) $R_1 > R$ (B) $R_1 < R$ (C) $R_2 > R$ (D) $R_2 < R$

CE0055

15. An energy source will supply a constant current into the load, if its internal resistance is-

[AIEEE - 2005]

(A) equal to the resistance of the load (B) very large as compared to the load resistance
(C) zero (D) non-zero but less than the resistance of the load

CE0056

16. When a current of 4 A flows within a battery from its positive to negative terminal, the potential difference across the battery is 12 volts. The potential difference across the battery is 9 volts when a current of 2 A flows within it from its negative to its positive terminal. The internal resistance and the e.m.f. of the battery are :-

(A) $0.1\ \Omega$, 4V (B) $0.2\ \Omega$, 5V (C) $0.5\ \Omega$, 10V (D) $0.7\ \Omega$, 10V

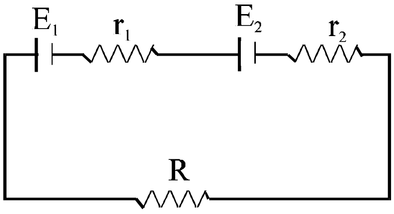
CE0057

17. Two resistances of equal magnitude R and having temperature coefficient α_1 and α_2 respectively are connected in parallel. The temperature coefficient of the parallel combination is, approximately

(A) $2(\alpha_1+\alpha_2)$ (B) $\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}$ (C) $\frac{\alpha_1-\alpha_2}{2}$ (D) $\frac{\alpha_1+\alpha_2}{2}$

CE0058

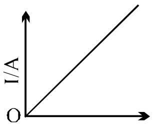
18. Under what condition current passing through the resistance R can be increased by short circuiting the battery of emf E_2. The internal resistances of the two batteries are r_1 and r_2 respectively.

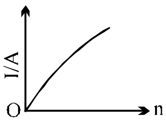


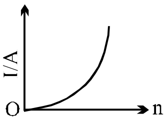
(A) $E_2r_1 > E_1(R + r_2)$ (B) $E_1r_2 > E_2(R + r_1)$
(C) $E_2r_2 > E_1(R + r_2)$ (D) $E_1r_1 > E_2(R + r_1)$

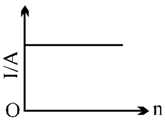
CE0059

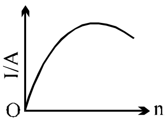
19. A battery consists of a variable number n of identical cells having internal resistance connected in series. The terminals of the battery are short circuited and the current I measured. Which one of the graph below shows the relationship between I and n?

(A) 

(B) 

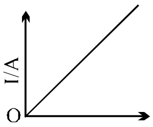
(C) 

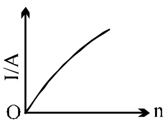
(D) 

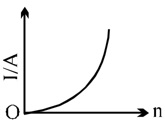
(E) 

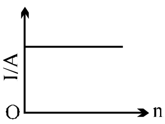
CE0060

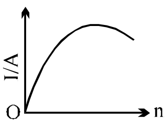
20. In **previous problem**, if the cell had been connected in parallel (instead of in series) which of the above graphs would have shown the relationship between total current I and n?

(A) 

(B) 

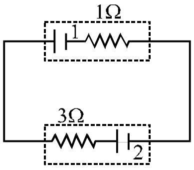
(C) 

(D) 

(E) 

CE0061

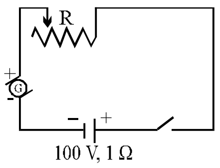
21. In the figure shown, battery 1 has emf = 6 V and internal resistance = 1 Ω. Battery 2 has emf = 2V and internal resistance = 3 Ω. The wires have negligible resistance. What is the potential difference across the terminals of battery 2 ?



(A) 4 V (B) 1.5 V (C) 5 V (D) 0.5 V

CE0062

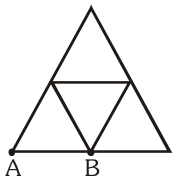
22. The battery in the diagram is to be charged by the generator G. The generator has a terminal voltage of 120 volts when the charging current is10 amperes. The battery has an emf of 100 volts and an internal resistance of 1 ohm. In order to charge the battery at 10 amperes charging current, the resistance R should be set at :-



(A) 0.1 Ω (B) 0.5 Ω (C) 1.0 Ω (D) 5.0 Ω

CE0063

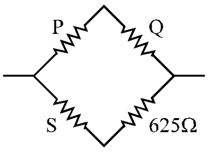
23. In the diagram resistance between any two junctions is R. Equivalent resistance across terminals A and B is :-



(A) $\frac{11R}{7}$ (B) $\frac{18R}{11}$ (C) $\frac{7R}{11}$ (D) $\frac{11R}{18}$

CE0064

24. A Wheatstone's bridge is balanced with a resistance of 625 Ω in the third arm, where P, Q and S are in the 1st, 2nd and 4th arm respectively. If P and Q are interchanged, the resistance in the third arm has to be increased by 51Ω to secure balance. The unknown resistance in the fourth arm is :-



(A) 625 Ω (B) 650 Ω (C) 676 Ω (D) 600 Ω

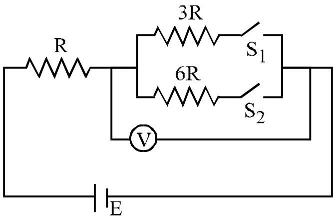
CE0065

25. One end of a Nichrome wire of length 2L and cross-sectional area A is attatched to an end of another Nichrome wire of length L and cross-sectional area 2A. If the free end of the longer wire is at an electric potential of 8.0 volts, and the free end of the shorter wire is at an electric potential of 1.0 volt, the potential at the junction of the two wires is equal to :-

(A) 2.4 V (B) 3.2 V (C) 4.5 V (D) 5.6 V

CE0066

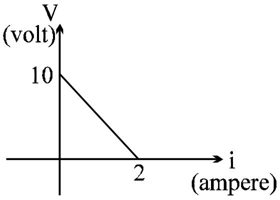
26. In the circuit shown in figure reading of voltmeter is V_1 when only S_1 is closed, reading of voltmeter is V_2 when only S_2 is closed. The reading of voltmeter is V_3 when both S_1 and S_2 are closed then



(A) $V_2 > V_1 > V_3$ (B) $V_3 > V_2 > V_1$ (C) $V_3 > V_1 > V_2$ (D) $V_1 > V_2 > V_3$

CE0067

27. A battery of emf E and internal resistance r is connected across a resistance R. Resistance R can be adjusted to any value greater than or equal to zero. A graph is plotted between the current (i) passing through the resistance and potential difference (V) across it. Select the correct alternative.



(A) internal resistance of battery is 5Ω

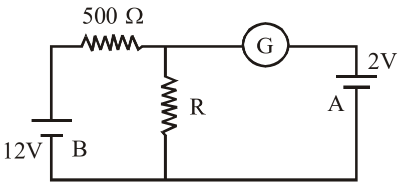
(B) emf of the battery is 20V

(C) maximum current which can be taken from the battery is 4A

(D) V- i graph can never be a straight line as shown in figure.

CE0068

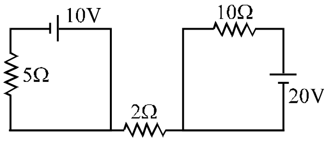
28. In the circuit, the galvanometer G shows zero deflection. If the batteries A and B have negligible internal resistance, the value of the resistor R will be- **[AIEEE - 2005]**



(A) 200 Ω (B) 100 Ω (C) 500 Ω (D) 1000 Ω

CE0069

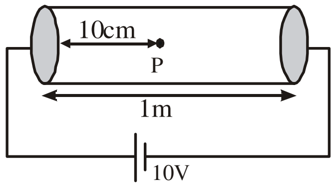
29. In the figure shown the current through 2Ω resistor is **[IIT-JEE 2005 (Scr)]**



(A) 2 A (B) 0 A (C) 4 A (D) 6 A

CE0070

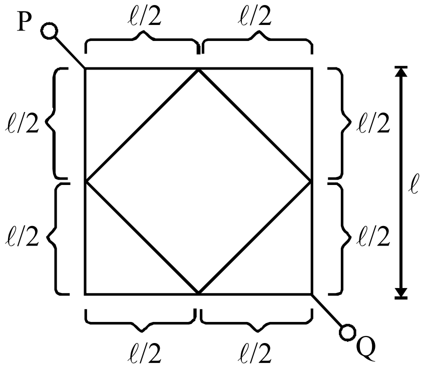
30. A cylindrical solid of length 1m and radius 1 m is connected across a source of emf 10V and negligible internal resistance shown in figure. The resistivity of the rod as a function of x (x measured from left end) is given by $\rho = bx$ [where b is a positive constant]. Find the electric field (in SI unit) at point P at a distance 10 cm from left end.



(A) 1 (B) 2 (C) 3 (D) 4

CE0071

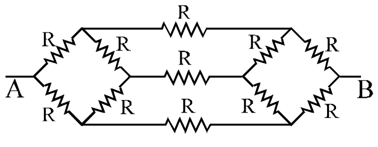
31. If the wire has resistivity ρ and cross sectional area A, the equivalent resistance between P and Q is :-



(A) $\frac{\rho\ell}{\sqrt{2}A}$ (B) $\frac{\sqrt{2}\rho\ell}{A}$ (C) $\frac{2\rho\ell}{A}$ (D) $\frac{\rho\ell}{A}$

CE0072

32. The equivalent resistance between the terminal points A and B in the network shown in figure is :-



(A) $\frac{7R}{5}$ (B) $\frac{5R}{6}$ (C) $\frac{7R}{12}$ (D) $\frac{5R}{12}$

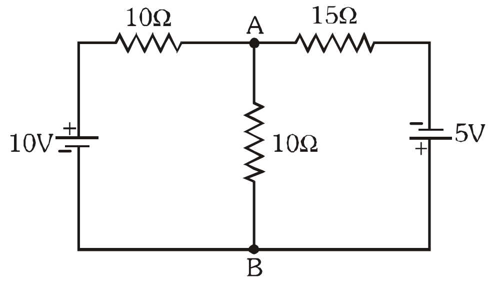
CE0073

33. An electric bell has a resistance of 5Ω and requires a current of 0.25 A to work it. Assuming that the resistance of the bell wire is 1Ω per 15m and that the bell push is 90m distance from the bell. How many cells each of emf 1.4V and internal resistance 2Ω, will be required to work the circuit-

(A) 3 (B) 4 (C) 5 (D) Can't be determined

CE0074

34. A circuit is arranged as shown. Then, the current from A to B is



(A) +500 mA (B) +250 mA (C) –250 mA (D) –500 mA

CE0075

Joule heating

35. Power generated across a uniform wire connected across a supply is H. If the wire is cut into n equal parts and all the parts are connected in parallel across the same supply, the total power generated in the wire is

(A) $\frac{H}{n^2}$ (B) n^2H (C) nH (D) $\frac{H}{n}$

CE0076

36. When electric bulbs of same power, but different marked voltage are connected in series across the power line, their brightness will be :

(A) proportional to their marked voltage

(B) inversely proportional to their marked voltage

(C) proportional to the square of their marked voltage

(D) inversely proportional to the square of their marked voltage

CE0077

37. Two bulbs rated (25 W – 220V) and (100W – 220V) are connected in series to a 440 V line. Which one is likely to fuse?

(A) 25 W bulb (B) 100 W bulb (C) both bulbs (D) none

CE0078

38. Rate of dissipation of Joule's heat in resistance per unit volume is (symbols have usual meaning)

(A) σ E (B) σ J (C) J E (D) None

CE0079

39. If the length of the filament of a heater is reduced by 10%, the power of the heater will

(A) increase by about 9% (B) increase by about 11%

(C) increase by about 19% (D) decrease by about 10%

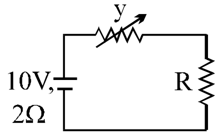
CE0080

40. Two bulbs one of 200 volts, 60 watts & the other of 200 volts, 100 watts are connected in series to a 200 volt supply. The power consumed will be

(A) 37.5 watt (B) 160 watt (C) 62.5 watt (D) 110 watt

CE0081

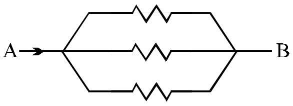
41. In the figure shown the power generated in y is maximum when y = 5Ω. Then R is :-



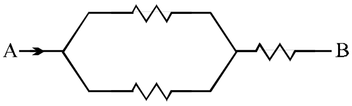
(A) 2 Ω (B) 6 Ω (C) 5 Ω (D) 3 Ω

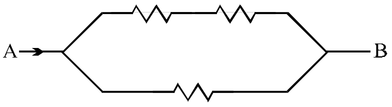
CE0082

42. Arrange the order of power dissipated in the given circuits, if the same current is passing through all circuits and each resistor is 'r' **[IIT-JEE' 2003 (Scr)]**

(1) 

(2) 

(3) 

(4) 

(A) $P_2 > P_3 > P_4 > P_1$ (B) $P_3 > P_2 > P_4 > P_1$

(C) $P_4 > P_3 > P_2 > P_1$ (D) $P_1 > P_2 > P_3 > P_4$

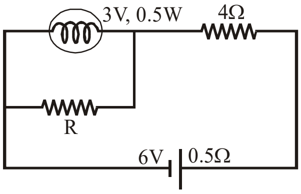
CE0083

43. A rigid container with thermally insulated walls contains a coil of resistance 100 Ω, carrying current 1 A. Change in internal energy after 5 min will be **[IIT-JEE 2005]**

(A) zero (B) 10 kJ (C) 20 kJ (D) 30 kJ

CE0084

44. The value of the resistance R in the circuit shown below so that electric bulb consumes the rated power is :-



(A) 4 ohm (B) 6 ohm (C) 8 ohm (D) 10 ohm

CE0085

45. A variable load R is connected to a voltage source of internal resistance r. Then choose the **INCORRECT** statement out of the following :-

(A) If R = r, maximum power is transferred to the load

(B) If current is maximum, power transfer to load is also maximum

(C) If R << r, the voltage source supplied a fixed current to the load

(D) Power supplied to load is minimum if load is either too low or too high

CE0086

46. The variation of current (I) and voltage (V) is as shown in figure A. The variation of power P with current *I* is best shown by which of the following graph

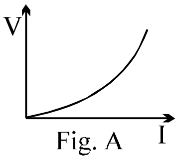
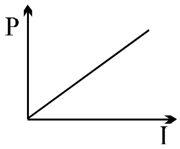
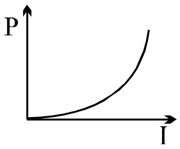
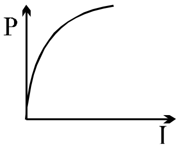
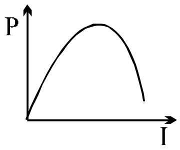


Fig. A

(A) 

(B) 

(C) 

(D) 

CE0087

Instruments

47. A galvanometer has a resistance of 20Ω and reads full-scale when 0.2 V is applied across it. To convert it into a 10 A ammeter, the galvanometer coil should have a

(A) 0.01Ω resistor connected across it
(B) 0.02Ω resistor connected across it
(C) 200Ω resistor connected in series with it
(D) 2000 Ω resistor connected in series with it

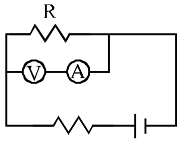
CE0088

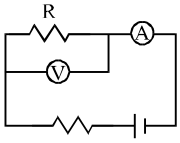
48. A galvanometer coil has a resistance 90 Ω and full scale deflection current 10 mA. A 910Ω resistance is connected in series with the galvanometer to make a voltmeter. If the least count of the voltmeter is 0.1V, the number of divisions on its scale is

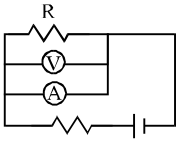
(A) 90
(B) 91
(C) 100
(D) none

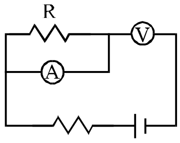
CE0089

49. Which of the following wiring diagrams could be used to experimentally determine R using ohm's law? Assume an ideal voltmeter and an ideal ammeter.

(A) 

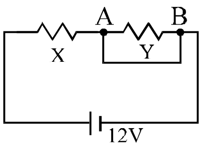
(B) 

(C) 

(D) 

CE0090

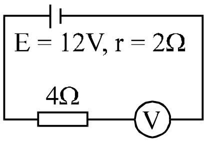
50. When an ammeter of negligible internal resistance is inserted in series with circuit it reads 1A. When the voltmeter of very large resistance is connected across X it reads 1V. When the point A and B are shorted by a conducting wire, the voltmeter measures 10 V across the battery. The internal resistance of the battery is equal to :-



(A) zero
(B) 0.5 Ω
(C) 0.2 Ω
(D) 0.1 Ω

CE0091

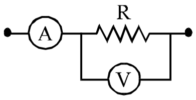
51. By error, a student places moving-coil voltmeter V (nearly ideal) in series with the resistance in a circuit in order to read the current, as shown. The voltmeter reading will be



(A) 0
(B) 4V
(C) 6V
(D) 12V

CE0092

52. In the circuit shown the readings of ammeter and voltmeter are 4A and 20V respectively. The meters are non ideal, then R is :



(A) 5Ω (B) less than 5Ω

(C) greater than 5Ω (D) between 4Ω & 5Ω

CE0093

53. ln a balanced wheat stone bridge, current in the galvanometer is zero. It remains zero when:

[1] battery emf is increased

[2] all resistances are increased by 10 ohms

[3] all resistances are made five times

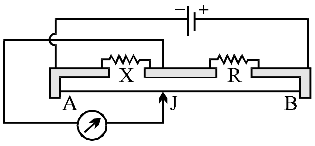
[4] the battery and the galvanometer are interchanged

(A) only [1] is correct (B) [1], [2] and [3] are correct

(C) [1], [3] and [4] are correct (D) [1] and [3] are correct

CE0094

54. The figure shows a metre-bridge circuit, with AB = 100 cm, X = 12Ω and R = 18Ω, and the jockey J in the position of balance. If R is now made 8Ω, through what distance will J have to be moved to obtain balance?



(A) 10 cm (B) 20 cm (C) 30 cm (D) 40 cm

CE0095

55. In a metre bridge experiment, null point is obtained at 20 cm from one end of the wire when resistance X is balanced against another resistance Y. If X < Y, then where will be the new position of the null point from the same end, if one decides to balance a resistance of 4X against Y? **[AIEEE - 2004]**

(A) 50 cm (B) 80 cm (C) 40 cm (D) 70 cm

CE0096

56. A resistance *R* is to be measured using a meter bridge. Student chooses the standard resistance *S* to be 100Ω. He finds the null point at $l_1 = 2.9$ cm. He is told to attempt to improve the accuracy.

Which of the following is a useful way?

(A) He should measure l_l more accurately.

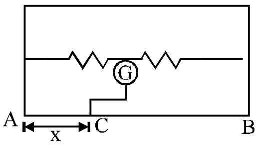
(B) He should change *S* to 1000Ω and repeat the experiment.

(C) He should change *S* to 3Ω and repeat the experiment.

(D) He should give up hope of a more accurate measurement with a meter bridge.

CE0097

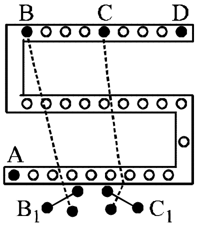
57. In the given circuit, no current is passing through the galvanometer. If the cross-sectional diameter of AB is doubled then for null point of galvanometer the value of AC would **[IIT-JEE' 2003 (Scr)]**



(A) x (B) x/2 (C) 2x (D) None

CE0098

58. For the post office box arrangement to determine the value of unknown resistance, the unknown resistance should be connected between **[IIT-JEE' 2004 (Scr)]**



(A) *B* and *C* (B) *C* and *D* (C) *A* and *D* (D) B_1 and C_1

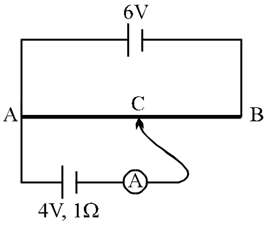
CE0099

59. A potentiometer wire has length 10 m and resistance 10Ω. It is connected to a battery of EMF 11 volt and internal resistance 1 Ω, then the potential gradient in the wire is :-

(A) 10 V/m (B) 1 V/m (C) 0.1 V/m (D) none

CE0100

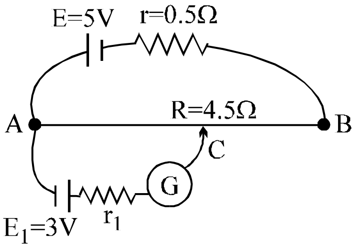
60. A 6 V battery of negligible internal resistance is connected across a uniform wire of length 1 m. The positive terminal of another battery of emf 4V and internal resistance 1 Ω is joined to the point A as shown in figure. The ammeter shows zero deflection when the jockey touches the wire at the point C. The AC is equal to :-



(A) 2/3 m (B) 1/3 m (C) 3/5 m (D) 1/2 m

CE0101

61. In the given potentiometer circuit length of the wire AB is 3 m and resistance is R = 4.5 Ω. The length AC for no deflection in galvanometer is :-



(A) 2 m (B) 1.8 m (C) dependent on r_1 (D) none of these

CE0102

62. The length of a potentiometer wire is ℓ. A cell of emf E is balanced at a length $\ell/3$ from the positive end of the wire. If the length of the wire is increased by $\ell/2$. At what distance will the same cell give a balance point.

(A) $\frac{2\ell}{3}$ (B) $\frac{\ell}{2}$ (C) $\frac{\ell}{6}$ (D) $\frac{4\ell}{3}$

CE0103

63. Two cells of emf's approximately 5V and 10V are to be accurately compared using a potentiometer of length 400cm.

(A) The battery that runs the potentiometer should have voltage of 8V.

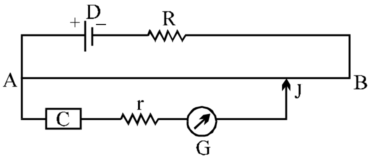
(B) The battery of potentiometer can have a voltage of 15V and *R* adjusted so that the potential drop across the wire slightly exceeds 10V.

(C) The first portion of 50 cm of wire itself should have a potential drop of 10V.

(D) Potentiometer is usually used for comparing resistances and not voltages.

CE0104

64. In the given potentiometer circuit, the resistance of the potentiometer wire AB is R_0. C is a cell of internal resistance r. The galvanometer G does not give zero deflection for any position of the jockey J. Which of the following cannot be a reason for this ?



(A) $r > R_0$ (B) $R >> R_0$

(C) emf of C > emf of D (D) The negative terminal of C is connected to A.

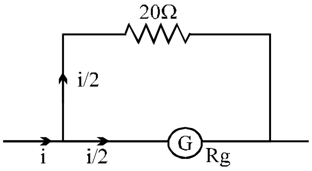
CE0105

65. A resistor has a color code of green, blue; brown and silver. What is its resistance?

(A) 56 Ω ± 5% (B) 560 Ω ± 10% (C) 560 Ω ± 5% (D) 5600 Ω ± 10%

CE0106

66. In a galvanometer, the deflection becomes one half when the galvanometer is shunted by a 20Ω resistor. The galvanometer resistance is

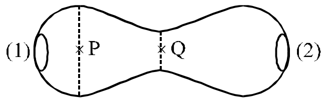


(A) 5Ω (B) 10Ω (C) 40Ω (D) 20Ω

CE0107

MULTIPLE CORRECT TYPE QUESTIONS

67. A metallic conductor of irregular cross-section is as shown in the figure. A constant potential difference is applied across the ends (1) and (2). Then :



(A) the current at the cross-section P equals the current at the cross-section Q

(B) the electric field intensity at P is less than that at Q.

(C) the rate of heat generated per unit time at Q is greater than that at P

(D) the number of electrons crossing per unit area of cross-section at P is less than that at Q.

CE0108

68. A current passes through an ohmic conductor of nonuniform cross section. Which of the following quantities are independent of the cross-section?

(A) the charge crossing in a given time interval.

(B) drift speed

(C) current density

(D) free-electron density

CE0109

69. A battery is of emf E is being charged from a charger such that positive terminal of the battery is connected to terminal A of charger and negative terminal of the battery is connected to terminal B of charger. The internal resistance of the battery is r.

(A) Potential difference across points A and B must be more than E.

(B) A must be at higher potential than B

(C) In battery, current flows from positive terminal to the negative terminal

(D) No current flows through battery

CE0110

70. Two identical fuses are rated at 10A. If they are joined

(A) in parallel, the combination acts as a fuse of rating 20A

(B) in parallel, the combination acts as a fuse of rating 5A

(C) in series, the combination acts as a fuse of rating 10A.

(D) in series, the combination acts as a fuse of rating 20A.

CE0111

71. A micrometer has a resistance of 100Ω and a full scale range of 50μA. It can be used as a voltmeter or a higher range ammeter provided a resistance is added to it. Pick the correct range and resistance combination(s).

(A) 50 V range with 10 kΩ resistance in series.

(B) 10 V range with 200 kΩ resistance in series.

(C) 5 mA range with 1 Ω resistance in parallel.

(D) 10 mA range with 1 kΩ resistance in parallel.

CE0112

72. Mark out the correct options.

(A) An ammeter should have small resistance.

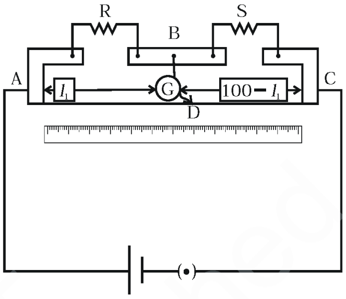
(B) An ammeter should have large resistance.

(C) A voltmeter should have small resistance.

(D) A voltmeter should have large resistance.

CE0113

73. In a meter bridge the point D is a neutral point as shown in figure.



(A) The meter bridge can have no other neutral point for this set of resistances.

(B) When the jockey contacts a point on meter wire left of D, current flows to B from the wire through galvanometer.

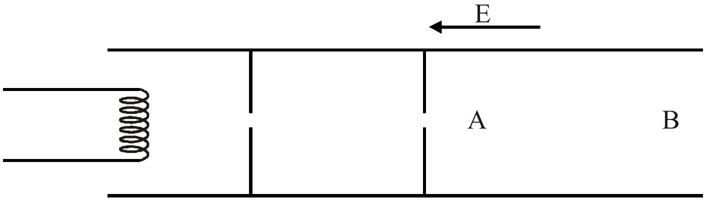
(C) When the jockey contacts a point on the meter wire to the right of D, current flows from B to the wire through galvanometer.

(D) When *R* is increased, the neutral point shifts to left.

CE0114

MATRIX MATCH TYPE QUESTION

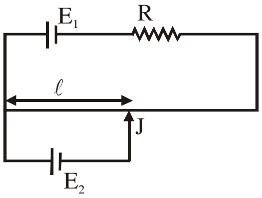
74. Electrons are emitted by a hot filament and are accelerated by an electric field as shown in figure. The two stops at the left ensure that the electron beam has a uniform cross-section. Match the entries of column-I with column-II as electron move from A to B :



	Column-I		**Column-II**
(A)	Speed of an electron	(P)	Inreases
(B)	Number of free electrons per unit volume	(Q)	Decreases
(C)	Current density	(R)	Remains same
(D)	Electric potential	(S)	any of the above is possible

CE0115

75. In the potentiometer arrangement shown in figure, null point is obtained at length ℓ.



	Column-I		**Column-II**
(A)	If E_1 is increased	(P)	ℓ should increase
(B)	If R is increased	(Q)	ℓ should decrease
(C)	If E_2 is increased	(R)	ℓ should remain the same to again get the null point

CE0116

EXERCISE (O-2)

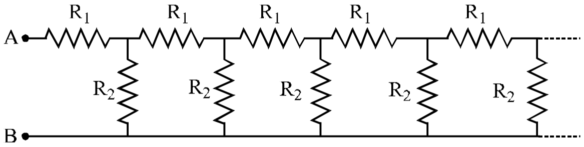
SINGLE CORRECT TYPE QUESTIONS

1. Which of the following quantities do not change when an ohmic resistor connected to a battery is heated due to the current?

(A) drift speed
(B) resistivity
(C) resistance
(D) number of free electrons

CE0117

2. Consider an infinte ladder network shown in figure. A voltage V is applied between the points A and B. This applied value of voltage is halved after each section.



(A) $R_1/R_2 = 1$
(B) $R_1/R_2 = 1/2$
(C) $R_1/R_2 = 2$
(D) $R_1/R_2 = 3$

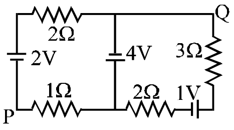
CE0118

3. A wire of length L and 3 identical cells of negligible internal resistances are connected in series. Due to the current, the temperature of the wire is raised by ΔT in time t. N number of similar cells is now connected in series with a wire of the same material and cross section but of length 2L. The temperature of the wire is raised by the same amount ΔT in the same time t. The value of N is :

(A) 4
(B) 6
(C) 8
(D) 9

CE0119

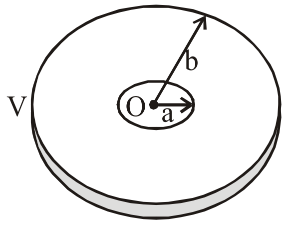
4. In the circuit shown, what is the potential difference V_{PQ}?



(A) + 3V
(B) + 2V
(C) –2V
(D) none

CE0120

5. A circular portion is cut of a disc of thickness t, its resistivity is ρ and radii of disc are a and b (b > a). A potential difference is maintained between outer and inner cylindrical surfaces of the disc. What is resistance of the disc ?



(A) $\frac{\rho}{2\pi t}\ln\left(\frac{b}{a}\right)$
(B) $\rho\left(\frac{1}{a}-\frac{1}{b}\right)$
(C) $2\pi\rho t\left(\frac{1}{a^2}-\frac{1}{b^2}\right)$
(D) $\frac{\rho}{2\pi t}\left(\frac{b^2-a^2}{ab}\right)$

CE0121

6. The wire shown in figure has a uniform cross-section A.

Resistivity of the material of wire is given by $\rho = \rho_0\left(\frac{L}{L+x}\right)$. A potential difference V is applied across the wire :-

(A) Resistance of wire is $\frac{\rho_0 L}{A}.\ln(2)$

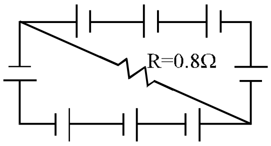
(B) Current density is variable inside the wire

(C) Electric field at x = 0 is $\frac{2V}{(\ln 2).L}$

(D) Electric field at x = L is $\frac{V}{(\ln 2)L}$

CE0122

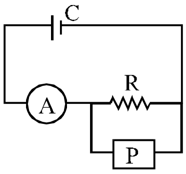
7. A circuit is comprised of eight identical batteries and a resistor R = 0.8Ω. Each battery has an emf of 1.0 V and internal resistance of 0.2Ω. The voltage difference across any of the battery is



(A) 0.5V (B) 1.0V (C) 0 V (D) 2 V

CE0123

8. An ammeter A of finite resistance, and a resistor R are joined in series to an ideal cell C. A potentiometer P is joined in parallel to R. The ammeter reading is I_0 and the potentiometer reading is V_0. P is now replaced by a voltmeter of finite resistance. The ammeter reading now is I and the voltmeter reading is V.



(A) $I > I_0$, $V < V_0$ (B) $I > I_0$, $V = V_0$

(C) $I = I_0$, $V < V_0$ (D) $I < I_0$, $V = V_0$

CE0124

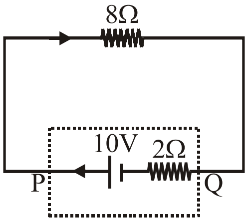
9. The sensitivity of post–office box for determination of resistance of 5Ω is maximum when, P and Q both are

(A) 1 Ω (B) 10 Ω (C) 100 Ω (D) 1000 Ω

CE0125

MULTIPLE CORRECT TYPE QUESTIONS

10. A battery of emf 10 volt and internal resistance 2Ω is connected to an external resistance 8Ω as shown in the figure :-



(A) Work done due to conservative electric field while a unit positive charge passes through battery from Q to P (along the arrow) is 8 Joule.

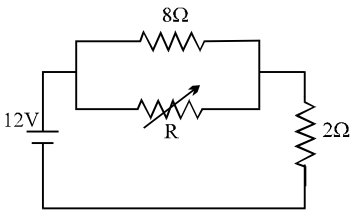
(B) Work done due to conservative electric field while a unit positive charge passes through battery from Q to P (along the arrow) is – 8 Joule.

(C) Work done due to conservative electric field while a unit positive charge passes through 8Ω along the arrow is –8 Joule.

(D) Work done due to non conservative electric field while a unit positive charge moves from Q to P (along the arrow) is 10 Joule.

CE0126

11. The value of the resistance R in figure is adjusted such that power dissipated in the 2Ω resistor is maximum. Under this condition



(A) R = 0

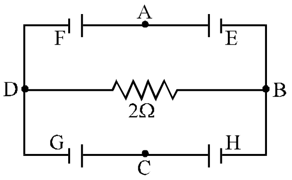
(B) R = 8Ω

(C) power dissipated in the 2 Ω resistor is 72 W.

(D) power dissipated in the 2 Ω resistor is 8 W.

CE0127

12. In the circuit shown E, F, G and H are cells of e.m.f. 2V, 1V, 3V and 1V respectively and their internal resistances are 2Ω, 1Ω, 3Ω and 1Ω respectively.



(A) $V_D - V_B = -2/13$ V

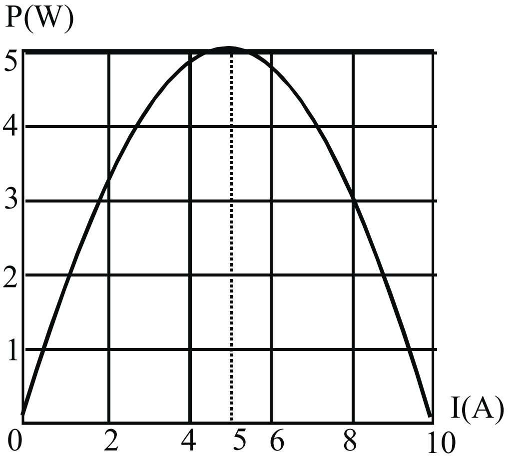
(B) $V_D - V_B = 2/13$ V

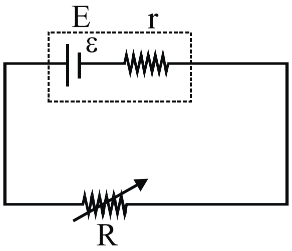
(C) $V_G = 21/13$ V = potential difference across G.

(D) $V_H = 19/13$ V = potential difference across H.

CE0128

13. Figure shows the net power dissipated in R versus the current in a simple circuit shown.





(A) The internal resistance of battery is 0.2Ω
(B) The emf of battery is 2V
(C) R at which power is 5W is 2.5 Ω
(D) At i = 2A, power is 3.2 W

CE0129

14. In a potentiometer wire experiment the emf of a battery in the primary circuit is 20V and its internal resistance is 5Ω. There is a resistance box in series with the battery and the potentiometer wire, whose resistance can be varied from 120Ω to 170Ω. Resistance of the potentiometer wire is 75Ω. The following potential differences can be measured using this potentiometer.

(A) 5V
(B) 6V
(C) 7V
(D) 8V

CE0130

EXERCISE-JM

1. Two conductors have the same resistance at 0°C but their temperature coefficients of resistance are α_1 and α_2. The respective temperature coefficients of their series and parallel combinations are nearly : **[AIEEE-2010]**

(1) $\frac{\alpha_1+\alpha_2}{2}, \frac{\alpha_1+\alpha_2}{2}$ (2) $\frac{\alpha_1+\alpha_2}{2}, \alpha_1+\alpha_2$ (3) $\alpha_1+\alpha_2, \frac{\alpha_1+\alpha_2}{2}$ (4) $\alpha_1+\alpha_2, \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}$

CE0131

2. If a wire is stretched to make it 0.1 % longer its resistance will :- **[AIEEE-2011]**

(1) decrease by 0.2% (2) decrease by 0.05% (3) increase by 0.05% (4) increase by 0.2%

CE0132

3. If 400 Ω of resistance is made by adding four 100 Ω resistance of tolerance 5%, then the tolerance of the combination is : **[AIEEE - 2011]**

(1) 20% (2) 5% (3) 10% (4) 15%

CE0133

4. The current in the primary circuit of a potentiometer is 0.2 A. The specific resistance and cross-section of the potentiometer wire are 4×10^{-7} ohm metre and 8×10^{-7} m^2 respectively. The potential gradient will be equal to :- **[AIEEE - 2011]**

(1) 0.2 V/m (2) 1 V/m (3) 0.5 V/m (4) 0.1 V/m

CE0134

5. Two electric bulbs marked 25W-220 V and 100 W-220 V are connected in series to a 440 V supply. Which of the bulbs will fuse ? **[AIEEE - 2012]**

(1) Neither (2) Both (3) 100 W (4) 25 W

CE0135

6. The supply voltage to a room is 120V. The resistance of the lead wires is 6Ω. A 60 W bulb is already switched on. What is the decrease of voltage across the bulb, when a 240 W heater is switched on in parallel to the bulb? **[JEE-Main2013]**

(1) zero Volt (2) 2.9 Volt (3) 13.3 Volt (4) 10.04 Volt

CE0136

7. This question has Statement I and Statement II. Of the four choice given after the Statements, choose the one that best describes the two Statements. **[JEE-Main 2013]**

Statement-I : Higher the range, greater is the resistance of ammeter.

Statement-II : To increase the range of ammeter, additional shunt needs to be used across it.

(1) Statement-I is true, Statement-II is true, Statement-II is the **correct** explanation of Statement-I

(2) Statement-I is true, Statement-II is true, Statement-II is **not** the correct explanation of Statement-I.

(3) Statement-I is **true,** Statement-II is false.

(4) Statement-I is **false,** Statement-II is true.

CE0137

8. In a large building, there are 15 bulbs of 40 W, 5 bulbs of 100 W, 5 fans of 80 W and 1 heater of 1kW. The voltage of the electric mains is 220 V. The minimum capacity of the main fuse of the building will be : **[JEE-Main 2014]**

(1) 12 A (2) 14 A (3) 8 A (4) 10 A

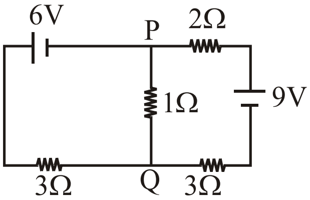
CE0138

9. When 5V potential difference is applied across a wire of length 0.1 m, the drift speed of electrons is 2.5×10^{-4} ms^{-1}. If the electron density in the wire is 8×10^{28} m^{-3}, the resistivity of the material is close to :- **[JEE-Main 2015]**

(1) $1.6 \times 10^{-6}\ \Omega$m (2) $1.6 \times 10^{-5}\ \Omega$m (3) $1.6 \times 10^{-8}\ \Omega$m (4) $1.6 \times 10^{-7}\ \Omega$m

CE0139

10. In the circuit shown, the current in the 1Ω resistor is :- **[JEE-Main 2015]**



(1) 0.13 A, from Q to P (2) 0.13 A, from P to Q

(3) 1.3 A, from P to Q (4) 0A

CE0140

11. A galvanometer having a coil resistance of 100Ω gives a full scale deflection, when a current of 1 mA is passed through it. The value of the resistance, which can convert this galvanometer into ammeter giving a full scale deflection for a current of 10A, is :- **[JEE-Main 2016]**

(1) 3Ω (2) 0.01Ω (3) 2Ω (4) 0.1Ω

CE0141

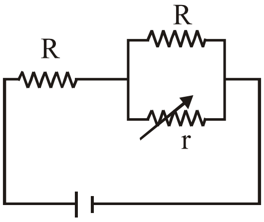
12. A 50Ω resistance is connected to a battery of 5V. A galvanometer of resistance 100 Ω is to be used as an ammeter to measure current through the resistance, for this a resistance r_s is connected to the galvanometer. Which of the following connections should be employed if the measured current is within 1% of the current without the ammeter in the circuit ? **[JEE-Mains (Online) 2016]**

(1) r_s = 1Ω in series with galvanometer (2) r_s = 0.5 Ω in parallel with the galvanometer

(3) r_s = 0.5 Ω in series with the galvanometer (4) r_s = 1 Ω in parallel with galvanometer

CE0142

13. In the circuit shown, the resistance r is a variable resistance. If for r = f R, the heat generation in r is maximum then the value of f is : **[JEE-Mains (Online) 2016]**



(1) 1 (2) $\frac{1}{4}$ (3) $\frac{1}{2}$ (4) $\frac{3}{4}$

CE0143

14. Which of the following statements is false ? **[JEE-Main 2017]**

(1) A rheostat can be used as a potential divider

(2) Kirchhoff's second law represents energy conservation

(3) Wheatstone bridge is the most sensitive when all the four resistances are of the same order of magnitude.

(4) In a balanced wheatstone bridge if the cell and the galvanometer are exchanged, the null point is disturbed.

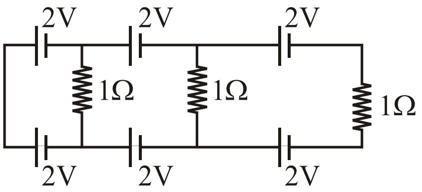
CE0146

15. When a current of 5 mA is passed through a galvanometer having a coil of resistance 15 Ω, it shows full scale deflection. The value of the resistance to be put in series with the galvanometer to convert it into to voltmeter of range 0 – 10 V is :- **[JEE-Main 2017]**

(1) 2.535×10^3 Ω	(2) 4.005×10^3 Ω
(3) 1.985×10^3 Ω	(4) 2.045×10^3 Ω

CE0147

16. In the above circuit the current in each resistance is :- **[JEE-Main 2017]**



(1) 0.5 A (2) 0 A (3) 1 A (4) 0.25 A

CE0148

17. In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 52 cm of the potentiometer wire. If the cell is shunted by a resistance of 5 Ω, a balance is found when the cell is connected across 40 cm of the wire. Find the internal resistance of the cell. **[JEE-Main 2018]**

(1) 1.5 Ω (2) 2 Ω (3) 2.5 Ω (4) 1 Ω

CE0149

18. On interchanging the resistances, the balance point of a meter bridge shifts to the left by 10 cm. The resistance of their series combination is 1 kΩ. How much was the resistance on the left slot before interchanging the resistances ? **[JEE-Main 2018]**

(1) 505 kΩ	(2) 550 kΩ
(3) 910 kΩ	(4) 990 kΩ

CE0150

19. Two batteries with e.m.f 12 V and 13 V are connected in parallel across a load resistor of 10 Ω. The internal resistances of the two batteries are 1 Ω and 2 Ω respectively. The voltage across the load lies between. **[JEE-Main 2018]**

(1) 11.5 V and 11.6 V (2) 11.4 V and 11.5 V

(3) 11.7 V and 11.8 V (4) 11.6 V and 11.7 V

CE0151

SELECTED PROBLEMS FROM JEE-MAINS ONLINE PAPERS

20. A carbon resistance has a following colour code. What is the value of the resistance ?

[JEE-Main-2019_Jan]

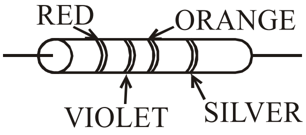
(1) 1.64 MΩ ± 5% (2) 530 kΩ ± 5%

(3) 64 kΩ ± 10% (4) 5.3 MΩ ± 5%

CE0196

21. A resistance is shown in the figure. Its value and tolerance are given respectively by:

[JEE-Main-2019_Jan]



(1) 27 KΩ, 20% (2) 270 KΩ, 5%

(3) 270 KΩ, 10% (4) 27 KΩ, 10%

CE0197

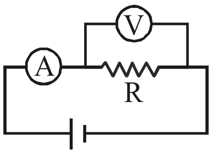
22. A copper wire is stretched to make it 0.5% longer. The percentage change in its electrical resistance if its volume remains unchanged is: **[JEE-Main-2019_Jan]**

(1) 2.5% (2) 0.5% (3) 1.0% (4) 2.0%

CE0198

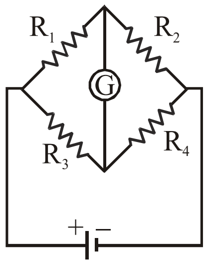
23. The actual value of resistance R, shown in the figure is 30Ω. This is measured in an experiment as shown using the standard formula $R = \frac{V}{I}$, where V and I are the readings of the voltmeter and ammeter, respectively. If the measured value of R is 5% less, then the internal resistance of the voltmeter is :

[JEE-Main-2019_Jan]
CE0199



(1) 350Ω (2) 570Ω (3) 35 Ω (4) 600 Ω

24. The Wheatstone bridge shown in Fig. here, gets balanced when the carbon resistor used as R_1 has the colour code (Orange, Red, Brown). The resistors R_2 and R_4 are 80Ω and 40Ω, respectively. Assuming that the colour code for the carbon resistors gives their accurate values, the colour code for the carbon resistor, used as R_3, would be : **[JEE-Main-2019_Jan]**



(1) Red, Green, Brown
(2) Brown, Blue, Brown
(3) Grey, Black, Brown
(4) Brown, Blue, Black

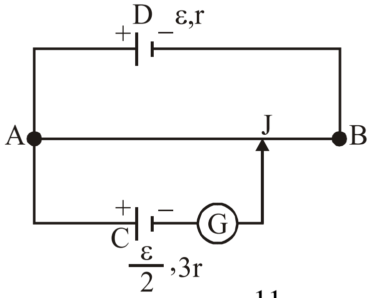
CE0200

25. A 2 W carbon resistor is color coded with green, black, red and brown respectively. The maximum current which can be passed through this resistor is : **[JEE-Main-2019_Jan]**

(1) 63 mA
(2) 0.4 mA
(3) 100 mA
(4) 20 mA

CE0201

26. A potentiometer wire AB having length L and resistance 12 r is joined to a cell D of emf ε and internal resistance r. A cell C having emf ε/2 and internal resistance 3r is connected. The length AJ at which the galvanometer as shown in fig. shows no deflection is : **[JEE-Main-2019_Jan]**



(1) $\frac{5}{12}L$
(2) $\frac{11}{24}L$
(3) $\frac{11}{12}L$
(4) $\frac{13}{24}L$

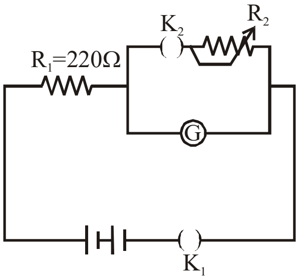
CE0202

27. Two equal resistance when connected in series to a battery, consume electric power of 60 W. If these resistances are now connected in parallel combination to the same battery, the electric power consumed will be : **[JEE-Main-2019_Jan]**

(1) 60 W
(2) 240 W
(3) 30 W
(4) 120 W

CE0203

28. The galvanometer deflection, when key K_1 is closed but K_2 is open, equals θ_0 (see figure). On closing K_2 also and adjusting R_2 to 5Ω, the deflection in galvanometer becomes $\frac{\theta_0}{5}$. The resistance of the galvanometer is, then, given by [Neglect the internal resistance of battery]: **[JEE-Main-2019_Jan]**

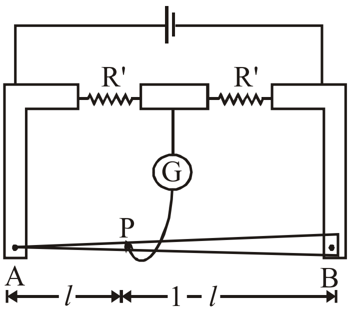


(1) 12Ω (2) 25Ω (3) 5Ω (4) 22Ω

CE0204

29. In a meter bridge, the wire of length 1 m has a non-uniform cross-section such that, the variation $\frac{dR}{d\ell}$ of its resistance R with length ℓ is $\frac{dR}{d\ell} \propto \frac{1}{\sqrt{\ell}}$. Two equal resistances are connected as shown in the figure. The galvanometer has zero deflection when the jockey is at point P. What is the length AP?

[JEE-Main-2019_Jan]

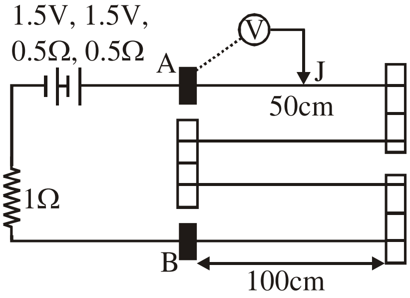


(1) 0.25 m (2) 0.3m (3) 0.35 m (4) 0.2 m

CE0205

30. In the circuit shown, a four-wire potentiometer is made of a 400 cm long wire, which extends between A and B. The resistance per unit length of the potentiometer wire is r = 0.01 Ω/cm. If an ideal voltmeter is connected as shown with jockey J at 50 cm from end A, the expected reading of the voltmeter will be :-

[JEE-Main-2019_April]



(1) 0.20 V (2) 0.25 V (3) 0.75 V (4) 0.50V

CE0206

31. In a conductor, if the number of conduction electrons per unit volume is 8.5×10^{28} m^{-3} and mean free time is 25*f*s (femto second), it's approximate resistivity is :- ($m_e = 9.1 \times 10^{-31}$ kg)

[JEE-Main-2019_April]

(1) 10^{-5} Ωm (2) 10^{-6} Ωm (3) 10^{-7} Ωm (4) 10^{-8} Ωm

CE0207

32. Space between two concentric conducting spheres of radii a and b (b > a) is filled with a medium of resistivity ρ. The resistance between the two spheres will be : **[JEE-Main-2019_April]**

(1) $\frac{\rho}{4\pi}\left(\frac{1}{a}-\frac{1}{b}\right)$ (2) $\frac{\rho}{2\pi}\left(\frac{1}{a}-\frac{1}{b}\right)$ (3) $\frac{\rho}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$ (4) $\frac{\rho}{4\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$

CE0208

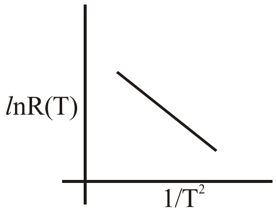
33. A current of 5 A passes through a copper conductor (resistivity = 1.7×10^{-8} Ωm) of radius of cross-section 5 mm. Find the mobility of the charges if their drift velocity is 1.1×10^{-3} m/s.

[JEE-Main-2019_April]

(1) 1.3 m^2/Vs (2) 1.5 m^2/Vs (3) 1.8 m^2/Vs (4) 1.0 m^2/Vs

CE0209

34. In an experiment, the resistance of a material is plotted as a function of temperature (in some range). As shown in the figure, it is a straight line. One may conclude that : **[JEE-Main-2019_April]**



(1) $R(T) = \frac{R_0}{T^2}$

(2) $R(T) = R_0 e^{-T^2/T_0^2}$

(3) $R(T) = R_0 e^{-T_0^2/T^2}$

(4) $R(T) = R_0 e^{T^2/T_0^2}$

CE0210

35. A moving coil galvanometer, having a resistance G, produces full scale deflection when a current I_g flows through it. This galvanometer can be converted into (i) an ammeter of range 0 to I_0 ($I_0 > I_g$) by connecting a shunt resistance R_A to it and (ii) into a voltmeter of range 0 to V(V = GI_0) by connecting a series resistance R_V to it. Then, **[JEE-Main-2019_April]**

(1) $R_AR_V = G^2\left(\frac{I_g}{I_0 - I_g}\right)$ and $\frac{R_A}{R_V} = \left(\frac{I_0 - I_g}{I_g}\right)^2$

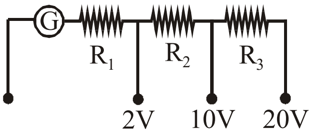
(2) $R_AR_V = G^2$ and $\frac{R_A}{R_V} = \left(\frac{I_g}{I_0 - I_g}\right)^2$

(3) $R_AR_V = G^2$ and $\frac{R_A}{R_V} = \frac{I_g}{(I_0 - I_g)}$

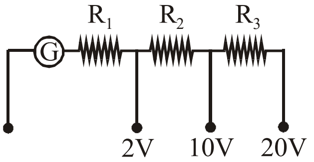
(4) $R_AR_V = G^2\left(\frac{I_0 - I_g}{I_g}\right)$ and $\frac{R_A}{R_V} = \left(\frac{I_g}{I_0 - I_g}\right)^2$

CE0211

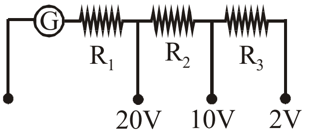
36. A galvanometer of resistance 100Ω has 50 divisions on its scale and has sensitivity of 20 μA/division. It is to be converted to a voltmeter with three ranges, of 0–2 V, 0–10 V and 0–20 V. The appropriate circuit to do so is : **[JEE-Main-2019_April]**

(1)


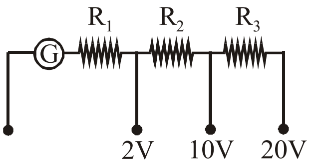
$R_1 = 1900\ \Omega$
$R_2 = 9900\ \Omega$
$R_3 = 19900\ \Omega$

(2)


$R_1 = 2000\ \Omega$
$R_2 = 8000\ \Omega$
$R_3 = 10000\ \Omega$

(3)


$R_1 = 19900\ \Omega$
$R_2 = 9900\ \Omega$
$R_3 = 1900\ \Omega$

(4)


$R_1 = 1900\ \Omega$
$R_2 = 8000\ \Omega$
$R_3 = 10000\ \Omega$

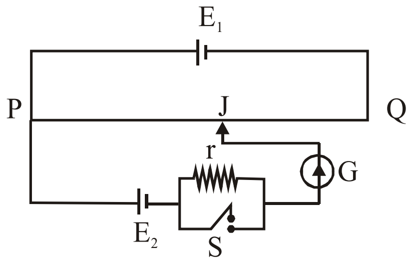
CE0212

37. The series combination of two batteries, both of the same emf 10 V, but different internal resistance of 20Ω and 5Ω, is connected to the parallel combination of two resistors 30 Ω and R Ω. The voltage difference across the battery of internal resistance 20Ω is zero, the value of R (in Ω) is : _______

[JEE-Main-2020_Jan]
CE0213

38. A potentiometer wire PQ of 1 m length is connected to a standard cell E_1. Another cell E_2 of emf 1.02 V is connected with a resistance 'r' and switch S (as shown in figure). With switch S open, the null position is obtained at a distance of 49 cm from Q. The potential gradient in the potentiometer wire is:

[JEE-Main-2020_Sep]

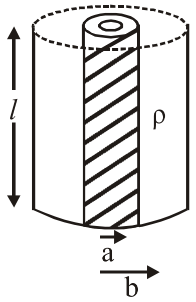


(1) 0.02 V/cm (2) 0.04 V/cm (3) 0.01 V/cm (4) 0.03 V/cm

CE0214

39. Model a torch battery of length l to be made up of a thin cylindrical bar of radius 'a' and a concentric thin cylindrical shell of radius 'b' filled in between with an electrolyte of resistivity ρ (see figure). If the battery is connected to a resistance of value R, the maximum Joule heating in R will take place for :-

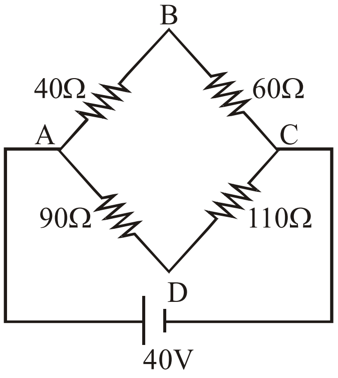
[JEE-Main-2020_Sep]



(1) $R = \frac{2\rho}{\pi l} ln\left(\frac{b}{a}\right)$ (2) $R = \frac{\rho}{\pi l} ln\left(\frac{b}{a}\right)$ (3) $R = \frac{\rho}{2\pi l}\left(\frac{b}{a}\right)$ (4) $R = \frac{\rho}{2\pi l} ln\left(\frac{b}{a}\right)$

CE0215

40.



Four resistances 40Ω, 60Ω, 90Ω and 110Ω make the arms of a quadrilateral ABCD. Across AC is a battery of emf 40V and internal resistance negligible. The potential difference across BD is V is _______.

[JEE-Main-2020_Sep]
CE0216

41. A galvanometer is used in laboratory for detecting the null point in electrical experiments. If, on passing a current of 6mA it produces a deflection of 2°, its figure of merit is close to :

[JEE-Main-2020_Sep]

(1) 3×10^{-3} A/div. (2) 333° A/div.

(3) 6×10^{-3} A/div. (4) 666° A/div.

CE0217

EXERCISE - JA

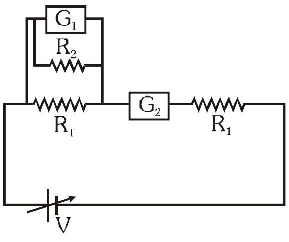
1. Incandescent bulbs are designed by keeping in mind that the resistance of their filament increases with the increase in temperature. If at room temperature, 100 W, 60 W and 40 W bulbs have filament resistance R_{100}, R_{60} and R_{40}, respectively, the relation between these resistances is **[IIT-JEE 2010]**

(A) $\frac{1}{R_{100}}=\frac{1}{R_{40}}+\frac{1}{R_{60}}$ (B) $R_{100} = R_{40} + R_{60}$ (C) $R_{100} > R_{60} > R_{40}$ (D) $\frac{1}{R_{100}}>\frac{1}{R_{60}}>\frac{1}{R_{40}}$

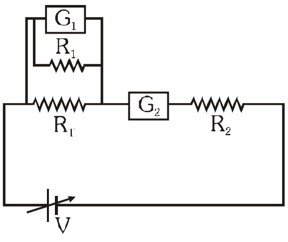
CE0152

2. To verify Ohm's law, a student is provided with a test resistor R_T, a high resistance R_1, a small resistance R_2, two identical galvanometers G_1 and G_2, and a variable voltage source V. The correct circuit to carry out the experiment is :– **[IIT-JEE 2010]**

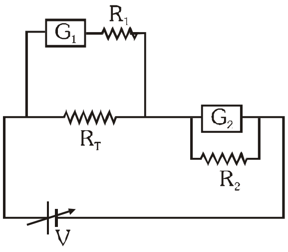
(A)



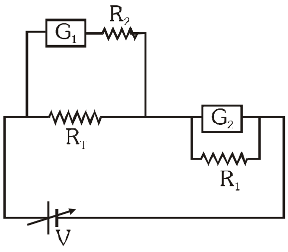
(B)



(C)

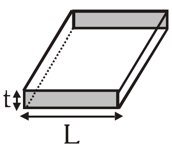


(D)



CE0153

3. Consider a thin square sheet of side L and thickness t, made of a material of resistivity ρ. The resistance between two opposite faces, shown by the shaded areas in the figure is **[IIT-JEE 2010]**



(A) directly proportional to L (B) directly proportional to t

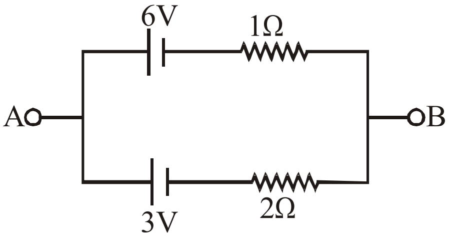
(C) independent of L (D) independent of t

CE0154

4. When two identical batteries of internal resistance 1Ω each are connected in series across a resistor R, the rate of heat produced in R is J_1. When the same batteries are connected in parallel across R, the rate is J_2. If $J_1 = 2.25\ J_2$ then the value of R in Ω is **[IIT-JEE 2010]**

CE0155

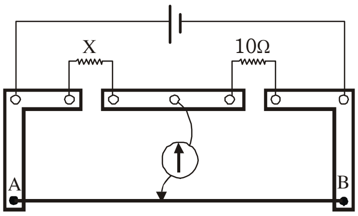
5. Two batteries of different emfs and different internal resistances are connected as shown. The voltage across AB in volts is **[IIT-JEE 2011]**



CE0156

6. A meter bridge is set-up as shown, to determine an unknown resistance 'X' using a standard 10 ohm resistor. The galvanometer shows null point when tapping-key is at 52 cm mark. The end-corrections are 1 cm and 2 cm respectively for the ends A and B. The determined value of 'X' is

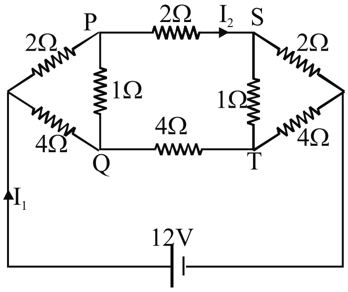
[IIT-JEE 2011]



(A) 10.2 ohm (B) 10.6 ohm (C) 10.8 ohm (D) 11.1 ohm

CE0157

7. For the resistance network shown in the figure, choose the correct option(s). **[IIT-JEE 2012]**



(A) the current through PQ is zero (B) $I_1 = 3A$

(C) The potential at S is less than that at Q (D) $I_2 = 2A$

CE0158

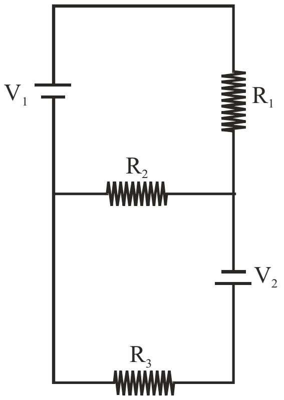
8. Heater of an electric kettle is made of a wire of length L and diameter d. It takes 4 minutes to raise the temperature of 0.5 kg water by 40 K. This heater is replaced by a new heater having two wires of the same material, each of length L and diameter 2d. The way these wires are connected is given in the options. How much time in minutes will it take to raise the temperature of the same amount of water by 40 K ? **[JEE Advanced 2014]**

(A) 4 if wires are in parallel (B) 2 if wires are in series

(C) 1 if wires are in series (D) 0.5 if wires are in parallel

CE0159

9. Two ideal batteries of emf V_1 and V_2 and three resistances R_1, R_2 and R_3 are connected as shown in the figure. The current in resistance R_2 would be zero if :- **[JEE Advanced 2014]**



(A) $V_1 = V_2$ and $R_1 = R_2 = R_3$ (B) $V_1 = V_2$ and $R_1 = 2R_2 = R_3$

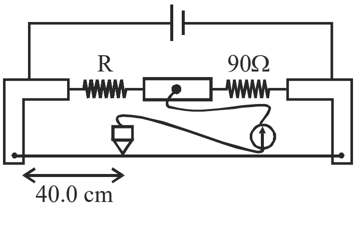
(C) $V_1 = 2V_2$ and $2R_1 = 2R_2 = R_3$ (D) $2V_1 = V_2$ and $2R_1 = R_2 = R_3$

CE0160

10. A galvanometer gives full scale deflection with 0.006 A current. By connecting it to a 4990 Ω resistance, it can be converted into a voltmeter of range 0 - 30 V. If connected to a $\frac{2n}{249}\Omega$ resistance, it becomes an ammeter of range 0 - 1.5 A. The value of n is :- **[JEE Advanced 2014]**

CE0161

11. During an experiment with a metre bridge, the galvanometer shows a null point when the jockey is pressed at 40.0 cm using a standard resistance of 90Ω, as shown in the figure. The least count of the scale used in the metre bridge is 1mm. The unknown resistance is :- **[JEE Advanced 2014]**

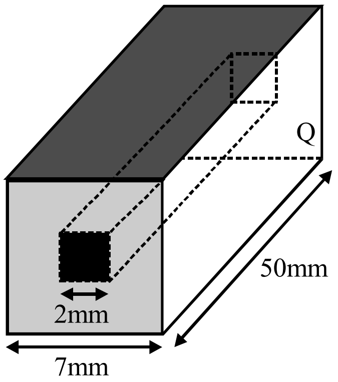


(A) 60 ± 0.15 Ω (B) 135 ± 0.56 Ω (C) 60 ± 0.25 Ω (D) 135 ± 0.23 Ω

CE0162

12. In an aluminum (Al) bar of square cross section, a square hole is drilled and is filled with iron (Fe) as shown in the figure. The electrical resistivities of Al and Fe are $2.7 \times 10^{-8}\ \Omega$ m and $1.0 \times 10^{-7}\Omega$m, respectively. The electrical resistance between the two faces P and Q of the composite bar is :

[JEE Advanced-2015]

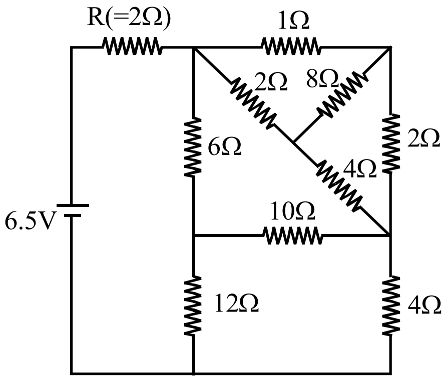


(A) $\frac{2475}{64}\mu\Omega$ (B) $\frac{1875}{64}\mu\Omega$ (C) $\frac{1875}{49}\mu\Omega$ (D) $\frac{2475}{132}\mu\Omega$

CE0163

13. In the following circuit, the current through the resistor R (= 2Ω) is I Amperes. The value of I is

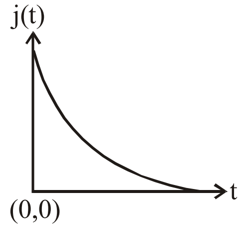
[JEE Advanced-2015]

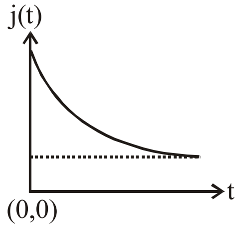


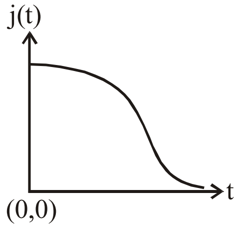
CE0164

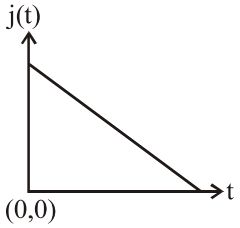
14. An infinite line charge of uniform electric charge density λ lies along the axis of an electrically conducting infinite cylindrical shell of radius R. At time t = 0, the space inside the cylinder is filled with a material of permittivity ε and electrical conductivity σ. The electrical conduction in the material follows Ohm's law. Which one of the following graphs best describes the subsequent variation of the magnitude of current density j(t) at any point in the material?

[JEE Advanced-2016]

(A)


(B)


(C)


(D)


CE0165

15. An incandescent bulb has a thin filament of tungsten that is heated to high temperature by passing an electric current. The hot filament emits black-body radiation. The filament is observed to break up at random locations after a sufficiently long time of operation due to non-uniform evaporation of tungsten from the filament. If the bulb is powered at constant voltage, which of the following statement(s) is(are) true? **[JEE Advanced-2016]**

(A) The temperature distribution over the filament is uniform

(B) The resistance over small sections of the filament decreases with time

(C) The filament emits more light at higher band of frequencies before it breaks up

(D) The filament consumes less electrical power towards the end of the life of the bulb

CE0166

16. Consider two identical galvanometers and two identical resistors with resistance R. If the internal resistance of the galvanometers $R_C < R/2$, which of the following statement(s) about any one of the galvanometers is(are) true ? **[JEE Advanced-2016]**

(A) The maximum voltage range is obtained when all the components are connected in series

(B) The maximum voltage range is obtained when the two resistors and one galvanometer are connected in series, and the second galvanometer is connected in parallel to the first galvanometer

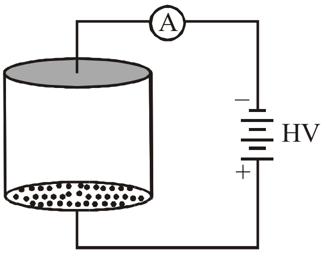
(C) The maximum current range is obtained when all the components are connected in parallel

(D) The maximum current range is obtained when the two galvanometers are connected in series and the combination is connected in parallel with both the resistors.

CE0167

Paragraph for Questions No. 17 and 18

Consider an evacuated cylindrical chamber of height h having rigid conducting plates at the ends and an insulating curved surface as shown in the figure. A number of spherical balls made of a light weight and soft material and coated with a conducting material are placed on the bottom plate. The balls have a radius r <<h. Now a high voltage source (HV) is connected across the conducting plates such that the bottom plate is at $+V_0$ and the top plate at $-V_0$. Due to their conducting surface, the balls will get charged, will become equipotential with the plate and are repelled by it. The balls will eventually collide with the top plate, where the coefficient of restitution can be taken to be zero due to the soft nature of the material of the balls. The electric field in the chamber can be considered to be that of a parallel plate capacitor. Assume that there are no collision between the balls and the interaction between them is negligible. (Ignore gravity) **[JEE Advanced-2016]**



17. Which of the following statements is correct ?

(A) The balls will bounce back to the bottom plate carrying the opposite charge they went up with

(B) the balls will execute simple harmonic motion between the two plates

(C) The balls will bounce back to the bottom plate carrying the same charge they went up with

(D) The balls will stick to the top plate and remain there

CE0168

18. The average current in the steady state registered by the ammeter in the circuit will be :

(A) Proportional to $V_0^{1/2}$ (B) Proportional to V_0^2

(C) Proportional to the potential V_0 (D) Zero

CE0168

19. Two identical moving coil galvanometer have 10 Ω resistance and full scale deflection at 2 μA current. One of them is converted into a voltmeter of 100 mV full scale reading and the other into an Ammeter of 1 mA full scale current using appropriate resistors. These are then used to measure the voltage and current in the Ohm's law experiment with R = 1000 Ω resistor by using an ideal cell. Which of the following statement(s) is/are correct ? **[JEE Advanced-2019]**

(1) The measured value of R will be $978\ \Omega < R < 982\ \Omega$.

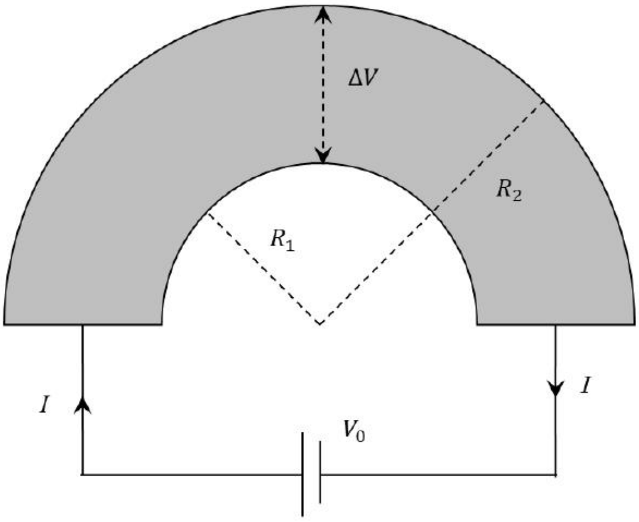
(2) The resistance of the Voltmeter will be 100 kΩ.

(3) The resistance of the Ammeter will be 0.02 Ω (round off to 2nd decimal place)

(4) If the ideal cell is replaced by a cell having internal resistance of 5 Ω then the measured value of R will be more than 1000 Ω.

CE0169

20. Shown in the figure is a semicircular metallic strip that has thickness t and resistivity ρ. Its inner radius is R_1 and outer radius is R_2. If a voltage V_0 is applied between its two ends, a current I flows in it. In addition, it is observed that a transverse voltage ΔV develops between its inner and outer surfaces due to purely kinetic effects of moving electrons (ignore any role of the magnetic field due to the current). Then (figure is schematic and not drawn to scale)- **[JEE Advanced-2020]**



(A) $I = \frac{V_0 t}{\pi\rho} \ln\left(\frac{R_2}{R_1}\right)$

(B) the outer surface is at a higher voltage than the inner surface

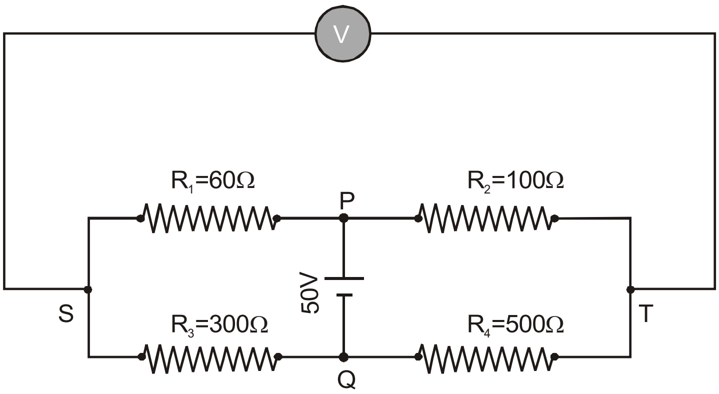
(C) the outer surface is at a lower voltage than the inner surface

(D) $\Delta V \propto I^2$

CE0218

21. In the balanced condition, the values of the resistances of the four arms of a Wheatstone bridge are shown in the figure below. The resistance R_3 has temperature coefficient 0.0004 °C^{-1}. If the temperature of R_3 is increased by 100°C, the voltage developed between S and T will be __________ volt.

[JEE Advanced-2020]



CE0219

CURRENT ELECTRICITY

(CBSE Previous Year's Questions)

1. Explain how does the resistivity of a conductor depend upon (i) number density ‘n’ of free electrons, and (ii) relaxation time ‘λ’. **[2; CBSE-2004]**

2. Two cells of emf 1.5 V and 2 V and internal resistance 1 ohm and 2 ohm respectively are connected in parallel to pass a current in the same direction through an external resistance of 5 ohm.

(i) Draw the circuit diagram.

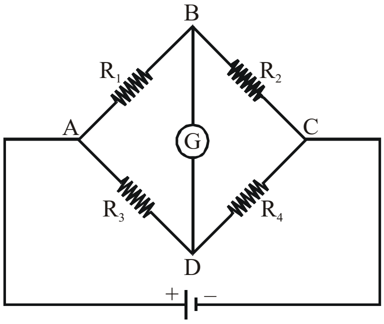
(ii) Using Kirchhoff’s laws, calculate the current through each branch of the circuit and potential difference across the 5 ohm resistor. **[3; CBSE-2005]**

3. You are given ‘n’ resistors, each of resistance ‘r’. These are first connected to get minimum possible resistance. In the second case, these are again connected differently to get maximum possible resistance. Compute the ratio between the minimum and maximum values of resistance so obtained. **[2; CBSE-2006]**

4. Draw a circuit diagram using a metre bridge and write the necessary mathematical relation used to determinethevalueofanunknownresistance. Whycannotsuchanarrangementbeusedformeasuring very low resistances? **[2; CBSE-2006]**

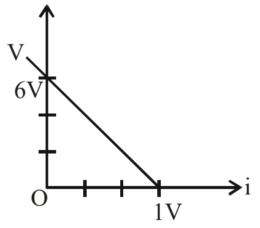
5. Which one of the two, an ammeter or a millianuneter, has a higher resistance and why? **[2; CBSE-2006]**

6. The given figure shows a network of resistances R_1, R_2, R_3 and R_4.



Using Kirchhoff’s laws, establish the balance condition for the network. **[2; CBSE-2007]**

7. The plot of the variation of potential difference across a combination of three identical cells in series, versus current is as shown below. What is the emf of each, cell **[1; CBSE-2008]**

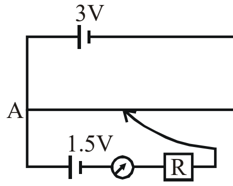


8. Prove that the current density of a metallic conductor is directly proportional to the drift speed of electrons. **[3; CBSE-2008]**

9. A number of identical cells, n each of emf E, internal resistance r connected in series are charged by a. d.c source of emf E', using a resistor R. (i) Draw the circuit arrangement. (ii) Deduce the expressions for (a) the charging current and (b) the potential difference across the combination of the cells.

[3; CBSE-2008]

10. A potentiometer wire of length 1m is connected to a drive cell of emf 3V as shown in the figure. When a cell of 1.5 V emf is used in the secondaiy circuit, the balance point is found to be 60cm. On replacing this cell and using a cell of unknown emf, the balance point shifts to 80cm



(i) Calculate unknown emf of the cell. **[3; CBSE-2008]**

(ii) Explain with reason, whether the circuit works, if the drive cell is replaced with a cell emf 1V.

(iii) Does the high resistance R, used in the secondary circuit affect the balance point ? Justify your answer.

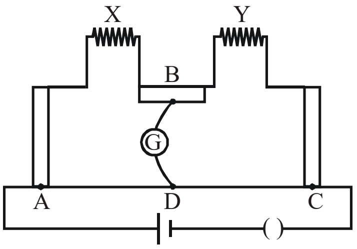
11. A cell of emf 'E' and internal resistance 'r' is connected across a variable resistor 'R'. Plot a graph showing the variation of terminal potential 'V' with Resistance R. **[2; CBSE-2009]**

12. Derive an expression for drift velocity of the free electrons in a conductor in terms of relaxation time.

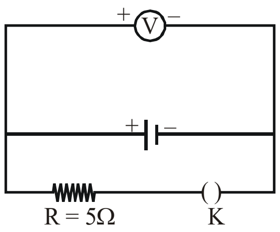
[2; CBSE-2009]

13. The figure shows experimental set up of a bridge. When the two unknown resistances X and Y are inserted, the null point D is obtained 40 cm from the end A. when a resistance of 10 is connected in series with X, the null point shifts by 10 cm. find the position of the null point when the 10 resistance is instead connected in series with resistance 'Y'. Determine the values of the resistances X and Y.

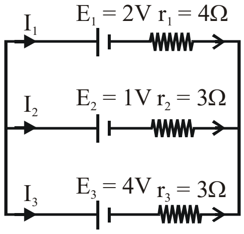
[3; CBSE-2009]



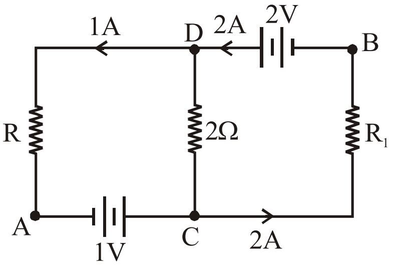
14. Write any two factors on which internal resistance of a cell depends. The reading on a high resistance voltmeter, when a cell is connected across it, it 2.2 V. When the terminals of the cell are also connected to a resistance of 5Ω as shown in the figure, the voltmeter reading.drops to 1.8 V. Find the internal resistance of the cell. **[3; CBSE-2010]**



15. State Kirchhoff's rules. Use these rules to write the expressions for the currents I_1, I_2 and I_3 in the circuit diagram shown. **[3; CBSE-2010]**



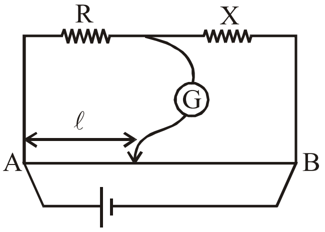
16. In the given circuit, assuming point A to be at zero potential, use Kirchhofi's rules to determine the potential at point B. **[2; CBSE-2011]**



17. In the meter bridge experiment, balance point was observed at J withAJ = *l*.

(i) The values of R and X were doubled and then interchanged. What would be the new position of balance point ?

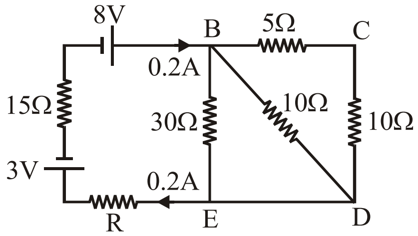
(ii) If the galvanometer and battery are interchanged at the balance position, how will balance point get affected ? **[2; CBSE-2011]**



18. Two heating elements of resistance R_1 and R_2 when operated at a constant supply of voltage, V, consume powers P_1 and P_2 respectively. Deduce the expressions for the power of their combination when they are, in turn, connected in (i) series and (ii) parallel across the same voltage supply.

[3; CBSE-2011]

19. Calculate the value of the resistance R in the circuit shown in the figure so that the current in the circuit is 0.2A. What would be the potential difference between points B and E? **[3; CBSE-2012]**



20. Define relaxation time of the free electrons drifting in a conductor. How is it related to the drift velocity of free electrons? Use this relation to deduce the expression for the electrical resistivity of the material. **[3; .CBSE-2012]**

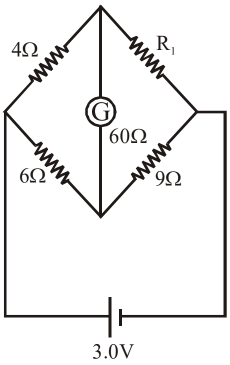
21. Two identical cells, each of emf E, having negligible internal resistance, are connected in parallel with each other across an external resistance R. What is the current through this resistance ? **[CBSE-2013]**

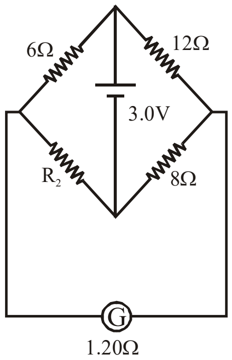
22. Explain the term 'drift velocity' of electrons in a conductor. Hence obtain the expression for the current through a conductor in terms of'drift velocity'. **[CBSE-2013]**

OR

Describe briefly, with the help of a circuit diagram, how a potentiometer is used to detennine the internal resistance of a cell.

23. Define the current sensitivity of a galvanometer. Write its S. **I.** unit. Figure shows two circuits each having a galvanometer and a battery of 3V. When the galvanometers in each arrangement do not show any deflection, obtain the ratio R_1/R_2. **[CBSE-2013]**

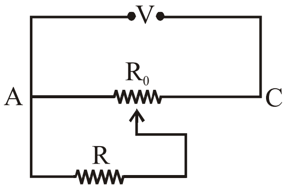




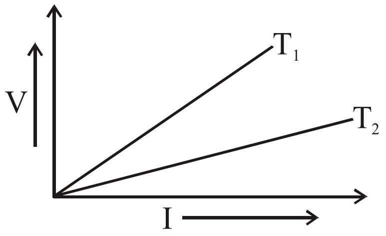
24. Estimate the average drift speed of conduction electrons in a copper wire of cross-sectional area 2.5×10^{-7} m^2 carrying a current of 1.8 A. Assume the density of conduction electrons to be 9×10^{28} m^{-3}. **[CBSE-2014]**

25. A cell of emf 'E' and internal resistance 'r' is connected across a variable resistor 'R'. Plot a graph showing variation of terminal voltage 'V' of the cell versus the current'I'. Using the plot, show how the emf of the cell and its internal resistance can be determined. **[CBSE-2014]**

26. A resistance of RΩ draws current from a potentiometer as shown in the figure. The potentiometer has a total resistance R_0Ω. A voltage V is supplied to the potentiometer. Derive an expression for the voltage across R when the sliding contact is in the middle of the potentiometer. **[CBSE-2014]**

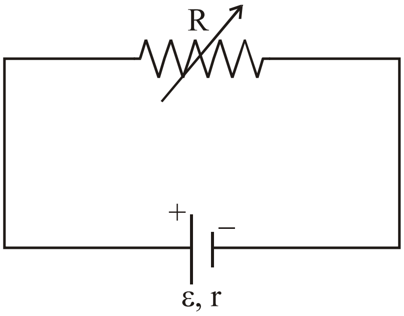


27. V – I graph for a metallic wire at two different termperatures T_1 and T_2 is as shown in the figure. Which of the two temperatures is higher and why ? **[1; CBSE-2015]**



28. A variable resistor R is connected across a cell of emf ε and internal resistance r as shown in the figure. Draw a plot showing the variation of (i) terminal voltage V and (ii) the current I, as a function of R.

[2; CBSE-2015]



29. A potential difference V is applied across a conductor of length L and diameter D. How is the drift velocity, vd, of charge carriers in the conductor affected when (i) V is halved, (ii) L is doubled and (iii) D is halved ? Justify your answer in each case. **[3; CBSE-2015]**

30. Define mobility of a charge carrier. What is its relation with relaxation time ? **[1; CBSE-2016]**

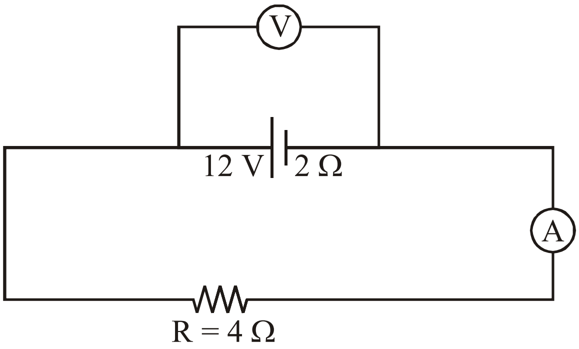
31. When 5V potential difference is applied across a wire of length 0.1m, the drift speed of electrons is 2.5×10^{-4} m/s. If the electron density in the wire is 8×10^{28} m^{-3}, calculate the resistivity of the material of wire. **[2; CBSE-2016]**

32. Two identical cells of emf 1.5 V each joined in parallel supply energy to an external circuit consisting of two resistances of 7Ω each joined in parallel. A very high resistance voltmeter reads the terminal voltage of cells to be 1.4 V. Calculate the internal resistance of each cell. **[3; CBSE-2016]**

33. Nichrome and copper wires of same length and same radius are connected in series. Current I is passed through them. Which wire gets heated up more ? Justify your answer. **[CBSE-2017]**

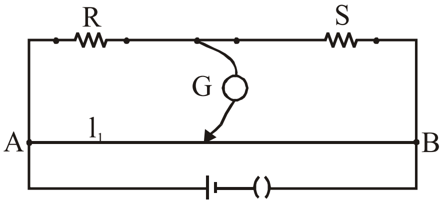
34. (a) The potential difference applied across a given resistor is altered so that the heat produced per second increases by a factor of 9. By what factor does the applied potential difference change ?

(b) In the figure shown, an ammeter A and a resistor of 4 Ω are connected to the terminals of the source. The emf of the source is 12 V having an internal resistance of 2 Ω. Calculate the voltmeter and ammeter readings. **[CBSE-2017]**



35. (a) Write the principle of working of a metre bridge.

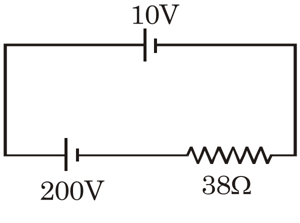
(b) In a metre bridge, the balance point is found at a distance l_1 with resistance R and S as shown in the figure. **[CBSE-2017]**



An unknown resistance X is now connected in parallel to the resistance S and the balance point is found at a distance l_2. Obtain a formula for X in terms of l_1, l_2 and S.

36. Two electric bulbs P and Q have their resistance in the ratio of 1 : 2. They are connected in series across a battery. Find the ratio of the power dissipation in these bulbs. **[CBSE-2018]**

37. A 10 V cell of negligible internal resistance is connected in parallel across a battery of emf 200 V and internal resistance 38Ω as shown in the figure. Find the value of current in the circuit. **[CBSE-2018]**



In a potentiometer arrangement for determining the emf of a cell, the balance point of the cell in open circuit is 350 cm. When a resistance of 9Ω is used in the external circuit of the cell, the balance point shifts to 300 cm. Determine the internal resistance of the cell.

38. (a) Define the term 'conductivity' of a metallic wire. Write its SI unit. **[CBSE-2018]**

(b) Using the concept of free electrons in a conductor, derive the expression for the conductivity of a wire in terms of number density and relaxation time. Hence obtain the relation between current density and the applied electric field E.

ANSWER KEY

EXERCISE - S-1

1. Ans. Iρ/A **2. Ans.** S = e n l <v> / j **3. Ans.** $p = I\,m_e\,l/e$ **4. Ans.** 3 : 1

5. Ans. (a) (i) in series, (ii) all in parallel: n^2. (b)
(i) Join 1Ω, 2Ω in parallel and the combination in series with 3Ω,
(ii) parallel combination of 2 Ω and 3 Ω in series with 1 Ω, (iii) all in series, (iv) all in parallel.
(c) (i) (16/3) Ω, (ii) 5 R.

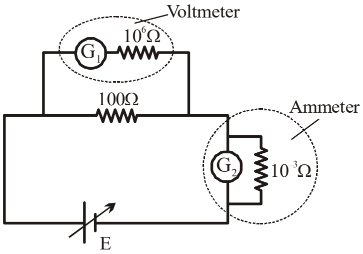
6. Ans. 600Ω **7. Ans.** 7.5 mA **8. Ans.** 1 V **9. Ans.** I = 2.5 A, V = 3.5 Volts

10. Ans. 20/3 V **11. Ans.** 3.5 A **13. Ans.** 1 Ω **14. Ans.** $x = \frac{4}{3}$ V, $12\frac{1}{3}$ V, $\frac{1}{15}$ A

15. Ans. 5 **16. Ans.** $\sqrt{R_1R_2}$ **17. Ans.** 25 **18. Ans.** 3 **19. Ans.** 12A, –20W

20. Ans. 4/9 kg/sec., 450 sec **21. Ans.** 20 ohm **22. Ans.** $R_1 = 0.0278\Omega$, $R_2 = 0.25\Omega$, $R_3 = 2.5\Omega$

23. Ans. 233.3Ω, 144V **24. Ans.**



25. Ans. Battery should be connected across A and B. Out put can be taken across the terminals A and C or B and C

26. Ans. This is true for $r_1 = r_2$; So R_2 given most accurate value **27. Ans.** $\frac{10}{3}$ Ω, 5 Ω

28. Ans. (a) +ve, $E_\ell > E$ (b) –ve **29. Ans.** 2.25 V **30. Ans.** 4 ohm **31. Ans.** 46.67 cm

EXERCISE - S-2

1. Ans. 5×10^{-7} A **2. Ans.** $\alpha_{eff} = \frac{5}{4}\alpha$ **3. Ans.** (i) $\frac{1}{2}i_0t_0$ (ii) $i = i_0\left(1-\frac{t}{t_0}\right)$ (iii) $\frac{Rt_0i_0^2}{3}$

4. Ans. $\frac{1}{R} = \frac{\pi r^2}{3l}(2\sigma_2 + \sigma_1)$ **5. Ans.** (a) $J_0A/3$; (b) $2J_0A/3$ **6. Ans.** 4 Ω

7. Ans. $-\frac{22}{9}$ V **8. Ans.** (i) 1.01Ω (ii) 0-5 A, 0-10V, (ii) 0.05 A

9. Ans. 7.5 m, 8.75m, 6.25m **10. Ans. 2**

EXERCISE - O-1

SINGLE CORRECT TYPE QUESTIONS

1. Ans. (C) **2. Ans. (C)** **3. Ans. (D)** **4. Ans. (B)**
5. Ans. (A) **6. Ans. (B)** **7. Ans. (C)** **8. Ans. (B)**
9. Ans. (C) **10. Ans. (D)** **11. Ans. (A)** **12. Ans. (D)**
13. Ans. (C) **14. Ans. (B,C)** **15. Ans. (B)** **16. Ans. (C)**
17. Ans. (D) **18. Ans. (B)** **19. Ans. (D)** **20. Ans. (A)**
21. Ans. (C) **22. Ans. (C)** **23. Ans. (D)** **24. Ans. (B)**
25. Ans. (A) **26. Ans. (A)** **27. Ans. (A)** **28. Ans. (B)**
29. Ans. (B) **30. Ans. (B)** **31. Ans. (A)** **32. Ans. (A)**
33. Ans. (C) **34. Ans. (B)** **35. Ans. (B)** **36. Ans. (C)**
37. Ans. (A) **38. Ans. (C)** **39. Ans. (B)** **40. Ans. (A)**
41. Ans. (D) **42. Ans. (A)** **43. Ans. (D)** **44. Ans. (B)**
45. Ans. (B) **46. Ans. (B)** **47. Ans. (B)** **48. Ans. (C)**
49. Ans. (B) **50. Ans. (C)** **51. Ans. (D)** **52. Ans. (C)**
53. Ans. (C) **54. Ans. (B)** **55. Ans. (A)** **56. Ans. (C)**
57. Ans. (A) **58. Ans. (C)** **59. Ans. (B)** **60. Ans. (A)**
61. Ans. (D) **62. Ans. (B)** **63. Ans. (B)** **64. Ans. (A)**
65. Ans. (B) **66. Ans. (D)**

MULTIPLE CORRECT TYPE QUESTIONS

67. Ans. (A,B,C,D) **68. Ans. (A,D)** **69. Ans. (A,B,C)** **70. Ans. (A,C)**
71. Ans. (B,C) **72. Ans. (A,D)** **73. Ans. (A,B, C)**

MATRIX MATCH TYPE QUESTION

74. Ans. A-P; B-Q; C-R; D-P **75. Ans. A-Q; B-P; C-P**

EXERCISE - O-2

SINGLE CORRECT TYPE QUESTIONS

1. Ans. (D) **2. Ans. (B)** **3. Ans. (B)** **4. Ans. (B)** **5. Ans. (A)**
6. Ans. (A) **7. Ans. (C)** **8. Ans. (A)** **9. Ans. (A)**

MULTIPLE CORRECT TYPE QUESTIONS

10. Ans. (B, D) **11. Ans. (A,C)** **12. Ans. (A,C,D)** **13. Ans. (A, B,D)** **14. Ans. (A,B,C)**

EXERCISE - JM

1. Ans. (1) **2. Ans. (4)** **3. Ans. (2)** **4. Ans. (4)** **5. Ans. (4)** **6. Ans. (4)**
7. Ans. (4) **8. Ans. (1)** **9. Ans. (2)** **10. Ans. (1)** **11. Ans. (2)** **12. Ans. (2)**
13. Ans. (3) **14. Ans. (4)** **15. Ans. (3)** **16. Ans. (2)** **17. Ans. (1)** **18. Ans. (2)**
19. Ans. (1) **20. Ans. (2)** **21. Ans. (4)** **22. Ans. (3)** **23. Ans. (2)** **24. Ans. (2)**
25. Ans. (4) **26. Ans. (4)** **27. Ans. (2)** **28. Ans. (4)** **29. Ans. (1)** **30. Ans. (2)**
31. Ans. (4) **32. Ans. (1)** **33. Ans. (4)** **34. Ans. (3)** **35. Ans. (2)** **36. Ans. (4)**
37. Ans. 30 **38. Ans. (1)** **39. Ans. (4)** **40. Ans. 2** **41. Ans. (1)**

EXERCISE - JA

1. Ans. (D) **2. Ans. (C)** **3. Ans. (C)** **4. Ans. 4** **5. Ans. 5**
6. Ans. (B) **7. Ans. (A,B,C,D)** **8. Ans. (B,D)** **9. Ans. (A,B,D)** **10. Ans. 5**
11. Ans. (C) **12. Ans. (B)** **13. Ans. 1** **14. Ans. (A)** **15. Ans. (C,D)**
16. Ans. (B,C) **17. Ans. (A)** **18. Ans. (B)** **19. Ans. (1,3)** **20. Ans. (A,C,D)**
21. Ans. 0.26 to 0.28

ALLEN CAREER INSTITUTE
KOTA (RAJASTHAN)
path to success

Chapter 04
Contents

CAPACITANCE

Important Notes

CAPACITANCE

KEY CONCEPTS

CONCEPT OF CAPACITANCE

When a conductor is charged then its potential rises. The increase in potential is directly proportional to the charge given to the conductor. $Q \propto V \Rightarrow Q = CV$

The constant C is known as the capacity of the conductor.

Capacitance is a scalar quantity with dimension $C = \frac{Q}{V} = \frac{Q^2}{W} = \frac{A^2T^2}{M^1L^2T^{-2}} = M^{-1}L^{-2}T^4A^2$

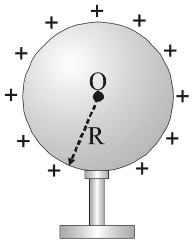
Unit :- farad, coulomb/volt

The capacity of a conductor is independent of the charge given or its potential raised. It is also independent of nature of material and thickness of the conductor. Theoretically infinite amount of charge can be given to a conductor. But practically the electric field becomes so large that it causes ionisation of medium surrounding it. The charge on conductor leaks reducing its potential.

THE CAPACITANCE OF A SPHERICAL CONDUCTOR

When a charge Q is given to a isolated spherical conductor then its potential rises.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} \Rightarrow C = \frac{Q}{V} = 4\pi\varepsilon_0 R$$



If conductor is placed in a medium then

$$C_{medium} = 4\pi\varepsilon R = 4\pi\varepsilon_0\varepsilon_r R$$

Capacitance depends upon :

(a) Size and Shape of Conductor

(b) Surrounding medium

(c) Presence of other conductors nearby

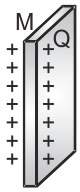
CONDENSER/CAPACITOR

The pair of conductor of opposite charges on which sufficient quantity of charge may be accommodated is defined as condenser.

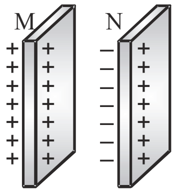
- **Principle of a Condenser**

It is based on the fact that capacitance can be increased by reducing potential keeping the charge constant.

Consider a conducting plate M which is given a charge Q such that its potential rises to V then

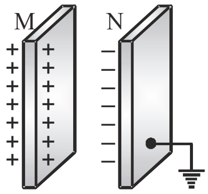


$$C = \frac{Q}{V}$$



Let us place another identical conducting plate N parallel to it such that charge is induced on plate N (as shown in figure). If V_- is the potential at M due to induced negative charge on N and V_+ is the potential at M due to induced positive charge on N, then

$$C' = \frac{Q}{V'} = \frac{Q}{V + V_+ - V_-}$$



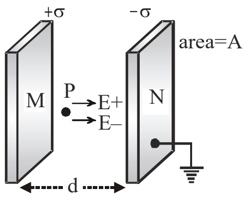
Since V' < V (as the induced negative charge lies closer to the plate M in comparison to induced positive charge). ⇒ C' > C Further, if N is earthed from the outer side (see figure) then $V'' = V_+ - V_-$ ($\because$ the entire positive charge flows to the earth)

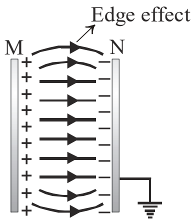
$$C'' = \frac{Q}{V''} = \frac{Q}{V - V_-} \Rightarrow C'' >> C$$

If an identical earthed conductor is placed in the vicinity of a charged conductor then the capacitance of the charged conductor increases appreciable. This is the principle of a parallel plate capacitor.

- **Parallel Plate Capacitor**

(i) Capacitance





It consists of two metallic plates M and N each of area A at separation d. Plate M is positively charged and plate N is earthed. If ε_r is the dielectric constant of the material medium and E is the field at a point P that exists between the two plates, then

I step : Finding electric field $E = E_+ + E_- = \frac{\sigma}{2\varepsilon} + \frac{\sigma}{2\varepsilon} = \frac{\sigma}{\varepsilon} = \frac{\sigma}{\varepsilon_0\varepsilon_r}$ $[\ \varepsilon = \varepsilon_0\ \varepsilon_r]$

II step : Finding potential difference $V = Ed = \frac{\sigma}{\varepsilon_0\varepsilon_r} d = \frac{qd}{A\varepsilon_0\varepsilon_r}$ $\left(\because E = \frac{V}{d} \text{ and } \sigma = \frac{q}{A}\right)$

III step : Finding capacitance $C = \frac{q}{V} = \frac{\varepsilon_r\varepsilon_0 A}{d}$

(ii) Force between the plates

The two plates of capacitor attract each other because they are oppositely charged.

Electric field due to positive plate $E = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2\varepsilon_0 A}$

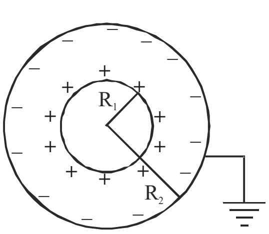
Force on negative charge –Q is $F = -QE = -\frac{Q^2}{2\varepsilon_0 A}$

Magnitude of force $F = \frac{Q^2}{2\varepsilon_0 A} = \frac{1}{2}\varepsilon_0 A E^2$

Force per unit area or energy density or electrostatic pressure $= \frac{F}{A} = u = p = \frac{1}{2}\varepsilon_0 E^2$

- **Spherical Capacitor**

Outer sphere is earthed



When a charge Q is given to inner sphere it is uniformly distributed on its surface A charge –Q is induced on inner surface of outer sphere. The charge +Q induced on outer surface of outer sphere flows to earth as it is grounded.

$E = 0$ for $r < R_1$ and $E = 0$ for $r > R_2$

$$\text{Potential of inner sphere } V_1 = \frac{Q}{4\pi\varepsilon_0 R_1} + \frac{-Q}{4\pi\varepsilon_0 R_2} \Rightarrow \frac{Q}{4\pi\varepsilon_0}\left(\frac{R_2 - R_1}{R_1R_2}\right)$$

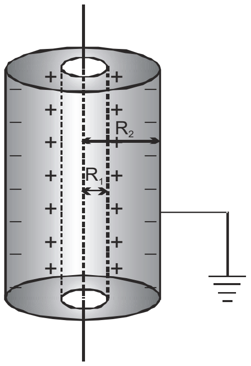
As outer surface is earthed so potential $V_2 = 0$

$$\text{Potential difference between plates } V = V_1 - V_2 = \frac{Q}{4\pi\varepsilon_0}\frac{(R_2 - R_1)}{R_1R_2}$$

$$\text{So } C = \frac{Q}{V} = 4\,\pi\,\varepsilon_0\,\frac{R_1R_2}{R_2 - R_1} \text{ (in air or vacuum)}$$

- **Cylindrical Capacitor**

When a charge Q is given to inner cylinder it is uniformly distributed on its surface. A charge –Q is induced on inner surface of outer cylinder. The charge +Q induced on outer surface of outer cylinder flows to earth as it is grounded



$$\text{Electrical field between cylinders } E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{Q/\ell}{2\pi\varepsilon_0 r}$$

$$\text{Potential difference between plates } V = \int_{R_1}^{R_2} \frac{Q}{2\pi\varepsilon_0 r\ell}dr = \frac{Q}{2\pi\varepsilon_0\ell}\,\ell n\left(\frac{R_2}{R_1}\right)$$

$$\text{Capacitance } C = \frac{Q}{V} = \frac{2\pi\varepsilon_0\ell}{\log_e(R_2/R_1)}$$

Ex. The stratosphere acts as a conducting layer for the earth. If the stratosphere extends beyond 50 km from the surface of earth, then calculate the capacitance of the spherical capacitor formed between stratosphere and earth's surface. Take radius of earth as 6400 km.

Sol. The capacitance of a spherical capacitor is $C = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$

b = radius of the top of stratosphere layer = 6400 km + 50 km = 6450 km = 6.45×10^6 m

a = radius of earth = 6400 km = 6.4×10^6 m

$$\therefore \quad C = \frac{1}{9\times10^9}\times\frac{6.45\times10^6\times6.4\times10^6}{6.45\times10^6 - 6.4\times10^6} = 0.092 \text{ F}$$

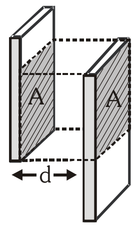
Ex. A cylindrical capacitor has two co-axial cylinders of length 15 cm and radii 1.5 cm and 1.4 cm. The outer cylinder is earthed and the inner cylinder is given a charge of 3.5 μC. Determine the capacitance of the system and the potential of the inner cylinder.

Sol. $\ell = 15 \text{ cm} = 15 \times 10^{-2}$ m; $a = 1.4\text{cm} = 1.4 \times 10^{-2}$ m ;
$b = 1.5 \text{ cm} = 1.5 \times 10^{-2}$m; $q = 3.5\ \mu\text{C} = 3.5 \times 10^{-6}$C

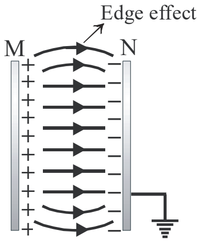
Capacitance $C = \dfrac{2\pi\varepsilon_0\ell}{2.303\log_{10}\left(\dfrac{b}{a}\right)} = \dfrac{2\pi\times 8.854\times10^{-12}\times15\times10^{-2}}{2.303\log_{10}\dfrac{1.5\times10^{-2}}{1.4\times10^{-2}}} = 1.21 \times 10^{-8}$ F

Since the outer cylinder is earthed, the potential of the inner cylinder will be equal to the potential difference between them. Potential of inner cylinder, is $V = \dfrac{q}{C} = \dfrac{3.5\times10^{-6}}{1.2\times10^{-10}} = 2.89\times10^4$ V

- If one of the plates of parallel plate capacitor slides relatively than C decrease (As overlapping area decreases).



- If both the plates of parallel plate capacitor are touched each other resultant charge and potential became zero.
- Electric field between the plates of a capacitor is shown in figure. Non-uniformity of electric field at the boundaries of the plates is negligible if the distance between the plates is very small as compared to the length of the plates.



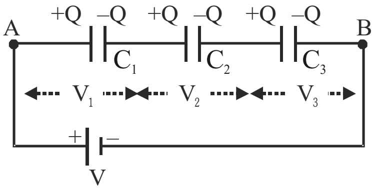
$\vec{E}$ = uniform in the centre

$\vec{E}$ = non-uniform at the edges

COMBINATION OF CAPACITOR

- **Capacitor in series**:

In this arrangement of capacitors the charge has no alternative path(s) to flow.



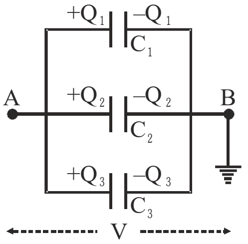
(i) The charges on each capacitor are equal
i.e. $Q = C_1V_1 = C_2V_2 = C_3V_3$

(ii) The total potential difference across AB is shared by the capacitors in the inverse ratio of the capacitances $V = V_1 + V_2 + V_3$
If C_S is the net capacitance of the series combination, then

$$\frac{Q}{C_S} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \Rightarrow \frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

- **Capacitors in parallel**

In such arrangement of capacitors the charge has an alternative path(s) to flow.



(i) The potential difference across each capacitor is same and equal the total potential applied. i.e. $V = V_1 = V_2 = V_3 \Rightarrow V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_3}{C_3}$

(ii) The total charge Q is shared by each capacitor in the direct ratio of the capacitances. $Q = Q_1 + Q_2 + Q_3$

If C_P is the net capacitance for the parallel combination of capacitors :

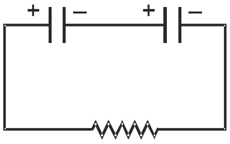
$$C_PV = C_1V + C_2V + C_3V \quad \Rightarrow C_P = C_1 + C_2 + C_3$$

- For a given voltage to store maximum energy capacitors should be connected in parallel.
- If N identical capacitors each having breakdown voltage V are joined in

(i) series then the break down voltage of the combination is equal to NV

(ii) parallel then the breakdown voltage of the combination is equal to V.

- Two capacitors are connected in series with a battery. Now battery is removed and loose wires connected together then final charge on each capacitor is zero.



- If N identical capacitors are connected then $C_{series} = \frac{C}{N}$, $C_{parallel} = NC$
- In DC capacitor's offers infinite resistance in steady state, so there will be no current flows through capacitor branch.

Ex. Capacitor C, 2C, 4C, ... ∞ are connected in parallel, then what will be their effective capacitance ?

Sol. Let the resultant capacitance be $C_{resultant} = C + 2C + 4C +... \infty = C[1 +2 + 4 +... \infty] = C \times \infty = \infty$

Ex. An infinite number of capacitors of capacitance C, 4C, 16C ... ∞ are connected in series then what will be their resultant capacitance ?

Sol. Let the equivalent capacitance of the combination = C_{eq}

$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{4C} + \frac{1}{16C} + ...\infty = \left[1 + \frac{1}{4} + \frac{1}{16} + ...\infty\right]\frac{1}{C}$$ (this is G. P.series)

$\Rightarrow S_\infty = \frac{a}{1-r}$ first term a = 1, common ratio $r = \frac{1}{4}$

$$\Rightarrow \frac{1}{C_{eq}} = \frac{1}{1-\frac{1}{4}} \times \frac{1}{C} \Rightarrow C_{eq} = \frac{3}{4}C$$

ENERGY STORED IN A CHARGED CONDUCTOR/CAPACITOR

Let C is capacitance of a conductor. On being connected to a battery. It charges to a potential V from zero potential. If q is charge on the conductor at that time then q = CV. Let battery supplies small amount of charge dq to the conductor at constant potential V. Then small amount of work done by the battery against the force exerted by exsiting charge is

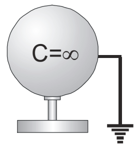
$$dW = Vdq = \frac{q}{C}dq \Rightarrow W = \int_0^Q \frac{q}{C}dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q \Rightarrow W = \frac{Q^2}{2C}$$

where Q is the final charge acquired by the conductor. This work done is stored as potential energy, so

$$U = \frac{Q^2}{2C} = \frac{1}{2}\frac{(CV)^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{Q}{V}\right)V^2 = \frac{1}{2}QV \quad \therefore \quad U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

- As the potential of the Earth is assumed to be zero, capacity of earth or a conductor

  connceted to earth will be infinite $C = \frac{q}{V} = \frac{q}{0} = \infty$



- Actual capacity of the Earth $C = 4\pi\varepsilon_0 R = \frac{1}{9\times10^9}\times 64\times10^5 = 711\ \mu F$
- Work done by battery W_b = (charge given by battery) × (emf) = QV but

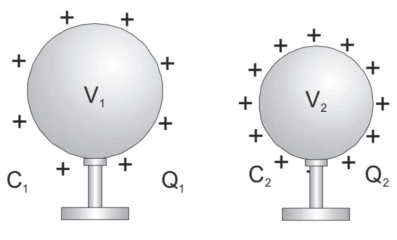
  Energy stored in conductor $= \frac{1}{2}QV$

  so 50% energy supplied by the battery is lost in form of heat.

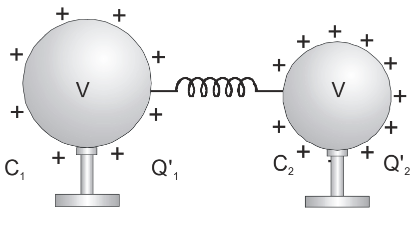
REDISTRIBUTION OF CHARGES AND LOSS OF ENERGY

When two charged conductors are connected by a conducting wire then charge flows from a conductor at higher potential to that at lower potential. This flow of charge stops when the potential of two conductors became equal.

Let the amounts of charges after the conductors are connected are Q_1' and Q_2' respectively and potential is V then



(Before connection)



(After connection)

- **Common potential**

  According to law of Conservation of charge $Q_{\text{before connection}} = Q_{\text{after connection}}$

  $\Rightarrow C_1V_1 + C_2V_2 = C_1V + C_2V$

  Common potential after connection $\boxed{V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}}$

- **Charges after connection**

$$Q_1' = C_1V = C_1\left(\frac{Q_1+Q_2}{C_1+C_2}\right)=\left(\frac{C_1}{C_1+C_2}\right)Q \qquad \text{(Q : Total charge on system)}$$

$$Q_2' = C_2V = C_2\left(\frac{Q_1+Q_2}{C_1+C_2}\right)=\left(\frac{C_2}{C_1+C_2}\right)Q$$

Ratio of the charges after redistribution $\boxed{\frac{Q_1'}{Q_2'}=\frac{C_1V}{C_2V}=\frac{R_1}{R_2}\text{ (in case of spherical conductors)}}$

- **Loss of energy in redistribution**

When charge flows through the conducting wire then **energy is lost mainly on account of Joule effect**, electrical energy is converted into heat energy, so change in energy of this system,

$$\Delta U = U_f - U_i \quad \Rightarrow\left(\frac{1}{2}C_1V^2+\frac{1}{2}C_2V^2\right)-\left(\frac{1}{2}C_1V_1^2+\frac{1}{2}C_2V_2^2\right) \Rightarrow \Delta U=-\frac{1}{2}\left(\frac{C_1C_2}{C_1+C_2}\right)(V_1-V_2)^2$$

Here negative sign indicates that energy of the system decreases in the process.

Ex. The plates of a capacitor are charged to a potential difference of 100 V and then connected across a resister. The potential difference across the capacitor decays exponentially with respect to time. After one second the potential difference between the plates of the capacitor is 80 V. What is the fraction of the stored energy which has been dissipated ?

Sol. Energy losses $\Delta U = \frac{1}{2}CV_0^2-\frac{1}{2}\ CV^2$

Fractional energy loss $\frac{\Delta U}{U_0}=\frac{\frac{1}{2}CV_0^2-\frac{1}{2}CV^2}{\frac{1}{2}CV_0^2}=\frac{V_0^2-V^2}{V_0^2}=\frac{(100)^2-(80)^2}{(100)^2}=\frac{20\times180}{(100)^2}=\frac{9}{25}$

Ex. Two uniformly charged spherical drops at potential V coalesce to form a larger drop. If capacity of each smaller drop is C then find capacity and potential of larger drop.

Sol. When drops coalesce to form a larger drop then total charge and volume remains conserved. If r is radius and q is charge on smaller drop then C = 4 π ε_0 r and q = CV

Equating volume we get $\frac{4}{3}\pi R^3 = 2\times\frac{4}{3}\pi r^3 \Rightarrow R = 2^{1/3}\ r$

Capacitance of larger drop $C' = 4\pi\varepsilon_0 R \quad = 2^{1/3}\ C$

Charge on larger drop $Q = 2q = 2CV$

Potential of larger drop $V' = \frac{Q}{C'}=\frac{2CV}{2^{1/3}C}=2^{2/3}\ V$

EFFECT OF DIELECTRIC

- The insulators in which microscopic local displacement of charges takes place in presence of electric field are known as **dielectrics**.
- Dielectrics are non conductors upto certain value of field depending on its nature. If the field exceeds this limiting value called **dielectric strength** they lose their insulating property and begin to conduct.

- **Dielectric strength** is defined as the maximum value of electric field that a dielectric can tolerate without breakdown. Unit is volt/metre. Dimensions $M^1L^1T^{-3}A^{-1}$

Polar dielectrics

- In absence of external field the centres of positive and negative charge do not coincide-due to asymmetric shape of molecules.
- Each molecule has permanent dipole moment.
- The dipole are randomly oriented so average dipole moment per unit volume of polar dielectric in absence of external field is nearly zero.
- In presence of external field dipoles tends to align in direction of field.

Ex. Water, Alcohol, CO_2, $HC\ell$, NH_3

Non polar dielectrics

- In absence of external field the centre of positive and negative charge coincides in these atoms or molecules because they are symmetric.
- The dipole moment is zero in normal state.
- In presence of external field they acquire induced dipole moment.

Ex. Nitrogen, Oxygen, Benzene, Methane

Polarisation :

The alignment of dipole moments of permanent or induced dipoles in the direction applied electric field is called polarisation.

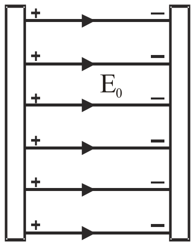
- **Polarisation vector** $\vec{P}$
  This is a vector quantity which describes the extent to which molecules of dielectric become polarized by an electric field or oriented in direction of field.

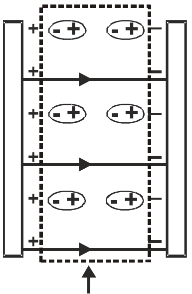
$\vec{P}$ = the dipole moment per unit volume of dielectric = $n\vec{p}$

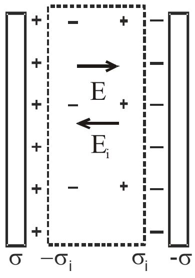
where n is number of atoms per unit volume of dielectric and $\vec{p}$ is dipole moment of an atom or molecule.

$$\vec{P} = n\vec{p} = \frac{q_i d}{Ad} = \left(\frac{q_i}{A}\right) = \sigma_i = \text{induced surface charge density.}$$

Unit of $\vec{P}$ is C/m^2 Dimension is $L^{-2}T^1A^1$







Let E_0, V_0, C_0 be electric field, potential difference and capacitance in absence of dielectric. Let E, V, C are electric field, potential difference and capacitance in presence of dielectric respectively.

Electric field in absence of dielectric $E_0 = \frac{V_0}{d} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$

Electric field in presence of dielectric $E = E_0 - E_i = \frac{\sigma - \sigma_i}{\varepsilon_0} = \frac{Q - Q_i}{\varepsilon_0} = \frac{V}{d}$

Capacitance in absence of dielectric $C_0 = \frac{Q}{V_0}$

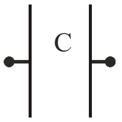
Capacitance in presence of dielectric $C = \frac{Q - Q_i}{V}$

The dielectric constant or relative permittivity K

or $\varepsilon_r = \frac{E_0}{E} = \frac{V_0}{V} = \frac{C}{C_0} = \frac{Q}{Q - Q_i} = \frac{\sigma}{\sigma - \sigma_i} = \frac{\varepsilon}{\varepsilon_0}$

From $K = \frac{Q}{Q - Q_i} \Rightarrow Q_i = Q\left(1 - \frac{1}{K}\right)$ and $K = \frac{\sigma}{\sigma - \sigma_i} \Rightarrow \sigma_i = \sigma\left(1 - \frac{1}{K}\right)$

CAPACITY OF DIFFERENT CONFIGURATION



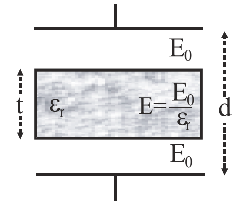
In case of parallel plate capacitor $C = \frac{\varepsilon_0 A}{d}$

If capacitor is partially filled with dielectric

When the dielectric is filed partially between plates, the thickness of dielatric slab is $t (t < d)$.

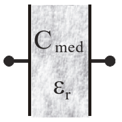
If no slab is introduced between the plates of the capacitor, then a field E_0 given by $E_0 = \frac{\sigma}{\varepsilon_0}$, exists in a space d.

On inserting the slab of thickness t, a field $E = \frac{E_0}{\varepsilon_r}$ exists inside the slab of thickness t and a field E_0 exists in remaining space $(d - t)$. If V is total potential then $V = E_0(d - t) + E\,t$



$$\Rightarrow V = E_0\left[d - t + \left(\frac{E}{E_0}\right)t\right] \quad \because \frac{E_0}{E} = \varepsilon_r = \text{Dielectric constant}$$

$$\Rightarrow V = \frac{\sigma}{\varepsilon_0}\left[d - t + \frac{t}{\varepsilon_r}\right] = \frac{q}{A\varepsilon_0}\left[d - t + \frac{t}{\varepsilon_r}\right] \Rightarrow C = \frac{q}{V} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{\varepsilon_r}\right)} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{\varepsilon_r}\right)} \quad \text{...(i)}$$

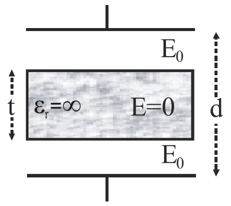


If medium is fully present between the space.

$\because \quad t = d$

Now from equation (i) $C_{\text{medium}} = \frac{\varepsilon_0 \varepsilon_r A}{d}$

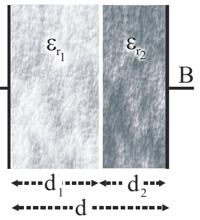
If capacitor is partialy filled by a conducting slab of thickness (t< d).



$\because \quad \varepsilon_r = \infty$ for conductor $C = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{\infty}\right)} = \frac{\varepsilon_0 A}{(d - t)}$

DISTANCE AND AREA DIVISION BY DIELECTRIC

- **Distance Division**



(i) Distance is divided and area remains same.

(ii) Capacitors are in series.

(iii) Individual capacitances are $C_1 = \frac{\varepsilon_0 \varepsilon_{r_1} A}{d_1}$, $C_2 = \frac{\varepsilon_0 \varepsilon_{r_2} A}{d_2}$

These two are in series $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow \frac{1}{C} = \frac{d_1}{\varepsilon_0 \varepsilon_{r_1} A} + \frac{d_2}{\varepsilon_0 \varepsilon_{r_2} A}$

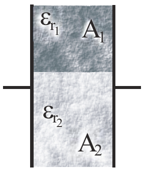
$$\Rightarrow \frac{1}{C} = \frac{1}{\varepsilon_0 A}\left[\frac{d_1 \varepsilon_{r_2} + d_2 \varepsilon_{r_1}}{\varepsilon_{r_1}\varepsilon_{r_2}}\right] \Rightarrow C = \varepsilon_0 A\left[\frac{\varepsilon_{r_1}\varepsilon_{r_2}}{d_1 \varepsilon_{r_2} + d_2 \varepsilon_{r_1}}\right]$$

Special case : If $d_1 = d_2 = \frac{d}{2} \Rightarrow C = \frac{\varepsilon_0 A}{d}\left[\frac{2\varepsilon_{r_1}\varepsilon_{r_2}}{\varepsilon_{r_1} + \varepsilon_{r_2}}\right]$

- **Area Division**

(i) Area is divided and distance remains same.

(ii) Capacitors are in parallel.



(iii) Individual capacitances are $C_1 = \frac{\varepsilon_0 \varepsilon_{r_1} A_1}{d}$ $C_2 = \frac{\varepsilon_0 \varepsilon_{r_2} A_2}{d}$

These two are in parallel so $C = C_1 + C_2 = \frac{\varepsilon_0 \varepsilon_{r_1} A_1}{d} + \frac{\varepsilon_0 \varepsilon_{r_2} A_2}{d} = \frac{\varepsilon_0}{d}(\varepsilon_{r_1} A_1 + \varepsilon_{r2} A_2)$

Special case : If $A_1 = A_2 = \frac{A}{2}$ Then $C = \frac{\varepsilon_0 A}{d}\left(\frac{\varepsilon_{r_1} + \varepsilon_{r2}}{2}\right)$

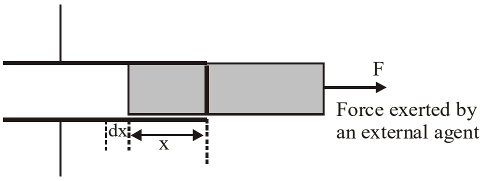
- **Variable Dielectric Constant :**

If the dielectric constant is variable, then equivalent capacitance can be obtained by selecting an element as per the given condition and then integrating.

(i) If different elements are in parallel, then $C = \int dC$, where dC = capacitance of selected differential element.

(ii) If different element are in series, then $\frac{1}{C} = \int d\left(\frac{1}{C}\right)$ is solved to get equivalent capacitance C.

FORCE ON A DIELECTRIC IN A CAPACITOR



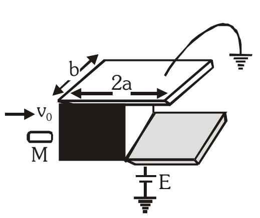
Consider a differential displacement dx of the dielectric as shown in figure always keeping the net force on it zero so that the dielectric moves slowly without acceleration. Then, $W_{\text{Electrostatic}} + W_F = 0$, where W_F denotes the work done by external agent in displacement dx

$W_F = -W_{\text{Electrostatic}}$ $W_F = \Delta U$

$$\Rightarrow -F.dx = \frac{Q^2}{2} d\left[\frac{1}{C}\right] \left[U = \frac{Q^2}{2C}\right] \Rightarrow -F.dx = \frac{-Q^2}{2C^2} dC \Rightarrow F = \frac{Q^2}{2C^2}\left(\frac{dC}{dx}\right)$$

This is also true for the force between the plates of the capacitor. If the capacitor has battery connected to it, then as the p.d. across the plates is maintained constant. $V = \frac{Q}{C} \Rightarrow F = \frac{1}{2} V^2 \frac{dC}{dx}$.

Ex. A parallel plate capacitor is half filled with a dielectric (K) of mass M. Capacitor is attached with a cell of emf E. Plates are held fixed on smooth insulating horizontal surface. A bullet of mass M hits the dielectric elastically and its found that dielectric just leaves out the capacitor. Find speed of bullet and the current as a function of time.



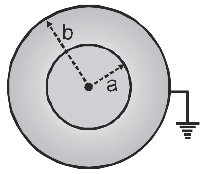
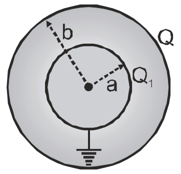
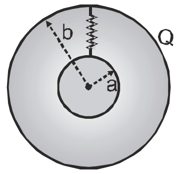
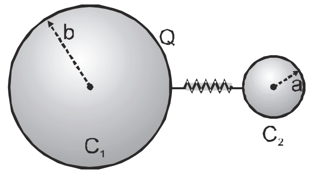
Sol. Since collision is elastic $\therefore$ Velocity of dielectric after collision is v_0.
Dielectric will move and when it is coming out of capacitor a force is applied on it by the capacitor $F = \frac{-dU}{dx} = \frac{-E^2\varepsilon_0 b(K-1)}{2d}$

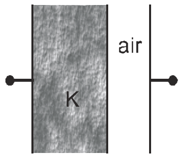
Which decreases its speed to zero, till it comes out it travels a distance a.

$$\frac{1}{2}Mv_0^2 = \frac{E^2\varepsilon_0 b(K-1)a}{2d} \Rightarrow v_0 = E\left[\frac{\varepsilon_0 ab(K-1)}{Md}\right]^{1/2}$$

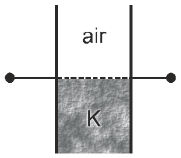
$$i = vE.\frac{dC}{dx}\left[\text{Since, } i = \frac{dq}{dt} = \frac{d}{dt}(EC) = E\frac{dC}{dt} = \frac{EdC}{dx}\frac{dx}{dt}\right]$$

$$i = \left(v_0 - \frac{F}{M}t\right)E\varepsilon_0 b\,\frac{(K-1)}{d} \quad \text{for } t_0 < t < \left(t_0 + \frac{v_0 M}{F}\right) \ (\text{where, } t_0 = \frac{a}{v_0})$$

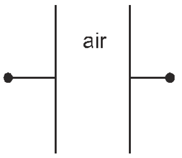
Spherical capacitor outer is earthed	Inner is earthed and outer is given a charge	Connected and outer is given a charge	Connected spheres
 $C = \frac{4\pi\varepsilon_0 ab}{b-a}$ (b > a)	 $C = \frac{4\pi\varepsilon_0 b^2}{b-a}$ (b > a)	 $C = 4\pi\varepsilon_0 b$	 $C = C_1 + C_2$ $C = 4\pi\varepsilon_0(a+b)$



$C_1 = \left[\frac{2K}{K+1}\right]C$



$C_2 = \left[\frac{K+1}{2}\right]C$



$C_3 = C$
when no dielectric is used

$$\boxed{C_2 > C_1 > C_3}$$

Ex. A capacitor has two circular plates whose radius are 8cm and distance between them is 1mm. When mica (dielectric constant = 6) is placed between the plates, calculate the capacitance of this capacitor and the energy stored when it is given potential of 150 volt.

Sol. Area of plate $\pi r^2 = \pi \times (8 \times 10^{-2})^2 = 0.0201\ m^2$ and $d = 1mm = 1 \times 10^{-3}\ m$

Capacity of capacitor $C = \frac{\varepsilon_0\varepsilon_r A}{d} = \frac{8.85\times10^{-12}\times6\times0.0201}{1\times10^{-3}} = 1.068 \times 10^{-9}\ F$

Potential difference $V = 150$ volt

Energy stored $U = \frac{1}{2}CV^2 = \frac{1}{2} \times (1.068 \times 10^{-9}) \times (150)^2 = 1.2 \times 10^{-5}$ J

Ex. A parallel-plate capacitor is formed by two plates, each of area 100 cm^2, separated by a distance of 1mm. A dielectric of dielectric constant 5.0 and dielectric strength 1.9×10^7 V/m is filled between the plates. Find the maximum charge that can be stored on the capacitor without causing any dielectric breakdown.

Sol. If the charge on the capacitor = Q

the surface charge density $\sigma = \frac{Q}{A}$ and the electric field = $\frac{Q}{KA\varepsilon_0}$.

This electric field should not exceed the dielectric strength 1.9×10^7 V/m.

$\therefore$ if the maximum charge which can be given is Q

then $\frac{Q}{KA\varepsilon_0} = 1.9 \times 10^7$ V/m, $\because A = 100 \text{ cm}^2 = 10^{-2} \text{ m}^2$

$\Rightarrow Q = (5.0) \times (10^{-2}) \times (8.85 \times 10^{-12}) \times (1.9 \times 10^7) = 8.4 \times 10^{-6}$ C.

Ex. The distance between the plates of a parallel-plate capacitor is 0.05 m. A field of 3×10^4 V/m is established between the plates of capacitor by connecting with battery. Now capacitor is disconnected from the battery and an uncharged metal plate of thickness 0.01 m is inserted between the plates of capacitor. Calculate new potential difference between the plates of capacitor.What would be the potential difference if a plate of same thickness and dielectric constant K = 2 is introduced in place of metal plate ?

Sol. (i) In case of a capacitor as E = (V/d), the potential difference between the plates before the introduction of metal plate

$$V = E \times d = 3 \times 10^4 \times 0.05 = 1.5 \text{ kV}$$

(ii) Now as after charging battery is removed , capacitor is isolated so q = constant. If C' and V' are the capacity and potential after the introduction of plate $q = CV = C'V'$ i.e., $V' = \frac{C}{C'}V$

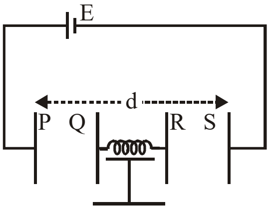
And as $C = \frac{\varepsilon_0 A}{d}$ and $C' = \frac{\varepsilon_0 A}{(d-t)+(t/K)}$, $V' = \frac{(d-t)+(t/K)}{d} \times V$

So in case of metal plate as $K = \infty$, $V_M = \left[\frac{d-t}{d}\right] \times V = \left[\frac{0.05-0.01}{0.05}\right] \times 1.5 = 1.2 \text{ kV}$

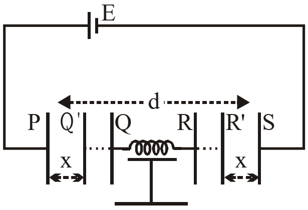
And if instead of metal plate, dielectric with K = 2 is introduced

$$V_D = \left[\frac{(0.05-0.01)+(0.01/2)}{0.05}\right] \times 1.5 = 1.35 \text{ kV}$$

Ex. Two parallel plate capacitors with area A are connected through a conducting spring of natural length ℓ in series as shown. Plates P and S have fixed positions at separation d. Now the plates are connected by a battery of emf E as shown. If the extension in the spring in equilibrium is equal to the separation between the plates, find the spring constant k.



Sol. At any time distance between plates P and Q, R and S is same because force acting on them is same. Let charge on capacitors be q and separation between plates P and Q, R and S be x



Capacitance of capacitor PQ, $C_1 = \frac{\varepsilon_0 A}{x}$

Capacitance of capacitor RS, $C_2 = \frac{\varepsilon_0 A}{x}$ From KVL $\frac{q}{C_1} + \frac{q}{C_2} = E \Rightarrow q = \frac{\varepsilon_0 AE}{2x}$

At this moment extension in spring, $y = d - 2x - \ell$.

Force on plate Q towards P, $F_1 = \frac{q^2}{2A\varepsilon_0} = \frac{\varepsilon_0^2 A^2 E^2}{8Ax^2\varepsilon_0} = \frac{A\varepsilon_0 E^2}{8x^2}$

Spring force on plate Q due to extension in spring, $F_2 = ky$

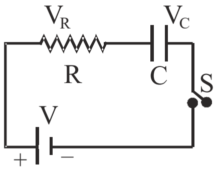
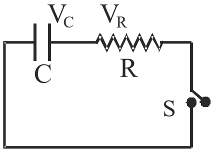
At equilibrium, separation between plates = extension in spring

Thus $x = y = d - 2x - \ell \Rightarrow x = \frac{d-\ell}{3}$ ...(i) and $F_1 = F_2$ ...(ii)

From eq. (i) and (ii), $\frac{A\varepsilon_0 E^2}{8x^2} = ky = kx \Rightarrow x = \left(\frac{A\varepsilon_0 E^2}{8k}\right)^{1/3}$ ...(iii)

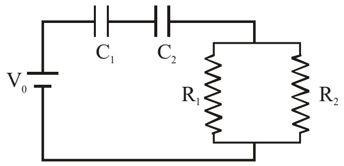
From eq. (i) and (iii), $\left(\frac{d-\ell}{3}\right) = \frac{A\varepsilon_0 E^2}{8k} \Rightarrow k = \frac{A\varepsilon_0 E^2 27}{8(d-\ell)^3}$

CHARGING & DISCHARGING OF A CAPACITOR

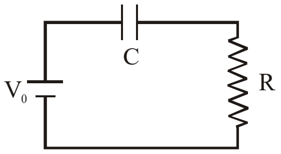
Charging	Discharging
- When a capacitor, resistance, battery, and key is conected in series and key is closed, then	- When a charged capacitor, resistance and keys is conected in series and key is closed. Then energy stored in capacitor is used to circulate current in the circuit.
	
- **Charge at any instant**	- **Charge at any instant**
$V = V_C + V_R = \frac{Q}{C} + IR = \frac{Q}{C} + \frac{dQ}{dt}R$	$V_C + V_R = 0$
$Q = CV\left[1 - e^{-t/RC}\right] = Q_0\left[1 - e^{-t/RC}\right]$	$Q = Q_0 e^{-t/RC}$
At $t = \tau = RC =$ time constant	At $t = \tau = RC =$ time constant
$Q = Q_0[1 - e^{-1}] = 0.632\ Q_0$	$Q = Q_0 e^{-1} = 0.368\ Q_0$
So, in charging, charge increases to 63.2% of charge in the time equal to τ.	So, in discharging, charge decreases to 36.8% of the initial charge in the time equal to τ.
- **Current at any instant**	- **Current at any instant**
$i = dQ/dt = i_0 e^{-t/RC}$ $\{i_0 = Q_0/RC\}$	$i = dQ/dt = -i_0 e^{-t/RC}$ $\{i_0 = Q_0/RC\}$
- **Potential at any instant**	- **Potential at any instant**
$V = V_0(1 - e^{-t/RC})$	$V = V_0\ e^{-t/RC}$

Ex. Find the time constant for given circuit if

$R_1 = 4\Omega$, $R_2 = 12\Omega$, $C_1 = 3\mu F$ and $C_2 = 6\mu F$.



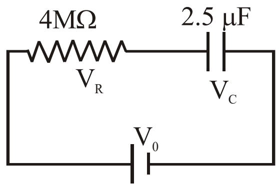
Sol. Given circuit can be reduced to :



$$C = \frac{C_1C_2}{C_1 + C_2} = \frac{3\times6}{3+6} = 2\mu F\ ,\ R = \frac{R_1R_2}{R_1+R_2} = \frac{4\times12}{4+12} = 3\Omega$$

Time constant = RC = (3) $(2 \times 10^{-6}) = 6\mu s$

Ex. A capacitor of 2.5 μF is charged through a series resistor of 4MΩ. In what time the potential drop across the the capacitor will become 3 times that of the resistor. (Given : $\ell n2 = 0.693$)



Sol. $V_C = V_0(1 - e^{-t/RC})$ $\because$ $V_C = 3V_R$ $\therefore V_0 = V_C + \frac{V_C}{3} \Rightarrow V_C = \frac{3}{4}V_0$

$$\Rightarrow \frac{3}{4}V_0 = V_0(1 - e^{-t/RC}) \Rightarrow \frac{3}{4} = 1 - e^{-t/RC} \Rightarrow \frac{1}{4} = e^{-t/RC} \Rightarrow 4 = e^{t/RC}$$

$$\Rightarrow \frac{t}{RC} = \ell n4 \Rightarrow t = RC\ell n4 = 2RC\ \ell n2 = 2 \times 4 \times 10^{6} \times 2.5 \times 10^{-6} \times 0.693 = 13.86\ s$$

EXERCISE (S-1)

Capacitance

1. Two large parallel conducting plates are 34 mm apart and carry equal but opposite charges on their facing surfaces. An electron placed midway between the plates experiences a force of 3.2×10^{-16} N. What is the potential difference (in volts) between the plates?

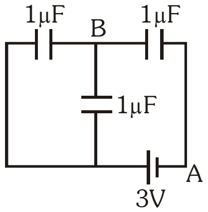
CP0001

2. 2 conducting objects one with charge of +Q and another with –Q are kept on x-axis at x = 0 and x = 1 respectively. The electric field on the x-axis is given by $3Q\left(x^2 + \frac{4}{3}\right)$. If the capacitance of this configuration of objects is C. Then fill $\frac{1}{C}$ (in F^{-1}) in OMR sheet.

CP0002

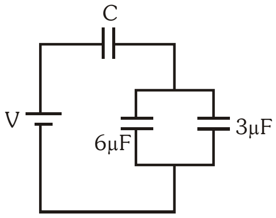
Capacitor Circuits

3. If potential of A is 5V, then potential of B in volt is



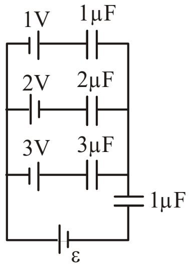
CP0003

4. If charge on 3μF capacitor is 3μC. Find the charge on capacitor of capacitance C in μC.



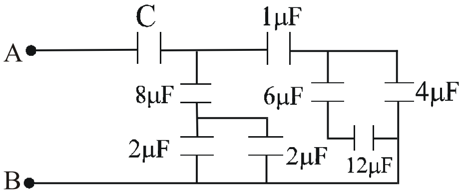
CP0004

5. In the figure shown, find the e.m.f. ε for which charge on 2μF capacitor is 4 μC.



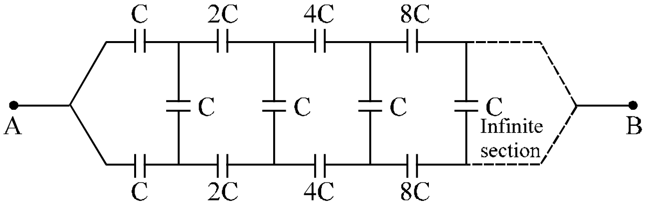
CP0005

6. In the following circuit, the resultant capacitance between A and B is 1 μF. Find the value of C.



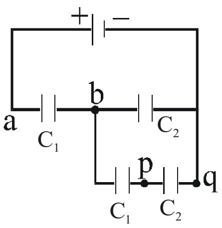
CP0006

7. Find the equivalent capacitance of the circuit between point A and B.



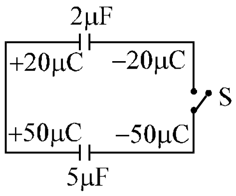
CP0007

8. In the given network if potential difference between p and q is 2V and $C_2 = 3C_1$. Then find the potential difference between a & b.



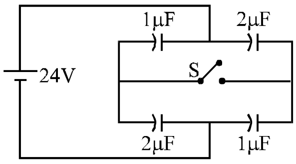
CP0008

9. Find heat produced in the circuit shown in figure on closing the switch S.



CP0009

10. The connections shown in figure are established with the switch S open. How much charge will flow through the switch if it is closed ?



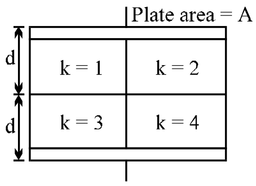
CP0010

11. The plates of a parallel plate capacitor are given charges +4Q and –2Q. The capacitor is then connected across an uncharged capacitor of same capacitance as first one (= C). Find the final potential difference between the plates of the first capacitor.

CP0011

Dielectrics

12. Find the capacitance of the system shown in figure.

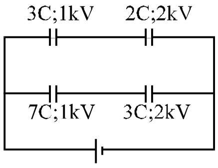


CP0012

13. The plates of a parallel plate capacitor are charged upto 100 volt. A 2 mm thick plate is inserted between the plates, then to maintain the same potential difference, the distance between the capacitor plates is increased by 1.6 mm. Find the dielectric constant of the plate.

CP0013

14. The diagram shows four capacitors with capacitances and break down voltages as mentioned. What should be the maximum value of the external emf source such that no capacitor breaks down?



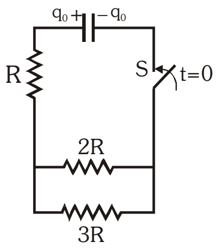
CP0014

15. Two square metallic plates of 1 m side are kept 0.01 m apart, like a parallel plate capacitor, in air in such a way that one of their edges is perpendicular, to an oil surface in a tank filled with an insulating oil. The plates are connected to a battery of e.m.f. 500 volt . The plates are then lowered vertically into the oil at a speed of 0.001 m/s. Calculate the current drawn from the battery during the process. [di-electric constant of oil = 11, $\in_0 = 8.85 \times 10^{-12}$ C^2/N^2 m^2]

CP0015

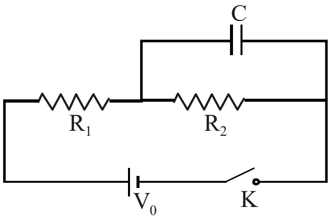
R-C Circuits

16. At t = 0 charge on capacitor is q_0. Now switch S is closed. Heat loss in 3R is x × 10^{-6} J. Then find the value of x. [Given q_0= 15μC, C = 6/55 μF]



CP0016

17. In the connection shown in the *figure* the switch K is open and the capacitor is uncharged. Then we close the switch and let the capacitor charge up to the maximum and open the switch again. Then (Use the following data : V_0=30 V, R_1=10 kΩ, R_2=5 kΩ.)

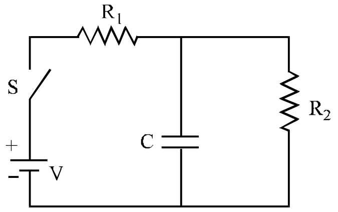


(i) the current through R_1 be I_1 immediately after closing the switch
(ii) the current through R_2 be I_2 a long time after the switch was closed
(iii) the current through R_2 be I_3 immediately after reopening the switch

Find the value of $\frac{I_1}{I_2 I_3}$ (in ampere^{-1}).

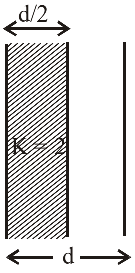
CP0017

18. In the given circuit, the switch S is closed at time t = 0. The charge Q on the capacitor at any instant t is given by $Q(t) = Q_0(1-e^{-\alpha t})$. Find the value of Q_0 and α in terms of given parameters shown in the circuit. **[IIT-JEE 2005]**



CP0018

19. A certain series RC circuit is formed using a resistance R, a capacitor without dielectric having a capacitance C = 2F and a battery of emf E = 3V. The circuit is completed and it is allowed to attain the steady state. After this, at t = 0, half the thickness of the capacitor is filled with a dielectric of constant K = 2 as shown in the figure. The system is again allowed to attain a steady state. What will be the heat generated (in joule) in the circuit between t = 0 and t = ∞ ?



CP0019

20. A capacitor filled with dielectric of permittivity $\varepsilon = 2.1$ loses half the charge acquired during a time interval τ = 3.0 min. Assuming the charge to leak only through the dielectric filler, calculate its resistivity.

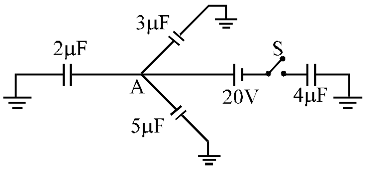
CP0020

EXERCISE (S-2)

1. Three capacitors of $2\mu F$, $3\mu F$ and $5\mu F$ are independently charged with batteries of emf's 5V, 20V and 10V respectively. After disconnecting from the voltage sources. These capacitors are connected as shown in figure with their positive polarity plates are connected to A and negative polarity is earthed. Now a battery of 20V and an uncharged capacitor of $4\mu F$ capacitance are connected to the junction A as shown with a switch S. When switch is closed, find :

(a) the potential of the junction A.

(b) final charges on all four capacitors.

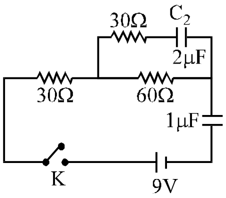


CP0021

2. For the arrangement shown in the figure, the key is closed at t = 0. C_2 is initially uncharged while C_1 has a charge of $2\mu C$.

(a) Find the current coming out of the battery just after switch is closed.

(b) Find the charge on the capacitors in the steady state condition.



CP0022

3. A potential difference of 300 V is applied between the plates of a parallel plate capacitor spaced 1 cm apart. A plane parallel glass plate with a thickness of 0.5 cm and a plane parallel paraffin plate with a thickness of 0.5 cm are placed in the space between the capacitor plates find :

(i) Intensity of electric field in each layer.

(ii) The drop of potential in each layer.

(iii) Surface charge density on the capacitor. Given that : $k_{glass} = 6$, $k_{paraffin} = 2$

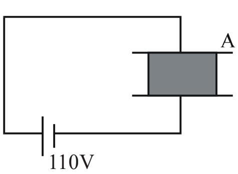
CP0023

4. Two parallel plate capacitors A & B have the same separation $d = 8.85 \times 10^{-4}$ m between the plates. The plate areas of A & B are $0.04\ m^2$ & $0.02\ m^2$ respectively. A slab of di-electric constant (relative permittivity) K=9 has dimensions such that it can exactly fill the space between the plates of capacitor B.

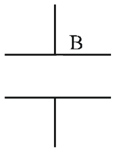
(i) The di-electric slab is placed inside A as shown in the figure (a) A is then charged to a potential difference of 110 volt. Calculate the capacitance of A and the energy stored in it.

(ii) The battery is disconnected & then the di-electric slab is removed from A . Find the work done by the external agency in removing the slab from A .

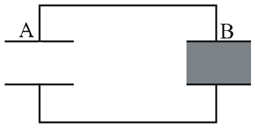
(iii) The same di-electric slab is now placed inside B, filling it completely. The two capacitors A & B are then connected as shown in figure (c). Calculate the energy stored in the system.



(a)



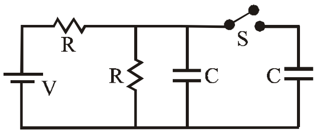
(b)



(c)

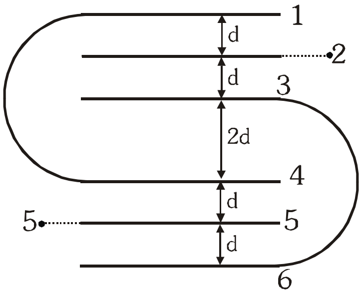
CP0024

5. In the figure shown initially switch is open for a long time. Now the switch is closed at t = 0. Find the charge on the rightmost capacitor as a function of time given that it was intially uncharged.



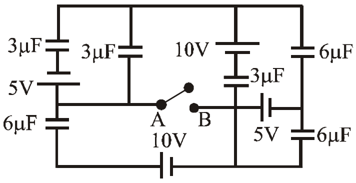
CP0025

6. There are six plates of equal area A and separation between the plates is d (d<<A) are arranged as shown in figure. The equivalent capacitance between points 2 and 5, is $\alpha\frac{\epsilon_0 A}{d}$. Then find the value of α.



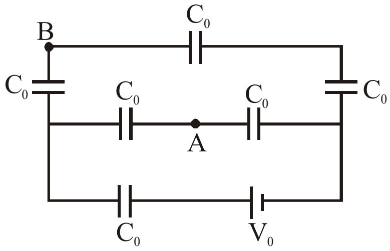
CP0026

7. Find the charge flown through the switch from A to B when it is closed.



CP0027

8. In the arrangement shown in figure, find the potential difference $V_B - V_A$ (in Volt). (Take $V_0 = 55$ V)



CP0028

EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

Capacitance

1. A parallel plate capacitor has d = 1mm and C = 1 F with no medium inside will have :-

(A) Area = $36\pi \times 10^6$
(B) Area = $4\pi \times 10^6$
(C) Area = $6\pi \times 10^6$
(D) Area = $\pi \times 10^6$

CP0029

2. A capacitor connected with battery is kept in a box so :-

(A) Net flux will come out from the box
(B) Net flux will get in the box
(C) Net flux is zero
(D) Net flux depends on polarity of battery

CP0030

3. Choose the CORRECT statement :-

(A) C will increase on increasing Q
(B) C will increase on decreasing Q
(C) C will increase on decreasing V
(D) C doesnot depend on Q & V

CP0031

4. The plate areas and plate separations of parallel plate capacitors are

Capacitor 1 : area A_0, separation d_0
Capacitor 2 : area $2A_0$, separation $2d_0$
Capacitor 3 : area $2A_0$, separation $d_0/2$
Capacitor 4 : area $A_0/2$, separation $2d_0$
Capacitor 5 : area A_0, separation $d_0/2$

Rank these according to their capacitances, least to greatest

(A) 1, 2, 3, 4, 5
(B) 5, 4, 3, 2, 1
(C) 5, 3 and 4 tie ; then 1, 2
(D) 4, 1 and 2 tie ; then 5, 3

CP0032

Capacitor circuits

5. A 2-μF and a 1-μF capacitor are connected in parallel and a potential difference is applied across the combination. The 2-μF capacitor has :

(A) Twice the charge of the 1-μF capacitor
(B) Half the charge of the 1-μF capacitor
(C) Twice the potential difference of the 1-μF
(D) Half the potential difference of the 1-μF capacitor

CP0033

6. A 2-μF and a 1-μF capacitor are connected in series and a potential difference is applied across the combination. The 2-μF capacitor has :

(A) Twice the charge of the 1-μF capacitor

(B) Half the charge of the 1-μF capacitor

(C) Twice the potential difference of the 1-μF

(D) Half the potential difference of the 1-μF capacitor

CP0034

7. Capacitor C_1 and C_2 are connected in parallel and a potential difference is applied to the combination. If the capacitor that is equivalent to the combination has the same potential difference, then the charge on the equivalent capacitor is the same as :

(A) The charge on C_1

(B) The sum of the charges on C_1 and C_2

(C) The difference of the charges on C_1 and C_2

(D) The product of the charges on C_1 and C_2

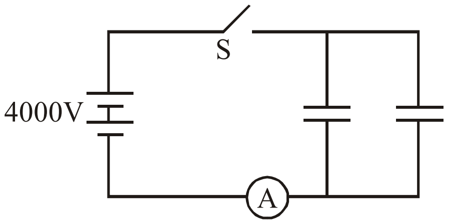
CP0035

8. A 2-μF and a 1-μF capacitor are connected in series and charged from a battery. They store charges P and Q, respectively. When disconnected and charged separately using the same battery, they have charges R and S, respectively. Then :

(A) $R > S > Q = P$ (B) $P > Q > R = S$ (C) $R > P = Q > S$ (D) $R = P > S = Q$

CP0036

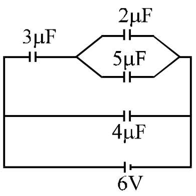
9. Each of the two 25-μF capacitors shown is initially unchanged. How many coulombs of charge pass through the ammeter A after the switch S is closed ?



(A) 0.10 (B) 0.20 (C) 10 (D) 0.05

CP0037

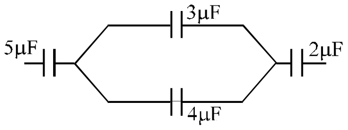
10. In the circuit shown in figure, the ratio of charges on 5μF and 4μF capacitor is :-



(A) 4/5 (B) 3/5 (C) 3/8 (D) 1/2

CP0038

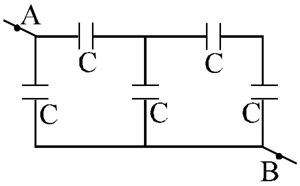
11. If charge on left plate of the 5μF capacitor in the circuit segment shown in the figure is –20μC, the charge on the right plate of 3μF capacitor is :-



(A) +8.57 μC (B) –8.57 μC (C) +11.42 μC (D) –11.42 μC

CP0039

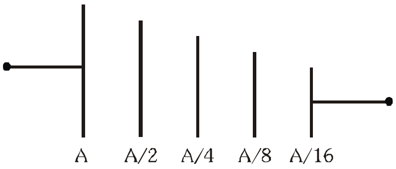
12. What is the equivalent capacitance of the system of capacitors between A & B as shown in the figure.



(A) $\frac{7}{6}$C (B) 1.6 C (C) C (D) None

CP0040

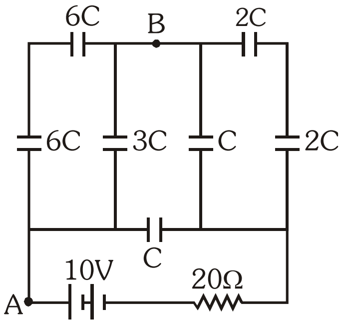
13. 5 Conducting plates each are placed face to face & equi-spaced at distance d. Area of each plate is half the previous plate. If area of first plate is A. Then the equivalent capacitance of the system shown is :-



(A) $\frac{\varepsilon_0 A}{d}$ (B) $\frac{\varepsilon_0 A}{10d}$ (C) $\frac{\varepsilon_0 A}{20d}$ (D) $\frac{\varepsilon_0 A}{30d}$

CP0041

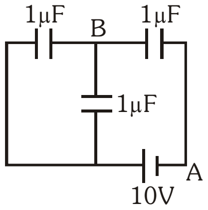
14. For the circuit shown here, the potential difference between points A and B is :-



(A) 2.5 V (B) 7.5 V (C) 10 V (D) Zero

CP0042

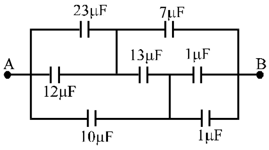
15. If potential of A is 10V, then potential of B is :-



(A) 25/3 V (B) 50/3 V (C) 100/3 V (D) 50 V

CP0043

16. Find the equivalent capacitance across A & B :-



(A) $\frac{28}{3}$ μf (B) $\frac{15}{2}$ μF (C) 15 μF (D) none

CP0044

17. A capacitor of capacitance C is charged to a potential difference V from a cell and then disconnected from it. A charge +Q is now given to its positive plate. The potential difference across the capacitor is now :-

(A) V (B) $V+\frac{Q}{C}$ (C) $V+\frac{Q}{2C}$ (D) $V-\frac{Q}{C}$, if V < CV

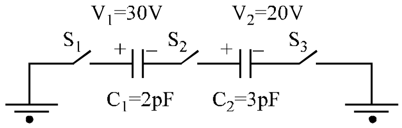
CP0045

18. A parallel plate capacitor is made by stacking n equally spaced plates connected alternatively. If the capacitance between any two adjacent plates is C, then the resultant capacitance is :-**[AIEEE-2005]**

(A) (n–1)C (B) (n+1)C (C) C (D) nC

CP0046

19. For the circuit shown, which of the following statements is true ? **[IIT-JEE 1999]**



(A) with S_1 closed, $V_1 = 15$ V, $V_2 = 20$ V

(B) with S_3 closed, $V_1 = V_2 = 25$ V

(C) with S_1 & S_2 closed, $V_1 = V_2 = 0$

(D) with S_1 & S_2 closed, $V_1 = 30$ V, $V_2 = 20$ V

CP0047

Force & energy

20. A 2-μF and a 1-μF capacitor are connected in series and charged by a battery. They store energies P and Q, respectively. When disconnected and charged separately using the same battery, they store energies R and S, respectively. Then

(A) R > P > S > Q (B) P > Q > R > S (C) R > P > Q > S (D) R > S > Q > P

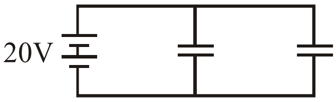
CP0048

21. Capacitors A and B are identical. Capacitor A is charged so it stores 4J of energy and capacitor B is uncharged. The capacitor are then connected in parallel. The total stored energy in the capacitors is now:

(A) 16 J (B) 8 J (C) 4 J (D) 2 J

CP0049

22. To store a total of 4×10^{-4} J of energy in the two identical capacitors shown, each should have a capacitance of :



(A) 0.10 μF (B) 0.50 μF, 0.10 μF (C) 1.0 μF (D) 1.5 μF

CP0050

23. A battery is used to charged a parallel-plate capacitor, after which it is disconnected. Then the plates are pulled apart to twice their original separation. This process will doubled the :

(A) capacitance (B) surface charge density on each plate

(C) stored energy (D) electric field between the two places3

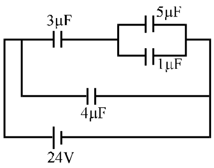
CP0051

24. A parallel-plate capacitor has a plate area of 0.3 m^2 and a plate separation of 0.1 mm. If the charge on each plate has a magnitude of 5×10^{-6} C then the force exerted by one plate on the other has a magnitude of about :

(A) 0 (B) 5 N (C) 1×10^4 N (D) 9×10^5 N

CP0052

25. In the circuit shown, the energy stored in 1μF capacitor is :-



(A) 40 μJ (B) 64 μJ (C) 32 μJ (D) none

CP0053

26. A parallel plate capacitor has an electric field of 10^5V/m between the plates. If the charge on the capacitor plate is 1μC, then the force on each capacitor plate is :-

(A) 0.1N (B) 0.05N (C) 0.02N (D) 0.01N

CP0054

27. Consider a capacitor connected with a battery, capacitor is in steady state. Now plates of capacitor are drawn apart so as to double the separation in two cases.

Case :

(i) Battery remains connected

(ii) Battery is disconnected

Mark the **CORRECT** statement.

(A) In case (i) energy of capacitor increases

(B) In case (i) work done by battery is positive

(C) In case (ii) energy of capacitor increases

(D) In case (ii) potential difference across capacitor decreases

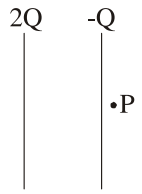
CP0055

28. Two identical capacitors, have the same capacitance C. One of them is charged to potential V_1 and the other to V_2. The negative ends of the capacitors are connected together. When the positive ends are also connected, the decrease in energy of the combined system is :- **[IIT-JEE 2002 (Scr)]**

(A) $\frac{1}{4}C\left(V_1^2 - V_2^2\right)$ (B) $\frac{1}{4}C\left(V_1^2 + V_2^2\right)$ (C) $\frac{1}{4}C\left(V_1 - V_2\right)^2$ (D) $\frac{1}{4}C\left(V_1 + V_2\right)^2$

CP0056

29. In the figure shown the plates of a parallel plate capacitor have unequal charges. Its capacitance is 'C'. P is a point outside the capacitor and close to the plate of charge–Q. The distance between the plates is 'd' then which statement is wrong :-



(A) A point charge at point 'P' will experience electric force due to capacitor

(B) The potential difference between the plates will be $\frac{3Q}{2C}$

(C) The energy stored in the electric field in the region between the plates is $\frac{9Q^2}{8C}$

(D) The force on one plate due to the other plate is $\frac{Q^2}{2\pi \in_0 d^2}$

CP0057

Dielectrics

30. A dielectric slab is slowly inserted between the plates of a parallel plate capacitor, while the potential difference between the plates is held constant by a battery. As it is being inserted :

(A) the capacitance, the potential difference between the plates, and the charge on the positive plate all increase

(B) the capacitance, the potential difference between the plates, and the charge on the positive plate all decrease

(C) the potential difference between the plates increases, the charge on the positive plate decreases, and the capacitance remains the same

(D) the capacitance and the charge on the positive plate increase but the potential difference between the plates remains the same

CP0058

31. An air-filled parallel-plate capacitor has a capacitance of 1 pF. The plate separation is then doubled and a wax dielectric is inserted, completely filling the space between the plates. As a result, the capacitance becomes 2 pF. The dielectric of the wax is

(A) 0.25 (B) 0.5 (C) 2.0 (D) 4.0

CP0059

32. One of materials listed below is to be placed between two identical metal sheets, with no, air gap, to form a parallel-plate capacitor. Which produces the greatest capacitance ?

(A) material of thickness 0.1 mm and dielectric constant 2

(B) material of thickness 0.2 mm and dielectric constant 3

(C) material of thickness 0.3 mm and dielectric constant 2

(D) material of thickness 0.4 mm and dielectric constant 8

CP0060

33. Two parallel-plate capacitors with the same plate area but different capacitance are connected in parallel to a battery. Both capacitors are filled with air. The quantity that is the same for both capacitors when they are fully charged is :

(A) potential difference

(B) energy density

(C) electric field between the plates

(D) charge on the positive plate

CP0061

34. A capacitor stores 60μC charge when connected across a battery. When the gap between the plates is filled with a dielectric, a charge of 120μC flows through the battery. The dielectric constant of the material inserted is :

(A) 1 (B) 2 (C) 3 (D) none

CP0062

35. Condenser A has a capacity of 15 μF when it is filled with a medium of dielectric constant 15. Another condenser B has a capacity 1 μF with air between the plates. Both are charged separately by a battery of 100V. After charging, both are connected in parallel without the battery and the dielectric material being removed. The common potential now is :-

(A) 400V (B) 800V (C) 1200V (D) 1600V

CP0063

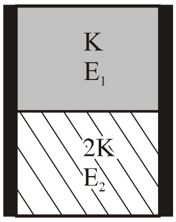
36. Three capacitors 2 μF, 3 μF and 5 μF can withstand voltages to 3V, 2V and 1V respectively. Their series combination can withstand a maximum voltage equal to :-

(A) 5 Volts (B) (31/6) Volts

(C) (26/5) Volts (D) None

CP0064

37. A parallel plate capacitor is connected from a cell and then isolated from it. Two dielectric slabs of dielectric constant K and 2K are now introduce in the region between upper half and lower half of the plate (as shown in figure). The electric field intensity in upper half of dielectric is E_1 and lower half is E_2 then



(A) $E_1 = 2E_2$
(B) Electrostatic potential energy of upper half is less than that of lower half
(C) Induced charges on both slabs are same
(D) Charge distribution on the plates remains same after insertion of dielectric

CP0065

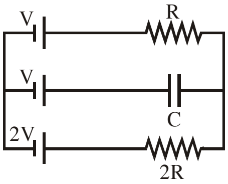
38. Two point charges exert a force F_0 on each other when placed in vacuum. Now the charges are increased to four times, separation between them is doubled and the system is placed is an insulating medium. Now they experience the same force. What should be the dielectric constant of the medium?

(A) 3 (B) 4 (C) 2 (D) 5

CP0066

R-C Circuits

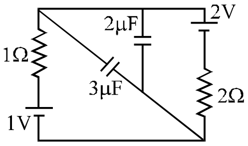
39. In the given circuit, with steady current the potential drop across the capacitor must be :-



(A) V (B) $\frac{V}{2}$ (C) $\frac{V}{3}$ (D) $\frac{2V}{3}$

CP0067

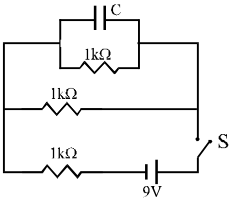
40. In the circuit shown, the charge on the 3μF capacitor at steady state will be



(A) 6 μC (B) 4 μC (C) $\frac{2}{3}$ μC (D) 3 μC

CP0068

41. A capacitor C = 100 μF is connected to three resistor each of resistance 1 kΩ and a battery of emf 9V. The switch S has been closed for long time so as to charge the capacitor. When switch S is opened, the capacitor discharges with time constant :-



(A) 33 ms (B) 5 ms (C) 3.3 ms (D) 50 ms

CP0069

42. An uncharged capacitor of capacitance 4μF, a battery of emf 12 volt and a resistor of 2.5 MΩ are connected in series. The time after which $v_c = 3v_R$ is (take $ln2 = 0.693$) **[IIT-JEE' 2005 (Scr)]**

(A) 6.93 sec. (B) 13.86 sec. (C) 20.52 sec. (D) none of these

CP0070

MULTIPLE CORRECT TYPE QUESTIONS

Capacitor Circuits

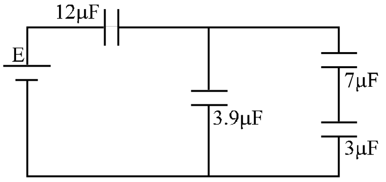
43. Four capacitors and a battery are connected as shown. The potential drop across the 7 μF capacitor is 6 V. Then the :

(A) potential difference across the 3 μF capacitor is 10 V

(B) charge on the 3 μF capacitor is 42 μC

(C) e.m.f. of the battery is 30 V

(D) potential difference across the 12 μF capacitor is 10 V.



CP0071

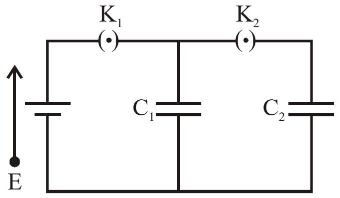
44. In the circuit shown in figure initially key K_1 is closed and key K_2 is open. Then K_1 is opened and K_2 is closed (order is important). [Take Q'_1 and Q'_2 as charges on C_1 and C_2 and V_1 and V_2 as voltage respectively. Then

(A) charge on C_1 gets redistributed such that $V_1 = V_2$

(B) charge on C_1 gets redistributed such that $Q'_1 = Q'_2$

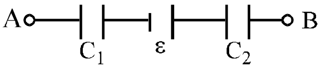
(C) charge on C_1 gets redistributed such that $C_1V_1 + C_2V_2 = C_1 E$

(D) charge on C_1 gets redistributed such that $Q'_1 + Q'_2 = Q$



CP0072

45. A circuit shown in the figure consists of a battery of emf 10 V and two capacitance C_1 and C_2 of capacitances 1.0 μF and 2.0 μF respectively. The potential difference $V_A - V_B$ is 5V



(A) charge on capacitor C_1 is equal to charge on capacitor C_2

(B) Voltage across capacitor C_1 is 5V.

(C) Voltage across capacitor C_2 is 10 V

(D) Energy stored in capacitor C_1 is two times the energy stored in capacitor C_2.

CP0073

Dielectrics

46. A parallel plate capacitor has a parallel slab of copper inserted between and parallel to the two plates, without touching the plates. The capacity of the capacitor after the introduction of the copper sheet is:

(A) minimum when the copper slab touches one of the plates.

(B) maximum when the copper slab touches one of the plates.

(C) invariant for all positions of the slab between the plates.

(D) greater than that before introducing the slab.

CP0074

47. A parallel plate air-core capacitor is connected across a source of constant potential difference. When a dielectric plate is introduced between the two plates then :

(A) some charge from the capacitor will flow back into the source.

(B) some extra charge from the source will flow back into the capacitor.

(C) the electric field intensity between the two plate does not change.

(D) the electric field intensity between the two plates will decrease.

CP0075

48. A parallel plate capacitor of plate area A and plate seperation d is charged to potential difference V and then the battery is disconnected. A slab of dielectric constant K is then inserted between the plates of the capacitor so as to fill the space between the plates. If Q, E and W denote, the magnitude of charge on each plate, the electric field between the plates (after the slab is inserted) and the work done on the system respectively in question, then in the process of inserting the slab

(A) $Q = \frac{\varepsilon_0 AV}{d}$

(B) $Q = \frac{\varepsilon_0 KAV}{d}$

(C) $E = \frac{V}{Kd}$

(D) $W = -\frac{\varepsilon_0 AV^2}{2d}\left(1-\frac{1}{K}\right)$

CP0076

49. The capacitance of a parallel plate capacitor is C when the region between the plate has air. This region is now filled with a dielectric slab of dielectric constant k. The capacitor is connected to a cell of emf E, and the slab is taken out

(A) charge CE(k – 1) flows through the cell

(B) energy $E^2C(k-1)$ is absorbed by the cell.

(C) the energy stored in the capacitor is reduced by $E^2C(k-1)$

(D) the external agent has to do $\frac{1}{2}E^2C(k-1)$ amount of work to take the slab out.

CP0077

50. A capacitor of capacity C_0 is connected to a battery of emf V_0. When steady state is attained a dielectric slab of dielectric constant K is slowly introduced in the capacitor. Mark the Correct statement(s), in final steady state :-

(A) Magnitude of induced charge on the each surface of slab is C_0V_0 (K – 1)

(B) Net electric force due to induced charges on the plate is zero.

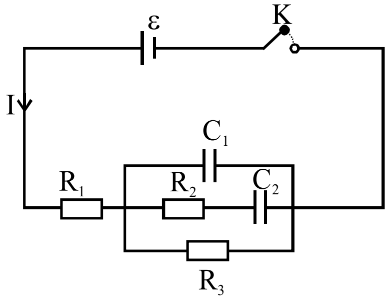
(C) Force of attraction between plates of capacitor is $\frac{K(C_0V_0)^2}{2\in_0 A}$

(D) Net field due to induced charges in dielectric slab is $\frac{8V_0(k-1)^2}{K\in_0 A}$

CP0078

R-C Circuits

51. Mark the **CORRECT** statement(s) regarding the current I through the battery in the circuit shown in figure.



(A) Immediately after the key K is closed, $I = \frac{\varepsilon}{R_1}$

(B) Immediately after the key K is closed, $I = \frac{\varepsilon}{R_1 + R_3}$

(C) Long time after key K is closed, $I = \frac{\varepsilon}{R_1 + R_3}$

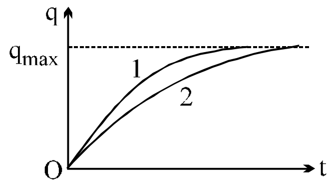
(D) Long time after key K is closed, $I = \frac{\varepsilon}{R_1 + R_2}$

CP0079

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 52 & 53

The charge across the capacitor in two different RC circuits 1 and 2 are plotted as shown in figure.



52. Choose the correct statement(s) related to the two circuits.

(A) Both the capacitors are charged to the same charge.

(B) The emf's of cells in both the circuit are equal.

(C) The emf's of the cells may be different.

(D) The emf E_1 is more than E_2

CP0080

53. Identify the correct statement(s) related to the R_1, R_2, C_1 and C_2 of the two RC circuits.

(A) $R_1 > R_2$ if $E_1 = E_2$ (B) $C_1 < C_2$ if $E_1 = E_2$

(C) $R_1C_1 > R_2C_2$ (D) $\frac{R_1}{R_2} < \frac{C_2}{C_1}$

CP0081

MATRIX MATCH TYPE QUESTION

54.

	Column-I		**Column-II**
(A)	Plates of an isolated, charged, parallel plate, air core capacitor are slowly pulled apart.	(P)	Electric energy stored inside capacitor increases in the process.
(B)	A dielectric is slowly inserted inside an isolated and charged parallel plate air cored capacitor to completely fill the space between plates.	(Q)	Force between the two plates of the capacitor remain unchanged.
(C)	Plates of a parallel plate capacitor connected across a battery are slowly pulled apart.	(R)	Electric field in the region between plates remain unchanged.
(D)	A dielectric slab is slowly inserted inside a parallel plate capacitor connected across a battery to completely fill the space between plates.	(S)	Total electric energy stored inside capacitor decreases in the process.
		(T)	Electric field in the region decreases.

CP0082

EXERCISE (O-2)

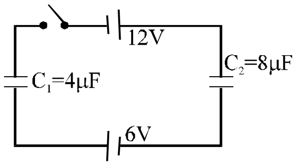
SINGLE CORRECT TYPE QUESTIONS

1. Three long concentric conducting cylindrical shells have radii R, 2R and $2\sqrt{2}$ R. Inner and outer shells are connected to each other. The capacitance across middle and inner shells per unit length is:

(A) $\frac{\frac{1}{3}\in_0}{\ln 2}$ (B) $\frac{6\pi\in_0}{\ln 2}$ (C) $\frac{\pi\in_0}{2\ln 2}$ (D) None

CP0083

2. In the circuit shown initially C_1 & C_2 are uncharged. After closing the switch



(A) The charge on C_2 is greater that on C_1

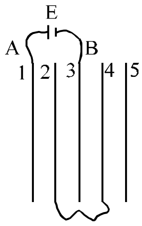
(B) The charge on C_1 and C_2 are the same

(C) The potential drops across C_1 and C_2 are the same

(D) The potential drops across C_2 is greater than that across C_1

CP0084

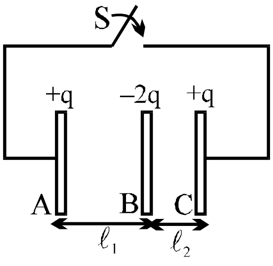
3. Five conducting parallel plates having area A and separation between them d, are placed as shown in the figure. Plate number 2 and 4 are connected wire and between point A and B, a cell of emf E is connected. The charge flown through the cell is :-



(A) $\frac{3}{4}\frac{\varepsilon_0 AE}{d}$ (B) $\frac{2}{3}\frac{\varepsilon_0 AE}{d}$ (C) $\frac{4\varepsilon_0 AE}{d}$ (D) $\frac{\varepsilon_0 AE}{2d}$

CP0085

4. Three long conducting plate A, B & C having charges +q, –2q & +q as shown in figure. Here plate A and C are fixed. If the switch S is closed. The middle plate (B) will start moving in



(here $\ell_1 > \ell_2$)

(A) Leftward direction (B) Rightward direction

(C) will not move (D) First move leftward & then rightward

CP0086

5. A capacitor of capacitance C is initially charged to a potential difference of V volt. Now it is connected to a battery of 2V Volt with opposite polarity. The ratio of heat generated to the final energy stored in the capacitor will be :-

(A) 1.75 (B) 2.25 (C) 2.5 (D) 1/2

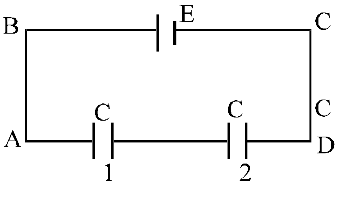
CP0087

6. A conducting body 1 has some initial charge Q, and its capacitance is C. There are two other conducting bodies, 2 and 3, having capacitances : $C_2 = 2C$ and $C_3 \to \infty$. Bodies 2 and 3 are initially uncharged. "Body 2 is touched with body 1. Then, body 2 is removed from body 1 and touched with body 3, and then removed." This process is repeated N times. Then, the charge on body 1 at the end must be

(A) $Q/3^N$ (B) $Q/3^{N-1}$ (C) Q/N^3 (D) None

CP0088

7. In the adjoining figure, capacitor (1) and (2) have a capacitance 'C' each. When the dielectric of dielectric consatnt K is inserted between the plates of one of the capacitor, the total charge flowing through battery is :-

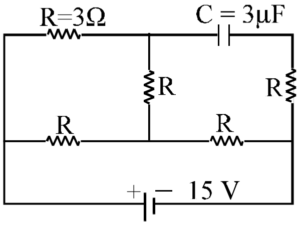


(A) $\frac{KCE}{K+1}$ from B to C (B) $\frac{KCE}{K+1}$ from C to B

(C) $\frac{(K-1)CE}{2(K+1)}$ from B to C (D) $\frac{(K-1)CE}{2(K+1)}$ from C to B

CP0089

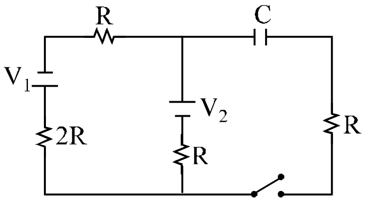
8. In the circuit shown, the cell is ideal, with emf = 15 V. Each resistance is of 3Ω. The potential difference across the capacitor is :-



(A) zero (B) 9 V (C) 12 V (D) 15 V

CP0090

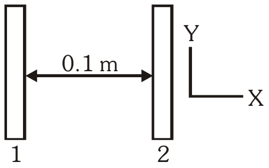
9. In the transient circuit shown the time constant of the circuit is :



(A) $\frac{5}{3}RC$ (B) $\frac{5}{2}RC$ (C) $\frac{7}{4}RC$ (D) $\frac{7}{3}RC$

CP0091

10. Two insulating plates are both uniformly charged in such a way that the potential difference between them is $V_2 - V_1 = 20$ V. (i.e., plate 2 is at a higher potential). The plates are separated by d = 0.1 m and can be treated as infinitely large. An electron is released from rest on the inner surface of plate 1. What is its speed when it hits plate 2 ? ($e = 1.6 \times 10^{-19}$ C, $m_0 = 9.11 \times 10^{-31}$ kg) **[AIEEE-2006]**



(A) 2.65×10^6 m/s (B) 7.02×10^{12} m/s (C) 1.87×10^6 m/s (D) 32×10^{-19} m/s

CP0092

11. A battery is used to charge a parallel plate capacitor till the potential difference between the plates becomes equal to the electromotive force of the battery. The ratio of the energy stored in the capacitor and the work done by the battery will be- **[AIEEE-2007]**

(A) 1 (B) 2 (C) 1/4 (D) 1/2

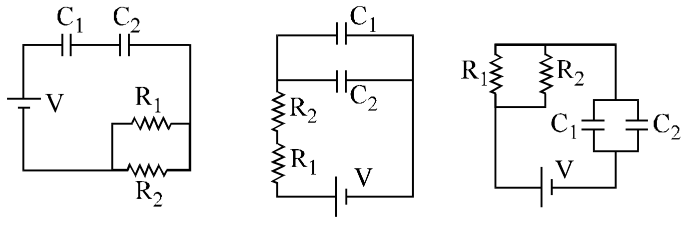
CP0093

12. A parallel plate condenser with a dielectric of dielectric constant K between the plates has a capacity C and is charged to a potential V volts. The dielectric slab is slowly removed from between the plates and then re-inserted. The net work done by the system in this process is- **[AIEEE-2007]**

(A) $\frac{1}{2}(K-1)CV^2$ (B) $CV^2(K-1)/K$ (C) $(K-1)CV^2$ (D) zero

CP0094

13. Given : $R_1 = 1\Omega$, $R_2 = 2\Omega$, $C_1 = 2\mu F$, $C_2 = 4\mu F$. The time constants (in μS) for the circuits I, II, III are respectively **[IIT-JEE 2006]**

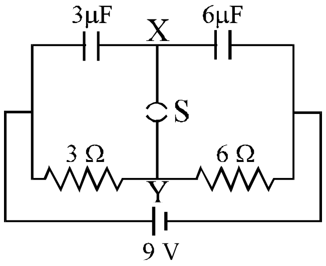


(I) (II) (III)

(A) 18, 8/9, 4 (B) 18, 4, 8/9 (C) 4, 8/9, 18 (D) 8/9, 18, 4

CP0095

14. A circuit is connected as shown in the figure with the switch S open. When the switch is closed, the total amount of charge that flows from Y to X is :- **[IIT-JEE 2007]**



(A) 0 (B) 54 μC (C) 27 μC (D) 81 μC

CP0096

MULTIPLE CORRECT TYPE QUESTIONS

15. A parallel-plate capacitor is connected to a cell. Its positive plate A and its negative plate B have charges +Q and –Q respectively. A third plate C, identical to A and B, with charge +Q, is now introduced midway between A and B, parallel to them. Which of the following are correct?

(A) The charge on the inner face of B is now $-\frac{3Q}{2}$

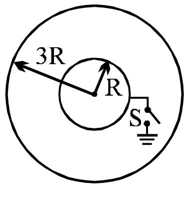
(B) There is no change in the potential difference between A and B.

(C) The potential difference between A and C is one-third of the potential difference between B and C.

(D) The charge on the inner face of A is now $Q/2$.

CP0097

16. Two thin conducting shells of radii R and 3R are shown in the figure. The outer shell carries a charge +Q and the inner shell is neutral. The inner shell is earthed with the help of a switch S.



(A) With the switch S open, the potential of the inner sphere is equal to that of the outer.

(B) When the switch S is closed, the potential of the inner sphere becomes zero.

(C) With the switch S closed, the charge attained by the inner sphere is – Q/3.

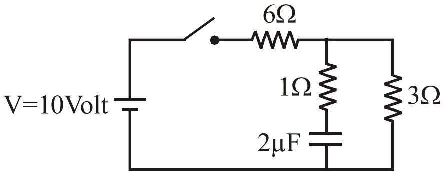
(D) By closing the switch the capacitance of the system increases.

CP0098

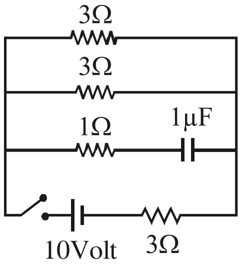
MATRIX MATCH TYPE QUESTION

17. Match the following. In each of the cases shown below, find the time constant of the circuit (in μs) after switch is closed.

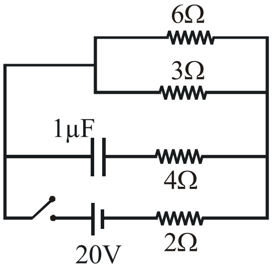
List-I **List-II**

(P) 

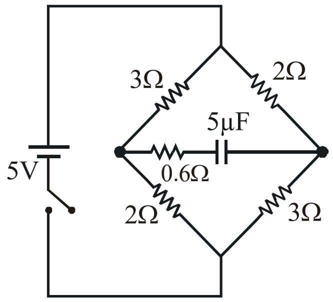
(1) 6

(Q) 

(2) 2

(R) 

(3) 5

(S) 

(4) 15

Codes :

	P	**Q**	**R**	**S**
(A)	1	2	1	4
(B)	3	1	4	2
(C)	1	2	3	4
(D)	2	4	1	3

CP0099

EXERCISE (J-M)

1. Let C be the capacitance of a capacitor discharging through a resistor R. Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then the ratio t_1/t_2 will be : **[AIEEE-2010]**

(1) 2 (2) 1 (3) 1/2 (4) 1/4

CP0100

2. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 30° with each other. When suspended in a liquid of density 0.8 g cm^{-3}, the angle remains the same. If density of the material of the sphere is 1.6 g cm^{-3}, the dielectric constant of the liquid is :

[AIEEE - 2010]

(1) 1 (2) 4 (3) 3 (4) 2

CP0101

3. A resistor 'R' and 2μF capacitor in series is connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R to make the bulb light up 5s after the switch has been closed. ($\log_{10} 2.5 = 0.4$) **[AIEEE-2011]**

(1) $2.7 \times 10^6\ \Omega$ (2) $3.3 \times 10^7\ \Omega$ (3) $1.3 \times 10^4\ \Omega$ (4) $1.7 \times 10^5\ \Omega$

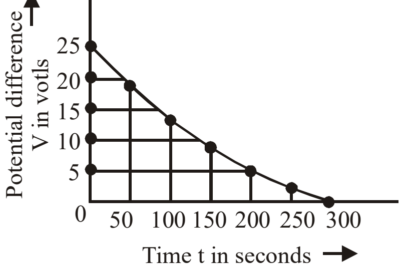
CP0102

4. Combination of two identical capacitors, a resistor R and a dc voltage source of voltage 6V is used in an experiment on (C–R) circuit. It is found that for a parallel combination of the capacitor the time in which the voltage of the fully charged combination reduces to half its original voltage is 10 second. For series combination the time needed for reducing the voltage of the fully charged series combination by half is :- **[AIEEE-2011]**

(1) 20 second (2) 10 second (3) 5 second (4) 2.5 second

CP0103

5. The figure shows an experimental plot for discharging of a capacitor in an R–C circuit. The time constant τ of this circuit lies between:- **[AIEEE 2012]**



(1) 100 sec and 150 sec (2) 150 sec and 200 sec

(3) 0 and 50 sec (4) 50 sec and 100 sec

CP0104

6. Two capacitors C_1 and C_2 are charged to 120V and 200V respectively. It is found that by connecting them together the potential on each one can be made zero. Then : **[JEE-Main-2013]**

(1) $5C_1 = 3C_2$ (2) $3C_1 = 5C_2$ (3) $3C_1 + 5C_2 = 0$ (4) $9C_1 = 4C_2$

CP0105

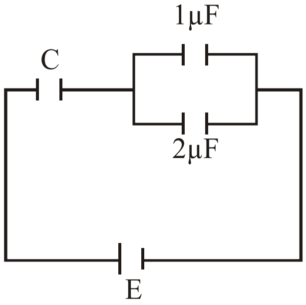
7. A parallel plate capacitor is made of two circular plates separated by a distance of 5 mm and with a dielectriic of dielectric constant 2.2 between them. When the electric field in the dielectric is 3×10^4 V/m, the charge density of the positive plate will be close to : **[JEE-Main-2014]**

(1) 3×10^4 C/m^2 (2) 6×10^4 C/m^2 (3) 6×10^{-7} C/m^2 (4) 3×10^{-7} C/m^2

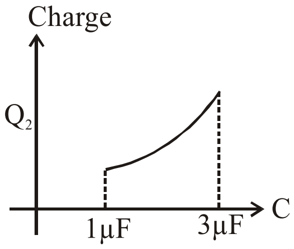
CP0106

8. In the given circuit, charge Q_2 on the 2μF capacitor changes as C is varied from 1μF to 3μF. Q_2 as a function of 'C' is given properly by : (figures are drawn schematically and are not to scale) :-

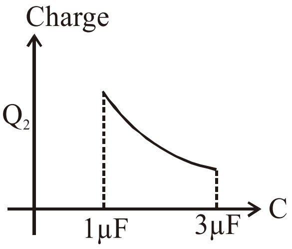
[JEE-Main-2015]



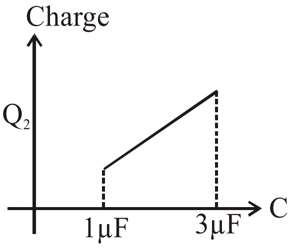
(1)



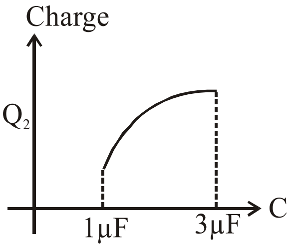
(2)



(3)

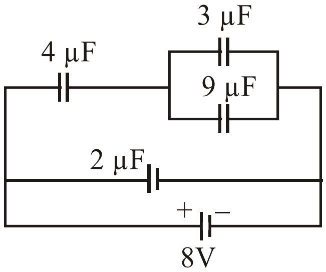


(4)



CP0107

9. A combination of capacitors is set up as shown in the figure. The magnitude of the electric field, due to a point charge Q (having a charge equal to the sum of the charges on the 4 μF and 9 μF capacitors), at a point 30 m from it, would equal: **[JEE-Main-2016]**



(1) 480 N/C (2) 240 N/C (3) 360 N/C (4) 420 N/C

CP0108

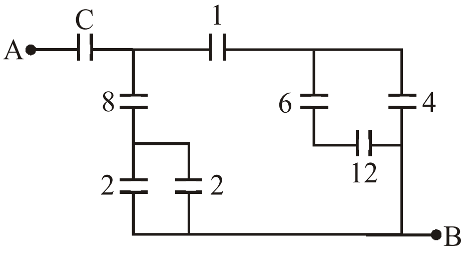
10. Three capacitors each of 4 μF are to be connected in such a way that the effective capacitance is 6 μF. This can be done by connecting them : **[JEE-Main online-2016]**

(1) two in parallel and one in series (2) all in parallel

(3) two in series and one in parallel (4) all in series

CP0109

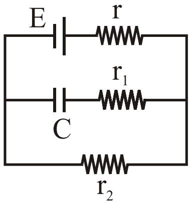
11. Figure shows a network of capacitors where the numbers indicates capacitances in micro Farad. The value of capacitance C if the equivalent capacitance between point A and B is to be 1 μF is :- **[JEE-Main online-2016]**



(1) $\frac{33}{23}\mu F$ (2) $\frac{34}{23}\mu F$ (3) $\frac{31}{23}\mu F$ (4) $\frac{32}{23}\mu F$

CP0110

12. In the given circuit diagram when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be : **[JEE-Main-2017]**



(1) $CE\frac{r_2}{(r+r_2)}$ (2) $CE\frac{r_1}{(r_1+r)}$ (3) CE (4) $CE\frac{r_1}{(r_2+r)}$

CP0111

13. A capacitance of 2 μF is required in an electrical circuit across a potential difference of 1.0 kV. A large number of 1 μF capacitors are available which can withstand a potential difference of not more than 300 V. The minimum number of capacitors required to achieve this is : **[JEE-Main-2017]**

(1) 24 (2) 32 (3) 2 (4) 16

CP0112

14. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20V. If a dielectric material of dielectric constant $K = \frac{5}{3}$ is inserted between the plates, the magnitude of the induced charge will be :- **[JEE-Main-2018]**

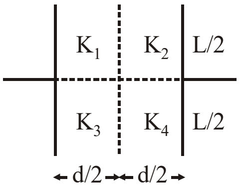
(1) 0.3 n C (2) 2.4 n C (3) 0.9 n C (4) 1.2 n C

CP0113

SELECTED PROBLEMS FROM JEE-MAINS ONLINE PAPERS

15. A parallel plate capacitor with square plates is filled with four dielectrics of dielectric constants K_1, K_2, K_3, K_4 arranged as shown in the figure. The effective dielectric constant K will be :

[JEE-Main-2019_Jan]

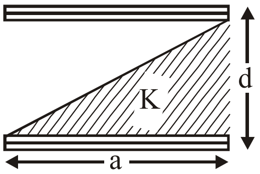


(1) $K = \dfrac{(K_1 + K_2)(K_3 + K_4)}{2(K_1 + K_2 + K_3 + K_4)}$ (2) $K = \dfrac{(K_1 + K_2)(K_3 + K_4)}{(K_1 + K_2 + K_3 + K_4)}$

(3) $K = \dfrac{(K_1 + K_4)(K_2 + K_3)}{2(K_1 + K_2 + K_3 + K_4)}$ (4) $K = \dfrac{(K_1 + K_3)(K_2 + K_4)}{K_1 + K_2 + K_3 + K_4}$

CP0134

16. A parallel plate capacitor is made of two square plates of side 'a', separated by a distance d (d<<a). The lower triangular portion is filled with a dielectric of dielectric constant K, as shown in the figure. Capacitance of this capacitor is : **[JEE-Main-2019_Jan]**



(1) $\dfrac{1}{2}\dfrac{k \in_0 a^2}{d}$ (2) $\dfrac{k \in_0 a^2}{d} \ln K$ (3) $\dfrac{k \in_0 a^2}{d(K-1)} \ln K$ (4) $\dfrac{k \in_0 a^2}{2d(K+1)}$

CP0135

17. A parallel plate capacitor has 1μF capacitance. One of its two plates is given +2μC charge and the other plate, +4μC charge. The potential difference developed across the capacitor is:-

[JEE-Main-2019_April]

(1) 5V (2) 2V (3) 3V (4) 1V

CP0136

18. The parallel combination of two air filled parallel plate capacitors of capacitance C and nC is connected to a battery of voltage, V. When the capacitors are fully charged, the battery is removed and after that a dielectric material of dielectric constant K is placed between the two plates of the first capacitor. The new potential difference of the combined system is :- **[JEE-Main-2019_April]**

(1) $\dfrac{V}{K+n}$ (2) V (3) $\dfrac{(n+1)V}{(K+n)}$ (4) $\dfrac{nV}{K+n}$

CP0137

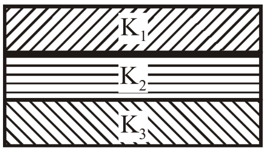
19. A capacitor with capacitance 5μF is charged to 5μC. If the plates are pulled apart to reduce the capacitance to 2μF, how much work is done ? **[JEE-Main-2019_April]**

(1) 3.75×10^{-6} J (2) 2.55×10^{-6} J (3) 2.16×10^{-6} J (4) 6.25×10^{-6} J

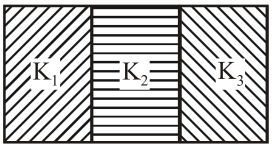
CP0138

20. Two identical parallel plate capacitors, of capacitance C each, have plates of area A, separated by a distance d. The space between the plates of the two capacitors, is filled with three dielectrics, of equal thickness and dielectric constants K_1, K_2 and K_3. The first capacitor is filled as shown in fig. I, and the second one is filled as shown in fig. II.

If these two modified capacitors are charged by the same potential V, the ratio of the energy stored in the two, would be (E_1 refers to capacitor (I) and E_2 to capacitor (II)) : **[JEE-Main-2019_April]**



(I)



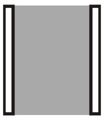
(II)

(1) $\frac{E_1}{E_2}=\frac{9K_1K_2K_3}{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}$ (2) $\frac{E_1}{E_2}=\frac{K_1K_2K_3}{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}$

(3) $\frac{E_1}{E_2}=\frac{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}{K_1K_2K_3}$ (4) $\frac{E_1}{E_2}=\frac{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}{9K_1K_2K_3}$

CP0139

21. A parallel plate capacitor has plates of area A separated by distance 'd' between them. It is filled with a dielectric which has a dielectric constant that varies as $k(x) = K(1 + \alpha x)$ where 'x' is the distance measured from one of the plates. If $(\alpha d) << 1$, the total capacitance of the system is best given by the expression : **[JEE-Main-2020_Jan]**



(1) $\frac{AK\varepsilon_0}{d}\left(1+\frac{\alpha d}{2}\right)$ (2) $\frac{A\varepsilon_0 K}{d}\left(1+\left(\frac{\alpha d}{2}\right)^2\right)$ (3) $\frac{A\varepsilon_0 K}{d}\left(1+\frac{\alpha^2 d^2}{2}\right)$ (4) $\frac{AK\varepsilon_0}{d}(1+\alpha d)$

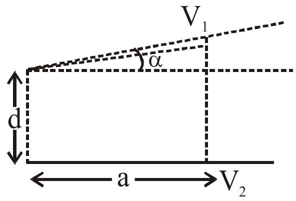
CP0140

22. Effective capacitance of parallel combination of two capacitors C_1 and C_2 is 10 μF. When these capacitors are individually connected to a voltage source of 1V, the energy stored in the capacitor C_2 is 4 times that of C_1. If these capacitors are connected in series, their effective capacitance will be : **[JEE-Main-2020_Jan]**

(1) 3.2 μF (2) 8.4 μF (3) 1.6 μF (4) 4.2 μF

CP0141

23. A capacitor is made of two square plates each of side 'a' making a very small angle α between them, as shown in figure. The capacitance will be close to : **[JEE-Main-2020_Jan]**



(1) $\frac{\varepsilon_0 a^2}{d}\left(1-\frac{3\alpha a}{2d}\right)$ (2) $\frac{\varepsilon_0 a^2}{d}\left(1-\frac{\alpha a}{4d}\right)$ (3) $\frac{\varepsilon_0 a^2}{d}\left(1+\frac{\alpha a}{d}\right)$ (4) $\frac{\varepsilon_0 a^2}{d}\left(1-\frac{\alpha a}{2d}\right)$

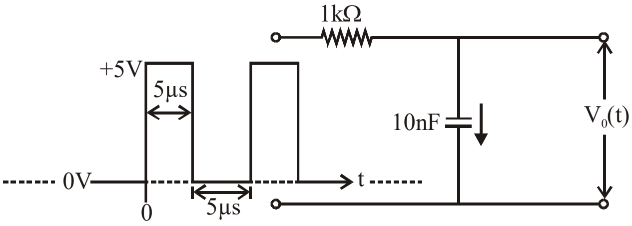
CP0142

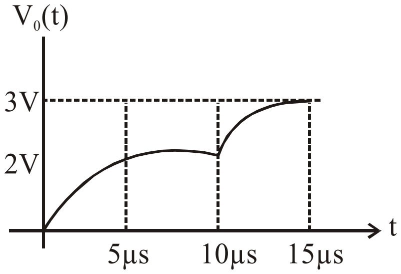
24. A parallel plate capacitor has plate of length 'l', width 'w' and separation of plates is 'd'. It is connected to a battery of emf V. A dielectric slab of the same thickness 'd' and of dielectric constant k = 4 is being inserted between the plates of the capacitor. At what length of the slab inside plates, will be energy stored in the capacitor be two times the initial energy stored? **[JEE-Main-2020_Sep]**

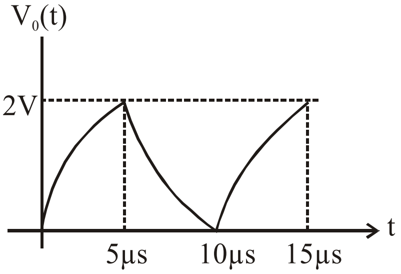
(1) $l/4$ (2) $l/2$ (3) $l/3$ (4) $2l/3$

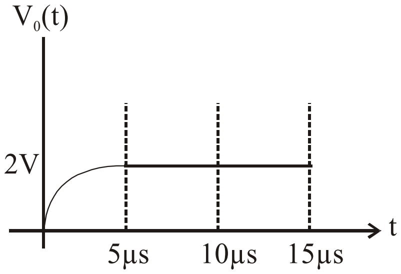
CP0143

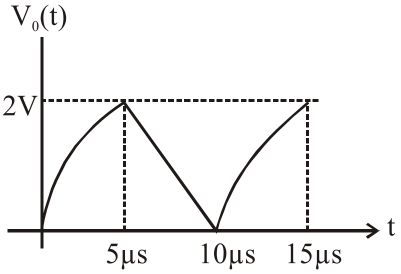
25. For the given input voltage waveform $V_{in}(t)$, the output voltage waveform $V_D(t)$, across the capacitor is correctly depicted by: **[JEE-Main-2020_Sep]**



(1)


(2)


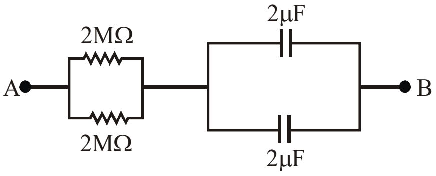
(3)


(4)


CP0144

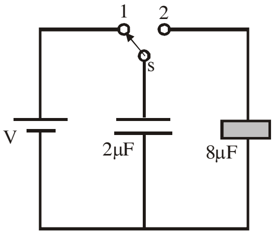
EXERCISE (J-A)

1. At time t = 0, a battery of 10V is connected across points A and B in the given circuit. If the capacitors have no charge initially, at what time (in seconds) does the voltage across them become 4V?
[Take : ℓn 5 = 1.6, ℓn3 = 1.1] **[IIT-JEE 2010]**



CP0114

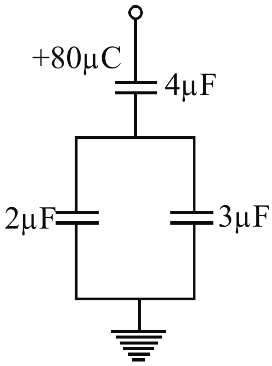
2. A 2 μF capacitor is charged as shown in figure. The percentage of its stored energy dissipated after the switch S is turned to position 2 is :- **[IIT-JEE 2011]**



(A) 0% (B) 20% (C) 75% (D) 80%

CP0115

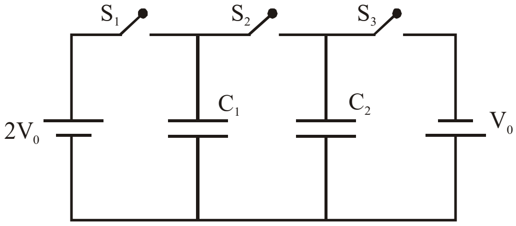
3. In the given circuit, a charge of +80 μC is given to the upper plate of the 4μF capacitor. Then in the steady state, the charge on the upper plate of the 3μF capacitor is :- **[IIT-JEE 2012]**



(A) + 32 μC (B) + 40μC (C) +48μC (D) + 80μC

CP0116

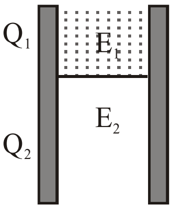
4. In the circuit shown in the figure, there are two parallel plate capacitors each of the capacitance C. The switch S_1 is pressed first to fully charge the capacitor C_1 and then released. The switch S_2 is then pressed to charge the capacitor C_2. After some time, S_2 is released and then S_3 is pressed, After some time, **[IIT-JEE 2013]**



(A) the charge on the upper plate of C_1 is $2CV_0$
(B) the charge on the upper plate of C_1 is CV_0
(C) the charge on the upper plate of C_2 is 0.
(D) the charge on the upper plate of C_2 is $-CV_0$

CP0117

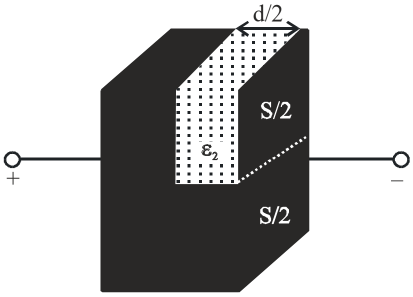
5. A parallel plate capacitor has a dielectric slab of dielectric constant K between its plates that covers 1/3 of the area of its plates, as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between is C_1. When the capacitor is charged, the plate area covered by the dielectric gets charge Q_1 and the rest of the area gets charge Q_2. The electric field in the dielectric is E_1 and that in the other portion is E_2. Choose the correct option/options, ignoring edge effects. **[JEE-Advance 2014]**



(A) $\frac{E_1}{E_2}=1$ (B) $\frac{E_1}{E_2}=\frac{1}{K}$ (C) $\frac{Q_1}{Q_2}=\frac{3}{K}$ (D) $\frac{C}{C_1}=\frac{2+K}{K}$

CP0118

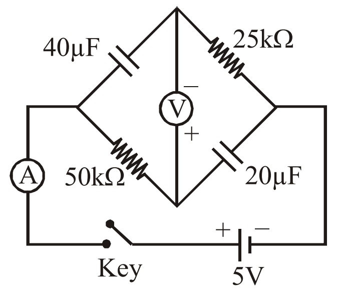
6. A parallel plate capacitor having plates of area S and plate separation d, has capacitance C_1 in air. When two dielectrics of different relative permittivities ($\varepsilon_1 = 2$ and $\varepsilon_2 = 4$) are introduced between the two plates as shown in the figure, the capacitance becomes C_2. The ratio $\frac{C_2}{C_1}$ is :- **[JEE-Advance 2015]**



(A) $\frac{6}{5}$ (B) $\frac{5}{3}$ (C) $\frac{7}{5}$ (D) $\frac{7}{3}$

CP0119

7. In the circuit shown below, the key is pressed at time t = 0. Which of the following statement(s) is(are) true? **[JEE-Advance 2016]**



(A) The voltmeter displays –5V as soon as the key is pressed, and displays +5V after a long time
(B) The voltmeter will display 0 V at time t = ln 2 seconds
(C) The current in the ammeter becomes 1/e of the initial value after 1 second
(D) The current in the ammeter becomes zero after a long time

PARAGRAPH–1

Consider a simple RC circuit as shown in figure 1.

Process 1 : In the circuit the switch S is closed at t = 0 and the capacitor is fully charged to voltage V_0 (i.e., charging continues for time T >> RC). In the process some dissipation (E_D) occurs across the resistance R. The amount of energy finally stored in the fully charged capacitor is E_C.

Process 2 : In a different process the voltage is first set to $\frac{V_0}{3}$ and maintained for a charging time T >> RC. Then the voltage is raised to $\frac{2V_0}{3}$ without discharging the capacitor and again maintained for a time T >> RC. The process is repeated one more time by raising the voltage to V_0 and the capacitor is charged to the same final voltage V_0 as in Process 1.

These two processes are depicted in Figure 2. **[JEE-Advance 2017]**

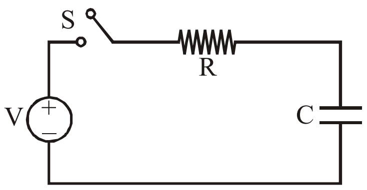


Figure 1

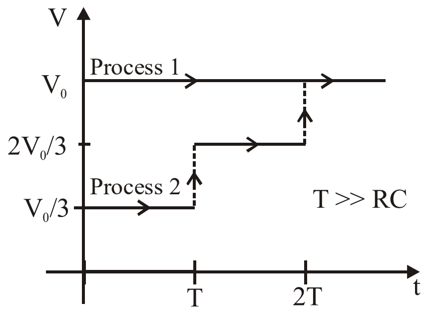


Figure 2

CP0120

8. In Process 1, the energy stored in the capacitor E_C and heat dissipated across resistance E_D are related by :-

(A) $E_C = E_D$ (B) $E_C = 2E_D$ (C) $E_C = \frac{1}{2}E_D$ (D) $E_C = E_D \ln 2$

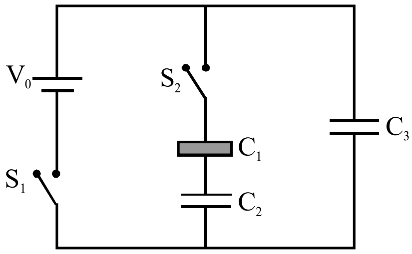
CP0121

9. In Process 2, total energy dissipated across the resistance E_D is :-

(A) $E_D = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$ (B) $E_D = 3\left(\frac{1}{2}CV_0^2\right)$ (C) $E_D = \frac{1}{2}CV_0^2$ (D) $E_D = 3CV_0^{\,2}$

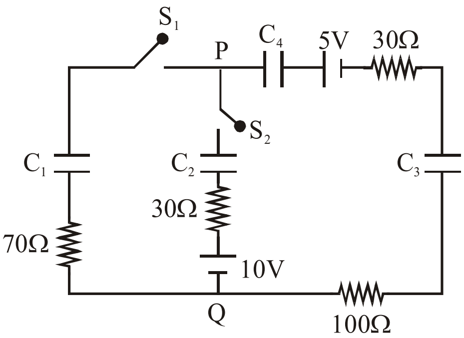
CP0122

10. Three identical capacitors C_1, C_2 and C_3 have a capacitance of 1.0 μF each and they are uncharged initially. They are connected in a circuit as shown in the figure and C_1 is then filled completely with a dielectric material of relative permittivity $\in_r$. The cell electromotive force (emf) $V_0 = 8V$. First the switch S_1 is closed while the switch S_2 is kept open. When the capacitor C_3 is fully charged, S_1 is opened and S_2 is closed simultaneously. When all the capacitors reach equilibrium, the charge on C_3 is found to be 5μC. The value of $\in_r$. **[JEE-Advance 2018]**



CP0123

11. In the circuit shown, initially there is no charge on capacitors and keys S_1 and S_2 are open. The values of the capacitors are C_1 = 10 μF, C_2 = 30 μF and $C_3 = C_4$ = 80 μF. **[JEE-Advance 2019]**



Which of the statement(s) is/are correct ?

(1) The keys S_1 is kept closed for long time such that capacitors are fully charged. Now key S_2 is closed, at this time, the instantaneous current across 30 Ω resistor (between points P and Q) will be 0.2 A (round off to 1st decimal place).

(2) If key S_1 is kept closed for long time such that capacitors are fully charged, the voltage difference between points P and Q will be 10 V.

(3) At time t = 0, the key S_1 is closed, the instantaneous current in the closed circuit will be 25 mA.

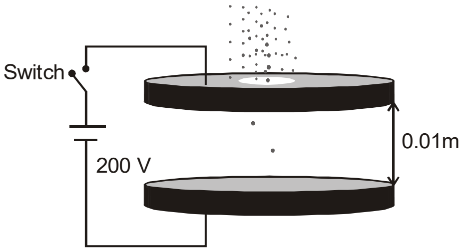
(4) If key S_1 is kept closed for long time such that capacitors are fully charged, the voltage across the capacitors C_1 will be 4V. **CP0124**

12. A parallel plate capacitor of capacitance C has spacing d between two plates having area A. The region between the plates is filled with N dielectric layers, parallel to its plates, each with thickness $\delta = \frac{d}{N}$. The dielectric constant of the m^{th} layer is $K_m = K\left(1+\frac{m}{N}\right)$. For a very large N (> 10^3), the capacitance C is $\alpha\left(\frac{K \in_0 A}{d\ell n2}\right)$. The value of α will be _____.

[$\in_0$ is the permittivity of free space] **[JEE-Advance 2019]**

CP0125

13. Two large circular discs separated by a distance of 0.01 m are connected to a battery via a switch as shown in the figure. Charged oil drops of density 900 kg m^{-3} are released through a tiny hole at the center of the top disc. Once some oil drops achieve terminal velocity, the switch is closed to apply a voltage of 200 V across the discs. As a result, an oil drop of radius 8×10^{-7} m stops moving vertically and floats between the discs. The number of electrons present in this oil drop is ________. (neglect the buoyancy force, take acceleration due to gravity =10 ms^{-2} and charge on an electron (e) = 1.6×10^{-19} C) **[JEE-Advance 2020]**



CP0145

CAPACITANCE

(CBSE Previous Year's Questions)

1. A parallel capacitor is to be designed with a voltage rating 1 kV using a material of dielectric constant 3 and dielectric strength about 10^7 Vm^{-1}. For safety we would like the field 'never to exceed say, 10% of the dipole strength. What minimum area of the plats is required to have a capacitance of 50 PF?

[2; CBSE-2005]

2. A 4μF capacitor is charged by a 200 V supply. The supply is then disconnected and the charged capacitor is connected to another uncharged 2 μF capacitor. How much electrostatic energy of the first capacitor is lost in the process of attaining the steady situation? **[2; CBSE-2005]**

3. Two capacitors of capacitance of 6μF and 12μF are connected in series with a battery. The voltage across the 6μF capacitor is 2 V. Compute the total battery voltage. **[2; CBSE-2006]**

4. A parallel plate capacitor with air between the plates has a capacitance of 8 pF. The separation between the pates is now reduced by half and the space between them is filled with a medium of dielectric constant 5. Calculate the value of capacitance of the capacitor in the second case.

[2; CBSE-2006]

5. The equivalent capacitance of the combination between A and B in the given figure is 4μF.

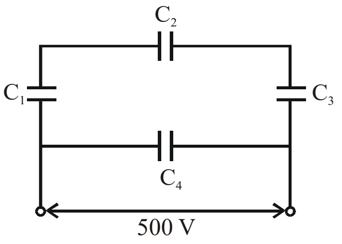
(i) Calculate capacitance of the capacitor C.

(ii) Calculate charge on each capacitor if a 12 V battery is connected across tem1inalsAand B.

(iii) What will be the potential drop across each capacitor ? **[3; CBSE-2009]**

6. A network of four capacitors each of 12μF capacitance is connected to a 500 V supply as shown in the figure. Determine (a) equivalent capacitance of the network and (b) charge on each capacitor.

[3; CBSE-2010]



7. A parallel plate capacitor is being charged by a time varying current. Explain briefly how Ampere's circuital law is generalized to incorporate the effect due to the displacement current.

[2; CBSE-2011]

8. Net capacitance of three identical capacitor in series is 1μF. What will be their net capacitance if connected in parallel ? Find the ratio of energy stored in the two configurations if they are both connected to the same source. **[2; CBSE-2011]**

9. A capacitor of capacitance 'C' is being charged by connecting it across a dc source along with an ammeter. Will the ammeter show a momentary deflection during the process of charging? If so, how would you explain this momentary deflection and the resulting continuity of current in the circuit? Write the expression for the current inside the capacitor. **[2; CBSE-2012]**

10. Deduce the expression for the electrostatic energy stored in a capacitor ofcapacitance 'C' and having charge'Q'.

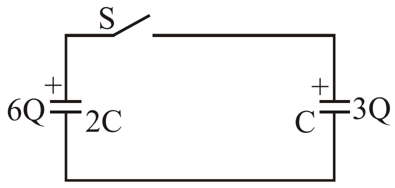
How will the (i) energy stored and (ii) the electric field inside the capacitor be affected when it is completely filled with a dielectric material of dielectric constant K? **[3; CBSE-2012]**

11. A slab of material of dielectric constant K has the same area as that of the plates of a parallel plate capacitor but bas the thickness d/2, where d is the separation between the plates. Find out the expression for its capacitance when the slab is inserted between the plates of the capacitor. **[CBSE-2013]**

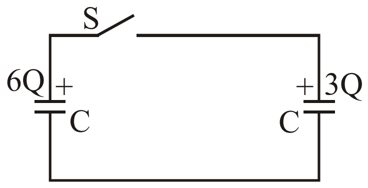
12. A parallel plate capacitor of capacitance C is charged to a potential V. It is then connected to another uncharged capacitor having the same capacitance. Find out the ratio of the energy stored in the combined system to that stored initially in the single capacitor. **[CBSE-2014]**

13. Define dielectric constant of a medium. What is its S.I. unit ? **[1; CBSE-2015]**

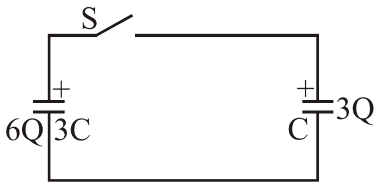
14. Three circuits, each consisting of a switch 'S' and two capacitors, are initially charged, as shown in the figure. After the switch has been closed, in which circuit will the charge on the left-hand capacitor (i) increase, (ii) decrease and (iii) remain same ? Given reasons. **[3; CBSE-2015]**



(a)



(b)



(c)

15. (a) Distinguish, with the help of a suitable diagram, the difference in the behaviour of a conductor and a dielectric plaed in an external electric field. How does polarised dielectric modify the original external field ? **[5; CBSE-2016]**

(b) A capacitor of capacitance C is charged fully by connecting it to a battery of emf E. It is then disconnected from the battery. If the separation between the plates of the capacitor is now doubled, how will the following change ?

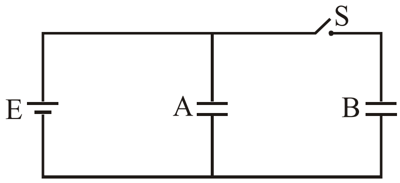
(i) charge stored by the capacitor.

(ii) field strength between the plates.

(iii) energy stored by the capacitor.

Justify your answer in each case.

16. Two identical parallel plate capacitors A and B are connected to a battery of V volts with the switch S closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant K. Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric. **[3; CBSE-2017]**



ANSWER KEY

EXERCISE (S-1)

1. Ans. 68

2. Ans. 5

3. Ans. 7

4. Ans. 9

5. Ans. 6V, 34V

6. Ans. $\frac{32}{23}$ μF

7. Ans. (C)

8. Ans. 30 V

9. Ans. 0

10. Ans. 12μC

11. Ans. 3Q/2C

12. Ans. $\frac{25}{24}\frac{\varepsilon_0 A}{d}$

13. Ans. 5

14. Ans. 2.5 kV

15. Ans. 4.425×10^{-9} Ampere

16. Ans. 225

17. Ans. 750

18. Ans. $Q_0 = \frac{CVR_2}{R_1+R_2}$ and $\alpha = \frac{R_1+R_2}{CR_1R_2}$

19. Ans. $\frac{3}{4}$ J

20. Ans. $\rho = \tau/\varepsilon_0\varepsilon \ln 2 = 1.4 \times 10^{13}\ \Omega.\text{m}$.

EXERCISE (S-2)

1. Ans. (a) $\frac{100}{7}$ volts; (b) 28.56 μC, 42.84 μC, 71.4 μC, 22.88 μC

2. Ans. (a) $\frac{7}{50}$A or $\frac{11}{50}$A, (b) $Q_1 = 9\mu C, Q_2 = 0$

3. Ans. (i) 1.5×10^4 V/m, 4.5×10^4 V/m, (ii) 75 V, 225 V, (iii) 8×10^{-7} C/m^2

4. Ans. (i) 0.2×10^{-8} F, 1.2×10^{-5} J; (ii) 4.84×10^{-5} J; (iii) 1.1×10^{-5} J

5. Ans. $q = \frac{CV}{2}\left(1 - \frac{1}{2}e^{-t/RC}\right)$

6. Ans. 1

7. Ans. 69 mC

8. Ans. 5

EXERCISE (O-1)

1. Ans. (A)

2. Ans. (C)

3. Ans. (D)

4. Ans. (D)

5. Ans. (A)

6. Ans. (D)

7. Ans. (B)

8. Ans. (A)

9. Ans. (B)

10. Ans. (C)

11. Ans. (A)

12. Ans. (B)

13. Ans. (D)

14. Ans. (A)

15. Ans. (B)

16. Ans. (B)

17. Ans. (C)

18. Ans. (A)

19. Ans. (D)

20. Ans. (D)

21. Ans. (D)

22. Ans. (C)

23. Ans. (C)

24. Ans. (B)

25. Ans. (C)

26. Ans. (B)

27. Ans. (C)

28. Ans. (C)

29. Ans. (D)

30. Ans. (D)

31. Ans. (D)

32. Ans. (A,D)

33. Ans. (A)

34. Ans. (C)

35. Ans. (B)

36. Ans. (B)

37. Ans. (B)

38. Ans. (B)

39. Ans. (C)

40. Ans. (B)

41. Ans. (D)

42. Ans. (B)

43. Ans. (B, C, D)

44. Ans. (A,C,D)

45. Ans. (A, D)

46. Ans. (C, D)

47. Ans. (B, C)

48. Ans. (A, C, D)

49. Ans. (A, B, D)

50. Ans. (A, B)

51. Ans. (A, C)

52. Ans. (A, C)

53. Ans. (D)

54. Ans. (A) → (PQR); (B) → (QST); (C) → (ST); (D) → (PR)

EXERCISE (O-2)

1. Ans. (B) **2. Ans. (B)** **3. Ans. (B)** **4. Ans. (B)** **5. Ans. (B)**

6. Ans. (A) **7. Ans. (D)** **8. Ans. (C)** **9. Ans. (C)** **10. Ans. (A)**

11. Ans. (D) **12. Ans. (D)** **13. Ans. (D)** **14. Ans. (C)**

15. Ans. (A,B,C,D) **16. Ans. (A,B,C,D)** **17. Ans. (C)**

EXERCISE (J-M)

1. Ans. (3) **2. Ans. (4)** **3. Ans. (1)** **4. Ans. (4)** **5. Ans. (1)**
6. Ans. (2) **7. Ans. (3)** **8. Ans. (4)** **9. Ans. (4)** **10. Ans. (3)**
11. Ans. (4) **12. Ans. (1)** **13. Ans. (2)** **14. Ans. (4)** **15. Ans. (Bonus)**
16. Ans. (3) **17. Ans. (4)** **18. Ans. (3)** **19. Ans. (1)** **20. Ans. (1)**
21. Ans. (1) **22. Ans. (3)** **23. Ans. (4)** **24. Ans. (3)** **25. Ans. (1)**

EXERCISE (J-A)

1. Ans. 2 **2. Ans. (D)** **3. Ans. (C)** **4. Ans. (B,D)** **5. Ans. (A,D)**

6. Ans. (D) **7. Ans. (A,B,C,D)** **8. Ans. (A)** **9. Ans. (A)** **10. Ans. 1.50**

11. Ans. (3,4) **12. Ans. (1.00)** **13. Ans. 6**

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Chapter 01 Contents

Magnetic Effect of Current

Important Notes

MAGNETIC EFFECT OF CURRENT

KEY CONCEPT

The branch of physics which deals with the magnetism due to electric current or moving charge (i.e. electric current is equivalent to the charges or electrons in motion) is called electromagnetism.

Biot-Savart Law

Currents which arise due to the motion of charges are the source of magnetic fields. When charges move in a conducting wire and produce a current *I*, the magnetic field at any point *P* due to the current can be calculated by adding up the magnetic field contributions, $d\vec{B}$, from small segments of the wire $d\vec{s}$ (Figure).

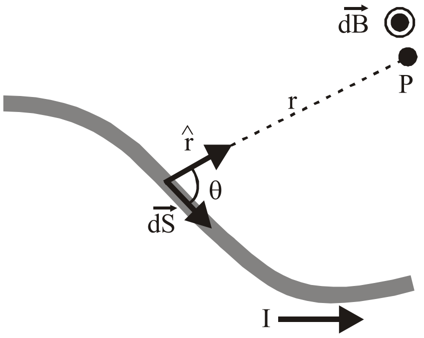


Figure : Magnetic field $d\vec{B}$ at point *P* due to a current-carrying element $Id\vec{s}$

These segments can be thought of as a vector quantity having a magnitude of the length of the segment and pointing in the direction of the current flow. The infinitesimal current source can then be written as $Id\vec{s}$.

Let *r* denote as the distance form the current source to the field point *P* and $\hat{r}$ the corresponding unit vector. The Biot-Savart law gives an expression for the magnetic field contribution, $d\vec{B}$, from the current source, $Id\vec{s}$,

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{s}\times\hat{r}}{r^2}$$

where μ_0 is a constant called the permeability of free space:

$\mu_0 = 4\pi \times 10^{-7}$ T.m/A here Tesla (T) is SI unit of $\vec{B}$

Adding up these contributions to find the magnetic field at the point *P* requires integrating over the current source,

$$\vec{B} = \int d\vec{B} = \frac{\mu_0 I}{4\pi}\int\frac{d\vec{s}\times\hat{r}}{r^2}$$

- According to $d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{\ell}\times\vec{r}}{r^3}$, direction of magnetic field vector $d\vec{B}$ is always perpendicular to the plane of vectors $(Id\vec{\ell})$ and ($\vec{r}$), where plane of $(Id\vec{\ell})$ and ($\vec{r}$) is the plane of wire.
- Magnetic field on the axis of current carrying conductor is always zero (θ = 0° or θ = 180°)

RIGHT HAND THUMB RULE

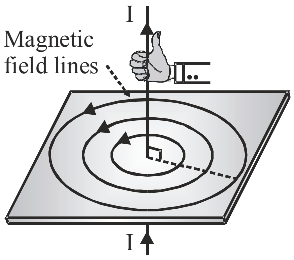
This rule gives the pattern of magnetic field lines due to current carrying wire.

(i) Straight current

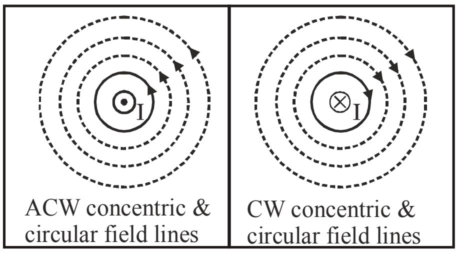
Thumb → In the direction of current

Curling fingers → Gives field line pattern

Case I : wire in the plane of the paper



Case II : Wire is ⊥ to the plane of the paper.

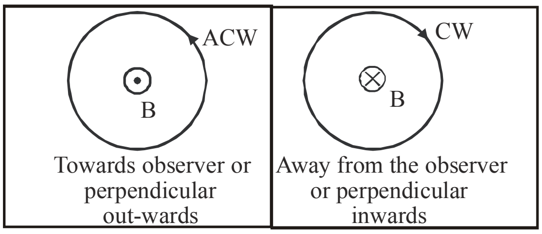


(ii) Circular current

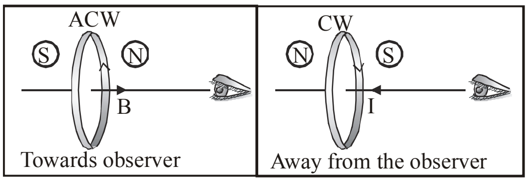
Curling fingers → In the direction of current,

Thumb → Gives field line pattern

Case I : wire in the plane of the paper



Case II : Wire is ⊥ to the plane of the paper



APPLICATION OF BIOT-SAVART LAW :

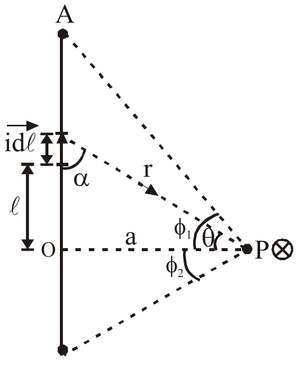
- **Magnetic field surrounding a thin straight current carrying conductor**

  AB is a straight conductor carrying current i from B to A.

  At a point P, whose perpendicular distance from AB is OP = a, the direction of field is perpendicular to the plane of paper, inwards (represented by a cross)

  $\ell = a\tan\theta \Rightarrow dl = a\sec^2\theta d\theta$...(i)

  $\alpha = 90° - \theta$ & $r = a\sec\theta$



- **By Biot-Savart's law**

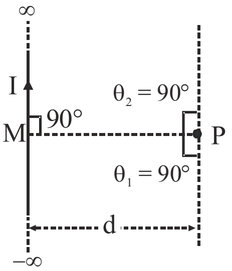
$$\overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{id\ell\sin\alpha}{r^2} \otimes \text{ (due to a current element } id\ell \text{ at point P)}$$

$$\Rightarrow B = \int dB = \int \frac{\mu_0}{4\pi}\frac{id\ell\sin\alpha}{r^2} \text{ (due to wire AB) } \therefore B = \frac{\mu_0 i}{4\pi}\int\cos\theta d\theta$$

Taking limits of integration as $-\phi_2$ to ϕ_1

$$B = \frac{\mu_0 i}{4\pi a}\int_{-\phi_2}^{\phi_1}\cos\theta d\theta = \frac{\mu_0 i}{4\pi a}\left[\sin\theta\right]_{-\phi_2}^{\phi_1} = \frac{\mu_0 i}{4\pi a}\left[\sin\phi_1 + \sin\phi_2\right] \text{ (inwards)}$$

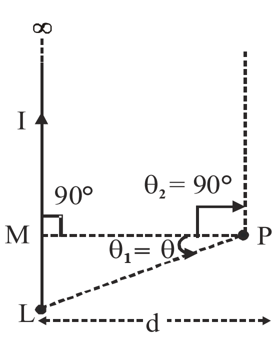
Ex. Magnetic field due to infinite length wire at point 'P'



Sol. $B_P = \dfrac{\mu_0 I}{4\pi d}[\sin 90° + \sin 90°]$

$B_P = \dfrac{\mu_0 I}{2\pi d}$

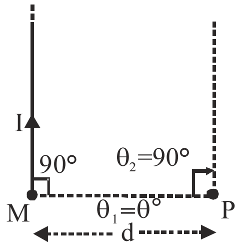
Ex. Magnetic field due to semi infinite length wire at point 'P'



Sol. $B_P = \dfrac{\mu_0 I}{4\pi d}[\sin\theta + \sin 90°]$

$$B_P = \frac{\mu_0 I}{4\pi d}[\sin\theta + 1]$$

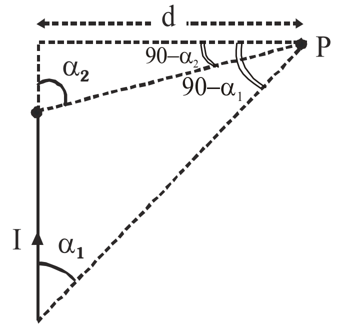
Ex. Magnetic field due to special semi infinite length wire at point 'P'



Sol. $B_P = \dfrac{\mu_0 I}{4\pi d}[\sin 0° + \sin 90°]$

$$B_P = \frac{\mu_0 I}{4\pi d}$$

Ex. If point 'P' lies out side the line of wire then magnetic field at point 'P' :

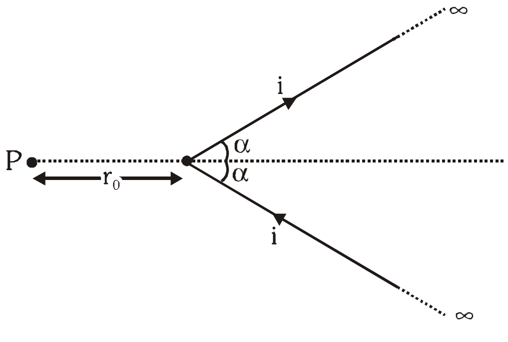


Sol. $B_P = \dfrac{\mu_0 I}{4\pi d}$

$$\left[\sin\left(90-\alpha_1\right)-\sin\left(90-\alpha_2\right)\right]$$

$$= \frac{\mu_0 I}{4\pi d}(\cos\alpha_1 - \cos\alpha_2)$$

Ex. A current carrying wire in the form of 'V' alphabet is kept as shown in the figure. Magnetic field intensity at point P which lies on the angular bisector of V is



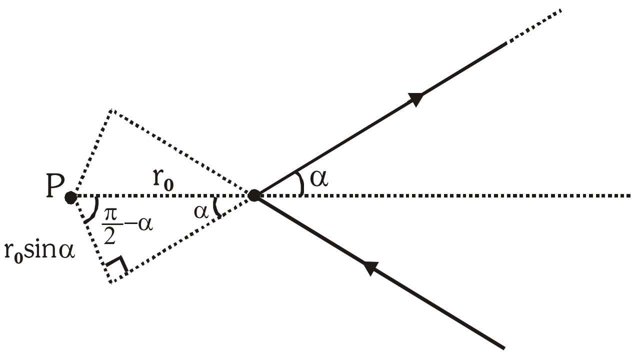
(A) $\dfrac{\mu_0 i}{4\pi r_0}[1-\cos\alpha]$ (B) $\dfrac{\mu_0 i}{2\pi r_0}[1-\cos\alpha]$

(C) $\dfrac{\mu_0 i}{4\pi r_0}\dfrac{[1-\cos\alpha]}{\sin\alpha}$ (D) $\dfrac{\mu_0 i}{2\pi r_0}\dfrac{[1-\cos\alpha]}{\sin\alpha}$

Ans. (D)

Sol. By using formula for stragith wire i.e.

$$B = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_1 + \sin\theta_2\right)$$

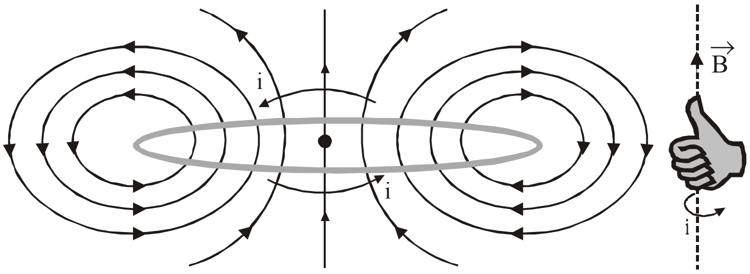


$$B = 2\left[\frac{\mu_0 i}{4\pi r_0 \sin\alpha}\left(\sin 90° - \sin\left(90° - \alpha\right)\right)\right]$$

$$= \frac{\mu_0 i}{2\pi r_0 \sin\alpha}[1-\cos\alpha]$$

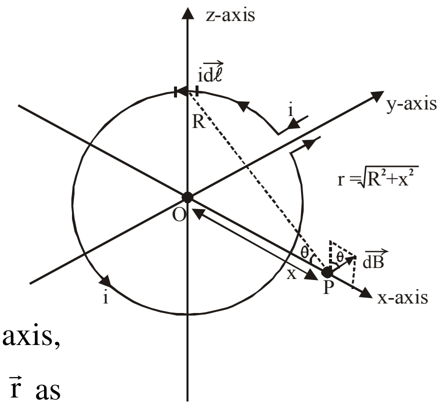
- **Magnetic field due to a loop of current**

Magnetic field lines due to a loop of wire are shown in the figure



The direction of magnetic field on the axis of current loop can be determined by right hand thumb rule. If fingers of right hand are curled in the direction of current, the stretched thumb is in the direction of magnetic field.

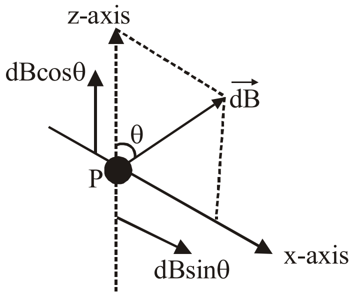
- **Calculation of magnetic field**



Consider a current loop placed in y-z plane carrying current i in anticlockwise sense as seen from positive x-axis. Due to a small current element $id\vec{\ell}$ shown in the figure, the magnetic field at P is given by $dB=\frac{\mu_0}{4\pi}\frac{id\ell\sin 90^0}{r^2}$.

The angle between $id\vec{\ell}$ and $\vec{r}$ is 90^0 because $id\vec{\ell}$ is along y-axis, while $\vec{r}$ lies in x-z plane. The direction of $\overrightarrow{dB}$ is perpendicular to $\vec{r}$ as shown. The vector $\overrightarrow{dB}$ can be resolved into two components, dBcosθ along z-axis and dB sinθ along x-axis.

For any two diametrically opposite current elements, the components along x-axis add up, while the other two components cancel out. Therefore, the field at P is due to x-component of field only. Hence, we have



$$\therefore\ B=\frac{\mu_0}{4\pi}\frac{i\times 2\pi R^2}{\left(R^2+x^2\right)^{3/2}}\quad\left(\because r=\sqrt{R^2+x^2}\right)$$

(a) At the centre, x = 0, $B_{\text{centre}}=\frac{\mu_0 i}{2R}$

(b) At points very close to centre, x << R $\Rightarrow B=\frac{\mu_0 i}{2}\left(1+\frac{x^2}{R^2}\right)^{-3/2}=\frac{\mu_0 i}{2}\left(1-\frac{3x^2}{2R^2}\right)$

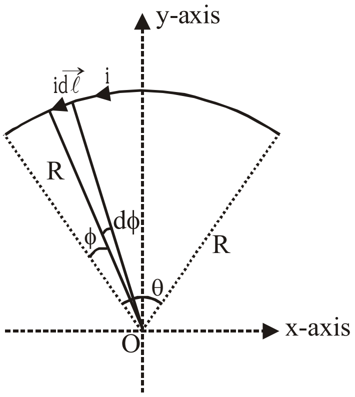
(c) At points far off from the centre, x >> R $\Rightarrow B=\frac{\mu_0\ell}{4\pi}\frac{2\pi R^2}{x^3}$

(d) The result in point (c) is also expressed as $B=\frac{\mu_0}{4\pi}\frac{2M}{x^3}$

where $M=\ell\times\pi R^2$, is called magnetic dipole moment.

Ex. Find the magnetic field at the centre of a current carrying conductor bent in the form of an arc subtending angle θ at its centre. Radius of the arc is R.

Sol. Let the arc lie in x-y plane with its centre at the origin.



Consider a small current element $id\vec{\ell}$ as shown.
The field due to this element at the centre is

$$dB = \frac{\mu_0}{4\pi}\frac{id\ell\sin 90^0}{R^2} \quad \left(\because id\vec{\ell} \text{ and R are perpendicular}\right)$$

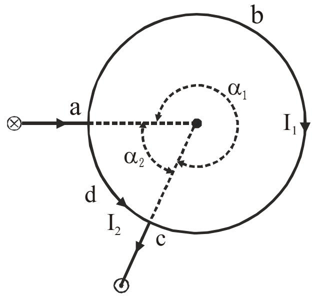
$$\text{Now } d\ell = Rd\phi \quad \therefore dB = \frac{\mu_0}{4\pi}\frac{iRd\phi}{R^2} \Rightarrow dB = \frac{\mu_0}{4\pi}\frac{i}{R}d\phi$$

The direction of field is outward perpendicular to plane of paper

$$\text{Total magnetic field } B = \int dB \therefore B = \frac{\mu_0 i}{4\pi R}\int_0^\theta d\phi = \frac{\mu_0 i}{4\pi R}[\phi]_0^\theta \quad \therefore B = \frac{\mu_0 i}{4\pi R}\theta$$

Ex. Find the magnetic field at the centre of a current carrying conductor bent in the form of an arc subtending angle α_1 and α_2 at the centre.

Sol. Magnetic field at the centre of arc abc and adc wire of circuit loop



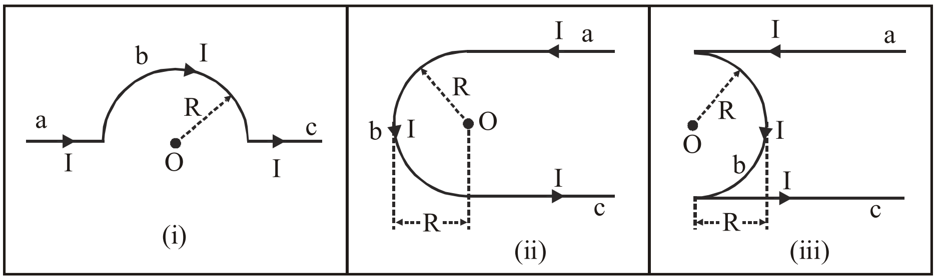
$$B_{abc} = \frac{\mu_0 I_1\alpha_1}{4\pi r} \text{ and } B_{adc} = \frac{\mu_0 I_2\alpha_2}{4\pi r} \Rightarrow \frac{B_{abc}}{B_{adc}} = \frac{I_1\alpha_1}{I_2\alpha_2}$$

$$\because \text{ angle} = \frac{\text{arc length}}{\text{radius}} \Rightarrow \frac{\alpha_1}{\alpha_2} = \frac{\ell_1}{\ell_2}$$

$$\therefore V = I_1R_1 = I_2R_2 \Rightarrow \frac{I_1}{I_2} = \frac{R_2}{R_1} \Rightarrow \frac{I_1}{I_2} = \frac{\ell_2}{\ell_1} \left(\because R = \frac{\rho\ell}{A} \Rightarrow R \propto \ell\right)$$

$$\therefore \frac{B_{abc}}{B_{adc}} = \left(\frac{\ell_2}{\ell_1}\right)\left(\frac{\ell_1}{\ell_2}\right) \Rightarrow \frac{B_{\alpha_1}}{B_{\alpha_2}} = \frac{1}{1}$$

Ex. Calculate the field at the centre of a semi-circular wire of radius R in situations depicted in figure (i), (ii) and (iii) if the straight wire is of infinite length.



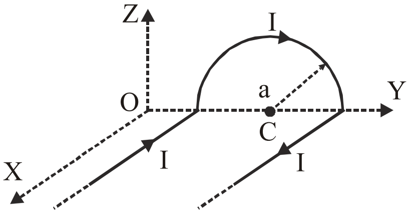
(i) (ii) (iii)

Sol. The magnetic field due to a straight current carring wire of infinite length, for a point at a distance R from one of its ends is zero if the point is along its length and $\frac{\mu_0 I}{4\pi R}$ if the point is on a line perpendicular to its length while at the centre of a semicircular coil is $\frac{\mu_0 I}{4R}$ so net magnetic field at the centre of semicircular wire is $\vec{B}_R = \vec{B}_a + \vec{B}_b + \vec{B}_c$

(i) $\vec{B}_R = 0 + \frac{\mu_0}{4}\frac{I}{R}\otimes + 0 = \frac{\mu_0 I}{4R}$ ⊗(into the page)

(ii) $\vec{B}_R = \frac{\mu_0}{4\pi}\frac{I}{R}\odot + \frac{\mu_0}{4}\frac{I}{R}\odot + \frac{\mu_0}{4\pi}\frac{I}{R}\odot = \frac{\mu_0}{4\pi}\frac{I}{R}[\pi + 2]$ ⊙ (out of the page)

(iii) $\vec{B}_R = \frac{\mu_0}{4\pi}\frac{I}{R}\odot + \frac{\mu_0}{4}\frac{I}{R}\otimes + \frac{\mu_0}{4\pi}\frac{I}{R}\odot = \frac{\mu_0}{4\pi}\frac{I}{R}[\pi - 2]\otimes$ (into the page)



Ex. A long wire bent as shown in the figure carries current I. If the radius of the semi-circular portion is "a" then find the magnetic induction at the centre C.

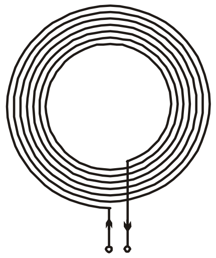
Sol. Due to semi circular part

$$\vec{B}_1 = \frac{\mu_0 I}{4a}\left(-\hat{i}\right)$$

due to parallel parts of currents

$$\vec{B}_2 = 2\times\frac{\mu_0 I}{4\pi a}(-\hat{k}),\ B_{net} = B_C = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0 I}{4a}(-\hat{i}) + \frac{\mu_0 I}{2\pi a}(-\hat{k})$$

magnitude of resultant field $B = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0 I}{4\pi a}\sqrt{\pi^2 + 4}$



Ex. A thin insulated wire forms a plane spiral of N = 100 tight turns carrying a current I = 8 mA. The radii of inside and outside turns (Fig.) are equal to a = 50 mm and b = 100 mm.
Find the magnetic induction at the centre of the spiral;

Ans. $B = \frac{\mu_0 IN\, ln\ (b/a)}{2(b-a)} = 7\ \mu T$;

Sol. From Biot-Savart's law, the magnetic induction due to a circular current carrying wire loop at its centre is given by,

$$B_r = \frac{\mu_0}{2r}i$$

The plane spiral is made up of concentric circular loops, having different radii, varying from a to b. Therefore, the total magnetic induction at the centre,

$$B_0 = \int\frac{\mu_0}{2r}dN$$

where $\frac{\mu_0}{2r}i$ is the contribution of one turn of radius r and dN is the number of turns in the interval (r, r + dr).

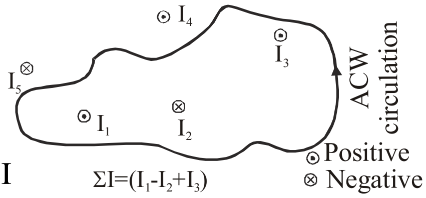
i.e. $dN = \frac{N}{b-a}dr$

Substituting in equation (1) and integrating the result over r between a and b, we obtain,

$$B_0 = \int_a^b\frac{\mu_0 i}{2r}\frac{N}{(b-a)}dr = \frac{\mu_0 iN}{2(b-a)}\ln\frac{b}{a}$$

AMPERE'S CIRCUITAL LAW

Ampere's circuital law state that line integral of the magnetic field around any closed path in free space or vacuum is equal to μ_0 times of net current or total current which crossing through the area bounded by the closed path. Mathematically $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \Sigma I$



This law independent of size and shape of the closed path.

Any current outside the closed path is not included in writing the right hand side of law

Note :

- This law suitable for infinite long and symmetrical current distribution.
- Radius of cross section of thick cylinderical conductor and current density must be given to apply this law.

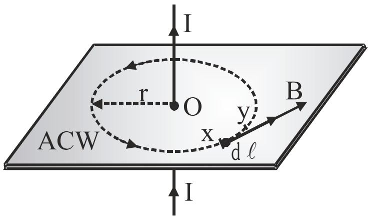
APPLICATION OF AMPERE'S CIRCUITAL LAW

- **Magnetic field due to infinite long thin current carrying straight conductor**

Consider a circle of radius 'r'. Let XY be the small element of length $d\ell$. $\vec{B}$ and $d\vec{\ell}$ are in the same direction because direction of along the tangent of the circle. By A.C.L.

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \Sigma I \ , \oint B\, d\ell \cos\theta = \mu_0 I \quad (\text{where } \theta = 0^\circ)$$

$$\oint B\, d\ell \cos 0^\circ = \mu_0 I \ \Rightarrow\ B \oint d\ell = \mu_0 I \quad (\text{where } \oint d\ell = 2\pi r\ )$$



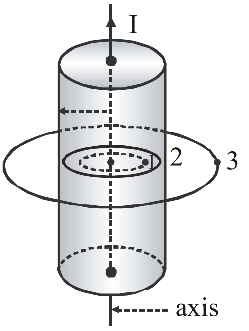
$$B\,(2\pi r) = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

- **Magnetic field due to infinite long solid cylinderical conductor**
- For a point inside the cylinder r < R, Current from area πR^2 is = I

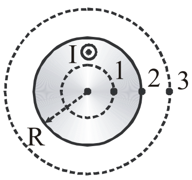
so current from area πr^2 is $= \dfrac{I}{\pi R^2}(\pi r^2) = \dfrac{I r^2}{R^2}$

By Ampere circuital law for circular path 1 of radius r

$$B_{in}\,(2\pi r) = \mu_0 I' = \mu_0 \frac{I r^2}{R^2} \Rightarrow B_{in} = \frac{\mu_0 I r}{2\pi R^2} \Rightarrow B_{in} \propto r$$



Cross-sectional view



- For a point on the axis of the cylinder (r = 0); $B_{axis} = 0$
- For a point on the surface of cylinder (r = R)

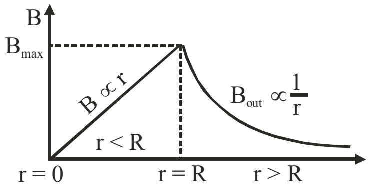
By Ampere circuital law for circular path 2 of radius R

$$B_s\,(2\pi R) = \mu_0 I \Rightarrow B_s = \frac{\mu_0 I}{2\pi R} \quad (\text{it is maximum})$$

- For a point outside the cylinder (r > R) :-

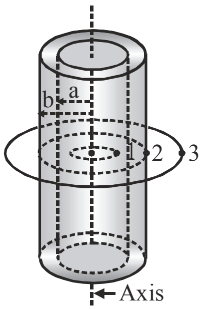
By Ampere circuital law for circular path 3 of radius r

$$B_{out}\,(2\pi r) = \mu_0 I \Rightarrow B_{out} = \frac{\mu_0 I}{2\pi r} \Rightarrow B_{out} \propto \frac{1}{r}$$

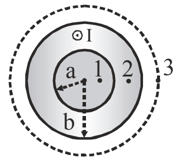


Magnetic field outside the cylinderical conductor does not depend upon nature (thick/thin or solid/hollow) of the conductor as well as its radius of cross section.

- **Magnetic field due to infinite long hollow cylinderical conductor**
- For a point at a distance r such that $r < a < b \qquad B_1 = 0$
- For a point at a distance r such that $a < r < b$



Side View



Cross sectional view

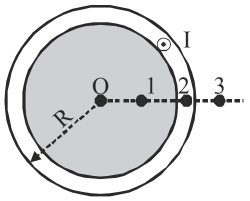
$$B_2(2\pi r) = \mu_0 I' \Rightarrow B_2(2\pi r) = \mu_0 I\left(\frac{r^2 - a^2}{b^2 - a^2}\right)$$

$$B_2 = \frac{\mu_0 I}{2\pi r}\left(\frac{r^2 - a^2}{b^2 - a^2}\right) \begin{cases} r = a \text{ (inner surface)} & \Rightarrow B_{IS} = 0 \\ r = b \text{ (outer surface)} & \Rightarrow B_{OS} = \dfrac{\mu_0 I}{2\pi b} \text{ (maximum)} \end{cases}$$

- For a point at a distance r such that $r > b > a$, $B_3(2\pi r) = \mu_0 I \Rightarrow B_3 = \dfrac{\mu_0 I}{2\pi r}$
- For a point at the axis of cylinder $r = 0 \qquad B_{axis} = 0$

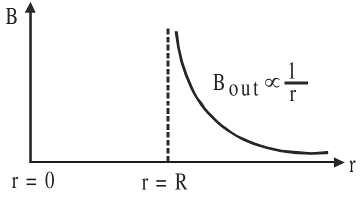
Magnetic field at specific positions for thin hollow cylinderical conductor

At point 1 $B_1 = 0$



At point 2 $B_2 = \dfrac{\mu_0 I}{2\pi R}$ (maximum) [outer surface] and

$B_2 = 0$ (minimum) [inner surface]



At point 3 $B_3 = \dfrac{\mu_0 I}{2\pi r}$ (for the point on axis $B_{axis} = 0$)

Ex. **Non-Uniform Current Density :** Consider an infinitely long, cylindrical conductor of radius *R* carrying a current *I* with a non-uniform current density $J = \alpha r$ where α is a constant. Find the magnetic field everywhere.

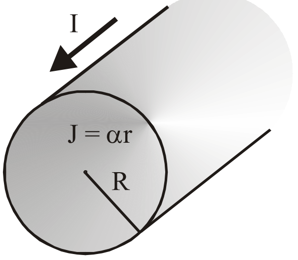


Figure : Non-uniform current density

Solution :

The problem can be solved by using the Ampere's law:

$$\oint \vec{B}.d\vec{s} = \mu_0 I_{enc}$$

Where the enclosed current I_{enc} is given by

$$I_{enc} = \int \vec{J} \, . \, d\vec{A} = \int (\alpha r')(2\pi r' dr')$$

(a) For r < R, the enclosed current is $I_{enc} = \int_0^r 2\pi\alpha r'^2\, dr' = \dfrac{2\pi\alpha r^3}{3}$

Applying Ampere's law, the magnetic field at P_1 is given by

$$B_1(2\pi r) = \frac{2\mu_0\pi\alpha r^3}{3} \text{ or } B_1 = \frac{\alpha\mu_0}{3}r^2$$

The direction of the magnetic field $\vec{B}_1$ is tangential to the Amperian loop which encloses the currect.

(b) For r > R, the enclosed current is : $I_{enc} = \int_0^R 2\pi\alpha r'^2 dr' = \dfrac{2\pi\alpha R^3}{3}$

which yields $B_2(2\pi r) = \dfrac{2\mu_0\pi\alpha R^3}{3}$

Thus, the magnetic field at a point P_2 outside the conductor is ;

$$B_2 = \frac{\alpha\mu_0 R^3}{3r}$$

A plot of B as a function of r is shown in figure.

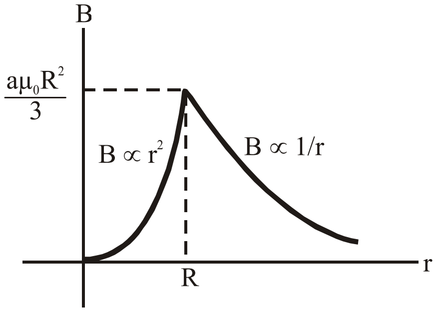
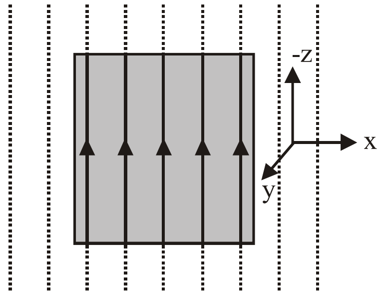
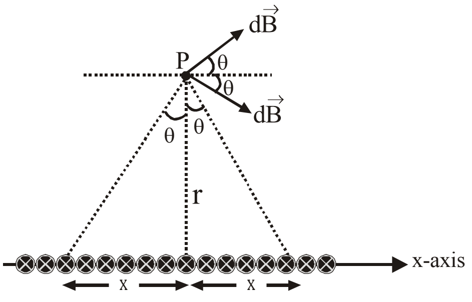
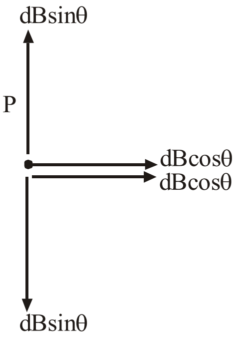


Figure : The magnetic field as a function of distance away from the conductor

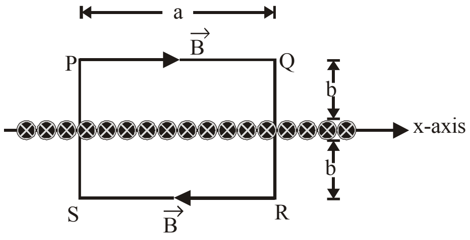
Magnetic field due to an infinite plane sheet of current







An infinite sheet of current lies in x-z plane, carrying current along-z axis. The field at any point P on y is along a line parallel to x-z plane. We can take a rectangular amperian loop as shown. If you traverse the loop in clockwise direction, inward current will be positive.



By Ampere circuital law, $\oint_{PQRS} \vec{B}.d\vec{\ell} = \mu_0 \ell_{enclosed}$ ....(i)

Let λ represents current per unit length.

The current enclosed is given by $\ell_{enclosed} = \lambda a$

Now, $\oint_{PQRS} \vec{B}.d\vec{\ell} = \int_{PQ} \vec{B}.d\vec{\ell} + \int_{QR} \vec{B}.d\vec{\ell} + \int_{RS} \vec{B}.d\vec{\ell} + \int_{SR} \vec{B}.d\vec{\ell}$

Now, $\int_{QR} \vec{B}.d\vec{\ell} = \int_{SP} \vec{B}.d\vec{\ell} = 0$ as $\vec{B} \perp \vec{d\ell}$

Also, $\int_{PQ} \vec{B}.d\vec{\ell} + \int_{RS} \vec{B}.d\vec{\ell} = 2 \times B \times a \left(\text{as } \vec{B} \| d\vec{\ell}\right)$ $\therefore$ $2B \times a = \mu_0 \lambda a \Rightarrow B = \frac{\mu_0 \lambda}{2}$

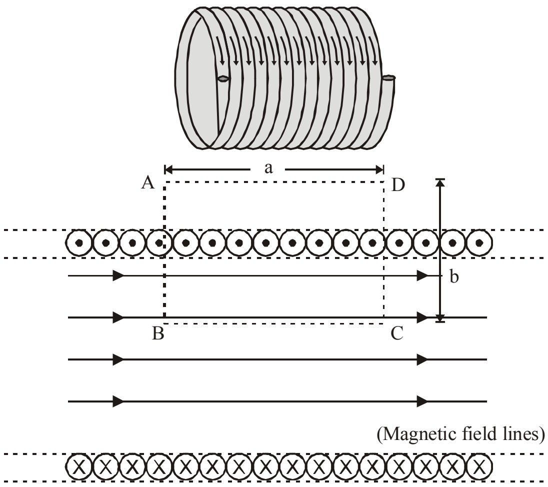
MAGNETIC FIELD DUE TO SOLENOID

It is a coil which has length and used to produce uniform magnetic field of long range. It consists a conducting wire which is tightly wound over a cylinderical frame in the form of helix. All the adjacent turns are electrically insulated to each other. The magnetic field at a point on the axis of a solenoid can be obtained by superposition of field due to large number of identical circular turns having their centres on the axis of solenoid.

Magnetic field due to a long solenoid

A solenoid is a tightly wound helical coil of wire. If length of solenoid is large, as compared to its radius, then in the central region of the solenoid, a reasonably uniform magnetic field is present. Figure shows a part of long solenoid with number of turns/length n.We can find the field by using Ampere circuital law.

Consider a rectangular loop ABCD. For this loop $\oint_{ABCD} \vec{B}.\vec{d\ell} = \mu_0 i_{enc}$



(Magnetic field lines)

Now

$$\oint_{ABCD} \vec{B}.\vec{d\ell} = \oint_{AB} \vec{B}.\vec{d\ell} + \oint_{BC} \vec{B}.\vec{d\ell} + \oint_{CD} \vec{B}.\vec{d\ell} + \oint_{DA} \vec{B}.\vec{d\ell} = B \times a$$

This is because $\oint_{AB} \vec{B}.\vec{d\ell} = \oint_{CD} \vec{B}.\vec{d\ell} = 0,\ \vec{B} \perp \vec{d\ell}$.

And, $\oint_{DA} \vec{B}.\vec{d\ell} = 0$

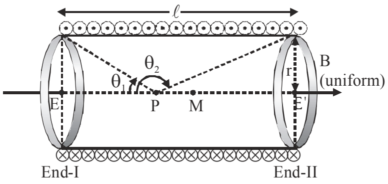
($\because$ $\vec{B}$ outside the solenoid is negligible

Now, $i_{enc} = (n \times a) \times i$ s $\Rightarrow B \times a = \mu_0 (n \times a \times i)$

$\Rightarrow B = \mu_0 n i$

Finite length solenoid :

Its length and diameter are comparable. By the concept of BSL magnetic field at the axial point 'P' obtained as :



$$B_P = \frac{\mu_0 n I}{2}(\cos\theta_1 - \cos\theta_2)$$

Angle θ_1 and θ_2 both measured in same sense from the axis of the solenoid to end vectors.

Infinite length solenoid :

Its length very large as compared to its diameter i.e. ends of solenoid tends to infinity.

(a) Magnetic field at axial point which is well inside the solenoid

$$\theta_1 \simeq 0^\circ \text{ and } \theta_2 \simeq 180^\circ \Rightarrow B \simeq \frac{\mu_0 nI}{2}[\cos 0^\circ - \cos 180^\circ] \simeq \frac{\mu_0 nI}{2}[(1)-(-1)] \simeq \mu_0 nI$$

(b) Magnetic field at both axial end points of solenoid

$$\theta_1 = 90^\circ \text{ and } \theta_2 \simeq 180^\circ \Rightarrow B \simeq \frac{\mu_0 nI}{2}[\cos 90^\circ - \cos 180^\circ] \simeq \frac{\mu_0 nI}{2}[(0)-(-1)] \simeq \frac{\mu_0 nI}{2}$$

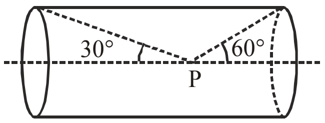
Ex. The length of solenoid is 0.1m. and its diameter is very small. A wire is wound over it in two layers. The number of turns in inner layer is 50 and that of outer layer is 40. The strength of current flowing in two layers in opposite direction is 3A. Then find magnetic induction at the middle of the solenoid.

Sol. Direction of magnetic field due to both layers is opposite, as direction of current is opposite so

$$B_{net} = B_1 - B_2 = \mu_0 n_1 I_1 - \mu_0 n_2 I_2 = \mu_0 \frac{N_1}{\ell} I - \mu_0 \frac{N_2}{\ell} I \quad (\because I_1 = I_2 = I)$$

$$= \frac{\mu_0 I}{\ell}(N_1 - N_2) = \frac{4\pi \times 10^{-7} \times 3}{0\cdot 1}(50-40) = 12\pi \times 10^{-5}\ \text{T}$$

Ex. Find out magnetic field at axial point 'P' of solenoid shown in figure (where turn density 'n' and current through it is I)

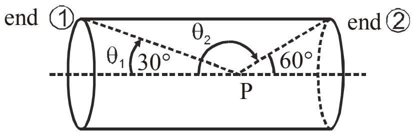


Sol. Magnetic field at point 'P' due to finite length solenoid

$$B_P = \frac{\mu_0 nI}{2}[\cos\theta_1 - \cos\theta_2],$$

where $\theta_1 = 30^\circ$ (CW),

$$\theta_2 = (180^\circ - 60^\circ) = 120^\circ \text{ (CW)} = \frac{\mu_0 nI}{2}[\cos 30^\circ - \cos 120^\circ]$$

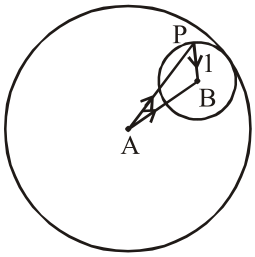


$$= \frac{\mu_0 nI}{2}\left[\frac{\sqrt{3}}{2} - \left(-\frac{1}{2}\right)\right] = \frac{\mu_0 nI}{4}(\sqrt{3}+1)$$

Ex. Inside a long straight uniform wire of round cross-section there is a long round cylindrical cavity whose axis is parallel to the axis of the wire and displaced from the latter by a distance I. A direct current of density j flows along the wire. Find the magnetic induction inside the cavity. Consider, in particular, the case I = 0.

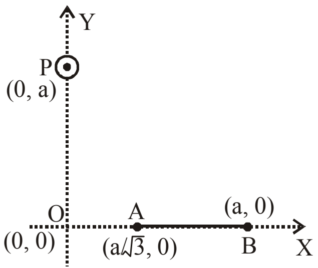
Sol. We can think of the given current which will be assumed uniform, as arising due to a negative current, flowing in the cavity, superimposed on the true current, everywhere including the cavity. Then from the previous problem, by superposition

$$\vec{B} = \frac{1}{2}\mu_0 \vec{j} \times (\vec{AP} - \vec{BP}) = \frac{1}{2}\mu_0 \vec{j} \times \vec{\ell}$$

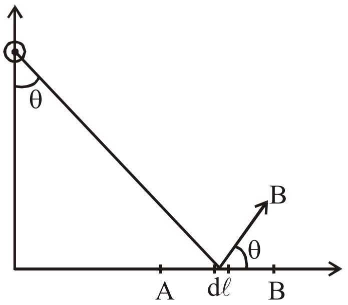


If $\vec{\ell}$ vanishes so that the cavity is concentric with the conductor, there is no magnetic field in the cavity.

Ex. An infinite current carrying conductor, parallel to z-axis is situated at point P as shown in the figure. Value of $\int_A^B \vec{B}\cdot d\vec{\ell}$ is given by $\alpha\frac{\mu_0 i}{96}$, then fill the value of α in OMR sheet ?



Ans. **4**



Sol. $d\ell = d(a\tan\theta) = a\sec^2\theta d\theta.$

$$B = \frac{\mu_0 i}{2\pi a}\cos\theta$$

$$\therefore \int B.d\ell = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\mu_0 i}{2\pi a}\cos\theta\ a\sec^2\theta d\theta.\ \cos\theta = \frac{\mu_0 i}{2\pi}\int_{\pi/6}^{\pi/4} d\theta = \frac{\mu_0 i}{2\pi}\cdot\frac{\pi}{12} = \frac{\mu_0 i}{24}.$$

MOTION OF A CHARGED PARTICLE IN A MAGNETIC FIELD

Motion of a charged particle when it is moving collinear with the field magnetic field is not affected by the field (i.e. if motion is just along or opposite to magnetic field) ($\because F = 0$). The following two cases are possible :

- **Case I :**

When the charged particle is moving perpendicular to the field.

The angle between $\vec{B}$ and $\vec{v}$ is $\theta=90°$. So the force will be maximum (= qvB) and always perpendicular to motion (and also field); Hence the charged particle will move along a circular path (with its plane perpendicular to the field). Centripetal force is provided by the force qvB, So $\frac{mv^2}{r} = qvB \Rightarrow r = \frac{mv}{qB}$

Angular frequency of circular motion, called cyclotron or gyro-frequency. $\omega = \frac{v}{r} = \frac{qB}{m}$

and the time period, $T = \frac{2\pi}{\omega} = 2\pi\frac{m}{qB}$ i.e., time period (or frequency) is independent of speed of particle and radius of the orbit. Time period depends only on the field B and the nature of the particle, i.e., specific charge (q/m) of the particle.

This principle has been used in a large number of devices such as cyclotron (a particle accelerator), bubble-chamber (a particle detector) or mass-spectrometer etc.

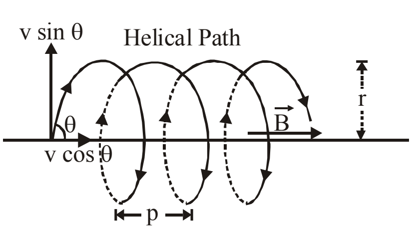
Note that $\vec{F}_B$ is always perpendicular to $\vec{v}$ and $\vec{B}$, and cannot change the particle's speed *v* (and thus the kinetic energy). In other words, magnetic force cannot speed up or slow down a charged particle. Consequently, $\vec{F}_B$ can do no work on the particle :

$$dW = \vec{F}_B \cdot d\vec{s} = q(\vec{v}\times\vec{B})\cdot\vec{v}dt = q(\vec{v}\times\vec{v})\cdot\vec{B}dt = 0$$

- **Case II :**

  The charged particle is moving at an angle θ to the field : ($\theta \neq 0°, 90°$ or $180°$)

  Resolving the velocity of the particle along and perpendicular to the field. The particle moves with constant velocity $v \cos\theta$ along the field



($\because$ no force acts on a charged particle when it moves parallel to the field).

And at the same time it is also moving with velocity $v \sin\theta$ perpendicular to the field due to which it will describe a circle (in a plane perpendicular to the field)

Radius of the circular path $r = \dfrac{m(v\sin\theta)}{qB}$ and Time period $T = \dfrac{2\pi r}{v\sin\theta} = \dfrac{2\pi m}{qB}$

So the resultant path will be a helix with its axis parallel to the field $\vec{B}$ as shown in fig.

The pitch p of the helix = linear distance travelled in one rotation $p = T(v\cos\theta) = \dfrac{2\pi m}{qB}(v\cos\theta)$

Ex. An electron emitted by a heated cathode and accelerated through a potential difference of 2.0 kV enters a region with uniform magnetic field of 0.15 T. Determine the radius of the trajectory of the electron if the field is –

(a) Transverse to its initial velocity (b) Makes an angle of 30° with the initial velocity

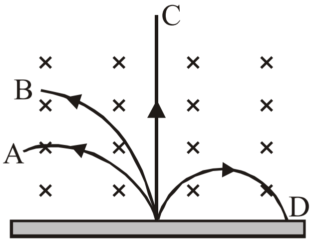
[Given : $m_e = 9 \times 10^{-31}$ kg]

Sol. $\dfrac{1}{2}mv^2 = eV \Rightarrow v = \sqrt{\dfrac{2eV}{m}} = \sqrt{\dfrac{2\times1.6\times10^{-19}\times2\times10^{3}}{9\times10^{-31}}} = \dfrac{8}{3} \times 10^7$ m/s

(a) Radius $r_1 = \dfrac{mv}{qB} = \dfrac{9\times10^{-31}\times(8/3)\times10^{7}}{1.6\times10^{-19}\times0.15} = 10^{-3}$ m = 1mm

(b) Radius $r_2 = \dfrac{mv\sin\theta}{qB} = r_1\sin\theta = 1\times\sin30° = 1\times\dfrac{1}{2} = 0.5$ mm

Ex. A neutron, a proton, an electron an α-particle enter a region of constant magnetic field with equal velocities. The magnetic field is along the inwards normal to the plane of the paper. The tracks of the particles are shown in fig. Relate the tracks to the particles.



Sol. Force on a charged particle in magnetic field $\vec{F} = q(\vec{v}\times\vec{B})$

For neutron q = 0, F = 0 hence it will pass undeflected.

i.e., tracks C corresponds to neutron.

If the particle is negatively charged, i.e. electron. $\vec{F} = -e(\vec{v}\times\vec{B})$

It will experience a force to the right; so track D corresponds to electron.

If the charge on particle is positive. It will experience a force to the left; so both tracks A and B corresponds to positively charged particles (i.e., protons and α-particles). When motion of charged particle perpendicular to the magnetic field the path is a circle with radius

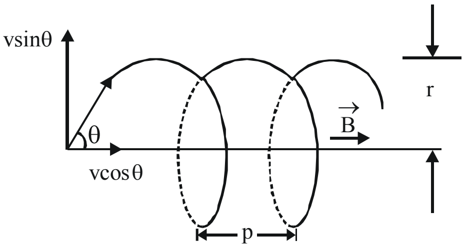
$r = \dfrac{mv}{qB}$ i.e. $r \propto \dfrac{m}{q}$ and as $\left(\dfrac{m}{q}\right)_\alpha = \left(\dfrac{4m}{2e}\right)$ while $\left(\dfrac{m}{q}\right)_p = \dfrac{m}{e} \Rightarrow \left(\dfrac{m}{q}\right)_\alpha > \left(\dfrac{m}{q}\right)_p$

So $r_\alpha > r_p \Rightarrow$ track B to α-particle and A corresponds to proton.

Ex. A beam of protons with velocity 4 × 10^5 m/s enters a uniform magnetic field of 0.3 tesla at an angle of 60° to the magnetic field. Find the radius of the helical path taken by the proton beam. Also find the pitch of helix. Mass of proton=1.67× 10^{-27}kg.

Sol. Radius of helix $r = \dfrac{mv\sin\theta}{qB}$ ($\therefore$ component of velocity $\perp$ to field is vsinθ)

$$= \frac{(1.67\times10^{-27})(4\times10^{5})\sqrt{\frac{3}{2}}}{(1.6\times10^{-19})0.3} = \frac{2}{\sqrt{3}}\times10^{-2}\text{m} = 1.2\text{cm}$$



Again, pitch p = vcosθ × T (where $T = \dfrac{2\pi r}{v\sin\theta}$)

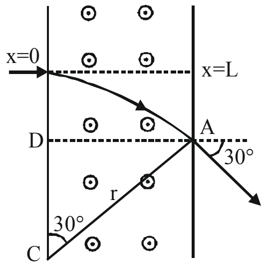
$$\therefore\ p = \frac{v\cos\theta\times2\pi r}{v\sin\theta} = \frac{\cos60°\times2\pi\times(1.2\times10^{-2})}{\sin60°} = 4.35\times10^{-2}\text{m} = 4.35\text{cm}$$

Ex. The region betwen x = 0 and x = L is filled with uniform, steady magnetic field $B_0\hat{k}$. A particle of mass m, positive charge q and velocity $v_0\hat{i}$ travels along X-axis and enters the region of magnetic field. Neglect the gravity throughout the question.

(a) Find the value of L if the particle emerges from the region of magnetic field with its final velocity at an angle 30° to its initial velocity.

(b) Find the final velocity of the particle and the time spent by it in the magnetic field, if the magnetic field now extends upto 2.1 L.

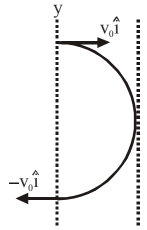
Sol. (a) The particle is moving with velocity $v_0\hat{i}$, perpendicular to magnetic field $B_0\hat{k}$. Hence the particle will move along a circular arc OA of radius $r = \dfrac{mv_0}{qB_0}$



Let the particle leave the magnetic field at A.

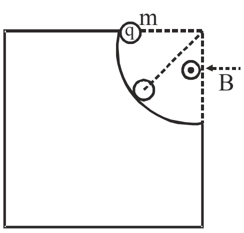
From ΔCDA, $\sin60° = \dfrac{AD}{CA} = \dfrac{L}{r} \Rightarrow L = r\sin30° = \dfrac{r}{2}\ \therefore L = \dfrac{mv_0}{2qB_0}$

(b) As the magnetic field extends upto 2.1 L i.e., L > 2r, so the particle completes half cycle before leaving the magnetic field, as shown in figure.



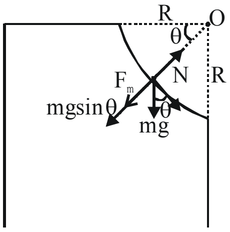
The magnetic field is always perpendicular to velocity vector, therefore the magnitude of velocity will remain the same.

$\therefore$ Final velocity $= v_0(-\hat{i}) = -v_0\hat{i}$ Time spent in magnetic field $= \dfrac{\pi r}{v_0} = \dfrac{\pi m}{qB_0}$



Ex. A charged sphere of mass m and charge q starts sliding from rest on a vertical fixed circular track of radius R from the position as shown in figure. There exists a uniform and constant horizontal magnetic field of induction B. Find the maximum force exerted by the track on the sphere.

Sol. Magnetic force on sphere $F_m = qvB$ (directed radially outward)



$$\because\ N - mg\sin\theta - qvB = \frac{mv^2}{R}$$

$$\Rightarrow N = \frac{mv^2}{R} + mg\sin\theta + qvB$$

Hence, at $\theta = \pi/2$ we get $N_{max} = \frac{2mgR}{R} + mg + qB\sqrt{2gR} = 3mg + qB\sqrt{2gR}$

Ex. A particle of charge q and mass m starts moving from the origin under the action of an electric field $\vec{E} = E_0\hat{i}$ and magnetic field $\vec{B} = B_0\hat{i}$ with velocity $\vec{v} = v_0\hat{j}$. The speed of the particle will become $2v_0$ after a time :-

(A) $t = \frac{2mv_0}{qE}$ (B) $t = \frac{2Bq}{mv_0}$ (C) $t = \frac{\sqrt{3}\,Bq}{mv_0}$ (D) $t = \frac{\sqrt{3}\,mv_0}{qE}$

Ans. (D)

Sol. Charged particle will move in a Helical path.

$$\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}$$

$\vec{a} = \frac{qE\hat{i}}{m}$ this part will increase x component of velocity

+

$\frac{qv_0 \times B}{m}$ (in y-z plane) this term will provide centripetal acceleration.

$$v_x = \frac{qE}{m}\cdot t$$

$$v = \sqrt{v_x^2 + v_0^2}$$

$$2v_0 = \sqrt{\left(\frac{qE}{m}t\right)^2 + v_0^2}$$

$$\sqrt{3}v_0 = \frac{qE}{m}t$$

$$t = \frac{\sqrt{3}mv_0}{qE}$$

(1) Velocity Selector :

In the presence of both electric field $\vec{E}$ and magnetic field $\vec{B}$, the total force on a charged particle is

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

This is known as the Lorentz force. By combining the two fields, particles which move with a certain velocity can be selected. This was the principle used by J. J. Thomson to measure the charge-to-mass ratio of the electrons. In Figure the schematic diagram of Thomson's apparatus is depicted.

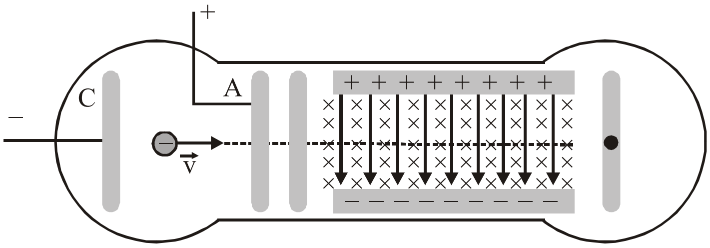


Figure : Thomson's apparatus

The electrons with charge *q* = –*e* and mass *m* are emitted from the cathode C and then accelerated toward slit A. Let the potential difference between A and C be $V_A - V_C = \Delta V$. The change in potential energy is equal to the external work done in accelerating the electrons: $\Delta U = W_{ext} = q\Delta V = -e\Delta V$. By energy conservation, the kinetic energy gained is $\Delta K = -\Delta U = mv^2/2$. Thus, the speed of the electrons is given by

$$v = \sqrt{\frac{2e\Delta V}{m}}$$

If the electrons further pass through a region where there exists a downward uniform electric field, the electrons, being negatively charged, will be deflected upward. However, if in addition to the electric field, a magnetic field directed into the page is also applied, then the electrons will experience an additional downward magnetic force $-e\vec{v} \times \vec{B}$. When the two forces exactly cancel, the electrons will move in a straight path. From Eq., we see that when the condition for the cancellation of the two forces is given by *eE* = *evB*. which implies

$$v = \frac{E}{B}$$

In other words, only those particles with speed *v* = *E* / *B* will be able to move in a straight line. Combining the two equations, we obtain

$$\frac{e}{m} = \frac{E^2}{2(\Delta V)B^2}$$

By measuring *E*, *ΔV* and *B*, the charge-to-mass ratio can be readily determined. The most precise measurement to date is $e/m = 1.758820174(71) \times 10^{11}$ C/kg.

(2) Mass Spectrometer :

Various methods can be used to measure the mass of an atom. One possibility is through the use of a mass spectrometer. The basic feature of a *Bainbridge* mass spectrometer is illustrated in Figure. A particle carrying a charge +*q* is first sent through a velocity selector.

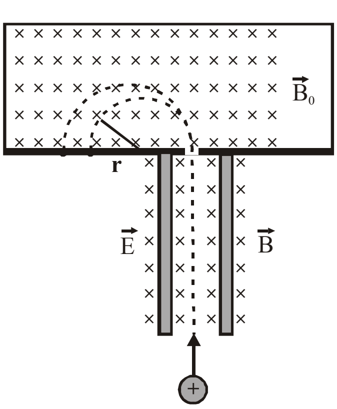


Figure : A Bainbridge mass spectrometer

The applied electric and magnetic fields satisfy the relation $E = vB$ so that the trajectory of the particle is a straight line. Upon entering a region where a second magnetic field $\vec{B}_0$ pointing into the page has been applied, the particle will move in a circular path with radius *r* and eventually strike the photographic plate. Using Eq., we have

$$r = \frac{mv}{qB_0}$$

Since v = E/B, the mass of the particle can be written as

$$m = \frac{qB_0r}{v} = \frac{qB_0Br}{E}$$

Ex. Particle *A* with charge *q* and mass m_A and particle *B* with charge 2*q* and mass m_B, are accelerated from rest by a potential difference ΔV, and subsequently deflected by a uniform magnetic field into semicircular paths. The radii of the trajectories by particle *A* and *B* are *R* and 2*R*, respectively. The direction of the magnetic field is perpendicular to the velocity of the particle. What is their mass ratio?

Sol. The kinetic energy gained by the charges is equal to

$$\frac{1}{2}mv^2 = q\Delta V$$

which yields $v = \sqrt{\frac{2q\Delta V}{m}}$

The charges move in semicircles, since the magnetic force points radially inward and provides the source of the centripetal force :

$$\frac{mv^2}{r} = qvB$$

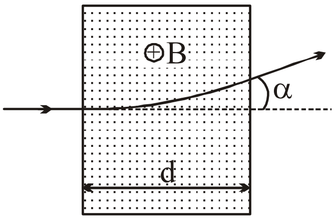
The radius of the circle can be readily obtained as :

$$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2q\Delta V}{m}} = \frac{1}{B}\sqrt{\frac{2m\Delta V}{q}}$$

which shows that r is proportional to $(m/q)^{1/2}$. The mass ratio can then be obtained from

$$\frac{r_A}{r_B} = \frac{(m_A/q_A)^{1/2}}{(m_B/q_B)^{1/2}} \Rightarrow \frac{R}{2R} = \frac{(m_A/q)^{1/2}}{(m_B/2q)^{1/2}}$$

which gives $\frac{m_A}{m_B} = \frac{1}{8}$



Ex. A proton accelerated by a potential difference V = 500 kV flies through a uniform transverse magnetic field with induction B = 0.51 T. The field occupies a region of space d = 10 cm in thickness (Fig.). Find the angle a through which the proton deviates from the initial direction of its motion.

Ans. $\alpha = \arcsin\left(dB\sqrt{\frac{q}{2mV}}\right) = 30^\circ$

Sol. From the figure,

$$\sin\alpha = \frac{d}{R} = \frac{dqB}{mv}$$

As radius of the arc $R = \frac{mv}{qB}$, where v is the velocity of the particle, when it enters into the field.

From initial condition of the problem,

$$qV = \frac{1}{2}mv^2 \qquad \text{or, } v = \sqrt{\frac{2qV}{m}}$$

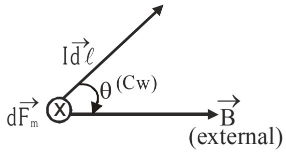
$$\text{Hence, } \sin\alpha = \frac{dqB}{m\sqrt{2qV/m}} = dB\sqrt{\frac{q}{2mV}}$$

and $a = \sin^{-1}\left(dB\sqrt{\frac{q}{2mV}}\right) = 30^\circ$, on putting the values.

CURRENT CARRYING CONDUCTOR IN MAGNETIC FIELD

When a current carrying conductor placed in magnetic field, a magnetic force exerts on each free electron which are present inside the conductor. The resultant of these forces on all the free electrons is called magnetic force on conductor.

- **Magnetic force on current element**

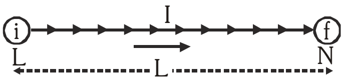
  

  Through experiments Ampere established that when current element $I d\vec{\ell}$ is placed in magnetic field $\vec{B}$, it experiences a magnetic force $d\vec{F}_m = I(d\vec{\ell} \times \vec{B})$
- Current element in a magnetic field does not experience any force if the current in it is parallel or anti–parallel with the field θ = 0° or 180°

  $dF_m = 0$ (min.)
- Current element in a magnetic field experiences maximum force if the current in it is perpendicular with the field θ = 90°

  $dF_m = BId\ell$ (max.)
- Magnetic force on current element is always perpendicular to the current element vector and magnetic field vector. $d\vec{F}_m \perp Id\vec{\ell}$ and $d\vec{F}_m \perp \vec{B}$ (always)
- Total magnetic force on straight current carrying conductor in uniform magnetic field given as

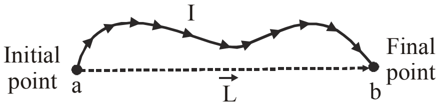
$$\vec{F}_m = \int_i^f d\vec{F}_m \left[\int_i^f d\vec{\ell}\right] = I \times \vec{B},\ \vec{F}_m = I(\vec{L} \times \vec{B})$$



Where $\vec{L} = \int_i^f \vec{d\ell}$, vector sum of all length elements from initial to final point, which is in accordance with the law of vector addition and $|\vec{L}|$ = length of the condutor.

- Total magnetic force on arbitrary shape current carrying conductor in uniform magnetic field $\vec{B}$ is

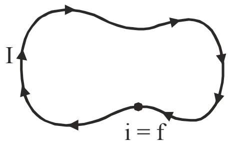
$$\int_i^f d\vec{F}_m = I\left[\int_i^f d\vec{\ell}\right] \times \vec{B},\ \vec{F}_m = I(\vec{L} \times \vec{B})\ (L = ab)$$



Where $\vec{L} = \int_i^f d\vec{\ell}$, vector sum of all length elements from initial to final point or displacement between free ends of an arbitrary conducter from initial to final point.

- A current carrying closed loop (or coil) of any shape placed in uniform magnetic field then no net magnetic force act on it (Torque may or may not be zero)

$$\vec{L} = \int_i^f d\vec{\ell} = 0 \text{ or } \oint d\vec{\ell} = 0$$



So net magnetic force acting on a current carrying closed loop $\vec{F}_m = 0$ (always)

- When a current carrying closed loop (or coil) of any shape placed in non uniform magnetic field then net magnetic force is always acts on it (Torque may or may not be zero)

Ex. : Magnetic Force on a Semi-Circular Loop

Consider a closed semi-circular loop lying in the *xy* plane carrying a current *I* in the counterclockwise direction, as shown in Figure.

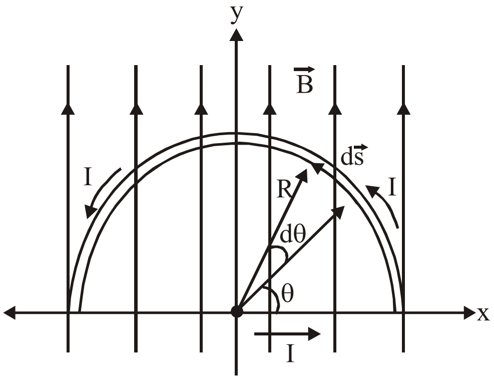


Figure Semi-circular loop carrying a current *I*

A uniform magnetic field pointing in the +*y* direction is applied. Find the magnetic force acting on the straight segment and the semicircular arc.

Sol. Let $\vec{B} = B\hat{j}$ and $\vec{F}_1$ and $\vec{F}_2$ the forces acting on the straight segment and the semicircular parts, respectively. Using Eq. and noting that the length of the straight segment is 2*R*, the magnetic force is

$$\vec{F}_1 = I(2R\hat{i}) \times (B\hat{j}) = 2IRB\hat{k}$$

where $\hat{k}$ is directed out of the page.

To evaluate $\vec{F}_2$, we first note that the differential length element $d\vec{s}$ on the semicircle can be written as $d\vec{s} = ds\hat{\theta} = Rd\theta(-\sin\theta\hat{i} + \cos\theta\hat{j})$. The force acting on the length element is $d\vec{s}$ is :

$$d\vec{s} = Id\vec{s}\times\vec{B} = IRd\theta(-\sin\theta\hat{i} + \cos\theta\hat{j})\times(B\hat{j}) = -IBR\sin\theta d\theta\hat{k}$$

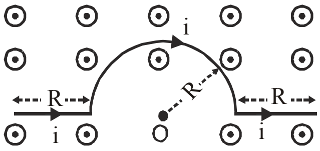
Here we see that $d\vec{F}_2$ points into the page. Integrating over the entire semi-circular arc, we have

Thus, the net force acting on the semi-circular wire is

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 = \vec{0}$$

This is consistent from our previous claim that the net magnetic force acting on a closed current-carrying loop must be zero.

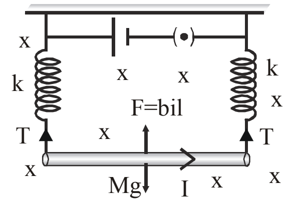
Ex. A wire bent as shown in fig carries a current i and is placed in a uniform field of magnetic induction $\vec{B}$ that emerges from the plane of the figure. Calculate the force acting on the wire.



Sol. The total force on the whole wire is

$F_m = I|\vec{L}|B = I(R + 2R + R)B = 4RIB$

Ex. A metal rod of mass 10 gm and length 25 cm is suspended on two springs as shown in figure. The springs are extended by 4 cm. When a 20 ampere current passes through the rod it rises by 1 cm. Determine the magnetic field assuming acceleration due to gravity to be 10 m/s².



Sol. Let tension in each spring is = T_0

Initially the rod will be in equilibrium if $2T_0 = Mg$

then $T_0 = kx_0$ ...(i)

Now when the current I is passed through the rod it will experience a force F = BIL vertically up; so in this situation for its equilibrium,

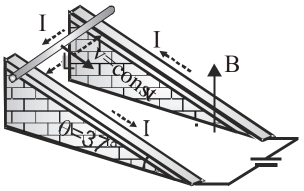
$2T + BIL = Mg$ with $T = kx$ ...(ii) $(x = 4 - 1 = 3\text{cm})$

So from eq. (i) and eq.(ii) $\dfrac{T}{T_0} = \dfrac{Mg - BIL}{Mg}$

$$\Rightarrow \frac{x}{x_0} = 1 - \frac{BIL}{Mg}$$

$$\Rightarrow B = \frac{Mg(x_0 - x)}{ILx_0} = \frac{10\times10^{-3}\times10\times3\times10^{-2}}{20\times25\times10^{-2}\times4\times10^{-2}} = 1.5\times10^{-2}\text{T}$$

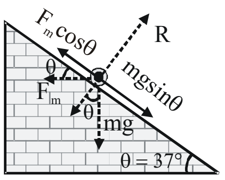
Ex. Two conducting rails are connected to a source of e.m.f. and form an incline as shown in fig. A bar of mass 50 g slides without friction down the incline through a vertical magnetic field B. If the length of the bar is 50 cm and a current of 2.5 A is provided by the battery, for what value of B will the bar slide at a constant velocity ? [g = 10 m/s²]



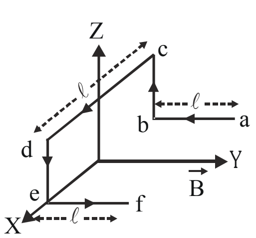
Sol. Force on current carrying wire F = BIL

The rod will move down the plane with constant velocity only if

$F\cos\theta = mg\sin\theta \Rightarrow BIL\cos\theta = mg\sin\theta$



$$\text{or,}\quad B = \frac{mg}{IL}\tan\theta = \frac{50\times10^{-3}\times10}{2.5\times50\times10^{-2}}\times\frac{3}{4} = 0.3\text{ T}$$



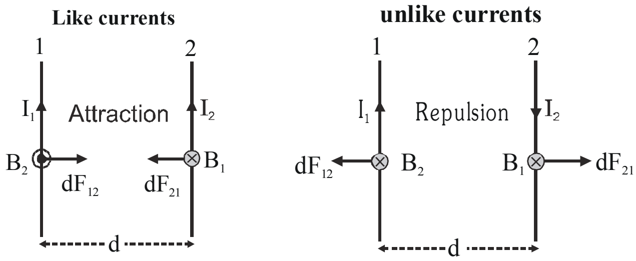
Ex. A wire abcdef with each side of length 'ℓ' bent as shown in figure and carrying a current I is placed in a uniform magnetic field B parallel to +y direction. What is the force experienced by the wire.

Sol. Magnetic force on wire abcdef in uniform magnetic field is $\vec{F}_m = I(\vec{L}\times\vec{B})$,

$\vec{L}$ is displacement between free ends of the conductor from initial to final point. $\vec{L} = (\ell)\,\hat{i}$ and $\vec{B} = (B)\,\hat{j}$

$F_m = I(\vec{L}\times\vec{B}) = BIL\,(\hat{i}\times\hat{j}) = BI\ell(\hat{k}) = BI\ell$, along +z direction

MAGNETIC FORCE BETWEEN TWO PARALLEL CURRENT CARRYING CONDUCTORS



The net magnetic force acts on a current carrying conductor due to its own field is zero. So consider two infinite long parallel conductors separated by distence 'd' carrying currents I_1 and I_2.

Magnetic field at each point on conductor (ii) due to current I_1 is $B_1 = \dfrac{\mu_0 I_1}{2\pi d}$

[uniform field for conductor (2)]

Magnetic field at each point on conductor (i) due to curent I_2 is $B_2 = \dfrac{\mu_0 I_2}{2\pi d}$

[Uniform field for conductor (1)]

consider a small element of length 'dℓ' on each conductor. These elements are right angle to the external magnetic field, so magnetic force experienced by elements of each conductor given as

$$dF_{12} = B_2 I_1 d\ell = \left(\frac{\mu_0 I_2}{2\pi d}\right) I_1\, d\ell \qquad \text{...(i)} \qquad (\text{Where } I_1\, d\ell \perp B_2)$$

$$dF_{21} = B_1 I_2 d\ell = \left(\frac{\mu_0 I_1}{2\pi d}\right) I_2\, d\ell \qquad \text{... (ii)} \qquad (\text{Where } I_2\, d\ell \perp B_1)$$

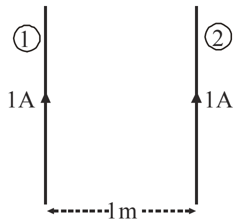
Where dF_{12} is magnetic force on element of conductor (i), due field of conductor (i) and dF_{21} is magnetic force on element of conductor (ii), due to field of conductor (i).

Magnetic force per unit length of each conductor is $\dfrac{dF_{12}}{d\ell} = \dfrac{dF_{21}}{d\ell} = \dfrac{\mu_0 I_1 I_2}{2\pi d}$

$f = \dfrac{\mu_0 I_1 I_2}{2\pi d}$ N/m (in S.I.) $\qquad$ $f = \dfrac{2 I_1 I_2}{d}$ dyne/cm (In C.G.S.)

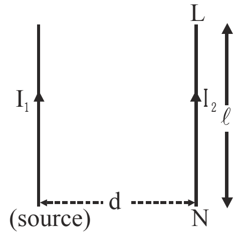
Definition of ampere :

Magnetic force/unit length for both infinite length conductor gives as



$$f = \frac{\mu_0 I_1 I_2}{2\pi d} = \frac{(4\pi\times10^{-7})(1)(1)}{2\pi(1)} = 2 \times 10^{-7}\ \text{N/m}$$

'Ampere' is the current which, when passed through each of two parallel infinite long straight conductors placed in free space at a distance of 1 m from each other, produces between them a force of 2 × 10^{-7} N/m



- Force scale $f = \dfrac{\mu_0 I_1 I_2}{2\pi d}$ is applicable when at least one conductor must be of infinite length so it behaves like source of uniform magnetic field for other conductor.

Magnetic force on conductor 'LN' is $F_{LN} = f \times \ell \Rightarrow F_{LN} = \left(\dfrac{\mu_0 I_1 I_2}{2\pi d}\right)\ell$

Torque on a Current Loop :

What happens when we place a rectangular loop carrying a current *I* in the *xy* plane and switch on a uniform magnetic field $\vec{B} = B\hat{i}$ which runs parallel to the plane of the loop, as shown in Figure (a)?

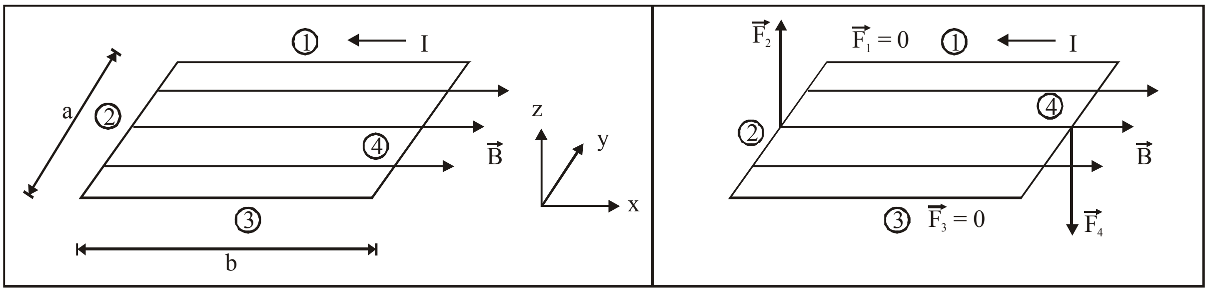


Figure : (a) A rectangular current loop placed in a uniform magnetic field.
(b) The magnetic forces acting on sides 2 and 4.

From Eq., we see the magnetic forces acting on sides 1 and 3 vanish because the length vectors $\vec{\ell}_1 = -b\hat{i}$ and $\vec{\ell}_3 = b\hat{i}$ are parallel and anti-parallel to $\vec{B}$ and their cross products vanish. On the other hand, the magnetic forces acting on segments 2 and 4 are non-vanishing:

$$\begin{cases} \vec{F}_2 = I(-a\hat{j})\times(B\hat{i}) = IaB\hat{k} \\ \vec{F}_4 = I(a\hat{j})\times(B\hat{i}) = -IaB\hat{k} \end{cases}$$

with $\vec{F}_2$ pointing out of the page and $\vec{F}_4$ into the page. Thus, the net force on the rectangular loop is

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = \vec{0}$$

as expected. Even though the net force on the loop vanishes, the forces $\vec{F}_2$ and $\vec{F}_4$ will produce a torque which causes the loop to rotate about the *y*-axis (Figure). The torque with respect to the center of the loop is

$$\vec{\tau} = \left(-\frac{b}{2}\hat{i}\right)\times\vec{F}_2 + \left(\frac{b}{2}\hat{i}\right)\times\left(\text{IaB}\hat{k} + \left(\frac{b}{2}\hat{i}\right)\times\left(-\text{IaB}\hat{k}\right)\right)$$

$$= \left(\frac{\text{IabB}}{2} + \frac{\text{IabB}}{2}\right)\hat{j} = \text{IabB}\hat{j} = \text{IAB}\hat{j}$$

where A = ab represents the area of the loop and the positive sign indicates that the rotation is clockwise about the *y*-axis. It is convenient to introduce the area vector $\vec{A} = A\hat{n}$ where $\hat{n}$ is a unit vector in the direction normal to the plane of the loop. The direction of the positive sense of $\hat{n}$ is set by the conventional right-hand rule. In our case, we have $\hat{n} = +\hat{k}$. The above expression for torque can then be rewritten as

$$\vec{\tau} = I\vec{A}\times\vec{B}$$

Notice that the magnitude of the torque is at a maximum when $\vec{B}$ is parallel to the plane of the loop (or perpendicular to).

Consider now the more general situation where the loop (or the area vector $\vec{A}$) makes an angle θ with respect to the magnetic field.

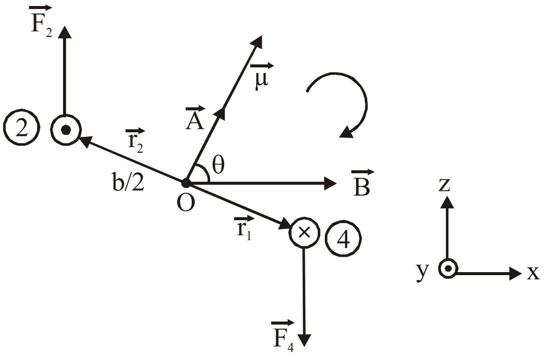


Figure : Rotation of a rectangular current loop

From Figure, the lever arms and can be expressed as:

$$\vec{r}_2 = \frac{b}{2}\left(-\sin\theta\hat{i} + \cos\theta\hat{k}\right) = -\vec{r}_4$$

and the net torque becomes

$$\vec{\tau} = \vec{r}_2\times\vec{F}_2 + \vec{r}_4 = 2\vec{r}_2\times\vec{F}_2 = 2.\frac{b}{2}\left(-\sin\theta\hat{i} + \cos\theta\hat{k}\right)\times\left(\text{IaB}\hat{k}\right)$$

$$\text{jIabB}\sin\theta\,\hat{j} = I\vec{A}\times\vec{B}$$

For a loop consisting of *N* turns, the magnitude of the toque is

$$\tau = \text{NIAB}\sin\theta$$

The quantity $\text{NI}\vec{A}$ is called the magnetic dipole moment $\vec{\mu}$

$$\vec{\mu} = \text{NI}\vec{A}$$

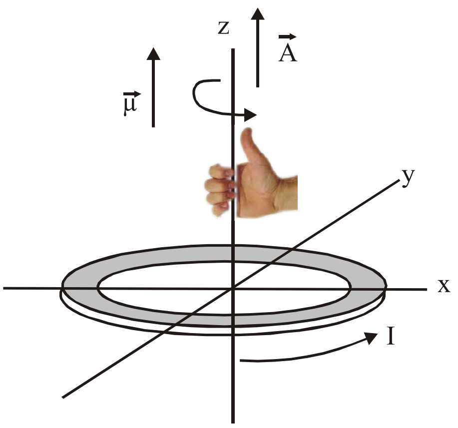


Figure : Right-hand rule for determining the direction of **μ**

The direction of $\vec{\mu}$ is the same as the area vector $\vec{A}$ (perpendicular to the plane of the loop) and is determined by the right-hand rule (Figure). The SI unit for the magnetic dipole moment is ampere-meter^2 (A•m^2).. Using the expression for $\vec{\mu}$, the torque exerted on a current-carrying loop can be rewritten as

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

The above equation is analogous to $\vec{\tau} = \vec{p} \times \vec{E}$ in Eq., the torque exerted on an electric dipole moment $\vec{p}$ in the presence of an electric field $\vec{E}$.

Configuratoin energy of current loop in uniform magnetic field.

Recalling that the potential energy for an electric dipole is $U = -\vec{p} \bullet \vec{E}$ [see Eq.], a similar form is expected for the magnetic case. The work done by an external agent to rotate the magnetic dipole from an angle θ_0 to θ is given by

$$W_{ext} = \int_{\theta_0}^{\theta} (\mu B \sin\theta')d\theta' = \mu B(\cos\theta_0 - \cos\theta)$$

$$= \Delta U = U - U_0$$

Once again, $W_{ext} = -W$, where W is the work done by the magnetic field. Choosing $U_0 = 0$ at $\theta_0 = \pi/2$, the dipole in the presence of an external field then has a potential energy os

$$U = -\mu B \cos\theta = -\vec{\mu} \bullet \vec{B}$$

The configuration is at a stable equilibrium when $\vec{\mu}$ is aligned parallel to $\vec{B}$, making U a minimum with $U_{min} = -\mu B$. On the other hand, when $\vec{\mu}$ and $\vec{B}$ are anti-parallel, $U_{max} = +\mu B$ is a maximum and the system is unstable.

Ex. A non-conducting thin disc of radius R charged uniformly over one side with surface density σ rotates about its axis with an angular velocity ω. Find:
(a) the magnetic induction at the centre of the disc;
(b) the magnetic moment of the disc.

Ans. (a) $B = 1/2\ \mu_0 \sigma \omega R$; (b) $p_m = 1/4\ \pi\sigma\omega R^4$

Sol. (a) Let us take a ring element of radius r and thickness dr, then charge on the ring element.,

$$dq = \sigma 2\pi\ r\ dr$$

and current, due to this element, $di = \frac{(\sigma 2\pi r\,dr)\omega}{2\pi} = \sigma\omega r\,dr$

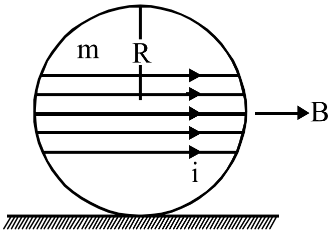
So, magnetic induction at the centre, due to this element : $dB = \frac{\mu_0}{2}\frac{di}{r}$

and hence, from symmetry : $B = \int dB = \int_0^R \frac{\mu_0\sigma\omega\, r\,dr}{r} = \frac{\mu_0}{2}\sigma\omega R$

(b) Magnetic moment of the element, considered,
$dp_m = (di)\,\pi r^2 = \sigma\omega dr\pi\, r^2 = \sigma\pi\omega r^3\,dr$
Hence, the sought magnetic moment,

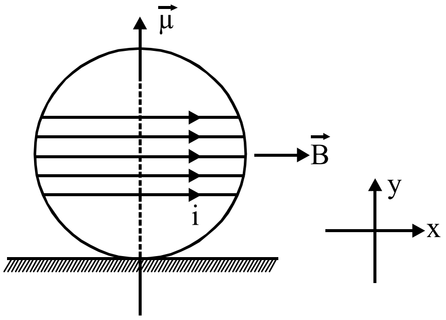
$$p_m = \int dp_m = \int_0^R \sigma\pi\omega r^3\,dr = \sigma\omega\pi\frac{R^4}{4}$$

Ex. A wire is wrapped N = 10 times over a solid sphere of mass m = 5kg, current I = 2A, which is placed on a smooth horizontal surface. A horizontal magnetic field of induction $|\vec{B}| = 10T$ is present. Find the angular acceleration experienced by the sphere. Assume that the mass of the wire is negligible compared to the mass of the sphere. If answer is 20nπ. Write value of n.



Ans. 5

Sol. (a) The net torque acting on the sphere is



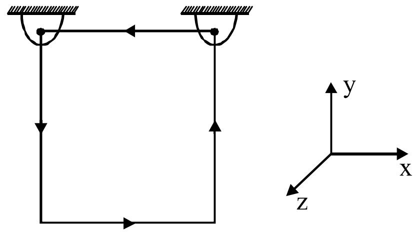
$\vec{\tau} = \vec{\mu}\times\vec{B} = (Ni A\hat{j})\times(B\hat{i}) = -NiAB\,\hat{k}$, where $A = \pi R^2$

or $\vec{\tau} = -N\pi R^2 iB\hat{k}$

(b) $\vec{\alpha} = \frac{\vec{\tau}}{I_C}$ (∵ the sphere is free to rotate, it must rotate about the centroidal axis)

$$= -\frac{N\pi R^2 iB}{\frac{2}{5}mR^2}\hat{k} \qquad \left(\because I_C = \frac{2}{5}mR^2\right) \qquad = \frac{5N\pi iB}{2m}\hat{k}$$

Ex. A current carrying uniform square frame is suspended from hinged supports as shown in the figure such that it can freely rotate about its upper side. The length and mass of each side of the frame is 2m and 4kg respectively. A uniform magnetic field $\vec{B} = \left(3\hat{i} + 4\hat{j}\right)$ is applied. When the wire frame is rotated to 45° from vertical and released it remains in equilibrium. If the magnitude of current (in A) in the wire frame is I then find $\left(\frac{3}{5}\right)I$.



Ans. 6

Sol. $\vec{\mu}$ (Magnetic moment of loop) when it is lifted by 45° = $i\ell^2\left(\frac{\hat{j}+\hat{k}}{\sqrt{2}}\right)$

$\therefore$ $\vec{\tau}$ due to magnetic field = $\vec{\mu}\times\vec{B}$ = $\frac{i\ell^2}{\sqrt{2}}\left[\left(\hat{j}+\hat{k}\right)\times\left(3\hat{i}+4\hat{j}\right)\right]$

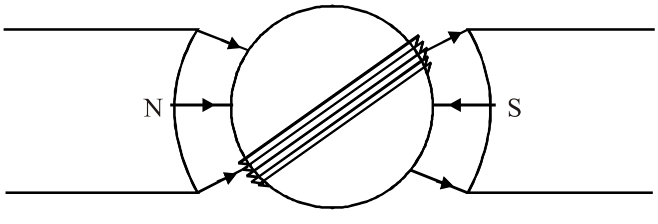
$\vec{\tau}$ due to mg (about top edge) = $4mg\frac{\ell}{2}\cos 45°\hat{i}$

$\therefore$ For equilibrium net torque along X-axis = 0

$$\therefore \quad \frac{4mg\ell}{2\sqrt{2}} = \frac{4i\ell^2}{\sqrt{2}} \Rightarrow i = \frac{mg}{2\ell} = 10A$$

MOVING COIL GALVANOMETER :

The main parts of a moving-coil galvanometer are shown in figure.



The current to be measured is passed through the galvanometer. As the coil is in the magnetic field $\vec{B}$ of the permanent magnet, a torque $\vec{\Gamma} = ni\vec{A}\times\vec{B}$ acts on the coil. Here n = number of turns, i = current in the coil $\vec{A}$ = area-vector of the coil and $\vec{B}$ = magnetic field at the site of the coil. This torque deflects the coil from its equilibrium position.

The pole pieces are made cylindrical. As a result, the magnetic field at the arms of the coil remains parallel to the plane of the coil everywhere even as the coil rotates. The deflecting torque is then $\Gamma = niAB$. As the upper end of the suspension strip W is fixed, the strip gets twisted when the coil rotates. This produces a restoring torque acting on the coil. If the deflection of the coil is θ and the torsional constant of the suspension strip is k, the restoring torque is kθ. The coil will stay at a deflection θ where

$$niAB = k\theta$$

or, $$i = \frac{k}{nAB}\theta$$

Hence, the current is proportional to the deflection. The constant $\frac{k}{nAB}$ is called the galvanometer constant.

We define the **current sensitivity** of the galvanometer as the deflection per unit current. From Eq. this current sensitivity is.

$$\frac{\phi}{I} = \frac{NAB}{k}$$

A convenient way for the manufacturer to increase the sensitivity is to increase the number of turns N. We choose galvanometers having sensitivities of value, required by our experiment.

We define the **voltage sensitivity** as the deflection per unit volt of applied potential difference

$$\frac{\phi}{I} = \left(\frac{NAB}{k}\right)\frac{I}{V} = \left(\frac{NAB}{k}\right)\frac{1}{R}$$

An interesting point to note is that increasing the current sensitivity may not necessarily increase the voltage sensitivity. If N → 2N, i.e., we double the number of turns, then

$$\frac{\phi}{I} \to 2\frac{\phi}{I}$$

Thus, the current sensitivity doubles. However, the resistance of the galvanometer is also likely to double, since it is proportional to the length of the wire. In eq. N → 2N, and R → 2R, thus the voltage sensitivity,

$$\frac{\phi}{V} \to \frac{\phi}{V}$$

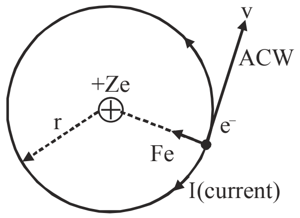
remains unchanged.

ATOMIC MAGNETISM

An atomic orbital electron, which doing bounded uniform circular motion around nucleus. A current constitues with this orbital motion and hence orbit behaves like current carrying loop. Due to this magnetism produces at nucleus position. This phenomenon called as 'atomic magnetism.

Bohr's postulates :

(i) $\frac{mv^2}{r} = \frac{kze^2}{r^2}$ (ii) $L = mvr = n\left(\frac{h}{2\pi}\right)$, where n = 1, 2,3 ....



Basic elements of atomic magnetism :

(a) **Orbital current :-** $I = ef = \frac{e}{T} = \frac{ev}{2\pi r} = \frac{e\omega}{2\pi}$

(b) **Magnetic induction at nucleus position :-** As circular orbit behaves like current carrying loop, so magnetic induction at nucleus position $B_N = \frac{\mu_0 I}{2r}$

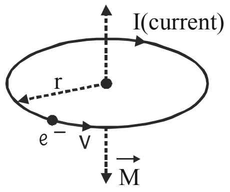
$$B_N = \frac{\mu_0 ef}{2r} = \frac{\mu_0 e}{2Tr} = \frac{\mu_0 ev}{4\pi r^2} = \frac{\mu_0 e\omega}{4\pi r}$$

(c) **Magnetic moment of circular orbit :-** Magnetic dipole moment of circular orbit

M = IA where A is area of circular orbit. $M = ef\left(\pi r^2\right) = \frac{\pi e r^2}{T} = \frac{evr}{2} = \frac{e\omega r^2}{2}$

- **Relation between magnetic moment and angular momentum of orbital electron**

Magnetic moment $M = \frac{evr}{2} \times \frac{m}{m} = \frac{eL}{2m}$ ($\because$ angular momentum L = mvr)



Vector form $\vec{M} = \frac{-e\vec{L}}{2m}$

For orbital electron its $\vec{M}$ and $\vec{L}$ both are antiparallel axial vectors.

A NONCONDUCTING CHARGED BODY IS ROTATED WITH SOME ANGULAR SPEED.

In this case the ratio of magnetic moment and angular momentum is constant which is equal to $\frac{q}{2m}$

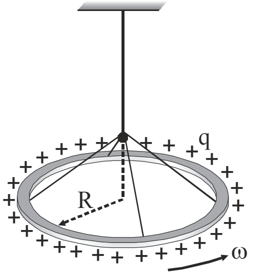
here q = charge and m = the mass of the body.

Ex. :- In case of a ring, of mass m, radius R and charge q distributed on it circumference.

Angular momentum $L = I\omega = (mR^2)(\omega)$ ... (i)

Magnetic moment $M = iA = (qf)(\pi R^2)$

$$M = (q)\left(\frac{\omega}{2\pi}\right)(\pi R^2) = q\frac{\omega R^2}{2} \quad ...(ii)$$



$\because f = \frac{\omega}{2\pi}$ From Eqs. (i) and (ii) $\frac{M}{L} = \frac{q}{2m}$

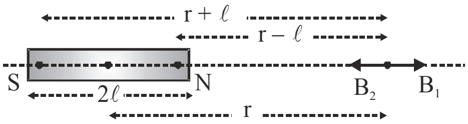
Although this expression is derived for simple case of a ring, it holds good for other bodies also. For example, for a disc or a sphere. $M = \frac{qL}{2m} \Rightarrow M = \frac{q(I\omega)}{2m}$, where $L = I\omega$

Rigid body	**Ring**	**Disc**	**Solid sphere**	**Spherical shell**
Moment of inertia (I)	mR^2	$\frac{mR^2}{2}$	$\frac{2}{5}mR^2$	$\frac{2}{3}mR^2$
Magnetic moment= $\frac{\mathbf{qI\omega}}{\mathbf{2m}}$	$\frac{q\omega R^2}{2}$	$\frac{q\omega R^2}{4}$	$\frac{q\omega R^2}{5}$	$\frac{q\omega R^2}{3}$

SUPPLEMENT FOR JEE-MAINS

MISCELLENEOUS

Magnetic field of long Bar magnet



(i) At Axial position :–

Magnetic field at point 'P' due to north pole $B_1 = \frac{\mu_0}{4\pi}\frac{m}{(r-\ell)^2}$ (away from north pole)

Magnetic field at point 'P' due to south pole $B_2 = \frac{\mu_0}{4\pi}\frac{m}{(r+\ell)^2}$ (towards north pole)

Net magnetic field at point 'P'

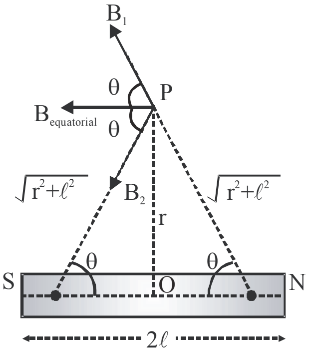
$$B_{axis} = B_1 - B_2\,,\ (\because\ B_1 > B_2) = \frac{\mu_0 m}{4\pi}\left[\frac{1}{(r-\ell)^2} - \frac{1}{(r+\ell)^2}\right] = \frac{\mu_0 m}{4\pi}\left[\frac{4r\ell}{(r^2-\ell^2)^2}\right]$$

$$B_{axis} = \frac{\mu_0}{4\pi}\frac{2Mr}{(r^2-\ell^2)^2}\text{, where } M = m\,(2\ell)$$

If magnet is short r >> ℓ, then $B_{axis} \simeq \frac{\mu_0}{4\pi}\frac{2M}{r^3}$

(ii) At equatorial position :–

Magnetic field at point 'P' due to north pole :–



$$B_1 = \frac{\mu_0}{4\pi}\frac{m}{\left(\sqrt{r^2+\ell^2}\right)^2}\ \ldots\ldots (1) \qquad \text{(along NP line)}$$

Magnetic field at point 'P' due to south pole :–

$$B_2 = \frac{\mu_0}{4\pi}\frac{m}{\left(\sqrt{r^2+\ell^2}\right)^2}\ \ldots\ldots (2) \qquad \text{(along PS line)}$$

From equation (1) & (2) $B_1 = B_2 = \frac{\mu_0}{4\pi}\cdot\frac{m}{r^2+\ell^2} = B$ (Let)

Net magnetic field at point 'P'

$$B_{eq} = 2\,B\cos\theta = 2\cdot\frac{\mu_0}{4\pi}\frac{m}{(r^2+\ell^2)}\cos\theta\,, \qquad \left[\text{where } \cos\theta = \frac{\ell}{\sqrt{(r^2+\ell^2)}}\right]$$

$$= 2\cdot\frac{\mu_0}{4\pi}\frac{m}{(r^2+\ell^2)}\frac{\ell}{\sqrt{(r^2+\ell^2)}}$$

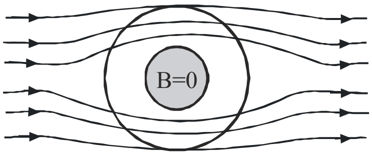
$$B_{eq.} = \frac{\mu_0}{4\pi}\cdot\frac{M}{(r^2+\ell^2)^{3/2}}, \qquad \text{where } M = m(2\ell)$$

If magnet is short r >> ℓ, then $B_{eq.} \simeq \frac{\mu_0}{4\pi}\cdot\frac{M}{r^3}$

Magnetic shielding

If a soft iron ring is placed in magnetic field, most of the lines are found to pass through the ring and no lines pass through the space inside the ring. The inside of the ring is thus protected against any external magnetic effect. This phenomenon is called magnetic screening or shielding and is used to protect costly wrist–watches and other instruments from

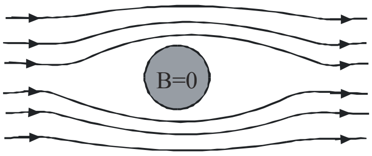
external magnetic fields by enclosing them in a soft–iron case or box.



Iron ring in a field

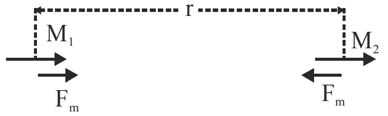
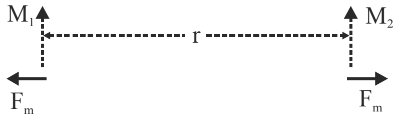
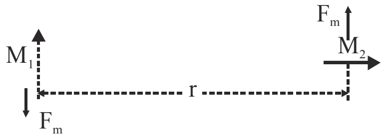
(i) Super conductors also provides perfect magnetic screening due to exclusion of lines of force. This effect is called **'Meissner effect'**

(ii) Relative magnetic premeability of super conductor is zero. So we can say that super conductors behaves like perfect dimagnetic.



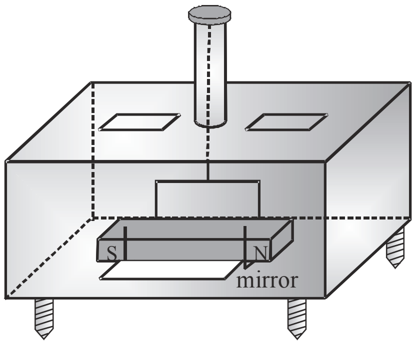
Super conductor in a field

Dipole - Dipole Interactions :

S.No.	Relative position of dipoles	Magnetic force (F_m)
(a)		$\frac{\mu_0}{4\pi}\cdot\frac{6M_1M_2}{r^4}$ (along r)
(b)		$\frac{\mu_0}{4\pi}\cdot\frac{3M_1M_2}{r^4}$ (along r)
(c)		$\frac{\mu_0}{4\pi}\cdot\frac{3M_1M_2}{r^4}$ (perpendicular to r)

VIBRATION MAGNETOMETER :

It is an instrument used to compare the horizontal components of magnetic field of earth of two different places, to compare magnetic fields and magetic moments of two bar magnets. It is also called oscillation magnetometer.

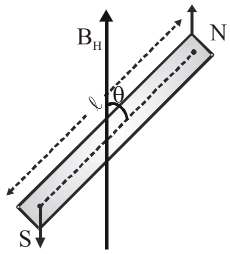


Principle **:** This device works on the principle, that whenever a freely suspended bar magent horizontal component in earth magnetic field (B_H) is slightly disturbed from its equilibrium position then, it will experience a torque and executes angular S.H.M. *Rotation is possible only in horizontal plane.

Angular S.H.M of magnetic dipole :- When a dipole is suspended in a uniform magnetic field it will align itself parallel to field. Now if it is given a small angular displacement θ about its equilibrium position. The restoring torque acts on it :

$\tau = -M B_H \sin\theta \Rightarrow I\alpha = -MB_H \sin\theta = -MB_H\,\theta, (\because \sin\theta \simeq \theta)$

$\Rightarrow \alpha = \dfrac{MB_H}{I}(-\theta) \Rightarrow \alpha = \omega^2(-\theta) \Rightarrow \omega^2 = \dfrac{MB_H}{I}$



The time period of angular S.H.M. $\Rightarrow T = 2\pi\sqrt{\dfrac{I}{MB_H}}$

M = magnetic moment of bar magnet

I = moment of inertia of bar magnet about its geometric axis

Comparision of magnetic moments of magnets of the same size

Let the two magnets of same size have moment of inertia I and magnetic moments M_1 and M_2. Suspend the two given magnets turn by turn in the metal stirrup of the vibration magnetometer and note the time period in each case.

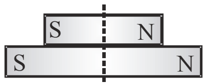
Then $T_1 = 2\pi\sqrt{\dfrac{I}{M_1B}}$ and $T_2 = 2\pi\sqrt{\dfrac{I}{M_2B}}$

Dividing, $\dfrac{T_1}{T_2} = \sqrt{\dfrac{M_2}{M_1}}$ or $\dfrac{M_1}{M_2} = \dfrac{T_2^2}{T_1^2}$

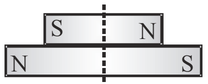
Since T_1 and T_2 are known therefore the ratio $\dfrac{M_2}{M_1}$ can be determined.

Comparision of magnetic moments of magnets of different sizes

Let the two magnets have moments of inertia I_1 and I_2 and magnetic moments M_1 and M_2 respectively. Place the two given magnets one upon the other as shown in Fig. (a). This combination is called sum combination'. It has moment of inertia $(I_1 + I_2)$ and magnetic moment $(M_1 + M_2)$. Put this combination in the magnetometer and set it into oscillations. The time period T_1 is determined.



(a) sum combination



(b) difference combination

$$T_1 = 2\pi\sqrt{\frac{I_1+I_2}{(M_1+M_2)B}} \quad ....(1)$$

Now, the two magnets are placed as shown in Fig. (b). This combination is called 'difference combination'. It has moment of inertia $(I_1 + I_2)$ and magnetic moment $(M_1 - M_2)$. This combination is put in the magneto meter and its time period T_2 is determined.

$$T_2 = 2\pi\sqrt{\frac{I_1+I_2}{(M_1-M_2)B}} \quad ......(2)$$

Dividing, $\dfrac{T_1}{T_2} = \sqrt{\dfrac{M_1-M_2}{M_1+M_2}}$ [from equation (1) and (2)]

knowing T_1 and T_2, we can determine $\dfrac{M_1}{M_2}$.

Comparison of earth's magnetic field at two different places

Let the vibrating magnet have moment of inertia I and magnetic moment M. Let it be vibrated in places where earth's magnetic field is B_{H_1} and B_{H_2}.

Then, $T_1 = 2\pi\sqrt{\dfrac{I}{MB_{H_1}}}$ and $T_2 = 2\pi\sqrt{\dfrac{I}{MB_{H_2}}}$

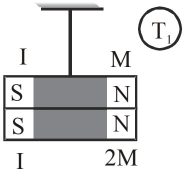
T_1 and T_2 are determined by placing magnetometer at two different places, turn by turn.

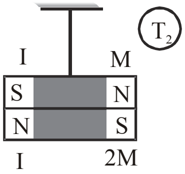
Dividing, $\dfrac{T_1}{T_2} = \sqrt{\dfrac{B_{H_2}}{B_{H_1}}}$ or $\dfrac{T_1^2}{T_2^2} = \dfrac{B_{H_2}}{B_{H_1}} = \dfrac{B_2\cos\theta_2}{B_1\cos\theta_1} \Rightarrow \dfrac{B_1}{B_2} = \dfrac{T_2^2\cos\theta_2}{T_1^2\cos\theta_1}$

Knowing T_1, T_2 and θ_1, θ_2 the ratio $\dfrac{B_1}{B_2}$ can be determined.

Ex. Magnetic moments of two identical magnets are M and 2M respectively. Both are combined in such a way that their similar poles are same side. The time period in this is case 'T_1'. If polrity of one of the magnets is reversed its period becomes 'T_2' then find out ratio of their time periods respectively.

Sol.





$M_{system} = 2M + M = 3M$
$I_{system} = 2I$

$M_{system} = 2M – M = M$
$I_{system} = 2I$

$T = 2\pi\sqrt{\dfrac{I}{MB_H}}$ ($I_{system} \rightarrow$ same, $B_H \rightarrow$ same) $\boxed{T \propto \dfrac{1}{\sqrt{M}}}$;

$$\frac{T_1}{T_2} = \sqrt{\frac{M_2}{M_1}} = \sqrt{\frac{M}{3M}} = \frac{1}{\sqrt{3}}$$

Ex. A magnet is suspended in such a way when it oscillates in the horizontal plane. It makes 20 oscillations per minute at a place where dip angle is 30° and 15 oscillations per min at a place where dip angle is 60°. Find the ratio of the total earth's magnetic field at the two places.

Sol. $f = \dfrac{1}{2\pi}\sqrt{\dfrac{MB_H}{I}}$

$\Rightarrow f^2 = \dfrac{1}{4\pi^2}\cdot\dfrac{MB\cos\theta}{I}$ I and M are same in given cases

$$\frac{B_1}{B_2} = \frac{f_1^2}{f_2^2}\times\frac{\cos\theta_2}{\cos\theta_1} = \frac{20\times20}{15\times15}\times\frac{\cos 60^\circ}{\cos 30^\circ} = \frac{16}{9\sqrt{3}}$$

NEUTRAL POINT

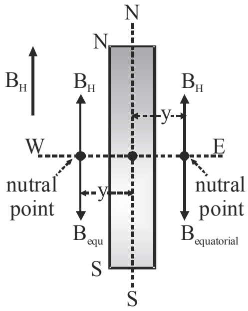
It is a point where net magnetic field is zero.

At this point magnetic field of bar magnet or current carrying coil or current carrying wire is just neutralised by magnetic field of earth. (B_H)

A compass needle placed at this neutral point can set itself in any direction.

Location of Neutral Points :

(a) **When N–pole of magnet directed towards North :-** Two neutral points symmetrically located on equatorial line of magnet. Let distance of each neutral point from centre of magnet is 'y' then

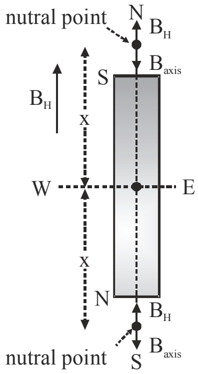


$$B_{eq} = B_H$$

$$B_H = \frac{\mu_0}{4\pi} \cdot \frac{M}{(y^2 + \ell^2)^{3/2}}$$

$$\frac{\mu_0}{4\pi} \cdot \frac{M}{y^3} = B_H \qquad (\text{If } y >>> \ell)$$

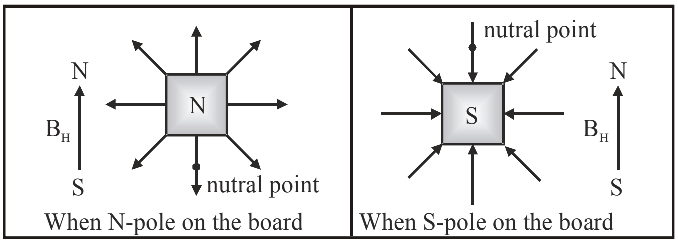
(b) **When S–pole of magnet directed towards North :-** Two neutral points symmetrically located on the axial line of magnet. Let distance of each neutral points from centre of the magnet is x, then



$$B_{axis} = B_H \Rightarrow B_H = \frac{\mu_0}{4\pi} \cdot \frac{2Mx}{(x^2 - \ell^2)^2}$$

$$\frac{\mu_0}{4\pi}\frac{2M}{x^3} = B_H \qquad (\text{If } x >>> \ell)$$

(c) If magnet is held vertically on the board, then only one neutral point is obtained on the horizontal board.



When N-pole on the board

When S-pole on the board

Ex. The magnetic field at a point x on the axis of a small bar magnet is equal to the field at a point y on the equator of the same magnet. Find the ratio of the distances of x and y from the centre of the magnet.

Sol. $B_{axis} = B_{equatorial} \Rightarrow \frac{\mu_0}{4\pi}\frac{2M}{x^3} = \frac{\mu_0}{4\pi}\frac{M}{y^3}$

$$\Rightarrow \frac{2}{x^3} = \frac{1}{y^3} \Rightarrow \frac{x^3}{y^3} = \frac{2}{1}$$

$$\Rightarrow \frac{x}{y} = 2^{1/3}$$

Ex. A coil of 0.1 m radius and 100 turns placed perpendicular magnetic meridian. When current of 2 ampere is flow through the coil then the neutral point is obtained at the centre. Find out magnetising field of earth.

Sol. Magnetic field at centre of coil $B = \dfrac{\mu_0 NI}{2R} = \mu_0 H_C$ $\quad (\because\ H_C = H_e)$

Magnetising field of earth $H_e = \dfrac{NI}{2R} = \dfrac{100 \times 2}{2 \times 0.1} = 1000$ A/m

Ex. A short magnet of moment 6.75 A–m^2 produces a neutral point on its axis. If the horizontal component of earth's magnetic field 5×10^{-5} Wb/m^2, Calculate the distance of the neutral point from the centre.

Sol. $B_H = \dfrac{2KM}{d^3} \Rightarrow d = \left(\dfrac{2KM}{B_H}\right)^{1/3} = \left(\dfrac{2 \times 10^{-7} \times 6.75}{5 \times 10^{-5}}\right)^{1/3} = 0.3\text{ m} = 30\text{ cm}$

GEO-MAGNETISM

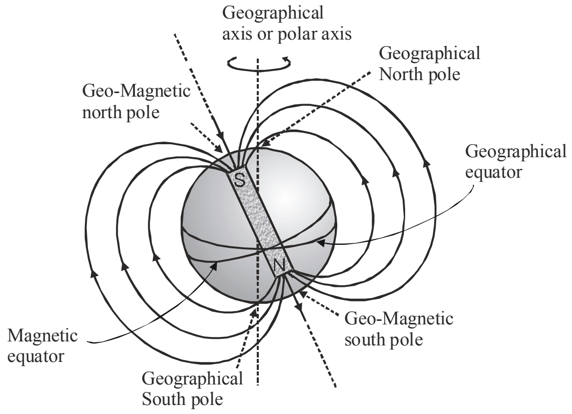
Important definitions :

(a) **Geographic axis** : It is a straight line passing through the geographical poles of the earth. It is also called axis of rotation or polar axis of the earth.

(b) **Geographic Meridian (GM)** : It is a vertical plane at any place which passing through geographical axis of the earth.

(c) **Geographic equator** : It is a great circle on the surface of the earth, in a plane perpendicular to the geographic axis. All the points on the geographic equator are at equal distance from the geographic poles.

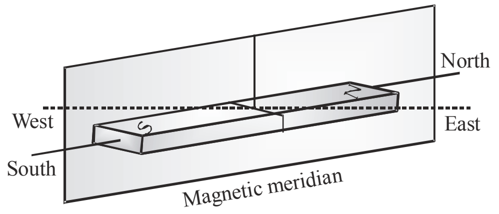
A great plane which passes through geographic equator and perpendicular to the geographic axis called **geographic equatorial plane**. This plane cuts the earth in two equal parts, a part has geographic north called **northen hemisphere** (NHS) and another part has geographic south called **southern hemi sphere** (SHS).



(d) **Magnetic axis** : It is a straight line passing through magnetic poles of the earth. It is inclined to the geographic axis at nearly 17°.

(e) **Magnetic Meridian (MM)** : (i) It is a vertical plane at any place which passing through magnetic axis of the earth. (ii) It is a vertical plane at any place which passing through axis of free suspended bar magnet or magetic needle.

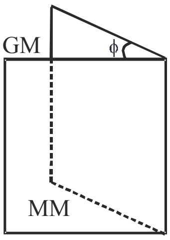
(iii) It is a vertical plane at any place which contains all the magnetic field lines of earth of that place.



(f) **Magnetic equator** : It is a great circle on the surface of the earth, in a plane perpendicular to the magnetic axis. All the points on the magnetic equator are at equal distance from the magnetic poles.

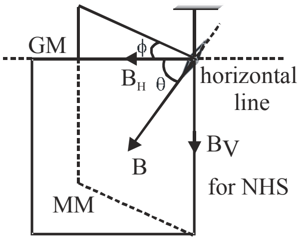
MAIN ELEMENTS OF EARTH'S MAGNETIC FIELD

Angle of declination (ϕ)



At a given place the acute angle between geographic meridian and the magnetic meridian is called angle of declination, i.e. at a given place it is the angle between the geographical north south direction and the direcation indicated by a agnetic compass needle in its equillibrium.

Angle of dip (θ)



(i) It is an angle which the direction of resultant magnetic field of the earth substends with the horizontal line in magnetic meridian at the given place.

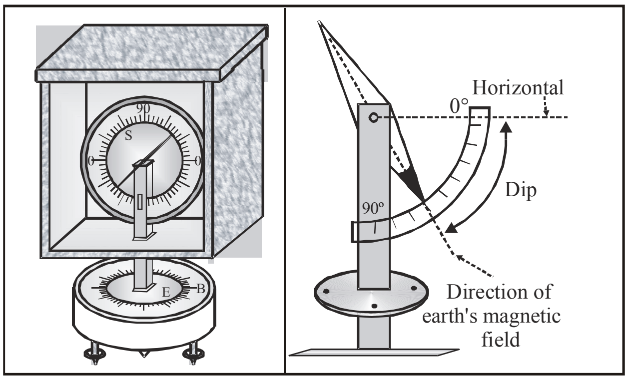
(ii) It is an angle which the axis of freely suspended magnetic needle (up or down) substends with the horizontal line in magnetic meridian at a given place.

In northen hemi sphere, north pole of freely suspended magnetic needle will dip downwards i.e. towards the earth surface. In southern hemi sphere, south pole of freely suspended magnetic needle will dip downwards i.e. towards the earth surface.

Dip circle : Angle of dip at a place is measured by the instrument called 'Dip-circle' in which a magnetic needle is free to rotate in vertical plane. About its horizontal axis. The ends of the needle move over a vertical scale graduated in degree.

Horizontal component of earth magnetic field (B_H)

Horizontal component of earth magnetic field at a given place is the component of resultant magnetic field of the earth along the horizontal line in magnetic meridian.



$$B_H = B\cos\theta \quad \text{and } B_v = B\sin\theta \qquad ....(1)$$

$$\text{so that } \tan\theta = \frac{B_V}{B_H} \text{ and } B = \sqrt{B_H^2 + B_V^2} \qquad ....(2)$$

At magnetic poles $\theta = 90°$ $\quad$ $B_H = 0$ and only B_V exist

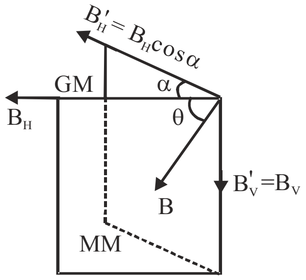
❖ At magnetic equator $\theta = 0°$ $\quad$ $B_V = 0$ and only B_H exist

ϕ decides the plane in which magnetic field lies at any place, (ϕ) and (θ) decides the direction of magnetic field and (θ) and (B_H) decides the magnitude of the field.

Apparent angle of dip (θ') :

When the plane of vertical scale of dip circle is in the magnetic meridian, the needle rest in the direction of earth's magnetic field. The angle made by the needle with the horizontal is called true dip or actual dip. If the plane of vertical scale of dip circle not kept in magnetic meridian, then the needle will not indicate the correct direction of earth magnetic field.

In this situation the angle made by the needle with the horizontal is called the apparent angle of dip.Suppose the dip circle is set at an angle α to the magnetic meridian. Effective horizontal component in this plane will be $B_H\cos\alpha$ and no effect on vertical component B_V



Apparent angle of dip $\tan\theta' = \dfrac{B'_V}{B'_H}$

$$\Rightarrow \tan\theta' = \frac{B_V}{B_H\cos\alpha} \Rightarrow \tan\theta' = \frac{\tan\theta}{\cos\alpha}$$

- ❖ For a vertical plane other than magnetic meridian

  $\alpha > 0 \Rightarrow \cos\alpha < 1 \Rightarrow \tan\theta' > \tan\theta \Rightarrow \theta' > \theta$,

  so apparent angle of dip is always more than actual angle of dip at any place.

- ❖ For a vertical plane perpendicular to magnetic meridian

  $$\alpha = 90° \Rightarrow \tan\theta' = \frac{\tan\theta}{\cos 90°} = \infty$$

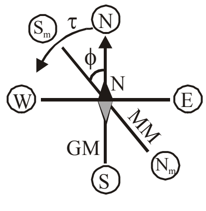
  $\theta' = 90°$, so in a plane perpendicular to magnetic meridian dip needle becomes just verticle.

Ex. At a certain place, the horizontal component of earth's magnetic field is $\sqrt{3}$ times of the vertical component. What the angle of dip at that place.

Sol. $B_H = \sqrt{3}\,B_V$, $\tan\theta = \dfrac{B_V}{B_H} = \dfrac{B_V}{\sqrt{3}B_v} = \dfrac{1}{\sqrt{3}} = \tan 30° \Rightarrow \theta = 30°$

Ex. A compass needle of magnetic moment is 60 A-m^2 pointing towards geographical north at a certain place where the horizontal component of earth's magnetic field is 40μT, experiences a torque 1.2×10^{-3} N-m. What is the declination of that place.

Sol. $\tau = MB\sin\phi$



$$\Rightarrow \sin\phi = \frac{\tau}{MB} = \frac{1.2\times10^{-3}}{24\times10^{-4}} = \frac{1}{2}$$

$\Rightarrow \phi = 30°$

Ex. If the dip circle is set at 45° to the magnetic meridian, then the apparent dip is 30°. Calculate the true dip.

Sol. $\tan\theta' = \dfrac{\tan\theta}{\cos\alpha}$

$$\tan\theta = \tan\theta'\cos\alpha = \tan 30^\circ \cos 45^\circ = \frac{1}{\sqrt{3}}\times\frac{1}{\sqrt{2}}$$

$$\tan\theta = \frac{1}{\sqrt{6}} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{\sqrt{6}}\right)$$

Ex. A magnetic needle is free to rotate in a vertical plane and that plane makes an angle of 60^0 with magnetic meridian. If the needle stays in a direction making an angle of $\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$ with the horizontal direction, what would be the actual dip at that place ?

Sol. $\tan\theta = \tan\theta'\cos\alpha \qquad \left(\because\ \theta' = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right),\ \alpha = 60^\circ\right)$

$$\therefore\ \tan\theta = \tan\left(\tan^{-1}\frac{2}{\sqrt{3}}\right)\cos 60^\circ$$

$$\tan\theta = \frac{2}{\sqrt{3}}\times\frac{1}{2} = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ$$

Ex. A 1-meter long narrow solenoid having 1000 turns is placed in magnetic meridian. Find the current in the solenoid which neutralises the earth's horizontal field of 0.36 oersted at the centre of the solenoid.

Sol. The magnetic field intensity at the centre of solenoid is $H = ni = 1000\ i$ A/m $= 4\pi i$ ovested

($\therefore$ 1 amp/meter $= 4\pi \times 10^{-3}$ oersted)

Since it neutralises the earth's field of 0.36 oersted, it is equal and opposite to the earth's field.

$\therefore\ 4\pi i = 0.36$

$$\Rightarrow i = \frac{0.36}{4\times 3.14} = 0.0286 \text{ ampere} = 28.6 \text{ milli-ampere or } 28.6 \text{ mA}$$

Ex. If θ_1 and θ_2 are angles of dip in two vertical planes at right angle to each other and θ is true dip then prove $\cot^2\theta = \cot^2\theta_1 + \cot^2\theta_2$.

Sol. If the vertical plane in which dip is θ_1 subtends an angle α with meridian than other vertical plane in which dip is θ_2 and is perpendicular to first will make an angle of $90 - \alpha$ with magnetic meridian. If θ_1 and θ_2 are apparent dips than

$$\tan\theta_1 = \frac{B_V}{B_H\cos\alpha}$$

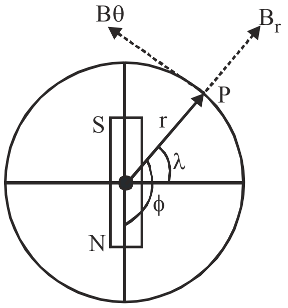
$$\tan\theta_2 = \frac{B_V}{B_H\cos(90-\alpha)} = \frac{B_V}{B_H\sin\alpha}$$

$$\cot^2\theta_1 + \cot^2\theta_2 = \frac{1}{(\tan\theta_1)^2} + \frac{1}{(\tan\theta_2)^2} = \frac{B_H^2\cos^2\alpha + B_H^2\sin^2\alpha}{B_V^2} = \frac{B_H^2}{B_V^2} = \left(\frac{B\cos\theta}{B\sin\theta}\right)^2 = \cot^2\theta$$

So $\cot^2\theta_1 + \cot^2\theta_2 = \cot^2\theta$

Ex. Considering earth as a short bar magnet show that the angle of dip θ is related to magnetic latitude λ as $\tan\theta = 2\tan\lambda$

Sol. For a magnetic dipole the field components at point P (r, φ) are given as



$$B_r = \frac{\mu_0}{4\pi}\frac{2M\cos\phi}{r^3} \quad B_\theta = \frac{\mu_0}{4\pi}\frac{M\sin\phi}{r^3}$$

$$\tan\theta = \frac{B_V}{B_H} = -\frac{B_r}{B_\theta} = -\frac{\mu_0}{4\pi}\frac{2M\cos\phi}{r^3}\frac{4\pi\ r^3}{\mu_0\ M\sin\phi}$$

or $\tan\theta = -2\cot\phi$

From figure $\phi = \frac{\pi}{2}+\lambda$ So $\tan\theta = -2\cot\left(\frac{\pi}{2}+\lambda\right)$ or $\tan\theta = 2\tan\lambda$

Ex. At a certain location in Africa, a compass points 12° west of the geographic north. The north tip of the magnetic needle of a dip circle placed in the plane of the magnetic meridian points 60° above the horizontal. The horizontal component of the earth's field is measured to be 0.16 G. Specify the direction and magnitude of the earth's field at the location.

Sol. From formula, $B_H = B\cos\theta$

$$B = \frac{B_H}{\cos\ \theta} = B_H \sec\theta = 0.16 \times 2 = 0.32\text{ G.}$$

The earth's field is 0.32 G, in direction 60° upwards from horizontal, in a plane (magnetic meridian) 12° West of geographical meridian.

Ex. A dip circle shows an apparent dip of 60° at a place where the true dip is 45°. If the dip circle is rotated through 90° what apparent dip will it show?

Sol. Let θ_1 and θ_2 be apparent dip shown by dip circle in two perpendicular positions then true dip θ is given by

$$\cot^2\theta = \cot^2\theta_1 + \cot^2\theta_2 \text{ or } \cot^2 45° = \cot^2 60° + \cot^2\theta_2$$

or $\cot^2\theta_2 = \frac{2}{3}$ or $\cot\theta_2 = 0.816$ giving $\theta_2 = 51°$

IMPORTANT DEFINATIONS AND RELATIONS

Magnetising field or Magnetic intensity $(\vec{H})$

Field in which a material is placed for magnetisation, called as magnetising field.

$$\text{Magnetising field } \vec{H} = \frac{\vec{B}_0}{\mu_0} = \frac{\text{magnetic field}}{\text{permeability of free space}}$$

SI Unit $\vec{H}$: ampere/meter

Intensity of magnetisation $(\vec{I})$

When a magnetic material is placed in magnetising field then induced dipole moment per unit volume of that material is known as intensity of magnetisation $\vec{I} = \frac{\vec{M}}{V}$

SI Unit : ampere/meter $[\because \frac{\vec{M}}{V} = \frac{I\vec{A}}{V} = \frac{\text{ampere} \times \text{meter}^2}{\text{meter}^3}]$

Magnetic susceptibility (χ_m)

$\chi_m = \frac{I}{H}$ [It is a scalar with no units and dimensions]

Physically it represent the ease with which a magnetic material can be magnetised
A material with more χ_m, can be change into magnet easily.

Magnetic permeability μ

$$\mu = \frac{B_m}{H} = \frac{\text{total magnetic field inside the material}}{\text{magnetising field}}$$

It measures the degree to which a magnetic material can be penetrated (or permeated) by the magnetic field lines

SI Unit of μ : $\mu = \frac{B_m}{H} \equiv \frac{\text{Wb}/\text{m}^2}{\text{A}/\text{m}} \equiv \frac{\text{Wb}}{\text{A}-\text{m}} \equiv \frac{\text{H}-\text{A}}{\text{A}-\text{m}} = \frac{\text{H}}{\text{m}}$

$[\because \phi = L\,I \therefore \text{ weber} \equiv \text{henry} - \text{ampere}]$

Relative permeability $\mu_r = \frac{\mu}{\mu_0}$

It has no units and dimensions.

Relation between permeability and susceptibility

When a magnetic material is placed in magnetic field $\vec{B}_0$ for magnetisation then total magnetic field in material $\vec{B}_m = \vec{B}_0 + \vec{B}_i$, where $\vec{B}_i$ = induced field.

$$\because \vec{B}_0 = \mu_0 \vec{H} ; \ \vec{B}_i = \mu_0 \vec{I}$$

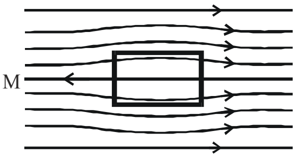
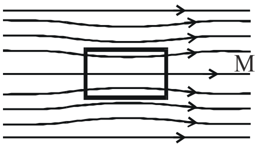
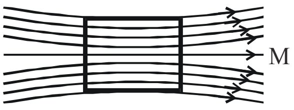
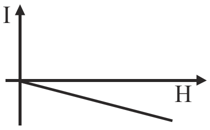
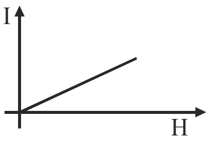
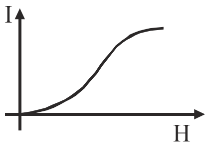
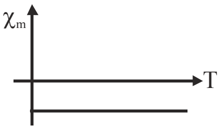
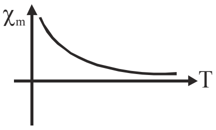
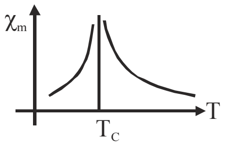
$$\therefore \vec{B}_m = \mu_0 \vec{H} + \mu_0 \vec{I} \Rightarrow \vec{B}_m = \mu_0(\vec{H} + \vec{I}) = \mu_0 \vec{H}\left(1 + \frac{I}{H}\right)$$

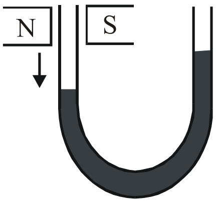
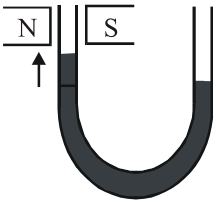
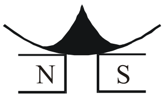
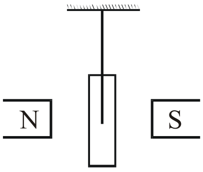
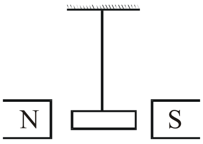
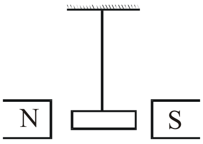
$$\Rightarrow \frac{B}{H} = \mu_0\left(1 + \frac{I}{H}\right) \Rightarrow \mu = \mu_0(1 + \chi_m) \Rightarrow \mu_r = 1 + \chi_m$$

for vacuum $\chi_m = 0, (\because \mu_r = 1)$
at STP for air $\chi_m = 0.04$
$(\because$ at S.T.P. for air $\mu_r = 1.04)$

CLASSIFICATION OF MAGNETIC MATERIALS

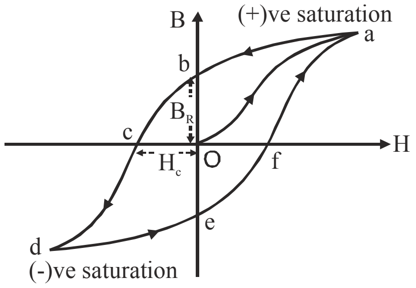
On the basis of magnetic properties of the materials [as magnetisation intensitily (I), Susceptibility (χ) and relative permeability (μ_r)] Faraday devide these materials in three classes –

PROPERTIES	**DIAMAGNETIC**	**PARAMAGNETIC**	**FERROMAGNETIC**
Cause of magnetism	Orbital motion of electrons	Spin motion of electrons	Formation of domains
Substance placed in uniform magnetic field.	Poor magnetisation in opposite direction. Here $B_m < B_0$  	Poor magnetisation in same direction. Here $B_m > B_0$  	Strong magnetisation in same direction. Here $B_m >>> B_0$ 
I – H curve	I → Small, negative, varies linearly with field  	I → Small, positive, varies linearly with field  	I → very large, positive & varies non–linearly with field 
χ_m – T curve	χ_m → small, negative & temperature independent $\chi_m \propto T^\circ$  	χ_m → small, positive & varies inversely with temp. $\chi_m \propto \frac{1}{T}$ (Curie law)  	χ_m → very large, positive & temp. dependent $\chi_m \propto \frac{1}{T - T_C}$ (Curie Weiss law) (for T > T_C) (T_C = Curie temperature)  $T_C(I_{iron})$=770°C or 1043K
μ_r	($\mu < \mu_0$) $1 > \mu_r > 0$	$2 > \mu_r > 1$ ($\mu > \mu_0$)	$\mu_r >>> 1$ ($\mu >>> \mu_0$)
Magnetic moment of single atom	Atoms donot have any permanent magnetic moment	Atoms have permanent megnetic moment which are randomly oriented. (i.e. in absence of external magnetic field the magnetic moment of whole material is zero)	Atoms have permanent megnetic moment which are organised in domains.

PROPERTIES	DIAMAGNETIC	PARAMAGNETIC	FERROMAGNETIC
Behaviour of substance in Nonuniform magnetic field	It moves from stronger to weaker magnetic field.  Weak Field  Strong Field  Level depressed in that limb	It moves with week force from weaker magnetic field to stronger magnetic field.  Weak Field  Strong Field  Level slightly rises	Strongly attract from weaker magnetic field to stronger magnetic field.  Weak Field  Strong Field
When rod of material is suspended between poles of magnet.	It becomes perpendicular to the direction of external magnetic field.  	If there is strong magnetic field in between the poles then rod becomes parallel to the magnetic field.  	Weak magnetic field between magnetic poles can made rod parallel to field direction. 
Magnetic moment of substance in presence of external magnetic field	Value $\vec{M}$ is very less and opposite to $\vec{H}$.	Value $\vec{M}$ is low but in direction of $\vec{H}$.	$\vec{M}$ is very high and in direction of $\vec{H}$.
Examples	Bi, Cu, Ag, Pb, H_2O, Hg, H_2, He, Ne, Au, Zn, Sb, NaCl, Diamond.(May be found in solid, liquid or gas).	Na, K, Mg, Mn, Sn, Pt, Al, O_2 (May be found in solid, liquid or gas.)	Fe,Co, Ni all their alloys, Fe_3O_4 Gd, Alnico, etc. (Normally found only in solids) (crystalline solids)

MAGNETIC HYSTERESIS

Only Ferromagnetic materials show magnetic hysteresis, when Ferromagnetic material is placed in external magnetic field for magnetisation then B increases with H non-linearly along Oa. If H is again bring to zero then it decreases along path ab. Due to lagging behind of B with H this curve is known as hysteresis curve. [Lagging of B behind H is called hysteresis]



Cause of hysteresis : By removing external magnetising field (H = 0), the magnetic moment of some domains remains aligned in the applied direction of previous magnetising field which results into a residual magnetism.

◎ **Residual magnetism (ob) = B_r** ≡ retentivity ≡ remanence
Retentivity of a specimen is a measure of the magnetic field remaining in the ferromagnetic specimen when the magnetising field is removed.

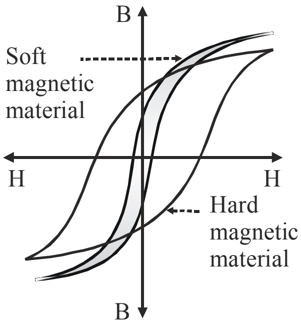
◎ **Coercivity (oc)** : Coercivity is an measure of magnetising field required to destroy the residual magnetism of the ferromagnetic specimen.

Ferromagnetic materials

Soft magnetic materials	**Hard magnetic materials**
Low retentivity, low coercivity and small hysteresis loss. suitable for making electromagnets, cores of transformers etc. Ex. Soft iron, (used in magnetic shielding)	High retentivity, high coercivity and large hysteresis loss suitable for permanent magnet Ex. Steel, Alnico

HYSTERESIS LOSS

(i) The area of hystersis loop for a ferromagnetic material is equal to the energy loss per cycle of magnetisation and demagnetisation per unit volume.



$$W_H = \oint B.dH = \mu_0 \oint I.dH$$

(ii) Its value is different for different materials.

(iii) The work done per cycle per unit volume of material is equal to the area of hysteresis loop.

$\therefore$ Total energy loss in material $W_H = V\,A\,n\,t$ joule $= \dfrac{V\,A\,n\,t}{J}$ calorie

i.e W_H = volume of material × area of hysteresis curve × frequency × time.

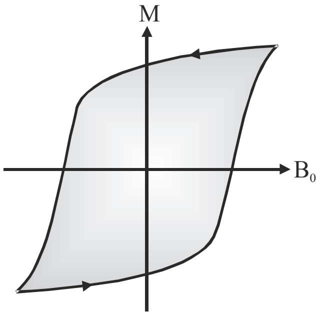


fig.(a)

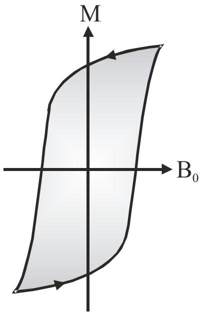


fig.(b)

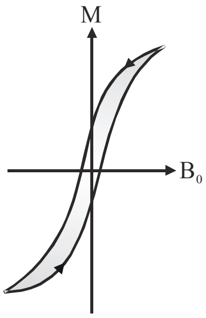


fig.(c)

The materials of both (a) and (b) remain strongly magnetized when B_0 is reduced to zero. The material of (a) is also hard to demagnetize, it would be good for permanent magnets.

The material of (b) magnetizes and demagnetizes more easily, it could be used as a computer memory material.

The material of (c) would be useful for transformers and other alternating-current devices where zero hysteresis would be optimal.

EXERCISE (S-1)

Biot savart law

1. Two long, straight wires, each carrying a current of 5 A, are placed along the X- and Y-axes respectively. The currents point along the positive directions of the axes. Find the magnetic field at the points (a) (1 m, 1 m), (b) (–1 m, 1 m), (c) (– 1 m, – 1 m) and (d) (1 m, – 1 m).

MG0001

2. A circular loop of radius 4.0 cm is placed in a horizontal plane and carries an electric current of 5.0 A in the clockwise direction as seen from above. Find the magnetic field

(a) At a point 3.0 cm above the centre of the loop.

(b) At a point 3.0 cm below the centre of the loop.

MG0002

3. Two concentric circular coils X and Y of radii 16 cm and 10 cm, respectively, lie in the same vertical plane containing the north to south direction. Coil X has 20 turns and carries a current of 16A ; coil Y has 25 turns and carries a current of 18 A. The sense of the current in X is anticlockwise, and clockwise in Y, for an observer looking at the coils facing west. Give the magnitude and direction of the net magnetic field due to the coils at their centre. ***(NCERT)***

MG0003

4. A current element $\Delta\vec{\ell} = \Delta x\,\hat{i} - \Delta y\,\hat{j}$ carries 10 A current. It is placed at origin. Calculate magnetic field at point 'P' which is at position vector $\vec{r} = (\hat{i} + \hat{j})$ m with respect to origin. (where $\Delta x = \Delta y = 1$mm)

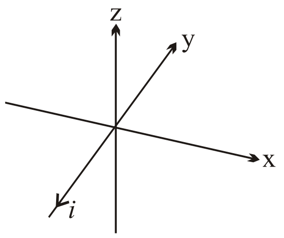
MG0004

5. A circular loop of radius r carries a current i. How should a long, straight wire carrying a current 4i be placed in the plane of the circle so that the magnetic field at the centre becomes zero ?

MG0005

6. A long straight wire carries a current of 10 A directed along the negative y-axis as shown in figure. A uniform magnetic field B_0 of magnitude 10^{-6} T is directed parallel to the x-axis. What is the resultant magnetic field at the following points?

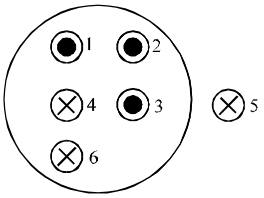
(a) x = 0, z = 2 m; (b) x = 2 m, z = 0; (c) x = 0, z = – 0.5 m



MG0006

Ampere's law

7. Six wires of current $I_1 = 1A$, $I_2 = 2A$, $I_3 = 3A$, $I_4 = 1A$, $I_5 = 5A$ and $I_6 = 4A$ cut the page perpendicularly at the points 1, 2, 3, 4, 5 and 6 respectively as shown in the figure. Find the value of the integral $\oint \vec{B}.d\vec{\ell}$ around the closed path.



MG0007

Motion of charged particle

8. A charged particle (charge q, mass m) has velocity v_0 at origin in +x direction. In space there is a uniform magnetic field B in – z direction. Find the y coordinate of particle when is crosses y axis.

MG0008

9. A beam of protons with a velocity 4×10^5 m/s enters a uniform magnetic field of 0.3 T at an angle of 60° to the magnetic field. Find the radius of the helical path taken by the proton beam. Also find the pitch of the helix (which is the distance travelled by a proton in the beam parallel to the magnetic field during one period of rotation).

MG0009

10. An electron emitted by a heated cathode and accelerated through a potential difference of 2.0 kV, enters a region with uniform magnetic field of 0.15 T. Determine the trajectory of the electron if the field (a) is transverse to its initial velocity, (b) makes an angle of 30°with the initial velocity.

(NCERT)

MG0010

Ampere force & torque

11. A straight horizontal conducting rod of length 0.45 m and mass 60g is suspended by two vertical wires at its ends. A current of 5.0 A is set up in the rod through the wires. ***(NCERT)***

(a) What magnetic field should be set up normal to the conductor in order that the tension in the wires is zero ?

(b) What will be the total tension in the wires if the direction of current is reversed keeping the magnetic field same as before ? [Ignore the mass of the wires). $g = 9.8\ ms^{-2}$.

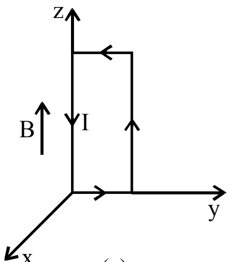
MG0011

12. (a) A circular coil of 30 turns and radius 8.0 cm carrying a current of 6.0 A is suspended vertically in a uniform horizontal magnetic field of magnitude 1.0 T. The field lines make an angle of 60° with the normal of the coil. Calculate the magnitude of the counter torque that must be applied to prevent the coil from turning. ***(NCERT)***

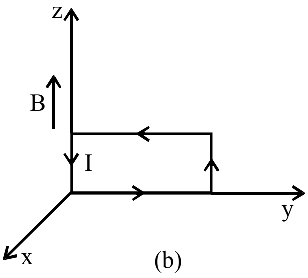
(b) Would your answer change, if the circular coil in (a) were replaced by a planar coil of some irregular shape that encloses the same area? (All other particulars are also unaltered.)

MG0012

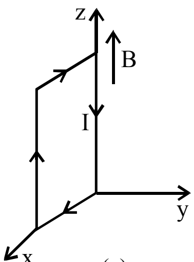
13. A uniform magnetic field of 3000 G is established along the positive z-direction. A rectangular loop of sides 10 cm and 5 cm carries a current of 12A. What is the torque on the loop in the different cases shown in figure. What is the force on each case ? Which case corresponds to stable equilibrium? ***(NCERT)***



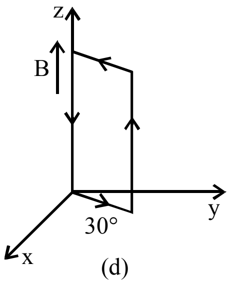
(a)



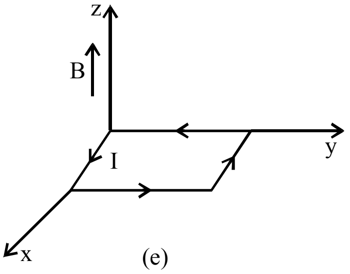
(b)



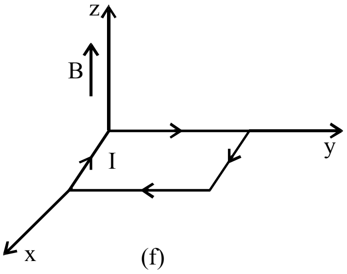
(c)



(d)



(e)



(f)

MG0013

14. A circular coil of 20 turns and radius 10 cm is placed in a uniform magnetic field of 0.10 T normal to the plane of the coil. If the current in the coil is 5.0 A, what is the ***(NCERT)***

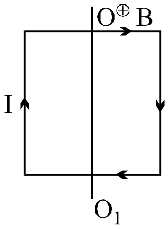
(a) total, torque on the coil,

(b) total force on the coil,

(c) average force on each electron in the coil due to the magnetic field ?

(The coil is made of copper wire of cross-sectional area 10^{-5} m^2, and the free electron density in copper is given to be about 10^{29} m^{-3}.) **MG0014**

15. A square current carrying loop made of thin wire and having a mass m = 10g can rotate without friction with respect to the vertical axis OO_1, passing through the centre of the loop at right angles to two opposite sides of the loop. The loop is placed in a homogeneous magnetic field with an induction $B = 10^{-1}$ T directed at right angles to the plane of the drawing. A current I = 2A is flowing in the loop. Find the period of small oscillations that the loop performs about its position of stable equilibrium.



MG0015

16. Two moving coil meters. M_1 and M_2 have the following particulars : ***(NCERT)***

$R_1 = 10\ \Omega$, $N_1 = 30$,

$A_1 = 3.6 \times 10^{-3}$ m^2. $B_1 = 0.25$ T

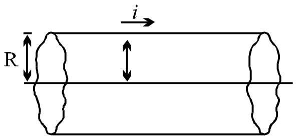
$R_2 = 14\ \Omega$, $N_2 = 42$ $A_2 = 1.8 \times 10^{-3}$ m^2, $B_2 = 0.50$ T

(The spring constants are identical for the two meters). Determine the ratio of (a) current sensitivity and (b) voltage sensitivity of M_2 and M_1.

MG0016

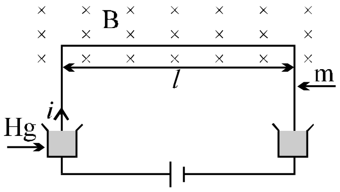
EXERCISE (S-2)

1. A cylindrical conductor of radius R carries a current along its length. The current density J, however, it is not uniform over the cross section of the conductor but is a function of the radius according to J = br, where b is a constant. Find an expression for the magnetic field B.
(a) at $r_1 < R$ & (b) at distance $r_2 > R$, measured from the axis



MG0017

2. A U-shaped wire of mass m and length l is immersed with its two ends in mercury (see figure). The wire is in a homogeneous field of magnetic induction **B**. If a charge, that is, a current pulse $q = \int i\,dt$, is sent through the wire, the wire will jump up.
Calculate, the height h that the wire reaches, the size of the charge or current pulse, assuming that the time of the current pulse is very small in comparision with the time of flight. Make use of the fact that impulse of force equals $\int F\,dt$,which equals mv. Evaluate q for B = 0.1 Wb/m^2, m = 10gm, ℓ = 20cm & h = 3 meters. [g = 10 m/s^2]



MG0018

3. A particle of mass 1×10^{-26} kg and charge $+1.6 \times 10^{-19}$ C travelling with a velocity 1.28×10^6 m/s in the +x direction enters a region in which a uniform electric field E and a uniform magnetic field of induction B are present such that $E_x = E_y = 0$, $E_z = -102.4$ kV/m and $B_x = B_z = 0$, $B_y = 8 \times 10^{-2}$ weber/m^2. The particle enters this region at the origin at time t = 0. Determine the location (x, y and z coordinates) of the particle at $t = 5 \times 10^{-6}$ s. If the electric field is switched off at this instant (with the magnetic field still present), what will be the position of the particle at $t = 7.45 \times 10^{-6}$ s ?

MG0019

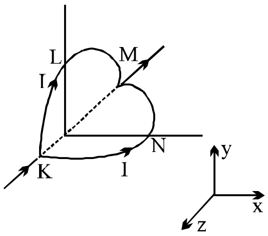
4. Two coils each of 100 turns are held such that one lies in the vertical plane with their centres coinciding. The radius of the vertical coil is 20 cm and that of the horizontal coil is 30 cm . How would you neutralize the magnetic field of the earth at their common centre ? What is the current to be passed through each coil ? Horizontal component of earth's magnetic induction = 3.49 x 10^{-5} T and angle of dip = 30º.

MG0020

5. A circular loop of radius R is bent along a diameter and given a shape as shown in the figure. One of the semicircles (KNM) lies in the $x-z$ plane and the other one (KLM) in the $y-z$ plane with their centers at the origin. Current I is flowing through each of the semicircles as shown in figure .

(i) A particle of charge q is released at the origin with a velocity $v = -v_0\hat{i}$. Find the instantaneous force f on the particle. Assume that space is gravity free.

(ii) If an external uniform magnetic field $B\hat{j}$ is applied, determine the forces F_1 and F_2 on the semicircles KLM and KNM due to this field and the net force F on the loop . **[JEE 2000]**

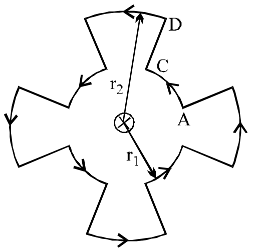


MG0021

6. A current of 10A flows around a closed path in a circuit which is in the horizontal plane as shown in the figure. The circuit consists of eight alternating arcs of radii r_1 = 0.08 m and r_2 = 0.12 m. Each arc subtends the same angle at the centre.

(a) Find the magnetic field produced by this circuit at the centre.

(b) An infinitely long straight wire carrying a current of 10A is passing through the centre of the above circuit vertically with the direction of the current being into the plane of the circuit. What is the force acting on the wire at the centre due to the current in the circuit? What is the force acting on the arc AC and the straight segment CD due to the current at the centre? **[JEE 2001]**



MG0022

7. A particle of charge +q and mass m moving under the influence of a uniform electric field $E\hat{i}$ and a magnetic field $B\hat{k}$ enters in I quadrant of a coordinate system at a point (0, a) with initial velocity $v\hat{i}$ and leaves the quadrant at a point (2a, 0) with velocity $-2v\hat{j}$. Find

(a) Magnitude of electric field

(b) Rate of work done by the electric field at point (0, a)

(c) Rate of work done by both the fields at (2a, 0).

MG0023

8. Electric charge q is uniformly distributed over a rod of length l. The rod is placed parallel to a long wire carrying a current i. The separation between the rod and the wire is a. Find the force needed to move the rod along its length with a uniform velocity v.

MG0024

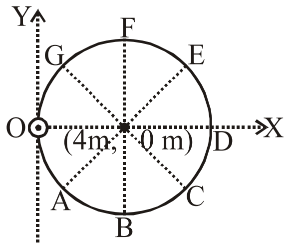
9. There exists a uniform magnetic and electric field of magnitude 1 T and 1 V/m respectively along positive y-axis. A charged particle of mass 1 kg and charge 1 C moving with velocity 1 m/sec along x-axis is at origin at t = 0. If the coordinates of particle at time **π seconds** is given as (X, Y, Z) in meter, then find the value of $(XY + YZ + ZX) \times \frac{5}{\pi^2}$.

MG0025

10. A proton beam passes without deviation through a region of space where there are uniform transverse mutually perpendicular electric and magnetic field with E and B. Then the beam strikes a grounded target. Find the force imparted by the beam on the target if the beam current is equal to I.

MG0026

11. An infinite uniform current carrying wire is kept along z-axis, carrrying current I_0 in the direction of the positive z-axis. OABCDEFG represents a circle (where all the points are equally spaced), whose centre at point (4m, 0 m) and radius 4 m as shown in the figure. If $\int_{DEF} \vec{B}.d\vec{\ell} = \frac{\mu_0 I_0}{k}$ in S.I. unit, then find the value of k.

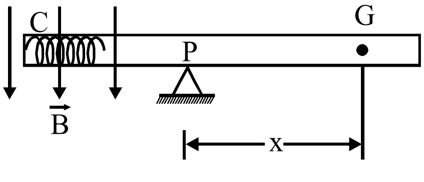


MG0027

12. A rod of length ℓ and total charge 'q' which is uniformly distributed is rotating with angular velocity ω about an axis passing through the centre of rod and perpendicular to rod. Find the magnitude of magnetic dipole moment (in Amp. m^2) of rod. (Take : q = 4C, ω = 6 rad/s and ℓ = 2m.)

MG0028

13. A uniform wooden bar of mass $\frac{\pi}{100}$ kg and radius of cross-section 10cm carries a light coil C of 100 turns. The bar is smoothly pivoted at P. If the coil carries a current 2A and subjected to an external magnetic field 10^{-2}T, the bar remains in equilibrium. Find the distance x (in cm) of the C.M. of the rod from the pivot.



MG0029

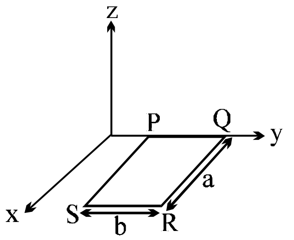
14. 3 infinitely long thin wires each carrying current *i* in the same direction , are in the x-y plane of a gravity free space . The central wire is along the y-axis while the other two are along x = ±d.

(i) Find the locus of the points for which the magnetic field B is zero .

(ii) If the central wire is displaced along the z-direction by a small amount & released, show that it will execute simple harmonic motion. If the linear density of the wires is λ , find the frequency of oscillation.

MG0030

15. A rectangular loop PQRS made from a uniform wire has length a, width b and mass m. It is free to rotate about the arm PQ, which remains hinged along a horizontal line taken as the y-axis (see figure). Take the vertically upward direction as the z-axis. A uniform magnetic field $\vec{B} = (3\hat{i} + 4\hat{k})B_0$ exists in the region. The loop is held in the x-y plane and a current I is passed through it. The loop is now released and is found to stay in the horizontal position in equilibrium. **[JEE 2002]**



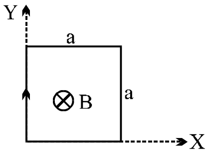
(a) What is the direction of the current I in PQ?

(b) Find the magnetic force on the arm RS.

(c) Find the expression for I in terms of B_0, a, b and m.

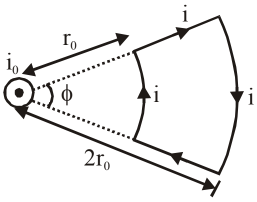
MG0031

16. A rectangular loop of wire is oriented with the left corner at the origin, one edge along X-axis and the other edge along Y-axis as shown in the figure. A magnetic field is into the page and has a magnitude that is given by $\beta = \alpha y$ where α is contant. Find the total magnetic force on the loop if it carries current i.



MG0032

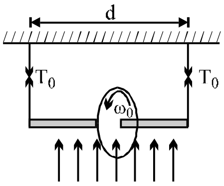
17. A closed loop carrying a current i is placed so that its plane is perpendicular to the long straight conductor which carries a current i_0 as shown in the figure. The torque acting on the current loop is $\alpha\mu_0$ N-m. Then find the value of α. (Given i = 2 A, i_0 = 2A, $r_0 = 2\pi$ m, $\phi = 60°$)



MG0033

18. A wheel of radius R having charge Q, uniformly distributed on the rim of the wheel is free to rotate about a light horizontal rod. The rod is suspended by light inextensible stringe and a magnetic field B is applied as shown in the figure. The initial tensions in the strings are T_0. If the breaking tension of the strings are $\frac{3T_0}{2}$, find the maximum angular velocity ω_0 with which the wheel can be rotate.

[JEE 2003]



MG0034

19. In a moving coil galvanometer, torque on the coil can be expressed as $\tau = ki$, where i is current through the wire and k is constant. The rectangular coil of the galvanometer having numbers of turns N, area A and moment of inertia I is placed in magnetic field B. Find

(a) k in terms of given parameters N, I, A and B.

(b) the torsional constant of the spring, if a current i_0 produces a deflection of $\pi/2$ in the coil in reaching equilibrium position.

(c) the maximum angle through which coil is deflected, id charge Q is passed through the coil almost instantaneously. (Ignore the damping in mechanical oscillations) **[JEE 2005]**

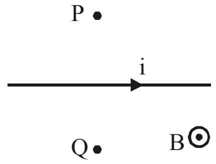
MG0035

EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

Biot savart law

1. A long, straight wire carrying a current of 1.0 A is placed horizontally in a uniform magnetic field $B = 1.0 \times 10^{-5}$ T pointing vertically upward (figure). The magnitude of the resultant magnetic field at the points P and Q, both situated at a distance of 2.0 cm from the wire in the same horizontal plane are respectively



(A) zero, 20 μT (B) 20 μT, zero (C) zero, zero (D) 20 μT, 20 μT

MG0036

2. A horizontal overhead powerline is at a height of 4m from the ground and carries a current of 100 A from east to west. The magnetic field directly below it one the ground is($\mu_0 = 4\pi \times 10^{-7}$TmA^{-1})

(A) 2.5×10^{-7} T southward (B) 5×10^{-6} T northward

(C) 5×10^{-6} T southward (D) 2.5×10^{-7} T northward

MG0037

3. Two parallel wires carry equal currents of 10 A along the same direction and are separated by a distance of 2.0 cm. Find the magnetic field at a point which is 2.0 cm away from each of these wires.

(A) 3.4×10^{-4} T in a direction parallel to the plane of the wires and perpendicular to the wires

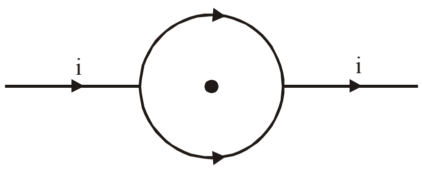
(B) 1.7×10^{-4} T in a direction parallel to the plane of the wires and parallel to the wires

(C) 1.7×10^{-4} T in a direction parallel to the plane of the wires and perpendicular to the wires

(D) 3.4×10^{-4} T in a direction parallel to the plane of the wires and parallel to the wires

MG0038

4. A conducting circular loop of radius a is connected to two long, straight wires. The straight wires carry a current i as shown in figure. Find the magnetic field B at the centre of the loop.



(A) zero (B) $\frac{\mu_0 I}{2a}$ (C) $\frac{\mu_0 I}{a}$ (D) $\frac{\mu_0 I}{a} + \frac{\mu_0 I}{2\pi a}$

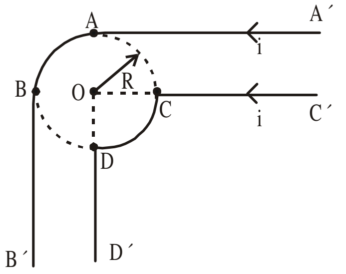
MG0039

5. A piece of wire carrying a current of 6.00 A is bent in the form of a circular arc of radius 10.0 cm, and it subtends an angle of 120° at the centre. Find the magnetic field B due to this piece of wire at the centre.

(A) zero (B) 1.26×10^{-5} T (C) 5×10^{-5} T (D) 7.2×10^{-5} T

MG0040

6. All straight wires are very long. Both AB and CD arc area of the same circle, both subtending right angles at the centre O. Then the magnetic field at O is



(A) $\frac{\mu_0 i}{4\pi R}$ (B) $\frac{\mu_0 i}{4\pi R}\sqrt{2}$ (C) $\frac{\mu_0 i}{2\pi R}$ (D) $\frac{\mu_0 i}{2\pi R}(\pi+1)$

MG0041

7. Two identical long conducting wires AOB and COD are placed at right angles to each other. The wire AOB carries an electric current I_1 and COD carries a current I_2. The magnetic field on a point lying at a distance d from O, in a direction perpendicular to the plane of the wires AOB and COD, will be given by-

(A) $\frac{\mu_0}{2\pi}\left(\frac{I_1+I_2}{d}\right)^{1/2}$ (B) $\frac{\mu_0}{2\pi d}\left(I_1^2+I_2^2\right)^{1/2}$ (C) $\frac{\mu_0}{2\pi d}(I_1+I_2)$ (D) $\frac{\mu_0}{2\pi d}\left(I_1^2+I_2^2\right)$

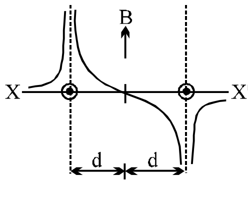
MG0042

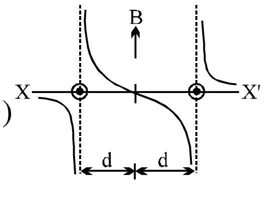
8. The magnetic field due to a current carrying circular loop of radius 3 cm at a point on the axis at a distance of 4 cm from the centre is 54 μT. What will be its value at the centre of the loop ?

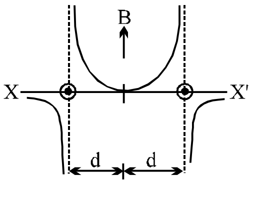
(A) 250 μT (B) 150 μT (C) 125 μT (D) 75 μT

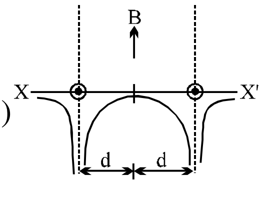
MG0043

9. Two long parallel wires are at a distance 2d apart. They carry steady equal currents flowing out of the plane of the paper, as shown. The variation of the magnetic field B along the XX' is given by

(A)


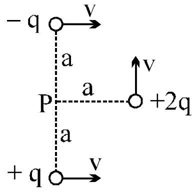
(B)


(C)


(D)


MG0044

10. A charge $+2q$ moves vertically upwards with speed v, a second charge $-q$ moves horizontally to the right with the same speed v, and a third charge $+q$ moves horizontally to the right with the same speed v. The point P is located a perpendicular distance a away from each charge as shown in the figure. The magnetic field at point P is



(A) Into the page with magnitude $\frac{\mu_0}{4\pi}\frac{2qv}{a^2}$

(B) Into the page with magnitude $\frac{\mu_0}{4\pi}\frac{4qv}{a^2}$

(C) Out of the page with magnitude $\frac{\mu_0}{4\pi}\frac{2qv}{a^2}$

(D) Out the page with magnitude $\frac{\mu_0}{4\pi}\frac{4qv}{a^2}$

MG0045

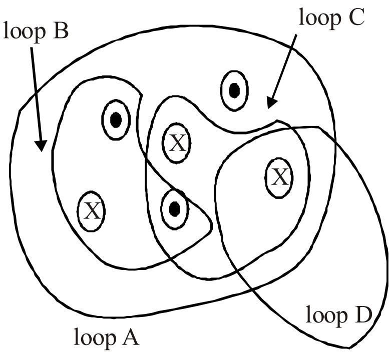
11. Considering magnetic field along the axis of a circular loop of radius 1 meter, at what distance from the centre of the loop the magnetic field is $\frac{1}{2\sqrt{2}}$ times of its value at the centre ?

(A) 1 m (B) 3 m (C) 5 m (D) 9 m

MG0046

Ampere's law

12. Consider six wires coming into or out of the page, all with the same current. Rank the line integral of the magnetic field (from most positive to most negative) taken counterclockwise around each loop shown.



(A) B > C > D > A (B) B > C = D > A (C) B > A > C = D (D) C > B = D > A

MG0047

13. **Statement-1 :** Ampere law can be used to find magnetic field due to finite length of a straight current carrying wire.

Statement-2 : The magnetic field due to finite length of a straight current carrying wire is symmetric about the wire.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

MG0048

14. A long, cylindrical wire of radius b carries a current i distributed uniformly over its cross-section. Find the magnitude of the magnetic field at a point inside the wire at a distance a from the axis.

(A) zero (B) $\frac{\mu_0 ib}{2\pi a^2}$ (C) $\frac{\mu_0 ia^2}{2\pi b^3}$ (D) $\frac{\mu_0 ia}{2\pi b^2}$

MG0049

15. A copper wire of diameter 1.6 mm carries a current of 20 A. Find the maximum magnitude of the magnetic field $\vec{B}$ due to this current.

(A) 5.0 mT (B) 10 mT (C) 15 mT (D) 15.5 mT

MG0050

16. A closely wound solenoid 80 cm long has 5 layers of windings of 400 turns each. The diameter of the solenoid is 1.8 cm. If the current carried is 8.0 A. estimate the magnitude of B inside the solenoid near its centre. ***(NCERT)***

(A) zero (B) $8\pi \times 10^{-3}$ T (C) $15\pi \times 10^{-3}$ T (D) $\pi \times 10^{-3}$ T

MG0051

17. A long solenoid has 200 turns per cm and carries a current i. The magnetic field at its centre is 6.28×10^{-2} weber/m^2. Another long solenoid has 100 turns per cm and it carries a current i/3. The value of the magnetic field at its centre is-

(A) 1.05×10^{-2} weber/m^2 (B) 1.05×10^{-5} weber/m^2

(C) 1.05×10^{-3} weber/m^2 (D) 1.05×10^{-4} weber/m^2

MG0052

18. A long straight wire of radius a carries a steady current i. The current is uniformly distributed across its cross-section. The ratio of the magnetic field at $\frac{a}{2}$ and 2a is-

(A) $\frac{1}{4}$ (B) 4 (C) 1 (D) $\frac{1}{2}$

MG0053

19. A current I flows along the length of an infinitely long, straight, thin walled pipe. Then-

(A) the magnetic field is zero only on the axis of the pipe

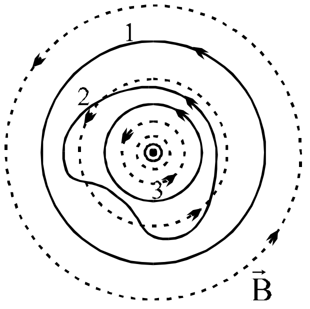
(B) the magnetic field is different at different points inside the pipe

(C) the magnetic field at any point inside the pipe is zero

(D) the magnetic field at all points inside the pipe is the same, but not zero

MG0054

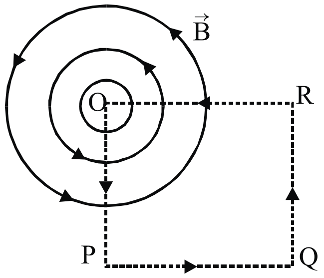
20. Consider the three closed loops drawn using solid line in the magnetic field (magnetic field lines are drawn using dotted line) of an infinite current-carrying wire normal to the plane of paper as shown. Rank the line integral of the magnetic field along each path in order of increasing magnitude



(A) 1 > 2 > 3 (B) 1 = 3 > 2 (C) 1 = 2 = 3 (D) 3 > 2 > 1

MG0055

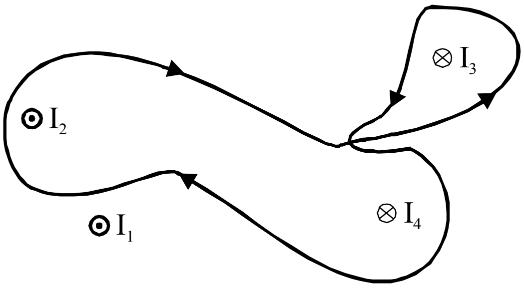
21. A current carrying wire (current = i) perpendicular to the plane of the paper produces a magnetic field, as shown in the figure. A square of side a is drawn with one of its vertices on the centre of the wire. The integral $\int \vec{B}.d\vec{r}$ along $OPQRO$ has the value



(A) $+\mu_0 i$ (B) $\frac{\mu_0 i}{8}$ (C) $\frac{\mu_0 i}{4}$ (D) $\frac{\mu_0 i}{2}$

MG0056

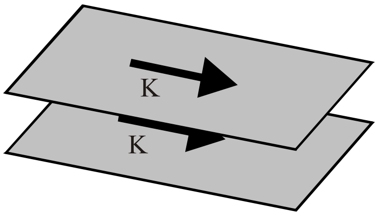
22. Four wires carrying current $I_1 = 2A$, $I_2 = 4A$, $I_3 = 6A$ and $I_4 = 8A$ respectively cut the page perpendicularly as shown in figure. The value of $\int \vec{B}.d\vec{\ell}$ for the loop shown would be :-



(A) $+2\ \mu_0$-wb/m (B) $-2\ \mu_0$-wb/m (C) $+ 10\ \mu_0$-wb/m (D) $- 10\ \mu_0$-wb/m

MG0057

23. What are the directions of the magnetic field between and outside a pair of two parallel large sheets carrying currents in the same directions, as illustrated in Figure (from the side shown)?



(A) towards us between the plates and away from us above and below the plates.
(B) toward us above the plates and away from us below plates and zero between plates.
(C) towards us above and below the plates and zero between the plates
(D) towards us between the plates and zero above and below the plates.

MG0058

Motion of charge particle

24. A charged particle enters a non-uniform uni-directional field such that initial velocity is parallel to magnetic field, then the radius of curvature of its path is (in standard notation):

(A) mV/qB (B) 0 (C) ∞ (D) qB/mV

MG0059

25. A charge particle moves in a uniform magnetic field such that initial velocity is perpendicular to the magnetic field . No other force acts on the particle .

(A) the motion is uniform rectilinear
(B) the motion can be non uniform circular motion
(C) the motion can be uniform circular motion
(D) the motion must be uniform circular motion.

MG0060

26. A tightly-wound, long solenoid carries a current of 2.00 A. An electron is found to execute a uniform circular motion inside the solenoid with a frequency of 1.00×10^8 rev/s. Find the number of turns per metre in the solenoid.

(A) 500 Turns/m (B) 1020 Turns/m (C) 1232 Turns/m (D) 1420 Turns/m

MG0061

27. In a region, steady and uniform electric and magnetic fields are present. These two fields are parallel to each other. A charged particle is released from rest in this region. The path of the particle will be a-

(A) helix (B) straight line (C) ellipse (D) circle

MG0062

28. An electron is moving along positive x-axis. A uniform electric field exists towards negative y-axis. What should be the direction of magnetic field of suitable magnitude so that net force of electron is zero

(A) positive z- axis (B) negative z-axis (C) positive y-axis (D) negative y-axis

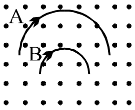
MG0063

29. An electron having kinetic energy T is moving in a circular orbit of radius R perpendicular to a uniform magnetic induction $\vec{B}$. If kinetic energy is doubled and magnetic induction tripled, the radius will become

(A) $\frac{3R}{2}$ (B) $\sqrt{\frac{3}{2}}\,R$ (C) $\sqrt{\frac{2}{9}}\,R$ (D) $\sqrt{\frac{4}{3}}\,R$

MG0064

30. Two particles A and B of masses m_A and m_B respectively and having the same charge are moving in a plane. A uniform magnetic field exists perpendicular to this plane. The speeds of the particles are v_A and v_B respectively and the trajectories are as shown in the figure. Then**[JEE, 2001 (Scr)]**



(A) $m_Av_A < m_Bv_B$ (B) $m_Av_A > m_Bv_B$

(C) $m_A < m_B$ and $v_A < v_B$ (D) $m_A = m_B$ and $v_A = v_B$

MG0065

31. A charged particle moves in a magnetic field $\vec{B} = 10\hat{i}$ with initial velocity $\vec{u} = 5\hat{i} + 4\hat{j}$. The path of the particle will be

(A) straight line (B) circle (C) helical (D) none

MG0066

32. An electron makes 3×10^5 revolutions per second in a circle of radius 0.5 angstrom. Find the magnetic field B at the centre of the circle.

(A) 6×10^{-10} T (B) 12×10^{-10} T (C) 18×10^{-10} T (D) 24×10^{-10} T

MG0067

33. Electrons moving with different speeds enter a uniform magnetic field in a direction perpendicular to the field. They will move along circular paths.

(A) of same radius

(B) with larger radii for the faster electrons

(C) with smaller radii for the faster electrons

(D) either (B) or (C) depending on the magnitude of the magnetic field

MG0068

34. In the previous question, time periods of rotation will be :

(A) same for all electrons

(B) greater for the faster electrons

(C) smaller for the faster electrons

(D) either (B) or (C) depending on the magnitude of the magnetic field

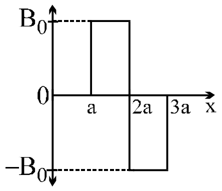
MG0069

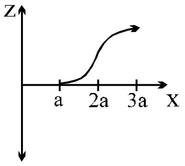
35. A particle of mass m and charge q moves with a constant velocity v along the positive x direction. It enters a region containing a uniform magnetic field B directed along the negative z direction, extending from x = a to x = b. The minimum value of v required so that the particle can just enter the region x > b is :- **JEE 2002 (screening)]**

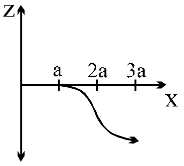
(A) q b B/m (B) q(b – a) B/m (C) q a B/m (D) q(b + a) B/2m

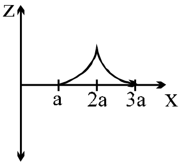
MG0070

36. A magnetic field $\vec{B} = B_0\hat{j}$ exists in the region a < x < 2a and $\vec{B} = -B_0\hat{j}$, in the region 2a < x < 3a, where B_0 is a positive constant. A positive point charge moving with a velocity $\vec{v} = v_0\hat{i}$, where v_0 is a positive constant, enters the magnetic field at x = a. The trajectory of the charge in this region can be like,



(A)


(B)


(C)


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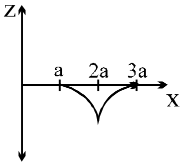
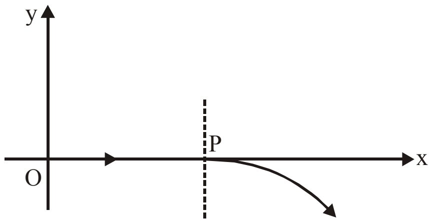


MG0071

37. For a positively charged particle moving in a x – y plane initially along the x–axis, there is a sudden change in it path due to the presence of electric and/or magnetic field beyond P. The curved path is shown in the x – y plane and is found to be non–circular. Which one of the following combinations is possible? **[JEE 2004]**



(A) $\vec{E} = 0; \vec{B} = b\hat{j} + c\hat{k}$

(B) $\vec{E} = a\hat{i}; \vec{B} = c\hat{k} + a\hat{i}$

(C) $\vec{E} = 0; \vec{B} = c\hat{j} + b\hat{k}$

(D) $\vec{E} = a\hat{i}; \vec{B} = c\hat{k} + b\hat{j}$

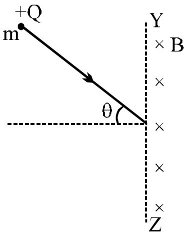
MG0072

38. A particle having charge of 1 C, mass 1 kg and speed 1 m/s enters a uniform magnetic field, having magnetic induction of 1 T, at an angle $\theta = 30°$ between velocity vector and magnetic induction. The pitch of its helical path is (in meters)

(A) $\frac{\sqrt{3}\pi}{2}$ (B) $\sqrt{3}\pi$ (C) $\frac{\pi}{2}$ (D) π

MG0073

39. A particle with charge +Q and mass m enters a magnetic field of magnitude B, existing only to the right of the boundary YZ. The direction of the motion of the particle is perpendicular to the direction of B. Let $T = 2\pi\frac{m}{QB}$. The time spent by the particle in the field will be



(A) $T\theta$ (B) $2T\theta$ (C) $T\left(\frac{\pi+2\theta}{2\pi}\right)$ (D) $T\left(\frac{\pi-2\theta}{2\pi}\right)$

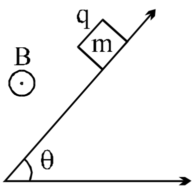
MG0074

40. In the previous question, if the particle has –Q charge, the time spend by the particle in the field will be :-

(A) $T\theta$ (B) $2T\theta$ (C) $T\left(\frac{\pi+2\theta}{2\pi}\right)$ (D) $T\left(\frac{\pi-2\theta}{2\pi}\right)$

MG0075

41. A block of mass m & charge q is released on a long smooth inclined plane magnetic field B is constant, uniform, horizontal and parallel to surface as shown. Find the time from start when block loses contact with the surface.



(A) $\frac{m\cos\theta}{qB}$ (B) $\frac{m\,\mathrm{cosec}\,\theta}{qB}$ (C) $\frac{m\cot\theta}{qB}$ (D) none

MG0076

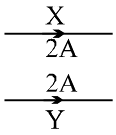
42. In a cyclotron, a charged particle

(A) undergoes acceleration all the time.
(B) speeds up between the dees because of the magnetic field.
(C) speeds up in a dee.
(D) slows down within a dee and speeds up between dees.

MG0077

Ampere force & torque

43. In given figure, X and Y are two long straight parallel conductors each carrying a current of 2 A. The force on each conductor is F newtons. When the current in each is changed to 1 A and reversed in direction, the force on each is now



(A) F/4 and unchanged in direction (B) F/2 and reversed in direction
(C) F/2 and unchanged in direction (D) F/4 and reversed in direction

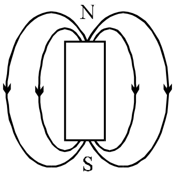
MG0078

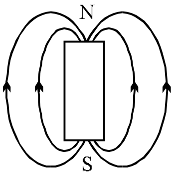
44. Two long and parallel straight wires A and B carrying currents of 8.0 A and 5.0 A in the same direction are separated by a distance of 4.0 cm. Estimate the force on a 10 cm section of wire A. ***(NCERT)***

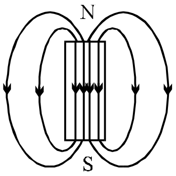
(A) 2×10^{-5} N ; attractive force normal to B towards A.
(B) 2×10^{-5} N ; attractive force normal to A towards B.
(C) 5×10^{-5} N ; attractive force normal to A towards B.
(D) 5×10^{-5} N ; attractive force normal to B towards A.

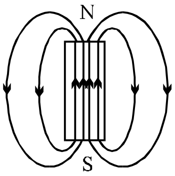
MG0079

45. The magnetic field lines due to a bar magnet are correctly shown in:- **[JEE 2002 (screening)]**

(A) 

(B) 

(C) 

(D) 

MG0080

46. A wire of mass 100 g carrying a current of 2A towards increasing x is in the form of $y = x^2$ $(-2\text{m} \le x \le +2\text{m})$. This wire is placed in a magnetic field $B = -0.02\hat{k}$ Tesla & gravity free region. The acceleration of the wire (in m/s^2) is :-

(A) $-1.6\hat{j}$ (B) $-3.2\hat{j}$ (C) $1.6\hat{j}$ (D) $2.4\hat{j}$

MG0081

47. A circular loop of radius R carries a current I. Another circular loop of radius r(<<R) carries a current i and is placed at the centre of the larger loop. The planes of the two circles are at right angle to each other. Find the torque acting on the smaller loop.

(A) zero (B) $\frac{\mu_0 \pi i I r^2}{4R}$ (C) $\frac{\mu_0 \pi i I r^2}{2R}$ (D) $\frac{\mu_0 \pi i I r^2}{R}$

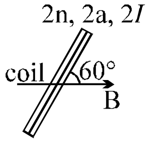
MG0082

48. A particle of charge q and mass m moves in a circular orbit of radius r with angular speed ω. The ratio of the magnitude of its magnetic moment to that of its angular momentum depends on.

(A) ω and q (B) ω, q and m (C) q and m (D) ω and m

MG0083

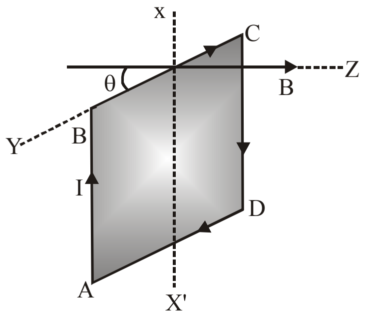
49. A rectangular coil PQ has 2n turns, an area 2a and carries a current 2*I*, (refer figure). The plane of the coil is at 60° to a horizontal uniform magnetic field of flux density B. The torque on the coil due to magnetic force is :-



(A) Bna*I* sin60° (B) 8Bna*I* cos60° (C) 4na*I* Bsin60° (D) none

MG0084

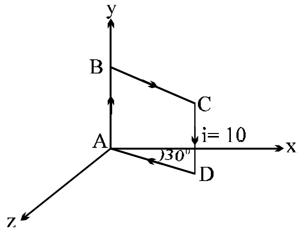
50. The square loop ABCD, carrying a current I, is placed in a uniform magnetic field B, as shown. The loop can rotate about the axis XX'. The plane of the loop makes an angle θ (θ < 90°) with the direction of B. Through what angle will the loop rotate by itself before the torque on it becomes zero–



(A) θ (B) 90° – θ (C) 90° + θ (D) 180° – θ

MG0085

51. Figure shows a square current carrying loop ABCD of side 10 cm and current i = 10A. The magnetic moment $\vec{M}$ of the loop is



(A) (0.05) $\left(\hat{i}-\sqrt{3}\hat{k}\right)A-m^2$

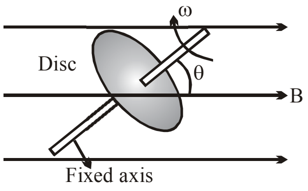
(B) (0.05) $\left(\hat{j}+\hat{k}\right)A-m^2$

(C) (0.05) $\left(\sqrt{3}\hat{i}+\hat{k}\right)A-m^2$

(D) $\left(\hat{i}+\hat{k}\right)A-m^2$

MG0086

52. A disc of radius r and carrying positive charge q is rotating with an angular speed ω in a uniform magnetic field B about a fixed axis as shown in figure, such that angle made by axis of disc with magnetic field is θ. Torque applied by axis on the disc is

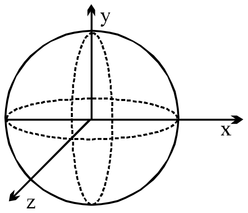


(A) $\frac{q\omega r^2 B\sin\theta}{2}$, clockwise (B) $\frac{q\omega r^2 B\sin\theta}{4}$, anticlockwise

(C) $\frac{q\omega r^2 B\sin\theta}{2}$, anticlockwise (D) $\frac{q\omega r^2 B\sin\theta}{4}$, clockwise

MG0087

53. Three rings, each having equal radius R, are placed mutually perpendicular to each other and each having its centre at the origin of co-ordinate system. If current I is flowing through each ring then the magnitude of the magnetic field at the common centre is



(A) $\sqrt{3}\frac{\mu_0 I}{2R}$ (B) zero (C) $\left(\sqrt{2}-1\right)\frac{\mu_0 I}{2R}$ (D) $\left(\sqrt{3}-\sqrt{2}\right)\frac{\mu_0 I}{2R}$

MG0088

54. Two mutually perpendicular long conducting wire carrying currents I_1 and I_2 lie in one plane. Locus of the point at which the magnetic induction is zero, is a

(A) circle with centre as the point of intersection of the wire.

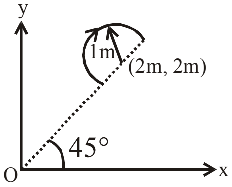
(B) parabola with vertex as the point of intersection of the wire

(C) straight line passing through the point of intersection of the wire

(D) rectangular hyperbola

MG0089

55. A uniform magnetic field $\vec{B}=\left(3\hat{i}+4\hat{j}+\hat{k}\right)$ Tesla exist in a region of space. A semicircular wire of radius 1 m carrying a current of 1A having its centre at (2,2,0) m is placed on the X-Y plane as shown. The force on the semicircular wire will be



(A) $\frac{1}{\sqrt{2}}\left(\hat{i}-\hat{j}+\hat{k}\right)$N (B) $\sqrt{2}\left(\hat{i}-\hat{j}+\hat{k}\right)$N (C) $\frac{1}{\sqrt{2}}\left(\hat{i}+\hat{j}-\hat{k}\right)$N (D) $\sqrt{2}\left(\hat{i}+\hat{j}-\hat{k}\right)$N

MG0090

56. A very long wire carrying current I is fixed along x-axis. Another parallel finite wire carrying a current in the opposite direction is kept at a distance d above the wire in xy plane. The second wire is free to move parallel to itself. The options available for its small displacements are in

(i) +ve x direction (ii) +ve y direction (iii) +ve z direction

Taking gravity in negative y direction, the nature of equilibrium of second wire is

(A) stable for movement in x direction, unstable for movement in y direction, neutral for movement in z direction

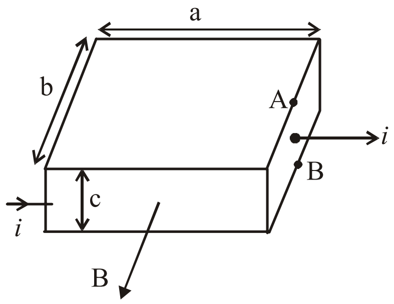
(B) stable for movement in y direction, unstable for movement in z direction, neutral for movement in x direction

(C) stable for movement in z direction, unstable for movement in y direction, neutral for movement in x direction

(D) stable for movement in y direction, unstable for movement in x direction, neutral for movement in z direction

MG0091

57. A current flows through a rectangular conductor in the presence of uniform magnetic field B pointing out of the page as shown. Then the potential difference $V_B - V_A$ is equal to
(Assume charge carries in the conductor to be positively charged moving with speed v)



(A) Bvc (B) –Bvc (C) Bvb (D) –Bvb

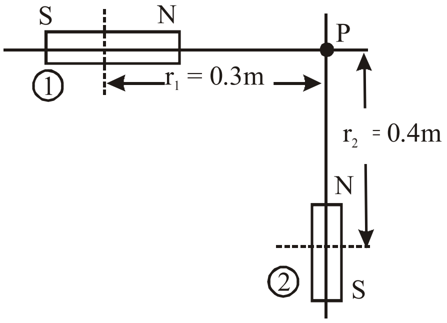
MG0092

58. A moving coil galvanometer has 150 equal divisions. Its current sensitivity is 10 divisions per milli ampere and voltage sensitivity is 2 divisions per millivolt. In order that each division reads 1 V, the resistance in Ohm's needed to be connected in series with the coil will be-**[AIEEE-2005]**
(A) 10^3 (B) 10^5 (C) 99995 (D) 9995

MG0093

SUPPLEMENT ON MAGNETIC PROPERTIES OF MATTER

59. Two short magnets of magnetic moment 2Am^2 and 5Am^2 respectively are placed along two lines drawn at right angle to each other on the sheet of paper as shown in the figure. What is the magnetic field at the point of intersection of their axis:



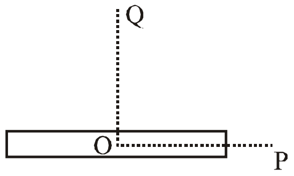
(A) 2.15×10^{-5} T (B) 215×10^{-5} T (C) 2.15×10^{-4} T (D) 21.5×10^{-5} T

MG0094

60. A steel wire of length L has a magnetic moment M. It is then bent into a semi-circular arc; the new magnetic moment will be :
(A) M (B) $2M/\pi$ (C) M/L (D) $M \times L$

MG0095

61. When magnetic field at P and Q is same then OP/OQ = ?



(A) $\sqrt[3]{2}$ (B) $\frac{1}{\sqrt[3]{2}}$ (C) $2\sqrt{2}$ (D) $\frac{3}{2\sqrt{2}}$

MG0097

62. Two identical poles each of pole strength 4×10^{-3} amp-m placed at the two corners of an equilateral triangle of side 20 cm. The magnetic field at the third corner is :-

(A) $\sqrt{3}\times10^{-8}$ N/amp-m (B) $\frac{\sqrt{3}}{4}\times 10^{-8}$N/amp-m

(C) zero (D) 10^{-8} N/amp-m

MG0099

63. The magnetic field at a point on the axis of a dipole is 20 Wb/m^2 Northwards. What will be the magnetic induction at the point if the dipole be rotated anticlockwise by 90° :-

(A) 10 Wb/m^2 Eastwards (B) 10 Web/m^2 Southwards

(C) 10 Wb/m^2 Northwards (D) 20 Wb/m^2 Southwards

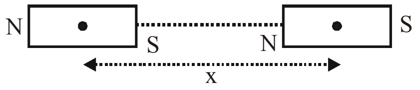
MG0100

64. A bar magnet of magnetic moment 2A-m^2 free to rotate about a vertical axis passing through its centre. The magnet is released from rest from east-west position. Then the kinetic energy of the magnet as it takes North-South position is (horizontal component of the earth's field is 25μT and angle of declination is zero :-

(A) 25 μJ (B) 50 μJ (C) 100 μJ (D) 125 μJ

MG0101

65. The mid points of two small magnetic dipoles of length d in end-on positions, are separated by a distance x, (x >> d). The force between them is proportional to x^{-n} where n is :-



(A) 3 (B) 4 (C) 2 (D) 1

MG0102

66. The angle of dip at a place is 30° and the intensity of the vertical component of the earth's magnetic field V = 6×10^{-5}T. The total intensity of the earth's magnetic field at this place is :

(A) 7×10^{-5} T (B) 6×10^{-5} T (C) 5×10^{-5} T (D) 12×10^{-5} T

MG0104

67. The total intensity of the earth's magnetic field at the magnetic equator is 5 units. What is its value at a magnetic latitude of 37 degree?

(A) $\sqrt{73}$ units (B) $\sqrt{52}$ units (C) 4 units (D) 3 units

MG0105

68. A magnet is suspended in the magnetic meridian with an untwisted wire. The upper end of wire is rotated through 180° to deflect the magnet by 30° from magnetic meridian. When the magnet is replaced by another magnet, the upper end of wire is rotated through 270° to deflect the magnet 30° from magnetic meridian. The ratio of magnetic moments of magnets is :-

(A) 1 : 5 (B) 1 : 8 (C) 5 : 8 (D) 8 : 5

MG0106

69. The imagined magnet makes an angle of 11.3° with earth's axis. On the earth, there are two points where magnetic equator and geographic equator meet let the angle of dip and angle of declination at these point be θ & ϕ respectively :-

(A) $\theta = \phi = 0°$ (B) $\theta = 11.3°,\ \phi = 0°$ (C) $\theta = 0°,\ \phi = 11.3°$ (D) $\theta = \phi = 11.3°$

MG0107

70. A specimen of iron of permeability 8×10^{-3} weber/ amp × metre is placed in a magnetic field of intensity 160 amp/metre. Then magnetic field in this iron is :

(A) 20×10^{3} Wb/m^2 (B) 1.28 Wb/m^2 (C) 5×10^{-5} Wb/m^2 (D) 0.8Wb/m^2

MG0108

71. Curie temperature is the temperature above which :

(A) a paramagnetic material becomes diamagnetic

(B) a ferromagnetic material becomes diamagnetic

(C) a paramagnetic material becomes ferromagnetic

(D) a ferromagnetic material becomes paramagnetic

MG0109

72. If a solution of ferromagnetic material is poured into a U-tube and one arm of this is placed between the poles of a strong magnet with the meniscus in line with the field, then the level of the solution will:

(A) Rise (B) Fall

(C) Rise till the liquid comes out (D) Remain unchanged

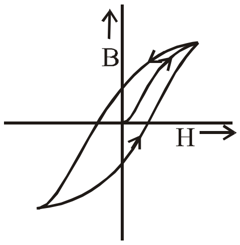
MG0110

73. A sample of paramagnetic salt contains 2.0×10^{24} atomic dipoles each of dipole moment 1.5×10^{-23}J T^{-1}. The sample is placed under a homogeneous magnetic field of 0.84T, and cooled to a temperature of 4.2K. The degree of magnetic saturation achieved is equal to 15%. What is the total dipole moment (approximate) of the sample for a magnetic field of 0.98 T and a temperature of 2.8K? (Assume Curie's law).

(A) 7.9 JT^{-1} (B) 52.5 J T^{-1} (C) 30 J T^{-1} (D) 4.6J T^{-1}

MG0111

74. The B-H curve for a certain specimen is schematically shown by the given diagram. Which one of the following is the correct magnetic nature of the specimen ?



(A) Diamagnetic and not ferromagnetic or paramagnetic

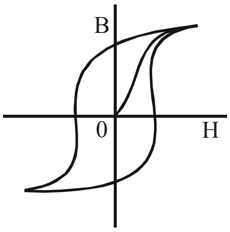
(B) Ferromagnetic and not diamagnetic or paramagnetic

(C) Paramagnetic and not diamagnetic or ferromagnetic

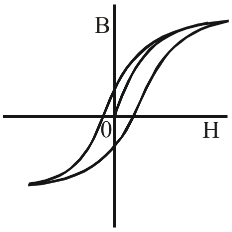
(D) Applicable to all the three types of magnetism mentioned above

MG0112

75. Two types of steel are characterized by the hysteresis loops shown in figure (a) and (b). The loops are obtained in the processes of magnetization and demagnetization of the steels. Which of the two types is best suited for using as the core of a transformer and which, for using as a permanent magnet ?



(a)



(b)

(A) Both are best suited for transformer

(B) Both are best suited for permanent magnet

(C) A is best suited for permanent magnet & B is best suited for core of transformer

(D) A is best suited for core of transformer & B is best suited for permanent magnet

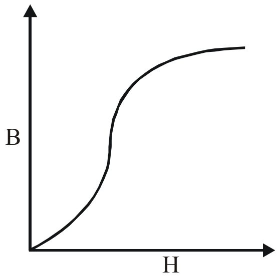
MG0113

76. A material satisfied the relation $\mu_0(H + M) = 0$, where H and M are magnetic intensity and magnetisation respectively, then material is :-

(A) nonmagnetic (B) paramagnetic (C) ferrormagnetic (D) diamagnetic

MG0114

77. The figure represent B-H curve for commercial iron. As H is increased, permeability :-



(A) increases and becomes constant
(B) increases, reaches a maximum and then decreases
(C) decreases continuously till it becomes very small
(D) decreases reaches a minimum and then increases

MG0115

78. **Statement-1 :** The sensitivity of a moving coil galvanometer is increased by placing a suitable magnetic material as a core inside the coil.

and

Statement-2 : Soft iron has a high magnetic permeability and cannot be easily magnetized or demagnetized.

(A) Statement-1 is True, Statement-2 is True ; statement-2 is a correct explanation for statement-1
(B) Statement-1 is True, Statement-2 is True ; statement-2 is NOT a correct explanation for statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

MG0116

79. Horizontal magnetic field of earth at Mumbai is 1.5×10^{-4} T in North direction. A small bar magnet of magnetic moment 10 Am^2 is kept on a horizontal table such that its North pole points due North. What is the magnetic field at 10 cm from centre of magnet at a point on its axis due North of it :-

(A) 3.5×10^{-4} T (B) 5×10^{-5} T (C) 2.15×10^{-3} T (D) 1.15×10^{-3} T

MG0117

80. A compass needle kept on a horizontal table points in direction 30° west of north. A line is marked showing this direction and compass is now kept in vertical plane containing this line. The needle points in direction 30° below horizontal. If the plane of compass is still vertical but contains true north south direction at what angle to horizontal will needle point :-

(A) $\tan^{-1}\left(\frac{3}{2}\right)$ (B) $\tan^{-1}\left(\frac{2}{3}\right)$ (C) $\tan^{-1}(2)$ (D) $\tan^{-1}\left(\frac{1}{2}\right)$

MG0118

81. The magnetic lines of force inside a bar magnet: **[AIEEE - 2003]**

(A) are from north-pole to south-pole of the magnet
(B) do not exist
(C) depend upon the area of cross-section of the bar magnet
(D) are from south-pole to north-pole of the magnet

MG0121

82. The length of a magnet is large compared to its width and breadth. The time period of its oscillation in a vibration magnetometer is 2s. The magnet is cut along its length into three equal parts and three parts are then placed on each other with their like poles together. The time period of this combination will be- **[AIEEE - 2004]**

(A) 2s (B) 2/3 s (C) $2\sqrt{3}$ s (D) $2/\sqrt{3}$ s

MG0122

83. The materials suitable for making electromagnets should have- **[AIEEE - 2004]**

(A) high retentivity and high coercivity

(B) low retentivity and low coercivity

(C) high retentivity and low coercivity

(D) low retentivity and high coercivity

MG0123

MULTIPLE CORRECT TYPE QUESTIONS

84. Consider three quantities x = E/B, y = $\sqrt{1/\mu_0\varepsilon_0}$ and z = $\frac{l}{CR}$. Here, l is the length of a wire, C is a capacitance and R is a resistance. All other symbols have standard meanings.

(A) x, y have the same dimensions

(B) y, z have the same dimensions

(C) z, x have the same dimensions

(D) none of the three pairs have the same dimensions.

MG0124

85. The magnetic lines of force due to a straight current carrying wire will be:

(A) circular for finite length of wire (B) circular for semi-infinite wire

(C) circular for infinite wire (D) Parabolic for finite wire

MG0125

86. A current I flows along the length of an infinitely long, straight, solid pipe. Then correct statement(s)-

(A) The magnetic field is zero only on the axis of the pipe

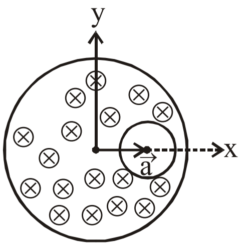
(B) The magnetic field is different at different points inside the pipe

(C) The magnetic field is maximum on surface

(D) The magnetic field at all points inside the pipe is the same, but not zero

MG0126

87. Figure shows cross-section view of a infinite cylindrical wire with a cylinderical cavity, current density is uniform $\vec{J} = -j_0\hat{k}$ as shown in figure.



(A) Field inside cavity is uniform

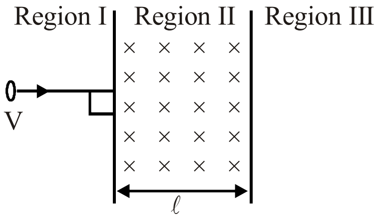
(B) Field inside cavity is along $\vec{a}$

(C) Field inside cavity is perpendicular to $\vec{a}$

(D) If an electron is projected with velocity $v_0\hat{j}$ it will move undeviated before colliding with cavity wall

MG0127

88. A particle of mass m and charge q, moving with velocity V enters Region II normal to the boundary as shown in the figure. Region II has a uniform magnetic field B perpendicular to the plane of the paper. The length of the Region II is ℓ. Choose the correct choice(s)



(A) The particle enters Region III only if its velocity $V > \frac{q\ell B}{m}$

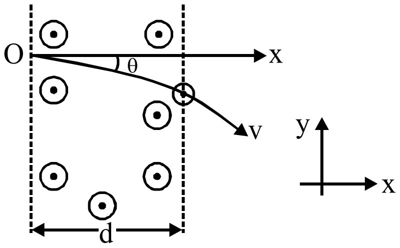
(B) The particle enters Region III only if its velocity $V < \frac{q\ell B}{m}$

(C) Path length of the particle in Region II is maximum when velocity $V = \frac{q\ell B}{m}$

(D) Time spent in Region II is same for any velocity V as long as the particle returns to Region I

MG0128

89. A particle of charge q is projected with a momentum $\vec{p} = p\hat{i}$ in the given region of magnetic field $\vec{B} = B\hat{k}$. It emerges from the magnetic field after deviating through an angle θ = 30°.



(A) The value of $\vec{p}$ is 2qBd

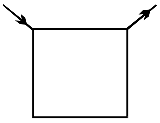
(B) The value of $\vec{p}$ is qBd

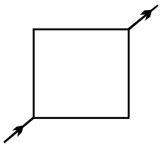
(C) Maximum change in momentum takes place for d $\geq \frac{P}{qB}$

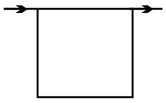
(D) Maximum change in momentum takes place for d $\geq \frac{P}{2qB}$

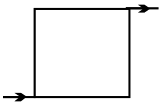
MG0129

90. Current flows through uniform, square frames as shown. In which case is the magnetic field at the centre of the frame not zero. The wire current carrying shown outside coil and infinitely long.?

(A) 

(B) 

(C) 

(D) 

MG0130

91. A long straight wire carries a current along the x-axis. Consider the points A(0, 1, 0), B(0, 1, 1), C(1, 0, 1) and D(1, 1, 1). Which of the following pairs of points will have magnetic fields of the same magnitude?

(A) A and B (B) A and C (C) B and C (D) B and D

MG0131

92. Two identical charged particles enter a uniform magnetic field with same speed but at angles 30° and 60° with field. Let a, b and c be the ratio of their time periods, radii and pitches of the helical paths than

(A) abc = 1 (B) abc > 1 (C) abc < 1 (D) a = bc

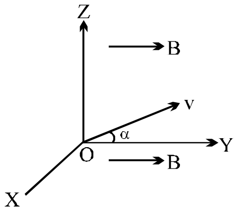
MG0132

93. An electron is moving along the positive X-axis. You want to apply a magnetic field for a short time so that the electron may reverse its direction and move parallel to the negative X–axis. This can be done by applying the magnetic field along :-

(A) Y-axis (B) Z-axis (C) Y-axis only (D) Z-axis only

MG0133

94. In a region of space, a uniform magnetic field B exists in the y-direction. A proton is fired from the origin, with its initial velocity v making a small angle α with the y-direction in the yz plane. In the subsequent motion of the proton,



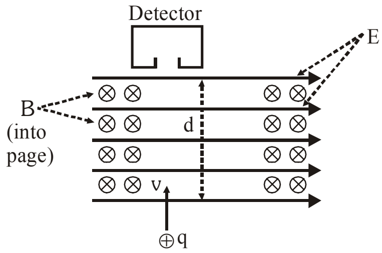
(A) its x-coordinate can never be positive
(B) its x- and z-coordinates cannot both be zero at the same time
(C) its z-coordinate can never be negative
(D) its y-coordinate will be proportional to the square of its time elapsed

MG0134

COMPREHENSION TYPE QUESTIONS

Paragraph for Questions no. 95 to 98

A velocity filter uses the properties of electric and magnetic fields to select charged particles that are moving with a specific velocity. Charged particles with varying speeds are directed into the filter as shown in figure. The filter consists of an electric field E and a magnetic field B, each of constant magnitude, directed perpendicular to each other as shown. The particles that move straight through the filter with their direction unaltered by the fields have the specific filter speed, v_0. Those with speeds to v_0 may experience sufficiently little deflection that they also enter the detector.



The charged particle will experience a force due to the electric field given by the relationship $\vec{F} = q\vec{E}$, where q is the charge of the particle and $\vec{E}$ is the electric field. The moving particle will also experience a force due to the magnetic field. This force acts to oppose the force due to the electric field. The strength of the force due to the magnetic field is given by the relationship $\vec{F} = q(\vec{v} \times \vec{B})$, where q is the charge of the particle, $\vec{v}$ is the speed of the particle, and $\vec{B}$ is the magnetic field strength. When the forces due to the two fields are equal and opposite, the net force on the particle will be zero, and the particle will pass through the filter with its path unaltered. The electric and magnetic field strengths can be adjusted to choose the specific velocity to be filtered. The effects of gravity can be neglected.

95. The electric and magnetic fields in the filter of figure are adjusted to detect particles with positive charge q of a certain speed, v_0. Which of the following expressions is equal to this speed ?

(A) $B/(q^2E)$ (B) $E/(q^2B)$ (C) B/E (D) E/B

MG0135

96. Which of the following is true about the velocity filter shown in figure ?

(A) It would not work with negatively charged particles

(B) The wider the detector entrance, the more narrow the range of speed detected

(C) The greater the distance d, the more narrow the range of speeds detected

(D) The detector may not detect a charged particle with the desired filter speed if its charge is too high

MG0136

97. Which of the following statements is true regarding a charged particle that is moving through the filter at a speed that is less than the filter speed ?

(A) It experiences a greater force due to the magnetic field than due to the electric field

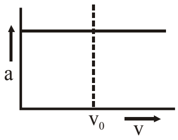
(B) It experiences a greater force due to the electric field than due to the magnetic field

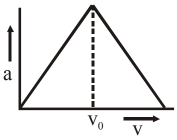
(C) It experiences equal force due to both fields but greater acceleration due to the electric field

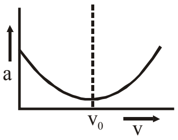
(D) It experiences equal force due to both fields but greater acceleration due to the magnetic field

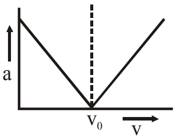
MG0137

98. Particles of identical mass and charge are sent through the filter at varying speeds, and the magnitude of acceleration of each particle is recorded as it first begins to be deflected. If the filter is set to detect particles of speed v_0, which one of the following is correct graph between acceleration and velocity of particle:

(A) 

(B) 

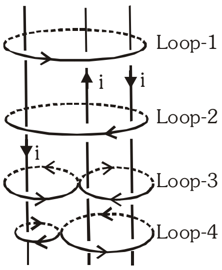
(C) 

(D) 

MG0138

MATRIX MATCH TYPE QUESTION

99. Three wires are carrying same constant current i in different directions. Four loops enclosing the wires in different manners are shown. The direction of $d\vec{\ell}$ is shown in figure.



	Column-I		**Column-II**
(A)	Along closed loop–1	(p)	$\oint \vec{B}.d\vec{\ell} = \mu_0 i$
(B)	Along closed loop–2	(q)	$\oint \vec{B}.d\vec{\ell} = -\mu_0 i$
(C)	Along closed loop–3	(r)	$\oint \vec{B}.d\vec{\ell} = 0$
(D)	Along closed loop–4	(s)	Net work done by the magnetic force to move a unit charge along the loop is zero

MG0139

100.

	Column-I (Magnetic moment of)		**Column-II**
(A)	a uniformly charged ring rotating uniformly about its axis	(p)	$\frac{q\omega r^2}{5}$
(B)	a charged particle rotating uniformly about a point	(q)	$\frac{q\omega r^2}{4}$
(C)	a uniformly charged disk rotating uniformly about its axis	(r)	$\frac{q\omega r^2}{3}$
(D)	a uniformly charged spherical shell rotating uniformly about one of its diameter	(s)	$\frac{q\omega r^2}{2}$
(E)	a uniformly charged sphere rotating	(t)	$q\omega r^2$

MG0140

EXERCISE (O-2)

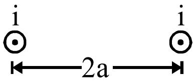
SINGLE CORRECT TYPE QUESTIONS

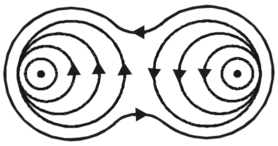
1. A ring like metallic conductor of resistance R and radius a, caries a constant current I. The ratio of the angular momentum L of the conduction electrons (about the axis of the ring) and the magnetic field B at the centre of the ring satisfy [where e and m represent the magnitudes of the electronic charge and mass]

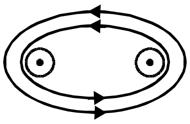
(A) $\frac{B}{L} \propto \frac{e^2}{m}$ (B) $\frac{B}{L} \propto e.m$ (C) $\frac{B}{L} \propto \frac{e}{m}$ (D) $\frac{B}{L} \propto \frac{m}{e}$

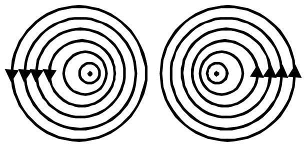
MG0141

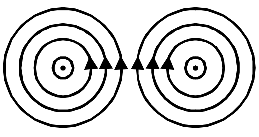
2. Which of the following field patterns is correct for two long straight equal parallel current carrying wires ?



(A) 

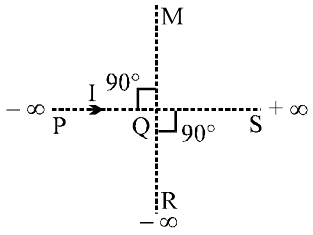
(B) 

(C) 

(D) 

MG0142

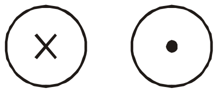
3. An infinitely long conductor PQR is bent to form a right angle as shown. A current I flows through PQR. The magnetic field due to this current at the point M is H_1. Now, another infinitely long straight conductor QS is connected at Q so that the current in PQ remaining unchanged. The magnetic field at M is now H_2. The ratio H_1/H_2 is given by :-



(A) 1/2 (B) 1 (C) 2/3 (D) 2

MG0143

4. Equal antiparallel currents are directed in two parallel wires so that one is out of the page and the other is into the page as shown. Compare the magnitude of the magnetic field B_2 at any arbitrary point equidistant from the wires to the magnitude of the field B_1 at that point from one wire alone :-



(A) $B_2 > B_1$ for all equidistant points

(B) $B_2 < B_1$ for all equidistant points

(C) $B_2 > B_1$ for closer equidistant points only

(D) $B_2 < B_1$ for closer equidistant points only

MG0144

5. A long straight wire along the z-axis carries a current I in the negative z direction. The magnetic vector field $\vec{B}$ at a point having coordinates (x, y) in the z = 0 plane is :- **[JEE 2002 (screening)]**

(A) $\frac{\mu_0 I}{2\pi}\frac{(y\hat{i}-x\hat{j})}{(x^2+y^2)}$

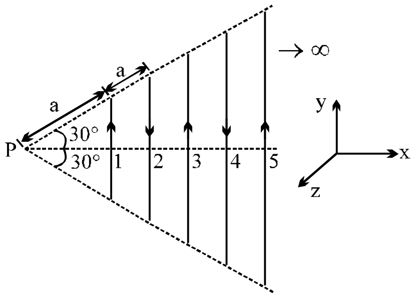
(B) $\frac{\mu_0 I}{2\pi}\frac{(x\hat{i}+y\hat{j})}{(x^2+y^2)}$

(C) $\frac{\mu_0 I}{2\pi}\frac{(x\hat{j}-y\hat{i})}{(x^2+y^2)}$

(D) $\frac{\mu_0 I}{2\pi}\frac{(x\hat{i}-y\hat{j})}{(x^2+y^2)}$

MG0145

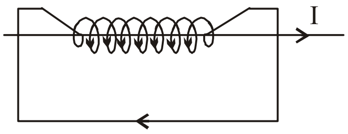
6. Infinite number of straight wires each carrying current I are equally placed as shown in the figure. Adjacent wires have current in opposite direction. Net magnetic field at point P is :-



(A) $\frac{\mu_0 I}{4\pi}\frac{\ln 2}{\sqrt{3}a}\hat{k}$ (B) $\frac{\mu_0 I}{4\pi}\frac{\ln 4}{\sqrt{3}a}\hat{k}$ (C) $\frac{\mu_0 I}{4\pi}\frac{\ln 4}{\sqrt{3}a}(-\hat{k})$ (D) Zero

MG0146

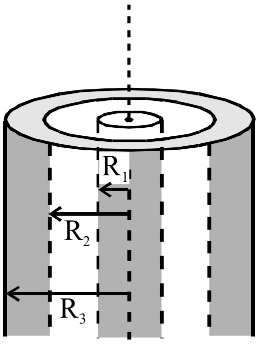
7. In the diagram shown, a wire carries current I. What is the value of the $\oint \vec{B} \cdot d\vec{s}$ (as in Ampere's law) on the helical loop shown in the figure? The integration in done in the sense shown. The loop has N turns and part of helical loop on which arrows are drawn is outside the plane of paper.



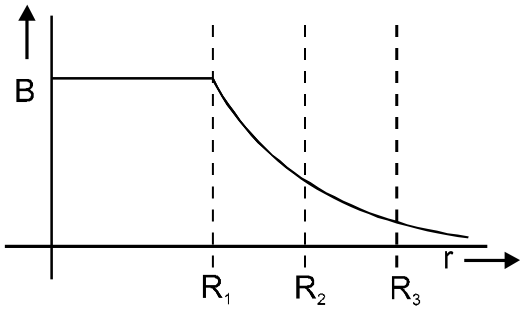
(A) $-\mu_0(NI)$ (B) $\mu_0(I)$ (C) $\mu_0(NI)$ (D) Zero

MG0147

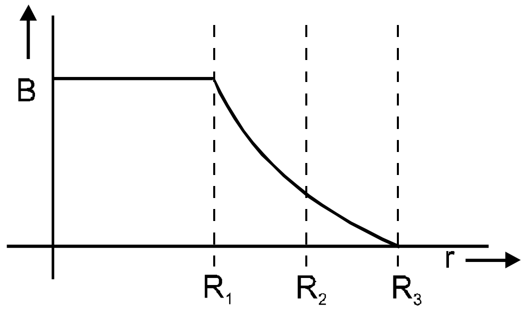
8. A coaxial cable is made up of two conductors. The inner conductor is solid and is of radius R_1 & the outer conductor is hollow of inner radius R_2 and outer radius R_3. The space between the conductors is filled with air. The inner and outer conductors are carrying currents of equal magnitudes and in opposite directions. Then the variation of magnetic field with distance from the axis is best plotted as:



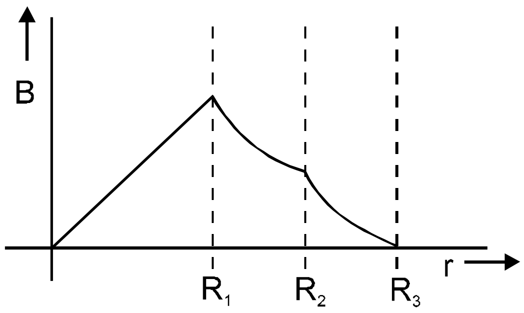
(A)



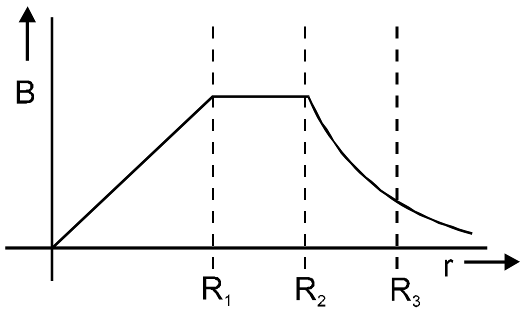
(B)



(C)

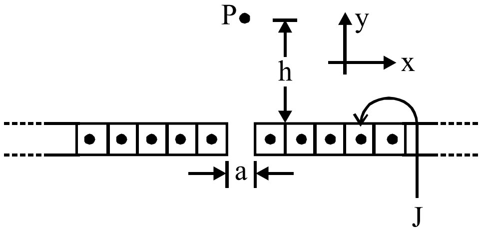


(D)



MG0148

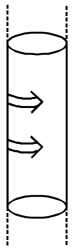
9. A small segment of length a is cut along z-axis from a infinite sheet having a surface current density J (current per unit width). The magnetic field at P is :-



(A) $2\mu_0 J\left(1-\frac{h}{a\pi}\right)\hat{i}$ (B) $\frac{\mu_0 Jh}{2a\pi}\hat{i}$ (C) $\frac{\mu_0 J}{2}\left(\frac{a}{h\pi}-1\right)\hat{i}$ (D) $-\frac{\mu_0 j}{2}\left(\frac{h}{a\pi}-1\right)\hat{i}$

MG0149

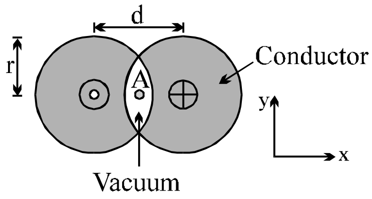
10. A hollow cylinder having infinite length and carrying uniform current per unit length λ along the circumference as shown. Magnetic field inside the cylinder is :-



(A) $\frac{\mu_0\lambda}{2}$ (B) $\mu_0\lambda$ (C) $2\mu_0\lambda$ (D) none

MG0150

11. Two long conductors are arranged as shown above to form overlapping cylinders, each of radius r, whose centers are separated by a distance d. Current of density J flows into the plane of the page along the shaded part of one conductor and an equal current flows out of the plane of the page along the shaded portion of the other, as shown. What are the magnitude and direction of the magnetic field at point A?



(A) $(\mu_0/2\pi)\pi dJ$, in the +y-direction (B) $(\mu_0/2\pi)d^2/r$, in the +y-direction

(C) $(\mu_0/2\pi)4d^2J/r$, in the –y-direction (D) $(\mu_0/2\pi)Jr^2/d$, in the –y-direction

MG0151

12. An ionized gas contains both positive and negative ions. If it is subjected simultaneously to an electric field along the +x direction and a magnetic field along the +z direction, then **[JEE 2000 (Scr)]**

(A) positive ions deflect towards +y direction and negative ions towards −y direction

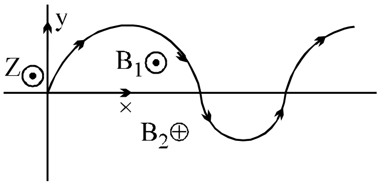
(B) all ions deflect towards +y direction.

(C) all ions deflect towards −y direction

(D) positive ions deflect towards −y direction and negative ions towards +y direction.

MG0152

13. At t = 0 a charge q is at the origin and moving in the y-direction with velocity $\vec{v} = v\hat{j}$. The charge moves in a magnetic field that is for y > 0 out of page and given by $B_1\hat{z}$ and for y < 0 into the page and given $-B_2\hat{z}$. The charge's subsequent trajectory is shown in the sketch. From this information, we can deduce that



(A) q > 0 and | B_1 | < | B_2 | (B) q < 0 and | B_1 | < | B_2 |

(C) q > 0 and | B_1 | > | B_2 | (D) q < 0 and | B_1 | > | B_2 |

MG0153

14. A particle of specific charge (charge/mass) α starts moving from the origin under the action of an electric field $\vec{E} = E_0\hat{i}$ and magnetic field $\vec{B} = B_0\hat{k}$. Its velocity at $(x_0, y_0, 0)$ is $(4\hat{i} - 3\hat{j})$. The value of x_0 is:

(A) $\frac{13}{2}\frac{\alpha E_0}{B_0}$ (B) $\frac{16\alpha B_0}{E_0}$ (C) $\frac{25}{2\alpha E_0}$ (D) $\frac{5\alpha}{2B_0}$

MG0154

15. A particle of specific charge (q/m) is projected from the origin of coordinates with initial velocity [u**i** – v**j**]. Uniform electric and magnetic fields exist in the region along the +y direction, of magnitude E and B. The particle will definitely return to the origin once if

(A) [vB/2πE] is an integer (B) $(u^2 + v^2)^{1/2}$ [B/πE] is an integer

(C) [vB/πE] in an integer (D) [uB/πE] is an integer

MG0155

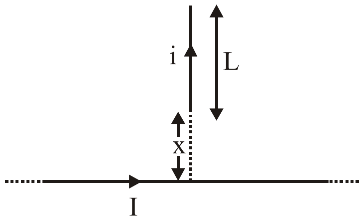
16. An electron moving with a velocity $\vec{V}_1 = 2\hat{i}$ m/s at a point in a magnetic field experiences a force $\vec{F}_1 = -2\hat{j}N$. If the electron is moving with a velocity $\vec{V}_2 = 2\hat{j}$ m/s at the same point, it experiences a force $\vec{F}_2 = +2\hat{i}\,N$. The force the electron would experience if it were moving with a velocity $\vec{V}_3 = 2\hat{k}$ m/s at the same point is

(A) zero (B) $2\hat{k}N$

(C) $-2\hat{k}N$ (D) information is insufficient

MG0156

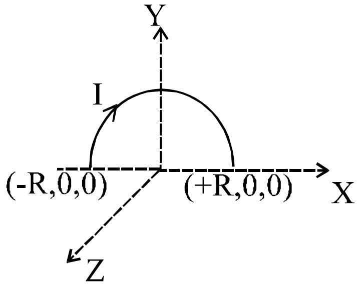
17. The magnetic force between wires as shown in figure is :-



(A) $\frac{\mu_0 iI^2}{2\pi}\ell n\left(\frac{x+\ell}{2x}\right)$ (B) $\frac{\mu_0 iI^2}{2\pi}\ell n\left(\frac{2x+\ell}{2x}\right)$ (C) $\frac{\mu_0 iI}{2\pi}\ell n\left(\frac{x+\ell}{x}\right)$ (D) None of these

MG0157

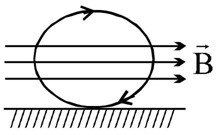
18. A semi circular current carrying wire having radius R is placed in x-y plane with its centre at origin 'O'. There is non-uniform magnetic field $\vec{B}=\frac{B_o x}{2R}\hat{k}$ (here B_o is +ve constant) is existing in the region. The magnetic force acting on semi circular wire will be along :-



(A) – x-axis (B) + y-axis (C) – y-axis (D) + x-axis

MG0158

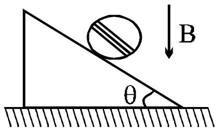
19. A conducting ring of mass 2 kg and radius 0.5 m is placed on a smooth horizontal plane. The ring carries a current i = 4A. A horizontal magnetic field B = 10T is switched on at time t = 0 as shown in figure. The initial angular acceleration of the ring will be :-



(A) 40 π rad/s^2 (B) 20 π rad/s^2 (C) 5 π rad/s^2 (D) 15 π rad/s^2

MG0159

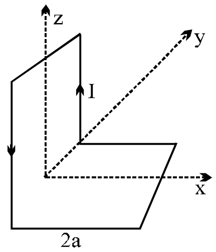
20. In the figure shown a coil of single turn is wound on a sphere of radius R and mass m. The plane of the coil is parallel to the plane and lies in the equatorial plane of the sphere. Current in the coil is i. The value of B if the sphere is in equilibrium is :-



(A) $\frac{mg\cos\theta}{\pi iR}$ (B) $\frac{mg}{\pi iR}$ (C) $\frac{mg\tan\theta}{\pi iR}$ (D) $\frac{mg\sin\theta}{\pi iR}$

MG0160

21. A non-planar loop of conducting wire carrying a current I is placed as shown in the figure. Each of the straight sections of the loop is of length 2a. The magnetic field due to this loop at the point P (a, 0, a) points in the direction :- **[JEE, 2001 (Scr)]**



(A) $\frac{1}{\sqrt{2}}(-\hat{j}+\hat{k})$ (B) $\frac{1}{\sqrt{3}}(-\hat{j}+\hat{k}+\hat{i})$ (C) $\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$ (D) $\frac{1}{\sqrt{2}}(\hat{i}+\hat{k})$

MG0161

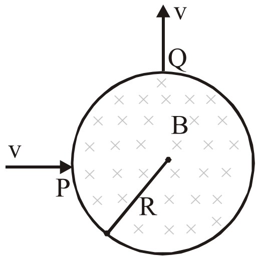
22. Two protons move parallel to each other, keeping distance r between them, both moving with same velocity $\vec{V}$. Then the ratio of the electric and magnetic force of interaction between them is :-

(A) c^2/V^2 (B) $2c^2/V^2$ (C) $c^2/2V^2$ (D) None

MG0162

MULTIPLE CORRECT TYPE QUESTIONS

23. A particle of charge 'q' and mass 'm' enters normally (at point P) in a region of magnetic field with speed v. It comes out normally from Q after time T as shown in figure. The magnetic field B is present only in the region of radius R and is uniform. Initial and final velocities are along radial direction and they are perpendicular to each other. For this to happen, which of the following expression(s) is/are **CORRECT** :-



(A) $B = \frac{mv}{qR}$ (B) $T = \frac{\pi R}{2v}$ (C) $T = \frac{\pi m}{2qB}$ (D) None of these

MG0163

24. A particle of mass m and charge q moving with velocity $\vec{v}$ enters a region of uniform magnetic field of induction $\vec{B}$. Then

(A) its path in the region of the field is always circular

(B) its path in the region of the field is circular if $\vec{v}.\vec{B}=0$

(C) its path in the region of the field is a straight line if $\vec{v}\times\vec{B}=0$

(D) distance travelled by the particle in time T does not depend on the angle between $\vec{v}$ and $\vec{B}$

MG0164

MATRIX MATCH TYPE QUESTION

25. A charged particle with some initial velocity is projected in a region where uniform electric and/ or magnetic fields are present. In Column-I information about the existence of electric and/or magnetic field and direction of initial velocity of charged particle are given, while in column-II the possible paths of charged particle is mentioned. Match the entries of Column I with the entries of Column-II.

Column–I

(A) $\vec{E} = 0, \vec{B} \neq 0$ and initial velocity is at an unknown angle with $\vec{B}$

(B) $\vec{E} \neq 0, \vec{B} = 0$ and initial velocity is at an unknown angle with $\vec{E}$

(C) $\vec{E} \neq 0, \vec{B} \neq 0, \vec{E} \parallel \vec{B}$ and initial velocity is perpendicular to $\vec{E}$

(D) $\vec{E} \neq 0, B \neq 0, \vec{E}$ is perpendicular to $\vec{B}$ and initial velocity is perpendicular to both $\vec{E}$ and $\vec{B}$

Column–II

(P) Straight line

(Q) Parabola

(R) Circular

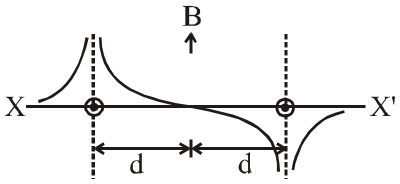
(S) Helical path with nonuniform pitch

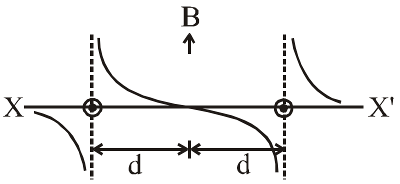
(T) Helical path with uniform pitch

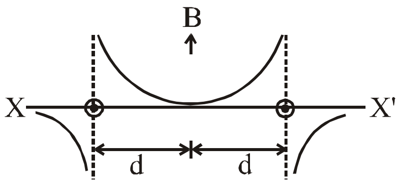
MG0165

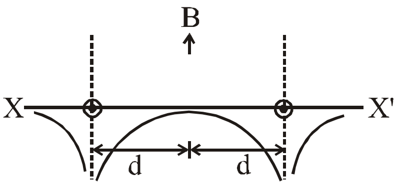
EXERCISE-JM

1. Two long parallel wires are at a distance 2d apart. They carry steady equal currents flowing out of the plane of the paper as shown. The variation of the magnetic field B along the line XX' is given by:- **[AIEEE - 2010]**

(1)


(2)


(3)


(4)


MG0166

2. A current I flows in an infinity long wire with cross section in the from of a semicircular ring of radius R. the magnitude of the magnetic induction along its axis is :- **[AIEEE - 2011]**

(1) $\frac{\mu_0 I}{2\pi R}$ (2) $\frac{\mu_0 I}{4\pi R}$ (3) $\frac{\mu_0 I}{\pi^2 R}$ (4) $\frac{\mu_0 I}{2\pi^2 R}$

MG0167

3. An electric charge + q moves with velocity $\vec{V}=3\hat{i}+4\hat{j}+\hat{k}$, in an electromagnetic field given by $\vec{E}=3\hat{i}+\hat{j}+2\hat{k}$ $\vec{B}=\hat{i}+\hat{j}-3\hat{k}$. The y-component of the force experienced by + q is :-

[AIEEE - 2010]

(1) 2 q (2) 11 q (3) 5 q (4) 3q

MG0168

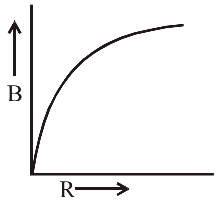
4. A thin circular disk of radius R is uniformly charged with density $\sigma > 0$ per unit area. The disk rotates about its axis with a uniform angular speed ω. The magnetic moment of the disk is :-

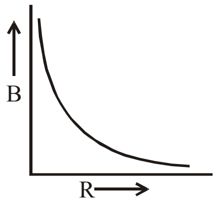
[AIEEE - 2011]

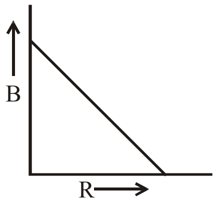
(1) $2\pi R^4\sigma\omega$ (2) $\pi R^4\sigma\omega$ (3) $\frac{\pi R^4}{2}\sigma\omega$ (4) $\frac{\pi R^4}{4}\sigma\omega$

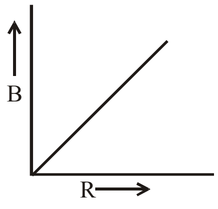
MG0169

5. A charge Q is uniformly distributed over the surface of non-conducting disc of radius R. The disc rotates about an axis perpendicular to its plane and passing through its centre with an angular velocity ω. As a result of this rotation a magnetic field of induction B is obtained at the centre of the disc. If we keep both the amount of charge placed on the disc and its angular velocity to be constant and vary the radius of the disc then the variation of the magnetic induction at the centre of the disc will be represented by the figure :- **[AIEEE-2012]**

(1)


(2)


(3)


(4)


MG0170

6. Two short bar magnets of length 1 cm each have magnetic moments 1.20 Am^2 and 1.00 Am^2 respectively. They are placed on a horizontal table parallel to each other with their N poles pointing towards the South. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic induction at the midpoint O of the line joining their centres is close to :- **[JEE(Mains) - 2013]**

(Horizontal component of earth's magnetic induction is 3.6×10^{-5} Wb/m^2)

(1) 3.6×10^{-5} Wb/m^2
(2) 2.56×10^{-4} Wb/m^2
(3) 3.50×10^{-4} Wb/m^2
(4) 5.80×10^{-4} Wb/m^2

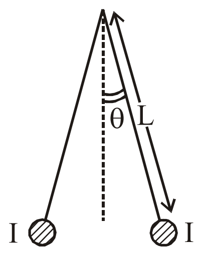
MG0171

7. The coercivity of a small magnet where the ferromagnet gets demagnetized is 3×10^3 $A\ m^{-1}$. The current required to be passed in a solenoid of length 10 cm and number of turns 100, so that the magnet gets demagnetized when inside the solenoid, is : **[JEE(Mains) - 2014]**

(1) 3A
(2) 6 A
(3) 30 mA
(4) 60 mA

MG0173

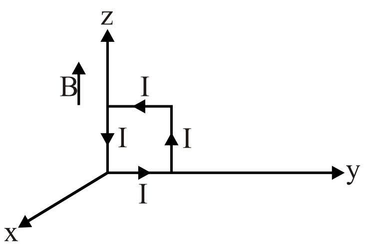
8. Two long current carrying thin wires, both with current I, are held by the insulating threads of length L and are in equilibrium as shown in the figure, with threads making an angle 'θ' with the vertical. If wires have mass λ per unit length then the value of I is :- (g = gravitational acceleration) **[JEE(Mains) - 2015]**

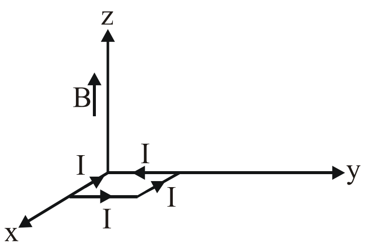


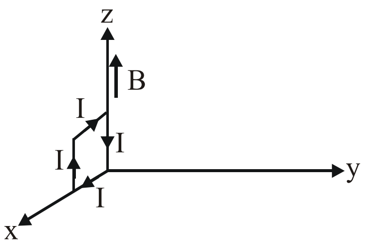
(1) $2\sqrt{\frac{\pi gL}{\mu_0}\tan\theta}$
(2) $\sqrt{\frac{\pi\lambda gL}{\mu_0}\tan\theta}$
(3) $\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0\cos\theta}}$
(4) $2\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0\cos\theta}}$

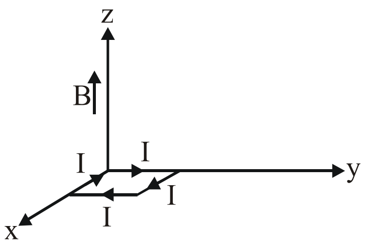
MG0174

9. A rectangular loop of sides 10 cm and 5 cm carrying a current I of 12 A is place in different orientations as shown in the figures below : **[JEE(Mains) - 2015]**

(a)


(b)


(c)


(d)


If there is a uniform magnetic field of 0.3 T in the positive z direction, in which orientations the loop would be in (i) stable equilibrium and (ii) unstable equilibrium ?

(1) (b) and (d), respectively
(2) (b) and (c), respectively
(3) (a) and (b), respectively
(4) (a) and (c), respectively

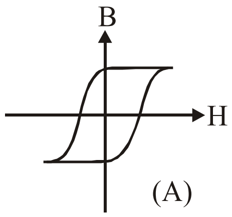
MG0175

10. Two coaxial solenoids of different radii carry current I in the same direction. Let $\vec{F_1}$ be the magnetic force on the inner solenoid due to the outer one and $\vec{F_2}$ be the magnetic force on the outer solenoid due to the inner one. Then : **[JEE Main-2015]**

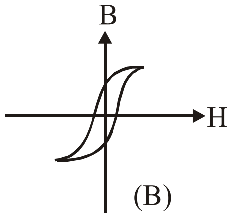
(1) $\vec{F_1}$ is radially inwards and $\vec{F_2} = 0$
(2) $\vec{F_1}$ is radially outwards and $\vec{F_2} = 0$
(3) $\vec{F_1} = \vec{F_2} = 0$
(4) $\vec{F_1}$ is radially inwards and $\vec{F_2}$ is radially outwards.

MG0176

11. Hysteresis loops for two magnetic materials A and B are given below :



(A)



(B)

These materials are used to make magnets for electric generators, transformer core and electromagnet core. Then it is proper to use ; **[JEE Main-2016]**

(1) B for electromagnets and transformers.
(2) A for electric generators and transformers.
(3) A for electromagnets and B for electric transformers.
(4) A for transformers and B for electric generators.

MG0177

12. Two identical wires A and B, each of length 'l', carry the same current I. Wire A is bent into a circle of radius R and wire B is bent to form a square of side 'a'. If B_A and B_B are the values of magnetic field at the centres of the circle and square respectively, then the ratio $\frac{B_A}{B_B}$ is :

[JEE Main-2016]

(1) $\frac{\pi^2}{8\sqrt{2}}$ (2) $\frac{\pi^2}{8}$ (3) $\frac{\pi^2}{16\sqrt{2}}$ (4) $\frac{\pi^2}{16}$

MG0178

13. A magnetic needle of magnetic moment 6.7×10^{-2} Am^2 and moment of inertia 7.5×10^{-6} kg m^2 is performing simple harmonic oscillations in a magnetic field of 0.01 T. Time taken for 10 complete oscillations is :

[JEE Main-2017]

(1) 6.98 s (2) 8.76 s (3) 6.65 s (4) 8.89 s

MG0179

14. An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii r_e, r_p, r_α respectively in a uniform magnetic field B. The relation between r_e, r_p, r_α is:-

[JEE Main-2018]

(1) $r_e < r_p = r_\alpha$ (2) $r_e < r_p < r_\alpha$ (3) $r_e < r_\alpha < r_p$ (4) $r_e > r_p = r_\alpha$

MG0180

15. The dipole moment of a circular loop carrying a current I, is m and the magnetic field at the centre of the loop is B_1. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2. The ratio $\frac{B_1}{B_2}$ is : **[JEE Main-2018]**

(1) $\sqrt{3}$ (2) $\sqrt{2}$ (3) $\frac{1}{\sqrt{2}}$ (4) 2

MG0181

SELECTED PROBLEMS FROM JEE-MAINS ONLINE PAPERS

16. A bar magnet is demagnetized by inserting it inside a solenoid of length 0.2 m, 100 turas, and carrying a current of 5.2 A. The coercivitv of the bar magnet is : **[JEE-Main-2019_Jan]**

(1) 1200 A/m (2) 2600 A/m (3) 520 A/jm (4) 285 A/m

MG0227

17. At some location on earth the horizontal component of earth's magnetic field is 18×10^{-6} T. At this location, magnetic neeedle of length 0.12 m and pole strength 1.8 Am is suspended from its mid-point using a thread, it makes 45° angle with horizontal in equilibrium. To keep this needle horizontal, the vertical force that should be applied at one of its ends is :

[JEE-Main-2019_Jan]

(1) 3.6×10^{-5} N (2) 6.5×10^{-5} N (3) 1.3×10^{-5} N (4) 1.8×10^{-5} N

MG0228

18. An insulating thin rod of length ℓ has a x linear charge density $p(x) = \rho_0 \frac{x}{\ell}$ on it. The rod is rotated about an axis passing through the origin (x = 0) and perpendicular to the rod. If the rod makes n rotations per second, then the time averaged magnetic moment of the rod is : **[JEE-Main-2019_Jan]**

(1) $\frac{\pi}{4} n\rho\ell^3$ (2) $n\rho\ell^3$ (3) $\pi n\rho\ell^3$ (4) $\frac{\pi}{3} n\rho\ell^3$

MG0229

19. A paramagnetic substance in the form of a cube with sides 1 cm has a magnetic dipole moment of 20×10^{-6} J/T when a magnetic intensity of 60×10^{3} A/m is applied. Its magnetic susceptibility is :- **[JEE-Main-2019_Jan]**

(1) 2.3×10^{-2} (2) 3.3×10^{-2} (3) 3.3×10^{-4} (4) 4.3×10^{-2}

MG0230

20. A galvanometer having a resistance of 20 Ω and 30 divisions on both sides has figure of merit 0.005 ampere/division. The resistance that should be connected in series such that it can be used as a voltmeter upto 15 volt, is :- **[JEE-Main-2019_Jan]**

(1) 80 Ω (2) 120 Ω (3) 125 Ω (4) 100 Ω

MG0231

21. A particle of mass m and charge q is in an electric and magnetic field given by

$\vec{E} = 2\hat{i} + 3\hat{j}$; $\vec{B} = 4\hat{j} + 6\hat{k}$.

The charged particle is shifted from the origin to the point P(x = 1 ; y = 1) along a straight path. The magnitude of the total work done is :- **[JEE-Main-2019_Jan]**

(1) (0.35)q (2) (0.15)q (3) (2.5)q (4) 5q

MG0232

22. There are two long co-axial solenoids of same length *l*. the inner and outer coils have radii r_1 and r_2 and number of turns per unit length n_1 and n_2 respectively. The rate of mutual inductance to the self-inductance of the inner-coil is : **[JEE-Main-2019_Jan]**

(1) $\frac{n_2}{n_1}.\frac{r_2^2}{r_1^2}$ (2) $\frac{n_2}{n_1}.\frac{r_1}{r_2}$ (3) $\frac{n_1}{n_2}$ (4) $\frac{n_2}{n_1}$

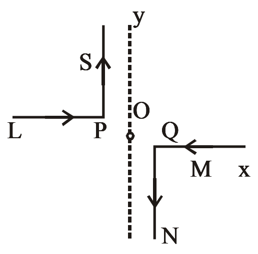
MG0233

23. A paramagnetic material has 10^{28}atoms/m^3. Its magnetic susceptibility at temperature 350 K is 2.8×10^{-4}. Its susceptibility at 300 K is : **[JEE-Main-2019_Jan]**

(1) 3.672×10^{-4} (2) 3.726×10^{-4} (3) 3.267×10^{-4} (4) 2.672×10^{-4}

MG0234

24. As shown in the figure, two infinitely long, identical wires are bent by 90° and placed in such a way that the segments LP and QM are along the x-axis, while segments PS and QN are parallel to the y-axis. If OP = OQ = 4cm, and the magnitude of the magnetic field at O is 10^{-4} T, and the two wires carry equal currents (see figure), the magnitude of the current in each wire and the direction of the magnetic field at O will be ($\mu_0 = 4\pi \times 10^{-7}$ NA^{-2}) : **[JEE-Main-2019_Jan]**

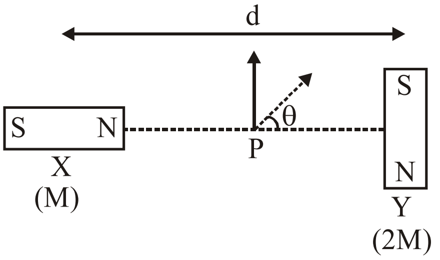


(1) 40 A, perpendicular into the page (2) 40 A, perpendicular out of the page

(3) 20 A, perpendicular out of the page (4) 20 A, perpendicular into the page

MG0235

25. Two magnetic dipoles X and Y are placed at a separation d, with their axes perpendicular to each other. The dipole moment of Y is twice that of X. A particle of charge q is passing, through their midpoint P, at angle $\theta = 45°$ with the horizontal line, as shown in figure. What would be the magnitude of force on the particle at that instant ? (d is much larger than the dimensions of the dipole) **[JEE-Main-2019_April]**



(1) $\sqrt{2}\left(\frac{\mu_0}{4\pi}\right)\frac{M}{(d/2)^3}\times qv$ (2) $\left(\frac{\mu_0}{4\pi}\right)\frac{2M}{(d/2)^3}\times qv$

(3) $\left(\frac{\mu_0}{4\pi}\right)\frac{M}{(d/2)^3}\times qv$ (4) 0 **MG0236**

26. Two coils 'P' and 'Q' are separated by some distance. When a current of 3 A flows through coil 'P', a magnetic flux of 10^{-3} Wb passes through 'Q'. No current is passed through 'Q'. When no current passes through 'P' and a current of 2 A passes through 'Q', the flux through 'P' is :-

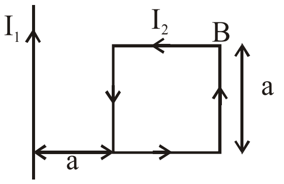
[JEE-Main-2019_April]

(1) 6.67×10^{-3} Wb (2) 6.67×10^{-4} Wb (3) 3.67×10^{-4} Wb (4) 3.67×10^{-3} Wb

MG0237

27. A rigid square loop of side 'a' and carrying current I_2 is lying on a horizontal surface near a long current I_1 carrying wire in the same plane as shown in figure. The net force on the loop due to wire will be :

[JEE-Main-2019_April]



(1) Attractive and equal to $\frac{\mu_0 I_1 I_2}{3\pi}$ (2) Repulsive and equal to $\frac{\mu_0 I_1 I_2}{4\pi}$

(3) Repulsive and equal to $\frac{\mu_0 I_1 I_2}{2\pi}$ (4) Zero

MG0238

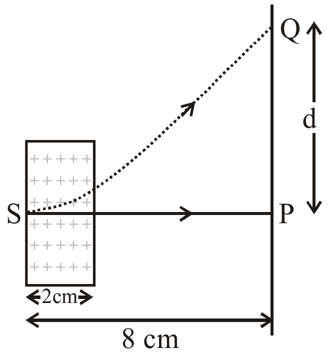
28. A rectangular coil (Dimension 5 cm × 2.5 cm) with 100 turns, carrying a current of 3 A in the clock-wise direction is kept centered at the origin and in the X-Z plane. A magnetic field of 1 T is applied along X-axis. If the coil is tilted through 45° about Z-axis, then the torque on the coil is : **[JEE-Main-2019_April]**

(1) 0.55 Nm (2) 0.27 Nm (3) 0.38 Nm (4) 0.42 Nm

MG0239

29. An electron, moving along the x-axis with an initial energy of 100 eV, enters a region of magnetic field $\vec{B} = (1.5 \times 10^{-3}\,T)\hat{k}$ at S (See figure). The field extends between x = 0 and x = 2 cm. The electron is detected at the point Q on a screen placed 8 cm away from the point S. The distance d between P and Q (on the screen) is : (electron's charge = 1.6×10^{-19} C, mass of electron = 9.1×10^{-31} kg)

[JEE-Main-2019_April]



(1) 12.87 cm (2) 1.22 cm (3) 11.65 cm (4) 2.25 cm

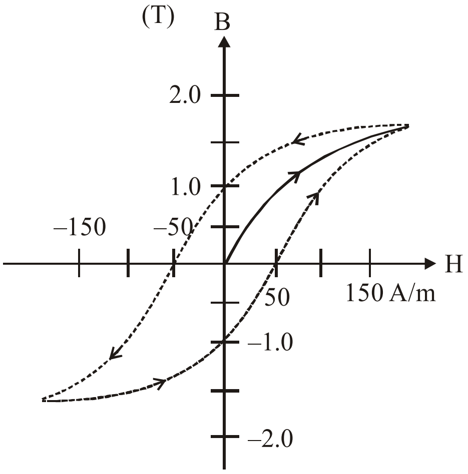
MG0240

30. A magnetic compass needle oscillates 30 times per minute at a place where the dip is 45°, and 40 times per minute where the dip is 30°. If B_1 and B_2 are respectively the total magnetic field due to the earth at the two places, then the ratio B_1/B_2 is best given by : **[JEE-Main-2019_April]**

(1) 2.2 (2) 1.8 (3) 0.7 (4) 3.6

MG0241

31. The figure gives experimentally measured B vs. H variation in a ferromagnetic material. The retentivity, co-ercivity and saturation, respectively, of the material are: **[JEE-Main-2020_Jan.]**



(1) 150 A/m, 1.0 T and 1.5 T (2) 1.0 T, 50 A/m and 1.5 T

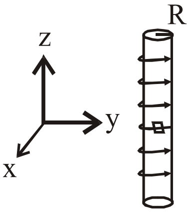
(3) 1.5 T, 50 A/m and 1.0 T (4) 1.5 T, 50 A/m and 1.0 T **MG0242**

32. Consider a circular coil of wire carrying constant current I, forming a magnetic dipole. The magnetic flux through an infinite plane that contains the circular coil and excluding the circular coil area is given by ϕ_i. The magnetic flux through the area of the circular coil area is given by ϕ_0. Which of the following option is correct ? **[JEE-Main-2020_Jan.]**

(1) $\phi_i = -\phi_0$ (2) $\phi_i = \phi_0$ (3) $\phi_i < \phi_0$ (4) $\phi_i > \phi_0$

MG0243

33. An electron gun is placed inside a long solenoid of radius R on its axis. The solenoid has n turns/length and carries a current I. The electron gun shoots an electron along the radius of the solenoid with speed v. If the electron does not hit the surface of the solenoid, maximum possible value of v is (all symbols have their standard meaning) : **[JEE-Main-2020_Jan.]**



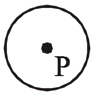
(1) $\frac{e\mu_0 nIR}{m}$ (2) $\frac{e\mu_0 nIR}{2m}$ (3) $\frac{2e\mu_0 nIR}{m}$ (4) $\frac{e\mu_0 nIR}{4m}$

MG0244

34. Magnetic materials used for making permanent magnets (P) and magnets in a transformer (T) have different properties of the following, which property best matches for the type of magnet required ? **[JEE-Main-2020_Sep.]**

(1) T : Large retentivity, small coercivity
(2) P : Small retentivity, large coercivity
(3) T : Large retentivity, large coercivity
(4) P : Large retentivity, large coercivity

35. A perfectly dimagnetic sphere has a small spherical cavity at its centre, which is filled with a paramagnetic substance. The whole system is placed in a uniform magnetic field $\vec{B}$. Then the field inside the paramagnetic substance is: **[JEE-Main-2020_Sep.]**



(1) Zero
(2) $\vec{B}$
(3) much large than $|\vec{B}|$ but opposite to $\vec{B}$
(4) much large than $|\vec{B}|$ and parallel to $\vec{B}$

MG0245

36. A galvanometer coil has 500 turns and each turn has an average area of 3×10^{-4} m^2. If a torque of 1.5 Nm is required to keep this coil parallel to magnetic field when a current of 0.5 A is flowing through it, the strength of the field (in T) is _______ . **[JEE-Main-2020_Sep.]**

MG0246

37. A paramagnetic sample shows a net magnetisation of 6 A/m when it is placed in an external magnetic field of 0.4 T at a temperature of 4 K. When the sample is placed in an external magnetic field of 0.3 T at a temperature of 24 K, then the magnetisation will be : **[JEE-Main-2020_Sep.]**

(1) 4 A/m (2) 0.75 A/m (3) 2.25 A/m (4) 1 A/m

MG0247

38. An iron rod of volume 10^{-3} m^3 and relative permeability 1000 is placed as core in a solenoid with 10 turns/cm. If a current of 0.5 A is passed through the solenoid, then the magnetic moment of the rod will be : **[JEE-Main-2020_Sep.]**

(1) 0.5×10^2 Am^2 (2) 50×10^2 Am^2 (3) 500×10^2 Am^2 (4) 5×10^2 Am^2

MG0248

39. A square loop of side 2a, and carrying current I, is kept in XZ plane with its centre at origin. A long wire carrying the same current I is placed parallel to the z-axis and passing through the point (0, b, 0), (b > > a). The magnitude of the torque on the loop about z-axis is given by:

[JEE-Main-2020_Sep.]

(1) $\frac{2\mu_0 I^2 a^2}{\pi b}$ (2) $\frac{\mu_0 I^2 a^3}{2\pi b^2}$ (3) $\frac{\mu_0 I^2 a^2}{2\pi b}$ (4) $\frac{2\mu_0 I^2 a^3}{\pi b^2}$

MG0249

40. A square loop of side 2a and carrying current I is kept in xz plane with its centre at origin. A long wire carrying the same current I is placed parallel to z-axis and passing through point (0, b, 0), (b >> a). The magnitude of torque on the loop about z-axis will be :

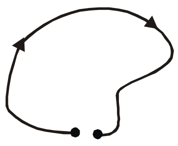
[JEE-Main-2020_Sep.]

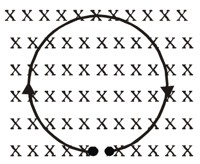
(1) $\frac{2\mu_0 I^2 a^2 b}{\pi(a^2+b^2)}$ (2) $\frac{\mu_0 I^2 a^2 b}{2\pi(a^2+b^2)}$ (3) $\frac{\mu_0 I^2 a^2}{2\pi b}$ (4) $\frac{2\mu_0 I^2 a^2}{\pi b}$

MG0250

EXERCISE-JA

1. A thin flexible wire of length L is connected to two adjacent fixed points and carries a current I in the clockwise direction, as shown in the figure. When the system is put in a uniform magnetic field of strength B going into the plane of the paper, the wire takes the shape of a circle. The tension in the wire is : **[2010, 3M]**



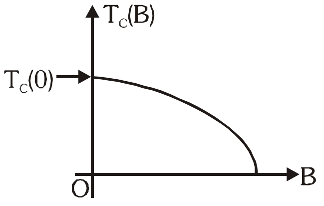


(A) IBL (B) $\frac{IBL}{\pi}$ (C) $\frac{IBL}{2\pi}$ (D) $\frac{IBL}{4\pi}$

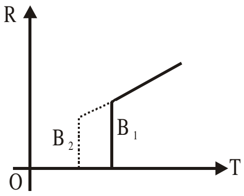
MG0182

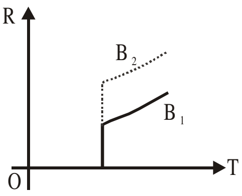
Paragraph for Question no. 2 and 3

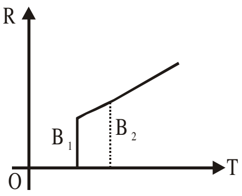
Electrical resistance of certain materials, known as superconductors, changes abruptly from a nonzero value to zero as their temperature is lowered below a critical temperature $T_C(0)$. An interesting property of superconductors is that their critical temperature becomes smaller than $T_C(0)$ if they are placed in a magnetic field, i.e., the critical temperature $T_C(B)$ is a function of the magnetic field strength B. The dependence of T_C (B) on B is shown in the figure. **[JEE2010]**

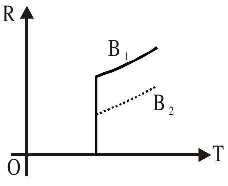


2. In the graphs below, the resistance R of a superconductor is shown as a function of its temperature T for two different magnetic fields B_1(sold line) and B_2 (dashed line). If B_2 is larger than B_1, which of the following graphs shows the correct variation of R with T in these fields?

(A)


(B)


(C)


(D)


MG0183

3. A superconductor has $T_C(0) = 100$ K. When a magnetic field of 7.5 Tesla is applied, its T_C decreases to 75 K. For this material one can definitely say that when

(A) B = 5 Tesla, T_C (B) = 80 K

(B) B = 5 Tesla, 75 K < T_C(B) < 100 K

(C) B = 10 Tesla, 75 K < T_C(B) < 100K

(D) B = 10 Tesla, T_C(B) = 70 K

MG0184

4. An electron and a proton are moving on straight parallel paths with same velocity. They enter a semi-infinite region of uniform magnetic field perpendicular to the velocity. Which of the following statement(s) is/are true? **[IIT-JEE 2011]**

(A) they will never come out of the magnetic field region

(B) they will come out travelling along parallel paths

(C) they will come out of the same time

(D) they will come out at different times

MG0185

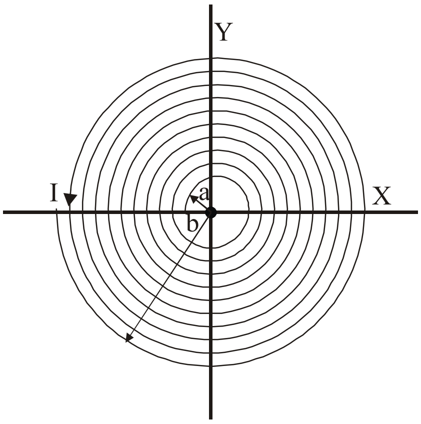
5. A long insulated copper wire is closely wound as a spiral of 'N' turns. The spiral has inner radius 'a' and outer radius 'b'. The spiral lies in the X-Y plane and a steady current '*I*' flows through the wire. The Z-component of the magnetic field at the center of the spiral is **[IIT-JEE 2011]**

(A) $\frac{\mu_0 NI}{2(b-a)}\ell n\left(\frac{b}{a}\right)$

(B) $\frac{\mu_0 NI}{2(b-a)}\ell n\left(\frac{b+a}{b-a}\right)$

(C) $\frac{\mu_0 NI}{2b}\ell n\left(\frac{b}{a}\right)$

(D) $\frac{\mu_0 NI}{2b}\ell n\left(\frac{b+a}{b-a}\right)$



MG0186

6. Consider the motion of a positive point charge in a region where there are simultaneous uniform electric and magnetic fields $\vec{E} = E_0\hat{j}$ and $\vec{B} = B_0\hat{j}$. At time t =0, this charge has velocity $\vec{v}$ in the x-y plane, making an angle θ with the x-axis. Which of the following option (s) is (are) correct for time t > 0? **[IIT-JEE 2012]**

(A) If θ = 0°, the charge moves in a circular path in the x-z plane.

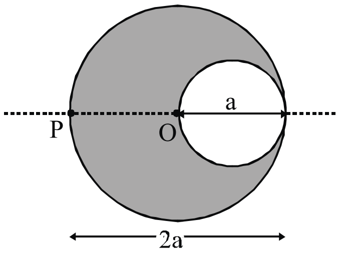
(B) If θ = 0°, the charge undergoes helical motion with constant pitch along the y-axis.

(C) If θ = 10°, the charge undergoes helical motion with its pitch increasing with time, along the y-axis

(D) If θ = 90°, the charge undergoes linear but accelerated motion along the y-axis

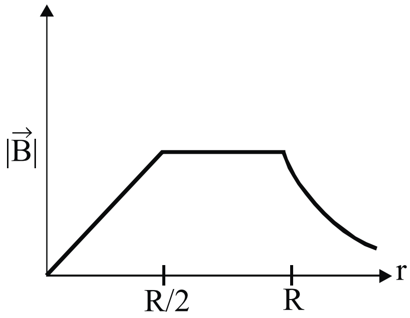
MG0187

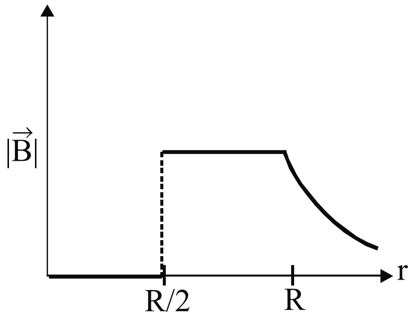
7. A cylindrical cavity of diameter a exists inside a cylinder of diameter 2a as shown in the figure. Both the cylinder and the cavity are infinitely long. A uniform current density J flows along the length. If the magnitude of the magnetic field at the point P is given by $\frac{N}{12}\mu_0 aJ$, then the value of N is : **[IIT-JEE 2012]**

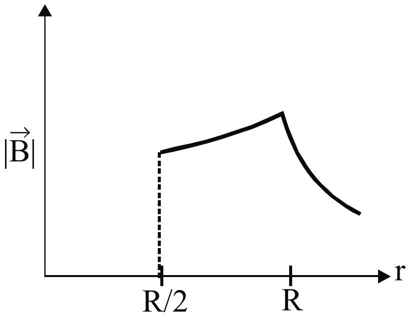


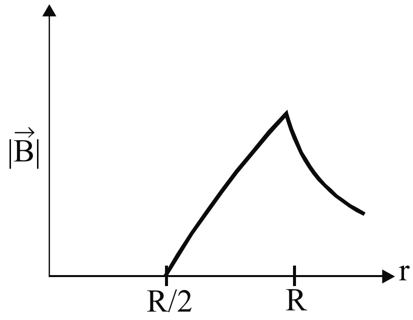
MG0188

8. An infinitely long hollow conducting cylinder with inner radius R/2 and outer radius R carries a uniform current density along its length. The magnitude of the magnetic field, $|\vec{B}|$ as a function of the radial distance r from the axis is best represented by **[IIT-JEE 2012]**

(A)


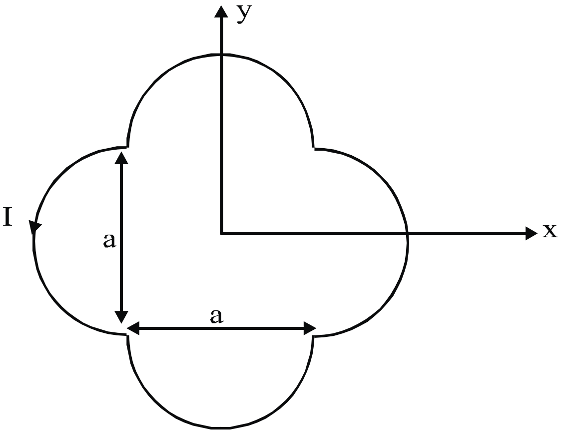
(B)


(C)


(D)


MG0189

9. A loop carrying current 'I' lies in the x-y plane as shown in the figure. The unit vector $\hat{k}$ is coming out of the plane of the paper. The magnetic moment of the current loop is **[JEE 2012]**



(A) $a^2I\hat{k}$ (B) $\left(\frac{\pi}{2}+1\right)a^2I\hat{k}$ (C) $-\left(\frac{\pi}{2}+1\right)a^2I\hat{k}$ (D) $(2\pi+1)a^2I\hat{k}$

MG0190

10. A particle of mass M and positive charge Q, moving with a constant velocity $\vec{u}_1 = 4\hat{i}\,\text{ms}^{-1}$, enters a region of uniform static magnetic field normal to the x-y plane. The region of the magnetic field extends from x = 0 to x = L from all values of y. After passing through this region, the particle emerges on the other side after 10 milliseconds with a velocity $\vec{u}_2 = 2\left(\sqrt{3}\hat{i}+\hat{j}\right)\text{ms}^{-1}$. The correct statements(s) is (are) :- **[IIT-JEE 2013]**

(A) The direction of the magnetic field is –z direction.

(B) The direction of the magnetic field is +z direction.

(C) The magnitude of the magnetic field $\frac{50\pi M}{3Q}$ units.

(D) The magnitude of the magnetic field is $\frac{100\pi M}{3Q}$ units.

MG0191

11. A steady current I flows along an infinitely long hollow cylindrical conductor of radius R. This cylinder is placed coaxially inside an infinite solenoid of radius 2R. The solenoid has n turns per unit length and carries a steady current I. Consider a point P at a distance r from the common axis. The correct statement(s) is (are) :- **[JEE-Advanced-2013]**

(A) In the region 0 < r < R, the magnetic field is non-zero

(B) In the region R < r < 2R, the magnetic field is along the common axis

(C) In the region R < r < 2R, the magnetic field is tangential to the circle of radius r, centred on the axis

(D) In the region r > 2R, the magnetic field is non-zero

MG0192

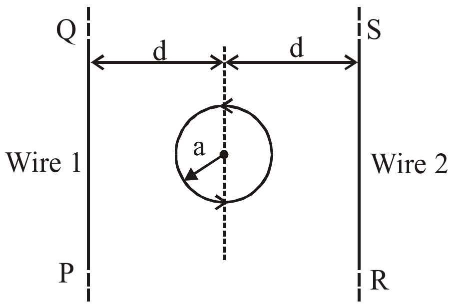
12. Two parallel wires in the plane of the paper are distance X_0 apart. A point charge is moving with speed u between the wires in the same plane at a distance X_1 from one of the wires. When the wires carry current of magnitude I in the same direction, the radius of curvature of the path of the point charge is R_1. In contrast, if the currents I in the two wires have directions opposite to each other, the radius of curvature of the path is R_2. If $\frac{X_0}{X_1} = 3$, and value of $\frac{R_1}{R_2}$ is:-

[JEE-Advanced-2014]

MG0193

Paragraph for Questions 13 & 14

The figure shows a circular loop of radius a with two long parallel wires (numbered 1 and 2) all in the plane of the paper. The distance of each wire from the centre of the loop is d. The loop and the wires are carrying the same current I. The current in the loop is in the counterclockwise direction if seen from above. **[JEE-Advanced-2014]**



13. When d ≈ a but wires are not touching the loop, it is found that the net magnetic field on the axis of the loop is zero at a height h above the loop. In that case

(A) current in wire 1 and wire 2 is the direction PQ and RS, respectively and h ≈ a

(B) current in wire 1 and wire 2 is the direction PQ and SR, respectively and h ≈ a

(C) current in wire 1 and wire 2 is the direction PQ and SR, respectively and h ≈ 1.2 a

(D) current in wire 1 and wire 2 is the direction PQ and RS, respectively and h ≈ 1.2 a

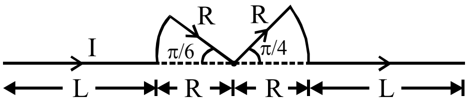
MG0194

14. Consider d >> a, and the loop is rotated about its diameter parallel to the wires by 30° from the position shown in the figure. If the currents in the wires are in the opposite directions, the torque on the loop at its new position will be (assume that the net field due to the wires is constant over the loop) **[JEE-Advanced-2014]**

(A) $\frac{\mu_0 I^2 a^2}{d}$ (B) $\frac{\mu_0 I^2 a^2}{2d}$ (C) $\frac{\sqrt{3}\mu_0 I^2 a^2}{d}$ (D) $\frac{\sqrt{3}\mu_0 I^2 a^2}{2d}$

MG0195

15. A conductor (shown in the figure) carrying constant current I is kept in the x-y plane in a unfirom magnetic field $\vec{B}$. If F is the magnitude of the total magnetic force acting on the conductor, then the **CORRECT** statement(s) is (are) **[JEE-Advanced-2015]**



(A) If $\vec{B}$ is along $\hat{z}$, $F \propto (L + R)$ (B) If $\vec{B}$ is along $\hat{x}$, $F = 0$

(C) If $\vec{B}$ is along $\hat{y}$, $F \propto (L + R)$ (D) If $\vec{B}$ is along $\hat{z}$, $F = 0$ **MG0196**

Answer Q.16, Q.17 and Q.18 by appropriately matching the information given in the three columns of the following table.

A charged particle (electron or proton) is introduced at the origin (x = 0, y = 0, z = 0) with a given initial velocity $\vec{v}$. A uniform electric field $\vec{E}$ and a uniform magnetic field $\vec{B}$ exist everywhere. The velocity $\vec{v}$, electric field $\vec{E}$ and magnetic field $\vec{B}$ are given in column 1, 2 and 3, respectively. The quantities E_0, B_0 are positive in magnitude. **[JEE-Advanced-2017]**

	Column-1		**Column-2**		**Column-3**
(I)	Electron with $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$	(i)	$\vec{E} = E_0\hat{z}$	(P)	$\vec{B} = -B_0\hat{x}$
(II)	Electron with $\vec{v} = \frac{E_0}{B_0}\hat{y}$	(ii)	$\vec{E} = -E_0\hat{y}$	(Q)	$\vec{B} = B_0\hat{x}$
(III)	Proton with $\vec{v} = 0$	(iii)	$\vec{E} = -E_0\hat{x}$	(R)	$\vec{B} = B_0\hat{y}$
(IV)	Proton with $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$	(iv)	$\vec{E} = E_0\hat{x}$	(S)	$\vec{B} = B_0\hat{z}$

16. In which case will the particle move in a straight line with constant velocity ?

(A) (II) (iii) (S) (B) (IV) (i) (S) (C) (III) (ii) (R) (D) (III) (iii) (P)

MG0197

17. In which case will the particle describe a helical path with axis along the positive z-direction ?

(A) (II) (ii) (R) (B) (IV) (ii) (R) (C) (IV) (i) (S) (D) (III) (iii) (P)

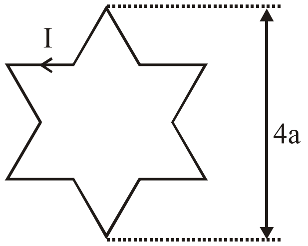
MG0198

18. In which case would the particle move in a straight line along the negative direction of y-axis (i.e., move along $-\hat{y}$) ?

(A) (IV) (ii) (S) (B) (III) (ii) (P) (C) (II) (iii) (Q) (D) (III) (ii) (R)

MG0199

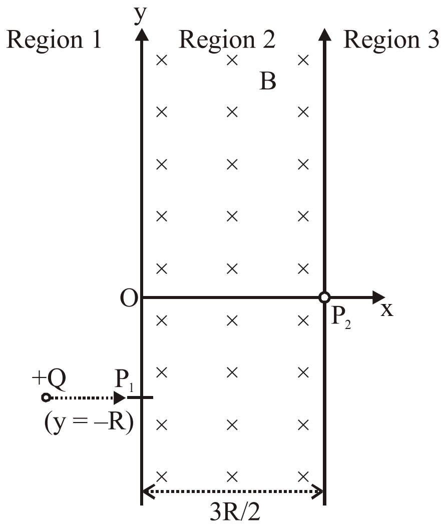
19. A symmetric star shaped conducting wire loop is carrying a steady state current I as shown in the figure. The distance between the diametrically opposite vertices of the star is 4a. The magnitude of the magnetic field at the center of the loop is : **[JEE-Advanced-2017]**



(A) $\frac{\mu_0 I}{4\pi a}3\left[\sqrt{3}-1\right]$ (B) $\frac{\mu_0 I}{4\pi a}6\left[\sqrt{3}-1\right]$ (C) $\frac{\mu_0 I}{4\pi a}6\left[\sqrt{3}+1\right]$ (D) $\frac{\mu_0 I}{4\pi a}3\left[2-\sqrt{3}\right]$

MG0200

20. A uniform magnetic field B exists in the region between x = 0 and $x = \frac{3R}{2}$ (region 2 in the figure) pointing normally into the plane of the paper. A particle with charge +Q and momentum p directed along x-axis enters region 2 from region 1 at point $P_1(y = -R)$. Which of the following options(s) is/are correct ? **[JEE-Advanced-2017]**



(A) For $B = \frac{8}{13}\frac{p}{QR}$, the particle will enter region 3 through the point P_2 on x-axis

(B) For $B > \frac{2}{3}\frac{p}{QR}$, the particle will re-enter region 1

(C) For a fixed B, particle of same charge Q and same velocity v, the distance between the point P_1 and the point of re-entry into region 1 is inversely proportional to the mass of the particle.

(D) When the particle re-enters region 1 through the longest possible path in region 2, the magnitude of the chage in its linear momentum between point P_1 and the farthest point from y-axis is $\frac{p}{\sqrt{2}}$.

MG0201

21. Two infinitely long straight wires lie in the xy-plane along the lines x = ±R. The wire located at x = +R carries a constant current I_1 and the wire located at x = –R carries a constant current I_2. A circular loop of radius R is suspended with its centre at (0, 0, $\sqrt{3}R$) and in a plane parallel to the xy-plane. This loop carries a constant current I in the clockwise direction as seen from above the loop. The current in the wire is taken to be positive if it is in the $+\hat{j}$ direction. Which of the following statements regarding the magnetic field $\vec{B}$ is (are) true ? **[JEE-Advanced-2018]**

(A) If $I_1 = I_2$, then $\vec{B}$ cannot be equal to zero at the origin (0, 0, 0)

(B) If $I_1 > 0$ and $I_2 < 0$, then $\vec{B}$ can be equal to zero at the origin (0, 0, 0)

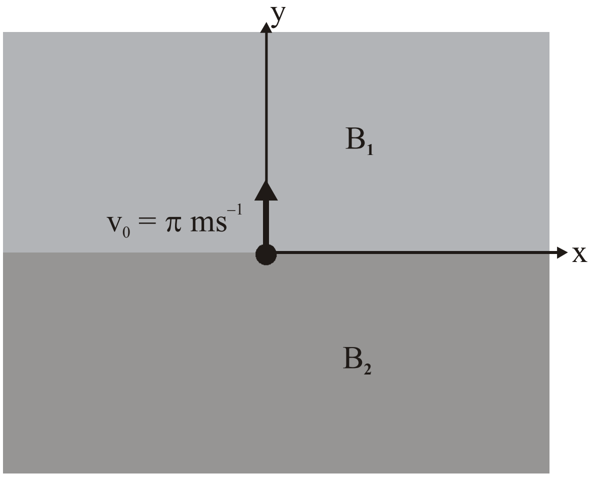
(C) If $I_1 < 0$ and $I_2 > 0$, then $\vec{B}$ can be equal to zero at the origin (0, 0, 0)

(D) If $I_1 = I_2$, then the z-component of the magnetic field at the centre of the loop is $\left(-\frac{\mu_0 I}{2R}\right)$

MG0202

22. In the x-y-plane, the region y > 0 has a uniform magnetic field $B_1\hat{k}$ and the region y < 0 has a another uniform magnetic field $B_2\hat{k}$. A positively charged particle is projected from the origin along the positive y-axis with speed $v_0 = \pi ms^{-1}$ at t = 0, as shown in the figure. Neglect gravity in this problem. Let t = T be the time when the particle crosses the x-axis from below for the first time. If $B_2 = 4B_1$, the average speed of the particle, in ms^{-1}, along the x-axis in the time interval T is _______.

[JEE-Advanced-2018]

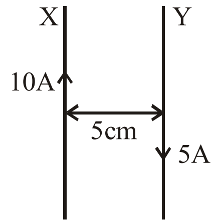


MG0203

MAGNETIC EFFECT OF CURRENT

(CBSE Previous Year's Questions)

1. Two long parallel straight wires X and Y separated by a distance of 5 cm in air carry currents of 10 A and 5 A respectively in opposite directions. Calculate the magnitude and direction of the force on a 20cm length of the wire Y.

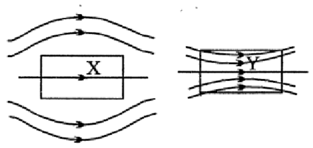


A circular coil of 100 turns, radius 10 cm carries a current of 5 A. it is suspended vertically in a uniform horizontal magnetic field of 0.5 T, the field lines making an angle of 60° with the of the plane of the coil. Calculate of the torque must be applied on it to prevent it from turning. **[2;CBSE-2004]**

2. Using Biot-Savart law, deduce an expression for the magnetic field on the axis of a circular current loop. Draw the magnetic field lines due to a circular current carrying loop. **[3; CBSE-2004]**

3. A hydrogen ion of mass 'm' and chargef 'q' travels with a speed 'v' in a circle of radius 'r' in a magnetic field of intensity 'B'. Write the equation in terms of these quantities only, relating the force on the ion to the required centripetal force. Hence derive an expression for its time period. **[3; CBSE-2004]**

4. A uniform magnetic field gets modified as shown below, when two specimens X and Y are placed in it



(i) Identify the two specimens X and Y.
(ii) State the reason for the behaviour of the field lines in X and Y. **[3; CBSE-2004]**

5. Two wires of equal lengths are bent in the form of two loops. One of the loops is square shaped whereas the other loop is circular. These are suspended in a uniform magnetic field and the same current is passed through them. Which loop will experience greater torque? Give reasons. **[1; CBSE-2005]**

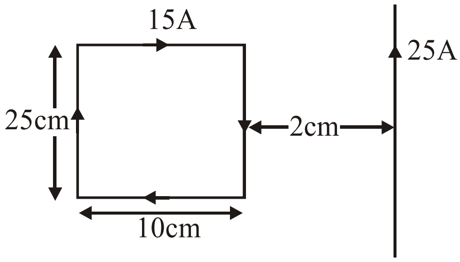
6. Write two characteristic properties to distinguish between diamagnetic and paramagnetic materials. **[2; CBSE-2005]**

7. Explain the principle and working of a cyclotron with the help of a Labeled diagram. A cyclotron's oscillator frequency is 10 Mhz. What should be the operating magnetic field for accelerating protons? If the radius of its 'dees' is 60 cm, what is the kinetic energy of the proton beam produced by the accelerator? Express your answer in units of Me V.
($e = 1.6 \times 10^{-19}$ C, $m_p = 1.67 \times 10^{-27}$ Kg, 1MeV = 1.602×10^{-13} J) **[5; CBSE-2005]**

8. Depict the magnetic field lines due to two straight, long, parallel conductors carrying currents I_1 and I_2 in the same direction. Hence deduce an expression for the force acting per unit length on one conductor due to the other. Is this force attractive or repulsive?

Figure shows a rectangular current-carrying loop placed 2 cm away from a long, straight, current-carrying conductor. What is the direction and magnitude of the net force acting on the loop?

[5; CBSE-2005]



9. Steel is preferred for making permanent magnets whereas soft iron is preferred for making electromagnets. Give one reason. **[1; CBSE-2006]**

10. Draw a neat and labelled diagram of a cyclotron. State the underlying principle and explain how a positively charged particle gets accelerated in this machine. Show mathematically that the cyclotron frequency does not depend upon the speed of the particle. **[5; CBSE-2006]**

11. State the Biot - Savart law for the magnetic field due to a current carrying element. Use this law to obtain a formula for magnetic field at the centre of a circular loop of radius R carrying a steady current I. Sketch the magnetic field lines for a current loop clearly indicating the direction of the field.

[5; CBSE-2006]

12. An electron is moving along +ve x–axis in the presence of uniform magnetic field along +ve y-axis. What is the direction of the force acting on it ? **[1; CBSE-2007]**

13. Explain, with the help of a labelled diagram, the principle and construction of a cyclotron. Deduce an expression for the cyclotron frequency and show that it does not depend on the speed of the charged particle. **[3; CBSE-2007]**

14. Distinguish the magnetic properties of dia, para- and ferro-magnetic substances in terms of (i) susceptibility, (ii) magnetic permeability and (iii) coercivity. Give one example of each of these materials. Draw the field lines due to an external magnetic field near a (i) diamagnetic, (ii) paramagnetic substance. **[3; CBSE-2007]**

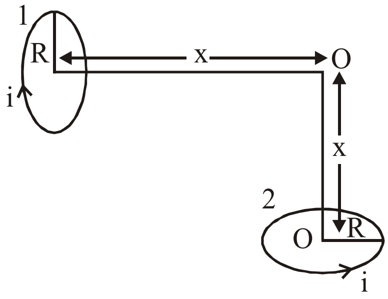
15. What is the direction of the force acting on a charged particle q, moving with a velocity $\vec{v}$ in a uniform magnetic field $\vec{B}$? **[1; CBSE-2008]**

16. Define magnetic susceptibility of a material Name two elements, one having positive susceptibility and the other having negative susceptibility. What does negative susceptibility signify ?

[2; CBSE-2008]

17. (a) Using Biot-Savart's law, derive an expression for the magnetic field at the centre of a circular coil of radius R, number of turns N, carrying current i.

(b) Two small identical circular coils marked 1,2 carry equal currents and are placed with their geometric axes perpendicular to each other as shown in the figure. Derive an expression for the resultant magnetic field at O.

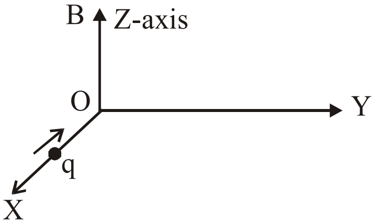


Draw a schematic diagram of a cyclotron. Explain its underlying principle and working, stating clearly the function of the electric and magnetic fields applied on a charged particle. Deduce an expression for the period of revolution and show that it does not depend on the speed of the charged particle.

[5; CBSE-2008]

18. Magnetic field lines can be entirely confined within the core of a toroid, but not within a straight solenoid. Why ? **[l; CBSE-2009]**

19. A charge 'q' moving along the X-axis with a velocity v is subjected to a uniformmagnetic field B acting along the Z-axis as it crosses the origin O.



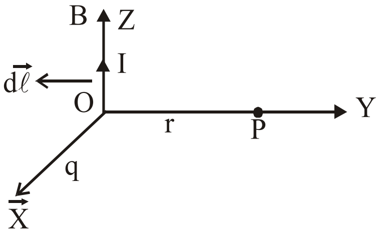
(i) Trace its trajectory.

(ii) Does the charge gain kinetic energy as it enters the magnetic field? Justify your answer.

[2; CBSE-2009]

20. State Biot-Savart law.

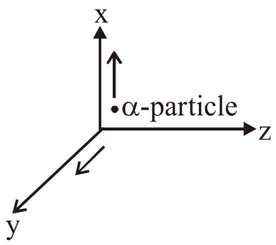
A current I flows in a conductor placed perpendicular to the plane of the paper. Indicate the direction of the magnetic field due to a small element $d\vec{\ell}$ at point P situated at a distance $\vec{r}$ from the element as shown in the figure.



21. Derive the expression for force per unit length between two long straight parallel current carrying conductors. Hence define one ampere. **[3; CBSE-2009]**

22. Explain the principle and working of a cyclotron with the help of a schematic diagram. Write the expression for cyclotron frequency. **[3; CBSE-2009]**

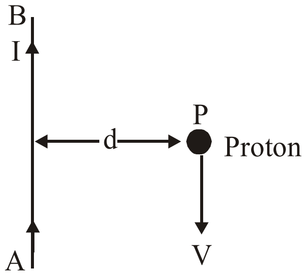
23. A beam of α particle projected along +x-axis, experiences a force due to a magnetic field along the +y – axis. What is the direction of the magnetic field? **[1; CBSE-2010]**



24. Deduce the expression for the magnetic dipole moment of an electron orbiting around the central nucleus. **[2; CBSE-2010]**

25. (a) With the help of a diagram, explain the principle and working of a moving coil galvanometer.
(b) What is the importance of a radial magnetic field and how is it produced ?
(c) Why is it that whil using a moving coil galvanometer as a voltmeter a high resistance in series is required whereas in an ammeter a shunt is used ? **[5; CBSE-2010]**

26. (a) Derive an expression for the force between two long parallel current carrying conductors.
(b) Use this expression to define S.I. unit of current.
(c) A long straight wire AB carries a current I. A proton P travels with a speed v, parallel to the wire, at a distance d from it in a direction opposite to the current as shown in the figure. What is the force experienced by the proton and what is its direction ? **[5; CBSE-2010]**



27. Where on the surface ofEarth is the angle of dip 90° ? **[1;CBSE-2011]**

28. Write the expression for Lorentz magnetic force on a particle of charge 'q' moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$. Show that no work is done by this force on the charged particle. **[2; CBSE-2011]**

29. A steady current (I_1) flows through a long straight wire. Another wire carrying steady current (I_2) in the same direction is kept close and parallel to the first wire. Show with the help of a diagram how the magnetic field due to the current I_1 exerts a magnetic force on the second wire. Write the expression for this force. **[2; CBSE-2011]**

30. (a) State the principle of the working of moving coil galvanometer, giving its labelled diagram.

(b) "Increasing the current sensitivity of a galvanometer may not necessarily increase its voltage sensitivity. "Justify this statement.

(c) Outline the necessary steps to convert a galvanometer of resistance R_G into an ammeter of a given range. **[5; CBSE-2011]**

31. (a) Using Ampere's circuital law, obtain the expression for the magnetic field due to a long solenoid at a point inside the solenoid on its axis.

(b) In what respect is a toroid different from a solenoid ? Draw and compare the pattern of the magnetic field lines in the two cases.

(c) How is the magnetic field inside a given solenoid made strong ? **[5; CBSE-2011]**

32. A circular coil of N turns and radius R carries a current I. It is unwound and rewound to make another coil of radius R/2, current I remaining the same. Calculate the ratio of the magnetic moments of the new coil and original coil. **[2; CBSE-2012]**

33. (a) Write the expression for the force, F, acting on a charged particle of charge 'q', moving with a velocity $\vec{v}$ in the presence of both electric field $\vec{E}$ and magnetic field $\vec{B}$. Obtain the condition under which the particle moves undeflected through the fields.

(b) A rectangular loop of size $l \times b$ carrying a steady current I is placed in uniform magnetic field $\vec{B}$. Prove that the torque $\vec{\tau}$ acting on the loop is given by $\vec{\tau} = \vec{m} \times \vec{B}$, where m is the magnetic moment of the loop.

OR

(a) Explain, giving reason, the basic difference in converting a galvanometer into (i) a voltmeter and (ii) an ammeter

(b) Two long straight parallel conductors carrying steady current I_1 and I_2 are separated by a distance 'd' Explain briefly, with the help of a suitable diagram, how the magnetic field due to one conductor acts on the other. Hence deduce the expression for the force acting between the two conductors. Mention the nature of this force. **[5; CBSE-2012]**

34. A wire AB is carrying a steady current of 12 A and is lying on the table. Another wire CD carrying 5 A is held directly above AB at a height of 1 mm. Find the mass per unit length of the wire CD so that it remains suspended at its position when left free. Give the direction of the current flowing in CD with respect to that in AB. [Take the value of $g = 10\ ms^{-2}$] **[CBSE-2013]**

35. (a) Using Biot - Savatfs law, derive the expression for the magnetic field in the vector form at a point on the axis of a circular current loop.

(b) What does a toroid consist of? Find out the expression for the magnetic field inside a toroid for N turns of the coil having the average radius r and carrying a current I. Show that the magnetic field in the open space inside and exterior to the toroid is zero.

OR

(a) Draw a schematic sketch of a cyclotron. Explain clearly the role of crossed electric and magnetic field in accelerating the charge. Hence derive the expression for the kinetic energy acquired by the particles.

(b) An α-particle and a proton are released from the centre of the cyclotron and made to accelerate.

(i) Can both be accelerated at the same cyclotron frequency ? Give reason to justify your answer.

(ii) When they are accelerated in turn, which of the two will have higher velocity at the exit slit of the dees? **[CBSE-2013]**

36. Using the concept of force between two infinitely long parallel current carrying conductors, define one ampere of current. **[CBSE-2014]**

37. Show diagrammatically the behaviour of magnetic field lines in the presence of (i) paramagnetic and (ii) diamagnetic substances. How does one explain this distinguishing feature ?**[CBSE-2014]**

38. (a) Deduce an expression for the frequency of revolution of a charged particle in a magnetic field and show that it is independent of velocity or energy of the particle.

(b) Draw a schematic sketch of a cyclotron. Explain, giving the essential details of its construction, how it is used to accelerate the charged particles.

OR

(a) Draw a labelled diagram of a moving coil galvanometer. Describe briefly its principle and working.

(b) Answer the following:

(i) Why is it necessary to introduce a cylindrical soft iron core inside the coil of a galvanometer ?

(ii) Increasing the current sensitivity of a galvanometer may not necessarily increase its voltage sensitivity. Explain, giving reason. **[CBSE-2014]**

39. A cyclotron's oscillator frequency is 10 MHz. What should be the operating magneticf field for accelerating protons ? If the radius of its 'dees' is 60 cm, calculate the kinetic energy (in MeV) of the proton beam produced by the accelerator. **[3; CBSE-2015]**

40. Deduce the expression for the torque $\vec{\tau}$ acting on a planar loop of area $\vec{A}$ and carryig current I placed in a uniform magnetic field $\vec{B}$. **[3; CBSE-2015]**
If the loop is free to rotate, what would be its orientation is stable equilibrium ?

41. What can be the cause of helical motion of a charged particle ? **[1; CBSE-2016]**

42. State Ampere's circuital law. Use this law to find magnetic field due to straight infinite current carrying wire. How are the magnetic field lines different from the electrostatic field lines ? **[3; CBSE-2016]**

OR

State the principle of a cyclotron. Show that the time period of revolution of particles in a cyclotron is independent of their speeds. Why is this property neccessary for the operation of a cyclotron ?

43. Seema's uncle was advised by his doctor to have an MRI (Magnetic Resonance Imaging) scan of his brain. Her uncle felt it to be expensive and wanted to postpone it. **[4; CBSE-2016]**
When Seema learnt about this, she took the help of her family and also approached the doctor, who also offered a substantial discount. She then convinced her uncle to undergo the test to enable the doctor to know the condition of his brain. The information thus obtained greatly helped the doctor to treat him properly.
Based on the above paragraph, answer the following questions :

(a) What according to you are the values displayed by Seems, her family and the doctor ?

(b) What could be the possible reason for MRI test to be so expensive ?

(c) Assuming that MRI test was performed using a magnetic field of 0.1 T., find the minimum and maximum values of the force that the magnetic field could exert on a proton (charge = 1.6×10^{-19} C) moving with a speed of 10^4 m/s.

44. (a) When a bar magnet is pushed towards (or away) from the coil connected to a galvanometer, the pointer in the galvanometer deflects. Identify the phenomenon causing this deflection and write the factors on which the amount and direction of the deflection depends. State the laws describing this phenomenon. **[5; CBSE-2016]**

45. Find the condition under which the charged particles moving with different speeds in the presence of electric and magnetic field vectors can be used to select charged particles of a particular speed. **[2; CBSE-2017]**

46. Write two properties of a material suitable for making (a) a permanent magnet, and (b) an electromagnet. **[2; CBSE-2017]**

47. (a) State Biot – Savart law and express this law in the vector form. **[3; CBSE-2017]**

(b) Two identical circular coils, P and Q each of radius R, carrying currents 1A and $\sqrt{3}$ A respectively, are placed concentrically and perpendicular to each other lying in the XY and YZ planes. Find the magnitude and direction of the net magnetic field at the centre of the coils.

48. A proton and an electron travelling along parallel paths enter a region of uniform magnetic field, acting perpendicular to their paths. Which of them will move in a circular path with higher frequency? **[3; CBSE-2018]**

49. A bar magnet of magnetic moment of 6J/T is aligned at 60° with a uniform external magnetic field of 0.44 T. Calculate (a) the work done in turning the magnet to align its magnetic moment (i) normal to the magnetic field, (ii) opposite to the magnetic field, and (b) the torque on the magnet in the final orientation in case (ii). **[3; CBSE-2018]**

50. (a) An iron ring of relative permeability μ_r has windings of insulated copper wire of n turns per metre. When the current in the windings is I, find the expression for the magnetic field in the ring.

(b) The susceptibility of a magnetic material is 0.9853. Identify the type of magnetic material. Draw the modification of the field pattern on keeping a piece of this material in a uniform magnetic field. **[3; CBSE-2018]**

ANSWER KEY

EXERCISE (S-1)

1. Ans. (a) zero (b) 2μT along the Z-axis (c) zero and (d) 2μT along the negative Z-axis

2. Ans. 4.0×10^{-5} T, downwards in both the cases

3. Ans. $5\pi \times 10^{-4}$ T = 1.6×10^{-3} T towards west. **4. Ans.** $7.07 \times 10^{-10}\ \hat{k}$T

5. Ans. At a distance of $\frac{4r}{\pi}$ from the centre in such a way that the direction of the current in it is opposite to that in the nearest part of the circular wire.

6. Ans. (a) 0 (b) 1.41×10^{-6} T, 45° in xz-plane, (c) 5×10^{-6} T, +x-direction]

7. Ans. μ_0 weber.m^{-1} **8. Ans.** $\frac{2mv_0}{qB}$ **9. Ans.** 1.2×10^{-2}m, 4.37×10^{-2}m

10. Ans. (a) Circular trajectory of radius 1.0 mm normal to B.
(b) Helical trajectory of radius 0.5 mm with velocity component 2.3×10^{7} ms^{-1} along B.

11. Ans. (a) A horizontal magnetic field of magnitude 0.26 T normal to the conductor in such a direction that Fleming a left-hand rule gives a magnetic force upward.
(b) 1.176 N.

12. Ans. (a) 3.1 Nm, (b) No, the answer is unchanged because the formula $\tau = N I A \times B$ is true for a planar loop of any shape.

13. Ans. Use $\tau = IA \times B$ and $F = I1 \times B$
(a) 1.8×10^{-2} N m along –y direction
(b) same as in (a)
(c) 1.8×10^{-2} N m along –x direction
(d) 1.8×10^{-2} N m at an angle of 240° with the +x direction
(e) zero
(f) zero
Force is zero in each case. Case (e) corresponds to stable, and case (f) corresponds to unstable equilibrium

14. Ans. (a) Zero, (b) zero, (c) force on each electron is $evB = IB/(nA) = 5 \times 10^{-25}$N. Note : Answer (c) denotes only the magnetic force.

15. Ans. $T_0 = 2\pi\sqrt{\frac{m}{6IB}} = 0.57$ s **16. Ans.** (a) 1.4, (b) 1

EXERCISE (S-2)

1. Ans. $B_1 = \frac{\mu_0 b r_1^2}{3}$, $B_2 = \frac{\mu_0 b R^3}{3 r_2}$ **2. Ans.** $\sqrt{15}$ C

3. Ans. (6.4 m, 0,0) (6.4m, 0, 2m) **4. Ans.** $i_1 = 0.1110$ A, $i_2 = 0.096$ A

5. Ans. (i) $-\frac{\mu_0 I}{4R} q v_0 \hat{k}$ (ii) $F_1 = 2IRB$ $F_2 = 2IRB$, Net force = $F_1 + F_2 = 4IRB\ \hat{i}$

6. Ans. (a) 6.6×10^{-5}T, (b) 0, 0, $2 \times 10^{-5}\,\ell n\left(\frac{3}{2}\right)$N **7. Ans.** (a) $\frac{3mv^2}{4qa}$, (b) $\frac{3mv^3}{4a}$, (c) zero

8. Ans. $\frac{\mu_0 iqv}{2\pi a}$ **9. Ans.** 5 **10. Ans.** $\frac{mEI}{Be}$ **11. Ans.** 8

12. Ans. 4 **13. Ans.** 020 **14. Ans.** $z = 0$, $x = \pm\frac{d}{\sqrt{3}}$, (ii) $\frac{I}{2\pi d}\sqrt{\frac{\mu_0}{\pi\lambda}}$

15. Ans. (a) current in loop PQRS is clockwise from P to QRS., (b) $\vec{F} = BI_0b\,(3\hat{k}-4\hat{i})$, (c) $I = \frac{mg}{6bB_0}$

16. Ans. $F = \alpha a^2 i\;\hat{j}$ **17. Ans.** 4 **18. Ans.** $\omega = \frac{dT_0}{QR^2B}$

19. Ans. (a) $k = NAB$, (b) $C = \frac{2i_0NAB}{\pi}$, (c) $Q \times \sqrt{\frac{NAB\pi}{2li_0}}$

EXERCISE (O-1)

1. Ans. (B) **2. Ans. (C)** **3. Ans. (C)** **4. Ans. (A)** **5. Ans. (B)**
6. Ans. (C) **7. Ans. (B)** **8. Ans. (A)** **9. Ans. (B)** **10. Ans. (D)**
11. Ans. (A) **12. Ans. (C)** **13. Ans. (D)** **14. Ans. (D)** **15. Ans. (A)**
16. Ans. (B) **17. Ans. (A)** **18. Ans. (C)** **19. Ans. (C)** **20. Ans. (C)**
21. Ans. (C) **22. Ans. (B)** **23. Ans. (B)** **24. Ans. (C)** **25. Ans. (D)**
26. Ans. (D) **27. Ans. (B)** **28. Ans. (B)** **29. Ans. (C)** **30. Ans. (B)**
31. Ans. (C) **32. Ans. (A)** **33. Ans. (B)** **34. Ans. (A)** **35. Ans. (B)**
36. Ans. (A) **37. Ans. (B)** **38. Ans. (B)** **39. Ans. (C)** **40. Ans. (D)**
41. Ans. (C) **42. Ans. (A)** **43. Ans. (A)** **44. Ans. (B)** **45. Ans. (D)**
46. Ans. (C) **47. Ans. (C)** **48. Ans. (C)** **49. Ans. (B)** **50. Ans. (C)**
51. Ans. (A) **52. Ans. (D)** **53. Ans. (A)** **54. Ans. (C)** **55. Ans. (B)**
56. Ans. (B) **57. Ans. (A)** **58. Ans. (D)** **59. Ans. (A)** **60. Ans. (B)**
61. Ans. (A) **62. Ans. (A)** **63. Ans. (A)** **64. Ans. (B)** **65. Ans. (B)**
66. Ans. (D) **67. Ans. (B)** **68. Ans. (C)** **69. Ans. (C)** **70. Ans. (B)**
71. Ans. (D) **72. Ans. (A)** **73. Ans. (A)** **74. Ans. (B)** **75. Ans. (C)**
76. Ans. (D) **77. Ans. (B)** **78. Ans. (C)** **79. Ans. (C)** **80. Ans. (B)**
81. Ans. (D) **82. Ans. (A)** **83. Ans. (B)** **84. Ans. (A,B,C)** **85. Ans. (A,B,C)**
86. Ans. (A,B,C) **87. Ans. (A, C, D)** **88. Ans. (A,C,D)** **89. Ans. (A, C)** **90. Ans.(C)**
91. Ans. (B,D) **92. Ans. (A,D)** **93. Ans. (A,B)** **94. Ans. (A)** **95. Ans. (D)**
96. Ans. (C) **97. Ans. (B)** **98. Ans. (D)**
99. Ans. (A)-q,s; (B)-p,s; (C)-q,s; (D)-p,s
100. Ans. (A)-s; (B)-s; (C)-q; (D)-r; (E)-p

EXERCISE (O-2)

1. Ans. (C) **2. Ans. (A)** **3. Ans. (C)** **4. Ans. (C)** **5. Ans. (A)** **6. Ans. (B)**

7. Ans. (C) **8. Ans. (C)** **9. Ans. (C)** **10. Ans. (B)** **11. Ans. (A)** **12. Ans. (C)**

13. Ans. (A) **14. Ans. (C)** **15. Ans. (C)** **16. Ans. (A)** **17. Ans. (C)** **18. Ans. (A)**

19. Ans. (A) **20. Ans. (B)** **21. Ans. (D)** **22. Ans. (A)** **23. Ans. (A,B,C)**

24. Ans. (B,C,D) **25. Ans. (A) → (P,R, T) ; (B) → (P,Q) ; (C) → (S) ; (D) → (P)**

EXERCISE (JM)

1. Ans. (2) **2. Ans. (3)** **3. Ans. (2)** **4. Ans. (4)** **5. Ans. (2)** **6. Ans. (2)**

7. Ans. (1) **8. Ans. (4)** **9. Ans. (1)** **10. Ans. (2)** **11. Ans. (1)** **12. Ans. (1)**

13. Ans. (3) **14. Ans. (1)** **15. Ans. (2)** **16. Ans. (2)** **17. Ans. (2)** **18. Ans. (1)**

19. Ans.(3) **20. Ans.(1)** **21. Ans.(4)** **22. Ans.(4)** **23. Ans. (3)** **24. Ans. (4)**

25. Ans. (4) **26. Ans. (2)** **27. Ans. (2)** **28. Ans. (2)** **29. Ans. (1)** **30. Ans. (3)**

31. Ans.(2) **32. Ans. (1)** **33. Ans.(2)** **34. Ans.(4)** **35. Ans.(1)** **36. Ans.(20)**

37. Ans.(2) **38. Ans.(4)** **39. Ans.(1)** **40. Ans.(1)**

EXERCISE (JA)

1. Ans. (C) **2. Ans. (A)** **3. Ans. (B)** **4. Ans. (BC,BD, BCD)** **5. Ans. (A)**

6. Ans. (C,D) **7. Ans. 5** **8. Ans. (D)** **9. Ans. (B)** **10. Ans. (A, C)**

11. Ans. (A, D) **12. Ans. 3** **13. Ans. (C)** **14. Ans. (B)** **15. Ans. (A,B,C)**

16. Ans. (A) **17. Ans. (C)** **18. Ans. (D)** **19. Ans. (B)** **20. Ans. (A) (B)**

21. Ans. (A,B,D) **22. Ans. 2 [1.99, 2.01]**

Important Notes

Chapter Contents 02

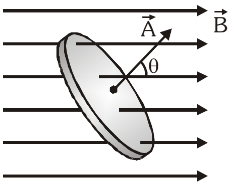
ELECTROMAGNETIC INDUCTION & ALTERNATING CURRENT

Important Notes

ELECTROMAGNETIC INDUCTION & ALTERNATING CURRENT

KEY CONCEPT

MAGNETIC FLUX



The magnetic flux (ϕ) linked with a surface held in a magnetic field (B) is defined as the number of magnetic lines of force crossing that area (A). If θ is the angle between the direction of the field and normal to the area, (area vector) then $\phi = \vec{B}.\vec{A} = BA\cos\theta$

FLUX LINKAGE

If a coil has more than one turn, then the flux through the whole coil is the sum of the fluxes through the individual turns. If the magnetic field is uniform, the flux through one turn is $\phi = BA\cos\theta$ If the coil has N turns, the total flux linkage $\phi = NBA\cos\theta$

- Magnetic lines of force are imaginary, magnetic flux is a real scalar physical quantity with dimensions

$$[\phi] = B \times \text{area} = \left[\frac{F}{IL}\right][L^2] \qquad \because\ B = \frac{F}{IL\sin\theta} \qquad [\because F = B\,I\,L\sin\theta]$$

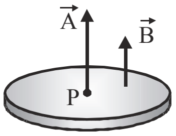
$$\therefore \quad [\phi] = \left[\frac{MLT^2}{AL}\right][L^2] = [ML^2T^{-2}A^{-1}]$$

SI UNIT of magnetic flux :

$\because$ $[ML^2T^{-2}]$ corresponds to energy

$$\frac{\text{joule}}{\text{ampere}} = \frac{\text{joule} \times \text{second}}{\text{coulomb}} = \text{ weber (Wb)}$$

or T–m² (as tesla = Wb/m²) $\left[\text{ampere} = \dfrac{\text{coulomb}}{\text{sec ond}}\right]$



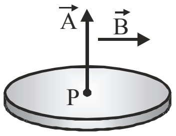
$\phi_{max} = B\,A$

- For a given area flux will be maximum :

when magnetic field $\vec{B}$ is normal to the area

$\theta = 0°$ $\Rightarrow$ $\cos\theta$ = maximum = 1 $\phi_{max} = B\,A$

For a given area flux will be minimum :

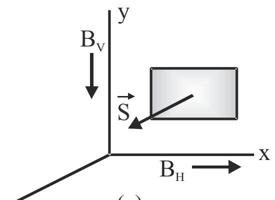


$\phi_{min} = 0$

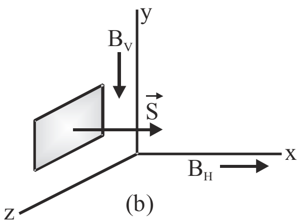
when magnetic field $\vec{B}$ is parallel to the area

$\theta = 90°$ $\Rightarrow \cos\theta$ = minimum = 0 $\phi_{min} = 0$

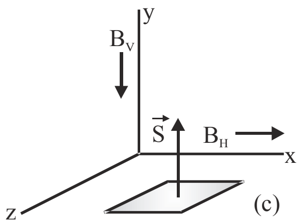
Ex. At a given plane, horizontal and vertical components of earth's magnetic field B_H and B_V are along x and y axes respectively as shown in figure. What is the total flux of earth's magnetic field associated with an area S, if the area S is in (a) x-y plane (b) y-z plane and (c) z-x plane ?



(a)



(b)



(c)

Sol. $\vec{B} = \hat{i}B_H - \hat{j}B_V = \text{constant}$, so $\phi = \vec{B}.\vec{S}$ [$\vec{B}$ = constant]

(a) For area in x-y plane $\vec{S} = S\hat{k}$, $\phi_{xy} = (\hat{i}B_H - \hat{j}B_V).(\hat{k}S) = 0$

(b) For area S in y-z plane $\vec{S} = S\hat{i}$, $\phi_{yz} = (\hat{i}B_H - \hat{j}B_V).(\hat{i}S) = B_H S$

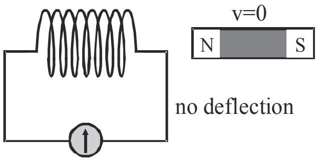
(c) For area S in z-x plane $\vec{S} = S\hat{j}$, $\phi_{zx} = (\hat{i}B_H - \hat{j}B_V).(\hat{j}S) = -B_V S$

Negative sign implies that flux is directed vertically downwards.

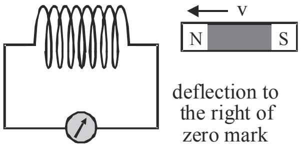
FARADAY'S EXPERIMENT

Faraday performed various experiments to discover and understand the phenomenon of electromagnetic induction. Some of them are :

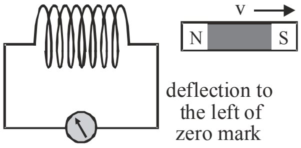
- When the magnet is held stationary anywhere near or inside the coil, the galvanometer does not show any deflection.



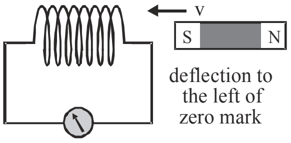
- When the N-pole of a strong bar magnet is moved towards the coil, the galvanometer shows a deflection right to the zero mark.



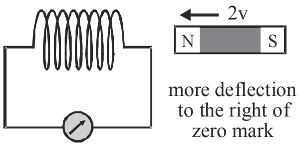
- When the N-pole of a strong bar magnet is moved away from the coil, the galvanometer shows a deflection left to the zero mark.



- If the above experiments are repeated by bringing the S-pole of the, magnet towards or away from the coil, the direction of current in the coil is opposite to that obtained in the case of N-pole.



- The deflection in galvanometer is more when the magnet moves faster and less when the magnet moves slow.



CONCLUSIONS

Whenever there is a relative motion between the source of magnetic field (magnet) and the coil, an emf is induced in the coil. When the magnet and coil move towards each other then the flux linked with the coil increases and emf is induced. When the magnet and coil move away from each other the magnetic flux linked with the coil decreases, again an emf is induced. This emf lasts so long the flux is changing.

Due to this emf an electric current start to flow and the galvanometer shows deflection.

The deflection in galvanometer last as long the relative motion between the magnet and coil continues.

Whenever relative motion between coil and magnet takes place an induced emf produced in coil. If coil is in closed circuit then current and charge is also induced in the circuit. This phenomenon is called electro magnetic induction.

FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION

Faraday’s law of induction may be stated as follows:

The induced emf ε in a coil is proportional to the negative of the rate of change of magnetic flux:

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

For a coil that consists of *N* loops, the total induced emf would be *N* times as large:

$$\varepsilon = -N\frac{d\Phi_B}{dt}$$

Thus, we see that an emf may be induced in the following ways:

(i) by varying the magnitude of $\vec{B}$ with time (illustrated in Figure)

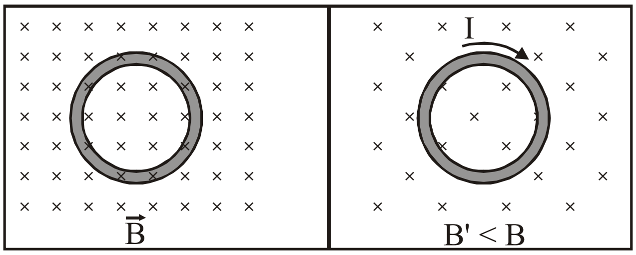


Figure : Inducing emf by varying the magnetic field strength

(ii) by varying the magnitude of $\vec{A}$, i.e., the area enclosed by the loop with time (illustrated in Figure)

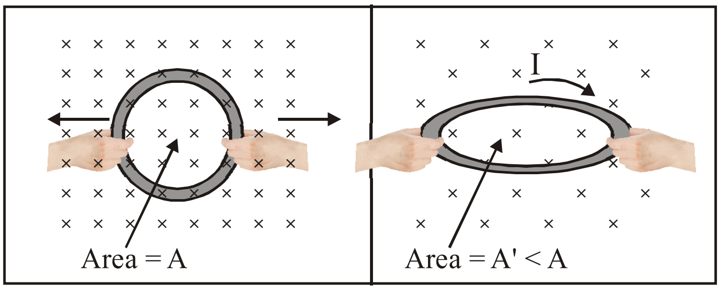


Figure : Inducing emf by changing the area of the loop

(iii) varying the angle between $\vec{B}$ and the area vector $\vec{A}$ with time (illustrated in Figure)

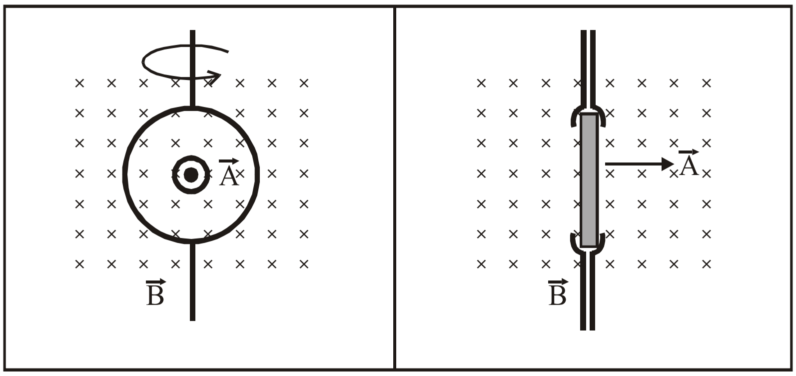


Figure : Inducing emf by varying the angle between **B** and **A**

LENZ'S LAW

Lenz's Law :

The direction of the induced current is determined by Lenz's law:

The induced current produces magnetic fields which tend to oppose the change in magnetic flux that induces such currents.

To illustrate how Lenz's law works, let's consider a conducting loop placed in a magnetic field. We follow the procedure below:

1. Define a positive direction for the area vector $\vec{A}$.

2. Assuming that $\vec{B}$ is uniform, take the dot product of $\vec{B}$ and $\vec{A}$. This allows for the determination of the sign of the magnetic flux Φ_B.

3. Obtain the rate of flux change $d\Phi_B/dt$ by differentiation. There are three possibilities:

$$\frac{d\Phi_B}{dt} : \begin{cases} > 0 \Rightarrow \text{induced emf } \varepsilon < 0 \\ < 0 \Rightarrow \text{induced emf } \varepsilon > 0 \\ = 0 \Rightarrow \text{induced emf } \varepsilon = 0 \end{cases}$$

4. Determine the direction of the induced current using the right-hand rule. With your thumb pointing in the direction of $\vec{A}$, curl the fingers around the closed loop. The induced current flows in the same direction as the way your fingers curl if $\varepsilon > 0$, and the opposite direction if $\varepsilon < 0$, as shown in Figure.

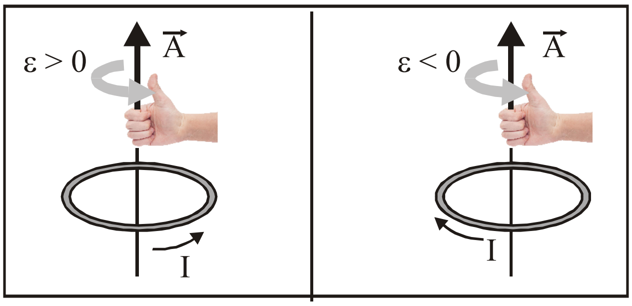


Figure : Determination of the direction of induced current by the right-hand rule

In Figure we illustrate the four possible scenarios of time-varying magnetic flux and show how Lenz's law is used to determine the direction of the induced current *I*.

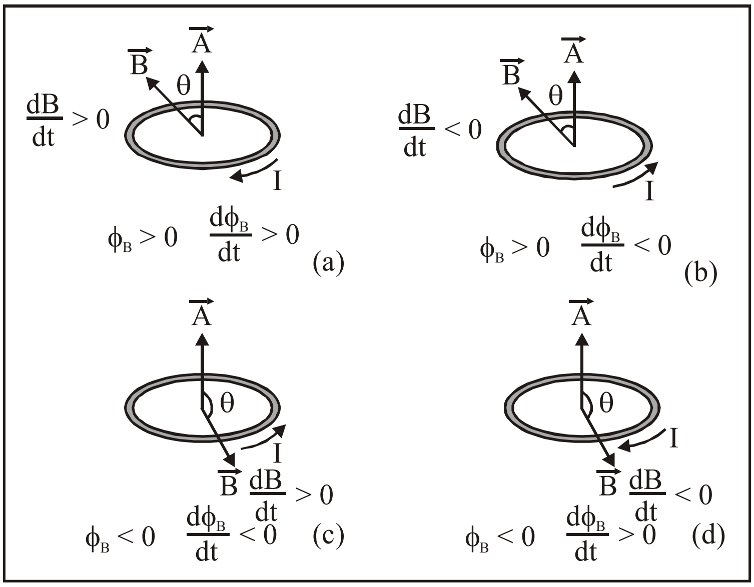


Figure : Direction of the induced current using Lenz's law

The above situations can be summarized with the following sign convention:

Φ_B	$d\Phi_B/dt$	ε	I
+	+	−	−
	−	+	+
−	+	−	−
	−	+	+

The positive and negative signs of *I* correspond to a counter clockwise and clockwise currents, respectively.

Ex. The radius of a coil decreases steadily at the rate of 10^{-2} m/s. A constant and uniform magnetic field of induction 10^{-3} Wb/m^2 acts perpendicular to the plane of the coil. What will be the radius of the coil when the induced e.m.f. in the 1μV

Sol. Induced emf $e = \frac{d(BA)}{dt} = \frac{Bd(\pi r^2)}{dt} = 2B\pi r\frac{dr}{dt}$ radius of coil $r = \frac{e}{2B\pi\left(\frac{dr}{dt}\right)} = \frac{1\times10^{-6}}{2\times10^{-3}\times\pi\times10^{-2}} = \frac{5}{\pi}$ cm

Ex. The ends of a search coil having 20 turns, area of cross-section 1 cm^2 and resistance 2 ohms are connected to a ballistic galvanometer of resistance 40 ohms. If the plane of search coil is inclined at 30° to the direction of a magnetic field of intensity 1.5 Wb/m^2, coil is quickly pulled out of the field to a region of zero magnetic field, calculate the charge passed through the galvanometer.

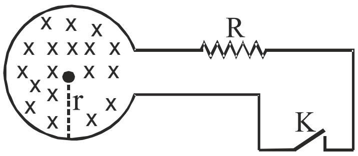
Sol. The total flux linked with the coil having turns N and area A is

$$\phi_1 = N(\vec{B}.\vec{A}) = NBA\cos\theta = NBA\cos(90^\circ - 30^\circ) = \frac{NBA}{2}$$

when the coil is pulled out, the flux becomes zero, $\phi_2 = 0$ so change in flux is $\Delta\phi = \frac{NBA}{2}$

the charge flowed through the circuit is $q = \frac{\Delta\phi}{R} = \frac{NBA}{2R} = \frac{20\times1.5\times10^{-4}}{2\times42} = 0.357\times10^{-4}$ C

Ex. Shown in the figure is a circular loop of radius r and resistance R. A variable magnetic field of induction $B = B_0e^{-t}$ is established inside the coil. If the key (K) is closed. Then calculate the electrical power developed right after closing the key.



Sol. Induced emf $\quad e = \frac{d\phi}{dt} = \frac{d}{dt}(BA) = A\frac{dB}{dt} = \pi r^2 B_0\frac{d}{dt}(e^{-t}) = -\pi r^2 B_0 e^{-t}$

At t = 0, $\quad e_0 = B_0 e^{-0}\,.\,\pi r^2 = B_0\,\pi r^2$

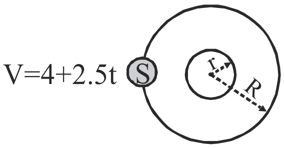
The electric power developed in the resistor R just at the instant of closing the key is

$$P = \frac{e_0^2}{R} = \frac{B_0^2\pi^2 r^4}{R}$$

Ex. Two concentric coplanar circular loops made of wire, resistance per unit length 10^{-4} Ωm^{-1}, have diameters 0.2 m and 2 m. A time-varying potential difference (4 + 2.5 t) volt is applied to the larger loop. Calculate the current in the smaller loop.

Sol. The magnetic field at the centre O due to the current in the larger loop is $B = \frac{\mu_0 I}{2R}$

If ρ is the resistance per unit length, then



$$I = \frac{\text{potential difference}}{\text{resistance}} = \frac{4+2.5t}{2\pi R \cdot \rho}$$

$$\therefore \quad B = \frac{\mu_0}{2R} \cdot \frac{4+2.5t}{2\pi R \rho}$$

$\because$ r << R, so the field B can be taken almost constant over the entire area of the smaller loop.

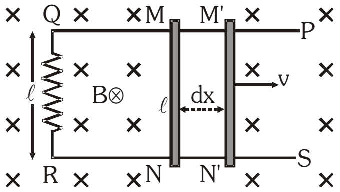
$\therefore$ the flux linked with the smaller loop is $\phi = B \times \pi r^2 = \frac{\mu_0}{2R} \cdot \frac{4+2.5t}{2\pi R \rho} \cdot \pi r^2$

Induced emf $e = \frac{d\phi}{dt} = \frac{\mu_0 r^2}{4R^2 \rho} \times 2.5$

The corresponding current in the smaller loop is I' then

$$I' = \frac{e}{R} = \frac{\mu_0 r^2}{4R^2 \rho} \times 2.5 \times \frac{1}{2\pi r \rho} = \frac{2.5 \mu_0 r}{8\pi R^2 \rho^2} = \frac{2.5 \times 4\pi \times 10^{-7} \times 0.1}{8\pi \times (1)^2 \times (10^{-4})^2} = 1.25 A$$

Induced emf by changing the area of the coil



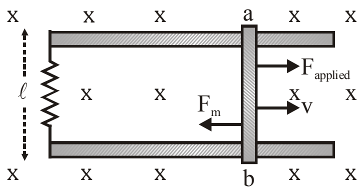
A U shaped frame of wire, PQRS is placed in a uniform magnetic field B perpendicular to the plane and vertically inward. A wire MN of length ℓ is placed on this frame. The wire MN moves with a speed v in the direction shown. After time dt the wire reaches to the position M'N' and distance covered = dx. The change in area ΔA = Length × area = ℓdx

Change in the magnetic flux linked with the loop in the dt is $d\phi = B \times \Delta A = B \times \ell dx$

induced emf $\quad e = \frac{d\phi}{dt} = B\ell\frac{dx}{dt} = B\,\ell\,v \;\because \left[v = \frac{dx}{dt}\right]$

If the resistance of circuit is R and the circuit is closed then the current through the circuit

$$I = \frac{e}{R} \qquad \Rightarrow I = \frac{Bv\ell}{R}$$



A magnetic force acts on the conductor in opposite direction of velocity is

$$F_m = i\ell B = \frac{B^2 \ell^2 v}{R}.$$

So, to move the conductor with a constant velocity v an

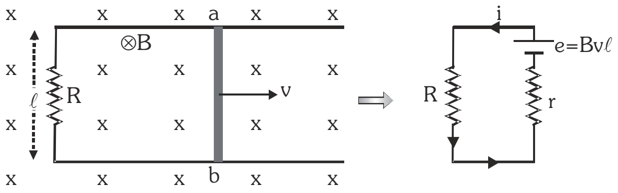
equal and opposite force F has to be applied in the conductor.

$$F = F_m = \frac{B^2 \ell^2 v}{R}$$

The rate at which work is done by the applied force is, $P_{\text{applied}} = Fv = \frac{B^2 \ell^2 v^2}{R}$

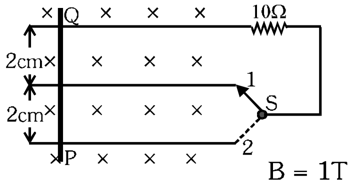
and the rate at which energy is dissipated in the circuit is, $P_{\text{dissipated}} = i^2 R = \left[\frac{Bv\ell}{R}\right]^2 R = \frac{B^2 \ell^2 v^2}{R}$

This is just equal to the rate at which work is done by the applied force.



- In the figure shown, we can replaced the moving rod ab by a battery of emf Bvℓ with the positive terminal at a and the negative terminal at b. The resistance r of the rod ab may be treated as the internal resistance of the battery.

Hence, the current in the ciruit is $i = \frac{e}{R+r} = \frac{Bv\ell}{R+r}$



Ex. Wire PQ with negligible resistance slides on the three rails with 5 cm/sec. Calculate current in 10 Ω resistance when switch S is connected to (a)position 1 (b)position 2

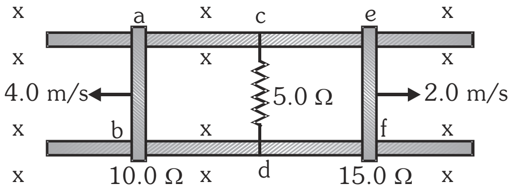
Sol. (a) For position 1

Induced current $I = \frac{e}{R} = \frac{Bv\ell}{R} = \frac{1 \times 5 \times 10^{-2} \times 2 \times 10^{-2}}{10} = 0.1 \text{ mA}$

(b) For position 2

Induced current $I = \frac{e}{R} = \frac{Bv(2\ell)}{R} = \frac{1 \times 5 \times 10^{-2} \times 4 \times 10^{-2}}{10} = 0.2 \text{ mA}$

Ex. Two parallel rails with negligible resistance are 10.0 cm apart. They are connected by a 5.0 Ω resistor. The circuit also contains two metal rods having resistance of 10.0Ω and 15.0 Ω along the rails (fig). The rods are pulled away from the resistor at constantspeeds 4.00 m/s and 2.00 m/s respectively. A uniform magnetic field of magnitude 0.01 T is applied perpendicular to the plane of the rails. Determine the current in the 5.0 Ω resistor.



Sol. Two conductors are moving in uniform magnetic field, so motional emf will induced across them.

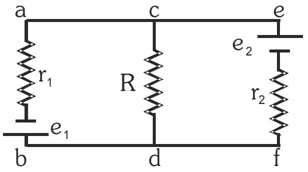
The rod ab will act as a source of emf $e_1 = Bv\ell = (0.01)(4.0)(0.1) = 4 \times 10^{-3}$ V

and internal resistance $r_1 = 10.0\ \Omega$

Similarly, rod ef will also act as source of emf $e_2 = (0.01)(2.0)(0.1) = 2 \times 10^{-3}$ V

and internal resistance $r_2 = 15.0\ \Omega$

From right hand rule : $V_b > V_a$ and $V_e > V_f$ Also $R = 5.0\ \Omega$,

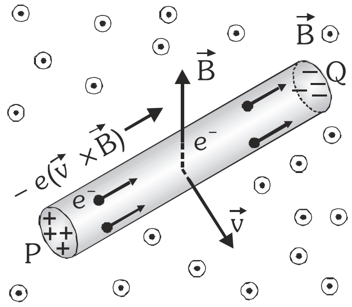


$$E_{eq} = \frac{e_1 r_2 - e_2 r_1}{r_1 + r_2} = \frac{6\times10^{-3} - 20\times10^{-3}}{15+10} = \frac{40}{25}\times10^{-3} = 1.6\times10^{-3} \text{ volt}$$

$$r_{eq} = \frac{15\times10}{15+10} = 6\Omega \quad \text{and}$$

$$I = \frac{E_{eq}}{r_{eq}+R} = \frac{1.6\times10^{-3}}{6+6} = \frac{1.6}{11}\times10^{-3} = \frac{8}{55}\times10^{-3} \text{ amp from d to c}$$

MOTIONAL EMF FROM LORENTZ FORCE



A conductor PQ is placed in a uniform magnetic field B, directed normal to the plane of paper outwards. PQ is moved with a velocity v, the free electrons of PQ also move with the same velocity. The electrons experience a magnetic Lorentz force, $\vec{F}_m = (\vec{v}\times\vec{B})$. According to Fleming's left hand rule, this force acts in the direction PQ and hence the free electrons will move towards Q. A negative chagre accumulates at Q and a positive charge at P. An electric field E is setup in the conductor from P to Q. Force exerted by electric field on the free electrons is, $\vec{F}_e = e\vec{E}$

The accumulation of charge at the two ends continues till these two forces balance each other.

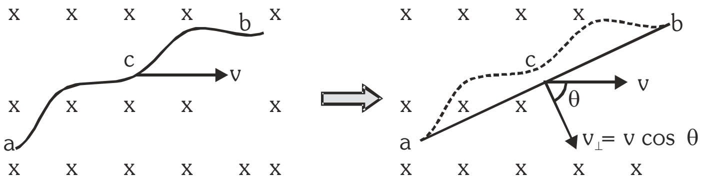
so $\vec{F}_m = -\vec{F}_e \Rightarrow e(\vec{v}\times\vec{B}) = -e\vec{E} \Rightarrow \vec{E} = -(\vec{v}\times\vec{B})$

The potential difference between the ends P and Q is $V = \vec{E}.\vec{\ell} = (\vec{v}\times\vec{B}).\vec{\ell}$ .It is the magnetic force on the moving free electrons that maintains the potential difference and produces the emf $\mathcal{E} = B\ell v$ (for $\vec{B}\perp\vec{v}\perp\vec{\ell}$)

As this emf is produced due to the motion of a conductor, so it is called a motional emf.

The concept of motional emf for a conductor can be generalized for any shape moving in any magnetic field uniform or not. For an element $\vec{d\ell}$ of conductor the contribution de to the emf is the magnitude $d\ell$ multiplied by the component of $\vec{v}\times\vec{B}$ parallel to $\vec{d\ell}$, that is $de = (\vec{v}\times\vec{B}).\vec{d\ell}$

For any two points a and b the motional emf in the direction from b to a is,

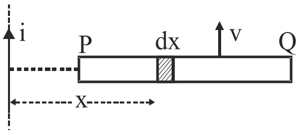


$$e = \int_b^a (\vec{v}\times\vec{B}).\vec{d\ell}$$

Motional emf in wire acb in a uniform magnetic field is the motional emf in an imaginary wire ab. Thus, $e_{acb} = e_{ab}$ = (length of ab) $(v_\perp)$ (B), $v_\perp$ = the component of velocity perpendicular to both $\vec{B}$ and ab. From right hand rule : b is at higher potential and a at lower potential. Hence, $V_{ba} = V_b - V_a$ = (ab) (v cosθ) (B)

Ex. A rod PQ of length L moves with a uniform velocity v parallel to a long straight wire carrying a current i, the end P remaining at a distance r from the wire. Calculate the emf induced across the rod. Take v = 5.0 m/s, i = 100 amp, r = 1.0 cm and L = 19 cm.

Sol. The rod PQ is moving in the magnetic field produced by the current-carrying long wire. The field is not uniform throughout the length of the rod (changing with distance). Let us consider a small element of length dx at distance x from wire. if magnetic field at the position of dx is B then emf induced



$$d\mathcal{E} = B\ v\ dx = \frac{\mu_0}{2\pi}\frac{i}{x} v\,dx$$

∴ emf $\mathcal{E}$ is induced in the entire length of the rod PQ is $\mathcal{E} = \int d\mathcal{E} = \int_P^Q \frac{\mu_0}{2\pi}\frac{i}{x} v\,dx$

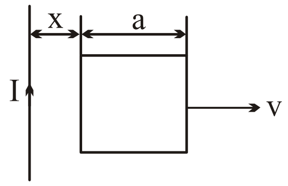
Now x = r at P, and x = r + L at Q. hence

$$\mathcal{E} = \frac{\mu_0\, i\, v}{2\pi} = \int_r^{r+L} \frac{dx}{x} = \frac{\mu_0\, i\, v}{2\pi}\left[\log_e x\right]_r^{r+L} = \frac{\mu_0\, i\, v}{2\pi}[\log_e (r+L) - \log_e r] = \frac{\mu_0\, i\, v}{2\pi}\log\frac{r+L}{r}$$

Putting the given values :

$$\mathcal{E} = (2 \times 10^{-7})\,(100)\,(5.0)\,\log_e \frac{1.0+19}{1.0} = 10^{-4}\log_e 20 \text{ Wb/s} = 3 \times 10^{-4} \text{ volt}$$

Ex. A square frame with side a and a long straight wire carrying a current I are located in the same plane as shown in Fig. The frame translates to the right with a constant velocity v. Find the emf induced in the frame as a function of distance x.



Ans. $\xi_i = \frac{\mu_0}{4\pi}\ \frac{2Ia^2v}{x\ (x\ +\ a)}$

Sol. Field, due to the current carrying wire, at a perpendicular distance x from it is given by,

$$B(x) = \frac{\mu_0}{2\pi}\frac{i}{x}$$

Motional emf is given by $\left|\int -(\vec{v}\times\vec{B})\,.\,d\vec{\ell}\right|$

There will be no induced emf in the segments (2) and (4) as, $\vec{v} \uparrow\uparrow d\vec{\ell}$ and magnitude of emf induced 1 and 3, will be

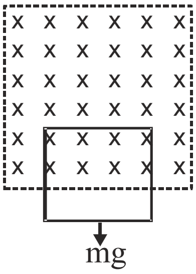
$$\xi_1 = v\left(\frac{\mu_0}{2\pi}\frac{i}{x}\right) a \text{ and } \xi_2 = v\left(\frac{\mu_0}{2\pi}\frac{i}{(a+x)}\right) a,$$

respectively, and their sense will be in the direction of $(\vec{v}\times\vec{B})$. So, emf induced in the network

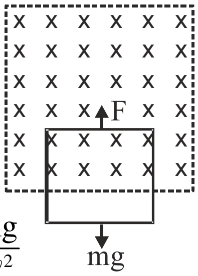
$= \xi_1 - \xi_2$ [as $\xi_1 > \xi_2$]

$$= \frac{av\mu_0 i}{2\pi}\left[\frac{1}{x} - \frac{1}{a+x}\right] = \frac{va^2\mu_0 i}{2\pi x(a+x)}$$

Ex. A horizontal magnetic field B is produced across a narrow gap between square iron pole-pieces as shown. A closed square wire loop of side ℓ, mass m and resistance R is allowed of fall with the top of the loop in the field. Show that the loop attains a terminal velocity given by $v = \dfrac{Rmg}{B^2\ell^2}$ while it is between the poles of the magnet.



Sol. As the loop falls under gravity, the flux passing through it decreases and so an induced emf is set up in it. Then a force F which opposes its fall. When this force becomes equal to the gravity force mg, the loop attains a terminal velocity v.

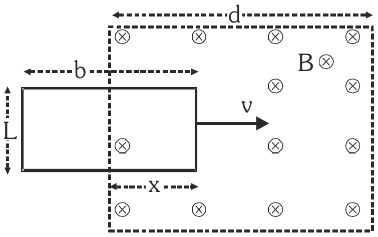


The induced emf e = B v ℓ, and the induced current is $i = \dfrac{e}{R} = \dfrac{Bv\ell}{R}$

The force experienced by the loop due to this current is $F = B\ell i = \dfrac{B^2v\ell^2}{R}$

When v is the terminal (constant) velocity F = mg or $\dfrac{B^2v\ell^2}{R} = mg$ or $v = \dfrac{R\,mg}{B^2\ell^2}$

Ex. Figure shows a rectangular conducting loop of resistance R, width L, and length b being pulled at constant speed v through a region of width d in which a uniform magnetic field B is set up by an electromagnet.Let L = 40 mm, b = 10 cm, d = 15 cm, R = 1.6 Ω, B = 2.0 T and v = 1.0 m/s



(i) Plot the flux ϕ through the loop as a function of the position x of the right side of the loop.

(ii) Plot the induced emf as a function of the positioin of the loop.

(iii) Plot the rate of production of thermal energy in the loop as a function of the position of the loop.

Sol. (i) When the loop is not in the field :

The flux linked with the loop $\phi = 0$

When the loop is entirely in the field :

Magnitic flux linked with the loop

$\phi = B\,L\,b$

$= 2 \times 40 \times 10^{-3} \times 10^{-1} = 8$ mWb

When the loop is entering the field :

The flux linked with the loop $\phi = B\,L\,x$

When the loop is leaving the field :

The flux $\phi = B\,L\,[b - (x - d)]$

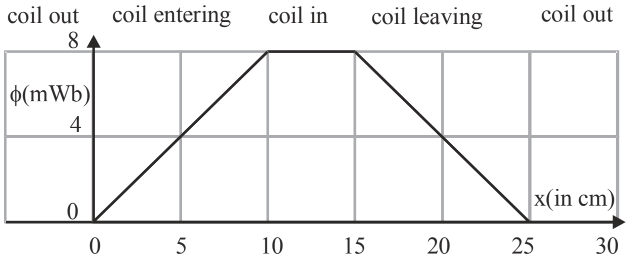


fig. (i)

(ii) Induced emf is $e = -\dfrac{d\phi}{dt} = -\dfrac{d\phi}{dx}\dfrac{dx}{dt} = -\dfrac{d\phi}{dx}v$

= – slope of the curve of figure (i) × v

The emf for 0 to 10 cm :

$$e = -\frac{(8-0)\times10^{-3}}{(10-0)\times10^{-2}}\times1 = -80\,\text{mV}$$

The emf for 10 to 15 cm : e = 0 × 1 = 0

The emf for 15 to 25 cm :

$$e = -\frac{(0-8)\times10^{-3}}{(25-15)\times10^{-2}}\times1 = +80\,\text{mV}$$

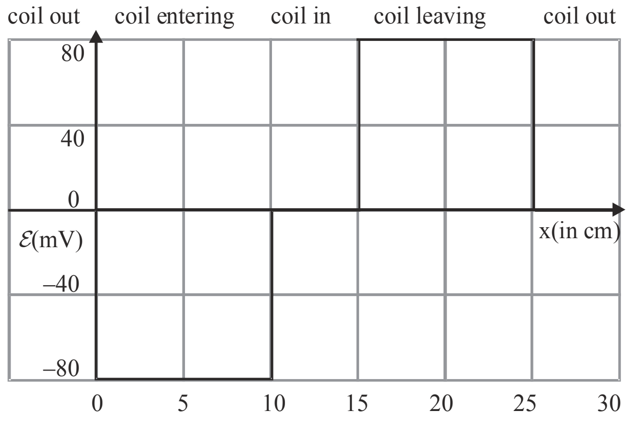


fig. (ii)

(iii) The rate of thermal energy production is $P=\dfrac{e^2}{R}$

for 0 to 10 cm : $P=\dfrac{(80\times10^{-3})^2}{1.6}=4\,\text{mW}$

for 10 to 15 cm : $P=0$

for 15 to 25 cm : $P=\dfrac{(80\times10^{-3})^2}{1.6}=4\,\text{mW}$

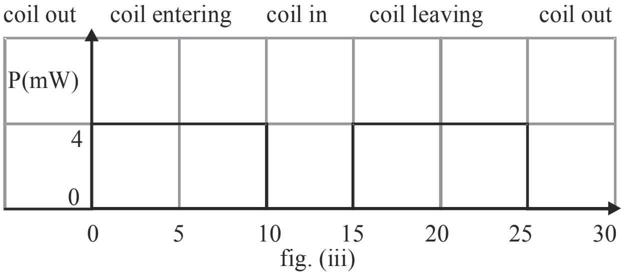
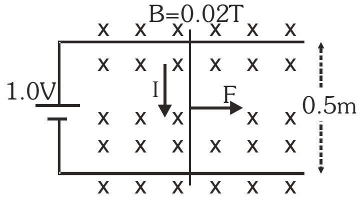


fig. (iii)

Ex. Two long parallel wires of zero resistance are connected to each other by a battery of 1.0 V. The separation between the wires is 0.5 m. A metallic bar, which is perpendicular to the wires and of resistance 10 Ω, moves on these wires. When a magneticfield of 0.02 testa is acting perpendicular to the plane containing the bar and the wires. Find the steady-state veclocity of the bar. If the mass of the bar is 0.002 kg then find its velocity as a function of time.



Sol. The current in the 10Ω bar is $I=\dfrac{1.0\,\text{V}}{10\,\Omega}=0.1\,\text{A}$

The current carrying bar is placed in the magnetic field $\vec{B}$ (0.2 T) perpendicular to the plane of paper and directed downwards.

The magnetic force of the bar is $F = B\,I\,\ell = 0.02\times 0.5\times 0.10 = 1\times 10^{-3}$ N

The moving bar cuts the lines of force of $\vec{B}$. If v be the instantaneous velocity of the bar, then the emf induced in the bar is $\mathcal{E}=B\ell v = 0.02\times 0.5\times v = 0.01\,v$ volt. By Lenz's law, $\mathcal{E}$ will oppose the motion of the bar which will ultimately attain a steady velocity. In this state, the induced emf $\mathcal{E}$ will be equal to the applied emf (1.0 volt). $\therefore\ 0.01\,v = 1.0$ or $v=\dfrac{1.0}{0.01}=100\ \text{ms}^{-1}$

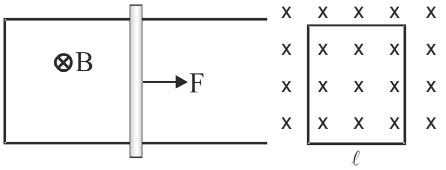
Again, a magnetic force F acts on the bar. If m be the mass of the bar, the acceleration of the rod is

$$\frac{dv}{dt}=\frac{F}{m}\Rightarrow dv=\frac{F}{m}.dt \qquad \text{Integrating, } \int dv=\int\frac{F}{m}dt\Rightarrow v=\frac{F}{m}t+C \text{ (constant)}$$

If at t = 0, v = 0 then C = 0. $\therefore\ v=\dfrac{F}{m}t$ But $F = 1\times 10^{-3}$ N, m = 0.002 kg

$$\therefore\quad v=\frac{1\times10^{-3}}{0.002}t=0.5\,t$$

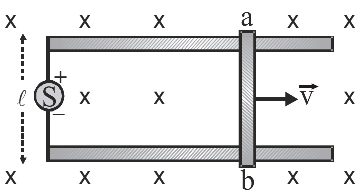
Ex. In figure, a rod closing the circuit moves along a U-shaped wire at a constant speed v under the action of the force F. The circuit is in a uniform magnetic field perpendicualr to its plane. Calculate F if the rate generation of heat is P.



Sol. The emf induced across the ends of the rod, $\mathcal{E}=B\ell v$

Current in the circuit, $I=\dfrac{\mathcal{E}}{R}=\dfrac{B\ell v}{R}$ Magnetic force on the conductor, $F' = I\ell B$, towards left

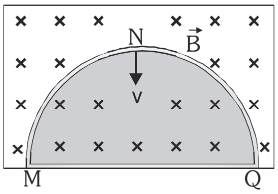
$\because$ acceleration is zero $F' = F \Rightarrow BI\ell = F$ or $I=\dfrac{F}{B\ell}$ $\because\ P=\mathcal{E}I=B\,\ell\,v\times\dfrac{F}{B\ell}=Fv$ $\therefore\ F=\dfrac{P}{v}$



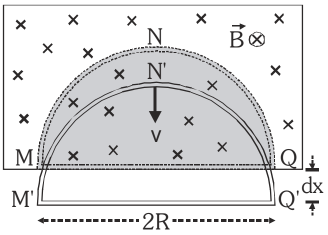
Ex. The diagram shows a wire ab of length ℓ and resistance R sliding on a smooth pair of rails with a velocity v towards right. A uniform magnetic field of induction B acts normal to the plane containing the rails and the wire inwards. S is a current source providing a constant I in the circuit. Determine the potential difference between a and b.

Sol. The wire ab which is moving with a velocity v is equivalent to an emf source of value B v ℓ with its positive terminal towards a.

$\therefore$ Potential difference $V_a - V_b = Bv\ell - IR$



Ex. A thin semicircular conduting ring of radius R is falling with its plane vertical in a horizontal magnetic induction $\vec{B}$ (fig.). At the position MNQ, the speed of the ring is v. What is the potential difference developed across the ring at the position MNQ ?



Sol. Let semiconductor ring falls through an infinitesimally small distance dx from its initial position MNQ to M'Q'N' in time dt (fig).

decrease in area of the ring inside the magnetic field,

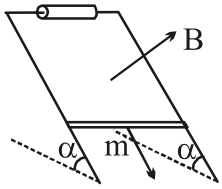
$dA = -$ MQQ'M' $= -$ M'Q' $\times$ QQ' $= -2R\,dx$

$\therefore$ change in magnetic flux linked with the ring,

$d\phi = B \times dA = B \times (-R\,dx) = -2BR\,dx$

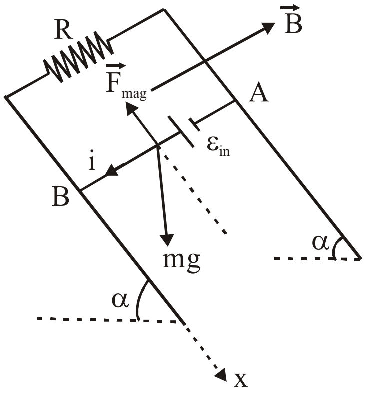
The potential difference developed across the ring, $e = -\dfrac{d\phi}{dt} = -\left[-2BR\dfrac{dx}{dt}\right] = 2BRv$

the speed with which the ring is falling $v = \dfrac{dx}{dt}$



Ex. A copper connector of mass m slides down two smooth copper bars, set at an angle α to the horizontal, due to gravity (Fig.). At the top the bars are interconnected through a resistance R. The separation between the bars is equal to l. The system is located in a uniform magnetic field of induction B, perpendicular to the plane in which the connector slides. The resistances of the bars, the connector and the sliding contacts, as well as the self-inductance of the loop, are assumed to be negligible. Find the steady-state velocity of the connector.

Ans. $v = \dfrac{mgR\sin\alpha}{B^2 l^2}$



Sol. From Lenz's law, the current through the connector is directed from A to B. Here $\xi_{in} = vB\ell$ between A and B.

where v is the velocity of the rod at any moment.

For the rod, from $F_x = mw_x$

or, $mg\sin\alpha - i\ell B = mw$

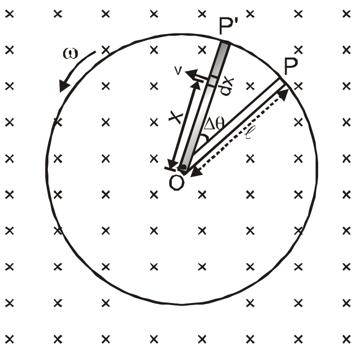
For steady state, acceleration of the rod must be equal to zero.

Hence, $mg\sin\alpha = i\ell B$ .... (1)

But, $i = \dfrac{\xi_{in}}{R} = \dfrac{vB\ell}{R}$

from (1) and (2) $v = \dfrac{mg\sin\alpha R}{B^2\ell^2}$

INDUCED E.M.F. DUE TO ROTATION OF A CONDUCTOR ROD IN A UNIFORM MAGNETIC FIELD



Let a conducting rod is rotating in a magnetic field around an axis passing through its one end, normal to its plane.

Consider an small element dx at a distance x from axis of rotation.

Suppose velocity of this small element = v

So, according to Lorent's formula induced e.m.f. across this small element

$$d\varepsilon = B\,v.\,dx$$

$\because$ This small element dx is at distance x from O (axis of rotation)

$\therefore$ Linear velocity of this element dx is $v = \omega x$

substitute of value of v in eq^n (i) $d\varepsilon = B\,\omega\,x\,dx$

Every element of conducting rod is normal to magnetic field and moving in perpendicular direction to the field

So, net induced e.m.f. across conducting rod $\varepsilon = \int d\varepsilon = \int_0^{\ell} B\omega\ x\ dx = \omega\ B\left(\dfrac{x^2}{2}\right)_0^{\ell}$

or $\varepsilon = \dfrac{1}{2}B\omega\ell^2$ $\varepsilon = \dfrac{1}{2}B\times 2\pi f\times\ell^2$ [f = frequency of rotation]

$= B\,f\,(\pi\ell^2)$ area traversed by the rod $A = \pi\,\ell^2$ or $\varepsilon = BAf$

Ex. A wheel with 10 metallic spokes each 0.5 m long is rotated with angular speed of 120 revolutions per minute in a plane normal to the earth's mangetic field. If the earth's magnetic field at the given place is 0.4 gauss, find the e.m.f. induced between the axle and the rim of the wheel.

Sol. $\omega = 2\pi n = 2\pi\times\dfrac{120}{60} = 4\pi$, $B = 0.4\ G = 4\times 10^{-5}$ T, length of each spoke = 0.5 m

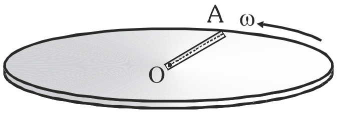
induced emf $e = \dfrac{1}{2}B\,\omega\,\ell^2 = \dfrac{1}{2}\times 4\times 10^{-5}\times 4\pi\times(0.5)^2 = 6.28\times 10^{-5}$ volt

As all the ten spokes are connected with their one end at the axle and the other end at the rim, so they are connected in parallel and hence emf across each spoke is same. The number of spokes is immaterial.

Ex. A horizontal copper disc of diameter 20 cm, makes 10 revolutions/sec about a vertical axis passing through its centre. A uniform magnetic field of 100 gauss acts perpendicular to the plane of the disc. Calculate the potential difference its centre and rim in volts.

Sol. $B = 100$ gauss $= 100\times 10^{-4}$ Wb/m^2 $= 10^{-2}$,

r = 10 cm = 0.10 m, frequency of rotaion = 10 rot/sec



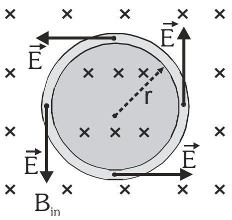
The emf induced between centre and rim $\mathcal{E} = \dfrac{1}{2}B\,\omega\,\ell^2 = \dfrac{1}{2}B\,\omega\,r^2\ (\because r = \ell)$

$\omega = 2\pi f = 2\times 3.14\times 10 = 62.8\ s^{-1}$

$\therefore\ \mathcal{E} = \dfrac{1}{2}\times 10\times 62.8\times(0.1)^2 = 3.14\times 10^{-3}$ V = 3.14 mV.

INDUCED ELECTRIC FIELD

When the magnetic field changes with time (let it increases with time) there is an induced electric field in the conductor caused by the changing magnetic flux.



Important properties of induced electric field :

(i) It is non conservative in nature. The line integral of $\vec{E}$ around a closed path is not zero. When a charge q goes once around the loop, the total work done on it by the electric field is equal to q times the emf.

Hence $\oint \vec{E}.\vec{d\ell} = e = -\frac{d\phi}{dt}$ ...(i)

This equation is valid only if the path around which we integrate is stationary.

(ii) Due to of symmetry, the electric field $\vec{E}$ has the same magnitude at every point on the circle and it is tangential at each point (figure).

(iii) Being nonconservative field, so the concept of potential has no meaning for such a field.

(iv) This field is different from the conservative electrostatic field produced by stationary charges.

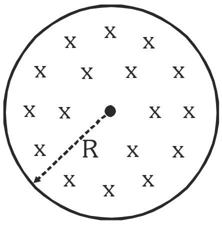
(v) The relation $\vec{F} = q\vec{E}$ is still valid for this field. (vi) This field can vary with time.

- For symmetrical situations $E\ell = \left|\frac{d\phi}{dt}\right| = A\left|\frac{dB}{dt}\right|$

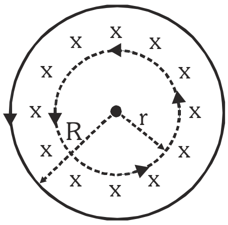
ℓ = the length of closed loop in which electric field is to be calculated
A = the area in which magnetic field is changing.

Direction of induced electric field is the same as the direction of included current.

Ex. The magnetic field at all points within the cylindrical region whose cross-section is indicated in the figure start increasing at a constant rate $\alpha\frac{\text{tesla}}{\text{sec ond}}$. Find the magnitude of electric field as a function of r, the distance from the geomatric centreof the region.



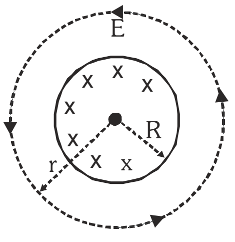
Sol. For r $\leq$ **R :**



$\because \quad E\ell = A\left|\frac{dB}{dt}\right|$

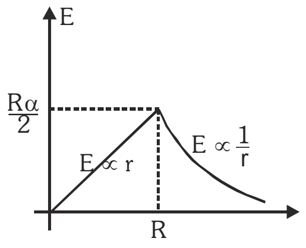
$\therefore \quad E(2\pi r) = (\pi r^2)\,\alpha \Rightarrow E = \frac{r\alpha}{2} \Rightarrow E \propto r$

E-r graph is straight line passing through origin.



At $\quad r = R, \qquad E = \dfrac{R\alpha}{2}$

For r $\geq$ R :



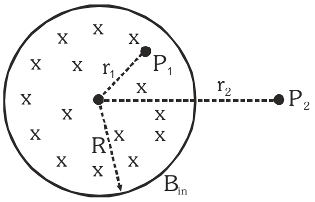
$$\because \qquad E\ell = A\left|\frac{dB}{dt}\right|$$

$$\therefore \qquad E\,(2\pi r) = (\pi R^2)\,\alpha \qquad \Rightarrow \; E = \frac{\alpha R^2}{2r} \;\Rightarrow\; E \propto \frac{1}{r}$$

Ex. For the situation described in figure the magnetic field changes with time according to, $B = (2.00\,t^3 - 4.00\,t^2 + 0.8)$ T and $r_2 = 2R = 5.0$ cm

(a) Calculate the force on an electron located at P_2 at t = 2.00 s

(b) What are the magnetude and direction of the electric field at P_1 when t = 3.00 s and $r_1 = 0.02$m.

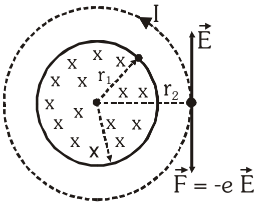


Sol. $E\ell = A\left|\dfrac{dB}{dt}\right| \;\Rightarrow\; E = \dfrac{\pi R^2}{2\pi r_2}\dfrac{d}{dt}(2t^3 - 4t^2 + 0.8) \;= \dfrac{R^2}{2r_2}(6t^2 - 8t)$

(a) Force on electron at P_2 is F = eE

$$\therefore \text{ at } t = 2\text{ s } \; F = \frac{1.6\times10^{-19}\times(2.5\times10^{-2})^2}{2\times5\times10^{-2}}\times[6(2)^2 - 8(2)]$$

$$= \frac{1.6}{4}\times2.5\times10^{-21}\times(24-16) = 8\times10^{-21}\text{N at } t = 2\text{s},$$



$\dfrac{dB}{dt}$ is positive so it is increasing.

$\therefore$ direction of induced current and E are as shown in figure and hence force of electron having charge –e is right perpendicular to r_2 downwards

(b) For $r_1 = 0.02$ m and at t = 3s, $E = \dfrac{\pi r_1^2}{2\pi r_1}(6t^2 - 8t) \;= \dfrac{0.02}{2}\times[6(3)^2 - 8(3)]$

$= 0.3$ V/m at t = 3sec, $\dfrac{dB}{dt}$

is positive so B is increasing and hence direction of E is same as in case (a) and it is left perpendicular to r_1 upwards.

Generators :

One of the most important applications of Faraday's law of induction is to generators and motors. A generator converts mechanical energy into electric energy, while a motor converts electrical energy into mechanical energy.

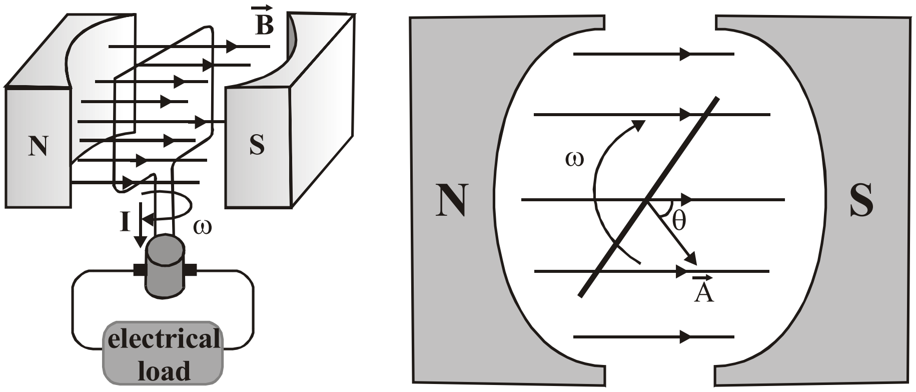


Figure : (a) A simple generator. (b) The rotating loop as seen from above.

Figure (a) is a simple illustration of a generator. It consists of an *N*-turn loop rotating in a magnetic field which is assumed to be uniform. The magnetic flux varies with time, thereby inducing an emf. From Figure (b), we see that the magnetic flux through the loop may be written as

$$\Phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta = BA\cos\omega t$$

The rate of change of magnetic flux is

$$\frac{d\Phi_B}{dt} = -BA\,\omega\sin\omega t$$

Since there are N turns in the loop, the total induced emf across the two ends of the loop is

$$\varepsilon = -N\frac{d\Phi_B}{dt} = NB\,A\omega\sin\omega t$$

If we connect the generator to a circuit which has a resistance R, then the current generated in the circuit is given by $I = \frac{|\varepsilon|}{R} = \frac{NBA\omega}{R}\sin\omega t$

The current is an alternating current which oscillates in sign and has an amplitude $I_0 = NBA\omega / R$. The power delivered to this circuit is

$$P = I|\varepsilon| = \frac{(NBA\omega)^2}{R}\sin^2\omega t$$

On the other hand, the torque exerted on the loop is

$$\tau = \mu B\sin\theta = \mu B\sin\omega t$$

Thus, the mechanical power supplied to rotate the loop is

$$P_m = \tau\omega = \mu B\,\omega\sin\omega t$$

Since the dipole moment for the N-turn current loop is

$$\mu = NIA = \frac{N^2A^2B\omega}{R}\sin\omega t$$

the above expression becomes

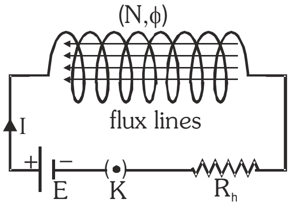
$$P_m = \left(\frac{N^2A^2B\omega}{R}\sin\omega t\right)B\omega\sin\omega t = \frac{(NAB\omega)^2}{R}\sin^2\omega t$$

As expected, the mechanical power put in is equal to the electric power output.

SELF INDUCTION

When the current through the coil changes, the magnetic flux linked with the coil also changes. Due to this change of flux a current induced in the coil itself according to lenz concept it opposes the change in magnetic flux. This phenomenon is called self induction and a factor by virtue of coil shows opposition for change in magnetic flux called cofficient of self inductance of coil.

Considering this coil circuit in two cases.



Case - I Current through the coil is constant

If $I \rightarrow B \rightarrow \phi$ (constant) $\Rightarrow$ No EMI

total flux of coil ($N\phi$) $\propto$ current through the coil

$$N\phi \propto I \Rightarrow N\phi = LI \qquad L = \frac{N\phi}{I} = \frac{NBA}{I} = \frac{\phi_{total}}{I}$$

where L = coefficient of self inductance of coil

S I unit of L : $1\,\frac{\text{Wb}}{\text{amp}} = 1 \text{ Henry} = 1\,\frac{\text{N.m}}{\text{A}^2} = 1\,\frac{\text{J}}{\text{A}^2}$ **Dimensions :** $[M^1L^2T^{-2}A^{-2}]$

Note : L is constant of coil it **does not depends on current** flow through the coil.

Case - II Current through the coil changes w.r.t. time

$$\text{If } \frac{dI}{dt} \rightarrow \frac{dB}{dt} \rightarrow \frac{d\phi}{dt} \Rightarrow \text{Static EMI} \qquad \Rightarrow N\phi = LI$$

$$-N\frac{d\phi}{dt} = -L\frac{dI}{dt}, \qquad \left(-N\frac{d\phi}{dt}\right) \text{ called total self induced emf of coil '}e_s\text{'}$$

$$e_s = -L\frac{dI}{dt} \qquad \textbf{S.I. unit of L} \rightarrow \frac{V.\,s}{A}$$

SELF-INDUCTANCE OF A PLANE COIL

Total magnetic flux linked with N turns,

$$\phi = NBA = N\left(\frac{\mu_0 NI}{2r}\right)A = \frac{\mu_0 N^2 I}{2r}A = \frac{\mu_0 N^2 I}{2r} \times \pi r^2 = \frac{\mu_0 \pi N^2 r}{2} I \text{ But } \quad \phi = LI \therefore L = \frac{\mu_0 \pi N^2 r}{2}$$

Ex. Self-Inductance of a Solenoid :

Compute the self-inductance of a solenoid with turns, length ℓ, and radius *NR* with a current *I* flowing through each turn, as shown in Figure.

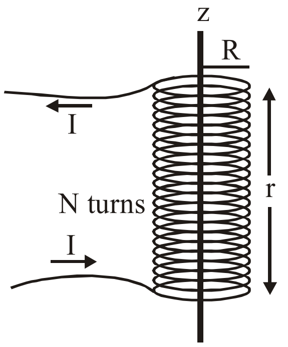


Figure : Solenoid

Solution:

Ignoring edge effects and applying Ampere's law, the magnetic field inside a solenoid is given by Eq. :

$$\vec{B} = \frac{\mu_0 NI}{\ell}\hat{k} = \mu_0 nI \quad \hat{k}$$

where n = N / ℓ is the number of turns per unit length. The magnetic flux through each turn is

$$\Phi_B = BA = \mu_0 nI \,.\, (\pi R_2) = \mu_0 nI\pi R^2$$

Thus, the self-inductance is $L = \dfrac{N\Phi_B}{I} = \mu_0 n^2 \pi R^2 \ell$

We see that *L* depends only on the geometrical factors (*n*, *R* and ℓ) and is independent of the current *I*.

Ex. **Self-Inductance of a Toroid :**

Calculate the self-inductance of a toroid which consists of *N* turns and has a rectangular cross section, with inner radius *a*, outer radius *b* and height *h*, as shown in Figure (a).

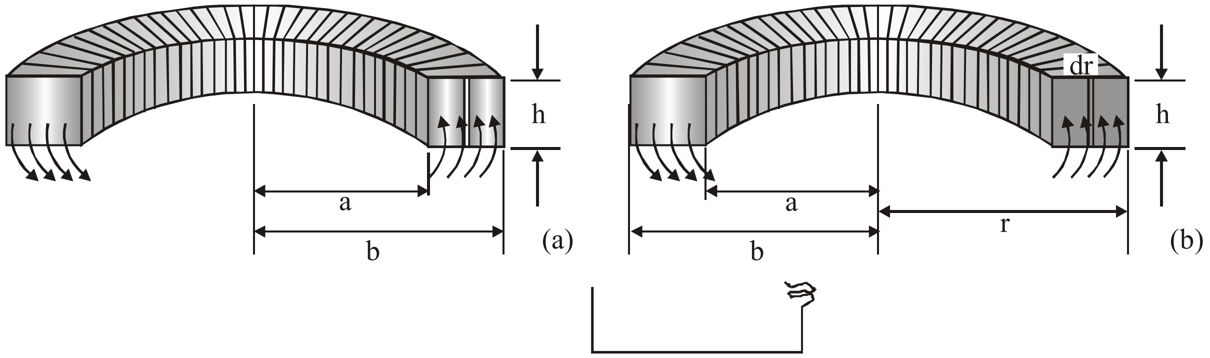


Figure : A toroid with *N* turns

Solution : According to Ampere's law discussed in section, the magnetic field is given by

$$\oint \vec{B}.d\vec{s} \oint Bds = B\oint ds = B\,(2\pi r) = \mu 0NI$$

or $\quad B = \dfrac{\mu_0 NI}{2\pi r}$

The magnetic flux through one turn of the torid may be obtained by integrating over the rectangular cross section,k with dA = hdr as the differential area element (figure-b)

$$\Phi_B = \iint \vec{B}\ .\ d\vec{A} = \int_a^b \left(\frac{\mu_0 NI}{2\pi r}\right) hdr = \frac{\mu_0 NIh}{2\pi}\ln\left(\frac{b}{a}\right)$$

The total flux is $N\Phi_B$. Therefore, the self-inductance is

$$L = \frac{N\Phi_B}{I} = \frac{\mu_0 N^2 h}{2\pi}\ln\left(\frac{b}{a}\right)$$

Again, the self-inductance L depends only on the geometrical factors. Let's consider the situation where a >>b – a. In this limit, the logarithmic term in the equation above may be expanded as

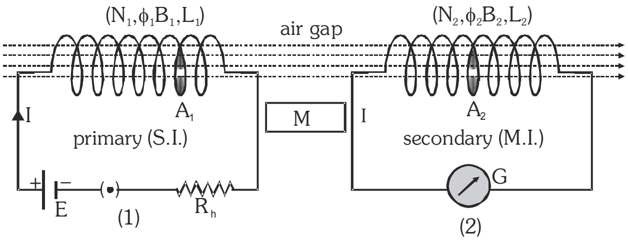
$$\ln\left(\frac{b}{a}\right) = \ln\left(1+\frac{b-a}{a}\right) \approx \frac{b-a}{a}$$

and the self-inductance becomes

$$L \approx \frac{\mu_0 N^2 h}{2\pi} \cdot \frac{b-a}{a} = \frac{\mu_0 N^2 A}{2\pi a} = \frac{\mu_0 N^2 A}{\ell}$$

where A = h(b – a) is the cross-sectional area, and $\ell = 2\pi a$. We see that the self inductance of the torid in this limit has the same form as that of a solenoid.

MUTUAL INDUCTION



Whenever the current passing through primary coil or circuit change then magnetic flux neighbouring secondary coil or circuit will also change. Acc. to Lenz for opposition of flux change, so an emf induced in the neighbouring coil or circuit.

This phenomenon called as 'Mutual induction'. In case of mutual inductance for two coils situated close to each other, flux linked with the secondary due to current in primary.

Due to Air gap always $\phi_2<\phi_1$ and $\phi_2 = B_1A_2$ $(\theta=0°)$.

Case - I When current through primary is constant

Total flux of secondary is directly proportional to current flow through the primary coil

$N_2\,\phi_2 \propto I_1 \Rightarrow N_2\,\phi_2 = MI_1$, $M = \frac{N_2\phi_2}{I_1} = \frac{N_2B_1A_2}{I_1} = \frac{(\phi_T)_s}{I_p}$ where M : is coefficient of mutual induction.

Case - II When current through primary changes with respect to time

$$\text{If} \quad \frac{dI_1}{dt} \rightarrow \frac{dB_1}{dt} \rightarrow \frac{d\phi_1}{dt} \rightarrow \frac{d\phi_2}{dt}$$

$\Rightarrow$ Static EMI $\quad N_2\phi_2 = MI_1 \; -N_2\frac{d\phi_2}{dt} = -M\frac{dI_1}{dt}, \left[-N_2\frac{d\phi}{dt}\right]$

Secondary ← $\boxed{e_m = -M\left(\frac{dI_1}{dt}\right)}$ → Primary

called total mutual induced emf of secondary coil e_m.

- The units and dimension of M are same as 'L'.
- The mutual inductance does not depends upon current through the primary and it is constant for circuit system.

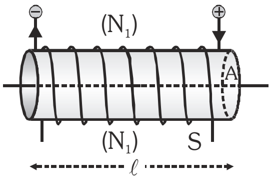
'M' depends on :

- Number of turns (N_1, N_2).
- Area of cross section.
- Distance between two coils (As $d\downarrow\; = M\uparrow$).
- **Coupling factor** 'K' between two coils.
- Cofficient of self inductance (L_1, L_2).
- Magnetic permeabibility of medium (μ_r).
- Orientation between two coils.

DIFFERENT COEFFICIENT OF MUTUAL INDUCTANCE

- In terms of their number of turns
- In terms of their coefficient of self inductances
- **In terms of their nos of turns (N_1, N_2)**

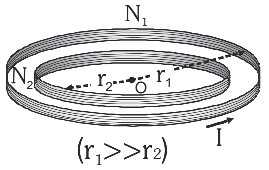
(a) Two co–axial solenoods :– ($M_{s_1s_2}$)



Coefficient of mutual inductance between two solenoids

$$M_{s_1s_2}=\frac{N_2B_1A}{I_1}=\frac{N_2}{I_1}\left[\frac{\mu_0N_1I_1}{\ell}\right]A \quad\Rightarrow M_{s_1s_2}=\left[\frac{\mu_0N_1N_2A}{\ell}\right]$$

(b) Two plane concentric coils ($M_{C_1C_2}$)

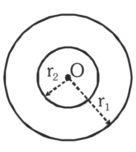


$$M_{c_1c_2}=\frac{N_2B_1A_2}{I_1}\text{ where }B_1=\frac{\mu_0N_1I_1}{2r_1},\ A_2=\pi r_2^2$$

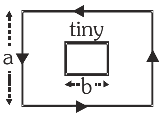
$$M_{c_1c_2}=\frac{N_2}{I_1}\left[\frac{\mu_0N_1I_1}{2r_1}\right](\pi r_2^2)\Rightarrow M_{c_1c_2}=\frac{\mu_0N_1N_2\pi r_2^2}{2r_1}$$

Two concentric loop : **Two concentric square loops :** **A square and a circular loop**

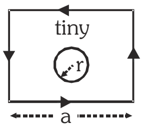
$M \propto \frac{r_2^2}{r_1}(r_1>>r_2)$



$M \propto \frac{b^2}{a}$



$M \propto \frac{r^2}{a}$

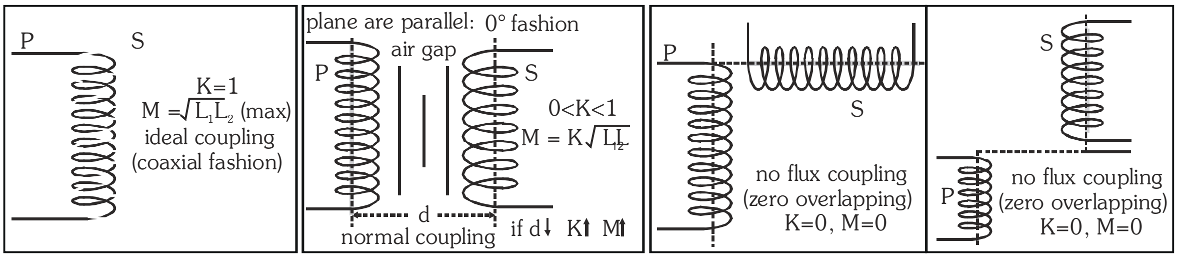


In terms of L_1 and L_2 : For two magnetically coupled coils :-

$M=K\sqrt{L_1L_2}$ here 'K' is coupling factor between two coils and its range $0\le K\le 1$

- For ideal coupling $K_{max}=1\Rightarrow$ $M_{max}=\sqrt{L_1L_2}$ (where M is geometrical mean of L_1 and L_2)
- For real coupling $(0<K<1)$ $M=K\sqrt{L_1L_2}$
- Value of coupling factor 'K' decided from fashion of coupling.

- **Different fashion of coupling**



'K' also defined as $K=\frac{\phi_s}{\phi_p}=\frac{\text{mag. flux linked with secondary (s)}}{\text{mag. flux linked with primary (p)}}$

INDUCTANCE IN SERIES AND PARALLEL

Two coil are connected in series : Coils are lying close together (M)

If M = 0, $L=L_1+L_2$ If $M\neq 0$ $L=L_1+L_2\pm 2M$

Two coils are connected in parallel :

(a) If M = 0 then $L=\frac{L_1L_2}{L_1+L_2}$ (b) If $M\neq 0$ then $L=\frac{L_1L_2-M^2}{L_1+L_2\pm 2M}$

Ex. A coil is wound on an iron core and looped back on itself so that the core has two sets of closely would wires in series carrying current in the opposite sense. What do you expect about its self-inductance ? Will it be larger or small ?

Sol. As the two sets of wire carry currents in opposite directions, their induced emf's also act in opposite directions. These induced emf's tend to cancel each other, making the self-inductance of the coil very small.

This situation is similar to two coils connected in series and producing fluxes in opposite directions. Therefore, their equivalent inductance must be $L_{eq} = L + L - 2M = L + L - 2L = 0$

Ex. A solenoid has 2000 turns wound over a length of 0.3 m. The area of cross-section is 1.2×10^{-3} m^2. Around its central section a coil of 300 turns is closely would. If an initial current of 2A is reversed in 0.25 s, find the emf induced in the coil.

Sol. $$M = \frac{\mu_0 N_1 N_2 A}{\ell} = \frac{4\pi\times10^{-7}\times2000\times300\times1.2\times10^{-3}}{0.3} = 3\times10^{-3}\,H$$

$$\mathcal{E} = -M\frac{dI}{dt} = -3\times10^{-3}\left[\frac{-2-2}{0.25}\right] = 48\times10^{-3}\,V = 48\text{ mV}$$

ENERGY STORED IN INDUCTOR

The energy of a capacitor is stored in the electric field between its plates. Similary, an inductor has the capability of storing energy in its magnetic field.An increasing current in an inductor causes an emf between its terminals.

Power P = The work done per unit time $= \frac{dW}{dt} = -ei = -\left[L\frac{di}{dt}\right]i = -L\,i\frac{di}{dt}$

here i = instanteneous current and L = inductance of the coil

From dW = – dU (energy stored) so $\frac{dW}{dt} = -\frac{dU}{dt}$ $\therefore$ $\frac{dU}{dt} = Li\frac{di}{dt} \Rightarrow dU = Li\,di$

The total energy U supplied while the current increases from zero to final value i is,

$$U = L\int_0^I i\,di = \frac{1}{2}L(i^2)_0^I \quad \therefore U = \frac{1}{2}L\,I^2$$

the energy stored in the magnetic field of an inductor when a current I is $= \frac{1}{2}L\,I^2$.

The source of this energy is the external source of emf that supplies the current.

- After the current has reached its final steady state value I, $\frac{di}{dt} = 0$ and no more energy is input to the inductor.
- When the current decreases from i to zero, the inductor acts as a source that supplies a total amount of energy $\frac{1}{2}Li^2$ to the external circuit. If we interrupt the circuit suddenly by opening a switch the current decreases very rapidly, the induced emf is very large and the energy may be dissipated in an arc the switch.

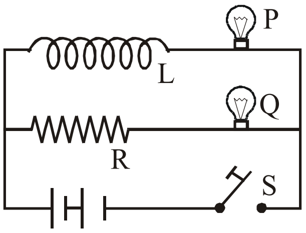
MAGNETIC ENERGY PER UNIT VOLUME OR ENERGY DENSITY

- The energy is an inductor is actually stored in the magnetic field within the coil. For a long solenoid its magnetic field can be assumed completely within the solenoid.

  The energy U stored in the solenoid when a current I is,

  $$U=\frac{1}{2}LI^2=\frac{1}{2}(\mu_0 n^2 V)I^2 \qquad (L=\mu_0 n^2 V) \qquad (V = \text{Volume} = A\ell)$$

  The energy per unit volume $u=\frac{U}{V}=\frac{1}{2}\mu_0 n^2 I^2=\frac{(\mu_0 n I)^2}{2\mu_0}=\frac{B^2}{2\mu_0}$ $\quad(B=\mu_0 n I)\ \therefore\ u=\frac{1}{2}\frac{B^2}{\mu_0}$



Ex. Figure shows an inductor L a resistor R connected in paralled to a battery through a switch. The resistance of R is same as that of the coil that makes L. Two identical bulb are put in each arm of the circuit.

(a) Which of two bulbs lights up earlier when S is closed?

(b) Will the bulbs be equally bright after some time?

Sol. (i) When switch is closed induced e.m.f. in inductor i.e. back e.m.f. delays the glowing of lamp P so lamp Q light up earlier.

(ii) Yes. At steady state inductive effect becomes meaningless so both lamps become equally bright after some time.

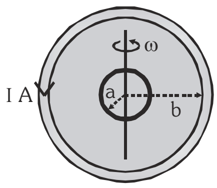
Ex. A very small circular loop of area 5×10^{-4} m², resistance 2 ohm and negligible inductance is initially coplanar and concentric with a much larger fixed circular loop of radius 0.1 m. A constant current of 1 ampere is passed in the bigger loop and the smaller loop is rotated with angular velocity ω rad/s about a diameter. Calculate (a) the flux linked with the smaller loop (b) induced emf and induced current in the smaller loop as a function of time.

Sol. (a) The field at the centre of larger loop $B_1=\frac{\mu_0 I}{2R}=\frac{2\pi\times10^{-7}}{0.1}=2\pi\times10^{-6}\ \text{Wb/m}^2$

is initially along the normal to the area of smaller loop. Now as the smaller loop (and hence normal to its plane) is rotating at angular velocity ω, with respect to $\vec{B}$ so the flux linked with the smaller loop at time t is, $\phi_2 = B_1 A_2 \cos\theta = (2\pi\times10^{-6})(5\times10^{-4})\cos\omega t$

i.e., $\phi_2=\pi\times10^{-9}\cos\omega t$ Wb

(b) The induced emf in the smaller loop



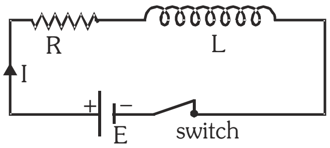
$$e_2=-\frac{d\phi_2}{dt}=-\frac{d}{dt}(\pi\times10^{-9}\cos\omega t)$$

$$=\pi\times10^{-9}\,\omega\sin\omega t\ \text{volt}$$

(c) The induced current in the smaller loop is, $I_2=\frac{e_2}{R}=\frac{1}{2}\pi\omega\times10^{-9}\sin\omega t$ ampere

R-L DC CIRCUIT

Current Growth



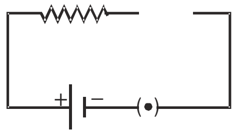
(i) emf equation $E = IR + L\frac{dI}{dt}$

(ii) Current at any instant

When key is closed the current in circuit increases exponentially with respect to time. The current in circuit at any instant 't' given by $I = I_0\left[1 - e^{\frac{-t}{\lambda}}\right]$

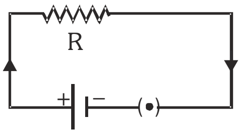
t = 0 (just after the closing of key) $\Rightarrow I = 0$

t = ∞ (some time after closing of key) $\Rightarrow I \rightarrow I_0$



(iii) Just after the closing of the key inductance behaves like open circuit and current in circuit is zero.

Open circuit, t = 0, I = 0 Inductor provide infinite resistence



(iv) Some time after closing of the key inductance behaves like simple connecting wire (short circuit) and current in circuit is constant.

Short circuit, $t \rightarrow \infty$, $I \rightarrow I_0$, Inductor provide zero resistence $I_0 = \frac{E}{R}$

(Final, steady, maximum or peak value of current) or ultimate current

Note : Peak value of current in circuit does not depends on self inductance of coil.

(v) Time constant of circuit (λ)

$\lambda = \frac{L}{R}$ sec. It is a time in which current increases up to 63% or 0.63 times of peak current value.

(vi) Half life (T)

It is a time in which current increases upto 50% or 0.50 times of peak current value.

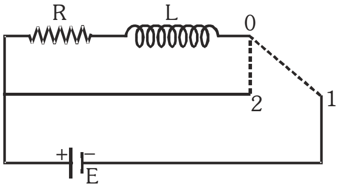
$$I = I_0(1 - e^{-t/\lambda}),\ t = T,\ I = \frac{I_0}{2} \Rightarrow \frac{I_0}{2} = I_0(1 - e^{-T/\lambda}) \Rightarrow e^{-T/\lambda} = \frac{1}{2} \Rightarrow e^{T/\lambda} = 2$$

$$\frac{T}{\lambda}\log_e e = \log_e 2 \quad \Rightarrow T = 0.693\lambda \quad \Rightarrow \quad T = 0.693\frac{L}{R}\ \text{sec.}$$

(vii) Rate of growth of current at any instant :–

$$\left[\frac{dI}{dt}\right] = \frac{E}{L}(e^{-t/\lambda}) \Rightarrow \quad t = 0 \Rightarrow \quad \left[\frac{dI}{dt}\right]_{max} = \frac{E}{L} \quad t = \infty \Rightarrow \quad \left[\frac{dI}{dt}\right] \rightarrow 0$$

Note : Maximum or initial value of rate of growth of current does not depends upon resistance of coil.



Current Decay

(i) Emf equation $IR + L\frac{dI}{dt} = 0$

(ii) Current at any instant

Once current acquires its final max steady value, if suddenly switch is put off then current start decreasing exponentially wrt to time. At switch put off condition t = 0, $I = I_0$, source emf E is cut off from circuit $I = I_0(e^{-t/\lambda})$

Just after opening of key t = 0 $\Rightarrow I = I_0 = \frac{E}{R}$

Some time after opening of key $t \rightarrow \infty$ $\Rightarrow I \rightarrow 0$

(iii) Time constant (λ)

It is a time in which current decreases up to 37% or 0.37 times of peak current value.

(iv) Half life (T)

It is a time in which current decreases upto 50% or 0.50 times of peak current value.

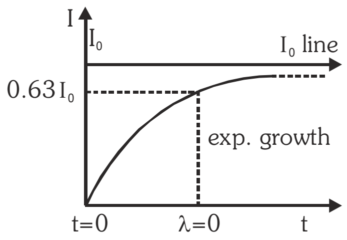
(v) Rate of decay of current at any instant

$$\left[-\frac{dI}{dt}\right]=\left[\frac{E}{L}\right]e^{-t/\lambda} \qquad t=0\ ,\ \left[-\frac{dI}{dt}\right]_{max.}=\frac{E}{L} \qquad t\to\infty \qquad \Rightarrow \qquad \left[-\frac{dI}{dt}\right]\to 0$$

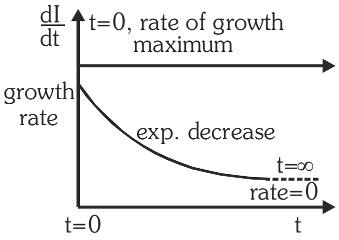
Graph for R–L circuit :–

Current Growth :–

(a)

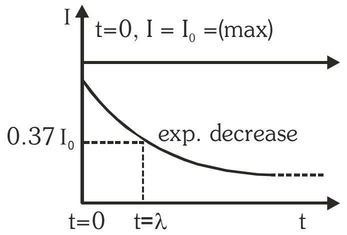


(b)

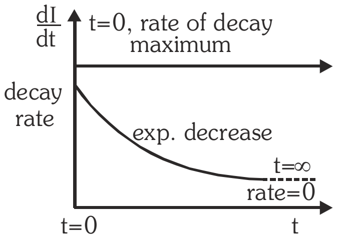


Current decay :–

(a)



(b)



LC Oscillations :

Consider an *LC* circuit in which a capacitor is connected to an inductor, as shown in Figure.

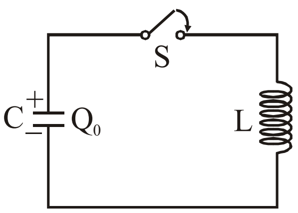


Figure *LC* Circuit

Suppose the capacitor initially has charge Q_0. When the switch is closed, the capacitor begins to discharge and the electric energy is decreased. On the other hand, the current created from the discharging process generates magnetic energy which then gets stored in the inductor. In the absence of resistance, the total energy is transformed back and forth between the electric energy in the capacitor and the magnetic energy in the inductor. This phenomenon is called electromagnetic oscillation.

The total energy in the *LC* circuit at some instant after closing the switch is

$$U=U_C+U_L=\frac{1}{2}\frac{Q^2}{C}+\frac{1}{2}LI^2$$

The fact that U remains constant implies that

$$\frac{dU}{dt}=\frac{d}{dt}\left(\frac{1}{2}\frac{Q^2}{C}+\frac{1}{2}LI^2\right)=\frac{Q}{C}\frac{dQ}{dt}+LI\frac{dI}{dt}=0$$

$$\frac{Q}{C}+L\,\frac{d^2Q}{dt^2}=0$$

where $I = -dQ / dt$ (and $dI / dt = -d^2Q/dt^2$). Notice the sign convention we have adopted here. The negative sign implies that the current I is equal to the rate of decrease of change in the capacitor plate immediately after the switch has been closed. The same equation can be obtained by applying the modified Kirchhoff's loop rule clockwise.

$$\frac{Q}{C}-L\frac{dI}{dt}=0$$

followed by our definition of current.

The general solution to equation is $Q(t) = Q_0 \cos(\omega_0 t + \phi)n$

where Q_0 is the amplitude of the charge and ϕ is the phase. The angular frequency ω_0 is given by

$$\omega_0=\frac{1}{\sqrt{LC}}$$

The corresponding current in the inductor is

$$I(t)=-\frac{dQ}{dt}=\omega_0Q_0\sin(\omega_0t+\phi)=I_0\sin(\omega_0t+\phi)$$

where $I_0 = \omega_0Q_0$. From the initial conditions $Q(t = 0) = Q_0$ and $I(t = 0) = 0$, the phase ϕ can be determined to $\phi = 0$. Thus, the solutions for the charge and the current in our LC circuit are

$Q(t) = Q_0 \cos \omega_0 t$

and $I(t) = I_0 \sin w_0 t$

The time dependence of Q(t) and I(t) are depicted in figure.

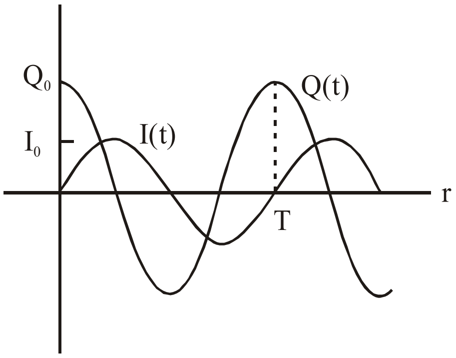


Figure : Charge and current in the *LC* circuit as a function of time

Using Eqs., we see that at any instant of time, the electric energy and the magnetic energies are given by

$$U_E=U_E=\frac{Q^2(t)}{2C}=\left(\frac{Q_0^2}{2C}\right)\cos^2\omega_0t$$

and $$U_B=\frac{1}{2}LI^2(t)=\frac{LI_0^2}{2}\sin^2\omega t=\frac{L(-\omega_0Q_0)^2}{2}\sin^2\omega_0t=\left(\frac{Q_0^2}{2C}\right)\sin^2\omega_0t=\frac{Q_0^2}{2C}$$

The electric and magnetic energy oscillation is illustrated in figure.

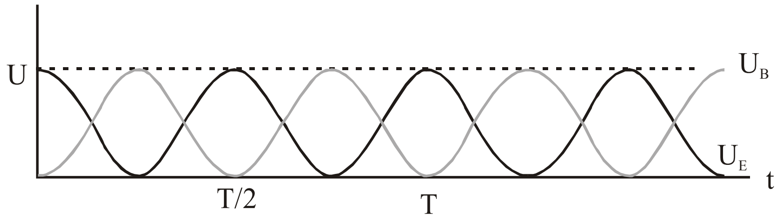
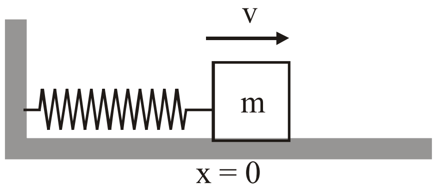


Figure : Electric and magnetic energy oscillations

The mechanical analog of the *LC* oscillations is the mass-spring system, shown in Figure.



x = 0

Figure : Mass-spring oscillations

If the mass is moving with a speed *v* and the spring having a spring constant *k* is displaced from its equilibrium by *x*, then the total energy of this mechanical system is

$$U = K + U_{sp} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

where K and U_{sp} are the kinetic energy of the mass and the potential energy of the spring, respectively. In the absence of friction, U is conserved and we obtain

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2 + \frac{1}{2}kx^2\right) = mv\frac{dv}{dt} + kx\frac{dx}{dt} = 0$$

Using v = dx/dt and dv/dt = d^2x/dt^2, the above equation may be rewritten as

$$m\frac{d^2x}{dt^2} + kx = 0$$

The general solution for the displacement is

$$x(t) = x_0 \cos(\omega_0 t + \phi)$$

where $\omega_0 = \sqrt{\frac{k}{m}}$

is the angular frequency and x_0 is the amplitude of the oscillations. Thus, at any instant in time, the energy of the system may be written as

$$U = \frac{1}{2}mx_0^2\omega_0^2 \sin^2(\omega_0 t + \phi) + \frac{1}{2}kx_0^2 \cos^2(\omega_0 t + \phi)$$

$$= \frac{1}{2}kx_0^2[\sin^2(\omega_0 t + \phi) + \cos^2(\omega_0 t + \phi) = \frac{1}{2}kx_0^2$$

In figure we illustrate the energy oscillations in the LC circuit and the mass spring system (harmonic oscillator).

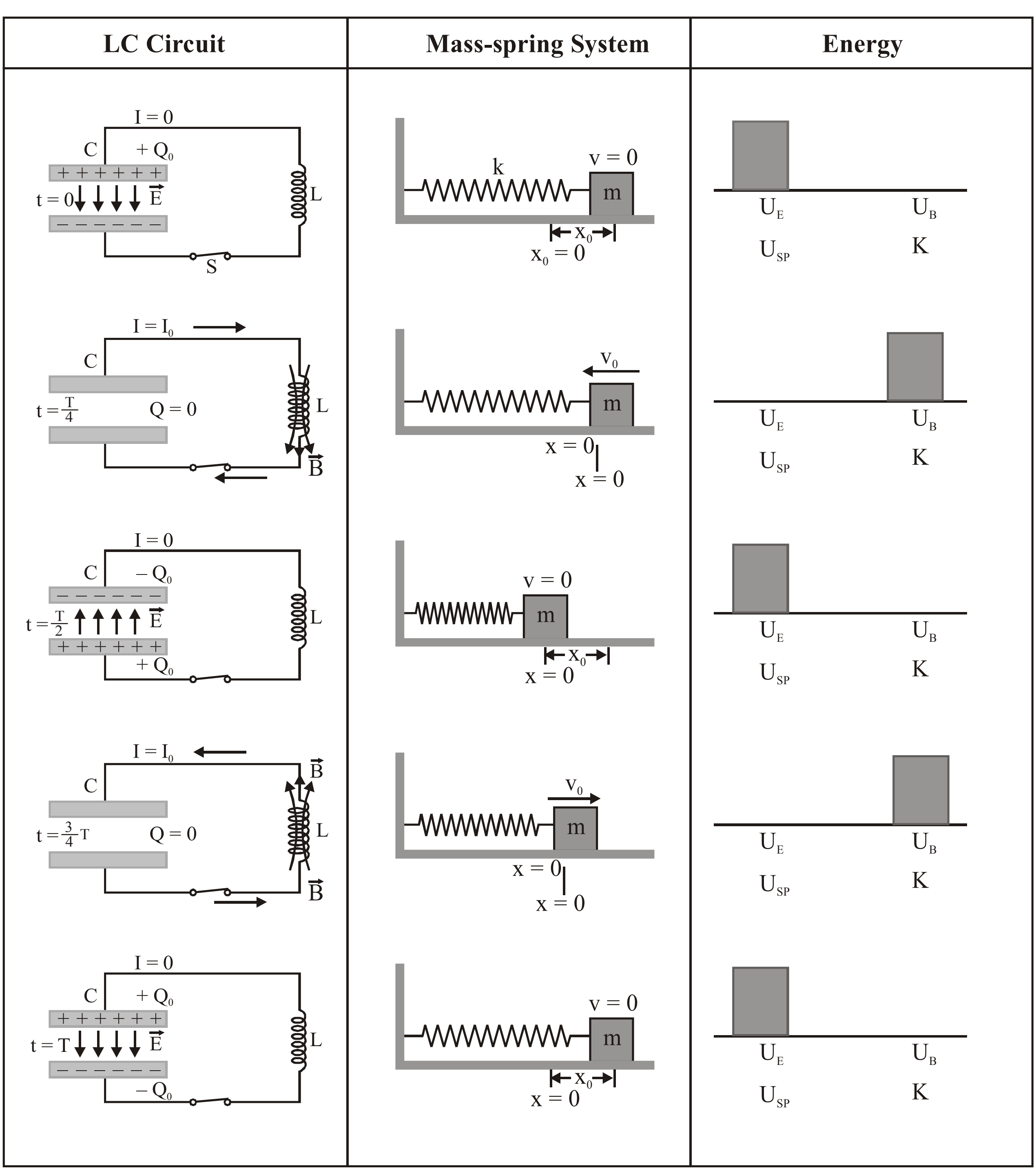


Figure : Energy oscillations in the *LC* Circuit and the mass-spring system

LC Circuit :

Ex. Consider the circuit shown in Figure. Suppose the switch which has been connected to point *a* for a long time is suddenly thrown to *b* at $t = 0$.

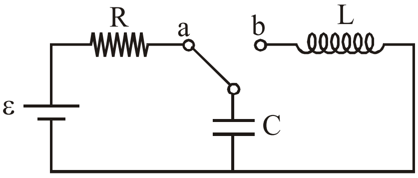


Figure : *LC* circuit

Find the following quantities :

(a) the frequency of oscillation of the LC circuit.

(b) the maximum charge that appears on the capacitor.

(c) the maximum current in the inductor.

(d) the total energy the circuit possesses at any time t.

Solution :

(a) The (angular) frequency of oscillation of the LC circuit is given by $\omega = 2\pi f = 1/\sqrt{LC}$. Therefore, the frequency is :

$$f = \frac{1}{2\pi\sqrt{LC}}$$

(b) The maximum charge stored in the capacitor before the switch is thrown to b is

$$Q = C\varepsilon$$

(c) The energy stored in the capacitor before the switch is thrown is :

$$U_E = \frac{1}{2}C\varepsilon^2$$

On the other hand, the magnetic energy stored in the inductor is :

$$U_B = \frac{1}{2}LI^2$$

Thus, when the current is at its maximum, all the energy originally stored in the capacitor is now in the inductor :

$$\frac{1}{2}C\varepsilon^2 = \frac{1}{2}LI_0^2$$

This implies a maximum current

$$I_0 = \varepsilon\sqrt{\frac{C}{L}}$$

(d) At any time, the total energy in the circuit would be equal to the initial energy that the capacitance stored, that is

$$U = U_E + U_B = \frac{1}{2}C\varepsilon^2$$

ALTERNATING CURRENT

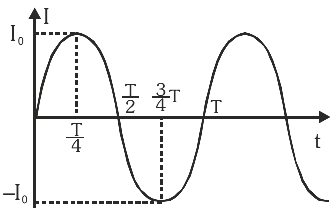
ALTERNATING CURRENT AND VOLTAGE

Voltage or current is said to be alternating if it is change continously in magnitude and perodically in direction with time. It can be represented by a sine curve or cosine curve

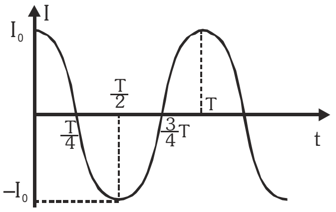
$I = I_0 \sin \omega t$ or $I = I_0 \cos \omega t$

where I = Instantaneous value of current at time t, I_0 = Amplitude or peak value

ω = Angular frequency $\omega = \frac{2\pi}{T} = 2\pi f$ T = time period f = frequency



I as a sine function of t



I as a cosine function of t

AMPLITUDE OF AC

The maximum value of current in either direction is called peak value or the amplitude of current. It is represented by I_0. Peak to peak value = $2I_0$

PERIODIC TIME

The time taken by alternating current to complete one cycle of variation is called periodic time or time period of the current.

FREQUENCY

The number of cycle completed by an alternating current in one second is called the frequency of the current.

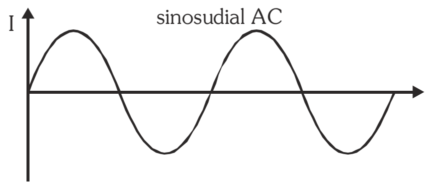
UNIT : cycle/s ; (Hz)

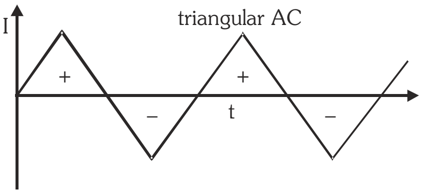
In India : f = 50 Hz ,supply voltage = 220 volt **In USA :** f = 60 Hz ,supply voltage = 110 volt

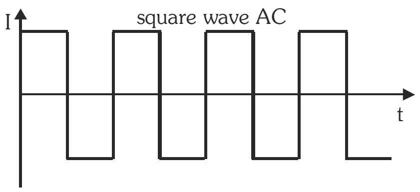
CONDITION REQUIRED FOR CURRENT/ VOLTAGE TO BE ALTERNATING

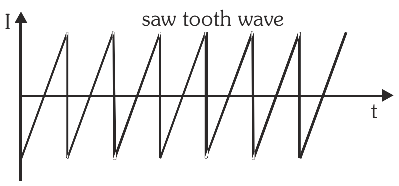
- Amplitude is constant
- Alternate half cycle is positive and half negative

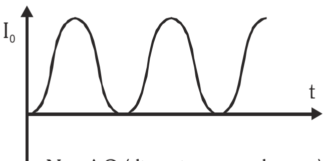
The alternating current continuously varies in magnitude and periodically reverses its direction.



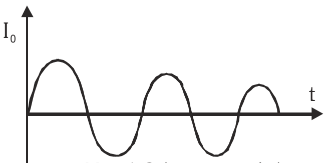




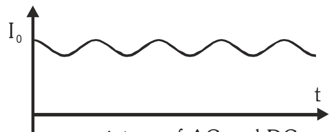




Not AC (direction not change)



Not AC (not periodic)



mixture of AC and DC

AVERAGE VALUE OR MEAN VALUE

The mean value of A.C over any half cycle (either positive or negative) is that value of DC which would send same amount of charge through a circuit as is sent by the AC through same circuit in the same time.

average value of current for half cycle $< I > = \dfrac{\int_0^{T/2} I dt}{\int_0^{T/2} dt}$

Average value of $I = I_0 \sin \omega t$ over the positive half cycle :

$$I_{av} = \frac{\int_0^{\frac{T}{2}} I_0 \sin \omega t \, dt}{\int_0^{\frac{T}{2}} dt} = \frac{2I_0}{\omega T}\left[-\cos \omega t\right]_0^{\frac{T}{2}} = \frac{2I_0}{\pi}$$

$< \sin\theta > = < \sin 2\theta > = 0$
$< \cos\theta > = < \cos 2\theta > = 0$
$< \sin\theta \cos\theta > = 0$
$< \sin^2\theta > = < \cos^2\theta > = \frac{1}{2}$

- For symmetric AC, average value over full cycle = 0,
  Average value of sinusoidal AC

Full cycle	(+ve) half cycle	(–ve) half cycle
0	$\frac{2I_0}{\pi}$	$\frac{-2I_0}{\pi}$

As the average value of AC over a complete cycle is zero, it is always defined over a half cycle which must be either positive or negative

MAXIMUM VALUE

- $I = a \sin\theta \Rightarrow I_{Max.} = a$
- $I = a + b \sin\theta \Rightarrow I_{Max.} = a + b$ (if a and b > 0)
- $I = a \sin\theta + b \cos\theta \Rightarrow I_{Max.} = \sqrt{a^2 + b^2}$
- $I = a \sin^2\theta \Rightarrow I_{Max.} = a \ (a > 0)$

ROOT MEAN SQUARE (rms) VALUE

It is value of DC which would produce same heat in given resistance in given time as is done by the alternating current when passed through the same resistance for the same time.

$$I_{rms} = \sqrt{\frac{\int_0^T I^2 dt}{\int_0^T dt}}$$

rms value = Virtual value = Apparent value

rms value of $I = I_0 \sin \omega t$:

$$I_{rms} = \sqrt{\frac{\int_0^T (I_0 \sin \omega t)^2 dt}{\int_0^T dt}} = \sqrt{\frac{I_0^2}{T}\int_0^T \sin^2 \omega t \, dt}$$

$$= I_0\sqrt{\frac{1}{T}\int_0^T \left[\frac{1-\cos 2\omega t}{2}\right] dt} = I_0\sqrt{\frac{1}{T}\left[\frac{t}{2} - \frac{\sin 2\omega t}{2\times 2\omega}\right]_0^T} = \frac{I_0}{\sqrt{2}}$$

- If nothing is mentioned then values printed in a.c circuit on electrical appliances, any given or unknown values, reading of AC meters are assumed to be RMS.

Current	**Average**	**Peak**	**RMS**	**Angular fequency**
$I_1 = I_0 \sin \omega t$	0	I_0	$\frac{I_0}{\sqrt{2}}$	ω
$I_2 = I_0 \sin \omega t \cos \omega t = \frac{I_0}{2}\sin 2\omega t$	0	$\frac{I_0}{2}$	$\frac{I_0}{2\sqrt{2}}$	2ω
$I_3 = I_0 \sin\omega t + I_0 \cos\omega t$	0	$\sqrt{2}\, I_0$	I_0	ω

- For above varieties of current rms = $\frac{\text{Peak value}}{\sqrt{2}}$

Ex. If I = 2 $\sqrt{t}$ ampere then calculate average and rms values over t = 2 to 4 s

Sol. $<I> = \frac{\int_2^4 2\sqrt{t}.dt}{\int_2^4 dt} = \frac{4}{3}\frac{(t^{\frac{3}{2}})_2^4}{(t)_2^4} = \frac{2}{3}\left[8-2\sqrt{2}\right]$ and $I_{rms} = \sqrt{\frac{\int_2^4 (2\sqrt{t})^2 dt}{\int_2^4 dt}} = \sqrt{\frac{\int_2^4 4t\,dt}{2}} = \sqrt{2\left[\frac{t^2}{2}\right]_2^4} = 2\sqrt{3}\text{ A}$

Ex. Find the time required for 50 Hz alternating current to change its value from zero to rms value.

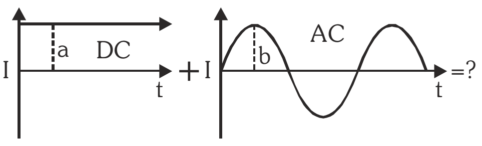
Sol. $\because I = I_0 \sin \omega t \;\therefore \frac{I_0}{\sqrt{2}} = I_0 \sin\omega t \Rightarrow \sin\omega t = \frac{1}{\sqrt{2}} \quad \omega t = \frac{\pi}{4}$

$$\Rightarrow \left(\frac{2\pi}{T}\right)t = \frac{\pi}{4} \Rightarrow t = \frac{T}{8} = \frac{1}{8\times 50} = 2.5\,\text{ms}$$

Ex. If E = 20 sin (100π t) volt then calculate value of E at $t = \frac{1}{600}$ s

Sol. At $t = \frac{1}{600}$ s E = 20 Sin $\left[100\pi\times\frac{1}{600}\right]$ = 20 sin $\left[\frac{\pi}{6}\right]$ = 20 × $\frac{1}{2}$ = 10V

Ex. If a direct current of value a ampere is superimposed on an alternating current I = b sinωt flowing through a wire, what is the effective value of the resulting current in the circuit ?



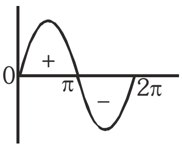
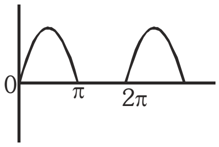
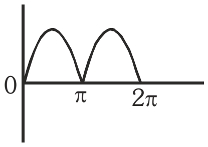
Sol. As current at any instant in the circuit will be,

$$I = I_{DC} + I_{AC} = a + b \sin\omega t$$

$$\therefore \quad I_{eff} = \sqrt{\frac{1}{T}\int_0^T I^2 dt} = \sqrt{\frac{1}{T}\int_0^T (a + b\sin\omega t)^2 dt} = \sqrt{\frac{1}{T}\int_0^T (a^2 + 2ab\sin\omega t + b^2\sin^2\omega t)dt}$$

but as $\frac{1}{T}\int_0^T \sin\omega t dt = 0$ and $\frac{1}{T}\int_0^T \sin^2\omega t dt = \frac{1}{2}$ $\quad\therefore\quad I_{eff} = \sqrt{a^2 + \frac{1}{2}b^2}$

SOME IMPORTANT WAVE FORMS AND THEIR RMS AND AVERAGE VALUE

Nature of wave form	Wave–form	RMS Value	Average or mean Value
Sinusoidal		$\frac{I_0}{\sqrt{2}}$ $= 0.707\ I_0$	$\frac{2I_0}{\pi}$ $= 0.637\ I_0$
Half wave rectifired		$\frac{I_0}{2}$ $= 0.5\ I_0$	$\frac{I_0}{\pi}$ $= 0.318\ I_0$
Full wave rectifired		$\frac{I_0}{\sqrt{2}}$ $= 0.707\ I_0$	$\frac{2I_0}{\pi}$ $0.637\ I_0$

PHASE AND PHASE DIFFERENCE

(a) Phase

$I = I_0 \sin(\omega t \pm \phi)$

Initial phase $= \phi$ (it does not change with time)

Instantaneous phase $= \omega t \pm \phi$ (it changes with time)

- Phase decides both value and sign. ◎ **UNIT:** radian

(b) Phase difference

Voltage $V = V_0 \sin(\omega t + \phi_1)$ Current $I = I_0 \sin(\omega t + \phi_2)$

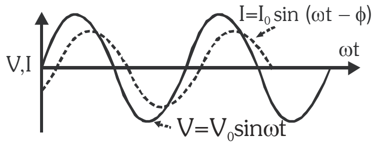
- Phase difference of I w.r.t. V $\phi = \phi_2 - \phi_1$
- Phase difference of V w.r.t. I $\phi = \phi_1 - \phi_2$

LAGGING AND LEADING CONCEPT

(a) V leads I or I lags V → It means, V reach maximum before I

Let if $V = V_0 \sin \omega t$ then $I = I_0 \sin(\omega t - \phi)$

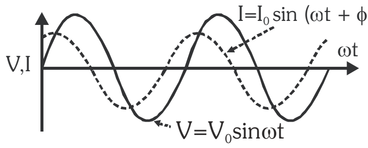
and if $V = V_0 \sin(\omega t + \phi)$ then $I = I_0 \sin \omega t$



(b) V lags I or I leads V → It means V reach maximum after I

Let if $V = V_0 \sin \omega t$ then $I = I_0 \sin(\omega t + \phi)$

and if $V = V_0 \sin(\omega t - \phi)$ then $I = I_0 \sin \omega t$



PHASOR AND PHASOR DIAGRAM

A diagram representing alternating current and voltage (of same frequency) as vectors (phasor) with the phase angle between them is called phasor diagram.

Let $V = V_0 \sin \omega t$ and $I = I_0 \sin(\omega t + \phi)$

In figure (a) two arrows represents phasors. The length of phasors represents the maximum value of quantity. The projection of a phasor on y-axis represents the instantaneous value of quantity

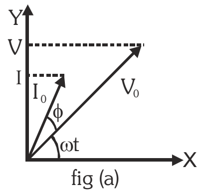


fig (a)

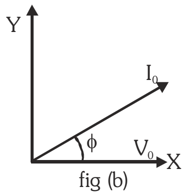


fig (b)

Ex. The Equation of current in AC circuit is $I = 4\sin\left[100\pi\, t + \frac{\pi}{3}\right]$ A. Calculate.

(i) RMS Value (ii) Peak Value (iii) Frequency (iv) Initial phase (v) Current at t = 0

Sol. (i) $I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$ A (ii) Peak value $I_0 = 4$A

(iii) $\because \omega = 100\,\pi$ rad/s $\therefore$ frequency $f = \frac{100\pi}{2\pi} = 50$ Hz

(iv) Initial phase $= \frac{\pi}{3}$ (v) At $t = 0$, $I = 4\sin\left[100\pi\times 0 + \frac{\pi}{3}\right] = 4\times\frac{\sqrt{3}}{2} = 2\sqrt{3}$ A

Ex. If $I = I_0 \sin \omega t$, $E = E_0 \cos\left[\omega t + \frac{\pi}{3}\right]$. Calculate phase difference between E and I

Sol. $I = I_0 \sin \omega t$ and $E = E_0 \sin\left[\frac{\pi}{2} + \omega t + \frac{\pi}{3}\right]$ $\therefore$ phase difference $= \frac{\pi}{2} + \frac{\pi}{3} = \frac{5\pi}{6}$

Ex. If $E = 500 \sin (100\,\pi t)$ volt then calculate time taken to reach from zero to maximum.

Sol. $\because \omega = 100\,\pi \Rightarrow T = \frac{2\pi}{100\pi} = \frac{1}{50}$ s, time taken to reach from zero to maximum $= \frac{T}{4} = \frac{1}{200}$ s

Ex. Show that average heat produced during a cycle of AC is same as produced by DC with $I = I_{rms}$.

Sol. For AC, $I = I_0 \sin\omega t$, the instantaneous value of heat produced (per second) in a resistance R is, $H = I^2R = I_0^2\sin^2\omega t \times R$ the average value of heat produced during a cycle is :

$$H_{av} = \frac{\int_0^T H\,dt}{\int_0^T dt} = \frac{\int_0^T (I_0^2 \sin^2 \omega t \times R)dt}{\int_0^T dt} = \frac{1}{2}I_0^2 R \quad \left[\because \int_0^T I_0^2 \sin^2 \omega t\, dt = \frac{1}{2}I_0^2 T\right]$$

$$\Rightarrow H_{av} = \left(\frac{I_0}{\sqrt{2}}\right)^2 R = I_{rms}^2 R \;.....(i)$$

However, in case of DC, $H_{DC} = I^2 R$...(ii) $\because I = I_{rms}$ so from equation (i) and (ii) $H_{DC} = H_{av}$ AC produces same heating effects as DC of value $I = I_{rms}$. This is also why AC instruments which are based on heating effect of current give rms value.

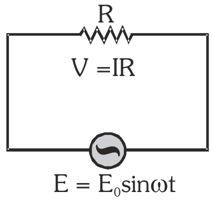
DIFFERENT TYPES OF AC CIRCUITS

In order to study the behaviour of A.C. circuits we classify them into two categories :

(a) Simple circuits containing only one basic element i.e. resistor (R) or inductor (L) or capacitor (C) only.

(b) Complicated circuit containing any two of the three circuit elements R, L and C or all of three elements.

AC CIRCUIT CONTAINING PURE RESISTANCE



Let at any instant t the current in the circuit = I.

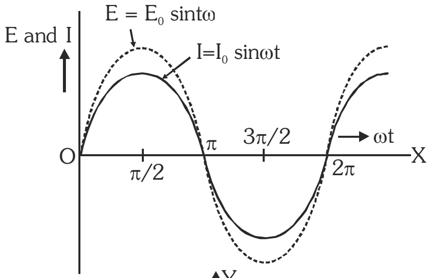
Potential difference across the resistance = I R.

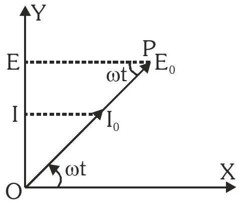
with the help of kirchoff's circuital law E – I R = 0

$\Rightarrow E_0 \sin \omega t = I R$

$$\Rightarrow I = \frac{E_0}{R}\sin \omega t = I_0 \sin \omega t \quad \left(I_0 = \frac{E_0}{R} = \text{peak or maximum value of current}\right)$$

Alternating current developed in a pure resistance is also of sinusoidal nature. In an a.c. circuits containing pure resistance, the voltage and current are in the same phase. The vector or phasor diagram which represents the phase relationship between alternating current and alternating e.m.f. as shown in figure. In the a.c. circuit having R only, as current and voltage are in the same phase, hence in fig. both phasors E_0 and I_0 are in the same direction, making an angle ωt with OX. Their projections on Y-axis represent the instantaneous values of alternating current and voltage.



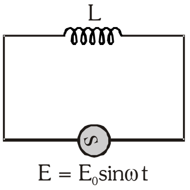


i.e. $I = I_0 \sin\omega t$ and $E = E_0 \sin\omega t$.

Since $I_0 = \dfrac{E_0}{R}$, hence $\dfrac{I_0}{\sqrt{2}} = \dfrac{E_0}{R\sqrt{2}}$ $\Rightarrow I_{rms} = \dfrac{E_{rms}}{R}$

AC CIRCUIT CONTAINING PURE INDUCTANCE

A circuit containing a pure inductance L (having zero ohmic resistance) connected with a source of alternating emf.



Let the alternating e.m.f. $E = E_0 \sin \omega t$

When a.c. flows through the circuit, emf induced across inductance $= -L\dfrac{dI}{dt}$

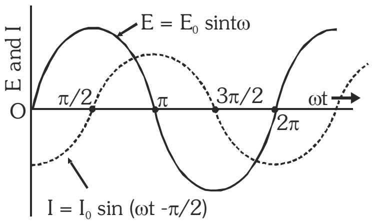
Negative sign indicates that induced emf acts in opposite direction to that of applied emf.

Because there is no other circuit element present in the circuit other then inductance so with the help of

Kirchoff's circuital law $E + \left(-L\dfrac{dI}{dt}\right) = 0$ $\Rightarrow E = L\dfrac{dI}{dt}$ so we get

$$I = \frac{E_0}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

Maximum current $I_0 = \dfrac{E_0}{\omega L} \times 1 = \dfrac{E_0}{\omega L}$, Hence, $I = I_0 \sin\left(\omega t - \dfrac{\pi}{2}\right)$



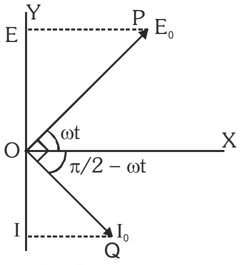
In a pure inductive circuit current always lags behind the emf by $\dfrac{\pi}{2}$.

or alternating emf leads the a. c. by a phase angle of $\dfrac{\pi}{2}$.

Expression $I_0 = \dfrac{E_0}{\omega L}$ resembles the expression $\dfrac{E}{I} = R$.

This non-resistive opposition to the flow of A.C. in a circuit is called the inductive reactance (X_L) of the circiut.

$X_L = \omega L = 2\pi f L$ where f = frequency of A.C.



E = E₀ sinωt
I=I₀ cosωt

Unit of X_L : ohm

(ωL) = Unit of L × Unit of ω = henry × $\sec^{-1}$

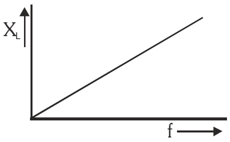
$$= \frac{\text{Volt}}{\text{Ampere / sec}} \times \sec^{-1} = \frac{\text{Volt}}{\text{Ampere}} = \text{ohm}$$

Inductive reactance $X_L \propto f$

Higher the frequency of A.C. higher is the inductive reactance offered by an inductor in an A.C. circuit.

For d.c. circuit, f = 0 $\therefore X_L = \omega L = 2\pi f L = 0$

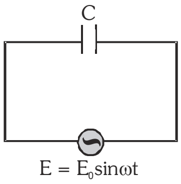
Hence, inductor offers no opposition to the flow of d.c. whereas a resistive path to a.c.



AC CIRCUIT CONTAINING PURE CAPACITANCE

A circuit containing an ideal capacitor of capacitance C connected with a source of alternating emf as shown in fig. The alternating e.m.f. in the circuit $E = E_0 \sin \omega t$

When alternating e.m.f. is applied across the capacitor a similarly varying alternating current flows in the circuit.



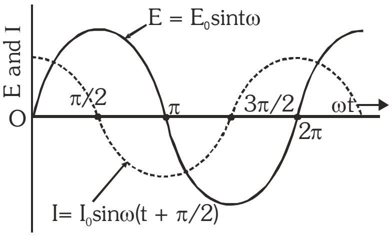
The two plates of the capacitor become alternately positively and negatively charged and the magnitude of the charge on the plates of the capacitor varies sinusoidally with time. Also the electric field between the plates of the capacitor varies sinusoidally with time.Let at any instant t charge on the capacitor = q

Instantaneous potential difference across the capacitor $E = \frac{q}{C}$

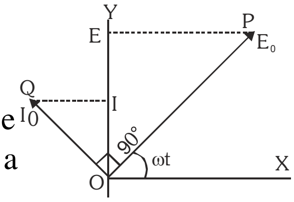
$\Rightarrow \quad q = C E \quad \Rightarrow q = CE_0 \sin \omega t$

The instantaneous value of current

$$I = \frac{dq}{dt} = \frac{d}{dt}\left(CE_0 \sin \omega t\right) = CE_0 \omega \cos \omega t$$

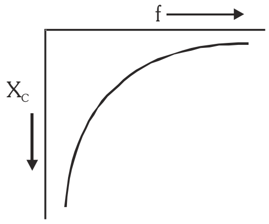


$$\Rightarrow \quad I = \frac{E_0}{(1/\omega C)} \sin\left(\omega t + \frac{\pi}{2}\right) = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) \text{ where } I_0 = \omega C V_0$$



In a pure capacitive circuit, the current always leads the e.m.f. by a phase angle of π/2. The alternating emf lags behinds the alternating current by a phase angle of π/2.

IMPORTANT POINTS



$\frac{E}{I}$ is the resistance R when both E and I are in phase, in present case they differ in phase by $\frac{\pi}{2}$, hence $\frac{1}{\omega C}$ is not the resistance of the capacitor, the capacitor offer opposition to the flow of A.C. This non-resistive opposition to the flow of A.C. in a pure capacitive circuit is known as capacitive reactance X_C. $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

Unit of X_C : ohm

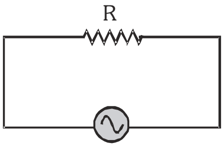
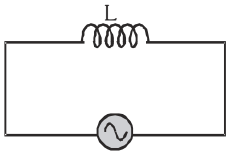
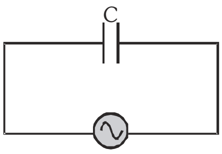
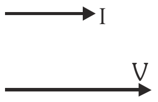
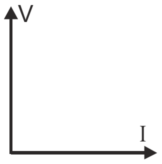
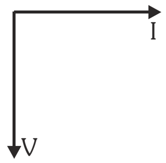
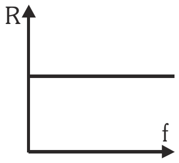
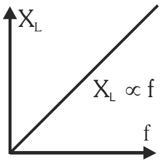
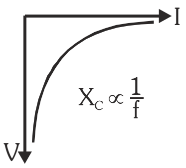
Capacitive reactance X_C is inversely proportional to frequence of A.C. X_C decreases as the frequency increases.

This is because with an increase in frequency, the capacitor charges and discharges rapidly following the flow of current.

For d.c. circuit f = 0 $\therefore X_C = \frac{1}{2\pi f C} = \infty$ but has a very small value for a.c.

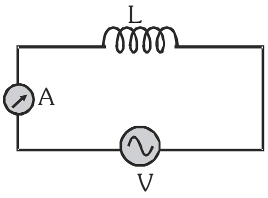
This shows that capacitor blocks the flow of d.c. but provides an easy path for a.c.

INDIVIDUAL COMPONENTS (R or L or C)

TERM	R	L	C
Circuit			
Supply Voltage	$V = V_0 \sin \omega t$	$V = V_0 \sin \omega t$	$V = V_0 \sin \omega t$
Current	$I = I_0 \sin \omega t$	$I = I_0 \sin (\omega t - \frac{\pi}{2})$	$I = I_0 \sin (\omega t + \frac{\pi}{2})$
Peak Current	$I_0 = \frac{V_0}{R}$	$I_0 = \frac{V_0}{\omega L}$	$I_0 = \frac{V_0}{1/\omega C} = V_0 \omega C$
Impedance (Ω)	$\frac{V_0}{I_0} = R$	$\frac{V_0}{I_0} = \omega L = X_L$	$\frac{V_0}{I_0} = \frac{1}{\omega C} = X_C$
$Z = \frac{V_0}{I_0} = \frac{V_{rms}}{I_{rms}}$	R = Resistance	X_L=Inductive reactance.	X_C=Capacitive reactance.
Phase difference	zero (in same phase)	$+\frac{\pi}{2}$ (V leads I)	$-\frac{\pi}{2}$ (V lags I)
Phasor diagram			
Variation of Z with *f*			
	R does not depend on *f*		
G, S_L, S_C (mho, seiman)	G=1/R = conductance.	$S_L = 1/X_L$ Inductive susceptance	$S_C = 1/X_C$ Capacitive susceptance
Behaviour of device in D.C. and A.C	Same in A C and D C	L passes DC easily (because $X_L = 0$) while gives a high impedance for the A.C. of high frequency ($X_L \propto f$)	C - blocks DC (because $X_C = \infty$) while provides an easy path for the A.C. of high frequency $\left[X_C \propto \frac{1}{f}\right]$
Ohm's law	$V_R = IR$	$V_L = IX_L$	$V_C = IX_C$

Ex. A capacitor of 50 pF is connected to an a.c. source of frequency 1kHz Calculate its reactance.

Sol. $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi\times10^3\times50\times10^{-12}} = \dfrac{10^7}{\pi}\ \Omega$



Ex. In given circuit applied voltage V = $50\sqrt{2}$ sin 100πt volt and ammeter reading is 2A then calculate value of L

Sol. $V_{rms} = I_{rms}X_L$ $\quad\because$ Reading of ammeter = I_{rms}

$$X_L = \frac{V_{rms}}{I_{rms}} = \frac{V_0}{\sqrt{2}\,I_{rms}} = \frac{50\sqrt{2}}{\sqrt{2}\times2} = 25\ \Omega \Rightarrow L = \frac{X_L}{\omega} = \frac{25}{100\pi} = \frac{1}{4\pi}\ H$$

Ex. A 50 W, 100 V lamp is to be connected to an AC mains of 200 V, 50 Hz. What capacitance is essential to be put in series with the lamp ?

Sol. $\because$ resistance of the lamp $R = \dfrac{V_s^2}{W} = \dfrac{(100)^2}{50} = 200\ \Omega$ and the maximum current $I = \dfrac{V}{R} = \dfrac{100}{200} = \dfrac{1}{2}A$

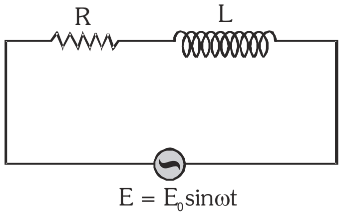
$\therefore$ when the lamp is put in series with a capacitance and run at 200 V AC, from V = IZ

$Z = \dfrac{V}{I} = \dfrac{200}{\frac{1}{2}} = 400\Omega$ Now as in case of C–R circuit $Z = \sqrt{R^2 + \dfrac{1}{(\omega C)^2}}$,

$$\Rightarrow R^2 + \frac{1}{(\omega C)^2} = (400)^2 \qquad \Rightarrow \frac{1}{(\omega C)^2} = 16\times10^4 - (200)^2 = 12\times10^4 \qquad \Rightarrow \frac{1}{\omega C} = \sqrt{12}\times10^2$$

$$\Rightarrow C = \frac{1}{100\pi\times\sqrt{12}\times10^2}F = \frac{100}{\pi\sqrt{12}}\mu F = 9.2\mu F$$

RESISTANCE AND INDUCTANCE IN SERIES (R-L CIRCUIT)

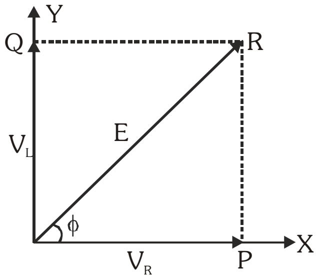


A circuit containing a series combination of a resistance R and an inductance L, connected with a source of alternating e.m.f. E as shown in figure.

PHASOR DIAGRAM FOR L-R CIRCUIT

Let in a L-R series circuit, applied alternating emf is $E = E_0 \sin\omega t$. As R and L are joined in series, hence current flowing through both will be same at each instant. Let I be the current in the circuit at any instant and V_L and V_R the potential differences across L and R respectively at that instant.

Then $V_L = IX_L$ and $V_R = IR$



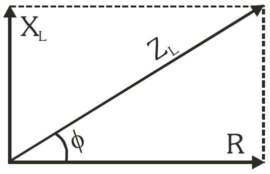
Now, V_R is in phase with the current while V_L leads the current by $\dfrac{\pi}{2}$.

So V_R and V_L are mutually perpendicular (Note : $E \neq V_R + V_L$)

The vector OP represents V_R (which is in phase with I), while OQ represents V_L (which leads I by 90°).

The resultant of V_R and V_L = the magnitude of vector OR $E = \sqrt{V_R^2 + V_L^2}$

Thus $E^2 = V_R^2 + V_L^2 = I^2(R^2 + X_L^2) \Rightarrow I = \dfrac{E}{\sqrt{R^2 + X_L^2}}$



The phasor diagram shown in fig. also shows that in L-R circuit the applied emf E leads the current I or conversely the current I lags behind the e.m.f. E. by a phase angle φ $\tan\phi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R} = \frac{\omega L}{R}$

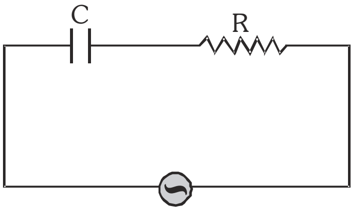
$$\Rightarrow \ \phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

Inductive Impedance Z_L :

In L-R circuit the maximum value of current $I_0 = \frac{E_0}{\sqrt{R^2+\omega^2L^2}}$ Here $\sqrt{R^2+\omega^2L^2}$ represents the effective opposition offered by L-R circuit to the flow of a.c. through it. It is known as impedance of L-R circuit and is represented by Z_L. $Z_L = \sqrt{R^2+\omega^2L^2} = \sqrt{R^2+(2\pi fL)^2}$ The reciprocal of impedance is called admittance $Y_L = \frac{1}{Z_L} = \frac{1}{\sqrt{R^2+\omega^2L^2}}$

RESISTANCE AND CAPACITOR IN SERIES (R-C CIRCUIT)

A circuit containing a series combination of a resistance R and a capacitor C, connected with a source of e.m.f. of peak value E_0 as shown in fig.



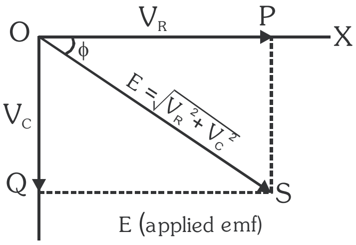
$E = E_0 \sin\omega t$

PHASOR DIAGRAM FOR R-C CIRCUIT

Current through both the resistance and capacitor will be same at every instant and the instantaneous potential differences across C and R are

$$V_C = I\,X_C \quad \text{and} \quad V_R = I\,R$$

where X_C = capacitive reactance and I = instantaneous current. Now, V_R is in phase with I, while V_C lags behind I by 90°.



The phasor diagram is shown in fig.

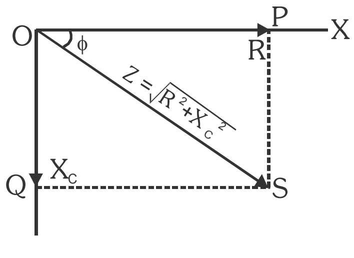
The vector OP represents V_R (which is in phase with I)

and the vector OQ represents V_C (which lags behind I by $\frac{\pi}{2}$).

The vector OS represents the resultant of V_R and V_C = the applied e.m.f. E.

Hence $V_R^2 + V_C^2 = E^2 \Rightarrow E = \sqrt{V_R^2+V_C^2}$

$$\Rightarrow E^2 = I^2 (R^2 + X_C^2) \Rightarrow I = \frac{E}{\sqrt{R^2+X_C^2}}$$



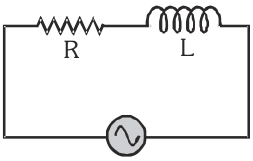
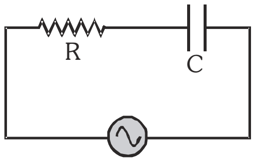
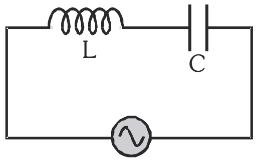
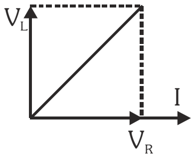
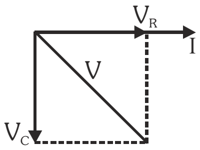
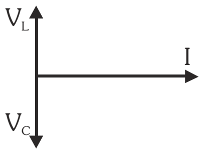
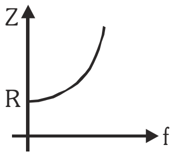
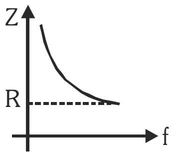
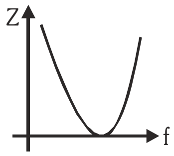
The term $\sqrt{(R^2+X_C^2)}$ represents the effective resistance of the R-C circuit and called the capacitive impedance Z_C of the circuit. Hence, in C-R circuit $Z_C=\sqrt{R^2+X_C^2}=\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}$

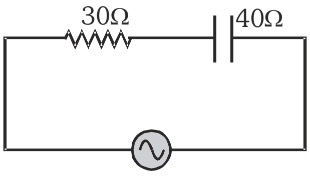
Capacitive Impedance Z_C :

In R-C circuit the term $\sqrt{R^2+X_C^2}$ effective opposition offered by R-C circuit to the flow of a.c. through it. It is known as impedance of R-C circuit and is represented by Z_C
The phasor diagram also shows that in R-C circuit the applied e.m.f. lags behind the current I (or the current I leads the emf E) by a phase angle ϕ given by

$$\tan\phi=\frac{V_C}{V_R}=\frac{X_C}{R}=\frac{1/\omega C}{R}=\frac{1}{\omega CR},\ \tan\ \phi=\frac{X_C}{R}=\frac{1}{\omega CR}\quad\Rightarrow\ \phi=\tan^{-1}\left(\frac{1}{\omega CR}\right)$$

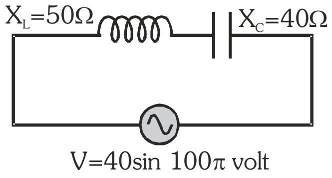
COMBINATION OF COMPONENTS (R-L or R-C or L-C)

TERM	R-L	R-C	L-C
Circuit			
	I is same in R & L	I is same in R & C	I is same in L & C
Phasor diagram			
	$V^2=V_R^2+V_L^2$	$V^2=V_R^2+V_C^2$	$V=V_L-V_C\ (V_L>V_C)$ $V=V_C-V_L\ (V_C>V_L)$
Phase difference	V leads I ($\phi=0$ to $\frac{\pi}{2}$)	V lags I ($\phi=-\frac{\pi}{2}$ to 0)	V lags I ($\phi=-\frac{\pi}{2}$, if $X_C>X_L$)
in between	V and I		V leads I($\phi=+\frac{\pi}{2}$, if $X_L>X_C$)
Impedance	$Z=\sqrt{R^2+X_L^2}$	$Z=\sqrt{R^2+(X_C)^2}$	$Z=\lvert X_L-X_c\rvert$
Variation of Z	as f↑, Z ↑	as f↑, Z↓	as f↑, Z first ↓ then↑
with f			
At very low f	$Z\simeq R\ (X_L\to 0)$	$Z\simeq X_C$	$Z\simeq X_C$
At very high f	$Z\simeq X_L$	$Z\simeq R\ (X_C\to 0)$	$Z\simeq X_L$



Ex. Calculate the impedance of the circuit shown in the figure.

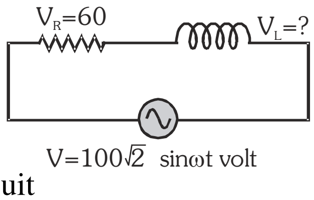
Sol. $Z = \sqrt{R^2+(X_c)^2} = \sqrt{(30)^2+(40)^2} = \sqrt{2500} = 50\ \Omega$



Ex. If $X_L = 50\ \Omega$ and $X_C = 40\ \Omega$ Calculate effective value of current in given circuit.

Sol. $Z = X_L - X_C = 10\ \Omega$

$$I_0 = \frac{V_0}{Z} = \frac{40}{10} = 4\text{A} \Rightarrow I_{rms} = \frac{4}{\sqrt{2}} = 2\sqrt{2}\ \text{A}$$

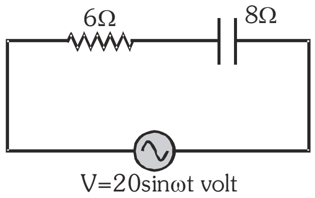


Ex. In given circuit calculate, voltage across inductor

Sol. $\because V^2 = V_R^2 + V_L^2 \qquad \therefore \qquad V_L^{\,2} = V^2 - V_R^{\,2}$

$$V_L \sqrt{V^2 - V_R^2} = \sqrt{(100)^2-(60)^2} = \sqrt{6400} = 80\ \text{V}$$

Ex. In given circuit find out (i) impedance of circuit (ii) current in circuit



Sol. (i) $Z = \sqrt{R^2+X_C^2} = \sqrt{(6)^2+(8)^2} = 10\ \Omega$

(ii) $V = IZ \Rightarrow I = \frac{V_0}{Z} = \frac{20}{10} = 2\text{A}$ so $I_{rms} = \frac{2}{\sqrt{2}} = \sqrt{2}\ \text{A}$

Ex. When 10V, DC is applied across a coil current through it is 2.5 A, if 10V, 50 Hz A.C. is applied current reduces to 2 A. Calculate reactance of the coil.

Sol. For 10 V D.C. $\because V = IR$ $\therefore$ Resistance of coil $R = \frac{10}{2.5} = 4\Omega$ For 10 V A.C. ↚$V = I\,Z$

$$\Rightarrow \quad Z = \frac{V}{I} = \frac{20}{10} = 5\Omega$$

$$\because \ Z = \sqrt{R^2+X_L^2} = 5 \ \Rightarrow R^2+X_L^2 = 25 \Rightarrow \ X_L^2 = 5^2 - 4^2 \qquad \Rightarrow X_L = 3\ \Omega$$

Ex. When an alternating voltage of 220V is applied across a device X, a current of 0.5 A flows through the circuit and is in phase with the applied voltage. When the same voltage is applied across another device Y, the same current again flows through the circuit but it leads the applied voltage by π/2 radians.

(a) Name the devices X and Y.

(b) Calculate the current flowing in the circuit when same voltage is applied across the series combination of X and Y.

Sol. (a) X is resistor and Y is a capacitor

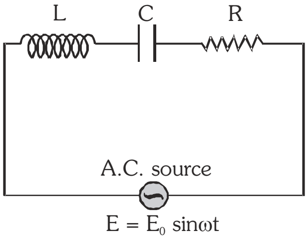
(b) Since the current in the two devices is the same (0.5A at 220 volt)

When R and C are in series across the same voltage then

$$R = X_C = \frac{220}{0.5} = 440\ \Omega \Rightarrow I_{rms} = \frac{V_{rms}}{\sqrt{R^2+X_C^2}} = \frac{220}{\sqrt{(440)^2+(440)^2}} = \frac{220}{440\sqrt{2}} = 0.35\text{A}$$

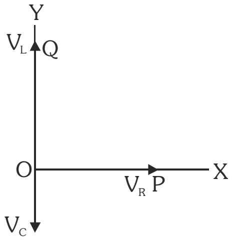
INDUCTANCE, CAPACITANCE AND RESISTANCE IN SERIES (L-C-R SERIES CIRCUIT)

A circuit containing a series combination of an resistance R, a coil of inductance L and a capacitor of capacitance C, connected with a source of alternating e.m.f. of peak value of E_0, as shown in fig.



PHASOR DIAGRAM FOR SERIES L-C-R CIRCUIT

Let in series LCR circuit applied alternating emf is $E = E_0 \sin \omega t$. As L,C and R are joined in series, therefore, current at any instant through the three elements has the same amplitude and phase. **However voltage across each element bears a different phase relationship with the current.**



Let at any instant of time t the current in the circuit is I

Let at this time t the potential differences across L, C, and R

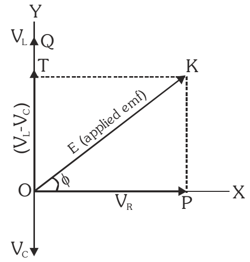
$V_L = I X_L$, $V_C = I X_C$ and $V_R = I R$

Now, V_R is in phase with current I but V_L leads I by 90°

While V_C legs behind I by 90°.

The vector OP represents V_R (which is in phase with I) the vector OQ represent VL (which leads I by 90°)

and the vector OS represents V_C (which legs behind I by 90°)



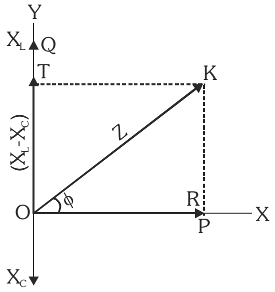
V_L and V_C are opposite to each other.

If $V_L > V_C$ (as shown in figure) the their resultant will be $(V_L - V_C)$ which is represented by OT.

Finally, the vector OK represents the resultant of V_R and $(V_L - V_C)$, that is, the resultant of all the three = applied e.m.f.

Thus $E = \sqrt{V_R^2 + (V_L - V_C)^2} = I\sqrt{R^2 + (X_L - X_C)^2} \Rightarrow I = \dfrac{E}{\sqrt{R^2 + (X_L - X_C)^2}}$

Impedance $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$



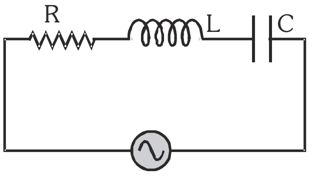
The phasor diagram also shown that in LCR circuit the applied e.m.f.

leads the current I by a phase angle ϕ $\tan\phi = \dfrac{X_L - X_C}{R}$

SERIES LCR AND PARALLEL LCR COMBINATION

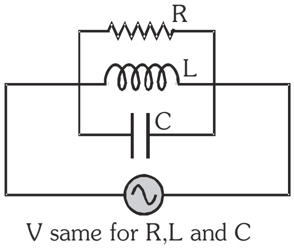
SERIES L-C-R CIRCUIT

1. Circuit diagram



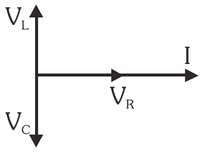
I same for R, L & C

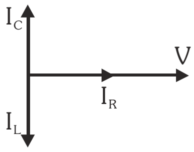
PARALLEL L-C-R CIRCUIT



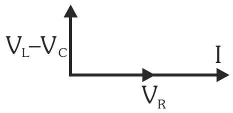
V same for R, L & C

2. Phasor diagram

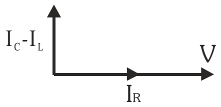




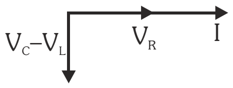
(i) If $V_L > V_C$ then



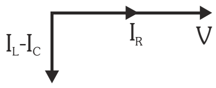
(i) if $I_C > I_L$ then



(ii) If $V_C > V_L$ then



(ii) if $I_L > I_C$ then



(iii) $V = \sqrt{V_R^2 + (V_L - V_C)^2}$

Impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$

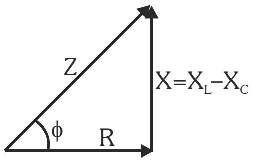
$$\tan\phi = \frac{X_L - X_C}{R} = \frac{V_L - V_C}{V_R}$$

(iii) $I = \sqrt{I_R^2 + (I_L - I_C)^2}$

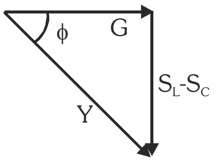
Admittance $Y = \sqrt{G^2 + (S_L - S_C)^2}$

$$\tan\phi = \frac{S_L - S_C}{G} = \frac{I_L - I_C}{I_R}$$

(iv) Impedance triangle



(iv) Admittance triangle

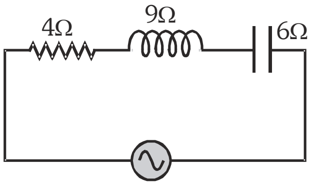


GOLDEN KEY POINTS

	Series		Parallel
(a)	if $X_L > X_C$ then V leads I,ϕ (positive) circuit nature inductive	(a)	if $S_L > S_C$ ($X_L < X_C$) then V leads I,ϕ (positive) circuit nature inductive
(b)	if $X_C > X_L$ then V lags I,ϕ (negative) circuit nature capacitive	(b)	if $S_C > S_L$ ($X_C < X_L$) then V lags I, ϕ (negative) circuit nature capacitive

- In A.C. circuit voltage for L or C may be greater than source voltage or current but it happens only when circuit contains L and C both and on R it never greater than source voltage or current.
- In parallel A.C.circuit phase difference between I_L and I_C is π

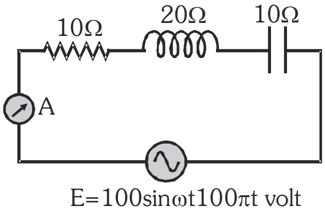
Ex. Find out the impedance of given circuit.



Sol. $Z=\sqrt{R^2+(X_L-X_C)^2}=\sqrt{4^2+(9-6)^2}=\sqrt{4^2+3^2}=\sqrt{25}=5\Omega$

($\because X_L > X_C \therefore$ Inductive)

Ex. Find out reading of A C ammeter and also calculate the potential difference across, resistance and capacitor.



Sol. $Z=\sqrt{R^2+(X_L-X_C)^2}=10\sqrt{2}\ \Omega \Rightarrow I_0=\frac{V_0}{Z}=\frac{100}{10\sqrt{2}}=\frac{10}{\sqrt{2}}A$

$\because$ ammeter reads RMS value, so its reading = $\frac{10}{\sqrt{2}\sqrt{2}}$ = 5A

so $V_R = 5 \times 10 = 50$ V and $V_C = 5 \times 10 = 50$ V

Ex. In LCR circuit with an AC source R = 300 Ω , C = 20 μF, L = 1.0 H, E_{rms} = 50V and f = 50/π Hz. Find RMS current in the circuit.

Sol. $$I_{rms}=\frac{E_{rms}}{Z}=\frac{E_{rms}}{\sqrt{R^2+\left[\omega L-\frac{1}{\omega C}\right]^2}}=\frac{50}{\sqrt{300^2+\left[2\pi\times\frac{50}{\pi}\times 1-\frac{1}{20\times10^{-6}\times2\pi\times\frac{50}{\pi}}\right]^2}}$$

$$\Rightarrow \quad I_{rms}=\frac{50}{\sqrt{(300)^2+\left[100-\frac{10^3}{2}\right]^2}}=\frac{50}{100\sqrt{9+16}}=\frac{1}{10}=0.1A$$

RESONANCE

A circuit is said to be resonant when the natural frequency of circuit is equal to frequency of the applied voltage. For resonance both L and C must be present in circuit.

There are two types of resonance : (i) Series Resonance (ii) Parallel Resonance

SERIES RESONANCE

(a) At Resonance

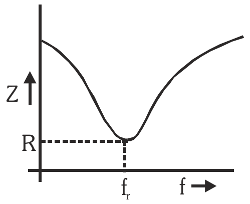
(i) $X_L = X_C$ (ii) $V_L = V_C$ (iii) $\phi = 0$ (V and I in same phase)

(iv) $Z_{\min} = R$ (impedance minimum) (v) $I_{\max} = \frac{V}{R}$ (current maximum)

(b) Resonance frequency

$$\because \quad X_L = X_C \Rightarrow \omega_r L = \frac{1}{\omega_r C} \Rightarrow \omega_r^2 = \frac{1}{LC} \Rightarrow \omega_r = \frac{1}{\sqrt{LC}} \Rightarrow f_r = \frac{1}{2\pi\sqrt{LC}}$$

(c) Variation of Z with f



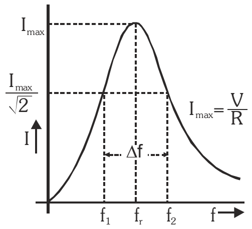
(i) If $f < f_r$ then $X_L < X_C$ circuit nature capacitive, ϕ (negative)

(ii) At $f = f_r$ then $X_L = X_C$ circuit nature, Resistive, ϕ = zero

(iii) If $f > f_r$ then $X_L > X_C$ circuit nature is inductive, ϕ (positive)

Variation of I with f as f increase, Z first decreases then increase

(d)



as f increase, I first increase then decreases

- At resonance impedance of the series resonant circuit is minimum so it is called 'acceptor circuit' as it most readily accepts that current out of many currents whose frequency is equal to its natural frequency. In radio or TV tuning we receive the desired station by making the frequency of the circuit equal to that of the desired station.

Half power frequencies

The frequencies at which, power become half of its maximum value called half power frequencies

Band width $= \Delta f = f_2 - f_1$

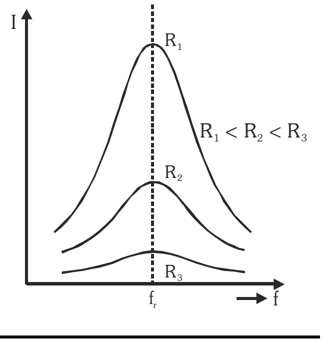
Quality factor Q : Q-factor of AC circuit basically gives an idea about stored energy & lost energy.

$$Q = 2\pi \frac{\text{maximum energy stored per cycle}}{\text{maximum energy loss per cycle}}$$

(i) It represents the sharpness of resonance.

(ii) It is unit less and dimension less quantity

(iii) $Q = \frac{(X_L)_r}{R} = \frac{(X_C)_r}{R} = \frac{2\pi f_r L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{f_r}{\Delta f} = \frac{f_r}{\text{band width}}$



Magnification

At resonance V_L or $V_C = QE$ (where E = supplied voltage)

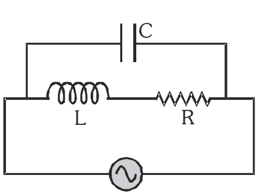
So at resonance Magnification factor = Q-factor

Sharpness

Sharpness ∝ Quality factor ∝ Magnification factor

R decrease ⇒ Q increases ⇒ Sharpness increases

PARALLEL RESONANCE



(a) At resonance

(i) $S_L = S_C$

(ii) $I_L = I_C$

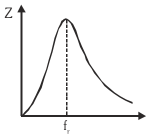
(iii) $\phi = 0$

(iv) $Z_{max} = R$ (impedance maximum)

(v) $I_{min} = \frac{V}{R}$ (current minimum)

(b) Resonant frequency $f_r = \frac{1}{2\pi\sqrt{LC}}$

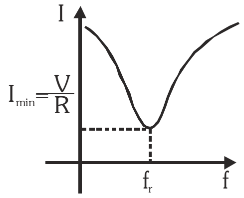
(c)



Variation of Z with f as f increases , Z first increases then decreases

- ◎ If $f < f_r$ then $S_L > S_C$, ϕ (positive), circuit nature is inductive
- ◎ If $f > f_r$ then $S_C > S_L$, ϕ (negative), circuit nature capacitive.

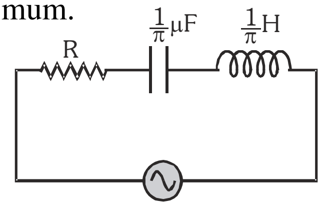
(d) Variation of I with f as f increases , I first decreases then increases



Note : For this circuit $f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \Rightarrow Z_{max} = \frac{L}{RC}$ For resonance $\frac{1}{LC} > \frac{R^2}{L^2}$

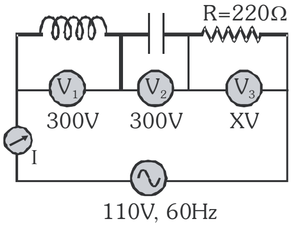
Ex. For what frequency the voltage across the resistance R will be maximum.

Sol. It happens at resonance



$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{\frac{1}{\pi}\times 10^{-6}\times\frac{1}{\pi}}} = 500 \text{ Hz}$$

Ex. A capacitor, a resistor and a 40 mH inductor are connected in series to an AC source of frequency 60Hz, calculate the capacitance of the capacitor, if the current is in phase with the voltage. Also calculate the value of X and I.



Sol. At resonance

$$\omega L = \frac{1}{\omega C},\ C = \frac{1}{\omega^2 L} = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\pi^2 \times (60)^2 \times 40 \times 10^{-3}} = 176\mu F$$

$V = V_R \Rightarrow X = 110$ V and $I = \frac{V}{R} = \frac{110}{220} = 0.5\,A$

Ex. A coil, a capacitor and an A.C. source of rms voltage 24 V are connected in series, By varying the frequency of the source, a maximum rms current 6 A is observed, If this coil is connected to a bettery of emf 12 V, and internal resistance 4Ω , then calculate the current through the coil.

Sol. At resonance current is maximum. $I = \frac{V}{R}$ $\Rightarrow$ Resistance of coil $R = \frac{V}{I} = \frac{24}{6} = 4\ \Omega$

When coil is connected to battery, suppose I current flow through it then

$$I = \frac{E}{R+r} = \frac{12}{4+4} = 1.5\ A$$

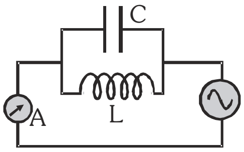
Ex. Radio receiver recives a message at 300m band, If the available inductance is 1 mH, then calculate required capacitance

Sol. Radio recives EM waves. (velocity of EM waves $c = 3 \times 10^8$ m/s)

$$\therefore \quad c = f\lambda \Rightarrow f = \frac{3\times10^8}{300} = 10^6\ Hz$$

$$\text{Now } f = \frac{1}{2\pi\sqrt{LC}} = 1\times 10^6 \Rightarrow C = \frac{1}{4\pi^2 L\times10^{12}} = 25\ pF$$

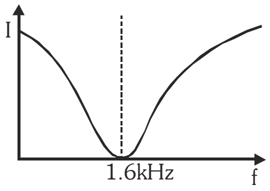
Ex. In a L–C circuit parallel combination of inductance of 0.01 H and a capacitor of 1 μF is connected to a variable frequency alternating current source as shown in figure. Draw a rough sketch of the current variation as the frequency is changed from 1kHz to 3kHz.



Sol. L and C are connected in parallel to the AC source,

$$\text{so resonance frequency } f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.01\times10^{-6}}} = \frac{10^4}{2\pi} \simeq 1.6\text{kHz}$$

In case of parallel resonance, current in L–C circuit at resonance is zero, so the I-f curve will be as shown in figure.



POWER IN AC CIRCUIT

The average power dissipation in LCR AC circuit

Let $V = V_0 \sin\omega t$ and $I = I_0 \sin(\omega t - \phi)$

Instantaneous power $P = (V_0 \sin\omega t)(I_0 \sin(\omega t - \phi)) = V_0 I_0 \sin\omega t\,(\sin\omega t\cos\phi - \sin\phi\cos\omega t)$

$$\text{Average power } \langle P\rangle = \frac{1}{T}\int_0^T (V_0I_0\sin^2\omega t\cos\phi - V_0I_0\sin\omega t\cos\omega t\sin\phi)dt$$

$$= V_0I_0\left[\frac{1}{T}\int_0^T \sin^2\omega t\cos\phi dt - \frac{1}{T}\int_0^T \sin\omega t\cos\omega t\sin\phi dt\right] = V_0I_0\left[\frac{1}{2}\cos\phi - 0\times\sin\phi\right]$$

$$\Rightarrow \quad \langle P\rangle = \frac{V_0I_0\cos\phi}{2} = V_{rms}\, I_{rm,s} \cos\phi$$

Instantaneous power	Average power/actual power/Virtual power/ apparent dissipated power/power loss	Power/rms	Peak power Power
$P = VI$	$P = V_{rms}\, I_{rms} \cos\phi$	$P = V_{rms}\, I_{rms}$	$P = V_0\, I_0$

- $I_{rms}\cos\phi$ is known as active part of current or wattfull current, workfull current. It is in phase with voltage.
- $I_{rms}\sin\phi$ is known as inactive part of current, wattless current, workless current. It is in quadrature (90°) with voltage.

Power factor :

Average power $\overline{P} = E_{rms}\, I_{rms} \cos\phi = \text{r m s power} \times \cos\phi$

Power factor $(\cos\phi) = \dfrac{\text{Average power}}{\text{r m s Power}}$ and $\cos\phi = \dfrac{R}{Z}$

Power factor : (i) is leading if I leads V (ii) is lagging if I lags V

GOLDEN KEY POINTS

- $P_{av} \le P_{rms}$.
- Power factor varies from 0 to 1
-

Pure/Ideal	ϕ	V	Power factor = $\cos\phi$	Average power
R	0	V, I same Phase	1 (maximum)	$V_{rms} . I_{rms}$
L	$+\frac{\pi}{2}$	V leads I	0	0
C	$-\frac{\pi}{2}$	V lags I	0	0
Choke coil	$+\frac{\pi}{2}$	V leads I	0	0

- At resonance power factor is maximum ($\phi = 0$ so $\cos\phi = 1$) and $P_{av} = V_{rms}\, I_{rms}$

Ex. A voltage of 10 V and frequency 10^3 Hz is applied to $\frac{1}{\pi}\,\mu F$ capacitor in series with a resistor of 500Ω. Find the power factor of the circuit and the power dissipated.

Sol. $$\because\ X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 10^3 \times \frac{10^{-6}}{\pi}} = 500\Omega \quad \therefore Z = \sqrt{R^2 + X_C^2} = \sqrt{(500)^2 + (500)^2} = 500\sqrt{2}\,\Omega$$

Power factor $$\cos\phi = \frac{R}{Z} = \frac{500}{500\sqrt{2}} = \frac{1}{\sqrt{2}}\ ,$$

Power dissipated $$= V_{rms}\, I_{rms} \cos\phi = \frac{V_{rms}^2}{Z}\cos\phi = \frac{(10)^2}{500\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{10}\ \text{W}$$

Ex. If V = 100 sin 100 t volt and I = 100 sin (100 t + $\frac{\pi}{3}$) mA for an A.C. circuit then find out

(a) phase difference between V and I (b) total impedance, reactance, resistance

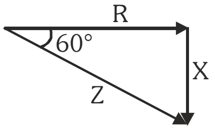
(c) power factor and power dissipated (d) components contains by circuits

Sol. (a) Phase difference $\phi = -\frac{\pi}{3}$ (I leads V)

(b) Total impedance $Z = \frac{V_0}{I_0} = \frac{100}{100\times10^{-3}} = 1k\Omega$

Now resistance $R = Z\cos 60° = 1000\times\frac{1}{2} = 500\Omega$

reactance $X = Z\sin 60° = 1000\times\frac{\sqrt{3}}{2} = \frac{500}{\sqrt{3}}\Omega$



(c) $\phi = -60°$ ⇒ Power factor = $\cos\phi = \cos(-60°) = 0.5$ (leading)

Power dissipated $P = V_{rms} I_{rms}\cos\phi = \frac{100}{\sqrt{2}}\times\frac{0.1}{\sqrt{2}}\times\frac{1}{2} = 2.5\,W$

(d) Circuit must contains R as $\phi \neq \frac{\pi}{2}$ and as ϕ is negative

so C must be their, (L may exist but $X_C > X_L$)

Ex. If power factor of a R-L series circuit is $\frac{1}{2}$ when applied voltage is V = 100 sin 100πt volt and resistance of circuit is 200Ω then calculate the inductance of the circuit.

Sol. $\cos\phi = \frac{R}{Z}$ ⇒ $\frac{1}{2} = \frac{R}{Z}$ ⇒ $Z = 2R$ ⇒ $\sqrt{R^2+X_L^2} = 2R$ ⇒ $X_L = \sqrt{3}R$

$\omega L = \sqrt{3}R$ ⇒ $L = \frac{\sqrt{3}R}{\omega} = \frac{\sqrt{3}\times200}{100\pi} = \frac{2\sqrt{3}}{\pi}H$

Ex. A circuit consisting of an inductance and a resistacne joined to a 200 volt supply (A.C.). It draws a current of 10 ampere. If the power used in the circuit is 1500 watt. Calculate the wattless current.

Sol. Apparent power = 200 × 10 = 2000 W

∴Power factor $\cos\phi = \frac{\text{True power}}{\text{Apparent power}} = \frac{1500}{2000} = \frac{3}{4}$

Wattless current = $I_{rms}\sin\phi = 10\sqrt{1-\left(\frac{3}{4}\right)^2} = \frac{10\sqrt{7}}{4}A$

Ex. A coil has a power factor of 0.866 at 60 Hz. What will be power factor at 180 Hz.

Sol. Given that $\cos\phi = 0.866$, $\omega = 2\pi f = 2\pi \times 60 = 120\pi$ rad/s,

$\omega' = 2\pi f' = 2\pi \times 180 = 360\pi$ rad/s

Now, $\cos\phi = R/Z \Rightarrow R = Z\cos\phi = 0.866\ Z$

But $Z = \sqrt{R^2 + (\omega L)^2} \Rightarrow \omega L = \sqrt{Z^2 - R^2} = \sqrt{Z^2 - (0.866\ Z)^2} = 0.5\ Z$

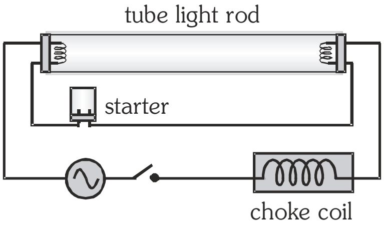
$$\therefore \quad L = \frac{0.5Z}{\omega} = \frac{0.5Z}{120\pi}$$

When the frequency is changed to $\omega' = 2\pi \times 180 = 3 \times 120\pi = 300$ rad/s, then inductive reactance $\omega' L = 3\ \omega L = 3 \times 0.5\ Z = 1.5\ Z$

$$\therefore \text{ New impedence } Z' = \sqrt{[R' + (\omega' L)^2]} = \sqrt{(0.866\ Z)^2 + (1.5\ Z)^2} = Z\ \sqrt{[(0.866)^2 + (1.5)^2]}$$
$$= 1.732Z$$

$$\therefore \text{ New power factor } = \frac{R}{Z'} = \frac{0.866\ Z}{1.732\ Z} = 0.5$$

CHOKE COIL



In a direct current circuit, current is reduced with the help of a resistance. Hence there is a loss of electrical energy $I^2 R$ per sec in the form of heat in the resistance. But in an AC circuit the current can be reduced by choke coil which involves very small amount of loss of energy. Choke coil is a copper coil wound over a soft iron laminated core. This coil is put in series with the circuit in which current is to be reduced. It also known as ballast.

Circuit with a choke coil is a series L-R circuit. If resistance of choke coil = r (very small)

The current in the circuit $I = \dfrac{E}{Z}$ with $Z = \sqrt{(R+r)^2 + (\omega L)^2}$ So due to large inductance L of the coil, the current in the circuit is decreased appreciably. However, due to small resistance of the coil r,

The power loss in the choke $P_{av} = V_{rms} I_{rms} \cos\phi \to 0 \qquad \because \cos\phi = \dfrac{r}{Z} = \dfrac{r}{\sqrt{r^2 + \omega^2 L^2}} \approx \dfrac{r}{\omega L} \to 0$

Ex. A choke coil and a resistance are connected in series in an a.c circuit and a potential of 130 volt is applied to the circuit. If the potential across the resistance is 50 V. What would be the potential difference across the choke coil.

Sol. $V = \sqrt{V_R^2 + V_L^2} \quad \Rightarrow \quad V_L = \sqrt{V^2 - V_R^2} = \sqrt{(130)^2 - (50)^2} = 120$ V

Ex. An electric lamp which runs at 80V DC consumes 10 A current. The lamp is connected to 100 V – 50 Hz ac source compute the inductance of the choke required.

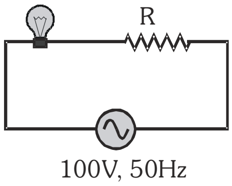
Sol. Resistance of lamp $R = \dfrac{V}{I} = \dfrac{80}{10} = 8\Omega$

Let Z be the impedance which would maintain a current of 10 A through the Lamp when it is run on 100 Volt a.c. then. $Z = \dfrac{V}{I} = \dfrac{100}{10} = 10\ \Omega$ but $Z = \sqrt{R^2 + (\omega L)^2}$

$\Rightarrow \quad (\omega L)^2 = Z^2 - R^2 = (10)^2 - (8)^2 = 36$

$$\Rightarrow \quad \omega L = 6 \quad \Rightarrow L = \frac{6}{\omega} = \frac{6}{2\pi \times 50} = 0.02\text{H}$$

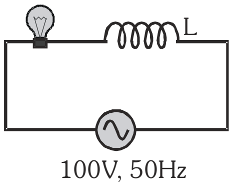
Ex. Calculate the resistance or inductance required to operate a lamp (60V, 10W) from a source of (100 V, 50 Hz)



Sol. (a) Maximum voltage across lamp = 60V

$\because$ $V_{\text{Lamp}} + V_R = 100$ $\therefore$ $V_R = 40V$

Now current througth Lamp is $= \dfrac{\text{Wattage}}{\text{voltage}} = \dfrac{10}{60} = \dfrac{1}{6}A$



But $V_R = IR$ $\Rightarrow$ $40 = \dfrac{1}{6}(R)$ $\Rightarrow R = 240\ \Omega$

(b) Now in this case $(V_{\text{Lamp}})^2 + (V_L)^2 = (V)^2$

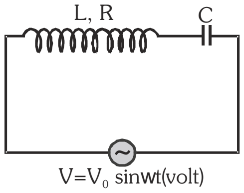
$(60)^2 + (V_L)^2 = (100)^2 \Rightarrow V_L = 80\ V$

Also $V_L = IX_L = \dfrac{1}{6}X_L$ so $X_L = 80 \times 6 = 480\ \Omega = L\,(2\pi f) \Rightarrow$ $L = 1.5\ H$

A capacitor of suitable capacitance replace a choke coil in an AC circuit, the average power consumed in a capacitor is also zero. Hence, like a choke coil, a capacitor can reduce current in AC circuit without power dissipation.

Cost of capacitor is much more than the cost of inductance of same reactance that's why choke coil is used.

Ex. A choke coil of resistance R and inductance L is connected in series with a capacitor C and complete combination is connected to a.c. voltage, Circuit resonates when angular frequency of supply is $\omega = \omega_0$.



(a) Find out relation betwen ω_0, L and C

(b) What is phase difference between V and I at resonance, is it changes when resistance of choke coil is zero.

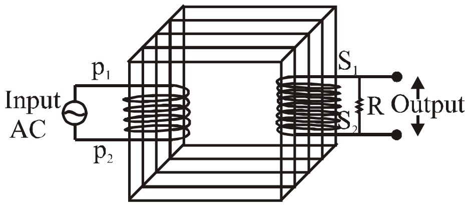
Sol. (a) At resonance condition $X_L = X_C \Rightarrow \omega_0 L = \dfrac{1}{\omega_0 C}$ $\Rightarrow$ $\omega_0 = \dfrac{1}{\sqrt{LC}}$

(b) $\because$ $\cos\phi = \dfrac{R}{Z} = \dfrac{R}{R} = 1$ $\therefore$ $\phi = 0°$ No, It is always zero.

Transformers

One of the great advantages of ac over dc for electric-power distribution is that it is much easier to step voltage levels up and down with ac than with dc. The necessary conversion is accomplished by a static device called transformer using the principle of mutual induction.

The figure shows an idealised transformer which consists of two coils or windings, electrically insulated from each other but wound on the same core. The winding to which power is supplied is called primary, the winding from which power is delivered is called the secondary.



Laminated core

The ac source causes an alternating current in the primary which sets up an alternating flux in the core and this induces an emf in each winding of secondary in accordance with Faraday's law. For ideal transformer we assume that primary has negligible resistance and all the flux in core links both primary and secondary.

The primary winding has N_1 turns and secondary has N_2 turns. When the magnetic flux changes because of changing currents in the two coils, the resulting induced emf are

$$E_1 = -N_1 \frac{d\phi_B}{dt} \text{ and } E_2 = -N_2 \frac{d\phi_B}{dt}$$

The flux per turn B is same in both primary and the secondary so that the emf per turn is same in each. The ratio of secondary emf E_2 to the primary emf E_1 is therefore equal at any instant to the ratio of secondary to primary turns.

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

If the windings have zero resistance, the induced emf 1 and 2 are equal to the terminal voltage across the primary and the secondary respectively, hence

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

If $N_2 > N_1$ then $V_2 > V_1$ and we have step up transformer, if $N_2 < N_1$ then $V_2 < V_1$ and we have a step down transformer.

If the transformer is assumed to be 100% efficient (no energy losses) the power input is equal to the power output i.e.

$$I_2V_2 = I_1V_1 \qquad \therefore \quad \frac{I_2}{I_1} = \frac{N_1}{N_2}$$

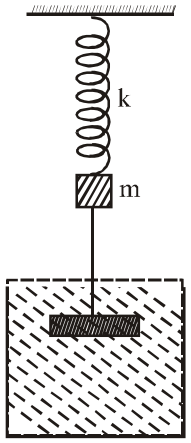
All the currents and voltages derived above have same frequency as that of source. The equations obtained above apply to ideal transformers, although some energy is lost but well designed transformers have efficiency more than 95%, this is a good approximation. The causes of energy losses and their rectification is given below

	Cause		**Rectification**
1	Due to poor design and air gaps in The core, all the flux due to primary does not pass through the secondary.	1	By winding the primary and secondary coil one over the other.
2	Resistance of windings causes I^2R loss.	2	In high current, low voltage, these are minimised by using thick wire.
3	The alternating magnetic flux induces eddy currents in the core and causes heating.	3	By using laminated core it can be reduced.
4	Alternating magnetisating of core causes hystersis loss.	4	It is kept minimum by using a magnetic material having low hystersis loss. (e.g. soft iron)

Damped Harmonic Oscillator:

In the previous section we have considered several examples of oscillatory systems. In each case the amplitude of the oscillation does not depend on time and the system, once set to oscillate, will continue to do so forever. Physical systems are never as ideal as that. Even in the most controlled case there will be dissipative elements.

In each of these cases the amplitudes of the oscillation will gradually decrease and the motion will be 'damped'.



Most of the time we are interested in the motion of an object through air or other viscous fluids. The motion of the object is then subject to a resistive force called aerodynamic or viscous drag. Experimentally the resistive force is found to be proportional to the velocity of the body for low relative speed with respect to the medium.

A laboratory model for a damped spring-mass system is shown in figure. Where a vertical spring has at its end a mass m which is connected to massless piston which moves through a liquid. For low speeds it is reasonable to write the frictional force as

$$F_{f_f} = -\alpha v \qquad (\alpha \text{ in kg/s})$$

where α is a positive constant known as damping constant. The equation of motion of a harmonic oscillator becomes

$$m = \frac{d^2x}{dt^2} = -kx - \alpha\frac{dx}{dt}$$

which can be rewritten as

$$\frac{d^2x}{dt^2} + 2\gamma\frac{dx}{dt} + \omega_0^2 x = 0$$

where $2\gamma = \alpha/m > 0$ and ω_0^2, as before is equal to k/m. γ is known as damping coefficient

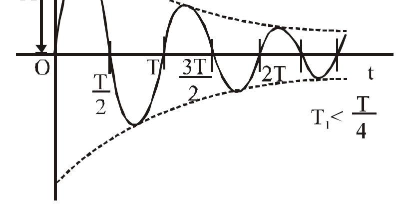
Damped Oscillations

If the dissipative forces are small, i.e. if $\gamma^2 < \omega_0^2$, the dominant force acting on the mass is the conservation restoring force. The system therefore shows oscillations, even though the amplitudes of oscillation continuously decrease with time.

we get $\quad x = Ae^{-\gamma t}\sin(\Omega t + \phi)$

where the new angular frequency of oscillation Ω is given by $\quad \Omega^2 = \omega_0^2 - \gamma^2$

i.e the frequency shift to a lower value due to damping. The amplitude of oscillation is no longer constant but decreases with time in an exponential fashion. The variation of x with time is shown in figure.



It may be seen that sin ($\Omega t + \phi$) is a periodic function with period $2\pi/\Omega$. when this is multiplied with $e^{-\gamma t}$ the zeroes of the product would still be at the same place where sin ($\Omega t + \phi$) becomes zero. The maxima however does not fall midway between the two minima because of the exponential $e^{-\gamma t}$ but occurs at a slightly earlier time.

Power Loss in a Damped Oscillation-Q-factor

Because of the work done against the nonconservative forces, the energy stored in an oscillator continuously decreases. The fraction of energy lost in a period with respect to its average value for that period is a measure of the quality ofthe oscillator. The less the value of this fraction is, the more is the ability ofthe oscillator to sustain periodic motion. Quantitatively, the quality of an oscillator is measured through a Q-factor defined as

$$Q = 2\pi \times \frac{\text{Average energy stored in one period}}{\text{average loss of energy in that period}}$$

If the damping is light i.e. $\gamma << \omega_0$, The instantaneous energy of the oscillator is

$$E(t) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$= \frac{1}{2}mA^2\omega_0^2 e^{-2\gamma t}$$

where we have replaced k by $m\omega_0^2$. At t = 0 the energy of the oscillator is $E_0 = mA^2\ \omega_0^2/2$ so that

$$E(t) = E_0 e^{-2\gamma t}$$

$$P(t) = \frac{dE}{dt} = -2\gamma E_0 e^{-2\gamma t} = 2\gamma\, E(t)$$

Energy lost in one period can be approximately written as

$$< p(t) > \frac{2\pi}{\Omega} = 2\gamma < E(t) > \frac{2\pi}{\Omega}$$

where the angular bracket < ..... > denotes the time average value during a period at time t. The quality or Q- factor for an oscillator is

$$2\pi \frac{< E(t) >}{2\gamma < E(t) > \frac{2\pi}{\Omega}} = \frac{\Omega}{2\pi} \cong \frac{\omega_0}{2\gamma}$$

It may be noted that Q is a dimensionless quantity, greater than unity by our assumption that $\gamma << \omega_0$.

Time to reach extreme:

The natural logarithm of two successive maxima is called logarithmic decrement and is denoted by λ. From Equation it follows that

$$\lambda = \frac{2\pi\gamma}{\Omega} = \frac{\pi}{Q}$$

Overdamped Motion

In equation if $\gamma > \omega_0$ we do not get oscillatory solution

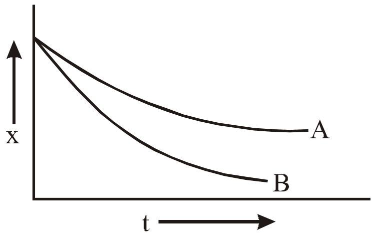
Critical Damping ($\gamma = \omega_0$)

The solution for the displacement is therefore

$$x = \left(A + Bt\right)e^{-\gamma t}$$

$$x' = -\frac{mg}{k}\left(1 + \gamma t\right)e^{-\gamma t}$$

It may be seen (hat once again there are no oscillation. As $t \to \infty$, $x \to 0$ so that the particle takes an infinite time before reaching the equilibrium position. The difference with the overdamped case is that the approach to the equilibrium is faster. The variation of displacement with time is shown in figure (curve B.)



Forced Oscillations and Resonance

The oscillations in a system can be indefinitely maintained by supplying energy continuously. In mechanical system this can be done by subjecting it to an external force which itself has harmonic time dependence.

An interesting phenomenon occurs if the frequency of the external source is equal or nearly equal to the natural frequency of the system. The amplitude of oscillations is found to increase many folds in such cases. This is called resonance.

Forced Damped Oscillations

In the presence of resistive forces proportional to velocity, equation be comes

$$\frac{d^2x}{dt^2} + 2\gamma\frac{dx}{dt} + \omega_0^2 x = \frac{F_0}{m}\cos\omega t$$

Let us therefore try a solution of the type

$x = A \cos \omega t + B \sin \omega t + Ce^{-\lambda t}$

The steady - state displacement is then given by

$x = C \cos (\omega t + \delta)$

with

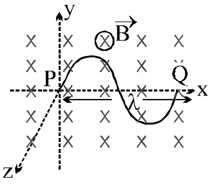
$$C = \frac{F_0}{m}\sqrt{\frac{1}{\left(\omega_0^2 - \omega^2\right)^2 + 4\omega^2\gamma^2}}$$

and $\tan\delta = \dfrac{2\omega\gamma}{\omega^2 - \omega_0^2}$

EXERCISE (S-1)

Faraday's law & Motional emf.

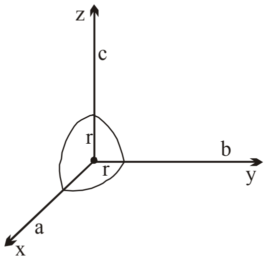
1. A wire forming one cycle of sine curve is moved in x-y plane with velocity $\vec{V} = V_x\hat{i} + V_y\hat{j}$. There exist a magnetic field $\vec{B} = -B_0\hat{k}$. Find the motional emf develop across the ends PQ of wire.



EM0001

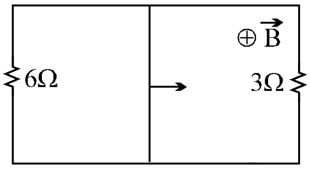
2. A wire is bent into 3 circular segments of radius r = 10 cm as shown in figure. Each segment is a quadrant of a circle, ab lying in the xy plane, bc lying in the yz plane & ca lying in the zx plane.

(i) if a magnetic field B points in the positive x direction, what is the magnitude of the emf developed in the wire, when B increases at the rate of 3 mT/s ?
(ii) what is the direction of the current in the segment bc.



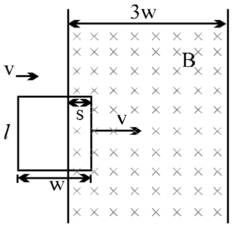
EM0002

3. A rectangular loop with a sliding connector of length *l* = 1.0 m is situated in a uniform magnetic field B = 2T perpendicular to the plane of loop. Resistance of connector is r = 2Ω. Two resistances of 6Ω and 3Ω are connected as shown in figure. Find the external force required to keep the connector moving with a constant velocity v = 2m/s.



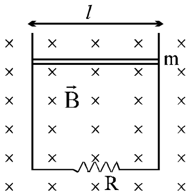
EM0003

4. A rectangular loop of dimensions l & w and resistance R moves with constant velocity V to the right as shown in the figure. It continues to move with same speed through a region containing a uniform magnetic field B directed into the plane of the paper & extending a distance 3 w. Sketch the flux, induced emf & external force acting on the loop as a function of the distance.



EM0004

5. A horizontal wire is free to slide on the vertical rails of a conducting frame as shown in figure. The wire has a mass m and length *l* and the resistance of the circuit is R. If a uniform magnetic field B is directed perpendicular to the frame, then find the terminal speed of the wire as it falls under the force of gravity.



EM0005

6. It is desired to measure the magnitude of field between the poles of a powerful loud speaker magnet. A small flat search coil of area 2 cm^2 with 25 closely wound turns, is positioned normal to the field direction, and then quickly snatched out of the field region. Equivalently, one can give it a quick 90° turn to bring its plane parallel to the field direction. The total charge flown In the coil (measured by a ballistic galvanometer connected to coil) is 7.5 mC. The combined resistance of the coil and the galvanometer is 0.50 Ω. Estimate the field strength of magnet.

(NCERT)

EM0006

Induced electric field

7. There exists a uniform cylindrically symmetric magnetic field directed along the axis of a cylinder but varying with time as B = kt. If an electron is released from rest in this field at a distance of ‘r’ from the axis of cylinder, its acceleration, just after it is released would be (e and m are the electronic charge and mass respectively)

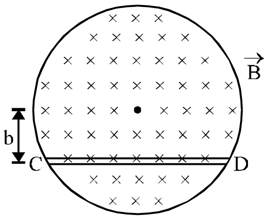
EM0007

8. An air -cored solenoid of length 30 cm. area of cross-section 25 cm^2 and number of turns 500, carries a current of 2.5 A. The current is suddenly switched off in a brief time of 10^{-3}s. How much is the average back emf induced across the ends of the open switch in the circuit? Ignore the variation in magnetic field near the ends of the solenoid. **(NCERT)**

EM0008

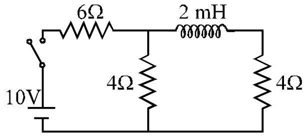
9. A uniform magnetic field $\vec{B}$ fills a cylindrical volumes of radius R. A metal rod CD of length l is placed inside the cylinder along a chord of the circular cross-section as shown in the figure. If the magnitude of magnetic field increases in the direction of field at a constant rate dB/dt, find the magnitude and direction of the EMF induced in the rod.

EM0009



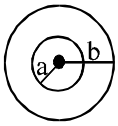
Inductance

10. In the given circuit, find the ratio of i_1 to i_2 where i_1 is the initial (at t = 0) current and i_2 is steady state (at t = ∞) current through the battery.



EM0010

11. Two concentric and coplanar circular coils have radii a and b(>>a)as shown in figure. Resistance of the inner coil is R. Current in the outer coil is increased from 0 to i, then find the total charge circulating the inner coil.

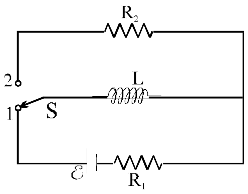


EM0011

12. Find the dimension of the quantity $\frac{L}{RCV}$, where symbols have usual meaning.

EM0012

13. In the circuit shown, initially the switch is in position 1 for a long time. Then the switch is shifted to position 2 for a long time. Find the total heat produced in R_2.

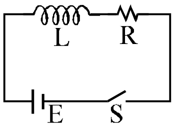


EM0013

14. An emf of 15 volt is applied in a circuit containing 5 H inductance and 10 Ω resistance. Find the ratio of the currents at time t = ∞ and t = 1 second.

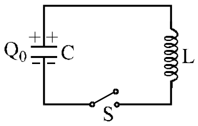
EM0014

15. In the circuit shown in figure switch S is closed at time t = 0. Find the charge which passes through the battery in one time constant.



EM0015

16. A capacitor C with a charge Q_0 is connected across an inductor through a switch S. If at t = 0, the switch is closed, then find the instantaneous charge q on the upper plate of capacitor.



EM0016

17. An inductor of inductance 2.0mH,is connected across a charged capacitor of capacitance 5.0μF and the resulting LC circuit is set oscillating at its natural frequency. Let Q denote the instantaneous charge on the capacitor, and I the current in the circuit .It is found that the maximum value of Q is 200μC.

(a) when Q = 100μC, what is the value of $|dI/dt|$?

(b) when Q = 200 μC, what is the value of I ?

(c) Find the maximum value of I.

(d) when I is equal to one half its maximum value, what is the value of $|Q|$

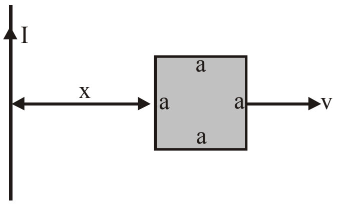
EM0017

18. A pair of adjacent coils has a mutual inductance of 1.5 H. If the current in one coil changes from 0 to 20 A in 0.5 s, what is the change of flux linkage with the other coil ? **(NCERT)**

EM0018

19. (a) Obtain an expression for the mutual inductance between a long straight wire and a square loop of side a as shown In figure.

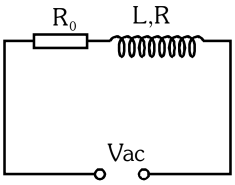
(b) Now assume that the straight wire carries a current of 50 A and the loop is moved to the right with a constant velocity, v = 10 m/s.Calculate the induced emf in the loop at the instant when x = 0.2m. Take a = 0.1 m and assume that the loop has a large resistance. **(NCERT)**



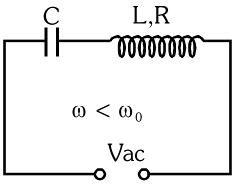
EM0019

Alternating current

20. Draw the approximate voltage vector diagrams in the electric circuits shown in Fig. a, b. The external voltage V is assumed to be alternating harmonically with frequency ω.



(a)



(b)

EM0020

21. A current of 4 A flows in a coil when connected to a 12 V dc source. If the same coil is connected to a 12V, 50 rad/s ac source a current of 2.4 A flows in the circuit. Determine the inductance of the coil. Also find the power developed in the circuit if a 2500 μF capacitor is connected in series with the coil.

EM0021

22. An LCR series circuit with 100Ω resistance is connected to an ac source of 200 V and angular frequency 300 rad/s. When only the capacitance is removed, the current lags behind the voltage by 60°. When only the inductance is removed, the current leads the voltage by 60°. Calculate the current and the power dissipated in the LCR circuit.

EM0022

23. A series LCR circuit containing a resistance of 120Ω has angular resonance frequency 4×10^5 rad s^{-1}. At resonance the voltages across resistance and inductance are 60 V and 40 V respectively. Find the values of L and C. At what frequency the current in the circuit lags the voltage by 45°?

EM0023

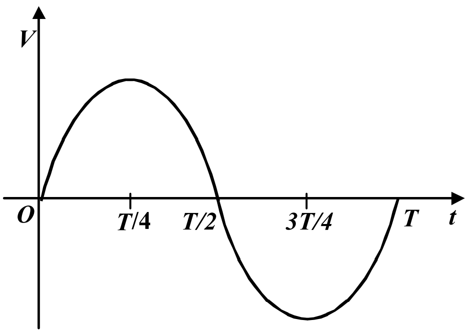
24. Find the value of an inductance which should be connected in series with a capacitor of 5μF, a resistance of 10Ω and an ac source of 50 Hz so that the power factor of the circuit is unity.

EM0024

25. In an L-R series A.C circuit the potential difference across an inductance and resistance joined in series are respectively 12 V and 16V. Find the total potential difference across the circuit.

EM0025

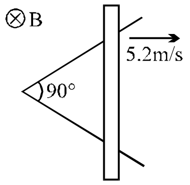
26. In an LR series circuit, a sinusoidal voltage $V = V_o \sin \omega t$ is applied. It is given that L = 35 mH, $R = 11\ \Omega$, $V_{rms} = 220$ V, $\frac{\omega}{2\pi} = 50$ Hz and $\pi = 22/7$. Find the amplitude of current in the steady state and obtain the phase difference between the current and the voltage. Also plot the variation of current for one cycle on the given graph. **[JEE 2004]**



EM0026

EXERCISE (S-2)

1. Two straight conducting rails form a right angle where their ends are joined. A conducting bar contact with the rails starts at vertex at the time t = 0 & moves symmetrically with a constant velocity of 5.2 m/s to the right as shown in figure. A 0.35 T magnetic field points out of the page. Calculate:



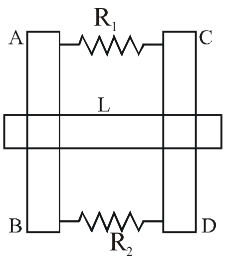
(i) The flux through the triangle by the rails & bar at t = 3.0 s.

(ii) The emf around the triangle at that time.

(iii) In what manner does the emf around the triangle vary with time .

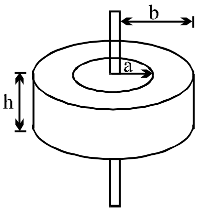
EM0027

2. Two parallel vertical metallic rails AB & CD are separated by 1 m. They are connected at the two ends by resistance R_1 & R_2 as shown in the figure. A horizontally metallic bar L of mass 0.2 kg slides without friction, vertically down the rails under the action of gravity. There is a uniform horizontal magnetic field of 0.6T perpendicular to the plane of the rails, it is observed that when the terminal velocity is attained, the power dissipated in R_1 & R_2 are 0.76 W & 1.2 W respectively. Find the terminal velocity of bar L & value R_1 & R_2.



EM0028

3. A long straight wire is arranged along the symmetry axis of a toroidal coil of rectangular cross–section, whose dimensions are given in the figure. The number of turns on the coil is N, and relative permeability of the surrounding medium is unity. Find the amplitude of the emf induced in this coil, if the current $i = i_m \cos \omega t$ flows along the straight wire.



EM0029

4. A metal rod of resistance 20Ω is fixed along a diameter of a conducting ring of radius 0.1 m and lies on x-y plane. There is a magnetic field $\vec{B} = (50T)\hat{k}$. The ring rotates with an angular velocity $\omega = 20$ rad/sec about its axis. An external resistance of 10Ω is connected across the centre of the ring and rim. Find the current through external resistance.

EM0030

5. A uniform but time varying magnetic field $B = Kt - C$; $(0 \le t \le C/K)$, where K and C are constants and t is time, is applied perpendicular to the plane of the circular loop of radius 'a' and resistance R. Find the total charge that will pass around the loop.

EM0031

6. A charged ring of mass m = 50 gm, charge 2 coulomb and radius R = 2m is placed on a smooth horizontal surface. A magnetic field varying with time at a rate of (0.2 t) Tesla/sec is applied on to the ring in a direction normal to the surface of ring. Find the angular speed attained in a time t_1 = 10 sec. Assume that the magnetic field is cylindrically symmetric and covering the entire ring.

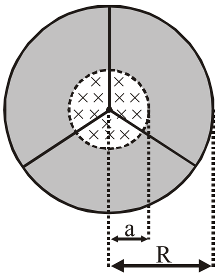
EM0032

7. A line charge λ per unit length is lodged uniformly onto the rim of a wheel of mass M and radius R. The wheel has light nonconducting spokes and is free to rotate without friction about its axis see figure. A uniform magnetic field extends over a circular region within the rim. It is given by,

$\mathbf{B} = -B_0\mathbf{k}$ $(r \le a\ ;\ a < R)$

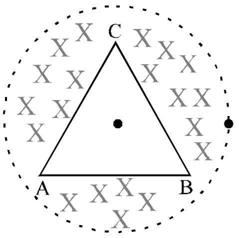
$= 0$ (otherwise)

What is the angular velocity of the wheel after the field is suddenly switched off ? **(NCERT)**



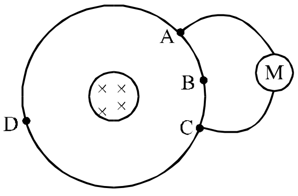
EM0033

8. A triangular wire frame (each side = 2m) is placed in a region of time variant magnetic field having $dB/dt = \sqrt{3}$ T/s. The magnetic field is perpendicular to the plane of the triangle. The base of the triangle AB has a resistance 1 Ω while the other two sides have resistance 2Ω each. The magnitude of potential difference between the points A and B will be



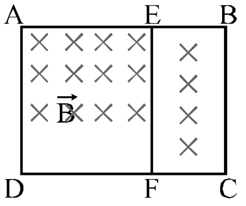
EM0034

9. A variable magnetic field creates a constant emf E in a conductor ABCDA. The resistances of portion ABC, CDA and AMC are R_1, R_2 and R_3 respectively. What current will be shown by meter M? The magnetic field is concentrated near the axis of the circular conductor.



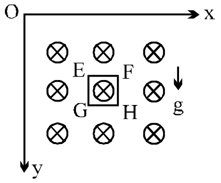
EM0035

10. A rectangular frame ABCD made of a uniform metal wire has a straightconnection between E & F made of the same wire as shown in the figure. AEFD is a square of side 1 m & EB = FC = 0.5 m. The entire circuit is placed in a steadily increasing uniform magnetic field directed into the plane of the paper & normal to it . The rate of change of the magnetic field is 1 T/s, the resistance per unit length of the wire is 1 Ω/m. Find the current in segments AE, BE & EF.



EM0036

11. A magnetic field $B = (B_0 y / a)\ \hat{k}$ is into the plane of paper in the +z direction. B_0 and a are positive constants. A square loop EFGH of side a, mass m and resistance R, in x-y plane, starts falling under the influence of gravity. Note the directions of x and y axes in the figure. Find



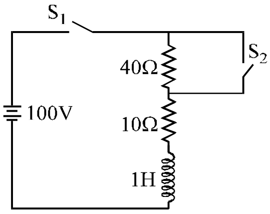
(a) the induced current in the loop and indicate its direction,

(b) the total Lorentz force acting on the loop and indicate its direction,

(c) an expression for the speed of the loop, v(t) and its terminal value.

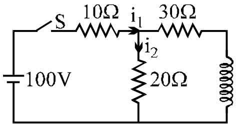
EM0037

12. In the circuit shown in the figure the switched S_1 and S_2 are closed at time t = 0. After time t = (0.1) ln 2 sec, switch S_2 is opened. Find the current in the circuit at time t = (0.2) ln 2 sec.



EM0038

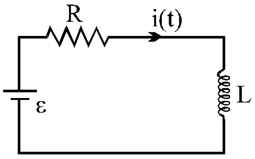
13. Find the values of i_1 and i_2



(i) immediately after the switch S is closed.
(ii) long time later, with S closed.
(iii) immediately after S is open.
(iv) long time after S is opened.

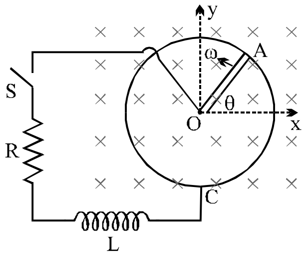
EM0039

14. Suppose the emf of the battery in the circuit shown varies with time t so the current is given by i(t) = 3 + 5t, where i is in amperes & t is in seconds. Take R = 4Ω, L = 6H & find an expression for the battery emf as function of time.



EM0040

15. A metal rod OA of mass m & length r is kept rotating with a constant angular speed ω in a vertical plane about a horizontal axis at the end O. The free end A is arranged to slide without friction along a fixed conducting circular ring in the same plane as that of rotation. A uniform & constant magnetic induction $\vec{B}$ is applied perpendicular & into the plane of rotation as shown in figure. An inductor L and an external resistance R are connected through a switch S between the point O & a point C on the ring to form an electrical circuit. Neglect the resistance of the ring and the rod. Initially, the switch is open.



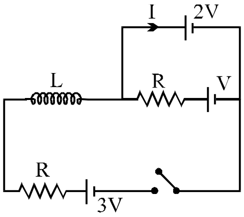
(a) What is the induced emf across the terminals of the switch ?
(b) (i) Obtain an expression for the current as a function of time after switch S is closed.
(ii) Obtain the time dependence of the torque required to maintain the constant angular speed, given that the rod OA was along the positive X-axis at t = 0.

EM0041

16. A zero resistance coil of inductance L connects the upper ends of two vertical parallel long conductors. A horizontal sliding conductor, free to slide up and down, always maintaining contact with the vertical conductors, starts falling from rest at t = 0, due to its own weight mg. A uniform magnetic field of magnitude B exists in the region horizontally and perpendicular to the plane of the conductors. The distance between the vertical conductors is '*l*'. After what time does the conductor come back to its starting position? Also find maximum speed achieved.

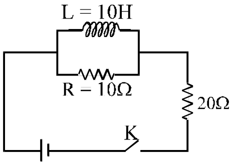
EM0042

17. In the LR circuit shown, what is the variation of the current I as a function of time? The switch is closed at time t = 0 sec.



EM0043

18. Two resistors of 10Ω and 20Ω and an ideal inductor of 10H are connected to a 2V battery as shown. The key K is shorted at time t = 0. Find the initial (t = 0) and final (t → ∞) currents through battery.



EM0044

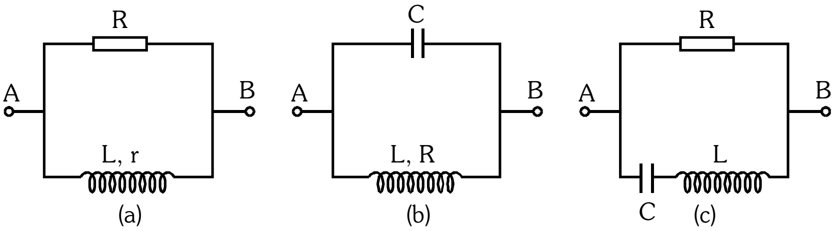
19. Two coils, 1 & 2, have a mutual inductance = M and resistances R each. A current flows in coil 1, which varies with time as: $I_1 = kt^2$, where k is a constant and 't' is time. Find the total charge that has flown through coil 2, between t = 0 and t = T.

EM0045

20. A box P and a coil Q are connected in series with an ac source of variable frequency. The emf of source is 10 V. Box P contains a capacitance of 1μF in series with a resistance of 32Ω while coil Q has a self-inductance 4.9 mH and a resistance of 68Ω series. The frequency is adjusted so that the maximum current flows in P and Q. Find the impedance of P and Q at this frequency. Also find the voltage across P and Q respectively.

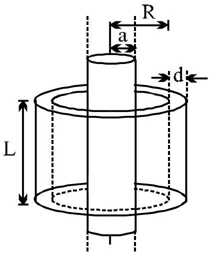
EM0046

21. Draw the approximate vector diagrams of currents in the circuits shown in Fig. The voltage applied across the points A and B is assumed to be sinusoidal; the parameters of each circuit are so chosen that the total current I_0 lags in phase behind the external voltage by an angle ϕ.



EM0047

22. A long solenoid of radius a and number of turns per unit length n is enclosed by cylindrical shell of radius R, thickness d (d <<R) and length L. A variable current $i = i_0 \sin \omega t$ flows through the coil. If the resistivity of the material of cylindrical shell is ρ, find the induced current in the shell. **[JEE 2005]**



EM0048

EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

Faraday's law & motional emf.

1. A square of side *2* meters lies in the *x-y* plane in a region, where the magnetic field is given by $\vec{B} = B_0\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)T$, where B_o is constant. The magnitude of flux passing through the square is:-

(A) 8 B_o Wb. (B) 12 B_o Wb. (C) 16 B_o Wb. (D) $\sqrt{4\times 29}B_0 Wb$

EM0049

2. **Statement-1 :** When a magnet is made to fall freely through a closed coil, its acceleration is always less than acceleration due to gravity.

and

Statement-2 : Current induced in the coil opposes the motion of the magnet, as per Lenz's law.

(A) Statement-1 is true, Statement-2 is true ; Statement-2 is a correct explanation for Statement-1

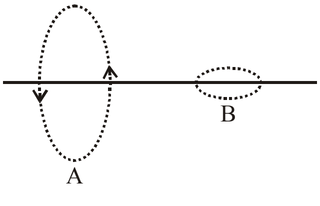
(B) Statement-1 is true, Statement-2 is true, Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

EM0050

3. In the given figure the centre of a small conducting circular loop B lies on the axis of bigger circular loop A and their axis are mutually perpendicular. An anticlockwise (when viewed from the side of B) current in the loop A start increasing then :-



(A) current induced in the loop B is in clockwise direction (when viewed from above the B)

(B) current induced in the loop B is in anti-clockwise direction (when viewed from above the B)

(C) current must induced in the loop B but its direction can not be predicted

(D) no current is induced in the loop B

EM0051

4. A vertical bar magnet is dropped from position on the axis of a fixed metallic coil as shown in fig - I. In fig.II the magnet is fixed and horizontal coil is dropped. The acceleration of the magnet and coil are a_1 and a_2 respectively then :-

Fig. (I)

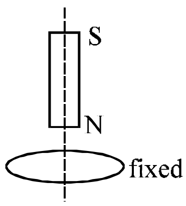
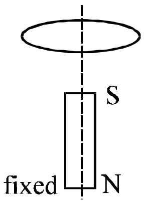


Fig. (II)



(A) $a_1 > g$, $a_2 > g$ (B) $a_1 > g$, $a_2 < g$ (C) $a_1 < g$, $a_2 < g$ (D) $a_1 < g$, $a_2 > g$

EM0052

5. Two identical coaxial circular loops carry a current *i* each circulating in the same direction. If the loops approach each other

(A) the current in each will decrease

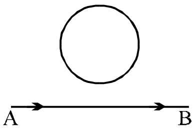
(B) the current in each will increase

(C) the current in each will remain the same

(D) the current in one will increase and in other will decrease

EM0053

6. In the arrangement shown in given figure current from A to B is increasing in magnitude. Induced current in the loop will

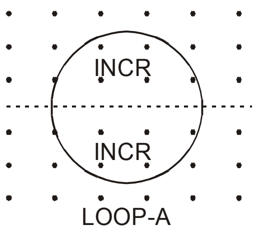


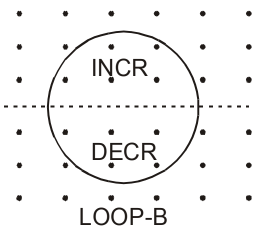
(A) have clockwise direction (B) have anticlockwise direction

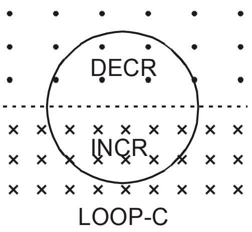
(C) be zero (D) oscillate between clockwise and anticlockwise

EM0054

7. Three identical conducting circular loops are placed in uniform magnetic fields. Inside each loop, there are two magnetic field regions, separated by dashed line that coincides with a diameter, as shown. Magnetic fields may either be increasing (marked as INCR) or decreasing (marked as DECR) in magnitude at the same rates. If I_A, I_B and I_C are the magnitudes of the induced currents in the loops A, B and C respectively then choose the **CORRECT** relation :-



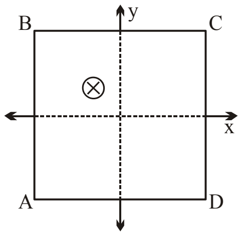




(A) $I_A > I_B = I_C$ (B) $I_A = I_C > I_B$ (C) $I_A = I_B = I_C$ (D) $I_C > I_A > I_B$

EM0055

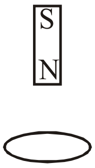
8. A square coil ABCD is placed in x-y plane with its centre at origin. A long straight wire, passing through origin, carries a current in negative z-direction. Current in this wire increases with time. The induced current in the coil is :



(A) clockwise (B) anticlockwise (C) zero (D) alternating

EM0056

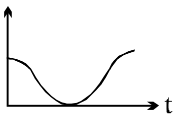
9. A short circuited coil is kept on the ground and a magnet is dropped on it as shown. The coil shows (when viewed from top)

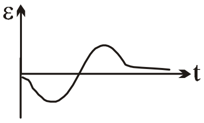


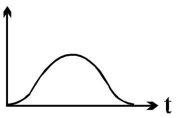
(A) anticlockwise current that increases in magnitude
(B) anticlockwise current that remains constant
(C) clockwise current that remains constant
(D) clockwise current that increases in magnitude

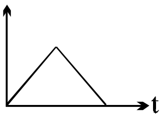
EM0057

10. The variation of induced emf (ε) with time (t) in a coil if a short bar magnet is moved along its axis with a constant velocity is best represented as **[JEE 2004(Scr)]**

(A)


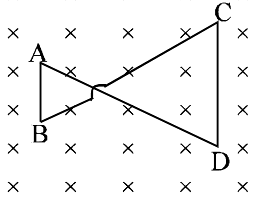
(B)


(C)


(D)


EM0058

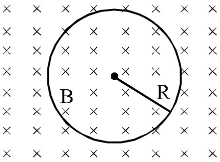
11. A conducting wire frame is placed in a magnetic field which is directed into the paper. The magnetic field is increasing at a constant rate. The directions of induced currents in wires AB and CD are



(A) B to A and D to C
(B) A to B and C to D
(C) A to B and D to C
(D) B to A and C to D

EM0059

12. A conducting loop of radius R is present in a uniform magnetic field B perpendicular to the plane of the ring. If radius R varies as a function of time 't', as $R = R_0 + t$. The e.m.f induced in the loop is



(A) $2\pi(R_0 + t)B$ clockwise (B) $\pi(R_0 + t)B$ clockwise

(C) $2\pi(R_0 + t)B$ anticlockwise (D) zero

EM0060

13. A thin wire of length 2m is perpendicular to the xy plane. It is moved with velocity $\vec{v} = (2\hat{i} + 3\hat{j} + \hat{k})\,m/s$ through a region of magnetic induction $\vec{B} = (\hat{i} + 2\hat{j})\ Wb/m^2$. Then potential difference induced between the ends of the wire :

(A) 2 volts (B) 4 volts (C) 0 volts (D) none of these

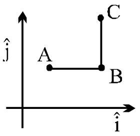
EM0061

14. A square loop of side a and resistance R is moved in the region of uniform magnetic field B (loop remaining completely inside field), with a velocity v through a distance x . The work done is :

(A) $\frac{B\ell^2 vx}{R}$ (B) $\frac{2B^2\ell^2 vx}{R}$ (C) $\frac{4B^2\ell^2 vx}{R}$ (D) 0

EM0062

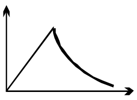
15. There is a uniform magnetic field B normal to the xy plane. A conductor ABC has length AB = l_1, parallel to the x-axis, and length BC = l_2, parallel to the y-axis. ABC moves in the xy plane with velocity $v_x\hat{i} + v_y\hat{j}$. The potential difference between A and C is proportional to :-

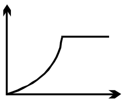


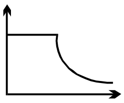
(A) $v_x l_1 + v_y l_2$ (B) $v_x l_2 + v_y l_1$ (C) $v_x l_2 - v_y l_1$ (D) $v_x l_1 - v_y l_2$

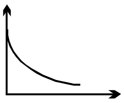
EM0063

16. A uniform but time variant magnetic field exists in a cylindrical region directed along the axis of cylinder of radius R. The graph of induced electric field at a given time v/s. r is (r = distance from axis)

(A) 

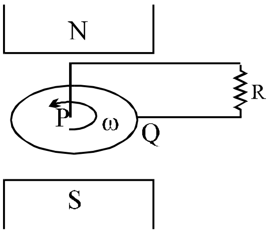
(B) 

(C) 

(D) 

EM0064

17. A metal disc rotates freely, between the poles of a magnet in the direction indicated. Brushes P and Q make contact with the edge of the disc and the metal axle. What current, if any, flows through R ?



(A) a current from P to Q

(B) a current from Q to P

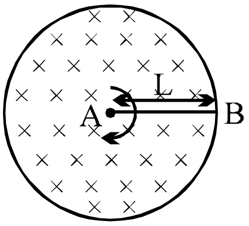
(C) no current, because the emf in the disc is opposed by the back emf

(D) no current, because the emf induced in one side of the disc is opposed by the emf induced in the other side.

(E) no current, because no radial emf is induced in the disc

EM0065

18. A copper rod AB of length L, pivoted at one end A, rotates at constant angular velocity ω, at right angles to a uniform magnetic field of induction B. The e.m.f developed between the mid point C of the rod and end B is



(A) $\frac{B\omega L^2}{4}$ (B) $\frac{B\omega L^2}{2}$ (C) $\frac{3B\omega L^2}{4}$ (D) $\frac{3B\omega L^2}{8}$

EM0066

19. The e.m.f. induced in a coil of wire, which is rotating in a magnetic field, does not depend on

(A) the angular speed of rotation (B) the area of the coil

(C) the number of turns on the coil (D) the resistance of the coil

EM0067

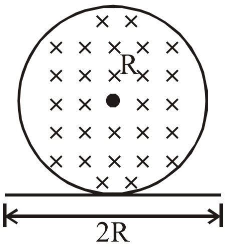
Induced electric field

20. A ring of resistance 10Ω, radius 10cm and 100 turns is rotated at a rate 100 revolutions per second about its diameter is perpendicular to a uniform magnetic field of induction 10mT. The amplitude of the current in the loop will be nearly (Take : $\pi^2 = 10$)

(A) 200A (B) 2A (C) 0.002A (D) none of these

EM0068

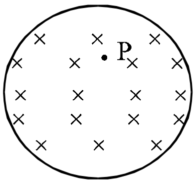
21. A uniform but time varying magnetic field is present in a circular region of radius R. The magnetic field is perpendicular and into the plane of the loop and the magnitude of field is increasing at a constant rate α. There is a straight conducting rod of length 2R placed as shown in figure. The magnitude of induced emf across the rod is



(A) $\pi R^2\alpha$ (B) $\frac{\pi R^2\alpha}{2}$ (C) $\frac{R^2\alpha}{\sqrt{2}}$ (D) $\frac{\pi R^2\alpha}{4}$

EM0069

22. Figure shows a uniform magnetic field B confined to a cylindrical volume and is increasing at a constant rate. The instantaneous acceleration experienced by an electron placed at P is



(A) zero (B) towards right (C) towards left (D) upwards

EM0070

23. **Statement–1 :** For a charged particle moving from point P to point Q the net work done by an induced electric field on the particle is independent of the path connecting point P to point Q.

Statement–2 : The net work done by a conservative force on an object moving along closed loop is zero.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1

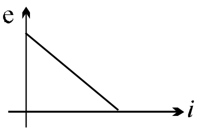
(C) Statement-1 is true, statement-2 is false.

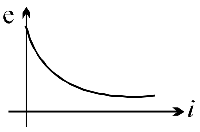
(D) Statement-1 is false, statement-2 is true.

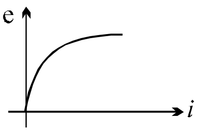
EM0071

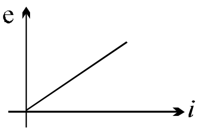
Inductance

24. In an L-R circuit connected to a battery of constant e.m.f. E switch S is closed at time t = 0. If e denotes the magnitude of induced e.m.f. across inductor and *i* the current in the circuit at any time t. Then which of the following graphs shows the variation of e with *i* ?

(A) 

(B) 

(C) 

(D) 

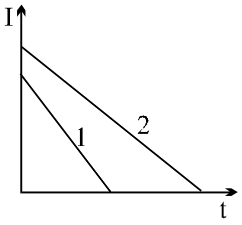
EM0072

25. A current of 2A is increasing at a rate of 4 A/s through a coil of inductance 2H. The energy stored in the inductor per unit time is :-

(A) 2 J/s (B) 1 J/s (C) 16 J/s (D) 4 J/s

EM0073

26. Two identical inductance carry currents that vary with time according to linear laws (as shown in figure). In which of two inductance is the self induction emf greater?

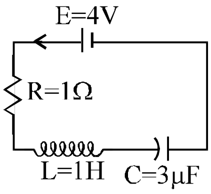


(A) 1 (B) 2

(C) same (D) data are insufficient to decide

EM0074

27. The current in the given circuit is increasing with a rate a = 4 amp/s. The charge on the capacitor at an instant when the current in the circuit is 2 amp will be :



(A) 4μC (B) 5μC (C) 6μC (D) none of these

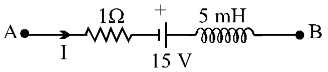
EM0075

28. A long solenoid of N turns has a self inductance L and area of cross section A. When a current i flows through the solenoid, the magnetic field inside it has magnitude B. The current i is equal to:

(A) BAN/L (B) BANL (C) BN/AL (D) B/ANL

EM0076

29. The network shown in the figure is part of a complete circuit. If at a certain instant, the current I is 5A and it is decreasing at a rate of 10^3 As^{-1} then $V_B - V_A$ equals



(A) 20 V (B) 15 V (C) 10 V (D) 5 V

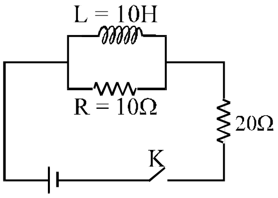
EM0077

30. In **Problem 29**, if I is reversed in direction, then $V_B - V_A$ equals

(A) 5 V (B) 10 V (C) 15 V (D) 20 V

EM0078

31. Two resistors of 10Ω and 20Ω and an ideal inductor of 10H are connected to a 2V battery as shown. The key K is shorted at time t = 0. Find the initial (t = 0) and final (t → ∞) currents through battery.



(A) $\frac{1}{15}A, \frac{1}{10}A$ (B) $\frac{1}{10}A, \frac{1}{15}A$ (C) $\frac{2}{15}A, \frac{1}{10}A$ (D) $\frac{1}{15}A, \frac{2}{25}A$

EM0079

32. An inductor coil stores U energy when i current is passed through it and dissipates energy at the rate of P. The time constant of the circuit, when this coil is connected across a battery of zero internal resistance is :-

(A) $\frac{4U}{P}$ (B) $\frac{U}{P}$ (C) $\frac{2U}{P}$ (D) $\frac{2P}{U}$

EM0080

33. A small coil of radius r is placed at the centre of a large coil of radius R, where R >> r. The coils are coplanar. The coefficient of mutual inductance between the coils is :-

(A) $\frac{\mu_0 \pi r}{2R}$ (B) $\frac{\mu_0 \pi r^2}{2R}$ (C) $\frac{\mu_0 \pi r^2}{2R^2}$ (D) $\frac{\mu_0 \pi r}{2R^2}$

EM0081

34. A long straight wire is placed along the axis of a circular ring of radius R. The mutual inductance of this system is :-

(A) $\frac{\mu_0 R}{2}$ (B) $\frac{\mu_0 \pi R}{2}$ (C) $\frac{\mu_0}{2}$ (D) 0

EM0082

35. Two coils are placed close to each other. The mutual inductance of the pair of coils depends upon- **[AIEEE - 2003]**

(A) the rates at which currents are changing in the two coils
(B) relative position and orientation of the two coils
(C) the materials of the wires of the coils
(D) the currents in the two coils

EM0083

36. A small square loop of wire of side l is placed inside a large square loop of wire of side $L (L >> l)$. The loop are coplanar & their centres coincide. The mutual inductance of the system is proportional to :

(A) $\frac{\ell}{L}$ (B) $\frac{\ell^2}{L}$ (C) $\frac{L}{\ell}$ (D) $\frac{L^2}{\ell}$

EM0084

37. L, C and R represent physical quantities inductance, capacitance and resistance. The combination which has the dimensions of frequency is

(A) $\frac{1}{RC}$ and $\frac{R}{L}$ (B) $\frac{1}{\sqrt{RC}}$ and $\sqrt{\frac{R}{L}}$ (C) $\sqrt{LC}$ (D) $\frac{C}{L}$

EM0085

38. An ideal coil of 10 H is connected in series with a resistance of 5Ω and a battery of 5 V.2s after the connection is made, the current flowing (in ampere) in the circuit is :-

(A) (1 – e) (B) e (C) e^{-1} (D) $(1 - e^{-1})$

EM0086

39. A coil of inductance 5H is joined to a cell of emf 6V through a resistance 10Ω at time t = 0. The emf across the coil at time t = $ln\sqrt{2}$ s is :

(A) 3V (B) 1.5 V (C) 0.75 V (D) 4.5 V

EM0087

40. For L-R circuit, the time constant is equal to

(A) twice the ratio of the energy stored in the magnetic field to the rate of dissipation of energy in the resistance

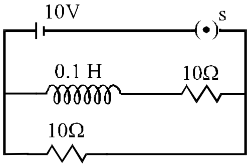
(B) ratio of the energy stored in the magnetic field to the rate of dissipation of energy in the resistance

(C) half the ratio of the energy stored in the magnetic field to the rate of dissipation of energy in the resistance

(D) square of the ratio of the energy stored in the magnetic field to the rate of dissipation of energy in the resistance

EM0088

41. In the adjoining circuit, initially the switch S is open. The switch 'S' is closed at t = 0. The difference between the maximum and minimum current that can flow in the circuit is

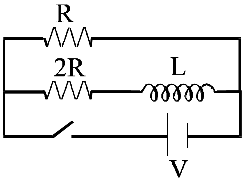


(A) 2 Amp (B) 3 Amp

(C) 1 Amp (D) nothing can be concluded

EM0089

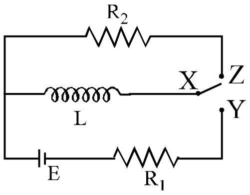
42. Find the ratio of time constant in build up and decay in the circuit as shown in figure :-



(A) 1 : 1 (B) 3 : 2 (C) 2 : 3 (D) 1 : 3

EM0090

43. In the circuit shown, X is joined to Y for a long time, and then X is joined to Z. The total heat produced in R_2 is :



(A) $\frac{LE^2}{2R_1^2}$ (B) $\frac{LE^2}{2R_2^2}$ (C) $\frac{LE^2}{2R_1R_2}$ (D) $\frac{LE^2R_2}{2R_1^2}$

EM0091

44. In a L–R decay circuit, the initial current at t = 0 is I. The total charge that has flown through the resistor till the energy in the inductor has reduced to one–fourth its initial value, is

(A) LI/R (B) $LI/2R$ (C) $LI\sqrt{2}/R$ (D) None

EM0092

45. The inductor in a L–C oscillation has a maximum potential difference of 16 V and maximum energy of 640 μJ. Find the value of capacitor in μF in L–C circuit.

(A) 5 (B) 4 (C) 3 (D) 2

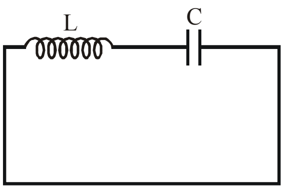
EM0093

46. A condenser of capacity 6 μF is fully charged using a 6-volt battery. The battery is removed and a resistanceless 0.2 mH inductor is connected across the condenser. The current which is flowing through the inductor when one-third of the total energy is in the magnetic field of the inductor is :-

(A) 0.1 A (B) 0.2 A (C) 0.4 A (D) 0.6 A

EM0094

47. In an LC circuit the capacitor has maximum charge q_0. The value of $\left(\frac{dI}{dt}\right)_{max}$ is :-



(A) $\frac{q_0}{LC}$ (B) $\frac{q_0}{\sqrt{LC}}$ (C) $\frac{q_0}{2LC}$ (D) $\frac{2q_0}{LC}$

EM0095

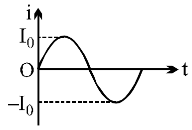
Alternating current

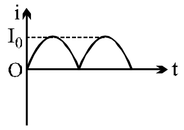
48. When 100 V DC is applied across a solenoid a current of 1 A flows in it. When 100 V AC is applied across the same coil, the current drops to 0.5 A. If the frequency of the AC source is 50 Hz, the impedance and inductance of the solenoid are:

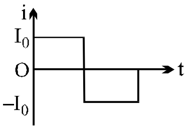
(A) 100Ω, 0.93 H (B) 200Ω, 1.0 H (C) 10Ω, 0.86H (D) 200Ω, 0.55 H

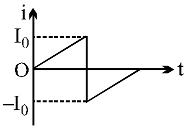
EM0096

49. If I_1, I_2, I_3 and I_4 are the respective r.m.s. values of the time varying currents as shown in the four cases I, II, III and IV. Then identify the correct relations.





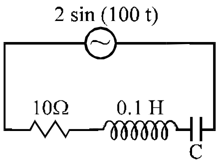




(A) $I_1 = I_2 = I_3 = I_4$ (B) $I_3 > I_1 = I_2 > I_4$ (C) $I_3 > I_4 > I_2 = I_1$ (D) $I_3 > I_2 > I_1 > I_4$

EM0097

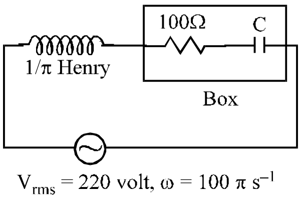
50. The power factor of the circuit is $1/\sqrt{2}$. The capacitance of the circuit is equal to



(A) 400 μF (B) 300 μF (C) 500 μF (D) 200 μF

EM0098

51. In the circuit, as shown in the figure, if the value of R.M.S current is 2.2 ampere, the power factor of the box is



(A) $\frac{1}{\sqrt{2}}$ (B) 1 (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{2}$

EM0099

52. The power in ac circuit is given by $P = E_{rms}I_{rms}\cos\phi$. The value of $\cos\phi$ in series LCR circuit at resonance is:

(A) zero (B) 1 (C) $\frac{1}{2}$ (D) $\frac{1}{\sqrt{2}}$

EM0100

53. In ac circuit when ac ammeter is connected it reads i current. If a student uses dc ammeter in place of ac ammeter the reading in the dc ammeter will be:

(A) $\frac{i}{\sqrt{2}}$ (B) $\sqrt{2}\,i$ (C) 0.637 i (D) zero

EM0101

54. The phase difference between current and voltage in an AC circuit is $\pi/4$ radian. If the frequency of AC is 50 Hz, then the phase difference is equivalent to the time difference :

(A) 0.78 s (B) 15.7 ms (C) 0.25 s (D) 2.5 ms

EM0102

55. The effective value of current $i = 2 \sin 100\pi t + 2 \sin(100\pi t + 30^\circ)$ is :

(A) $\sqrt{2}$ Amp (B) $2\sqrt{2+\sqrt{3}}$ Amp (C) 4 Amp (D) None

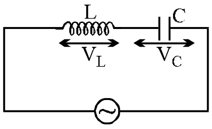
EM0103

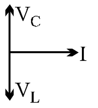
56. In series LR circuit $X_L = 3R$. Now a capacitor with $X_C = R$ is added in series. Ratio of new to old power factor is

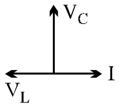
(A) 1 (B) 2 (C) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{2}$

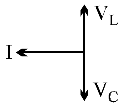
EM0104

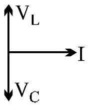
57. The current I, potential difference V_L across the inductor and potential difference V_C across the capacitor in circuit as shown in the figure are best represented vectorially as



(A) 

(B) 

(C) 

(D) 

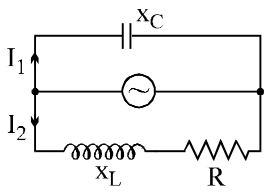
EM0105

58. In a series R-L-C circuit, the frequency of the source is half of the resonance frequency. The nature of the circuit will be

(A) capacitive (B) inductive (C) purely resistive (D) data insufficient

EM0106

59. In the shown AC circuit phase difference between currents I_1 and I_2 is



(A) $\frac{\pi}{2} - \tan^{-1}\frac{x_L}{R}$ (B) $\tan^{-1}\frac{x_L - x_C}{R}$ (C) $\frac{\pi}{2} + \tan^{-1}\frac{x_L}{R}$ (D) $\tan^{-1}\frac{x_L - x_C}{R} + \frac{\pi}{2}$

EM0107

60. In a transformer , number of turns in the primary are 140 and that in the secondary are 280. If current in primary is 4A, then that in the secondary is

(A) 4A (B) 2A (C) 6A (D) 10 A

EM0108

61. The primary of a 1 : 3 step - up transformer is connected to a source and the secondary is connected to a resistor R. The power dissipated by R in this situation is P. If R is connected directly to the source it will dissipate a power of :

(A) P/9 (B) P/3 (C) P (D) 3P

EM0109

62. An ideal efficient transformer has a primary power input of 10kW. The secondary current when the transformer is on load is 25A. If the primary : secondary turns ratio is 8 : 1, then the potential difference applied to the primary coil is

(A) $\frac{10^4 \times 8^2}{25}$V (B) $\frac{10^4 \times 8}{25}$V (C) $\frac{10^4}{25 \times 8}$V (D) $\frac{10^4}{25 \times 8^2}$V

EM0110

63. The core of any transformer is laminated so as to -

(A) Make it light weight (B) Make it robust and strong

(C) Increase the secondary voltage (D) Reduce the energy loss due to eddy current

EM0111

64. If the difference between the equivalent inductance in the following figures is nL then find the value of n. Given coupling coefficient is $C = \sqrt{2}$ (Where coupling coefficient is defined as $C = \frac{\sqrt{L_1L_2}}{M}$)

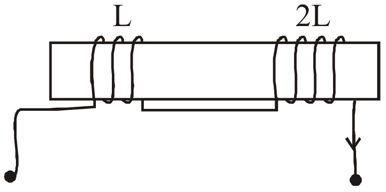


Figure (A)

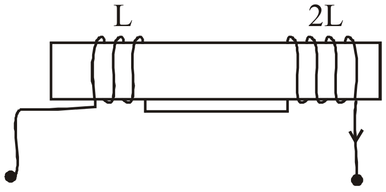


Figure (B)

(A) 2 (B) 3 (C) 4 (D) 5

EM0112

SUPPLEMENT FOR JEE-MAINS

65. Five particles undergodamped harmonic motion. Values for the spring constant k, the damping constant b, and the mass m are given below. Which leads to the smallest rate of loss of mechanical energy at the initial moment?

(A) k = 100N/m, m = 50g, b = 8g/s (B)k = 150N/m, m = 50g, b = 5g/s

(C) k = 150N/m, m = I0g,b = 8g/s (D) k = 200N/m, m = 8g, b - 6g/s

EM0113

66. An RLC circuit has a capacitance of 12μF, an inductance of 25mH, and a resistance of 60Ω. The current oscillates with an angular frequency of :

(A) 1.2×10^3 rad/s (B) 1.4×10^3 rad/s (C) 1.8×10^3 rad/s (D) 2.2×10^3 rad/s
(E) 2.6×10^3 rad/s

EM0114

67. An RLC circuit has an inductance of 25mH and a capacitance of 5.0μF. The charge on the capacitor does NOT oscillate but rather decays exponentially to zero. The resistance in the circuit must be:

(A) greater than or equal to $100\sqrt{2}\Omega$ (B) less than $100\sqrt{2}\Omega$ but greater than $50\sqrt{2}\Omega$
(C) less than $50\sqrt{2}\Omega$ but greater than $25\sqrt{2}\Omega$ (D) less than $25\sqrt{2}\Omega$ but greater than 0

EM0115

68. Two underdamped oscillators are known to have the same natural frequency ω_0. The mass and damping coefficient of the first oscillator are m_1 and b_1, and the mass and damping coeficient of the second oscillator are m_2 and b_2, respectively. A sinusoidal driving force of $F_{ext} = F_0 \cos \omega t$ is applied to each oscillator. Starting with ω far from ω_0 the driving force is tuned in order to observe resonant behavior. If $m_1 = 4\ m_2$ and $b_1 = 2b_2$, then which one of the following statements concerning the driven oscillations is correct ?

(A) The resonant peak of the first driven oscillator is higher and narrower than that ofthe second oscillator.
(B) The resonant peak of the first driven oscillator is higher and wider than that ofthe second oscillator.
(C) The resonant peak of the first driven oscillator is lower and wider than that ofthe second oscillator.
(D) The resonant peak of the first driven oscillator is lower and narrower than that ofthe second oscillator.

EM0116

69. A simple pendulum has a time period T if there is no air resistance. If a small air resistance is acting on the bob as it oscillates,

(A) The time period will be initially more than T and decreases with time.
(B) The time period will be less than T initially and increases with time
(C) The time period will be less than T and remains constant
(D) The time period will be more than T and remains constant.

EM0117

70. A block is executing damped harmonic oscillation with time period T. Choose correct statement

(1) Time taken to go from extreme to mean position is $\frac{T}{4}$

(2) Time taken to go from one extreme to another is —$\frac{T}{2}$

(3) Time taken to go from one extreme to another is less than $\frac{T}{2}$

(4) Time taken to go from one extreme to another is more than $\frac{T}{2}$

(A) 1,2 only (B) 1,2,3 only (C) 2 only (D) 1,2,4 only

EM0118

71. The angular frequecny of the damped oscillator is given by, $\omega = \sqrt{\frac{k}{m} - \frac{r^2}{4m^2}}$ where k is the spring constant, m is the mass of the oscillator and r is damping constant. If the ratio $\frac{r^2}{mk}$ is 8%, the change in time period composed to the undamped oscillator is approximately as follows:

(A) decreases by 8% (B) decreases by 1% (C) increases by 1% (D) increases by 8%

EM0119

72. Two spheres of the same diameter but of different masses are suspended by strings of equal length. If the spheres are deflected from their positions of equilibrium, which of the two will have a greater oscillation period and which will have a greater logarithmic decrement if their oscillations occur in a real medium with viscosity?

(A) Heavier mass has larger time period & greater logrithmic decrement

(B) Lighter mass has larger time period & greater logrithmic decrement

(C) Lighter mass has larger time period but lesser logrithmic decrement

(D) Heavier mass has larger time period but lesser logrithmic decrement

EM0120

73. The amplitude of a simple pendulum, oscillating in air with a small spherical bob, decreases from 10cm to 8 cm in 40 seconds. Assuming that Stokes law is valid, and ratio of the coefficient of viscosity of air to that of carbon dioxide is 1.3, the time in which amplitude of this pendulum will reduce from 10 cm to 5 cm in carbondioxide will be close to (ℓn 5 = 1.601, ℓn 2 = 0.693).

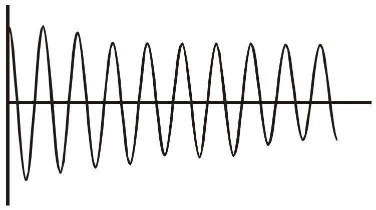
[JEE Main Online 2014]

(A) 231 s (B) 208 s (C) 142 s (D) 161 s

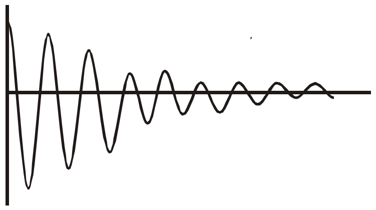
EM0121

74. Which graph has the highest Q factor ?

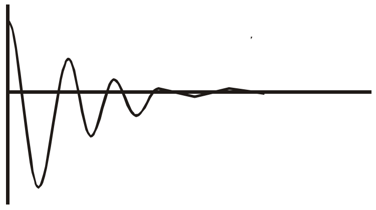
(A)



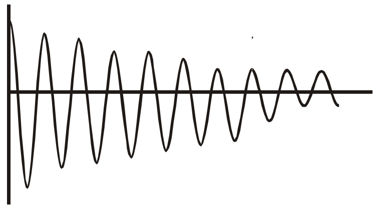
(B)



(C)

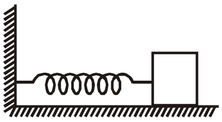


(D)



EM0122

75. In the situation shown, the block can execute free oscillation (no damping) with angular frequency ω_1. In presence of weak damping, it executes damped SHM with angular frequency ω_2. When it is subjected to a sinusoidal force, it executes forced oscillation with maximum amplitude at angular frequency ω_3 (assume damping is present) :-



(A) $\omega_1 > \omega_2 > \omega_3$ (B) $\omega_1 > \omega_2 = \omega_3$ (C) $\omega_1 = \omega_2 = \omega_3$ (D) $\omega_1 > \omega_3 > \omega_2$

EM0123

76. In forced oscillation of a particle the amplitude is maximum for a frequency ω_1 of the force, while the power is maximum for a frequency ω_2 of the force; then – **[AIEEE- 2004]**

(A) $\omega_1 = \omega_2$

(B) $\omega_1 > \omega_2$

(C) $\omega_1 < \omega_2$ when damping is small and $\omega_1 > \omega_2$ when damping is large

(D) $\omega_1 < \omega_2$

EM0124

77. A pendulum with time period of 1s is losing energy due to damping. At certain time its energy is 45 J. If after completing 15 oscillations, its energy has become 15 J, its damping constant (in s^{-1}) is:- **[JEE Mains On line 2015]**

(A) 2 (B) $\frac{1}{15}\ln 3$ (C) $\frac{1}{2}$ (D) $\frac{1}{30}\ln 3$

EM0125

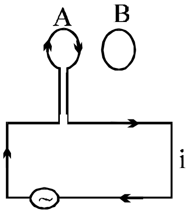
MULTIPLE CORRECT TYPE QUESTIONS

78. The dimension of the ratio of magnetic flux and the resistance is equal to that of :

(A) induced emf (B) charge (C) inductance (D) current

EM0126

79. Two circular coils A and B are facing each other as shown in figure. The current i through A can be altered



(A) there will be repulsion between A and B if i is increased

(B) there will be attraction between A and B if i is increased

(C) there will be neither attraction nor repulsion when i is changed

(D) attraction or repulsion between A and B depends on the direction of current. It does not depend whether the current is increased or decreased.

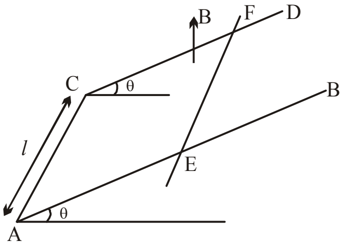
EM0127

80. A bar magnet is moved along the axis of copper ring placed far away from the magnet. Looking from the side of the magnet, an anticlockwise current is found to be induced in the ring. Which of the following may be true?

(A) The south pole faces the ring and the magnet moves towards it.
(B) The north pole faces the ring and the magnet moves towards it.
(C) The south pole faces the ring and the magnet moves away from it.
(D) The north pole faces the ring and the magnet moves away from it.

EM0128

81. AB and CD are smooth parallel rails, separated by a distance *l*, and inclined to the horizontal at an angle θ. A uniform magnetic field of magnitude B, directed vertically upwards, exists in the region. EF is a conductor of mass m, carrying a current *i*. For EF to be in equilibrium,



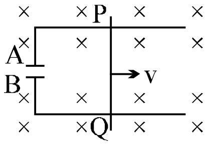
(A) *i* must flow from E to F (B) Bil = mg tan θ
(C) Bil = mg sin θ (D) Bil = mg

EM0129

82. In the **previous question**, if B is normal to the plane of the rails
(A) Bil = mg tan θ (B) Bil = mg sin θ
(C) Bil = mg cos θ (D) equilibrium cannot be reached

EM0130

83. A conducting rod PQ of length L = 1.0 m is moving with a uniform speed v = 20 m/s in a uniform magnetic field B = 4.0 T directed into the paper. A capacitor of capacity C = 10 μF is connected as shown in figure. Then



(A) $q_A = +800\mu C$ and $q_B = -800\mu C$
(B) $q_A = -800\mu C$ and $q_B = +800\mu C$
(C) $q_A = 0 = q_B$
(D) charge stored in the capacitor increases exponentially with time

EM0131

84. An LR circuit with a battery is connected at t = 0. Which of the following quantities is not zero just after the circuit
(A) current in the circuit (B) magnetic field energy in the inductor
(C) power delivered by the battery (D) emf induced in the inductor

EM0132

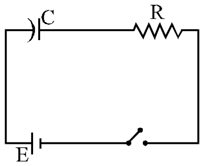
85. The switches in figures (a) and (b) are closed at t = 0

(A) The charge on C just after t = 0 is EC.

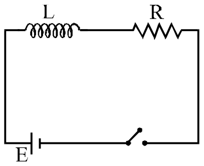
(B) The charge on C long after t = 0 is EC.

(C) The current in L just after t = 0 is E/R.

(D) The current in L long after t = 0 is E/R.



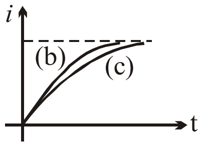
(a)



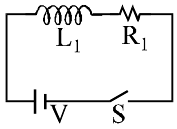
(b)

EM0133

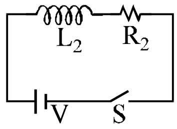
86. Current growth in two L-R circuits (b) and (c) as shown in figure (a). Let L_1, L_2, R_1 and R_2 be the corresponding values in two circuits. Then :-



(a)



(b)

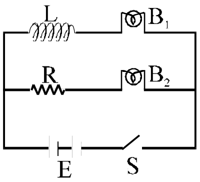


(c)

(A) $R_1 > R_2$ (B) $R_1 = R_2$ (C) $L_1 > L_2$ (D) $L_1 < L_2$

EM0134

87. An inductor L, a resistance R and two identical bulbs B_1 and B_2 are connected to a battery through a switch S as shown in the figure. The resistance of coil having inductance L is also R. Which of the following statement gives the correct description of the happenings when the switch S is closed ?



(A) The bulb B_2 lights up earlier than B_1 and finally both the bulbs shine equally bright.

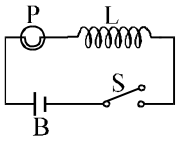
(B) B_1 lights up earlier and finally both the bulbs acquire equal brightness.

(C) B_2 lights up earlier and finally B_1 shines brighter than B_2.

(D) B_1 and B_2 light up together with equal brightness all the time.

EM0135

88. In figure, a lamp P is in series with an iron-core inductor L. When the switch S is closed, the brightness of the lamp rises relatively slowly to its full brightness than it would do without the inductor. This is due to



(A) the low resistance of P (B) the induced-emf in L

(C) the low resistance of L (D) the high voltage of the battery B

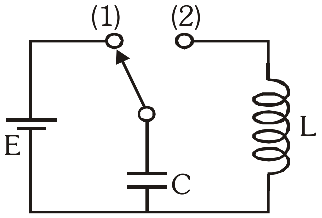
EM0136

89. Two different coils have self inductance 8mH and 2mH. The current in one coil is increased at a constant rate. The current in the second coil is also increased at the same constant rate. At a certain instant of time, the power given to the two coils is the same. At that time the current, the induced voltage and the energy stored in the first coil are I_1, V_1 and W_1 respectively. Corresponding values for the second coil at the same instant are I_2, V_2 and W_2 respectively. Then:

(A) $\frac{I_1}{I_2}=\frac{1}{4}$ (B) $\frac{I_1}{I_2}=4$ (C) $\frac{W_2}{W_1}=4$ (D) $\frac{V_2}{V_1}=\frac{1}{4}$

EM0137

90. Initially key was placed on (1) till the capacitor got fully charged. Key is placed on (2) at t = 0. The time when the energy in both capacitor and inductor will be same-



(A) $\frac{\pi\sqrt{LC}}{4}$ (B) $\frac{\pi\sqrt{LC}}{2}$ (C) $\frac{5\pi\sqrt{LC}}{4}$ (D) $\frac{5\pi\sqrt{LC}}{2}$

EM0138

COMPREHENSION TYPE QUESTIONS

Paragraph for Question Nos. 91 to 93

The fact that a changing magnetic flux produces an electric field is basic to the operation of many high energy particle accelerators. Since the principle was first successfully applied to the acceleration of electrons (or β particles) in a device called the betatron, this method of acceleration is often given that name. The general idea involved is shown in figure.

An electromagnet is used to produce a changing flux through a circular loop defined by the doughnut-shaped vacuum chamber. We see that there will be an electric field E along the circular length of the doughnut, i.e. circling the magnet poles, given by

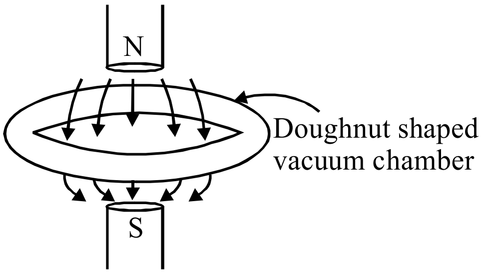
$$2\pi aE=\frac{d\phi}{dt}$$

Where a is the radius of the doughnut. Any charged particle inside the vacuum chamber will experience a force qE and will accelerate. Ordinarily, the charged particle would shoot out of the vacuum chamber and becomes lost.

However, if the magnetic field at the position of the doughnut is just proper to satisfy the relation Centripetal force = magnetic force

$$\text{or } \frac{mv^2}{a}=qvB$$

then the charge will travel in a circle within the doughnut. By proper shaping of the magnet pole pieces, this relation can be satisfied. As a result, the charge will move at high speed along the loop within the doughnut. Each time it goes around the loop, it has, in effect, fallen through a potential difference equal to the induced, emf, namely $\varepsilon = \frac{d\phi}{dt}$. Its energy after n trips around the loop will be q (nε)



91. Working of betatron is not based upon which of the following theories :-

(A) Changing magnetic flux induces electric field

(B) Charged particle at rest can be accelerated only by electric fields

(C) magnetic fields can apply a force on moving charges which is perpendicular to both magnetic field and motion of the particle

(D) Beta particles are emitted in radioactive decay process.

EM0139

92. Variable magnetic flux :-

(A) Can change sinusoidally

(B) Should increase all the time

(C) Must becomes zero when induced field is maximum

(D) None of these

EM0139

93. Magnetic field which keeps the particles in circular path must :-

(A) Remain a constant every where

(B) Increase gradually which is proportional to K.E. of the particle

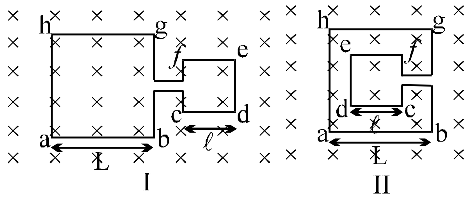
(C) Increase gradually which is proportional to speed of the particle

(D) None of these

EM0139

Paragraph for Question No. 94 to 97

The adjoining figure shows two different arrangements in which two square wire frames of same resistance are placed in a uniform constantly decreasing magnetic field B.



94. The value of magnetic flux in each case is given by

(A) Case I: $\Phi = \pi(L^2 + \ell^2)B$; Case II: $\Phi = \pi(L^2 - \ell^2)B$

(B) Case I: $\Phi = \pi(L^2 + \ell^2)B$; Case II: $\Phi = \pi(L^2 + \ell^2)B$

(C) Case I: $\Phi = (L^2 + \ell^2)B$; Case II: $\Phi = (L^2 - \ell^2)B$

(D) Case I: $\Phi = (L + \ell)^2B$; Case II: $\Phi = \pi(L - \ell)^2B$

EM0140

95. The direction of induced current in the case I is

(A) from a to b and from c to d (B) from a to b and from *f* to e

(C) from b to a and from d to c (D) from b to a and from e to *f*

EM0140

96. The direction of induced current in the case II is

(A) from a to b and from c to d (B) from b to a and from *f* to e

(C) from b to a and from c to d (D) from a to b and from d to c

EM0140

97. If I_1 and I_2 are the magnitudes of induced current in the cases I and II, respectively, then

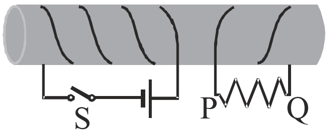
(A) $I_1 = I_2$ (B) $I_1 > I_2$ (C) $I_1 < I_2$ (D) nothing can be said

EM0140

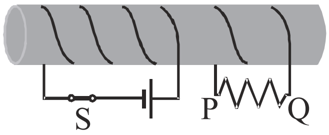
EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

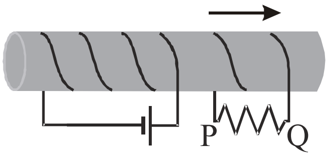
1. For each of the experiments (1, 2, 3, 4) shown in figure. Choose the **CORRECT** option(s) which shows the direction of current flow through the resistor PQ? Note that the wires are not always wrapped around the plastic tube in the same way.



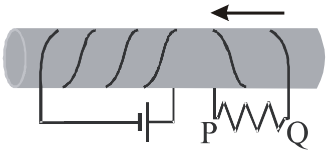
(1) S to be closed



(2) S to be opened



(3) Resistor coil PQ moves to right



(4) Resistor coil PQ moves to left

(1)	(2)	(3)	(4)
(A) P to Q	P to Q	P to Q	P to Q
(B) P to Q	Q to P	P to Q	Q to P
(C) Q to P	Q to P	Q to P	Q to P
(D) Q to P	Q to P	P to Q	P to Q

EM0141

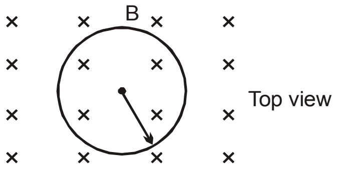
2. An electron is moving in a circular orbit of radius R with an angular acceleration α. At the centre of the orbit is kept a conducting loop of radius r, (r <<R). The e.m.f induced in the smaller loop due to the motion of the electron is :-

(A) zero, since charge on electron in constant (B) $\frac{\mu_0 e r^2}{4R}\alpha$

(C) $\frac{\mu_0 e r^2}{4\pi R}\alpha$ (D) none of these

EM0142

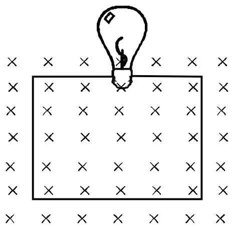
3. A non conducting ring (of mass m, radius r, having charge Q) is placed on a rough horizontal surface (in a region with transverse magnetic field). The field is increasing with time at the rate R and coefficient of friction between the surface and the ring is μ. For ring to remain in equilibrium μ should be greater than :-



(A) $\frac{QrR}{mg}$ (B) $\frac{QrR}{2mg}$ (C) $\frac{QrR}{3mg}$ (D) $\frac{2QrR}{mg}$

EM0143

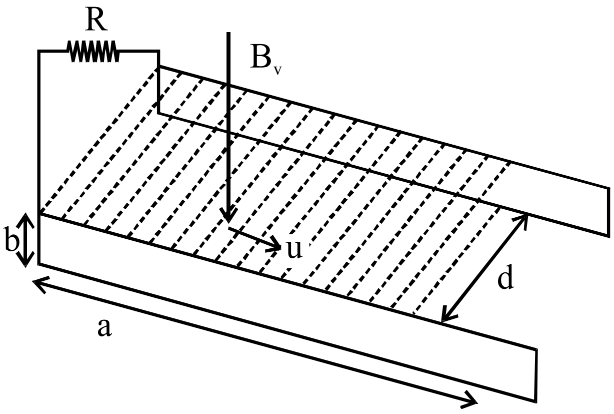
4. A square wire loop of 10.0 cm side lies at right angles to a uniform magnetic field of 20T. A 10 V light bulb is in a series with the loop as shown in the fig. The magnetic field is decreasing steadily to zero over a time interval Δt. The bulb will shine with full brightness if Δt is equal to :-



(A) 20 ms (B) 0.02 ms (C) 2 ms (D) 0.2 ms

EM0144

5. The figure shows an apparatus suggested by Faraday to generate electric current from a flowing river. Two identical conducting plates of length a and width b are placed parallel facing one another on opposite sides of the river following with velocity u at a distance d apart. Now both the plates are connected by a load resistance R. Then the current through the load R is :- (Consider vertical component of the magnetic field produced by earth is B_v and the resistivity of river water is ρ.)



(A) $\frac{B_v ub}{R}$ (B) $\frac{B_v ub}{R+\frac{\rho d}{ab}}$ (C) $\frac{B_v ud}{R+\frac{\rho d}{ab}}$ (D) None of the above

EM0145

6. The radius of a coil decreases steadily at the rate of 10^{-2} m/s. A constant and uniform magnetic field of induction 10^{-3} Wb/m^2 acts perpendicular to the plane of the coil. The radius of the coil when the induced e.m.f. in the coil is 1μV, is :-

(A) $\frac{2}{\pi}$ cm (B) $\frac{3}{\pi}$ cm (C) $\frac{4}{\pi}$ cm (D) $\frac{5}{\pi}$ cm

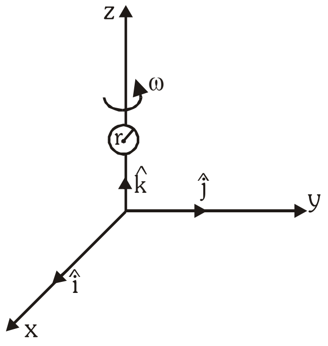
EM0146

7. A composite rod of length ℓ is one fourth insulator and remaining conductor is made to rotate freely with angular velocity ω, in a space free of any gravitational, electric & magnetic field. Then potential difference across the conducting region will be (rotation is about insulating end).

(A) $\frac{3m_e\omega^2\ell^2}{4e}$ (B) $\frac{1}{4}\frac{m_e\omega^2\ell^2}{e}$ (C) $\frac{1}{16}\frac{m_e\omega^2\ell^2}{e}$ (D) $\frac{15}{32}\frac{m_e\omega^2\ell^2}{e}$

EM0147

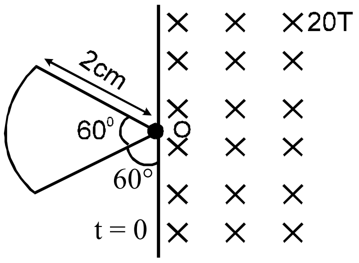
8. A circular loop wire of radius r rotates about the z–axis with angular velocity ω. The normal to the loop is always perpendicular to the z–axis. At time t = 0, the normal is parallel to the y–axis. An external magnetic field $\vec{B} = B_y\hat{j} + B_z\hat{k}$ is applied. The EMF ε(t) induced in the loop is



(A) $\pi r^2\omega B_y \sin\omega t$ (B) $\pi r^2\omega B_z \cos\omega t$ (C) $\pi r^2\omega B_z \sin\omega t$ (D) $\pi r^2\omega B_y \cos\omega t$

EM0148

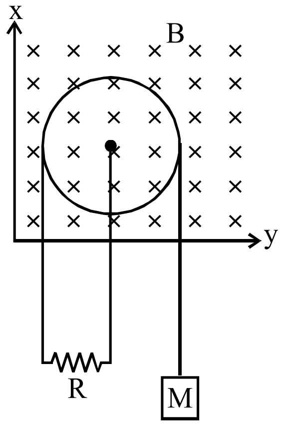
9. A uniform magnetic field 20 T exists on right side of the boundary in a gravity free space as shown in figure. The given circular arc of radius 2 cm made of conducting wire of total resistance 4Ω is rotated around point O at a constant angular speed 2 rad per second. Power required to maintain the constant angular velocity between time interval t = $\frac{\pi}{6}$s to t = $\frac{\pi}{3}$s is :-



(A) 64 μ W (B) 32 μ W (C) 128 μ W (D) 16 μ W

EM0149

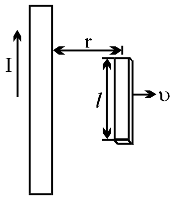
10. The block of mass (M) is connected by thread which is wound on a pulley, free to rotate about fixed horizontal axis as shown. A uniform magnetic field B exists in a horizontal plane. The disc is connected with the resistance R as shown. Calculate the terminal velocity of the block if it was released from rest. Treat pulley as uniform metallic disc of radius L.



(A) $\frac{4mgR}{B^2L^2}$ (B) $\frac{3mgR}{4B^2L^2}$ (C) $\frac{2mgR}{B^2L^2}$ (D) $\frac{3mgR}{2B^2L^2}$

EM0150

11. A conducting rod moves with constant velocity υ perpendicular to the long, straight wire carrying a current I as shown. Compute the emf generated between the ends of the rod.



(A) $\frac{\mu_0 \upsilon I l}{\pi r}$ (B) $\frac{\mu_0 \upsilon I l}{2\pi r}$ (C) $\frac{2\mu_0 \upsilon I l}{\pi r}$ (D) $\frac{\mu_0 \upsilon I l}{4\pi r}$

EM0151

12. A metallic rod of length L and mass M is moving under the action of two unequal forces F_1 and F_2 (directed opposite to each other) acting at its ends along its length. Ignore gravity and any external magnetic field. If specific charge of electrons is (e/m), then the potential difference between the ends of the rod in steady state must be

(A) $|F_1 - F_2| mL/eM$ (B) $(F_1 + F_2)$ mL/eM

(C) $[mL/eM]$ ln $[F_1/F_2]$ (D) None

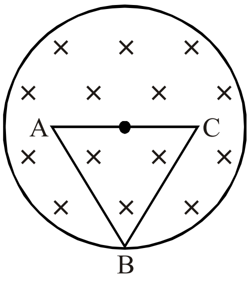
EM0152

13. Two parallel long straight conductors lie on a smooth surface. Two other parallel conductors rest on them at right angles so as to form a square of side a initially. A uniform magnetic field B exists at right angles to the plane containing the conductors. They all start moving out with a constant velocity v. If r is the resistance per unit length of the wire the current in the circuit will be

(A) $\frac{Bv}{r}$ (B) $\frac{Br}{v}$ (C) Bvr (D) Bv

EM0153

14. An equilateral triangle ABC of side a is placed in the magnetic field with side AC and its centre coinciding with the centre of the magnetic field. The magnetic field varies with time as B = ct. The emf induced across side AB is :-



(A) $\frac{\sqrt{3}}{4}a^2c$ (B) Zero (C) $\frac{\sqrt{3}}{8}a^2c$ (D) $\frac{(\sqrt{2}-1)}{2}a^2c$

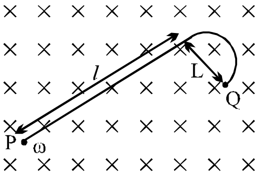
EM0154

15. The magnetic field in a region is given by $\vec{B} = B_0\left(1+\frac{x}{a}\right)\hat{k}$. A square loop of edge - length d is placed with its edge along x & y axis. The loop is moved with constant velocity $\vec{V} = V_0\hat{i}$. The emf induced in the loop is

(A) $\frac{V_0B_0d^2}{a}$ (B) $\frac{V_0B_0d^2}{2a}$ (C) $\frac{V_0B_0a^2}{d}$ (D) None

EM0155

16. When a 'J' shaped conducting rod is rotating in its own plane with constant angular velocity ω, about one of its end P, in a uniform magnetic field $\vec{B}$ directed normally into the plane of paper then magnitude of emf induced across it will be



(A) $B\omega\sqrt{L^2+l^2}$ (B) $\frac{1}{2}B\omega L^2$ (C) $\frac{1}{2}B\omega(L^2+l^2)$ (D) $\frac{1}{2}B\omega l^2$

EM0156

17. A rectangular coil of single turn, having area A, rotates in a uniform magnetic field B with an angular velocity ω about an axis perpendicular to the field. If initially the plane of coil is perpendicular to the field, then the average induced e.m.f. when it has rotated through 90° is :-

(A) $\frac{\omega BA}{\pi}$ (B) $\frac{\omega BA}{2\pi}$ (C) $\frac{\omega BA}{4\pi}$ (D) $\frac{2\omega BA}{\pi}$

EM0157

18. In figure (a) a solenoid produces a magnetic field whose strength increases into the plane of the page. An induced emf is established in a conduction loop surrounding the solenoid, and this emf lights bulbs A and B. In figure (b) point P and Q are shorted. After the short is inserted

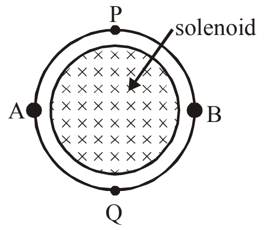


fig. (a)

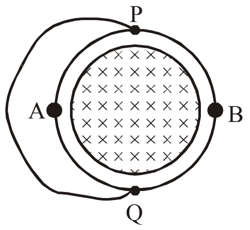
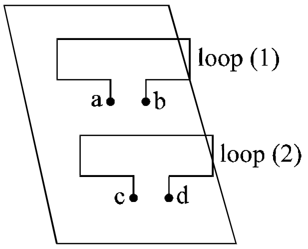


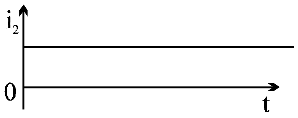
fig. (b)

(A) Bulb A goes out, bulb B gets brighter
(B) Bulb B goes out, bulb A gets brighter
(C) Bulb A goes out, bulb B gets dimmer
(D) Bulb B goes out, bulb A gets dimmer

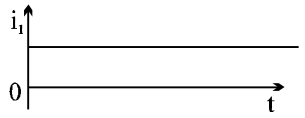
EM0158

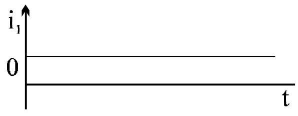
19. An electric current i_1 can flow either direction through loop (1) and induced current i_2 in loop (2). Positive i_1 is when current is from 'a' to 'b' in loop (1) and positive i_2 is when the current is from 'c' to 'd' in loop (2) In an experiment, the graph of i_2 against time 't' is as shown below

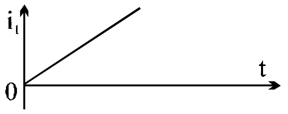


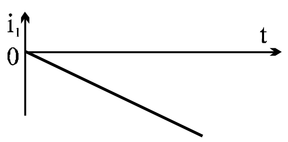


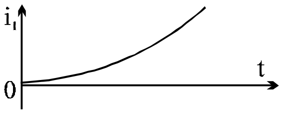
Which one(s) of the following graphs could have caused i_2 to behave as given above.

(A)


(B)


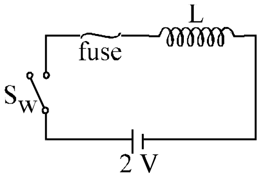
(C)


(D)


(E)


EM0159

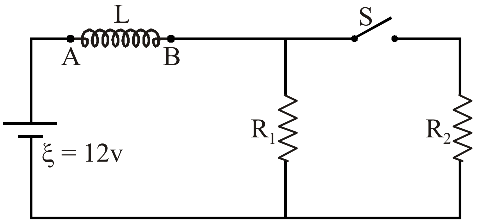
20. In the circuit shown, the cell is ideal. The coil has an inductance of 4H and zero resistance. F is a fuse of zero resistance and will blow when the current through it reaches 5A. The switch is closed at t = 0. The fuse will blow :



(A) just after t=0 (B) after 2s (C) after 5s (D) after 10s

EM0160

21. The circuit shown has been operating for a long time. The instant after the switch in the circuit labeled S is opened, what is the voltage across the inductor V_L and which labeled point (A or B) of the inductor is at a higher potential ? Take $R_1 = 4.0\ \Omega$, $R_2 = 8.0\ \Omega$, and L = 2.5 H.



(A) V_L = 12 V ; Point A is at the higher potential

(B) V_L = 12 V ; Point B is at the higher potential

(C) V_L = 6V ; Point A is at the higher potential

(D) V_L = 6V ; Point B is at the higher potential

EM0161

22. When a resistance R is connected in series with an element A, the electric current is found to be lagging behind the voltage by angle θ_1. When the same resistance is connected in series with element B, current leads voltage by θ_2. When R, A, B are connected in series, the current now leads voltage by θ. Assume same AC source is used in all cases, then

(A) $\theta = \theta_2 - \theta_1$ (B) $\tan\theta = \tan\theta_2 - \tan\theta_1$

(C) $\theta = \frac{\theta_1 + \theta_2}{2}$ (D) None of these

EM0162

23. An current is given by $I = I_0 + I_1 \sin \omega t$ then its rms value will be

(A) $\sqrt{I_0^2 + 0.5 I_1^2}$ (B) $\sqrt{I_0^2 + 0.5 I_0^2}$ (C) 0 (D) $I_0/\sqrt{2}$

EM0163

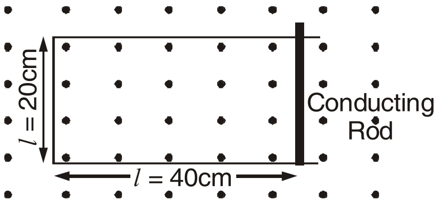
24. Power factor of an L-R series circuit is 0.6 and that of a C–R series circuit is 0.5. If the element (L, C, and R) of the two circuits are joined in series the power factor of this circuit is found to be 1. The ratio of the resistance in the L-R circuit to the resistance in the C–R circuit is

(A) 6/5 (B) 5/6 (C) $\frac{4}{3\sqrt{3}}$ (D) $\frac{3\sqrt{3}}{4}$

EM0164

MULTIPLE CORRECT TYPE QUESTIONS

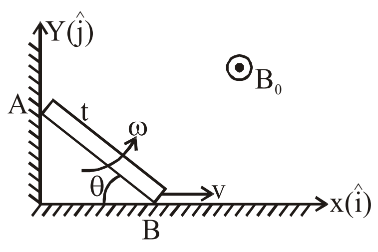
25. Figure shows a conducting rod of negligible resistance that can slide on smooth U-shaped rail made of wire of resistance 1 Ω/m. Position of the conducting rod at t = 0 is shown. A time dependent magnetic field B = 2t Tesla is switched on at t = 0. After the magnetic field is switched on, the conducting rod is moved to the left perpendicular to the rails at constant speed 5 cm/s by some external agent.



(A) The current in the loop at t = 0 due to induced emf is 0.16 A, clockwise
(B) At t = 2s, induced emf has magnitude 0.08 V
(C) The magnitude of the force required to move the conducting rod at constant speed 5 cm/s at t = 2s, is equal to 0.08 N
(D) The magnitude of the force required to move the conducting rod at constant speed 5 cm/s at t = 2s, is equal to 0.16 N

EM0165

26. A thin conducting rod of length ℓ is moved such that its end B moves along the X-axis while end A moves along the Y-axis. A uniform magnetic field $B = B_0\hat{k}$ exists in the region. At some instant, velocity of end B is v and the rod makes an angle of θ = 60° with the X-axis as shown in the figure. Then, at this instant

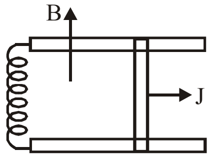


(A) angular speed of rod AB is $\omega = \frac{2v}{\sqrt{3}\ell}$ (B) angular speed of rod AB is $\omega = \frac{\sqrt{3}v}{2\ell}$

(C) e.m.f. induced in rod AB is $B\ell v\sqrt{3}$ (D) e.m.f. induced in rod AB is $B\ell v/2\sqrt{3}$

EM0166

27. Two parallel resistanceless rails are connected by an inductor of inductance L at one end as shown in the figure. A magnetic field B exists in the space which is perpendicular to the plane of the rails. Now a conductor of length ℓ and mass m is placed transverse on the rails and given an impulse J towards the rightward direction. Then choose the **CORRECT** option (s).



(A) Velocity of the conductor is half of the initial velocity after a displacement of the conductor

$d = \sqrt{\dfrac{3J^2L}{4B^2\ell^2 m}}$

(B) Current flowing through the inductor at the instant when velocity of the conductor is half of the initial velocity is $i = \sqrt{\dfrac{3J^2}{4Lm}}$

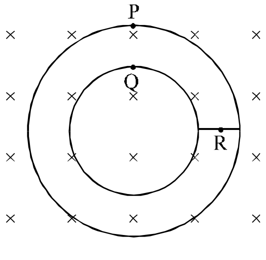
(C) Velocity of the conductor is half of the initial velocity after a displacement of the conductor

$d = \sqrt{\dfrac{3J^2L}{B^2\ell^2 m}}$

(D) Current flowing through the inductor at the instant when velocity of the conductor is half of the initial velocity is $i = \sqrt{\dfrac{3J^2}{mL}}$

EM0167

28. Figure shown plane figure made of a conductor located in a magnetic field along the inward normal to the plane of the figure. The magnetic field starts diminishing. Then the induced current



(A) at point P is clockwise

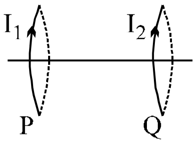
(B) at point Q is anticlockwise

(C) at point Q is clockwise

(D) at point R is zero

EM0168

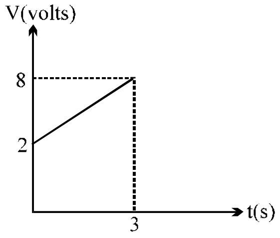
29. Two circular coils P & Q are fixed coaxially & carry currents I_1 and I_2 respectively



(A) if $I_2 = 0$ & P moves towards Q, a current in the same direction as I_1 is induced in Q
(B) if $I_1 = 0$ & Q moves towards P, a current in the opposite direction to that of I_2 is induced in P.
(C) when $I_1 \neq 0$ and $I_2 \neq 0$ are in the same direction then the two coils tend to move apart .
(D) when $I_1 \neq 0$ and $I_2 \neq 0$ are in opposite directions then the coils tends to move apart.

EM0169

30. A circuit element is placed in a closed box. At time t = 0, constant current generator supplying a current of 1 amp, is connected across the box. Potential difference across the box varies according to graph shown in figure. The element in the box is :



(A) resistance of 2Ω
(B) battery of emf 6V
(C) inductance of 2H
(D) capacitance of 0.5F

EM0170

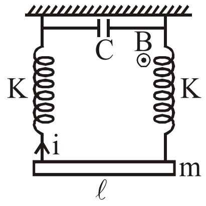
31. Two coils A and B have coefficient of mutual inductance M = 2H. The magnetic flux passing through coil A changes by 4 Weber in 10 seconds due to the change in current in B. Then
(A) the change in current in B in this time interval is 0.5 A
(B) the change in current in B in this time interval is 2A
(C) the change in current in B in this time interval is 8A
(D) a change in current of 1A in coil A will produce a change in flux passing through B by 4 Weber.

EM0171

COMPREHENSION TYPE QUESTIONS

Paragraph for Question Nos. 32 and 33

In the figure shown a uniform conducting rod of mass m and length ℓ is suspended in vertical plane by two conducting springs of spring constant K. Upper end of spring are connected to each other by capacitor of capacitance C. A uniform horizontal magnetic field (B_0) perpendicular to plane of spring exists in space. Initially rod is in equilibrium but if centre of rod is pulled down and released, it performs SHM. Assume that the spring is small and neglect the magnetic force of interaction between circular section of springs & self inductance of rod.



32. Find time period of oscillation of rod :-

(A) $2\pi\sqrt{\frac{m}{k}}$ (B) $2\pi\sqrt{\frac{B^2\ell^2C}{K}}$ (C) $\pi\sqrt{\frac{m+B^2\ell^2C}{K}}$ (D) $2\pi\sqrt{\frac{B^2\ell^2C+m}{2K}}$

EM0172

33. Choose correct options from following :-

(A) Electrical energy stored in capacitor is maximum when rod is at its lower extreme position

(B) Electrical energy stored in capacitor is maximum when rod is at its mean position

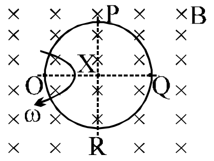
(C) Current in rod is maximum at mean position of rod

(D) If magnetic field is switched off then mean position of rod will change

EM0172

Paragraph for Question No. 34 to 36

A conducting ring of radius a is rotated about a point O on its periphery as shown in the figure in a plane perpendicular to uniform magnetic field B which exists everywhere. The rotational velocity is ω.



34. Choose the correct statement(s) related to the potential of the points P, Q and R

(A) $V_P - V_O > 0$ and $V_R - V_O < 0$ (B) $V_P = V_R > V_O$

(C) $V_O > V_P = V_Q$ (D) $V_Q - V_P = V_P - V_O$

EM0173

35. Choose the correct statement(s) related to the magnitude of potential differences

(A) $V_P - V_O = \frac{1}{2}B\omega a^2$ (B) $V_P - V_Q = \frac{1}{2}B\omega a^2$

(C) $V_Q - V_O = 2B\omega a^2$ (D) $V_P - V_R = 2B\omega a^2$

EM0173

36. Choose the correct statement(s) related to the induced current in the ring

(A) Current flows from Q ⟶ P ⟶ O ⟶ R ⟶ Q

(B) Current flows from Q ⟶ R ⟶ O ⟶ P ⟶ Q

(C) Current flows from Q ⟶ P ⟶ O and from Q ⟶ R ⟶ O

(D) No current flows

EM0173

Paragraph for question nos. 37 to 39

In a series L-R circuit, connected with a sinusoidal ac source, the maximum potential difference across L and R are respectively 3 volts and 4 volts.

37. At an instant the potential difference across resistor is 2 volts. The potential difference in volt, across the inductor at the same instant will be :

(A) 3 cos 30° (B) 3 cos 60° (C) 3 cos 45° (D) None of these

EM0174

38. At the same instant, the magnitude of the potential difference in volt, across the ac source may be

(A) $4+3\sqrt{3}$ (B) $\frac{4+3\sqrt{3}}{2}$ (C) $1+\frac{\sqrt{3}}{2}$ (D) $2+\frac{\sqrt{3}}{2}$

EM0174

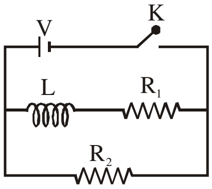
39. If the current at this instant is decreasing the magnitude of potential difference at that instant across the ac source is

(A) Increasing (B) Decreasing (C) Constant (D) Cannot be said

EM0174

EXERCISE-JM

1. In the circuit show below, the key K is closed at t = 0. The current through the battery is : **[AIEEE - 2010]**

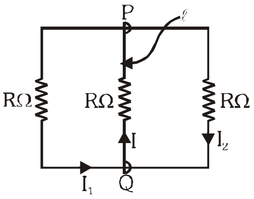


(1) $\frac{V(R_1+R_2)}{R_1R_2}$ at t = 0 and $\frac{V}{R_2}$ at $t=\infty$ (2) $\frac{VR_1R_2}{\sqrt{R_1^2+R_2^2}}$ at t = 0 and $\frac{V}{R_2}$ at $t=\infty$

(3) $\frac{V}{R_2}$ at t = 0 and $\frac{V(R_1+R_2)}{R_1R_2}$ at $t=\infty$ (4) $\frac{V}{R_2}$ at t = 0 and $\frac{VR_1R_2}{\sqrt{R_1^2+R_2^2}}$ at $t=\infty$

EM0175

2. A rectangular loop has a sliding connector PQ of length ℓ and resistance RΩ and it is moving with a speed v as shown. The set-up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1, I_2 and I are :- **[AIEEE - 2010]**



(1) $I_1 = I_2 = \frac{B\ell v}{6R}, I = \frac{B\ell v}{3R}$ (2) $I_1 = -I_2 = \frac{B\ell v}{R}, I = \frac{2B\ell v}{R}$

(3) $I_1 = I_2 = \frac{B\ell v}{3R},\ I = \frac{2B\ell v}{3R}$ (4) $I_1 = I_2 = I = \frac{B\ell v}{R}$

EM0176

3. In a series LCR circuit R = 200Ω and the voltage and the frequency of the main supply is 220 V and 50 Hz respectively. On taking out the capacitance from the circuit the current lages behind the voltage by 30°. On taking out the inductor from the circuit the current leads the voltage by 30°. The power dissipated in the LCR circuit is : **[AIEEE - 2010]**

(1) 242 W (2) 305 W (3) 210 W (4) 0 W

EM0177

4. A boat is moving due east in a region where the earth's magnetic field is $5.0\times10^{-5}NA^{-1}m^{-1}$ due north and horizontal. The boat carries a vertical aerial 2m long. If the speed of the boat is $1.50\ ms^{-1}$, the magnitude of the indueced emf in the wire of aerial is :- **[AIEEE - 2011]**

(1) 0.50 mV (2) 0.15 mV (3) 1 mV (4) 0.75 mV

EM0178

5. A horizontal straight wire 20 m long extending from east to west is falling with a speed of 5.0 m/s, at right angles to the horizontal component of the earth's magnetic field $0.30 \times 10^{-4}\ Wb/m^2$. The instantaneous value of the e.m.f. induced in the wire will be :- **[AIEEE - 2011]**

(1) 6.0 mV (2) 3 mV (3) 4.5 mV (4) 1.5 mV

EM0179

6. A fully charged capacitor C with intial charge q_0 is connected to a coil of self inductance L at t = 0. The time at which the energy is stored equally between the electric and the magnetic fields is :- **[AIEEE - 2011]**

(1) $2\pi\sqrt{LC}$ (2) $\sqrt{LC}$ (3) $\pi\sqrt{LC}$ (4) $\frac{\pi}{4}\sqrt{LC}$

EM0180

7. A coil is suspended in a uniform magnetic field, with the plane of the coil parallel to the magnetic lines of force. When a current is passed through the coil it starts oscillating; it is very difficult to stop. But if an aluminium plate is placed near to the coil, it stops. This is due to :-**[AIEEE - 2012]**

(1) Electromagnetic induction in the aluminium plate giving rise to electromagnetic damping
(2) Development of air current when the plate is placed
(3) Induction of electrical charge on the plate
(4) Shielding of magnetic lines of force as aluminium is a paramagnetic material

EM0181

8. If a simple pendulum has Significant amplitude (up to a factor of 1/ e of original) only in the period between t = 0s to t = τs, then τ may be called the average life ofthe pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity, with 'b' as the constant of proportionality, the average lifetime ofthe pendulum is (assuming damping is small) in seconds: **[AIEEE-2012]**

(1) $\frac{1}{b}$ (2) $\frac{2}{b}$ (3) $\frac{0.693}{b}$ (4) b

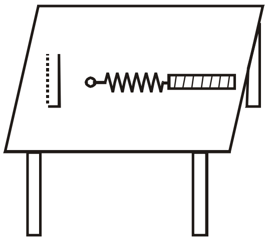
EM0182

9. The amplitude of adamped oscillator decreases to 0.9 times its original magnitude in 5s. In another 10s it will decrease to a times its original magnitude, where a equals:**[JEE Main-2013]**

(1) 0.81 (2) 0.729 (3) 0.6 (4) 0.7

EM0183

10. A metallic rod of length 'l' is tied to a string of length 2l and made to rotate with angular speed ω on a horizontal table with one end of the string fixed. If there is a vertical magnetic field 'B' in the region, the e.m.f. induced across the ends of the rod is : **[JEE Main-2013]**



(1) $\frac{2B\omega l^2}{2}$ (2) $\frac{3B\omega l^2}{2}$ (3) $\frac{4B\omega l^2}{2}$ (4) $\frac{5B\omega l^2}{2}$

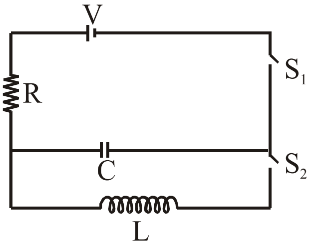
EM0184

11. A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the small loop is on the axis of the bigger loop. The distance between their centres is 15 cm. If a current of 2.0 A flows through the smaller loop, then the flux linked with bigger loop is :- **[JEE Main-2013]**

(1) 9.1×10^{-11} weber (2) 6×10^{-11} weber (3) 3.3×10^{-11} weber (4) 6.6×10^{-9} weber

EM0185

12. In an LCR circuit as shown below both switches are open initially. Now switch S_1 is closed, S_2 kept open, (q is charge on the capacitor and τ=RC is Capacitive time constant). Which of the following statement is correct? **[JEE Main-2013]**



(1) Work done by the battery is half of the energy dissipated in the resistor

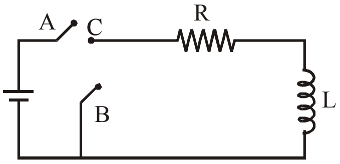
(2) At $t = \tau$, $q = CV/2$

(3) At $t = 2\tau$, $q = CV(1-e^{-2})$

(4) At $t = \frac{\tau}{2}$, $q = CV(1-e^{-1})$

EM0186

13. In the circuit shown here, the point 'C' is kept connected to point 'A' till the current flowing through the circuit becomes constant. Afterward, suddenly, point 'C' is disconnected from point 'A' and connected to point 'B' at time t = 0. Ratio of the voltage across resistance and the inductor at t = L/R will be equal to : **[JEE Main-2014]**

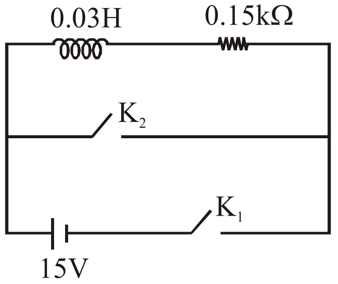


(1) –1 (2) $\frac{1-e}{e}$ (3) $\frac{e}{1-e}$ (4) 1

EM0187

14. An inductor (L = 0.03 H) and a resistor (R = 0.15 kΩ) are connected in series to a battery of 15V EMF in a circuit shown below. The key K_1 has been kept closed for a long time. Then at t = 0, K_1 is opened and key K_2 is closed simultaneously. At t = 1ms, the current in the circuit will be ($e^5 \cong 150$):-

[JEE Main-2015]

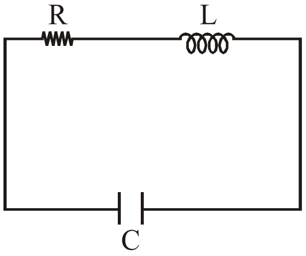


(1) 6.7 mA (2) 0.67 mA (3) 100 mA (4) 67 mA

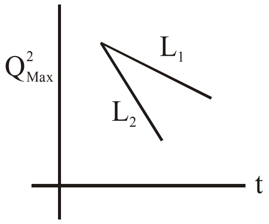
EM0188

15. An LCR circuit is equivalent to a damped pendulum. In an LCR circuit the capacitor is charged to Q_0 and then connected to the L and R as shown below. If a student plots graphs of the square of maximum charge (Q^2_{Max}) on the capacitor with time (t) for two different values L_1 and L_2 ($L_1 > L_2$) of L then which of the following represents this graph correctly ? (plots are schematic and not drawn to scale)

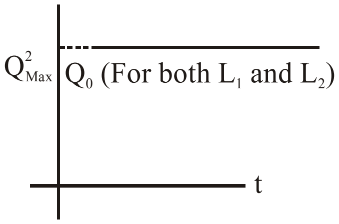
[JEE Main-2015]



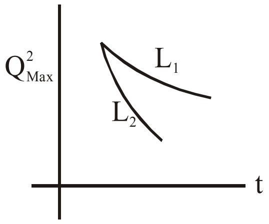
(1)



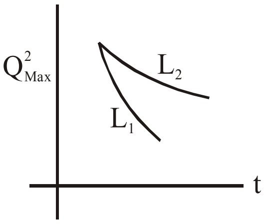
(2)



(3)



(4)



EM0189

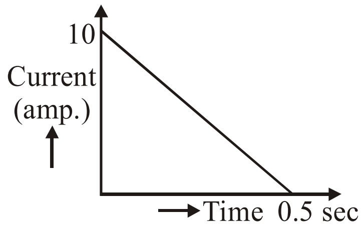
16. An arc lamp requires a direct current of 10A at 80V to function. If it is connected to a 220V (rms), 50Hz AC supply, the series inductor needed for it to work is close to :-

[JEE Main-2016]

(1) 0.065 H (2) 80 H (3) 0.08 H (4) 0.044 H

EM0190

17. In a coil of resistance 100 Ω, a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is :- **[JEE Main-2017]**



(1) 250 Wb (2) 275 Wb (3) 200 Wb (4) 225 Wb

EM0191

18. For an RLC circuit driven with voltage of amplitude v_m and frequency $w_0 = \frac{1}{\sqrt{LC}}$ the current exhibits resonance. The quality factor, Q is given by :- **[JEE Main-2018]**

(1) $\frac{\omega_0 R}{L}$ (2) $\frac{R}{(\omega_0 C)}$ (3) $\frac{CR}{\omega_0}$ (4) $\frac{\omega_0 L}{R}$

EM0192

19. In an a. c. circuit, the instantaneous e.m.f. and current are given by

$e = 100 \sin 30\,t$

$i = 20 \sin\left(30t - \frac{\pi}{4}\right)$

In one cycle of a.c., the average power consumed by the circuit and the wattless current are, respectively. **[JEE Main-2018]**

(1) $\frac{1000}{\sqrt{2}}$, 10 (2) $\frac{50}{\sqrt{2}}$, 0 (3) 50, 0 (4) 50, 10

EM0193

SELECTED PROBLEMS FROM JEE-MAINS ONLINE PAPERS

20. A series AC circuit containing an inductor (20 mH), a capacitor (120 μF) and a resistor (60Ω) is driven by an AC source of 24V/50Hz. The energy dissipated in the circuit in 60 s is :

[JEE Main-2019 Jan.]

(1) 2.26×10^3 J (2) 3.39×10^3 J (3) 5.65×10^2 J (4) 5.17×10^2 J

EM0239

21. A power transmission line feeds input power at 2300 V to a step down transformer with its primary windings having 4000 turns. The output power is delivered at 230 V bv the transformer. If the current in the primary of the transformer is 5A and its efficiency is 90%, the output current would be :

[JEE Main-2019 Jan.]

(1) 25 A (2) 50 A (3) 35 A (4) 45 A

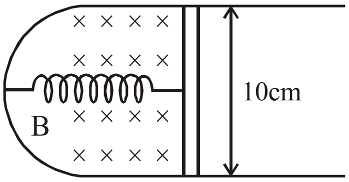
EM0240

22. A copper wire is wound on a wooden frame, whose shape is that of an equilateral triangle. If the linear dimension of each side of the frame is increased by a factor of 3, keeping the number of turns of the coil per unit length of the frame the same, then the self inductance of the coil : **[JEE Main-2019 Jan.]**

(1) Decreases by a factor of $9\sqrt{3}$
(2) Increases by a factor of 3
(3) Decreases by a factor of 9
(4) Increases by a factor of 27

EM0241

23. A thin strip 10 cm long is on a U shaped wire of negligible resistance and it is connected to a spring of spring constant 0.5 Nm^{-1} (see figure). The assembly is kept in a uniform magnetic field of 0.1 T. If the strip is pulled from its equilibrium position and released, the number of oscillation it performs before its amplitude decreases by a factor of e is N. If the mass of the strip is 50 grams, its resistance 10Ω and air drag negligible, N will be close to : **[JEE Main-2019 April]**



(1) 50000 (2) 5000 (3) 10000 (4) 1000

EM0242

24. An alternating voltage v(t) = 220 sin 100 πt volt is applied to a purely resistance load of 50 Ω. The time taken for the current to rise from half of the peak value to the peak value is : **[JEE Main-2019 April]**

(1) 2.2 ms (2) 5 ms (3) 3.3 ms (4) 7.2 ms

EM0243

25. The total number of turns and cross-section area in a solenoid is fixed. However, its length L is varied by adjusting the separation between windings. The inductance of solenoid will be proportional to :

[JEE Main-2019 April]

(1) $1/L^2$ (2) $1/L$ (3) L (4) L^2

EM0244

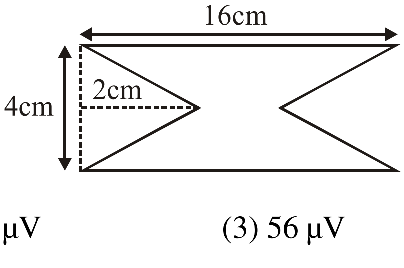
26. The displacement of a damped harmonic oscillator is given by
$x(t) = e^{-0.1t}\cos(10\pi t + \phi)$. Here t is in seconds.
The time taken for its amplitude of vibration to drop to half of its initial value is close to :

[JEE Main-2019 April]

(1) 13 s (2) 7 s (3) 27 s (4) 4 s

EM0245

27. At time t = 0 magnetic field of 100 Gauss is passing perpendicularly through the area defined by the closed loop shown in the figure. If the magnetic field reduces linearly to 500 Gauss, in the next 5s, then induced EMF in the loop is : **[JEE Main-2020_Jan.]**



(1) 36 μV (2) 48 μV (3) 56 μV (4) 28 μV

EM0246

28. A uniform magnetic field B exists in a direction perpendicular to the plane of a square loop made of a metal wire. The wire has a diameter of 4 mm and a total length of 30 cm. The magnetic field changes with time at a steady rate dB/dt = 0.032 Ts^{-1}.The induced current in the loop is close to (Resistivity of the metal wire is 1.23×10^{-8} Ωm) **[JEE Main-2020_Sep.]**

(1) 0.61 A (2) 0.34 A (3) 0.43 A (4) 0.53 A

EM0247

29. A 750 Hz, 20 V (rms) source is connected to a resistance of 100 Ω, an inductance of 0.1803 H and a capacitance of 10 μF all in series. The time in which the resistance (heat capacity 2J/°C) will get heated by 10°C. (assume no loss of heat to the surroundings) is close to : **[JEE Main-2020_Sep.]**

(1) 418 s (2) 245 s (3) 348 s (4) 365 s

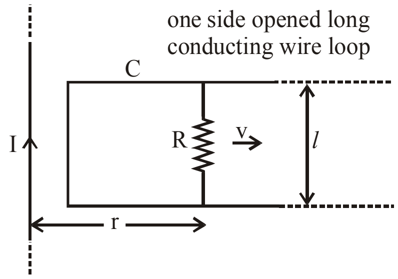
EM0248

30. A series L-R circuit is connected to a battery of emf V. If the circuit is switched on at t = 0, then the time at which the energy stored in the inductor reaches $\left(\frac{1}{n}\right)$ times of its maximum value, is : **[JEE Main-2020_Sep.]**

(1) $\frac{L}{R}\ln\left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)$ (2) $\frac{L}{R}\ln\left(\frac{\sqrt{n}}{\sqrt{n}+1}\right)$ (3) $\frac{L}{R}\ln\left(\frac{\sqrt{n}}{\sqrt{n}-1}\right)$ (4) $\frac{L}{R}\ln\left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)$

EM0249

31. An infinitely long straight wire carrying current I, one side opened rectangular loop and a conductor C with a sliding connector are located in the same plane, as shown in the figure. The connector has length *l* and resistance R. It slides to the right with a velocity v. The resistance of the conductor and the self inductance of the loop are negligible. The induced current in the loop, as a function of separation r, between the connector and the straight wire is : **[JEE Main-2020_Sep.]**



(1) $\frac{\mu_0}{\pi}\frac{Iv l}{Rr}$ (2) $\frac{\mu_0}{2\pi}\frac{Iv l}{Rr}$ (3) $\frac{2\mu_0}{\pi}\frac{Iv l}{Rr}$ (4) $\frac{\mu_0}{4\pi}\frac{Iv l}{Rr}$

EM0250

32. An AC circuit has R = 100 Ω, C = 2 μF and L = 80 mH, connected in series. The quality factor of the circuit is : **[JEE Main-2020_Sep.]**

(1) 0.5 (2) 2 (3) 20 (4) 400

EM0251

EXERCISE - JA

1. An AC voltage source of variable angular frequency ω and fixed amplitude V_0 is connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When ω is increased **[JEE 2010]**

(A) the bulb glows dimmer

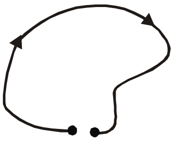
(B) the bulb glows brighter

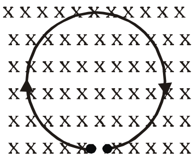
(C) total impedance of the circuit is unchanged

(D) total impedance of the circuit increases

EM0194

2. A thin flexible wire of length L is connected to two adjacent fixed points and carries a current I in the clockwise direction, as shown in the figure. When the system is put in a uniform magnetic field of strength B going into the plane of the paper, the wire takes the shape of a circle. The tension in the wire is :- **[JEE 2010]**





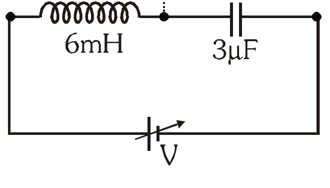
(A) IBL (B) $\frac{IBL}{\pi}$ (C) $\frac{IBL}{2\pi}$ (D) $\frac{IBL}{4\pi}$

EM0195

3. You are given many resistances, capacitors and inductors. These are connected to a variable DC voltage source (the first two circuits) or an AC voltage source of 50 Hz frequency (the next three circuits) in different ways as shown in **Column II**. When a current I (steady state for DC or rms for AC) flows through the circuit, the corresponding voltage V_1 and V_2 (indicated in circuits) are related as shown in **Column I**. Match the two **[JEE 2010]**

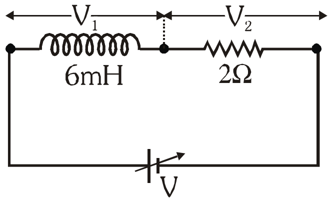
Column I **Column II**

(A) $I \neq 0$, V_1 is proportional to I (p)



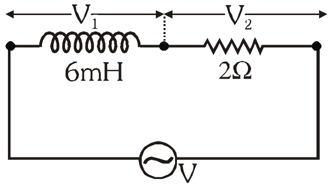
(B) $I \neq 0$, $V_2 > V_1$

(q)



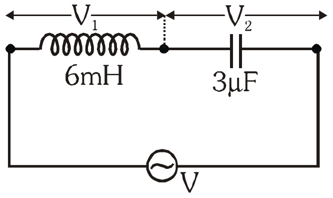
(C) $V_1 = 0$, $V_2 = V$

(r)

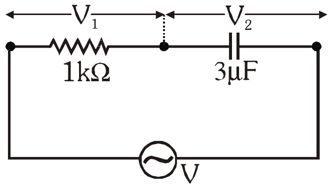


(D) $I \neq 0$, V_2 is proportional to I

(s)



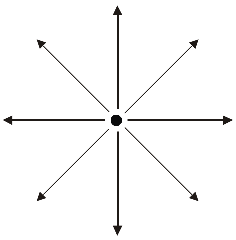
(t)



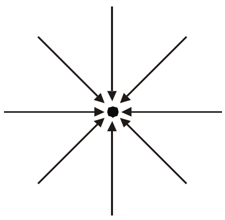
EM0196

4. Which of the field patterns given below is valid for electric field as well as for magnetic field? **[JEE 2011]**

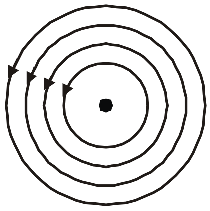
(A)



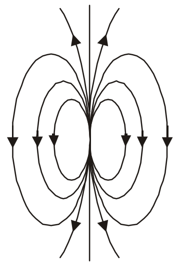
(B)



(C)



(D)



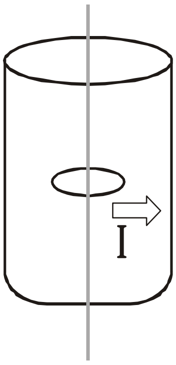
EM0197

5. A series R-C circuit is connected to AC voltage source. Consider two cases ; (A) when C is without a dielectric medium and (B) when C is filled with dielectric of constant 4. The current I_R through the resistor and voltage V_C across the capacitor are compared in the two cases. Which of the following is/are true? **[JEE 2011]**

(A) $I_R^A > I_R^B$ (B) $I_R^A < I_R^B$ (C) $V_C^A > V_C^B$ (D) $V_C^A < V_C^B$

EM0198

6. A long circular tube of length 10 m and radius 0.3 m carries a current I along its curved surface as shown. A wire-loop of resistance 0.005 ohm and of radius 0.1 m is placed inside the tube with its axis coinciding with the axis of the tube. The current varies as $I=I_0\cos(300t)$ where I_0 is constant. If the magnetic moment of the loop is $N\mu_0 I_0 \sin(300t)$, then 'N' is **[JEE 2011]**



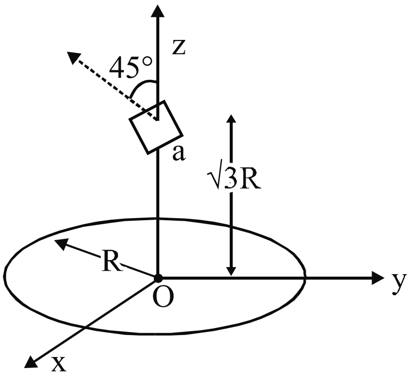
EM0199

7. A series R-C combination is connected to an AC voltage of angular frequency $\omega = 500$ radian/s. If the impedance of the R-C circuit is $R\sqrt{1.25}$, the time constant (in millisecond) of the circuit is :- **[JEE 2011]**

EM0200

8. A circular wire loop of radius R is placed in the x-y plane centred at the origin O. A square loop of side a (a<<R) having two turns is placed with its centre at z = $\sqrt{3}$ R along the axis of the circular wire loop, as shown in figure. The plane of the square loop makes an angle of 45° with respect to the z-axis. If the mutual inductance between the loops is given by $\dfrac{\mu_0 a^2}{2^{p/2}R}$, then the value of p is :-

[JEE 2012]



EM0201

9. A current carrying infinitely long wire is kept along the diameter of a circular wire loop, without touching it. The correct statement(s) is (are) **[JEE 2012]**

(A) The emf induced in the loop is zero if the current is constant.

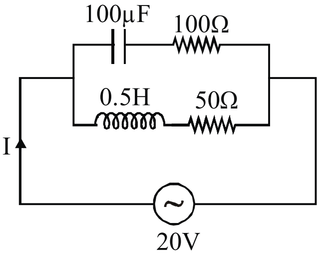
(B) The emf induced in the loop is infinite if the current is constant.

(C) The emf induced in the loop is zero if the current decreases at a steady rate.

(D) The emf induced in the loop is finite if the current decreases at a steady rate.

EM0202

10. In the given circuit, the AC source has $\omega = 100$ rad/s. Considering the inductor and capacitor to be ideal, the correct choice (s) is(are) **[JEE 2012]**



(A) The current through the circuit, I is 0.3 A. (B) The current through the circuit, i is $0.3\sqrt{2}$A.

(C) The voltage across 100Ω resistor = $10\sqrt{2}$V. (D) The voltage across 50Ω resistor = 10V.

EM0203

Paragraph for Questions 11 and 12

A point charge Q is moving in a circular orbit of radius R in the x-y plane with an angular velocity ω. This can be considered as equivalent to a loop carrying a steady current $\frac{Q\omega}{2\pi}$. A uniform magnetic field along the positive z-axis is now switched on, which increases at a constant rate from 0 to B in one second. Assume that the radius of the orbit remains constant. The application of the magnetic field induces an emf in the orbit. The induced emf is defined as the work done by an induced electric field in moving a unit positive charge around a closed loop. It is known that for an orbiting charge, the magnetic dipole moment is proportional to the angular momentum with a proportionality constant γ.

11. The change in the magnetic dipole moment associated with the orbit, at the end of the time interval of the magnetic field change is **[JEE Advance-2013]**

(A) $-\gamma BQR^2$ (B) $-\gamma\frac{BQR^2}{2}$ (C) $\gamma\frac{BQR^2}{2}$ (D) γBQR^2

EM0204

12. The magnitude of the induced electric field in the orbit at any instant of time during the time interval of the magnetic field change is

(A) $\frac{BR}{4}$ (B) $\frac{BR}{2}$ (C) BR (D) 2BR

EM0204

Paragraph for Questions 13 and 14

A thermal power plant produces electric power of 600 kW and 4000 V, which is to be transported to a place 20 km away from the power plant for consumers' usage. It can be transported either directly with a cable of large current carrying capacity or by using a combination of step-up and step-down transformers at the two ends. The drawback of the direct transmission is the large energy dissipation. In the method using transformers, the dissipation is much smaller. In this method, a step-up transformer is used at the plant side so that the current is reduced to a smaller value. At the consumers' end, a step-down transformer is used to supply power to the consumers at the specified lower voltage. It is reasonable to assume that the power cable is purely resistive and the transformers are ideal with a power factor unity. All the currents and voltages mentioned are rms values. **[JEE Advance-2013]**

13. In the method using the transformers, assume that the ratio of the number of turns in the primary to that in the secondary in the step-up transformer is 1 : 10. If the power to the consumers has to be supplied at 200 V, the ratio of the number of turns in the primary to that in the secondary in the step-down transformer is

(A) 200 : 1 (B) 150 : 1 (C) 100 : 1 (D) 50 : 1

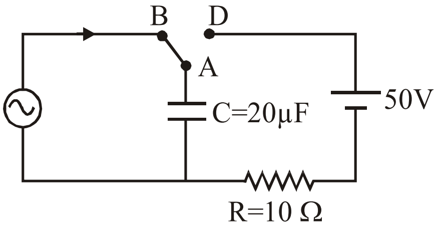
EM0205

14. If the direct transmission method with a cable of resistance 0.4 Ω km^{-1} is used, the power dissipation (in %) during transmission is

(A) 20 (B) 30 (C) 40 (D) 50

EM0205

15. At time t = 0, terminal A in the circuit shown in the figure is connected to B by a key and an alternating current $I(t) = I_0\cos(\omega t)$, with I_0 = 1A and ω = 500 rad s^{-1} starts flowing in it with the initial direction shown in the figure. At $t=\frac{7\pi}{6\omega}$, the key is switched from B to D. Now onwards only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If C = 20 μF, R = 10 Ω and the battery is ideal with emf of 50 V, identify the correct statement (s). **[JEE Advance-2014]**



(A) Magnitude of the maximum charge on the capacitor before $t=\frac{7\pi}{6\omega}$ is 1×10^{-3}C.

(B) The current in the left part of the circuit just before $t=\frac{7\pi}{6\omega}$ is clockwise.

(C) Immediately after A is connected to D, the current in R is 10A

(D) $Q = 2 \times 10^{-3}$ C

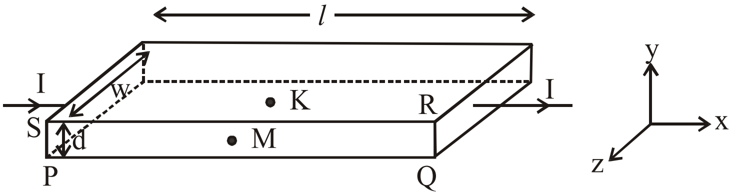
EM0206

Paragraph for Question No. 16 and 17

In a thin rectangular metallic strip a constant current I flows along the positive x-direction, as shown in the figure. The length, width the thickness of the strip are *l*, w and d, respectively.

A uniform magnetic field $\vec{B}$ is applied on the strip along the positive y-direction. Due to this, the charge carriers experience a net deflection along the z-direction. This results in accumulation of charge carriers on the surface PQRS and appearance of equal and opposite charges on the face opposite to PQRS. A potential difference along the z-direction is thus developed. Charge accumulation continues until the magnetic force is balanced by the electric force. The current is assumed to be uniformly distributed on the cross section of the strip and carried by electrons.

[JEE Advance-2015]



16. Consider two different metallic strips (1 and 2) of the same material. Their lengths are the same, widths are w_1 and w_2 and thicknesses are d_1 and d_2, respectively. Two points K and M are symmetrically located on the opposite faces parallel to the x-y plane (see figure). V_1 and V_2 are the potential differences between K and M in strips 1 and 2, respectively. Then, for a given current I flowing through them in a given magnetic field strength B, the correct statement(s) is(are)

(A) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = 2V_1$　(B) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = V_1$

(C) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = 2V_1$　(D) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = V_1$

EM0207

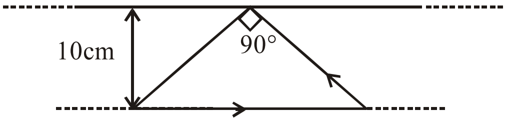
17. Consider two different metallic strips (1 and 2) of same dimensions (length *l*, width w and thickness d) with carrier densities n_1 and n_2, respectively. Strip 1 is placed in magnetic field B_1 and strip 2 is placed in magnetic field B_2, both along positive y-direction. Then V_1 and V_2 are the potential differences developed between K and M in strips 1 and 2, respectively. Assuming that the current I is the same for both the strips, the correct option(s) is(are)

(A) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = 2V_1$　(B) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = V_1$

(C) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = 0.5V_1$　(D) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = V_1$

EM0207

18. A conducting loop in the shape of right angled isosceles triangle of height 10 cm is kept such that the 90° vertex is very close to an infinitely long conducting wire (see the figure). The wire is electrically insulated from the loop. The hypotenuse of the triangle is parallel to the wire. The current in the triangular loop is in counterclockwise direction and increased at constant rate of 10 A s^{-1}. Which of the following statement(s) is(are) true? **[JEE Advance-2016]**



(A) The induced current in the wire is in opposite direction to the current along the hypotenuse.

(B) There is a repulsive force between the wire and the loop

(C) If the loop is rotated at a constant angular speed about the wire, an additional emf of $\left(\frac{\mu_0}{\pi}\right)$ volt is induced in the wire

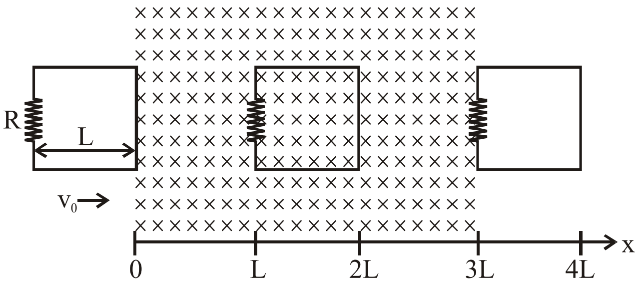
(D) The magnitude of induced emf in the wire is $\left(\frac{\mu_0}{\pi}\right)$ volt.

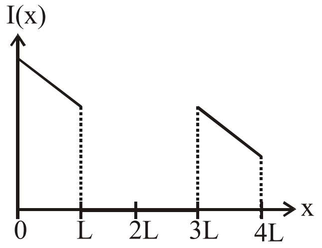
EM0208

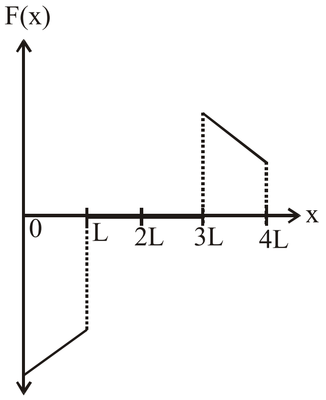
19. Two inductors L_1 (inductance 1 mH, internal resistance 3 Ω) and L_2 (inductance 2mH, internal resistance 4Ω), and a resistor R (resistance 12 Ω) are all connected in parallel across a 5V battery. The circuit is switched on at time t = 0. The ratio of the maximum to the minimum current (I_{max}/I_{min}) drawn from the battery is. **[JEE Advance-2016]**

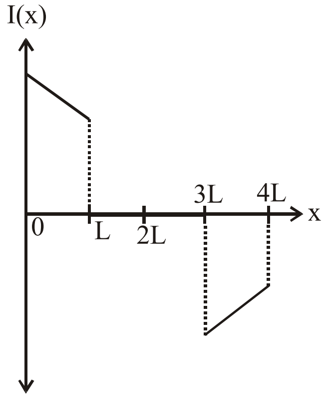
EM0209

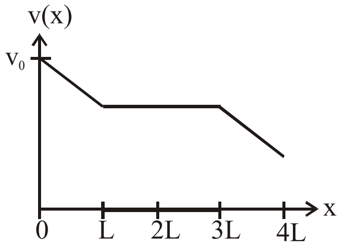
20. A rigid wire loop of square shape having side of length L and resistance R is moving along the x-axis with a constant velocity v_0 in the plane of the paper. At t = 0, the right edge of the loop enters a region of length 3L where there is a uniform magnetic field B_0 into the plane of the paper, as shown in the figure. For sufficiently large v_0, the loop eventually crosses the region. Let x be the location of the right edge of the loop. Let v(x), I(x) and F(x) represent the velocity of the loop, current in the loop, and force on the loop, respectively, as a function of x. Counter-clockwise current is taken as positive. Which of the following schematic plot(s) is(are) correct ? (Ignore gravity) **[JEE Advance-2016]**



(A)


(B)


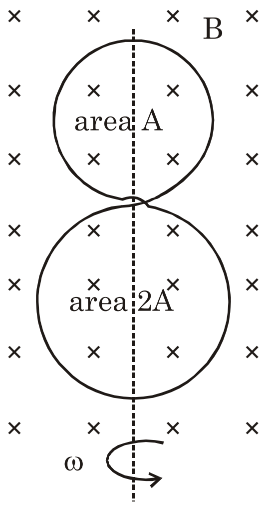
(C)


(D)


EM0210

21. A circular insulated copper wire loop is twisted to form two loops of area A and 2A as shown in the figure. *At the point of crossing the wires remain electrically insulated from each other.* The entire loop lies in the plane (of the paper). A uniform magnetic field $\vec{B}$ points into the plane of the paper. At t = 0, the loop starts rotating about the common diameter as axis with a constant angular velocity ω in the magnetic field. Which of the following options is/are correct?

[JEE Advance-2017]



(A) The rate of change of the flux is maximum when the plane of the loops is perpendicular to plane of the paper

(B) The net emf induced due to both the loops is proportional to cos ωt

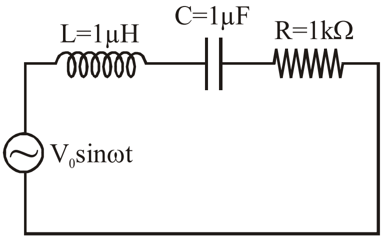
(C) The emf induced in the loop is proportional to the sum of the areas of the two loops

(D) The amplitude of the maximum net emf induced due to both the loops is equal to the amplitude of maximum emf induced in the smaller loop alone

EM0211

22. In the circuit shown, L = 1 μH, C = 1 μF and R = 1 kΩ. They are connected in series with an a.c. source $V = V_0 \sin \omega t$ as shown. Which of the following options is/are correct ?

[JEE Advance-2017]



(A) The frequency at which the current will be in phase with the voltage is independent of R.

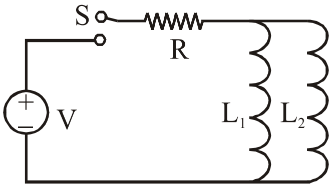
(B) At ω ~ 0 the current flowing through the circuit becomes nearly zero

(C) At $\omega >> 10^6$ rad.s^{-1}, the circuit behaves like a capacitor.

(D) The current will be in phase with the voltage if $\omega = 10^4$ rad.s^{-1}.

EM0212

23. A source of constant voltage V is connected to a resistance R and two ideal inductors L_1 and L_2 through a switch S as shown. There is no mutual inductance between the two inductors. The switch S is initially open. At t = 0, the switch is closed and current begins to flow. Which of the following options is/are correct? **[JEE Advance-2017]**



(A) The ratio of the currents through L_1 and L_2 is fixed at all times (t > 0)

(B) After a long time, the current through L_1 will be $\frac{V}{R}\frac{L_2}{L_1+L_2}$

(C) After a long time, the current through L_2 will be $\frac{V}{R}\frac{L_1}{L_1+L_2}$

(D) At t = 0, the current through the resistance R is $\frac{V}{R}$

EM0213

24. The instantaneous voltages at three terminals marked X, Y and Z are given by

$$V_X = V_0 \sin \omega t$$

$$V_Y = V_0 \sin\left(\omega t + \frac{2\pi}{3}\right) \text{ and}$$

$$V_Z = V_0 \sin\left(\omega t + \frac{4\pi}{3}\right)$$

An ideal voltmeter is configured to read rms value of the potential difference between its terminals. It is connected between points X and Y and then between Y and Z. The reading(s) of the voltmeter will be:- **[JEE Advance-2017]**

(A) $V_{XY}^{rms} = V_0$

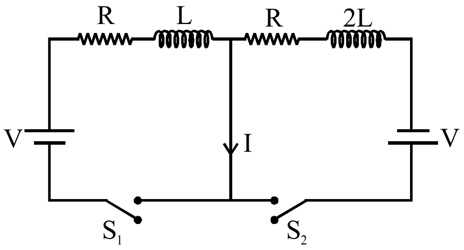
(B) $V_{YZ}^{rms} = V_0\sqrt{\frac{1}{2}}$

(C) Independent of the choice of the two terminals

(D) $V_{XY}^{rms} = V_0\sqrt{\frac{3}{2}}$

EM0214

25. In the figure below, the switches S_1 and S_2 are closed simultaneously at t = 0 and a current starts to flow in the circuit. Both the batteries have the same magnitude of the electromotive force (emf) and the polarities are as indicated in the figure. Ignore mutual inductance between the inductors. The current I in the middle wire reaches its maximum magnitude I_{max} at time t = τ. Which of the following statement(s) is (are) true? **[JEE Advance-2018]**

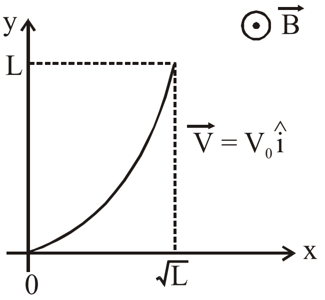


(A) $I_{max} = \frac{V}{2R}$ (B) $I_{max} = \frac{V}{4R}$ (C) $\tau = \frac{L}{R}\ell n2$ (D) $\tau = \frac{2L}{R}\ell n2$

EM0215

26. A conducting wire of parabolic shape, initially $y = x^2$, is moving with velocity $\vec{V} = V_0\hat{i}$ in a non-uniform magnetic field $\vec{B} = B_0\left(1+\left(\frac{y}{L}\right)^{\beta}\right)\hat{k}$, as shown in figure. If V_0, B_0, L and β are positive constants and $\Delta\phi$ is the potential difference developed between the ends of the wire, then the correct statement(s) is/are:

[JEE Advance-2019]



(1) $|\Delta\phi|$ remains the same if the parabolic wire is replaced by a straight wire, y = x initially, of length $\sqrt{2}L$

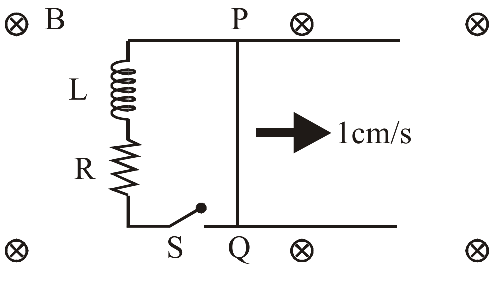
(2) $|\Delta\phi|$ is proportional to the length of the wire projected on the y-axis.

(3) $|\Delta\phi| = \frac{1}{2}B_0V_0L$ for $\beta = 0$

(4) $|\Delta\phi| = \frac{4}{3}B_0V_0L$ for $\beta = 2$

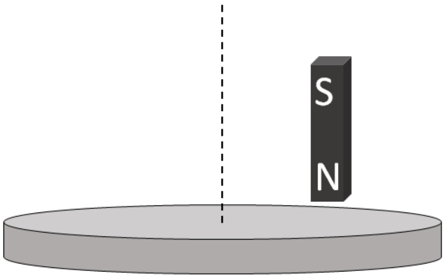
EM0216

27. A 10 cm long perfectly conducting wire PQ is moving, with a velocity 1cm/s on a pair of horizontal rails of zero resistance. One side of the rails is connected to an inductor L = 1 mH and a resistance R = 1Ω as shown in figure. The horizontal rails, L and R lie in the same plane with a uniform magnetic field B = 1 T perpendicular to the plane. If the key S is closed at certain instant, the current in the circuit after 1 millisecond is x × 10^{-3}A, where the value of x is_______.
[Assume the velocity of wire PQ remains constant (1 cm/s) after key S is closed.
Given : $e^{-1} = 0.37$, where e is base of the natural logarithm] **[JEE Advance-2019]**



EM0217

28. A light disc made of aluminium (a nonmagnetic material) is kept horizontally and is free to rotate about its axis as shown in the figure. A strong magnet is held vertically at a point above the disc away from its axis. On revolving the magnet about the axis of the disc, the disc will (figure is schematic and not drawn to scale)- **[JEE Advance-2020]**

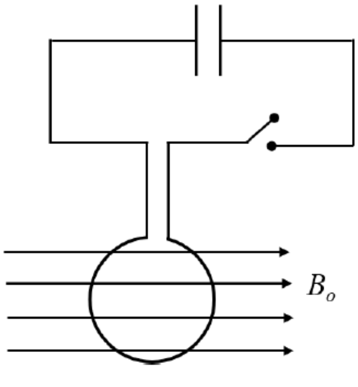


(A) rotate in the direction opposite to the direction of magnet's motion
(B) rotate in the same direction as the direction of magnet's motion
(C) not rotate and its temperature will remain unchanged
(D) not rotate but its temperature will slowly rise

EM0252

29. A circular coil of radius R and N turns has negligible resistance. As shown in the schematic figure, its two ends are connected to two wires and it is hanging by those wires with its plane being vertical. The wires are connected to a capacitor with charge Q through a switch. The coil is in a horizontal uniform magnetic field B_0 parallel to the plane of the coil. When the switch is closed, the capacitor gets discharged through the coil in a very short time. By the time the capacitor is discharged fully, magnitude of the angular momentum gained by the coil will be (assume that the discharge time is so short that the coil has hardly rotated during this time)-

[JEE Advance-2020]

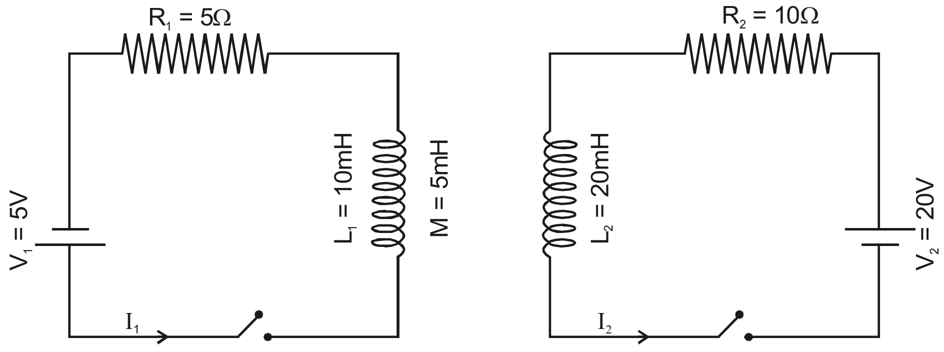


(A) $\frac{\pi}{2}NQB_0R^2$ (B) πNQB_0R^2 (C) $2\pi NQB_0R^2$ (D) $4\pi NQB_0R^2$

EM0253

30. The inductors of two LR circuits are placed next to each other, as shown in the figure. The values of the self-inductance of the inductors, resistances, mutual-inductance and applied voltages are specified in the given circuit. After both the switches are closed simultaneously, the total work done by the batteries against the induced EMF in the inductors by the time the currents reach their steady state values is________ mJ.

[JEE Advance-2020]



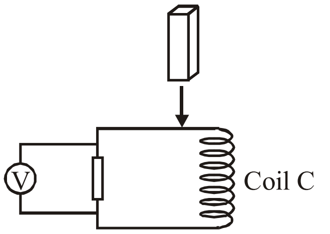
EM0254

ELECTROMAGNETIC INDUCTION & ALTERNATING CURRENT
CBSE Previous Year's Questions

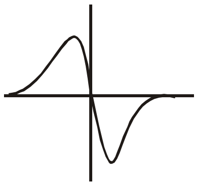
1. A solenoid with an iron core and a bulb are connected to a dc. source. How does the brightness of the bulb change, when the imp core is removed from the solenoid? **[1; CBSE-2004]**

2. Peak value of emf of an a.c. source is E_0. What is its r.m.s. value? **[1; CBSE-2004]**

3. A bar magnet M is dropped so that it falls vertically through the coil C. The graph obtained for voltage produced across the coil vs time is shown in figure (b).
(i) Explain the shape of the graph.
(ii) Why is the negative peak longer than the positive peak? **[2; CBSE-2004]**



(a)


(b)

4. What is induced emf? Write Faraday's law of electromagnetic induction Express it mathematically. A conducting rod of length ℓ with one end pivoted is rotated with a uniform angular speed in a vertical plane, normal to a uniform magnetic field 'B'. Deduce an expression for the emf induced in this rod. In India, domestic power supply is at 220 V, 50 Hz, while in USA it is 110 V, 50 Hz. Give one advantage and one disadvantage of 220 V supply over 110 V supply. **[5; CBSE-2004]**

5. A bulb and a capacitor are connected in series to an a.c. source of variable frequency. How will the brightness of the bulb change on increasing the frequency of the a.c. source? Give reason. **[1; CBSE-2005]**

6. A circular coil of radius 8 cm and 20 turns rotates about its vertical diameter with an angular speed of 50 s^{-1} in a uniform horizontal magnetic field of magnitude 3×10^{-2} T. Find the maximum and average value of the emf induced in the coil. **[2; CBSE-2005]**

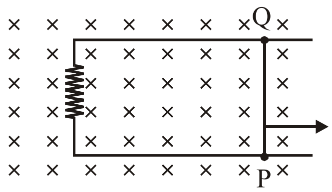
7. State the condition under which the phenomenon of resonance occurs in a series LCR circuit. Plot a graph showing variation of current with frequency of a.c. source in a series LCR circuit **[2; CBSE-2005]**

8. Define self-inductance and give its S. I. unit. Derive an expression for self- inductance of a long, air-cored solenoid of length ℓ, radius r, and having N number of turns. **[3; CBSE-2005]**

9. An alternating voltage of frequency f is applied across a series LCR circuit. Let f_r be the resonance frequency for the circuit. Will the current in the circuit lag, lead or remain in phase with the applied voltage when (i) $f > f_r$, (ii) $f < f_r$? Explain your answer in each case. **[2; CBSE-2006]**

10. When an inductor L and a resistor R in series are connected across a 12 V, 50Hz supply, a current of 0.5 A flows in the circuit. The current differs in phase from applied voltage by $\pi/3$ radian. Calculate the value of R. **[3; CBSE-2006]**

11. A 0.5 long metal rod PQ completes the circuit as shown in the figure. The area of the circuit is perpendicular to the magnetic field of flux density 0.15 T. If the resistance of the total circuit is 3Ω, calculate the force needed to move the rod in the direction as indicated with a constant speed of 2 ms^{-1}. **[3 ; CBSE-2006]**



12. What are eddy currents. How are these produced? in what sense are eddy currents considered undesirable in a transformer and how are these reduced in such a device ? **[3 ; CBSE-2006]**

13. In a series LCR circuit, the voltages across an inductor, a capacitor and a resistor are 30 V, 30 V and 60 V respectively. What is the phase difference between the applied voltage and the current in the circuit ?

14. Calculate the current drawn by the primary of a transformer which steps down 200 V to 20 V to operate a device of resistance 20Ω. Assume the efficiency of the transformer to be 80%. **[1 ; CBSE-2007]**

15. An a.c. voltage of 100 V, 50 Hz is connected across a 20 ohm resistor and mH inductor in series. Calculate (i) impedance of the circuit, (ii) rms current in the circuit. **[2 ; CBSE-2007]**

16. Explain the term 'inductive reactance'. Show graphically the variation of inductive reactance with frequency of the applied alternating voltage. An a.c. voltage $E = E_0 \sin \omega t$ is applied across a pure inductor of inductance L. Show mathematically that the current flowing through it lags behind the applied voltage by a phase angle of $\pi/2$. **[3 ; CBSE-2007]**

17. Explain the term 'capacitive reactance'. Show graphically the variation of capacitive reactance with frequency of the applied alternating voltage. An a.c. voltage $E = E_0 \sin \omega t$ is applied across a pure capacitor of capacitance C. Show mathematically that the current flowing through it leads the applied voltage by a phase angle of $\pi/2$. **[3 ; CBSE-2007]**

18. Prove that an ideal capacitor, in an a. c. circuit does not dissipate power. **[2 ; CBSE-2008]**

19. Derive an expression for the impedance of a.c. circuit consisting of an inductor and a resistor. **[2 ; CBSE-2008]**

20. A metallic rod of length ℓ is rotated at a constant angular speed ω, normal to a uniform magnetic field B. Derive an expression for the current induced in the rod, if the resistance of the rod is R. **[3 ; CBSE-2008]**

21. An inductor 200mH, capacitor 500 μF, resistor 10Ω are connected in series with a 100 V, variable frequency ac. source. Calculate the **[3 ; CBSE-2008]**

(i) frequency at which the power factor of the circuit is unity

(ii) current amplitude at this frequency

(iii) Q-factor

22. (a) Define self inductance. Write is S.I, units.

(b) Derive an expression for self inductance of a long solenoid of length ℓ, cross-sectional area A having N number of turns. **[3 ; CBSE-2009]**

23. (a) Derive an expression for the average power consumed in a series LCR circuit connected to a.c. source, in which the phase difference between the voltage and the current in the circuit is ϕ. (b) Define the quality factor in an ac. circuit. Why should the quality factor have high value in receiving circuits? Name the factors on which it depends. **[5; CBSE-2009]**

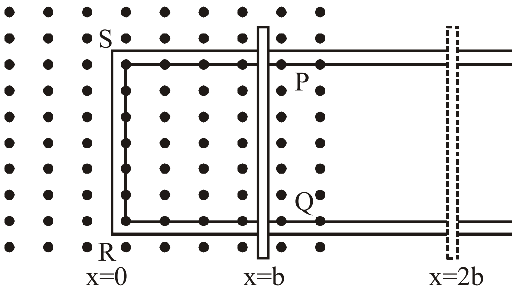
24. (a) Derive the relationship between the peak, and the rms value of current in an ac. circuit, (b) Describe briefly, with the help of a labeled diagram, working of a step - up transformer. A step - up transformer converts a low voltage into high voltage. Does it not violate the principle of conservation of energy? Explain.

25. Define self-inductance of a coil. Write its S.I. units. **[1; CBSE-2010]**

26. Two identical loops, one of copper and the other of aluminium, are rotated with the same angular speed in the same magnetic field. Compare (i) the induced emf and (ii) the current produced in the two coils. Justify your answer. **[2; CBSE-2010]**

27. State Faraday's law of electromagnetic induction.

Figure shows a rectangular conductor PQRS in which the conductor PQ is free to move in a uniform magnetic field B perpendicular to the plane of the paper. The field extends from x = 0 to x = b and is zero for x > b. Assume that only the arm PQ possesses resistance r. When the arm PQ is pulled outward from x=0 with constant speed v, obtain the expressions for the flux and the induced emf. Sketch the variations of these quantities with distance $0 \leq X \leq 2b$.**[5; CBSE-2010]**

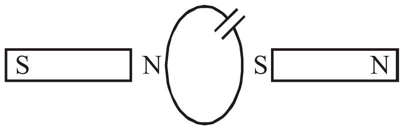


28. Draw a schematic diagram of a step-up transformer. Explain its working principle. Deduce the expression for the secondary to primary voltage in terms of the number of turns in the two coils. In an ideal transformer, how is this ratio related to the currents in the two coils ? How is the transformer used in large scale transmission and distribution of electrical energy over long distances ? **[5; CBSE-2010]**

29. What are eddy currents ? Write any two applications of eddy currents. **[2; CBSE-2011]**

30. State the working of a.c. generator with the help of a labelled diagram. The coil of an a.c. generator having N turns, each of area A, is rotated with a constant angular velocity ω. Deduce the expression for the alternating e.m.f. generated in the coil. What is the source of energy generation in this device? **[5; CBSE-2011]**

31. Two bar magnetic are quickly moved towards a metallic loop connected across a capacitor 'C as shown in the figure. Predict the polarity of the capacitor. **[1; CBSE-2011]**



32. (a) Show that in an a.c. circuit containing a pure inductor, the voltage is ahead of current by $\pi/2$ in phase.

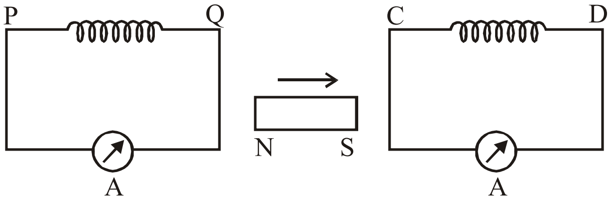
(b) A horizontal straight wire of length L extending from east to west is falling with speed v at right angles to the horizontal component of Earth's magnetic field B.

(i) Write the expression for the instantaneous value of the e.m.f. induced in the wire,

(ii) What is the direction of the e.m.f. ?

(iii) Which end of the wire is at the higher potential? **[5; CBSE-2011]**

33. A bar magnetic is moved in the direction indicated by the arrow between two coils PQ and CD. Predict the directions of induced current in each coil. **[1;CBSE-2012]**



34. Mention the two characteristic properties of the material suitable for making core of a transformer. **[1; CBSE-2012]**

35. State the underlying principle of a transformer. How is the large scale transmission of electric energy over a long distances done with the use of transformers? **[2; CBSE-2012]**

36. A light bulb is rated 100 W for 220 V ac supply of 50 Hz. Calculate **[2; CBSE-2012]**

(i) The resistance of the bulb (ii) The rms current through the bulb

OR

An alternative voltage given by V = 140 sin 314t is connected across a pure resistor of 50Ω. Find

(i) the frequency of the source (ii) the rms current through the resistor

[2; CBSE-2012]

37. A series LCR circuit is connected to an ac source. Using the phasor diagram, derive the expression for the source, explaining the nature of its variation. **[3; CBSE-2012]**

38. How does the mutual inductance of a pair of coils change when **[CBSE-2013]**

(i) distance between the coil s is increased and

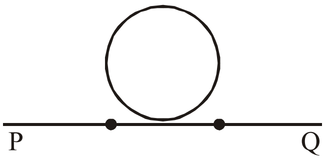
(ii) number of turns in the coils is increased?

39. The motion of copper plate is damped when it is allowed to oscillate between the two poles of a magnet. What is the cause of this damping? **[CBSE-2013]**

40. (a) For a given a,c, $i = i_m \sin \omega t$, show that the average power dissipated in a resistor R over a complete cycle is $\frac{1}{2} i_m^2 R$ **[CBSE-2013]**

(b) A light bulb is rated at 100 W for a 220V a.c. supply. Calculate the resistance of the bulb.

41. A conducting loop is held above a current carrying wire 'PQ' as shown in the figure. Depict the direction of the current induced in the loop when the current in the wire PQ is constantly increasing. **[CBSE-2014]**



42. Why is the use of ac. voltage preferred over dc. voltage ? Give two reasons. **[CBSE-2014]**

43. A voltage $V = V_0 \sin \omega t$ is applied to a series LCR circuit. Derive the expression for the average power dissipated over a cycle. **[CBSE-2014]**
Under what condition is (i) no power dissipated even though the current flows through the circuit, (ii) maximum power dissipated in the circuit ? **[CBSE-2014]**

44. Define the term self-inductance of a solenoid. Obtain the expression for the magnetic energy stored in an inductor of self-inductance L to build up a current I through it. **[CBSE-2014]**

45. A planar loop of rectangular shape is moved within the region of a uniform magnetic field acting perpendicular to its plane. What is the direction and magnitude of the current induced in it ? **[1; CBSE-2015]**

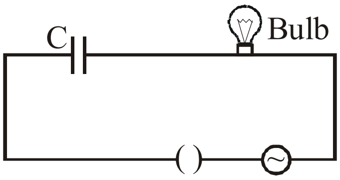
46. Sunita and her friends visited an exhibition. The policeman asked them to pass through a metal detector. Sunita's friends were initially scared of it. Sunita, however, explained to them the purpose and working of the metal detector. **[4; CBSE-2015]**
Answer the following questions :
(a) On what principle does a metal detector work ?
(b) Why does the detector emit sound when a person carrying any metallic object walks through it ?
(c) State any two qualities which Sunita displayed while explaining the purpose of walking through the detector. **[4; CBSE-2015]**

47. (a) State Faraday's law of electromagnetic induction. **[5; CBSE-2015]**
(b) Explain, with the help of a suitable example, how we can show that Lenz's law is a consequence of the principle of conservation of energy.
(c) Use the expression for Lorentz force acting on the charge carriers of a conductor to obtain the expression for the induced emf across the conductor of length l moving with velocity υ through a magnetic field B acting perpendicular to its length. **[5 ; CBSE-2015]**

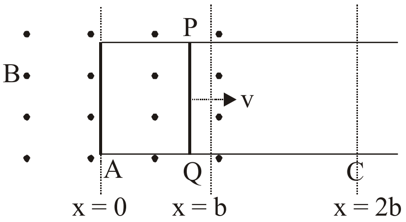
OR

(a) Using phasor diagram, derive the expression for the current flowing in an ideal inductor connected to an a.c. source of voltage, $v = v_0 \sin \omega t$. Hence plot graphs showing variation of (i) applied voltage and (ii) the current as a function of ωt.
(b) Derive an expression for the average power dissipated in a series LCR circuit.

48. (i) When an AC source is connected to an ideal capacitor, show that the average power supplied by the source over a complete cycle is zero. **[3 ; CBSE-2016]**
(ii) A bulb is connected in series with a variable capacitor and an A.C. source as shown. What happens to the brightness of the bulb when the key is plugged in and capacitance of the capacitor is gradually reduced ?



49. (b) Sketch the change in flux, emf and force when a conducting rod PQ of resistance R and length ℓ moves freely to and fro between A and C with speed v on a rectangular conductor placed in uniform magnetic field as shown in the figure. **[5 ; CBSE-2016]**

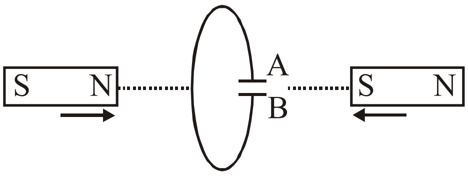


OR

In a series LCR circuit connected to an a.c. source of voltage $v = v_m \sin\omega t$, use phasor diagram to derive an expression for the current in the circuit.

Hence obtain the expression for the power dissipated in the circuit. Show that power dissipated at resonance is maximum.

50. Predict the polarity of the capacitor in the situation described below : **[2; CBSE-2017]**

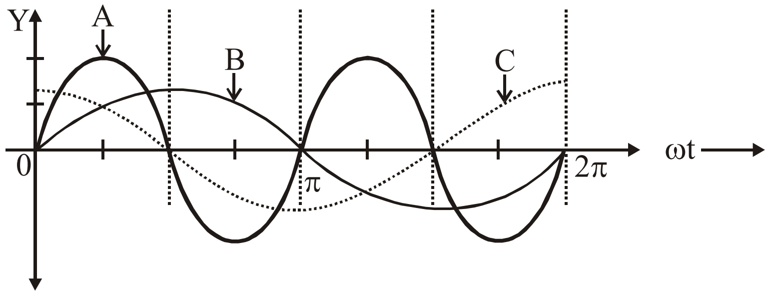


51. Define mutual inductance between a pair of coils. Derive an expression for the mutual inductance of two long coaxial solenoids of same length wound one over the other. **[3; CBSE-2017]**

OR

Define self-inductance of a coil. Obtain the expression for the energy stored in an inductor L connected across a source of emf.

52. A device 'X' is connected to an ac source $V = V_0 \sin \omega t$. The variation of voltage, current and power in one cycle is shown in the following graph : **[5; CBSE-2017]**



(a) Identify the device 'X'.

(b) Which of the curves A, B and C represent the voltage, current and the power consumed in the circuit? Justify your answer.

(c) How does its impedance vary with frequency of the ac source? Show graphically.

(d) Obtain an expression for the current in the circuit and its phase relation with ac voltage.

OR

(a) Draw a labelled diagram of an ac generator. Obtain the expression for the emf induced in the rotating coil of N turns each of cross-sectional area A, in the presence of a magnetic field $\vec{B}\cdot$

(b) A horizontal conducting rod 10 m long extending from east to west is falling with a speed 5.0 ms^{-1} at right angles to the horizontal component of the Earth's magnetic field, 0.3×10^{-4} Wb m^{-2}. Find the instantaneous value of the emf induced in the rod.

53. The teachers of Geeta's school took the students on a study trip to a power generating station, located nearly 200 km away from the city. The teacher explained that electrical energy is transmitted over such a long distance to their city, in the form of alternating current (ac) raised to a high voltage. At the receiving end in the city, the voltage is reduced to operate the devices. As a result, the power loss is reduced. Geeta listened to the teacher and asked questions about how the ac is converted to a higher or lower voltage.

(a) Name the device used to change the alternating voltage to a higher or lower value. State one cause for power dissipation in this device.

(b) Explain with an example, how power loss is reduced if the energy is transmitted over long distances as an alternating current rather than a direct current.

(c) Write two values each shown by the teachers and Geeta. **[4; CBSE-2018]**

54. (a) State the principle of an ac generator and explain its working with the help of a labelled diagram. Obtain the expression for the emf induced in a coil having N turns each of cross-sectional area A, rotating with a constant angular speed 'ω' in a magnetic field $\vec{B}$, directed perpendicular to the axis of rotation.

(b) An aeroplane is flying horizontally from west to east with a velocity of 900 km/hour. Calculate the potential difference developed between the ends of its wings having a span of 20 m. The horizontal component of the Earth's magnetic field is 5×10^{-4} T and the angle of dip is 30°.

OR

A device X is connected across an ac source of voltage $V = V_0 \sin \omega t$. the current through X is given as $I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$.

(a) Identify the device X and write the expression for its reactance.

(b) Draw graphs showing variation of voltage and current with time over one cycle of ac, for X.

(c) How does the reactance of the device X vary with frequency of the ac? Show this variation graphically.

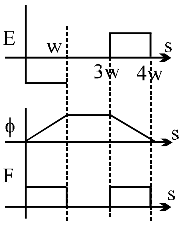
(d) Draw the phasor diagram for the device X. **[5; CBSE-2018]**

ANSWER-KEY

EXERCISE (S-1)

1. Ans. $\lambda V_y B_0$ **2. Ans.** (i) 2.4×10^{-5} V (ii) from c to b **3. Ans.** 2 N

4. Ans.



5. Ans. $\frac{mgR}{B^2\ell^2}$ **6. Ans.** 0.75 T

7. Ans. $\frac{erk}{2m}$ directed along tangent to the circle of radius r, whose centre lies on the axis of cylinder.

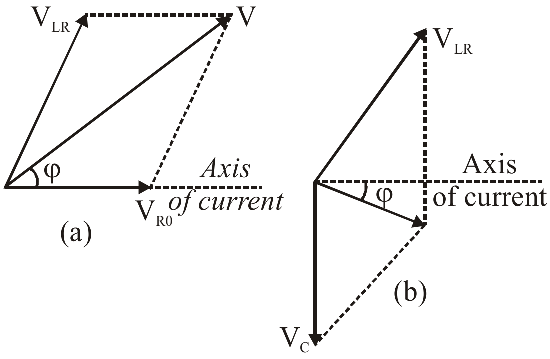
8. Ans. 6.5 V **9. Ans.** $\frac{l}{2}\frac{dB}{dt}\sqrt{R^2-\frac{l^2}{4}}$ **10. Ans.** 0.8 **11. Ans.** $\frac{\mu_0 i a^2 \pi}{2Rb}$

12. Ans. I^{-1} **13. Ans.** $\frac{LE^2}{2R_1^2}$ **14. Ans.** $\frac{e^2}{e^2-1}$ **15. Ans.** $\frac{EL}{eR^2}$

16. Ans. $q = Q_0 \sin\left(\sqrt{\frac{1}{LC}}\,t+\frac{\pi}{2}\right)$ **17. Ans.** (a) 10^4 A/s (b) 0 (c) 2A (d) $100\sqrt{3}$ μC

18. Ans. 30 Wb. **19. Ans.** $\varepsilon = 1.7 \times 10^{-5}$ V

20. Ans.



(a) (b)

21. Ans. 0.08 H, 17.28 W

22. Ans. 2A, 400W **23. Ans.** 0.2 mH, $\frac{1}{32}$ μF, 8×10^5 rad/s **24. Ans.** $\frac{20}{\pi^2} \cong 2H$

25. Ans. 20 V **26. Ans.** 20 A, π/4,

EXERCISE (S-2)

1. Ans. (i) 85.22 Tm^2; (ii) 56.8 V; (iii) linearly

2. Ans. $V = 1\ ms^{-1}$, $R_1 = 0.47\ \Omega$, $R_2 = 0.30\ \Omega$

3. Ans. $\frac{\mu_0 h\omega i_m N}{2\pi} \ln \frac{b}{a}$

4. Ans. $\frac{1}{3}A$

5. Ans. $C\pi a^2/R$

6. Ans. 200 rad/sec

7. Ans. $-\frac{B\pi a^2\lambda}{MR}\hat{k}$

8. Ans. 0.4 V

9. Ans. $\frac{E\ R_1}{R_1R_2 + R_2R_3 + R_3R_1}$

10 Ans. $I_{EA} = \frac{7}{22}$ A ; $I_{BE} = \frac{3}{11}$ A ; $I_{FE} = \frac{1}{22}$ A

11. Ans. (a) $i = \frac{B_0 av}{R}$ in anticlockwise direction, v = velocity at time t, (b) $F_{nett} = B_0^2 a^2 V/R$,

(c) $V = \frac{mgR}{B_0^2 a^2}\left(1 - e^{-\frac{B_0^2 a^2 t}{mR}}\right)$

12. Ans. 67/32 A

13. Ans. (i) $i_1 = i_2 = 10/3$ A, (ii) $i_1 = 50/11$ A ; $i_2 = 30/11$ A, (iii) $i_1 = 0$, $i_2 = 20/11$ A, (iv) $i_1 = i_2 = 0$

14. Ans. 42 + 20t volt

15. Ans. (a) $E = \frac{1}{2} B\omega r^2$ (b) (i) $I = \frac{B\omega r^2\left[1 - e^{-Rt/L}\right]}{2R}$, (ii) $\tau = \frac{mgr}{2}\cos\omega t + \frac{\omega B^2 r^4}{4R}(1 - e^{-Rt/L})$

16. Ans. $2\pi\frac{\sqrt{mL}}{l B}, g\frac{\sqrt{mL}}{l B}$

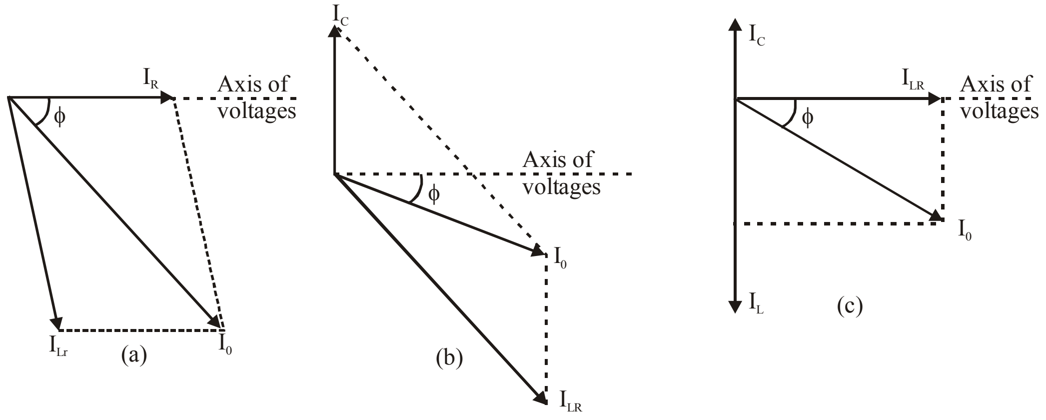
17. Ans. $-\frac{V}{R}e^{-\frac{Rt}{L}}$

18. Ans. $\frac{1}{15}A, \frac{1}{10}A$

19. Ans. $kMT^2/(R)$

20. Ans. 77Ω, 97.6Ω, 7.7 V, 9.76 V

21. Ans.



22. Ans. $I = \frac{(\mu_0 n i_0 \omega\cos\omega t)\pi a^2(Ld)}{\rho 2\pi R}$

EXERCISE (O-1)

1. Ans. (C) 2. Ans. (A) 3. Ans. (D) 4. Ans. (C) 5. Ans. (A) 6. Ans. (A)

7. Ans. (B) 8. Ans. (C) 9. Ans. (A) 10. Ans. (B) 11. Ans. (A) 12. Ans. (C)

13. Ans. (A) 14. Ans. (D) 15. Ans. (C) 16. Ans. (A) 17. Ans. (A) 18. Ans. (D)

19. Ans. (D) 20. Ans. (B) 21. Ans. (D) 22. Ans. (B) 23. Ans. (D) 24. Ans. (A)

25. Ans. (C) 26. Ans. (A) 27. Ans. (C) 28. Ans. (A) 29. Ans. (B) 30. Ans. (C)

31. Ans. (A) 32. Ans. (C) 33. Ans. (B) 34. Ans. (D) 35. Ans. (B) 36. Ans. (B)

37. Ans. (A) 38. Ans. (D) 39. Ans. (A) 40. Ans. (A) 41. Ans. (C) 42. Ans. (B)

43. Ans. (A) 44. Ans. (B) 45. Ans. (A) 46. Ans. (D) 47. Ans. (A) 48. Ans. (D)

49. Ans. (B) 50. Ans. (C) 51. Ans. (A) 52. Ans. (B) 53. Ans. (D) 54. Ans. (D)

55. Ans. (D) 56. Ans. (D) 57. Ans. (D) 58. Ans. (A) 59. Ans. (C) 60. Ans. (B)

61. Ans. (A) 62. Ans. (B) 63. Ans. (D) 64. Ans. (C)

SUPPLEMENT FOR JEE-MAINS

65. Ans. (B) 66. Ans. (B) 67. Ans. (A) 68. Ans. (D) 69. Ans. (D) 70. Ans. (C)

71. Ans. (C) 72. Ans. (B) 73. Ans. (D) 74. Ans. (A) 75. Ans. (A) 76. Ans. (D)

77. Ans. (D)

MULTIPLE CORRECT TYPE QUESTIONS

78. Ans. (B) 79. Ans. (A) 80. Ans. (B, C) 81. Ans. (A, B) 82. Ans. (B) 83. Ans. (A)

84. Ans. (D) 85. Ans. (B, D) 86. Ans. (B,D) 87. Ans. (A) 88. Ans. (B)

89. Ans. (A, C, D) 90. Ans. (A,C) 91. Ans. (D) 92. Ans. (B) 93. Ans. (C)

94. Ans. (C) 95. Ans. (C) 96. Ans. (B) 97. Ans. (B)

EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

1. Ans. (A) 2. Ans. (B) 3. Ans. (B) 4. Ans. (A) 5. Ans. (C) 6. Ans. (D)

7. Ans. (D) 8. Ans. (A) 9. Ans. (D) 10. Ans. (A) 11. Ans. (B) 12. Ans. (A)

13. Ans. (A) 14. Ans. (C) 15. Ans. (A) 16. Ans. (C) 17. Ans. (D) 18. Ans. (A)

19. Ans. (D) 20. Ans. (D) 21. Ans. (D) 22. Ans. (B) 23. Ans. (A) 24. Ans. (D)

MULTIPLE CORRECT TYPE QUESTIONS

25. Ans. (A, B, C) 26. Ans. (A, D) 27. Ans. (A, B) 28. Ans. (A, C, D)

29. Ans. (B, D) 30. Ans. (D) 31. Ans. (B)

COMPREHENSION TYPE QUESTIONS

32. Ans. (D) 33. Ans. (B) 34. Ans. (B, D) 35. Ans. (C) 36. Ans. (D) 37. Ans. (A)

38. Ans. (B) 39. Ans. (A)

EXERCISE-JM

1. Ans. (3) **2. Ans. (3)** **3. Ans. (1)** **4. Ans. (2)** **5. Ans. (2)** **6. Ans. (4)**
7. Ans. (1) **8. Ans. (2)** **9. Ans. (2)** **10. Ans. (4)** **11. Ans. (1)** **12. Ans. (3)**
13. Ans. (1) **14. Ans. (2)** **15. Ans. (3)** **16. Ans. (1)** **17. Ans. (1)** **18. Ans. (4)**
19. Ans. (1)

SELECTED PROBLEMS FROM JEE-MAINS ONLINE PAPERS

20. Ans. (4) **21. Ans. (4)** **22. Ans. (2)** **23. Ans. (2)** **24. Ans. (3)** **25. Ans. (2)**
26. Ans. (2) **27. Ans. (3)** **28. Ans. (1)** **29. Ans. (3)** **30. Ans. (3)** **31. Ans. (2)**
32. Ans. (2)

EXERCISE-JA

1. Ans. (B) **2. Ans. (C)** **3. Ans. (A)-R,S,T; (B)-Q,R,S,T; (C)-P,Q; (D)-Q,R,S,T**
4. Ans. (C) **5. Ans. (B,C)** **6. Ans. 6** **7. Ans. 4** **8. Ans. 7** **9. Ans. (A,C)**
10. Ans. (C) or (A,C) **11. Ans. (B)** **12. Ans. (B)** **13. Ans. (A)** **14. Ans. (B)**
15. Ans. (C,D) **16. Ans. (A,D)** **17. Ans. (A,C)** **18. Ans. (B,D)** **19. Ans. 8**
20. Ans. (C, D) **21. Ans. (A, D)** **22. Ans. (A,B)** **23. Ans. (A), (B), (C)** **24. Ans. (C), (D)**
25. Ans. (B,D) **26. Ans. (1,2,4)** **27. Ans. (0.63)** **28. Ans. (B)** **29. Ans. (B)** **30. Ans. 55.00**

Important Notes

Important Notes

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Chapter Contents

MODERN PHYSICS-1

MODERN PHYSICS-1

KEY CONCEPT

PHOTO ELECTRIC EFFECT

PHOTOELECTRIC EFFECT

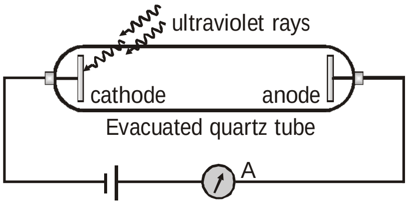
It was discovered by Hertz in 1887. He found that when the negative plate of an electric discharge tube was illuminated with ultraviolet light, the electric discharge took place more readily. Further experiments carried out by Hallwachs confirmed that certain negatively charged particles are emitted, when a Zn plate is illuminated with ultraviolet light. These particles were identified as electrons.

The phenomenon of emission of electrons from the surface of certain substances, when suitable radiations of certain frequency or wavelength are incident upon it is called photoelectric effect.

EXPLANATION OF PHOTOELECTRIC EFFECT

- **On the basis of wave theory** :According to wave theory, light is an electromagnetic radiation consisting of oscillating electric field vectors and magnetic field vectors. When electromagnetic radiations are incident on a metal surface, the free electrons [free electrons means the electrons which are loosely bound and free to move inside the metal] absorb energy from the radiation. This occurs by the oscillations of electron under the action of electric field vector of electromagnetic radiation. When an electron acquires sufficiently high energy so that it can overcome its binding energy, it comes out from the metal.
- **On the basis of photon theory:** According to photon theory of light, light consists of particles (called photons). Each particle carries a certain amount of energy with it. This energy is given by E=hν, where h is the Plank's constant and ν is the frequency. When the photons are incident on a metal surface, they collide with electrons. In some of the collisions, a photon is absorbed by an electron. Thus an electron gets energy hν. If this energy is greater than the binding energy of the electron, it comes out of the metal surface. The extra energy given to the electron becomes its kinetic energy.

EXPERIMENTS

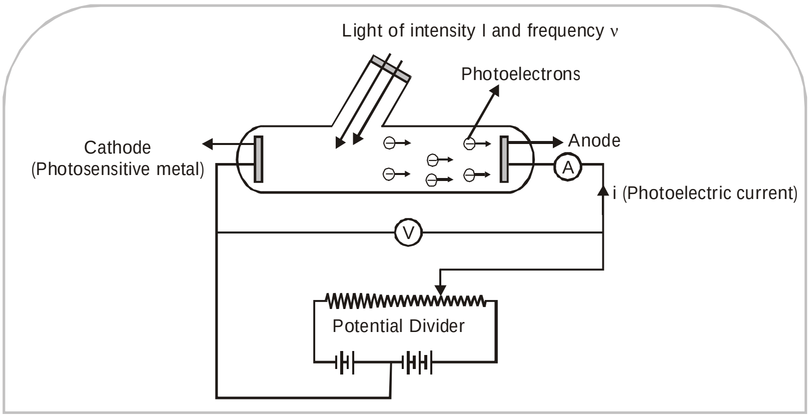


- **Hertz Experiment**

  Hertz observed that when ultraviolet rays are incident on negative plate of electric discharge tube then conduction takes place easily in the tube.
- **Hallwach experiment :** Hallwach observed that if negatively charged Zn plate is illuminated by U.V. light, its negative charge decreases and it becomes neutral and after some time it gains positive charge. It means in the effect of light, some negative charged particles are emitted from the metal.
- **Lenard Explanation :** He told that when ultraviolet rays are incident on cathode, electrons are ejected. These electrons are attracted by anode and circuit is completed due to flow of electrons and current flows. When U.V. rays incident on anode, electrons are ejected but current does not flow.

  For the photo electric effect the light of short wavelength (or high frequency) is more effective than the light of long wavelength (or low frequency)

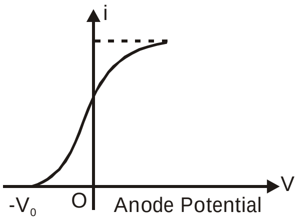
- **Experimental study of photoelectric Effect :** When light of frequency ν and intensity I falls on the cathode, electrons are emitted from it. The electrons are collected by the anode and a current flows in the circuit. This current is called photoelectric current. This experiment is used to study the variation of photoelectric current with different factors like intensity, frequency and the potential difference between the anode and cathode.



(i) Variation of photoelectric current with potential difference

With the help of the above experimental setup, a graph is obtained between current and potential difference. The potential difference is varied with the help of a potential divider. The graph obtained is shown below.

The main points of observation are :



(a) At zero anode potential, a current exists. It means that electrons are emitted from cathode with some kinetic energy.

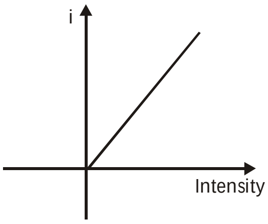
(b) As anode potential is increased, current increases. This implies that different electrons are emitted with different kinetic energies.

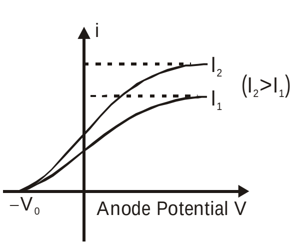
(c) After a certain anode potential, current acquires a constant value called saturation current. Current acquires a saturation value because the number of electrons emitted7 per second from the cathode are fixed.

(d) At a certain negative potential, the photoelectric current becomes zero. This is called stopping potential (V_0). Stopping potential is a measure of maximum kinetic energy of the emitted electrons. Let KE_{max} be the maximum kinetic energy of an emitted electron, then $KE_{max} = eV_0$.

(ii) Variation of current with intensity

The photoelectric current is found to be directly proportional to intensity of incident radiation.

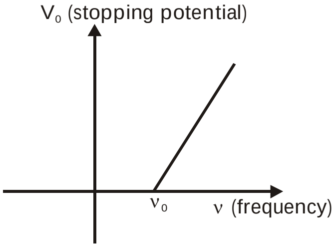




(iii) Effect of intensity on saturation current and stopping potential

(a) Saturation current increases with increase in intensity.

(b) Stopping potential (and therefore maximum kinetic energy) is independent of intensity.



(iv) Effect of frequency

(a) Stopping potential is found to vary with frequency of incident light linearly. Greater the frequency of incident light, greater the stopping potential.

(b) There exists a certain minimum frequency ν_0 below which no stopping potential is required as no emission of electrons takes place. This frequency is called threshold frequency. For photoelectric emission to take place, $\nu > \nu_0$.

IMPORTANT POINTS

- Photo electric effect is an instantaneous process, as soon as light is incident on the metal, photo electrons are emitted.
- Stopping potential does not depend on the distance between cathode and anode.
- The work function represented the energy needed to remove the least tightly bounded electrons from the surface. It depends only on nature of the metal and independent of any other factors.

- **Failure of wave theory of light**

(i) According to wave theory when light incident on a surface, energy is distributed continuously over the surface. So that electron has to wait to gain sufficient energy to come out. But in experiment there is no time lag. Emission of electrons takes place in less than 10^{-9} s. This means, electron does not absorb energy. They get all the energy once.

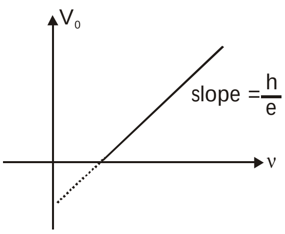
(ii) When intensity is increased, more energetic electrons should be emitted. So that stopping potential should be intensity dependent. But it is not observed.

(iii) According to wave theory, if intensity is sufficient then, at each frequency, electron emission is possible. It means there should not be existence of threshold frequency.

- **Einstein's Explanation of Photoelectric Effect**

Einstein explained photoelectric effect on the basis of photon–electron interaction. The energy transfer takes place due to collisions between an electrons and a photon. The electrons within the target material are held there by electric force. The electron needs a certain minimum energy to escape from this pull. This minimum energy is the property of target material and it is called the work function. When a photon of energy E=hν collides with and transfers its energy to an electron, and this energy is greater than the work function, the electron can escape through the surface.

- **Einstein's Photoelectric Equation** $h\nu = \phi + KE_{max}$



Here hν is the energy transferred to the electron. Out of this, ϕ is the energy needed to escape.

The remaining energy appears as kinetic energy of the electron.

Now $KE_{max} = eV_0$ (where V_0 is stopping potential)

$$\therefore\ h\nu = \phi + eV_0 \Rightarrow\ V_0 = \left(\frac{h}{e}\right)\nu - \frac{\phi}{e}$$

Thus, the stopping potential varies linearly with the frequency of incident radiation.

Slope of the graph obtained is $\frac{h}{e}$. This graph helps in determination of Planck's constant.

IMPORTANT POINTS

- Einstein's Photo Electric equation is based on conservation of energy.
- Einstein explained P.E.E. on the basis of quantum theory, for which he was awarded noble prize.
- According to Einstein one photon can eject one e^- only. But here the energy of incident photon should greater or equal to work function.
- In photoelectric effect all photoelectrons do not have same kinetic energy. Their KE range from zero toE_{max} which depends on frequency of incident radiation and nature of cathode.
- The photo electric effect takes place only when photons strike bound electrons because for free electrons energy and momentum conservations do not hold together.

Ex. Calculate the possible velocity of a photoelectron if the work function of the target material is 1.24 eV and wavelength of light is 4.36×10^{-7} m. What retarding potential is necessary to stop the emission of electrons?

Sol. As $KE_{max} = h\nu - \phi \Rightarrow \frac{1}{2}mv_{max}^2 = h\nu - \phi = \frac{hc}{\lambda} - \phi$

$$v_{max} = \sqrt{\frac{2\left(\frac{hc}{\lambda}-\phi\right)}{m}} = \sqrt{\frac{2\left(\frac{6.63\times10^{-34}\times3\times10^{8}}{4.36\times10^{-7}}-1.24\times1.6\times10^{-19}\right)}{9.11\times10^{-31}}} = 7.523 \times 10^5 \text{ m/s}$$

$\therefore$ The speed of a photoelectron can be any value between 0 and 7.43×10^5 m/s

If V_0 is the stopping potential, then $eV_0 = \frac{1}{2}mv_{max}^2$

$$\Rightarrow V_0 = \frac{1}{2}\frac{mv_{max}^2}{e} = \frac{hc}{e\lambda} - \frac{\phi}{e} = \frac{12400}{4360} - 1.24 = 1.60 \text{ V} \quad \left[\because \frac{hc}{e} = 12400 \times 10^{-10}\,\text{V}-\text{m}\right]$$

Ex. The surface of a metal of work function ϕ is illuminated by light whose electric field component varies with time as $E = E_0\,[1 + \cos\omega t]\sin\omega_0 t$. Find the maximum kinetic energy of photoelectrons emitted from the surface.

Sol. The given electric field component is $E = E_0 \sin\omega_0 t + E_0 \sin\omega_0 t \cos\omega t = E_0 \sin\omega_0 t + \frac{E_0}{2}[\sin(\omega_0 + \omega)t + \sin(\omega_0 - \omega)t]$

$\therefore$ The given light comprises three different frequencies viz. ω, $\omega_0 + \omega$, $\omega_0 - \omega$

The maximum kinetic energy will be due to most energetic photon.

$$\therefore KE_{max} = h\nu - \phi = \frac{h(\omega + \omega_0)}{2\pi} - \phi \qquad \left(\because \omega = 2\pi\nu \text{ or } \nu = \frac{\omega}{2\pi}\right)$$

Ex. When light of wavelength λ is incident on a metal surface, stopping potential is found to be x. When light of wavelength $n\lambda$ is incident on the same metal surface, stopping potential is found to be $\frac{x}{n+1}$. Find the threshold wavelength of the metal.

Sol. Let λ_0 is the threshold wavelength. The work function is $\phi = \frac{hc}{\lambda_0}$.

Now, by photoelectric equation ex = $\frac{hc}{\lambda} - \frac{hc}{\lambda_0}$ ...(i) $\frac{ex}{n+1} = \frac{hc}{n\lambda} - \frac{hc}{\lambda_0}$ ...(ii)

From (i) and (ii) $\frac{hc}{\lambda} - \frac{hc}{\lambda_0} = (n+1)\frac{hc}{n\lambda} - (n+1)\frac{hc}{\lambda_0} \Rightarrow \frac{nhc}{\lambda_0} = \frac{hc}{n\lambda} \Rightarrow \lambda_0 = n^2\lambda$

PHOTON THEORY OF LIGHT

- A photon is a particle of light moving with speed 299792458 m/s in vacuum.
- The speed of a photon is independent of frame of reference. This is the basic postulate of theory of relativity.
- The rest mass of a photon is zero. i.e. photons do not exist at rest.
- Effective mass of photon m = $\frac{E}{c^2} = \frac{hc}{c^2\lambda} = \frac{h}{c\lambda}$ i.e. $m \propto \frac{1}{\lambda}$

  So mass of violet light photon is greater than the mass of red light photon. ($\because \lambda_R > \lambda_V$)
- According to Planck the energy of a photon is directly proportional to the frequency of the radiation.

  $E \propto \nu$ or $E = h\nu$

  $E = \frac{hc}{\lambda}$ joule ($\because c = \nu\lambda$) or $E = \frac{hc}{\lambda e} = \frac{12400}{\lambda}$ eV – Å

  $\left[\because \frac{hc}{e} = 12400(\text{Å} - \text{eV})\right]$

  Here E = energy of photon, c = speed of light, h = Planck's constant, e = charge of electron

  $h = 6.62 \times 10^{-34}$ J–s, ν = frequency of photon, λ = wavelength of photon
- Linear momentum of photon p = $\frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$
- A photon can collide with material particles like electron. During these collisions, the total energy and total momentum remain constant.
- Energy of light passing through per unit area per unit time is known as intensity of light.

  Intensity of light $I = \frac{E}{At} = \frac{P}{A}$ ...(i)

  Here P = power of source, A = Area, t = time taken

  E = energy incident in t time = Nhν, N = number of photon incident in t time

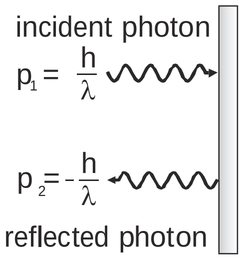
  Intensity $I = \frac{N(h\nu)}{At} = \frac{n(h\nu)}{A}$ ...(ii)

  $\left[\because n = \frac{N}{t} = \text{no. of photon per sec.}\right]$

  From equation (i) and (ii), $\frac{P}{A} = \frac{n(h\nu)}{A} \Rightarrow n = \frac{P}{h\nu} = \frac{P\lambda}{hc} = 5 \times 10^{24} J^{-1}m^{-1} \times P \times \lambda$

- When photons fall on a surface, they exert a force and pressure on the surface. This pressure is called radiation pressure.
- **Force exerted on perfectly reflecting surface**

  Let 'N' photons are there in time t,



Momentum before striking the surface $(p_1) = \frac{Nh}{\lambda}$

Momentum after striking the surface $(p_2) = -\frac{Nh}{\lambda}$

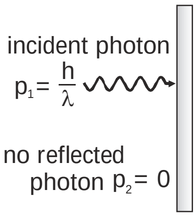
Change in momentum of photons $= p_2 - p_1 = \frac{-2Nh}{\lambda}$

But change in momentum of surface $= \Delta p = \frac{2Nh}{\lambda}$

So that force on surface $F = \frac{2Nh}{t\lambda} = n\left[\frac{2h}{\lambda}\right]$ but $n = \frac{P\lambda}{hc}$

$\therefore$ $F = \frac{2h}{\lambda} \times \frac{P\lambda}{hc} = \frac{2P}{c}$ and Pressure $= \frac{F}{A} = \frac{2P}{cA} = \frac{2I}{c}$ $\left[\because I = \frac{P}{A}\right]$

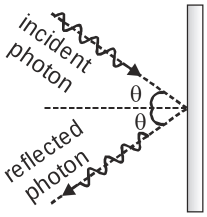
- **Force exerted on perfectly absorbing surface**



$$F = \frac{p_1 - p_2}{t} = \frac{\frac{Nh}{\lambda} - 0}{t} = \frac{Nh}{t\lambda} = n\frac{h}{\lambda}$$

$F = \frac{P}{c}$ $\left(\because n = \frac{P\lambda}{hc}\right)$ and Pressure $= \frac{F}{A} = \frac{P}{Ac} = \frac{I}{c}$

- **When a beam of light is incident at angle θ on perfectly reflector surface**



$F = \frac{2P}{c}\cos\theta = n\left[\frac{2h}{\lambda}\right]\cos\theta = \frac{2IA\cos\theta}{c}$

Pressure $= \frac{F}{A} = \frac{2I\cos^2\theta}{c}$

Ex. The intensity of sunlight on the surface of earth is 1400 W/m^2. Assuming the mean wavelength of sunlight to be 6000 Å, calculate:–

(a) The photon flux arriving at 1 m^2 area on earth perpendicular to light radiations and

(b) The number of photons emitted from the sun per second (Assuming the average radius of Earth's orbit to be 1.49×10^{11} m)

Sol. (a) Energy of a photon $E = \frac{hc}{\lambda} = \frac{12400}{6000}$ =2.06 eV = 3.3×10^{-19} J

Photon flux $= \frac{IA}{E} = \frac{1400 \times 1}{3.3 \times 10^{-19}} = 4.22 \times 10^{21}$ photons/sec.

(b) Number of photons emitted per second $n = \frac{P}{E} = \frac{IA}{E} = \frac{1400 \times 4\pi \times (1.49 \times 10^{11})^2}{3.3 \times 10^{-19}} = 1.18 \times 10^{45}$

Ex. In a photoelectric setup, a point source of light of power 3.2×10^{-3} W emits monochromatic photons of energy 5.0 eV. The source is located at a distance 0.8 m from the centre of a stationary metallic sphere of work function 3.0 eV and radius 8×10^{-3} m. The efficiency of photoelectron emission is one for every 10^6 incident photons. Assuming that the sphere is isolated and initially neutral and that photoelectrons are instantly swept away after emission, Find (i) the number of photoelectrons emitted per second. (ii) the time t after light source is switched on, at which photoelectron emission stops.

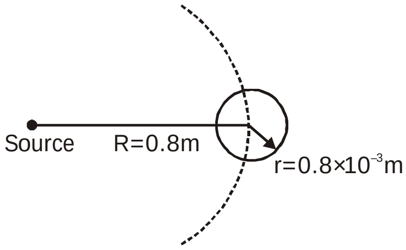
Sol. Energy of a single photon $E = 5.0 \text{ eV} = 8 \times 10^{-19}$ J

Power of source $P = 3.2 \times 10^{-3}$ W

$\therefore$ number of photons emitted per second $n = \frac{P}{E} = \frac{3.2 \times 10^{-3}}{8 \times 10^{-19}} = 4 \times 10^{15}$/s

The number of photons incident per second on metal surface is $n_0 = \frac{n}{4\pi R^2} \times \pi r^2$

$$n_0 = \frac{4 \times 10^{15}}{4\pi (0.8)^2} \times \pi \left(8 \times 10^{-3}\right)^2 = 1.0 \times 10^{11} \text{ photon/s}$$



Number of electrons emitted $= \frac{1.0 \times 10^{11}}{10^6} = 10^5$ /s

$$KE_{max} = h\nu - \phi = 5.0 - 3.0 = 2.0 \text{ eV}$$

The photoelectron emission stops, when the metallic sphere acquires stopping potential.

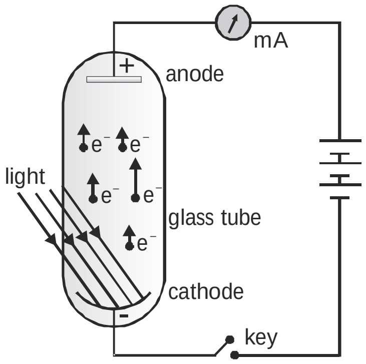
As $KE_{max} = 2.0$ eV $\Rightarrow$ Stopping potential $V_0 = 2V \Rightarrow 2 = \frac{q}{4\pi\varepsilon_0 r} \Rightarrow q = 1.78 \times 10^{-12}$ C

Now charge q = (number of electrons/second) × t× e $\Rightarrow t = \frac{1.78 \times 10^{-12}}{10^5 \times 1.6 \times 10^{-19}} = 111$s

PHOTO CELL

A photo cell is a practical application of the phenomenon of photo electric effect, with the help of photo cell light energy is converted into electrical energy.

- **Construction :** A photo cell consists of an evacuated sealed glass tube containing anode and a concave cathode of suitable emitting material such as Cesium (Cs).



- **Working:** When light of frequency greater than the threshold frequency of cathode material falls on the cathode, photoelectrons emitted are collected by the anode and an electric current starts flowing in the external circuit. The current increase with the increase in the intensity of light. The current would stop, if the light does not fall on the cathode.
- **Application**
  (i) In television camera.
  (ii) In automatic door
  (iii) Burglar's alarm
  (iv) Automatic switching of street light and traffic signals.

MATTER WAVES THEORY

DUAL NATURE OF LIGHT

Experimental phenomena of light reflection, refraction, interference, diffraction are explained only on the basis of wave theory of light. These phenomena verify the wave nature of light. Experimental phenomena of light photoelectric effect and Crompton effect, pair production and positron inhalational can be explained only on the basis of the particle nature of light. These phenomena verify the particle nature of light.

It is inferred that light does not have any definite nature, rather its nature depends on its experimental phenomenon. This is known as the dual nature of light. The wave nature and particle nature both can not be possible simultaneously.

De-Broglie HYPOTHESIS

De Broglie imagined that as light possess both wave and particle nature, similarly matter must also posses both nature, particle as well as wave. De Broglie imagined that despite particle nature of matter, waves must also be associated with material particles. Wave associated with material particles, are defined as matter waves.

- **De Broglie wavelength associated with moving particles**

If a particle of mass m moving with velocity v

Kinetic energy of the particle $E=\frac{1}{2}mv^2=\frac{p^2}{2m}$ momentum of particle p = mv = $\sqrt{2mE}$ the wave length associated with the particles is $\lambda=\frac{h}{p}=\frac{h}{mv}=\frac{h}{\sqrt{2mE}}$ $\lambda\propto\frac{1}{p}\Rightarrow\lambda\propto\frac{1}{v}\Rightarrow\lambda\propto\frac{1}{\sqrt{E}}$

The order of magnitude of wave lengths associated with macroscopic particles is 10^{-24} Å.

The smallest wavelength whose measurement is possible is that of γ – rays ($\lambda\simeq10^{-5}$ Å). This is the reason why the wave nature of macroscopic particles is not observable.

The wavelength of matter waves associated with the microscopic particles like electron, proton, neutron, α – particle, atom, molecule etc. is of the order of 10^{-10} m, it is equal to the wavelength of X–rays, which is within the limit of measurement. Hence the wave nature of these particles is observable.

- **De Broglie wavelength associated with the charged particles**

Let a charged particle having charge q is accelerated by potential difference V.

Kinetic energy of this particle $E=\frac{1}{2}mv^2=qV$ Momentum of particle $p=mv=\sqrt{2mE}=\sqrt{2mqV}$

The De Broglie wavelength associated with charged particle $\lambda=\frac{h}{p}=\frac{h}{\sqrt{2mE}}=\frac{h}{\sqrt{2mqV}}$

- **For an Electron** $m_e=9.1\times10^{-31}$ kg, $q=1.6\times10^{-19}$ C, $h=6.62\times10^{-34}$ J–s

De Broglie wavelength associated with electron $\lambda=\frac{6.62\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.6\times10^{-19}V}}$

$$\lambda=\frac{12.27\times10^{-10}}{\sqrt{V}}\text{ meter}=\frac{12.27}{\sqrt{V}}\text{ Å} \quad \text{so} \quad \lambda\propto\frac{1}{\sqrt{V}}$$

Potential difference required to stop an electron of wavelength λ is $V = \frac{150.6}{\lambda^2}$ volt (Å)^2

- **For Proton** $m_p = 1.67 \times 10^{-27}$ kg
  De Broglie wavelength associated with proton

$$\lambda_p = \frac{6.62\times10^{-34}}{\sqrt{2\times1.67\times10^{-27}\times1.6\times10^{-19}V}}\ ;\ \lambda_p = \frac{0.286\times10^{-10}}{\sqrt{V}}\text{ meter} = \frac{0.286}{\sqrt{V}}\text{ Å}$$

- **For Deuteron** $m_d = 2 \times 1.67 \times 10^{-27}$ kg, $q_d = 1.6 \times 10^{-19}$ C

$$\lambda_d = \frac{6.62\times10^{-34}}{\sqrt{2\times2\times1.67\times10^{-27}\times1.6\times10^{-19}V}} = \frac{0.202}{\sqrt{V}}\overset{\circ}{A}$$

- **For α Particles** $q_\alpha = 2 \times 1.6 \times 10^{-19}$ C, $m_\alpha = 4 \times 1.67 \times 10^{-27}$ kg

$$\therefore \lambda_\alpha = \frac{6.62\times10^{-34}}{\sqrt{2\times4\times1.67\times10^{-27}\times2\times1.6\times10^{-19}V}} = \frac{0.101}{\sqrt{V}}\overset{\circ}{A}$$

DE BROGLIE WAVELENGTH ASSOCIATED WITH UNCHARGED PARTICLES

- **Kinetic energy of uncharged particle** $E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

  m = mass of particle, v = velocity of particle, p = momentum of particle.

- **Velocity of uncharged particle** $v = \sqrt{\frac{2E}{m}}$

- **Momentum of particle** $p = mv = \sqrt{2mE}$

wavelength associated with the particle $\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$

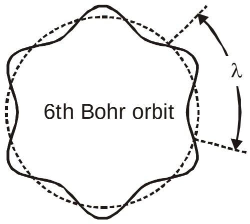
Kinetic energy of the particle in terms of its wavelength $E = \frac{h^2}{2m\lambda^2} = \frac{h^2}{2m\lambda^2 \times 1.6 \times 10^{-19}}\text{eV}$

For a neutron $m_n = 1.67 \times 10^{-27}$ kg

$$\therefore\ \lambda = \frac{6.62\times10^{-34}}{\sqrt{2\times1.67\times10^{-27}\times E}} = \frac{0.286\times10^{-10}}{\sqrt{E}}\text{meter}\sqrt{eV} = \frac{0.286}{\sqrt{E}}\overset{\circ}{A}\sqrt{eV}$$

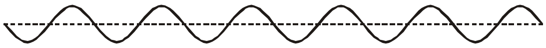
EXPLANATION OF BOHR QUANTIZATION CONDITION

According to De Broglie electron revolves round the nucleus in the form of stationary waves (i. e. wave packet) in the similar fashion as stationary waves in a vibrating string. Electron can stay in those circular orbits whose circumference is an integral multiple of De–Broglie wavelength associated with the electron, $2\pi r = n\lambda$



$\because\ \lambda = \frac{h}{mv}$ and $2\pi r = n\lambda \therefore\ mvr = \frac{nh}{2\pi}$

This is the Bohr quantizations condition.



equivalent straightened orbit

Ex. Find the initial momentum of electron if the momentum of electron is changed by p_m and the De Broglie wavelength associated with it changes by 0.50 %

Sol. $\frac{d\lambda}{\lambda}\times 100 = 0.5 \Rightarrow \frac{d\lambda}{\lambda}=\frac{0.5}{100}=\frac{1}{200}$ and $\Delta p = p_m$

$\because\ p=\frac{h}{\lambda}$, differentiating $\frac{dp}{d\lambda}=-\frac{h}{\lambda^2}=-\frac{h}{\lambda}\times\frac{1}{\lambda}=-\frac{p}{\lambda} \Rightarrow \frac{|dp|}{p}=\frac{d\lambda}{\lambda}\ \therefore\ \frac{p_m}{p}=\frac{1}{200} \Rightarrow p = 200\ p_m$

Ex. An α–particle moves in circular path of radius 0.83 cm in the presence of a magnetic field of 0.25 Wb/m^2. Find the De Broglie wavelength associated with the particle.

Sol. $\lambda=\frac{h}{p}=\frac{h}{qBr}=\frac{6.62\times10^{-34}}{2\times1.6\times10^{-19}\times0.25\times83\times10^{-4}}$ meter = 0.01 Å $\left[\because \frac{mv^2}{r}=qvB\right]$

Ex. A proton and an α–particle are accelerated through same potential difference. Find the ratio of their de- Broglie wavelength.

Sol. $\lambda=\frac{h}{mv}=\frac{h}{\sqrt{2mE}}=\frac{h}{\sqrt{2mqV}}\quad[\because E=qV]$ For proton m_p = m, q = e

For α–particle m_α =4 m, q = 2e, $\frac{\lambda_\alpha}{\lambda_p}=\sqrt{\frac{m_p q_p}{m_\alpha q_\alpha}}=\frac{1}{2\sqrt{2}}$

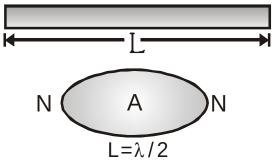
Ex. A particle of mass m is confined to a narrow tube of length L.

(a) Find the wavelengths of the de–Broglie wave which will resonate in the tube.

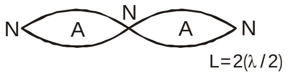
(b) Calculate the corresponding particle momenta, and

(c) Calculate the corresponding energies.

Sol. (a) The de–Broglie waves will resonate with a node at each end of the tube.

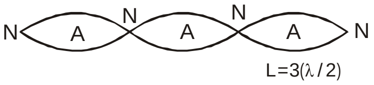


Few of the possible resonance forms are as follows : $\lambda_n=\frac{2L}{n}$, n=1,2,3.....

(b) 

Since de–Broglie wavelengths are $\lambda_n=\frac{h}{p_n}$

$p_n=\frac{h}{\lambda_n}=\frac{nh}{2L}$, $n = 1,2,3....$



(c) The kinetic energy of the particles are $K_n=\frac{p_n^2}{2m}=\frac{n^2h^2}{8L^2m}$, n = 1, 2, 3,....

ATOMIC STRUCTURE

VARIOUS MODELS FOR STRUCTURE OF ATOM

- **Dalton's Theory**

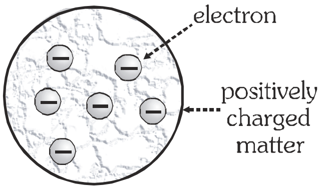
  Every material is composed of minute particles known as atom. Atom is indivisible i.e. it cannot be subdivided. It can neither be created nor be destroyed.

  All atoms of same element are identical physically as well as chemically, whereas atoms of different elements are different in properties.

  The atoms of different elements are made up of hydrogen atoms. (The radius of the heaviest atom is about 10 times that of hydrogen atom and its mass is about 250 times that of hydrogen).

  The atom is stable and electrically neutral.

- **Thomson's Atom Model**

  The atom as a whole is electrically neutral because the positive charge present on the atom (sphere) is equal to the negative charge of electrons present in the sphere.

  Atom is a positively charged sphere of radius 10^{-10} m in which electrons are embedded in between.

  The positive charge and the whole mass of the atom is uniformly distributed throughout the sphere.

- **Shortcomings of Thomson's model**

  (i) The spectrum of atoms cannot be explained with the help of this model

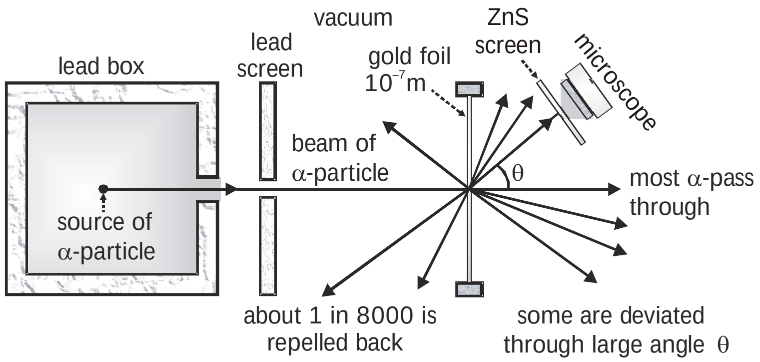
  (ii) Scattering of α–particles cannot be explained with the help of this model

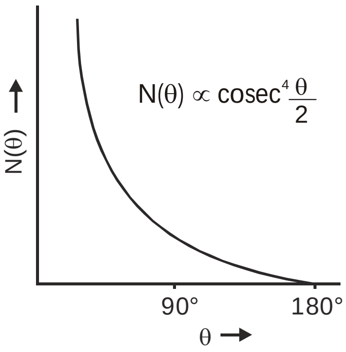
  (iii) Angular frequency of electron in nth orbit $\omega_n = \dfrac{8\pi^2k^2Z^2e^4m}{n^3h^3} \Rightarrow \omega_n \propto \dfrac{Z^2m}{n^3}$

RUTHERFORD ATOM MODEL

- **Rutherford experiments on scattering of α – particles by thin gold foil**

  The experimental arrangement is shown in figure. α–particles are emitted by some radioactive material (polonium), kept inside a thick lead box. A very fine beam of α–particles passes through a small hole in the lead screen. This well collimated beam is then allowed to fall on a thin gold foil. While passing through the gold foil, α–particles are scattered through different angles. A zinc sulphide screen was placed out the other side of the gold foil. This screen was movable, so as to receive the α–particles, scattered from the gold foil at angles varying from 0 to 180°. When an α–particle strikes the screen, it produces a flash of light and it is observed by the microscope. It was found that :





- Most of the α – particles went straight through the gold foil and produced flashes on the screen as if there were nothing inside gold foil. Thus the atom is hollow.
- Few particles collided with the atoms of the foil which have scattered or deflected through considerable large angles. Few particles even turned back towards source itself.
- The entire positive charge and almost whole mass of the atom is concentrated in small centre called a nucleus.
- The electrons could not deflected the path of a α – particles i.e. electrons are very light.
- Electrons revolve round the nucleus in circular orbits. So, Rutherford 1911, proposed a new type of model of the atom. According to this model, the positive charge of the atom, instead of being uniformly distributed throughout a sphere of atomic dimension is concentrated in a very small volume (Less than 10^{-13}m is diameter) at it centre. This central core, now called nucleus, is surrounded by clouds of electron makes.

  The entire atom electrically neutral. According to Rutherford scattering formula, the number of α – particle scattered at an angle θ by a target are given by

$$N_\theta = \frac{N_0 nt(2Ze^2)^2}{16(4\pi\varepsilon_0)^2 r^2 (mv_0^2)^2} \times \frac{1}{\sin^4\frac{\theta}{2}}$$

Where N_0 = number of α – particles that strike the unit area of the scatter

n = Number of target atom per m^3

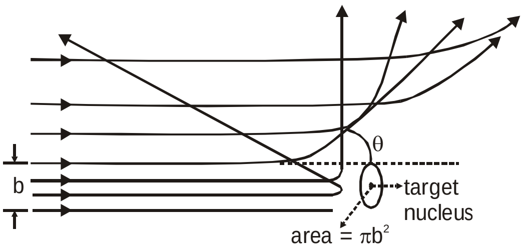
t = Thickness of target

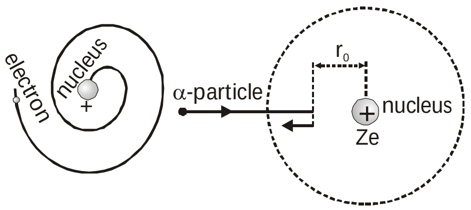
Ze = Charge on the target nucleus

2e = Charge on α – particle

r = Distance of the screen from target

v_0 = Velocity of α – particles at nearer distance of approach the size of a nucleus or the distance of nearer approach is given by.





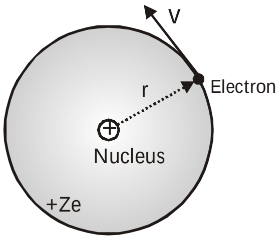
$$r_0 = \frac{1}{4\pi\varepsilon_0} \times \frac{(2Ze)^2}{\left[\frac{1}{2}mv_0^2\right]} = \frac{1}{4\pi\epsilon_0}\frac{(2Ze)^2}{E_K}$$ where E_K = K.E. of α–particle

Bohr's Atomic Model

In 1913 Neils Bohr, a Danish Physicist, introduced a revolutionary concept i.e., the quantum concept to explain the stability of an atom. He made a simple but bold statement that "The old classical laws which are applicable to bigger bodies cannot be directly applied to the sub–atomic particles such as electrons or protons.

Bohr incorporated the following new ideas now regarded as postulates of Bohr's theory.

1. The centripetal force required for an encircling electron is provided by the electrostatic attraction between the nucleus and the electron i.e. $\frac{1}{4\pi\varepsilon_0}\frac{(Ze)e}{r^2}=\frac{mv^2}{r}$ ...(i)



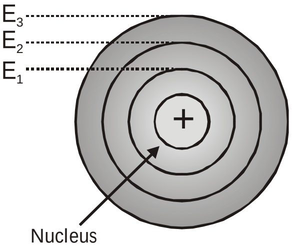
ε_0 =Absolute permittivity of free space = 8.85 × 10^{-12} C^2 N^{-1} m^{-2}
m = Mass of electron
v = Velocity (linear) of electron
r = Radius of the orbit in which electron is revolving.
Z = Atomic number of hydrogen like atom.

2. Electrons can revolve only in those orbits in which angular momentum of electron about nucleus is an integral multiple of $\frac{h}{2\pi}$. i.e., mvr = $\frac{nh}{2\pi}$ ...(ii)

n = Principal quantum number of the orbit in which electron is revolving.

3. Electrons in an atom can revolve only in discrete circular orbits called stationary energy levels (shells). An electron in a shell is characterised by a definite energy, angular momentum and orbit number. While in any of these orbits, an electron does not radiate energy although it is accelerated.

4. Electrons in outer orbits have greater energy than those in inner orbits. The orbiting electron emits energy when it jumps from an outer orbit (higher energy states) to an inner orbit (lower energy states) and also absorbs energy when it jumps from an inner orbit to an outer orbit. $E_n - E_m = h\nu_{n,m}$



where, E_n = Outer energy state
E_m = Inner energy state
$\nu_{n,m}$ = Frequency of radiation

5. The energy absorbed or released is always in the form of electromagnetic radiations.

MATHEMATICAL ANALYSIS OF BOHR'S THEORY

From above equation (i) and (ii) i.e., $\frac{1}{4\pi\varepsilon_0}\frac{(Ze)e}{r^2}=\frac{mv^2}{r}$ and mvr = $\frac{nh}{2\pi}$ ...(ii)

We get the following results.

1. **Velocity of electron in nth orbit :** By putting the value of mvr in equation (i) from (ii) we get

$$\frac{1}{4\pi\varepsilon_0}Ze^2=\left(\frac{nh}{2\pi}\right)\times v\ \Rightarrow v=\frac{Z}{n}\left[\frac{e^2}{2\varepsilon_0 h}\right]=\frac{Z}{n}v_0\ ....(iii)$$

Where, $v_0=\frac{\left(1.6\times10^{-19}\right)^2}{2\times8.85\times10^{-12}\times6.625\times10^{-34}}$ = 2.189 × 10^6 ms^{-1} = $\frac{c}{137}$ = 2.2 × 10^6 m/s

where c = 3 × 10^8 m/s = speed of light in vacuum

2. **Radius of the nth orbit :**

From equation (iii), putting the value of v in equation (ii), we get

$$m\left(\frac{Z}{n}\times\frac{e^2}{2\varepsilon_0 h}\right)r=\frac{nh}{2\pi}\Rightarrow r=\frac{n^2}{Z}\left[\frac{\varepsilon_0 h^2}{\pi me^2}\right]=\frac{n^2}{Z}r_0\ ...(iv)$$

where $r_0 = \dfrac{8.85 \times 10^{-12} \times \left(6.625 \times 10^{-34}\right)^2}{3.14 \times 9.11 \times 10^{-31} \times \left(1.6 \times 10^{-19}\right)^2} = 0.529 \times 10^{-10}$ m ≈ 0.53Å

3. Total energy of electron in nth orbit :

From equation (i) K.E. = $\dfrac{1}{2}mv^2 = \dfrac{Ze^2}{8\pi\varepsilon_0 r}$ and PE = $\dfrac{1}{4\pi\varepsilon_0}\dfrac{(Ze)(-e)}{r} = -2\text{K.E.}$ $\therefore$ |PE| = 2 KE

Total energy of the system E = KE + PE = –2KE + KE = – KE = $\dfrac{-Ze^2}{8\pi\varepsilon_0 r}$

By putting the value of r from the equation (iv), we get E = $\dfrac{Z^2}{n^2}\left(-\dfrac{me^4}{8\varepsilon_0^2 h^2}\right) = \dfrac{Z^2}{n^2}\cdot E_0$ ...(v)

where $E_0 = \dfrac{-\left(9.11 \times 10^{-3}\right)\left(1.6 \times 10^{-19}\right)^4}{8 \times \left(8.85 \times 10^{-12}\right)^2 \times \left(6.625 \times 10^{-34}\right)^2} = -\ 13.6$ eV

4. Time period of revolution of electron in nth orbit : $T = \dfrac{2\pi r}{v}$

By putting the values of r and v, from (iii) and (iv) $T = \dfrac{n^3}{Z^2} \times \left(\dfrac{4\,\epsilon_0^2\,h^3}{me^4}\right) = \dfrac{n^3}{Z^2}\cdot T_0$

where, $T_0 = \dfrac{4 \times \left(8.85 \times 10^{-12}\right)^2 \times \left(6.625 \times 10^{-34}\right)^3}{9.11 \times 10^{-31} \times \left(1.6 \times 10^{-19}\right)^4} = 1.51 \times 10^{-16}$ second

5. Frequency of revolution in nth orbit :

$f = \dfrac{1}{T} = \dfrac{Z^2}{n^3} \times \dfrac{me^4}{4\varepsilon_0^2 h^3} = \dfrac{Z^2}{n^3}\cdot f_0$ where, $f_0 = \dfrac{9.11 \times 10^{-31} \times \left(1.6 \times 10^{-19}\right)^4}{4 \times \left(8.85 \times 10^{-12}\right)^2 \left(6.625 \times 10^{-34}\right)^3} = 6.6 \times 10^{15}$ Hz

6. Wavelength of photon

$$\Delta E = E_{n_2} - E_{n_1} = \frac{me^4}{8\varepsilon_0^2 h^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]Z^2 = 13.6\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]Z^2 \Rightarrow \Delta E = \frac{hc}{\lambda} \Rightarrow \frac{1}{\lambda} = \bar{\nu} = \frac{me^4}{8\varepsilon_0^2 h^3 c}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]Z^2$$

$= R_\infty\left[\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right]Z^2$ where, $\bar{\nu}$ is called wave number. $R_\infty = R_H$ = Rydberg constant

$= \dfrac{9.11 \times 10^{-31} \times \left(1.6 \times 10^{-19}\right)^4}{8 \times \left(8.85 \times 10^{-12}\right)^2 \left(6.625 \times 10^{-34}\right)^3 \times 3 \times 10^8} = 1.097 \times 10^7\ \text{m}^{-1} = 1.097 \times 10^{-3}$ Å^{-1} (for stationary nucleus).

If nucleus is not stationary (i.e., mass of nucleus is not much greater than the mass of the revolving particle like electron), then $R = \dfrac{R_\infty}{1 + m/M}$ where, m = mass of revolving particle and M = mass of nucleus

SPECTRAL SERIES OF HYDROGEN ATOM

It has been shown that the energy of the outer orbit is greater than the energy of the inner ones. When the Hydrogen atom is subjected to external energy, the electron jumps from lower energy state i.e. the hydrogen atom is excited. The excited state is unstable hence the electron return to its ground state in about 10^{-8} sec. The excess of energy is now radiated in the form of radiations of different wavelength. The different wavelength constitute spectral series. Which are characteristic of atom emitting, then the wave length of different members of series can be found from the following relations

$$\bar{v} = \frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

This relation explains the complete spectrum of hydrogen. A detailed account of the important radiations are listed below.

- **Lyman Series** : The series consist of wavelength which are emitted when electron jumps from an outer orbits to the first orbit i. e., the electronic jumps to K orbit give rise to lyman series. Here $n_1 = 1$ & $n_2 = 2, 3, 4, ...\infty$

  The wavelengths of different members of Lyman series are :
- **First member** : In this case $n_1 = 1$ and $n_2 = 2$

  hence $\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{3R}{4} \Rightarrow \lambda = \frac{4}{3R} \Rightarrow \lambda = \frac{4}{3 \times 10.97 \times 10^6} = 1216 \times 10^{-10}$ m = 1216Å
- **Second member :** In this case $n_1 = 1$ and $n_2 = 3$ hence

  $\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{3^2}\right] = \frac{8R}{9} \Rightarrow \lambda = \frac{9}{8R} \Rightarrow \lambda = \frac{9}{8 \times 10.97 \times 10^6} = 1026 \times 10^{-10}$ m =1026Å

  Similarly the wavelength of the other members can be calculated.
- **Limiting members** : In this case $n_1 = 1$ and $n_2 = \infty$, hence

  $\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right] = R \Rightarrow \lambda = \frac{1}{R} \Rightarrow \lambda = \frac{1}{10.97 \times 10^6} = 912 \times 10^{-10}$m = 912Å

  This series lies in ultraviolet region.
- **Balmer Series :** This series is consist of all wave lengths which are emitted when an electron jumps from an outer orbit to the second orbit i. e., the electron jumps to L orbit give rise to Balmer series. Here $n_1 = 2$ and $n_2 = 3, 4, 5 ....\infty$ The wavelength of different members of Balmer series.
- **First member** : In this case $n_1 = 2$ and $n_2 = 3$, hence

  $\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5R}{36} \Rightarrow \lambda = \frac{36}{5R} \Rightarrow \lambda = \frac{36}{5 \times 10.97 \times 10^6} = 6563 \times 10^{-10}$m = 6563Å
- **Second member** : In this case $n_1 = 2$ and $n_2 = 4$, hence.

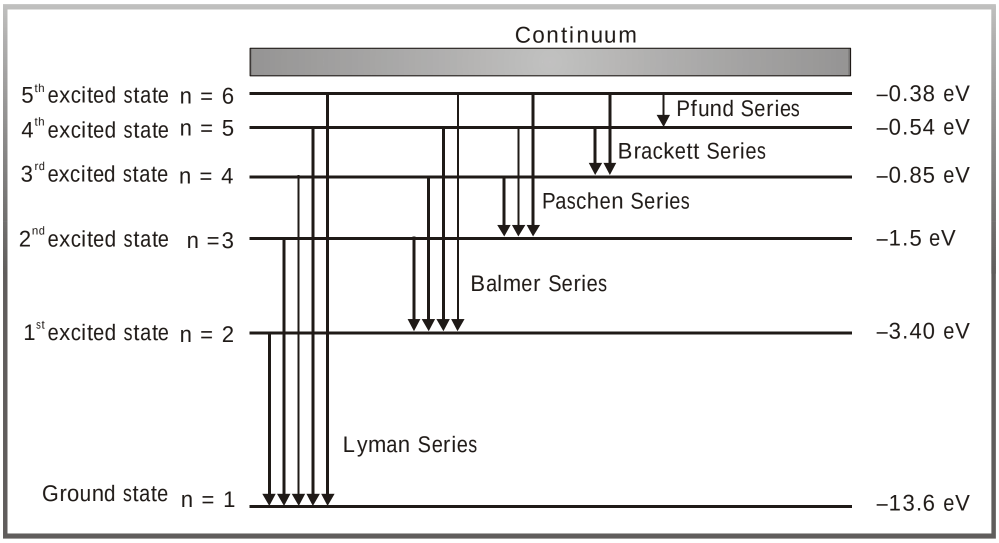
  $\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{4^2}\right] = \frac{3R}{16} \Rightarrow \lambda = \frac{16}{3R} \Rightarrow \lambda = \frac{16}{3 \times 10.97 \times 10^6} = 4861 \times 10^{-10}$m = 4861Å
- **Limiting members**: In this case $n_1 = 2$ and $n_2 = \infty$, hence $\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{\infty}\right] = \frac{R}{4} \Rightarrow \lambda = \frac{4}{R}$ = 3646Å

  This series lies in visible and near ultravoilet region.

- **Paschen Series :** This series consist of all wavelength are emitted when an electron jumps from an outer orbit to the third orbit i. e., the electron jumps to M orbit give rise to paschen series. Here $n_1=3$ & $n_2=4$, 5, 6 ....∞. The different wavelengths of this series can be obtained from the formula $\frac{1}{\lambda}=R\left[\frac{1}{3^2}-\frac{1}{n_2^2}\right]$
  where n_2 = 4, 5, 6 ....∞
  For the first member, the wavelengths is 18750Å. This series lies in infra–red region.
- **Brackett Series :** This series is consist of all wavelengths which are emitted when an electron jumps from an outer orbits to the fourth orbit i. e., the electron jumps to N orbit give rise to Brackett series. Here n_1 = 4 & n_2 = 5, 6, 7, ...∞.
  The different wavelengths of this series can be obtained from the formula $\frac{1}{\lambda}=R\left[\frac{1}{4^2}-\frac{1}{n_2^2}\right]$
  where n_2 = 5, 6, 7 ....∞
  This series lies in infra–red region of spectrum.
- **Pfund series :** The series consist of all wavelengths which are emitted when an electron jumps from an outer orbit to the fifth orbit i. e., the electron jumps to O orbit give right to Pfund series. Here n_1 = 5 and n_2 = 6, 7, 8 ....∞.
  The different wavelengths of this series can be obtained from the formula $\frac{1}{\lambda}=R\left[\frac{1}{5^2}-\frac{1}{n_2^2}\right]$
  where n_2 = 6, 7, 8 ....∞
  This series lies in infra–red region of spectrum.

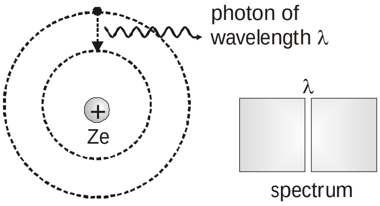
The result are tabulated below

S. No.	Series Observed	Value of n_1	Value of n_2	Position in the Spectrum
1.	Lyman Series	1	2,3,4...∞	Ultra Voilet
2.	Balmer Series	2	3,4,5...∞	Visible
3.	Paschen Series	3	4,5,6....∞	Infra–red
4.	Brackett Series	4	5,6,7....∞	Infra–red
5.	Pfund Series	5	6,7,8....∞	Infra–red

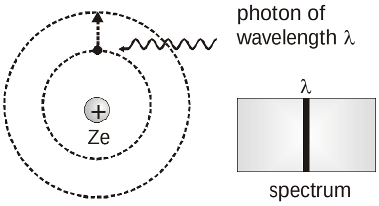


EXCITATION AND IONISATION OF ATOMS

Consider the case of simplest atom i. e., hydrogen atom, this has one electron in the innermost orbit i.e., (n = 1) and is said to be in the unexcited or normal state. If by some means, sufficient energy is supplied to the electron. It moves to higher energy states. When the atom is in a state of a high energy it is said to be excited. The process of raising or transferring the electron from lower energy state is called excitation. When by the process of excitation, the electron is completely removed from the atom. The atom is said to be ionized. Now the atom has left with a positive charge. Thus the process of raising the atom from the normal state to the ionized state is called ionisation. The process of excitation and ionisation both are absorption phenomena. The excited state is not stationary state and lasts in a very short interval of time (10^{-8} sec) because the electron under the attractive force of the nucleus jumps to the lower permitted orbit. This is accompanied by the emission of radiation according to BOHR'S frequency condition.



origin of emission spectra



origin of absorption spectra

The energy necessary to excite an atom can be supplied in a number of ways. The most commonly kinetic energy (Wholly or partly) of the electrons is transferred to the atom. The atom is now in a excited state. The minimum potential V required to accelerate the bombarding electrons to cause excitation from the ground state is called the resonance potential. The various values of potential to cause excitation of higher state called **excitation potential**. The potential necessary to accelerate the bombarding electrons to cause ionisation is called the **ionization potential**. The term critical potential is used to include the resonance, excitation and ionisation potential. We have seen that the energy required to excite the electron from first to second state is 13.6 – 3.4 = 10.2 eV. from first to third state is 13.6 – 1.5 = 12.1 eV., and so on. The energy required to ionise hydrogen atom is 0 – (– 13.6) = 13.6 eV. Hence ionization potential of hydrogen atom is 13.6 volt.

SUCCESSES AND LIMITATIONS

Bohr showed that Planck's quantum ideas were a necessary element of the atomic theory. He introduced the idea of quantized energy levels and explained the emission or absorption of radiations as being due to the transition of an electron from one level to another. As a model for even multielectron atoms, the Bohr picture is still useful. It leads to a good, simple, rational ordering of the electrons in larger atoms and quantitatively helps to predict a greater deal about chemical behavior and spectral detail.

Bohr's theory is unable to explain the following facts :

- The spectral lines of hydrogen atom are not single lines but each one is a collection of several closely spaced lines.
- The structure of multielectron atoms is not explained.
- No explanation for using the principles of quantization of angular momentum.
- No explanation for Zeeman effect. If a substance which gives a line emission spectrum is placed in a

magnetic field, the lines of the spectrum get splitted up into a number of closely spaced lines. This phenomenon is known as Zeeman effect.

Ex. A hydrogen like atom of atomic number Z is in an excited state of quantum number 2n. It can emit a maximum energy photon of 204 eV. If it makes a transition to quantum state n, a photon of energy 40.8 eV is emitted. Find n, Z and the ground state energy (in eV) for this atom. Also, calculate the minimum energy (in eV) that can be emitted by this atom during de–excitation. Ground state energy of hydrogen atom is –13.6 eV.

Sol. The energy released during de–excitation in hydrogen like atoms is given by :

$$E_{n_2}-E_{n_1}=\frac{me^4}{8\varepsilon_0^2h^2}\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]Z^2$$

Energy released in de–excitation will be maximum if transition takes place from nth energy level to ground state i.e.,

$$E_{2n}-E_1=13.6\left[\frac{1}{1^2}-\frac{1}{(2n)^2}\right]Z^2=204\text{ eV} \;\ldots\text{(i)} \;\&\; \text{also } E_{2n}-E_n=13.6\left[\frac{1}{n^2}-\frac{1}{(2n)^2}\right]Z^2=40.8\text{ eV}\;\ldots\text{(ii)}$$

Taking ratio of (i) to (ii), we will get $\frac{4n^2-1}{3}=5 \Rightarrow n^2=4 \Rightarrow n=2$

Putting n=2 in equation (i) we get

$$Z^2=\frac{204\times16}{13.6\times15} \Rightarrow Z=4 \because E_n=-13.6\frac{Z^2}{n^2} \Rightarrow E_1=-13.6\times\frac{4^2}{1^2}=-217.6\text{ eV}=\text{ground state energy}$$

ΔE is minimum if transition will be from 2n to 2n–1 i.e. between last two adjacent energy levels.

$$\therefore \Delta E_{\min}=E_{2n}-E_{2n-1}=13.6\left[\frac{1}{3^2}-\frac{1}{4^2}\right]4^2=10.57\text{ eV}$$

is the minimum amount of energy released during de–excitation.

Ex. A single electron orbits around a stationary nucleus of charge +Ze where Z is a constant and e is the magnitude of electronic charge. It requires 47.2 eV to excite the electron from the second orbit to third orbit. Find

(i) The value of Z

(ii) The energy required to excite the electron from the third to the fourth Bohr orbit

(iii) The wavelength of electronic radiation required to remove the electron from first Bohr orbit to infinity

(iv) Find the K.E., P.E. and angular momentum of electron in the 1st Bohr orbit.

[The ionization energy of hydrogen atom = 13.6 eV, Bohr radius = 5.3 × 10^{-11} m, Velocity of light = 3 × 10^8 m/s. Planck's constant = 6.6 × 10^{-34} J–s]

Sol. The energy required to excite the electron from n_1 to n_2 orbit revolving around the nucleus with charge +Ze is given by $E_{n_2}-E_{n_1}=\frac{me^4}{8\varepsilon_0^2h^2}\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]Z^2 \Rightarrow E_{n_2}-E_{n_1}=Z^2\times(13.6)\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$

(i) Since 47.2 eV energy is required to excite the electron from $n_1=2$ to $n_2=3$ orbit

$$47.2 = Z^2 \times 13.6\left[\frac{1}{2^2}-\frac{1}{3^2}\right] \Rightarrow Z^2 = \frac{47.2\times 36}{13.6\times 5} = 24.988 \approx 25 \Rightarrow Z=5$$

(ii) The energy required to excite the electron from $n_1=3$ to $n_2=4$ is given by

$$E_4 - E_3 = 13.6\ Z^2{=}5\left[\frac{1}{3^2}-\frac{1}{4^2}\right] = \frac{25\times 13.6\times 7}{144} = 16.53\text{ eV}$$

(iii)The energy required to remove the electron from the first Bohr orbit to infinity (∞) is given by

$$E_\infty - E_3 = 13.6\times Z^2\left[\frac{1}{1^2}-\frac{1}{\infty^2}\right] = 13.6\times 25\text{eV} = 340\text{eV}$$

In order to calculate the wavelength of radiation, we use Bohr's frequency relation

$$hf = \frac{hc}{\lambda} = 13.6\times 25\times(1.6\times 10^{-19})\text{J} \Rightarrow \lambda = \frac{\left(6.6\times10^{-34}\right)\times 10^{8}\times 3}{13.6\times 25\times\left(1.6\times 10^{-19}\right)} = 36.397\text{Å}$$

(iv)K.E. $= \frac{1}{2}mv_1^2 = \frac{1}{2}\times\frac{Ze^2}{4\pi g\,\epsilon_0\,r_1} = 543.4\times 10^{-19}$ J

P.E. $= -2\times$ K.E. $= -1086.8\times 10^{-19}$ J

Angular momentum $= mv_1r_1 = \frac{h}{2\pi} = 1.05\times 10^{-34}$ Js

The radius r_1 of the first Bohr orbit is given by

$$r_1 = \frac{\varepsilon_0 h^2}{\pi m e^2}\cdot\frac{1}{Z} = \frac{0.53\times 10^{-10}}{5}\left(\because \frac{\varepsilon_0 h^2}{\pi m e^2} = 0.53\times10^{-10}\,\text{m}\right) = 1.106\times 10^{-10}\text{ m} = 0.106\text{ Å}$$

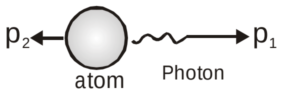
Ex. An isolated hydrogen atom emits a photon of 10.2 eV.

(i) Determine the momentum of photon emitted (ii) Calculate the recoil momentum of the atom

(iii) Find the kinetic energy of the recoil atom [Mass of proton= $m_p = 1.67\times 10^{-27}$ kg]

Sol. (i) Momentum of the photon is $p_1 = \frac{E}{c} = \frac{10.2\times 1.6\times 10^{-19}}{3\times 10^{8}} = 5.44\times 10^{-27}$ kg–m/s

(ii) Applying the momentum conservation



$p_2 = p_1 = 5.44\times 10^{-27}$ kg–m/s

(iii) $K = \frac{1}{2}mv^2$ (v = recoil speed of atom, m = mass of hydrogen atom) $K = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \frac{p^2}{2m}$

Substituting the value of the momentum of atom, we get $K = \frac{\left(5.44\times10^{-27}\right)^2}{2\times 1.67\times 10^{-27}} = 8.86\times 10^{-27}$J

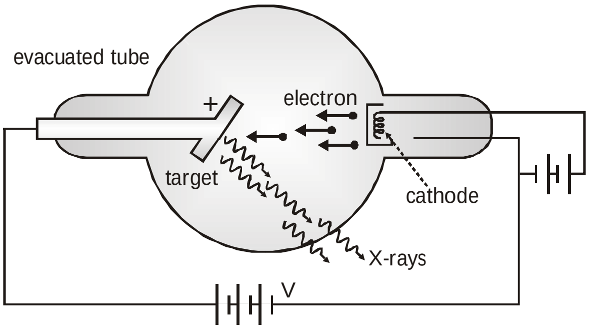
Physical quantity	**Formula**	**Ratio Formulae of hydrogen atom**

Radius of Bohr orbit (r_n)	$r_n = \frac{n^2h^2}{4\pi^2mkZe^2}$; $r_n = 0.53\frac{n^2}{Z}$ Å	$r_1 : r_2 : r_3 ... r_n = 1:4:9...n^2$
Velocity of electron in nth Bohr orbit (v_n)	$v_n = \frac{2\pi kZe^2}{nh}$; $v_n = 2.2 \times 10^6 \frac{Z}{n}$	$v_1 : v_2 : v_3 ... v_n = 1 : \frac{1}{2} : \frac{1}{3} ... \frac{1}{n}$
Momentum of electron (p_n)	$p_n = \frac{2\pi mke^2z}{nh}$; $p_n \propto \frac{Z}{n}$	$p_1 : p_2 : p_3 ... p_n = 1 : \frac{1}{2} : \frac{1}{3} ... \frac{1}{n}$
Angular velocity of electron(ω_n)	$\omega_n = \frac{8\pi^3k^2Z^2mc^4}{n^3h^3}$; $\omega_n \propto \frac{Z^2}{n^3}$	$\omega_1 : \omega_2 : \omega_3 ... \omega_n = 1 : \frac{1}{8} : \frac{1}{27} ... \frac{1}{n^3}$
Time Period of electron (T_n)	$T_n = \frac{n^3h^3}{4\pi^2k^2Z^2me^4}$; $T_n \propto \frac{n^3}{Z^2}$	$T_1 : T_2 : T_3 ... T_n = 1:8:27: ... :n^3$
Frequency (f_n)	$f_n = \frac{4\pi^2k^2Z^2e^4m}{n^3h^3}$; $f_n \propto \frac{Z^2}{n^3}$	$f_1 : f_2 : f_3 ... f_n = 1 : \frac{1}{8} : \frac{1}{27} ... \frac{1}{n^3}$
Orbital current (I_n)	$I_n = \frac{4\pi^2k^2Z^2me^5}{n^3h^3}$; $I_n \propto \frac{Z^2}{n^3}$	$I_1 : I_2 : I_3 ... I_n = 1 : \frac{1}{8} : \frac{1}{27} ... \frac{1}{n^3}$
Angular momentum (J_n)	$J_n = \frac{nh}{2\pi}$; $J_n \propto n$	$J_1 : J_2 : J_3 ... J_n = 1 : 2 : 3...n$
Centripetal acceleration (a_n)	$a_n = \frac{16\pi^4k^3Z^3me^6}{n^4h^4}$; $a_n \propto \frac{Z^3}{n^4}$	$a_1 : a_2 : a_3 ... a_n = 1 : \frac{1}{16} : \frac{1}{81} ... \frac{1}{n^4}$
Kinetic energy (E_{k_n})	$E_{K_n} = \frac{RchZ^2}{n^2}$; $E_{K_n} \propto \frac{Z^2}{n^2}$	$E_{K_1} : E_{K_2} ... E_{K_n} = 1 : \frac{1}{4} : \frac{1}{9} ... \frac{1}{n^2}$
Potential energy (U_n)	$U_n = \frac{-2RchZ^2}{n^2}$; $U_n \propto \frac{Z^2}{n^2}$	$U_1 : U_2 : U_3 ... U_n = 1 : \frac{1}{4} : \frac{1}{9} ... \frac{1}{n^2}$
Total energy (E_n)	$E_n = \frac{-RchZ^2}{n^2}$; $E_n \propto \frac{Z^2}{n^2}$	$E_1 : E_2 : E_3 ... E_n = 1 : \frac{1}{4} : \frac{1}{9} ... \frac{1}{n^2}$

X–RAYS

ROENTGEN EXPERIMENT

Roentgen discovered X–ray. While performing experiment on electric discharge tube Roentgen observed that when pressure inside the tube is 10^{-3}mm of Hg and applied potential is kept 25kV then some unknown radiation are emitted by anode. These are known as X–ray. X–rays are produced by bombarding high speed electrons on a target of high atomic weight and high melting point.



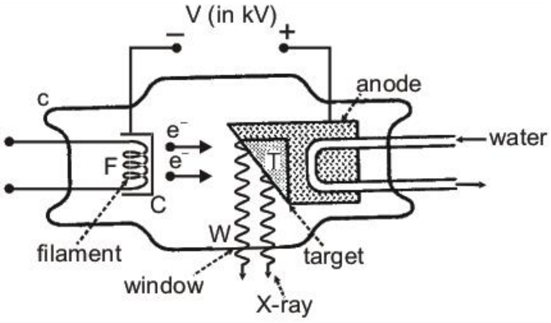
To Produce X–ray Three Things are Required

(i) Source of electron (ii) Means of accelerating these electron to high speed

(iii) Target on which these high speed electron strike

COOLIDGE METHOD

Coolidge developed thermoionic vacuum X–ray tube in which electron are produced by thermoionic emission method. Due to high potential difference electrons (emitted due to thermoionic method) move towards the target and strike from the atom of target due to which X–ray are produced. Experimentally it is observed that only 1% or 2% kinetic energy of electron beam is used to produce X–ray. Rest of energy is wasted in form of heat.



Characteristics of target

(a) Must have high atomic number to produce hard X–rays.

(b) High melting point to withstand high temperature produced.

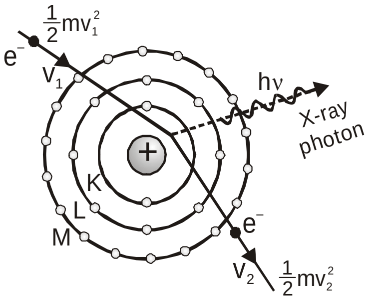
(c) High thermal conductivity to remove the heat produced

(d) Tantalum, platinum, molybdenum and tungsten serve as target materials

- **Control of intensity** : The intensity of X–ray depends on number of electrons striking the target and number of electron depend on temperature of filament which can be controlled by filament current. Thus intensity of X–ray depends on current flowing through filament.

- **Control of Penetrating Power**: The Penetrating power of X–ray depends on the energy of incident electron. The energy of electron can be controlled by applied potential difference. Thus penetrating power of X–ray depend on applied potential difference. Thus the intensity of X–ray depends on current flowing through filament while penetrating power depends on applied potential difference

	Soft X–ray	**Hard X–ray**
Wavelength	10 Å to 100 Å	0.1 Å – 10 Å
Energy	$\frac{12400}{\lambda}$ eV–Å	$\frac{12400}{\lambda}$ eV–Å
Penetrating power	Less	More
Use	Radio photography	Radio therapy



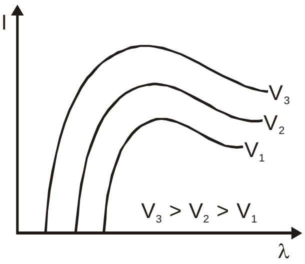
- **Continuous spectrum of X–ray** : When high speed electron collides from the atom of target and passes close to the nucleus.There is coulomb attractive force due to this electron is deaccelerated i.e. energy is decreased. The loss of energy during deacceleration is emitted in the form of X–rays. X–ray produced in this way are called Braking or Bremstralung radiation and form continuous spectrum. In continuous spectrum of X–ray all the wavelength of X–ray are present but below a minimum value of wavelength there is no X–ray. It is called cut off or threshold or minimum wavelength of X–ray. The minimum wavelength depends on applied potential.
- **Loss in Kinetic Energy**

$$\frac{1}{2}mv_1^2 - \frac{1}{2}mv_2^2 = h\nu + \text{heat energy} \quad \text{if } v_2 = 0 \text{ , } v_1 = v$$

(In first collision, heat = 0)

$$\frac{1}{2}mv^2 = h\nu_{max} \qquad \text{...(i)}$$

$$\frac{1}{2}mv^2 = eV \qquad \text{...(ii) [here V is applied potential]}$$

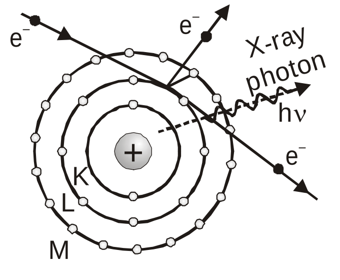


from (i) and (ii) $h\nu_{max} = eV \Rightarrow \frac{hc}{\lambda_{min}} = eV \Rightarrow \lambda_{min} = \frac{12400}{V} \times \text{ volt} = \frac{12400}{V} \times 10^{-10} \text{ m} \times \text{ volt}$

Continuous X–rays also known as white X–ray. Minimum wavelength of these spectrum only depends on applied potential and doesn' t depend on atomic number.

- **Characteristic Spectrum of X–ray**

When the target of X–ray tube is collide by energetic electron it emits two type of X–ray radiation. One of them has a continuous spectrum whose wavelength depend on applied potential while other consists of spectral lines whose wavelength depend on nature of target. The radiation forming the line spectrum is called characteristic X–rays. When highly accelerated electron strikes with the atom of target then it knockout the electron of orbit, due to this a vacancy is created. To fill this vacancy electron jumps from higher energy level and electromagnetic radiation are emitted which form characteristic spectrum of X–ray. Whose wavelength depends on nature of target and not on applied potential.



From Bohr Model

$n_1 = 1$, $n_2 = 2, 3, 4$.......K series

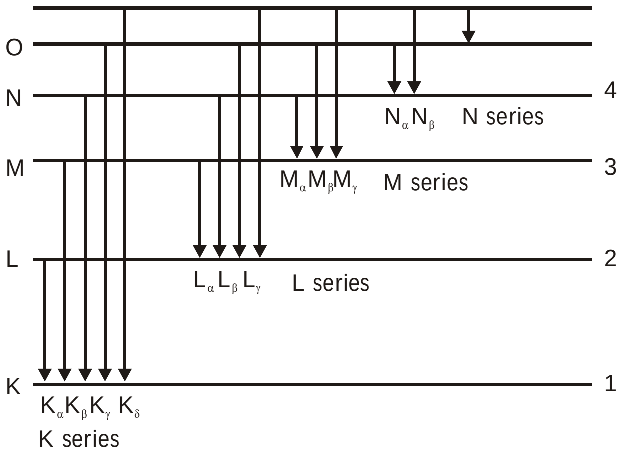
$n_1 = 2$, $n_2 = 3, 4, 5$.......L series

$n_1 = 3$, $n_2 = 4, 5, 6$.......M series

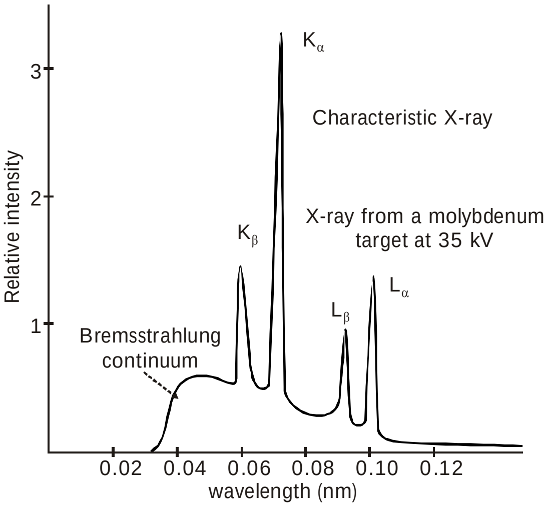
First line of series = α

Second line of series = β

Third line of series = γ



Transition	Wave–length	Energy	Energy difference	Wavelength
L → K	$\lambda_{K\alpha}$	$h\nu_{K\alpha}$	$-(E_K-E_L)$	$\lambda_{K\alpha}=\dfrac{hc}{(E_K-E_L)}$
(2 →1)			$=h\nu_{K\alpha}$	$=\dfrac{12400}{(E_K-E_L)}$ eVÅ
M → K	$\lambda_{K\beta}$	$h\nu_{K\beta}$	$-(E_K-E_M)$	$\lambda_{K\beta}=\dfrac{hc}{(E_K-E_M)}$
(3 →1)			$=h\nu_{K\beta}$	$=\dfrac{12400}{(E_K-E_M)}$ eVÅ
M → L	$\lambda_{L\alpha}$	$h\nu_{L\alpha}$	$-(E_L-E_M)$	$\lambda_{L\alpha}=\dfrac{hc}{(E_L-E_M)}$
(3 →2)			$=h\nu_{L\alpha}$	$=\dfrac{12400}{(E_L-E_M)}$ eVÅ



MOSELEY'S LAW

Moseley studied the characteristic spectrum of number of many elements and observed that the square root of the frequency of a K– line is closely proportional to atomic number of the element. This is called Moseley's law.

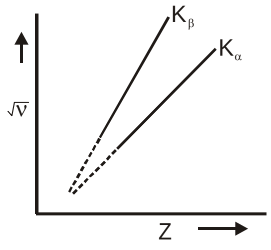
$$\sqrt{\nu} \propto (Z-b) \Rightarrow \nu \propto (Z-b)^2 \Rightarrow \nu = a\,(Z-b)^2 \quad ...(i)$$

Z = atomic number of target

ν = frequency of characteristic spectrum

b = screening constant (for K– series b=1, L series b=7.4)

a = proportionality constant



From Bohr Model $\nu = RcZ^2\left[\dfrac{1}{n_1^2}-\dfrac{1}{n_2^2}\right]$ ...(ii)

Comparing (i) and (ii) $a = Rc\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

- Thus proportionality constant 'a' does not depend on the nature of target but depend on transition.

Bohr model		**Moseley's correction**	
1.	For single electron species	**1.**	For many electron species
2.	$\Delta E = 13.6Z^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$ eV	**2.**	$\Delta E = 13.6\ (Z-1)^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$ eV
3.	$\nu = RcZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$	**3.**	$\nu = Rc(Z-1)^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$
4.	$\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$	**4.**	$\frac{1}{\lambda} = R\ (Z-1)^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

- For X–ray production, Moseley formulae are used because heavy metal are used.

When target is same $\lambda \propto \dfrac{1}{\frac{1}{n_1^2} - \frac{1}{n_2^2}}$ **When transition is same** $\lambda \propto \dfrac{1}{(Z-b)^2}$

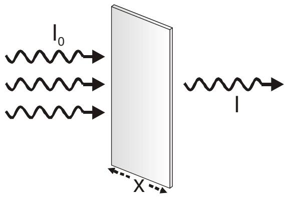
ABSORPTION OF X–RAY

When X–ray passes through x thickness then its intensity $I = I_0 e^{-\mu x}$

I_0 = Intensity of incident X–ray

I = Intensity of X–ray after passing through x distance

μ = absorption coefficient of material



- Intensity of X–ray decrease exponentially.
- Maximum absorption of X–ray → Lead
- Minimum absorption of X–ray → Air

Half thickness ($x_{1/2}$)

The distance travelled by X–ray when intensity become half the original value $x_{1/2} = \frac{\ell n_2}{\mu}$

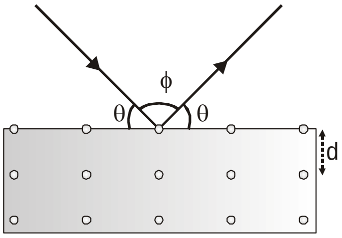
Ex. When X–rays of wavelength 0.5Å pass through 10 mm thick Al sheet then their intensity is reduced to one sixth. Find the absorption coefficient for Aluminium .

Sol. $\mu = \frac{2.303}{x} \log\left(\frac{I_o}{I}\right) = \frac{2.303}{10}\log_{10} 6 = \frac{2.303 \times 0.7781}{10} = 0.1752 / \text{mm}$

DIFFRACTION OF X–RAY

Diffraction of X–ray is possible by crystals because the interatomic spacing in a crystal lattice is order of wavelength of X–rays it was first verified by Lauve.

Diffraction of X–ray take place according to Bragg's law $2d \sin\theta = n\lambda$



d = spacing of crystal plane or lattice constant or distance between adjacent atomic plane

θ = Bragg's angle or glancing angle

ϕ = Diffracting angle n = 1, 2, 3 ....

For Maximum Wavelength

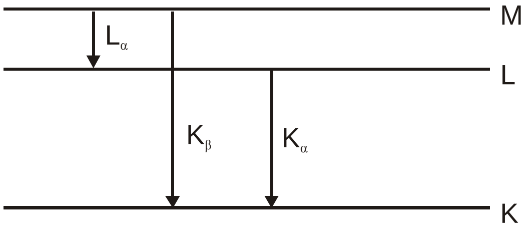
$\sin\theta = 1, n = 1 \Rightarrow \lambda_{max} = 2d$

so if $\lambda > 2d$ diffraction is not possible i.e. solution of Bragg's equation is not possible.

PROPERTIES OF X–RAY

- X–ray always travel with the velocity of light in straight line because X–rays are em waves
- X–ray is electromagnetic radiation it show particle and wave both nature
- In reflection, diffraction, interference, refraction X–ray shows wave nature while in photoelectric effect it shows particle nature.
- There is no charge on X–ray thus these are not deflected by electric field and magnetic field.
- X–ray are invisible.
- X–ray affects the photographic plate
- When X–ray incidents on the surface of substance it exerts force and pressure and transfer energy and momentum
- Characteristic X–ray can not obtained from hydrogen because the difference of energy level in hydrogen is very small.

Ex. Show that the frequency of K_β X–ray of a material is equal to the sum of frequencies of K_α and L_α X–rays of the same material.



Sol. The energy level diagram of an atom with one electron knocked out is shown above.

Energy of K_α X–ray is $E_{K\alpha} = E_L - E_K$

and of K_β X–ray is $E_{K\beta} = E_M - E_K$

and of L_α X–rays is $E_{L\alpha} = E_M - E_L$

thus $E_{K\beta} = E_{K\alpha} + E_{L\alpha}$ or $\nu_{K\beta} = \nu_{K\alpha} + \nu_{L\alpha}$

EXERCISE (S-1)

Photoelectric Effect :

1. In an experiment on photoelectric effect, the slope of the cut-off voltage versus frequency of incident light is found to be 4.12×10^{-15} V s. Calculate the value of Planck's constant. **NCERT**

MP0001

2. A 100W sodium lamp radiates energy uniformly in all directions. The lamp is located at the centre of a large sphere that absorbs all the sodium light which is incident on it. The wavelength of the sodium light is 589 nm. (a) What is the energy per photon associated with the sodium light ? (b) At what rate are the photons delivered to the sphere? **NCERT**

MP0002

3. In a photoelctric experiment set up, photons of energy 5 eV fall on the cathode having work function 3 eV.

(a) If the saturation current is $i_A = 4\mu A$ for intensity 10^{-5} W/m^2, then plot a graph between anode potential and current.

(b) Also draw a graph for intensity of incident radiation of 2×10^{-5} W/m^2 . **[JEE' 2003]**

MP0003

4. Monochromatic radiation of wavelength 640.2 nm (1nm = 10^{-9} m) from a neon lamp irradiates photosensitive material made of cesium. The stopping voltage is measured to be 0.54 V. The source is replaced by an iron source and its 427.2 nm line irradiates the same photo-cell. Predict the new stopping voltage. **NCERT**

MP0004

5. Monochromatic light of wavelength 632.8 nm is produced by a helium-neon laser. The power emitted is 9.42 mW. **NCERT**

(a) Find the energy and momentum of each photon in the light beam,

(b) How many photons per second, on the average, arrive at a target irradiated by this beam? (Assume the beam to have uniform cross-section which is less than the target area), and

(c) How fast does a hydrogen atom have to travel in order to have the same momentum as that of the photon ?

MP0005

Wave Nature of Matter

6. Calculate the **NCERT**

(a) momentum, and

(b) de-Broglie wavelength of the electrons accelerated through a potential difference of 56 V.

MP0006

7. The wavelength of light from the spectral emission line of sodium is 589 nm. Find the kinetic energy at which **NCERT**

(a) an electron, and

(b) a neutron, would have the same de-Broglie wavelength.

MP0007

8. (a) For what kinetic energy of a neutron will the associated de-Broglie wavelength be 1.40×10^{-10} m?

(b) Also find the de-Broglie wavelength of a neutron, in thermal equilibrium with matter, having an average kinetic energy of (3/2) k T at 300 K. **NCERT**

MP0008

9. The potential energy of a particle varies as $U(x) = E_0$ for $0 \leq x \leq 1$ and $U(x) = 0$ for $x > 1$

For $0 \leq x \leq 1$. de–Broglie wavelengths is λ_1 and for $x > 1$ the de–Broglie wavelength is λ_2.

Total energy of the particle is $2E_0$. Find $\frac{\lambda_1}{\lambda_2}$. **[JEE 2005]**

MP0009

Bohr's Theory

10. Determine the number of lines in Paschen series which have a wavelength greater than 1000 nm.

MP0010

11. In a photoelectric setup, the radiations from the Balmer series of hydrogen atom are incident on a metal surface of work function 2eV. The wavelength of incident radiations lies between 450 nm to 700 nm. Find the maximum kinetic energy of photoelectron emitted. (Given hc/e = 1242 eV-nm). **[JEE-2004]**

MP0011

12. An electron and a proton are separated by a large distance and the electron approaches the proton with a kinetic energy of 4.11 eV. If the electron is captured by the proton to form hydrogen atom in the ground state, the wavelength of photon given off is $\alpha \times 10^2$ Å? Fill the value of α in your OMR sheet. **MP0012**

13. A neutron moving through container filled with stationary deuterons. The neutron successively collides elastically and head on with stationary deuterons one at a time. The mass of the neutron is equal to half that of the deuteron. How many such collision would be required to slow the neutron down from 81 eV to 1 eV. [Neglect the relativistic effect] **MP0013**

14. What is the shortest wavelength present in the Paschen series of spectral lines? **NCERT**

MP0014

15. A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. What series of wavelengths will be emitted? **NCERT**

MP0015

16. The total energy of an electron in the first excited state of the hydrogen atom is about –3.4 eV.

(a) What is the kinetic energy of the electron in this state? **NCERT**

(b) What is the potential energy of the electron in this state?

(c) Which of the answers above would change if the choice of the zero of potential energy is changed?

MP0016

17. Obtain the first Bohr's radius and the ground state energy of a muonic hydrogen atom [i.e., an atom in which a negatively charged muon (μ^-) of mass about $207m_e$ orbits around a proton]. **NCERT**

MP0017

18. A potential difference of V volts is applied on two parallel electrodes separated by a distance of 4.0×10^{-2} m . The electrons of very low energy are injected into the region between the electrodes which contains argon at low pressure . The average distance the electrons travel between collisions with argon atoms is 8×10^{-5} m . The ionization energy of argon atom is 16 eV . Estimate the minimum value of V(in kV) such that the electrons will cause ionization in argon atoms by collision.

MP0018

19. In hydrogen-like atom (Z = 11), n^{th} line of Lyman series has wavelength λ equal to the de-Broglie's wavelength of electron in the level from which it originated. What is the value of n? [Take: Bohr radius (r_0) = 0.53 Å and Rydberg constant (R) = 1.1×10^7 m^{-1}] **[JEE 2006]**

MP0019

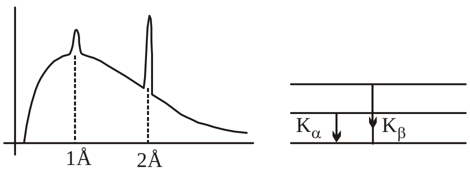
X-Rays

20. The wavelength of characteristic K_α-line emitted by a hydrogen like element is 2.5 Å. Find the wavelength of the K_γ-line emitted by the same element (in Å). [Assume the shielding effect to be same as of K_α] **MP0020**

21. In an accelerator experiment on high-energy collisions of electrons with positrons, a certain event is interpreted as annihilation of an electron-positron pair of total energy 10.2 BeV into two γ-rays of equal energy. What is the wavelength associated with each γ-ray? (1BeV = 10^9 eV) **NCERT**

MP0021

22. Figure shows K_α & K_β X-rays along with continuous X-ray. Find the energy of L_α X-ray. (Use hc = 12420 evÅ).



MP0022

23. A graph of $\sqrt{\nu}$ (where ν is the frequency of K_α line of the characteristic X-ray spectrum) is plotted against the atomic number Z of the elements emitting the characteristic X-ray .The intercept of the graph on the Z-axis is 1 and the slope of the graph is 0.5×10^8 S.I. units. The frequency of the K_α line for an element of atomic number 41 is given as $\alpha \times 10^{16}$ Hz. Find the value of α. **MP0023**

EXERCISE (S-2)

1. A cooling object was emitting radiations of time varying wavelength $\lambda = 3000 + 40t$, where λ is in Å and t is in second is incident on a metal sheet (of work function 2eV) such that the power incident on sheet is constant at 100 watt. This signal is switched on and off for time intervals of 2 minutes and 1 minute respectively. Each time the signal is switched on, λ again starts from fresh value of 3000 Å. If the metal plate is grounded so that it always remains neutral and electron clouding is negligible then find the maximum photocurrent(mA). The photoemission efficiency is 0.01% and remains constant.(Take hc=12400eV-Å) **MP0024**

2. A light of wavelength 3540 Å falls on a metal having work function of 2.5 eV. If ejected electron collides with another target metal inelastically and its total kinetic energy is utilized to raise the temperature of target metal. The mass of target metal is 10^{-3} kg and its specific heat is 160 J/kg/°C. If 10^{18} electrons are ejected per second, then find the rate of raise of temperature (in °C/s) of the metal [Assume there is no loss of energy of ejected electron by any other process, all the electron are reaching the target metal with max kinetic energy and take hc=12400 ev-Å] **MP0025**

3. A beam of light has three wavelengths 4144Å, 4972Å & 6216 Å with a total intensity of 3.6×10^{-3} W.m^{-2} equally distributed amongst the three wavelengths. The beam falls normally on an area 1.0 cm^2 of a clean metallic surface of work function 2.3 eV. Assume that there is no loss of light by reflection and that each energetically capable photon ejects one electron. Calculate the number of photoelectrons liberated in two seconds. **MP0026**

4. In a photo electric effect set-up, a point source of light of power 3.2×10^{-3} W emits mono energetic photons of energy 5.0 eV. The source is located at a distance of 0.8 m from the centre of a stationary metallic sphere of work function 3.0 eV & of radius 8.0×10^{-3}m . The efficiency of photo electrons emission is one for every 10^6 incident photons. Assume that the sphere is isolated and initially neutral, and that photo electrons are instantly swept away after emission.

(a) Calculate the number of photo electrons emitted per second.

(b) Find the ratio of the wavelength of incident light to the De - Broglie wave length of the fastest photo electrons emitted.

(c) It is observed that the photo electron emission stops at a certain time t after the light source is switched on. Why ?

(d) Evaluate the time t. **MP0027**

5. Two identical nonrelativistic particles move at right angles to each other, possessing De Broglie wavelengths, λ_1 & λ_2 . Find the De Broglie wavelength of each particle in the frame of their centre of mass. **MP0028**

6. Assume that the de-Broglie wave associated with an electron can form a standing wave between the atoms arranged in a one dimensional array with nodes at each of the atomic sites. It is found that one such standing wave is formed if the distance 'd' between the atoms of the array is 2 Å. A similar standing wave is again formed if 'd' is increased to 2.5 Å but not for any intermediate value of d. Find the energy of the electrons in electron volts and the least value of d for which the standing wave of the type described above can form. **MP0029**

7. A gas of identical hydrogen like atoms has some atoms in the lowest (ground) energy level A & some atoms in a particular upper (excited) energy level B & there are no atoms in any other energy level. The atoms of the gas make transition to a higher energy level by the absorbing monochromatic light of photon energy 2.55eV. Subsequently, the atoms emit radiation of only six different photon energies. Some of the emitted photons have energy 2.55 eV. Some have energy more and some have less than 2.55 eV.

(i) Find the principal quantum number of the initially excited level B.

(ii) Find the ionisation energy for the gas atoms.

(iii) Find the maximum and the minimum energies of the emitted photons.

MP0030

8. A monochromatic light source of frequency ν illuminates a metallic surface and ejects photoelectrons. The photoelectrons having maximum energy are just able to ionize the hydrogen atoms in ground state. When the whole experiment is repeated with an incident radiation of frequency (5/6)ν, the photoelectrons so emitted are able to excite the hydrogen atom beam which then emits a radiation of wavelength of 1215 Å. Find the work function of the metal and the frequency ν. **MP0031**

9. A neutron of kinetic energy 65 eV collides inelastically with a singly ionized helium atom at rest. It is scattered at an angle of 90° with respect of its original direction.

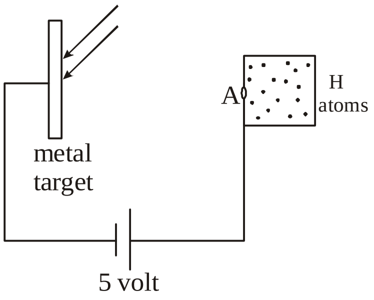
(Given : Mass of he atom = 4×(mass of neutron), ionization energy of H atom =13.6 eV)

(i) Find the allowed values of the energy of the neutron & that of the atom after collision.

(ii) If the atom gets de-excited subsequently by emitting radiation, find the frequencies of the emitted radiation. **MP0032**

10. A beam of ultraviolet light of wavelength 100 nm – 200 nm is passed through a box filled with hydrogen gas in ground state. The light coming out of the box is split into two beams 'A' and 'B'. A contains unabsorbed light from the incident light and B contains the emitted light by hydrogen atoms. The beam A is incident on the emitter in a photoelectric tube. The stopping potential in this case is 5 volts. Find the work function of the emitter. In the second case the beam B is incident on the same emitter. Find the stopping potential in this case. You can assume that the transition to higher energy states are not permitted from the excited states. Use hc = 12400 eVÅ. **MP0033**

11. Electromagnetic waves of wavelength 1242 Å are incident on a metal of work function 2eV. The target metal is connected to a 5 volt cell, as shown. The electrons pass through hole A into a gas of hydrogen atoms in their ground state. Find the number of spectral lines emitted when hydrogen atoms come back to their ground states after having been excited by the electrons. Assume all excitations in H-atoms from ground state only. (hc = 12420 eVÅ)



MP0034

12. A He^+ ion in ground state is fired towards a Hydrogen atom in ground state and at rest. What should be the minimum kinetic energy (in eV) of He^+ ion so that both single electron species may get excited. **MP0035**

13. Consider a universe in which the π-meson orbits around the nucleus instead of electron. Assuming a Bohr model for a π-meson of mass m_π and of the same charge as the electron is in a circular orbit of radius r about the nucleus with an orbital angular momentum $\frac{h}{2\pi}$. If the radius of a nucleus of atomic number Z is given by $R = 1.6 \times 10^{-15} Z^{1/3}$ m. The total number of elements in this universe that can exist bis given as 'N'. Fill $\left[\frac{N-1}{12}\right]$ in OMR sheet. [Given $\frac{\varepsilon_0 h^2}{\pi m_e e^2} = 0.53 Å$; $\frac{m_\pi}{m_e} = 265$; neglect any shielding effect for the havier atoms and assume non relativistic physics to be applicable and take $5^{1/4} \approx 1.5$] **MP0036**

14. The peak emission from a black body at a certain temperature occurs at a wavelength of 6000 Å. On increasing its temperature, the total radiation emitted is increased 16 times. These radiations are allowed to fall on a metal surface. Photoelectrons emitted by the peak radiation at higher temperature can be bought to rest by applying a potential equivalent to the excitation potential corresponding to the transition for the level n = 4 to n = 2 in the Bohr's hydrogen atom. The work function of the metal is given by $\frac{\alpha}{100}$ eV where α is the numerical constant. Find the value of α. [Take : hc = 12420 eV-Å]

MP0037

15. A neutron beam, in which each neutron has same kinetic energy, is passed through a sample of hydrogen like gas (but not hydrogen) in ground state. Due to collision of neutrons with the ions of the gas, ions are excited and then they emit photons. Six spectral lines are obtained in which one of the lines is of wavelength (6200/51) nm. What is the minimum possible value of kinetic energy of the neutrons for this to be possible. The mass of neutron and proton can be assumed to be nearly same. Find the answer in the form $25\alpha \times 10^{-2}$ eV and fill value of α. **MP0038**

16. Electrons in a hydrogen like atom (Z = 3) make transitions from the fourth excited state to the third excited state and from the third excited state to the second excited state. The resulting radiations are incident on a metal plate and eject photoelectrons. The stopping potential for photoelectrons ejected by shorter wavelength is 3.95 eV. Find the work function (in eV) of the metal plate. **MP0039**

17. A hydrogen atom at rest is in ground state. It is struck by a He^+ ion in first excited state. Assuming the collision to be head on and the mass of He^+ to be four times that of hydrogen atom, find the least value of kinetic energy of incoming particle which can excite both the particles to second excited state.

MP0040

18. In an X-ray tube the accelerating voltage is 20 KV. Two targets A and B are used one by one. For 'A' the wavelength of the K_α line is 62 pm. For 'B' the wavelength of the L_α line is 124 pm. The energy of the 'B' ion with vacancy in 'L' shell is 15.5 KeV higher than the atom of B.

[Take hc = 12400 eVÅ]

(i) Find λ_{min} in Å.

(ii) Can K_α – photon be emitted by 'A'? Explain with reason.

(iii) Can L – photons be emitted by 'B'? What is the minimum wavelength (in Å) of the characteristic X-ray that will be emitted by 'B'. **MP0041**

19. An X-rays tube is working at a potential difference of 38.08 kV. The potential difference is decreased to half its initial value. It is found that difference of the wavelength of K_α X-ray and the most energetic continuous X-rays becomes 1/4 times of the difference prior to the change of voltage. Assuming K_α line is present in both cases, find the atomic number of the target element. [Take Rch = 13.6 eV]

MP0042

EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

Photoelectric Effect

1. **Statement 1** : Photoelectric effect establishes quantum nature of light.

and

Statement 2 : There is negligible time lag between photon collisions with the material and photoelectron emission irrespective of intensity of incident light. (Assume incident light is of frequency greater than threshold frequency of the material).

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true. **MP0043**

2. **Statement-1 :** Work function of aluminum is 4.2 eV. If two photons each of energy 2.5 eV strikes on a piece of aluminum, the photo electric emission does not occur

Statement-2 : In photo electric effect a single photon interacts with a single electron and electron is emitted only if energy of each incident photon is greater then the work function.

(A) Statement-1 is True, Statement-2 is True, Statement-2 is a correct explanation for statement-1
(B) Statement-1 is True, Statement-2 is True, Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True **MP0044**

3. A photocell is illuminated by a small bright source placed 1 m away. When the same source of light is place $\frac{1}{2}$ m away, the number of electrons emitted by photocathode would- **[AIEEE - 2005]**

(A) decrease by a factor of 4 (B) increase by a factor of 4
(C) decrease by a factor of 2 (D) increase by a factor of 2

MP0045

4. Two monochromatic light sources, A and B, emit the same number of photons per second. The wavelength of A is $\lambda_A = 400$ nm, and that of B is $\lambda_B = 600$ nm. The power radiated by source B is

(A) equal to that of source A
(B) less than that of source A
(C) greater than that of source A
(D) cannot be compared to that from source A using the available data.

MP0046

5. The energy flux of sunlight reaching the surface of the earth is 1.388×10^3 W/m^2. How many photons (nearly) per square metre are incident on the Earth per second? Assume that the photons in the sunlight have an average wavelength of 550 nm. **NCERT**

(A) 8×10^{21} (B) 4×10^{21} (C) 4×10^{38} (D) 8×10^{38}

MP0047

node06\B0B0-BA\Kota\JEE(Advanced)\Enthuse\Physics\Sheet\Modern Physics-1\Eng\01_Exercise.p65

6. The threshold frequency for a certain metal is 3.3×10^{14} Hz. If light of frequency 8.2×10^{14} Hz is incident on the metal, the cutoff voltage for the photoelectric emission is. **NCERT**

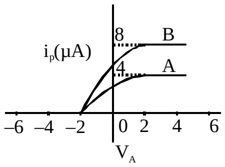
(A) 8 V (B) 6 V (C) 2 V (D) 4 **MP0048**

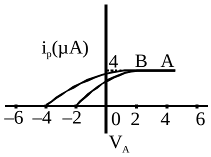
7. Light of frequency 7.21×10^{14} Hz is incident on a metal surface. Electrons with a maximum speed of 6.0×10^{5} m/s are ejected from the surface. The threshold frequency for photoemission of electrons is **NCERT**

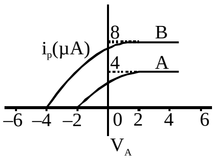
(A) 4.73×10^{14} Hz (B) 4.73×10^{10} Hz (C) 2.08×10^{10} Hz (D) None of these

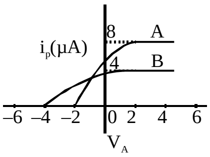
MP0049

8. In a photoelectric effect experiment, photons of energy 5 eV are incident on the photo-cathode of work function 3 eV. For photon intensity $I_A = 10^{15}$ m^{-2}s^{-1}, saturation current of 4.0 μA is obtained. Sketch of the variation of photocurrent i_p against the anode voltage V_a for photon intensity I_A (curve A) and $I_B = 2 \times 10^{15}$ m^{-2} s^{-1} (curve B) will be :

(A)


(B)


(C)


(D)


MP0050

9. **Statement-1 :** When ultraviolet light is incident on a photocell, its stopping potential is V_0 and the maximum kinetic energy of the photoelectrons is K_{max}. When the ultraviolet light is replaced by infrared light, both V_0 and K_{max} increase.

Statement-2 : Photoelectrons are emitted with speeds ranging from zero to a maximum value.

(A) Statement–1 is true, Statement–2 is false

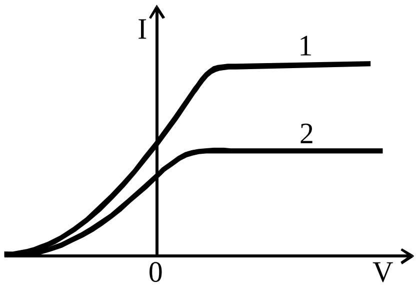
(B) Statement–1 is true, Statement–2 is true; Statement–2 is the correct explanation of Statement–1

(C) Statement–1 is true, Statement–2 is true; Statement–2 is not the correct explanation of Statement–1

(D) Statement–1 is false, Statement–2 is true

MP0051

10. A photocathode can be illuminated by the light from two sources, each of which emits monochromatic radiation. The sources are positioned at equal distances from the photocathode. The dependence of the photocurrent on the voltage between the cathode and the anode is depicted by curve 1 for one source and by curve 2 for the other. In what respect do these **sources** differ ?



(A) Highest frequency photon (B) Number of photons emmited per second

(C) Number of photoelectrons emmited per second

(D) None of these

MP0052

11. A radiation of energy E falls normally on a perfectly reflecting surface. The momentum transferred to the surface is- **[AIEEE - 2004]**

(A) E/c (B) 2E/c (C) Ec (D) E/c^2

MP0053

12. The threshold frequency for a metallic surface corresponds to an energy of 6.2 eV and the stopping potential for a radiation incident on this surface is 5V. The incident radiation lies in- **[AIEEE - 2006]**

(A) ultra-violet region (B) infra-red region (C) visible region (D) X-ray region

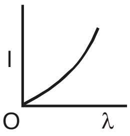
MP0054

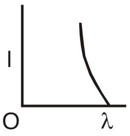
13. The time taken by a photoelectron to come out after the photon strikes is approximately- **[AIEEE - 2006]**

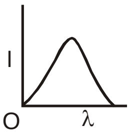
(A) 10^{-4} s (B) 10^{-10} s (C) 10^{-16} (D) 10^{-1} s

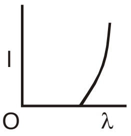
MP0055

14. The anode voltage of a photocell is kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current I of the photocell varies as follows : **[AIEEE - 2006]**

(A)


(B)


(C)


(D)


MP0056

15. Photon of frequency ν has a momentum associated with it. If c is the velocity of light, the momentum is- **[AIEEE - 2007]**

(A) ν/c (B) hνc (C) $h\nu/c^2$ (D) hν/c

MP0057

Wave Nature oF Matter

16. **Statement-1 :** If an electron has the same wavelength as a photon, they have the same energy.

Statement-2 : by debroglie hypothesis, p = h/λ for both the electron and the photon.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

MP0058

17. A particle of mass 4m at rest decays into two particles of masses m and 3m having non-zero velocities. The ratio of the de-Broglie wavelengths of the particles 1 and 2 is

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) 2 (D) 1

MP0059

18. A free particle with initial kinetic energy E and de–broglie wavelength λ enters a region in which it has potential energy U. What is the particle's new de–Broglie wavelength?

(A) $\lambda(1-U/E)^{-1/2}$ (B) $\lambda(1-U/E)$ (C) $\lambda(1-E/U)^{-1}$ (D) $\lambda(1+U/E)^{1/2}$

MP0060

19. Proton, deutron and α particles are accelerated through the same potential difference. Then the ratio of their de-Broglie wavelength as

(A) $1:\sqrt{2}:1$ (B) 1 : 1 : 1 (C) $1:2:2\sqrt{2}$ (D) $2\sqrt{2}:2:1$

MP0061

20. **Statement-1 :** An electron and a proton are accelerated through the same potential difference. The de-Broglie wavelength associated with the electron is longer.

Statement-2 : De-Broglie wavelength associated with a moving particle is $\lambda = \frac{h}{p}$ where, p is the linear momentum and both have same KE.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B)Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

MP0062

21. An α-particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. The distance of the closest approach is of the order of- **[AIEEE-2004]**

(A) 1 Å (B) 10^{-10} cm (C) 10^{-12} cm (D) 10^{-15} cm

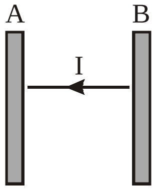
MP0063

22. After absorbing a slowly moving neutron of mass m_N (momentum ~0) a nucleus of mass M breaks into two nuclei of masses m_1 and $3m_1(4m_1 = M + m_N)$, respectively. If the de Broglie wavelength of the nucleus with mass m_1 is λ, then de-Broglie wavelength of the other nucleus will be:-

(A) 9λ (B) 3λ (C) $\frac{\lambda}{3}$ (D) λ

MP0064

23. A parallel beam of light of intensity I is incident normally on a plane surface A which absorbs 50% of the incident light. The reflected light falls on B which is perfect reflector, the light reflected by B is again partly reflected and partly absorbed and this process continues. For all absorption by A, asborption coefficient is 0.5. The pressure experienced by A due to light is :-



(A) $\frac{1.5\,I}{c}$ (B) $\frac{I}{c}$ (C) $\frac{3I}{2c}$ (D) $\frac{3I}{c}$

MP0065

24. **Statement-1 :** If the accelerating potential in an X-ray tube is increased, the wavelengths of the characteristic X-rays do not change. **[JEE 2007]**

because

Statement-2 : When an electron beam strikes the target in an X-ray tube, part of the kinetic energy is converted into X-ray energy

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

MP0066

25. Electrons with de-Broglie wavelength λ fall on the target in an X-ray tube. The cut-off wavelength of the emitted X-rays is **[JEE 2007]**

(A) $\lambda_0 = \frac{2mc\lambda^2}{h}$ (B) $\lambda_0 = \frac{2h}{mc}$ (C) $\lambda_0 = \frac{2m^2c^2\lambda^3}{h^2}$ (D) $\lambda_0 = \lambda$

MP0067

Bohr's Theory

26. The de-Broglie wavelength of an electron in the first Bohr orbit is

(A) equal to the circumference of first orbit (B) equal to $\frac{1}{2}$ × (circumference of first orbit)

(C) equal to $\frac{1}{4}$ × (circumference of first orbit) (D) equal to $\frac{3}{4}$ × (circumference of first orbit)

MP0068

27. **Statement–1** : When light is passed through a sample of hydrogen atoms in ground state, then wavelengths of absorption lines are same as wavelengths of lines of Lyman series in emission spectrum.

and

Statement–2 : In ground state hydrogen atom will absorb only those radiation which will excite to higher energy level.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

MP0069

28. **Statement-1 :** In a laboratory experiment, on emission from atomic hydrogen in a discharge tube, only a small number of lines are observed whereas a large number of lines are present in the hydrogen spectrum of a star.

Statement-2 : The temperature of discharge tube is much smaller than that of the star.

(A) Statement-1 is True, Statement-2 is True, Statement-2 is a correct explanation for statement-1

(B) Statement-1 is True, Statement-2 is True, Statement-2 is NOT a correct explanation for

Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

MP0070

29. If the binding energy of the electron in a hydrogen atom is 13.6 eV, the energy required to remove the electron from the first excited state of Li^{2+} is- **[AIEEE - 2003]**

(A) 30.6 eV (B) 13.6 eV (C) 3.4 eV (D) 122.4 eV

MP0071

30. A neutron having kinetic energy 13.6 eV collides with a hydrogen atom in ground state at rest. Assume that the mass of neutron and hydrogen atoms are same and the neutron does not leave its line of motion. Then which of the following is the possible KE of the neutron after the collision?

(A) zero (B) 3.4 eV (C) 1.5 eV (D) 6.8 eV.

MP0072

31. In the spectrum of single ionised helium, the wavelength of a line observed is almost the same as the first line of Balmer series of hydrogen. It is due to transition of electron from

(A) $n_1 = 6$ to $n_2 = 4$ (B) $n_1 = 5$ to $n_2 = 3$ (C) $n_1 = 4$ to $n_2 = 2$ (D) $n_1 = 3$ to $n_2 = 2$

MP0073

32. Light coming from a discharge tube filled with hydrogen falls on the cathode of the photoelectric cell. The work function of the surface of cathode is 4eV. Which one of the following values of the anode voltage (in Volts) with respect to the cathode will likely to make the photo current zero.

(A) – 4 (B) – 6 (C) – 8 (D) – 10

MP0074

33. According to Bohr model, magnetic field at the centre (at the nucleus) of a hydrogen atom due to the motion of the electron in nth orbit is proportional to

(A) $1/n^3$ (B) $1/n^5$ (C) n^5 (D) n^3

MP0075

34. A photon of 10.2 eV energy collides with a hydrogen atom in ground state inelastically. After few microseconds one more photon of energy 15 eV collides with the same hydrogen atom.Then what can be detected by a suitable detector. **[JEE' 2005 (Scr)]**

(A) one photon of 10.2 eV and an electron of energy 1.4 eV

(B) 2 photons of energy 10.2 eV

(C) 2 photons of energy 3.4 eV

(D) 1 photon of 3.4 eV and one electron of 1.4 eV

MP0076

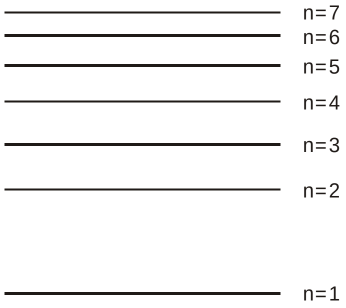
35. Two hydrogen atoms are in excited state with electrons residing in n = 2. First one is moving towards left and emits a photon of energy E_1 towards right. Second one is moving towards left with same speed and emits a photon of energy E_2 towards left. Taking recoil of nucleus into account during emission process

(A) $E_1 > E_2$ (B) $E_1 < E_2$

(C) $E_1 = E_2$ (D) information insufficient

MP0077

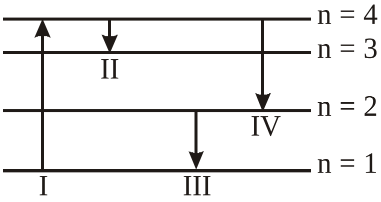
36. The maximum number of emission lines for atomic hydrogen that you would expect to see with naked eye if the only electronic levels involved are those shown in the figure, is



(A) 6 (B) 5 (C) 21 (D) ∞

MP0078

37. The diagram shows the energy levels for an electron in a certain atom. Which transition shown represents the emission of a photon with maximum wavelength?



(A) III (B) IV (C) I (D) II

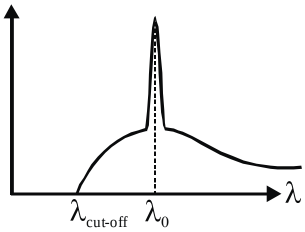
MP0079

38. The largest wavelength in the ultraviolet region of the hydrogen spectrum is 122 nm. The smallest wavelength in the infrared region of the hydrogen spectrum (to the nearest integer) is
[JEE 2007]
(A) 802 nm (B) 823 nm (C) 1882 nm (D) 1648 nm

MP0080

X-Rays

39. **Assertion (A):** Variation of intensity of X-rays is plotted against λ. On increasing the accelerating potential, $(\lambda_0 - \lambda_{\text{cut-off}})$ increases.



Reasoning (R): $\lambda_{\text{cut-off}}$ will decrease but λ_0 will be same, as wavelength of characteristic X-rays is independent of the accelerating potential.

Choose the correct statement from the following.

(A) A is correct and R is the correct explanation of A.

(B) Both A and R are correct but R is not the correct explanation of A.

(C) A is correct but R is wrong.

(D) Both A and R are wrong. **MP0081**

40. An X-ray tube is run at 50 kV. The current flowing in it is 20mA . The power of the tube is :

(A) 1000 W (B) 200 W (C) 20000 W (D) 20 W

MP0082

41. Choose the **INCORRECT** statement.

(A) Cut-off wavelength of X-rays is independent of filament voltage.

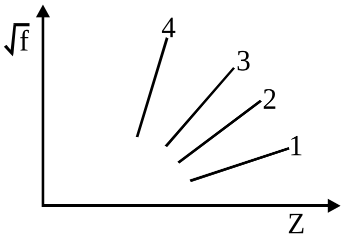
(B) Presence of K_α-line in X-ray spectrum means that L-series will also be present.

(C) Increase in filament current increases intensity of X-ray.

(D) Presence of L-series in X-ray spectrum means that K-series will also be present.

MP0083

42. The given graph shows the variation of $\sqrt{f}$ vs Z for characteristics X-rays. Lines 1, 2, 3, 4 shown in the graph corresponds to any one of k_α, k_β, L_α, L_β. Then L_β is represent by :-



(A) line 1 (B) line 2 (C) line 3 (D) line 4

MP0084

43. The X-ray beam coming from an X-ray tube will be

(A) monochromatic

(B) having all wavelengths smaller than a certain maximum wavelength

(C) having all wavelengths larger than a certain minimum wavelength

(D) having all wavelengths lying between a minimum and a maximum wavelength

MP0085

44. E_1 is energy of k_α photon of aluminium, E_2 is energy of k_β photon of aluminium and E_3 is energy of k_α photon from sodium, then the correct order of energies is given

(A) $E_1 > E_2 > E_3$ (B) $E_3 > E_2 > E_1$ (C) $E_3 > E_1 > E_2$ (D) $E_2 > E_1 > E_3$

MP0086

45. The K,L and M energy levels of platinum lie roughly at 78, 12 and 3 keV respectively. The ratio of wavelength of K_α line to that of K_β line in X-ray spectrum is-

(A) $\frac{22}{3}$ (B) $\frac{3}{22}$ (C) $\frac{22}{25}$ (D) $\frac{25}{22}$

MP0087

46. What is the essential distinction between X–rays and γ–rays

(A) γ–rays have shorter wavelength than X–rays

(B) γ–rays are extraterrestrial, X–rays are man–made

(C) γ–rays have less penetrating power than X–rays

(D) γ–rays originate from within an atomic nucleus, X–rays from outside an atomic nucleus.

MP0088

47. Mosley law relates :

(A) Frequency of emitted X-ray with applied voltage

(B) Wavelength and intensity of X-ray.

(C) Frequency of emitted X-ray with atomic number

(D) Wavelength and angle of scattering.

MP0089

48. The intensity of gamma radiation from a given source is I. On passing through 36 mm of lead, it is reduced to I/8. The thickness of lead, which will reduce the intensity to I/2 will be- **[AIEEE-2005]**

(A) 6 mm (B) 9 mm (C) 18 mm (D) 12 mm

MP0090

49. The wavelength of K_α X-ray of an element having atomic number Z = 11 is λ . The wavelength of K_α X-ray of another element of atomic number Z' is 4λ. Then Z' is **[JEE' 2005 (Scr)]**

(A) 11 (B) 44 (C) 6 (D) 4

MP0091

50. Characteristic X-ray

(A) Have only discrete wavelength which are characteristic of the target.

(B) Have all the possible wavelength.

(C) Are characteristic of speed of projectile electrons.

(D) None

MP0092

51. Which of the following transitions in hydrogen atoms emit photons of highest frequency ?

[AIEEE - 2007]

(A) n = 2 to n = 6 (B) n = 6 to n = 2 (C) n = 2 to n = 1 (D) n = 1 to n = 2

MP0093

MULTIPLE CORRECT TYPE QUESTIONS

52. Photoelectric effect supports quantum nature of light because

(A) there is minimum frequency of light below which no photoelectrons are emitted

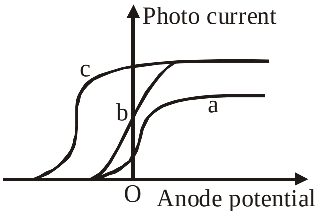
(B) the maximum kinetic energy of photo-electrons depends only on the frequency of light and not on its intensity

(C) even when the metal surface is faintly illuminated, the photoelectrons leave the surface immediately

(D) electric charge of photo-electrons is quantized

MP0094

53. The figure shows the variation of photo current with anode potential for a photosensitive surface for three different radiations. Let I_a, I_b and I_c be the intensities and f_a, f_b and f_c be the frequencies for the curves a, b and c respectively. Choose correct options



(A) $f_a = f_b$ (B) $I_a < I_b$ (C) $f_c < f_b$ (D) $I_c > I_b$

MP0095

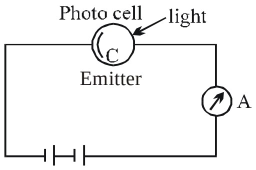
54. In photoelectric effect, stopping potential depends on
(A) frequency of the incident light
(B) intensity of the incident light by varies source distance
(C) emitter's properties
(D) frequency and intensity of the incident light

MP0096

55. Which of the following phenomena can be explained only on the basis of quantum theory of light?
(A) Energy spectrum of black body radiation (B) Atomic spectra
(C) Photoelectric effect (D) Doppler effect

MP0097

56. The figure shows a photo cell circuit. The cathode of the photo cell is illuminated by a monochromatic light. If the intensity is kept constant and the frequency of the incident light is increased, then the



(A) photo electric current in the circuit increases
(B) photo electric current in the circuit decreases
(C) maximum kinetic energy of the photo electrons increases
(D) photo electric current in the circuit can be reduced to zero, when the polarity of the terminals is reversed

MP0098

57. In a photoelectric effect, electrons are emitted
(A) at a rate that is proportional to the square of the amplitude of the incident radiation.
(B) with a maximum velocity proportional to the frequency of the incident radiation.
(C) at a rate that is independent of the emitter.
(D) only if the frequency of the incident radiation is above a certain threshold value
(E) only if the temperature of the emitter is high

MP0099

58. A small plate of area 1 cm^2 is placed at a distance of $\frac{1}{\sqrt{\pi}}$ m from an isotropic point source emitting light of frequency $\frac{1}{6.63}\times10^{14}$ Hz, at a power of 2.00 mW. Assume the plate to be normal to the incident photons. [where h = 6.63 × 10^{-34} J-s]. Select **CORRECT** alternative(s)
(A) Energy possessed by each photon is 10^{-20} J
(B) Photon emission rate is 2×10^{17} s^{-1}
(C) The fraction of area of beam intercepted by the plate is $\frac{1}{4}\times10^{-4}$
(D) The rate of photons striking the plate is 5×10^{12} per second.

MP0100

59. A metallic sphere of radius r remote from all other bodies is irradiated with a radiation of wavelength λ which is capable of causing photoelectric effect.

(A) the maximum potential gained by the sphere will be independent of its radius

(B) the net positive charge appearing on the sphere after a long time will depend on the radius of the sphere

(C) the maximum kinetic energy of the electrons emanating from the sphere will keep on declining with time

(D) the kinetic energy of the most energetic electrons emanating from the sphere initially will be independent of the radius of the sphere.

MP0101

60. Two electrons are moving with the same speed v. One electron enters a region of uniform electric field while the other enters a region of uniform magnetic field, then after sometime if the de–Broglie wavelengths of the two are λ_1 and λ_2, then select the possible option(s) :-

(A) $\lambda_1 = \lambda_2$ (B) $\lambda_1 > \lambda_2$

(C) $\lambda_1 < \lambda_2$ (D) $\lambda_1 > \lambda_2$ or $\lambda_1 < \lambda_2$

MP0102

61. A particle moves in a closed orbit around the origin, due to a central force which is directed towards the origin. The de Broglie wavelength of the particle varies cyclically between two values λ_1, λ_2 with $\lambda_1 > \lambda_2$. Which of the following statements is/are true ?

(A) The particle could be moving in a circular orbit with origin as centre

(B) The particle could be moving in an elliptic orbit with origin as its focus.

(C) When the de Broglie wave length is λ_1, the particle is nearer the origin than when its value is λ_2.

(D) When the de Broglie wavelength is λ_2, the particle is nearer the origin than when its value is λ_1.

MP0103

62. According to Bohr's theory of hydrogen atom, for the electron in the nth permissible orbit,

(A) linear momentum $\propto \frac{1}{n}$

(B) radius of orbit $\propto$ n

(C) kinetic energy $\propto \frac{1}{n^2}$

(D) angular momentum $\propto$ n

MP0104

63. The magnitude of angular momentum, orbit radius and frequency of an electron in hydrogen atom corresponding to the quantum number n are L, r and f respectively, then according to Bohr's theory of hydrogen atom.

(A) frL is constant for all orbits (B) Lf $\propto \frac{1}{n^2}$

(C) fr $\propto \frac{1}{n}$ (D) Lr $\propto \frac{1}{n^3}$

MP0105

64. A particular hydrogen like atom has its ground state binding “energy 122.4eV. Its is in ground state. Then:

(A) Its atomic number is 3

(B) An electron of 90eV can excite it.

(C) An electron of kinetic energy nearly 91.8eV can be brought to almost rest by this atom.

(D) An electron of kinetic energy 2.6eV may emerge from the atom when electron of kinetic energy 125eV collides with this atom.

MP0106

65. A beam ofultraviolet light of all wavelengths passes through hydrogen gas at room temperature, in the x-direction. Assume that all photons emitted due to electron transition inside the gas emerge in the y-direction. Let A and B denote the lights emerging from the gas in the x and y directions respectively.

(A) Some of the incident wavelengths will be absent in A.

(B) Only those wavelengths will be present in B which are absent in A.

(C) B will contain some visible light.

(D) B will contain some infrared light.

MP0107

66. In the hydrogen atom, if the reference level of potential energy is assumed to be zero at the ground state level. Choose the incorrect statement.

(A) The total energy of the shell increases with increase in the value of n

(B) The total energy of the shell decrease with increase in the value of n.

(C) The difference in total energy of any two shells remains the same.

(D) The total energy at the ground state becomes 13.6 eV.

MP0108

67. A neutron collides head-on with a stationary hydrogen atom in ground state. Which of the following statements are correct (Assume that the hydrogen atom and neutron has same mass) :

(A) If kinetic energy of the neutron is less than 20.4 eV collision must be elastic.

(B) If kinetic energy of the neutron is less than 20.4 eV collision may be inelastic.

(C) Inelastic collision may be take place only when initial kinetic energy of neutron is greater than 20.4 eV.

(D) Perfectly inelastic collision can not take place.

MP0109

68. A free hydrogen atom in ground state is at rest. A neutron of kinetic energy ‘K’ collides with the hydrogen atom. After collision hydrogen atom emits two photons in succession one of which has energy 2.55 eV. (Assume that the hydrogen atom and neutron has same mass)

(A) minimum value of ‘K’ is 25.5 eV.

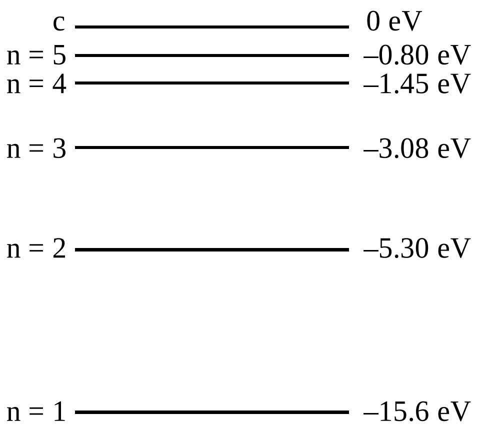
(B) minimum value of ‘K’ is 12.75 eV

(C) the other photon has energy 10.2 eV if K is minium.

(D) the upper energy level is of excitation energy 12.75 eV.

MP0110

69. The energy levels of a hypothetical one electron atom are shown in the figure



(A) The ionization potential of this atom is 15.6 V

(B) The short wavelength limit of the series terminating at n = 2 is 2339 Å

(C) The excitation potential for the state n = 3 is 12.52 V

(D) Wave number of the photon emitted for the transition n = 3 to n = 1 is $1.009 \times 10^7\ \text{m}^{-1}$

MP0111

70. Suppose frequency of emitted photon is f_0 when electron of a stationary hydrogen atom jumps from a higher state m to a lower state n. If the atom is moving with a velocity v (<< c) and emits a photon of frequency f during the same transition, then which of the following statements are possible :-

(A) f may be equal to f_0

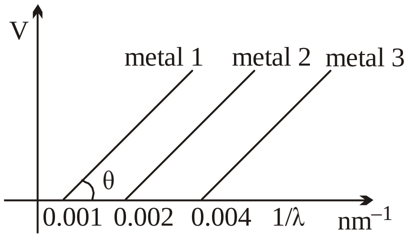
(B) f may be greater than f_0

(C) f may be less than f_0

(D) f cannot be equal to f_0

MP0112

71. The graph between 1/λ and stopping potential (V) of three metals having work functions ϕ_1, ϕ_2 and ϕ_3 in an experiment of photo-electric effect is plotted as shown in the figure. Which of the following statement(s) is/are correct? [Here λ is the wavelength of the incident ray]. **[JEE 2006]**



(A) Ratio of work functions $\phi_1 : \phi_2 : \phi_3 = 1 : 2 : 4$

(B) Ratio of work functions $\phi_1 : \phi_2 : \phi_3 = 4 : 2 : 1$

(C) tan θ is directly proportional to hc/e, where h is Planck's constant and c is the speed of light

(D) The violet colour light can eject photoelectrons from metals 2 and 3.

MP0113

MATRIX MATCH TYPE QUESTION

72. Some quantities related to the photoelectric effect are mentioned under Column I and Column II. Match each quantity in Column I with the corresponding quantities in Column II on which it depends

	Column-I		**Column-II**
(A)	Saturation current	(P)	Frequency of light
(B)	Stopping potential	(Q)	Work function
(C)	de-Broglie wavelength of photoelectron	(R)	Area of photosensitive plate
(D)	Force due to radiation falling on the photo–plate.	(S)	Intensity of light (at constant frequency)
		(T)	None of these

MP0114

73. When we write expression for energy of electron in nth orbit of helium ion (He^+) we take zero potential energy for $n = \infty$, but the potential energy depends on reference. If we take total energy of atom for $n = 1$ orbit as zero then

	Column–I		**Column–II**
(A)	Total energy of electron in $n = 2$	(P)	54.4 eV
(B)	Ionization energy from ground state	(Q)	40.8 eV
(C)	Energy required to exit electron from $n = 1$ to $n = 2$	(R)	depends on reference level
(D)	Negative of potential energy of electron in $n = 1$	(S)	independent of reference level.
		(T)	70.3 eV

MP0115

74. In each situation of column–I a physical quantity related to orbiting electron in a hydrogen like atom is given. The terms 'Z' and 'n' given in column–II have usual meaning in Bohr's theory. Match the quantities in column–I with the terms they depend on in column–II :-

	Column-I		**Column-II**
(A)	Frequency of orbiting electron	(P)	Is directly proportional to Z^2
(B)	Angular momentum of orbiting electron	(Q)	Is directly proportional to n
(C)	Magnetic moment of orbiting electron	(R)	Is inversely proportional to n^3
(D)	The average current due to orbiting of electron	(S)	Is independent of Z
		(T)	None of these

MP0116

75. Match the entries of column–I with the entries of column–II :-

	Column-I		**Column-II**
(I)	Characteristic X-ray	(P)	Inverse process of photoelectric effect
(II)	Photoelectric effect	(Q)	Emission of electrons
(III)	Thermo-ionic emission	(R)	Moseley's law
(IV)	Continuous X-ray	(S)	Emission of radiations

Then choose the correct matching.

(A) (i) →(RS) ; (ii) → (RS) ; (iii) →(S) ; (iv) →(PS)

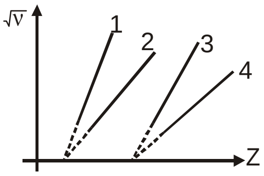
(B) (i) →(RS) ; (ii) → (Q) ; (iii) →(Q) ; (iv) →(PS)

(C) (i) →(RS) ; (ii) → (S) ; (iii) →(S) ; (iv) →(PRS)

(D) (i) →(RS) ; (ii) → (Q) ; (iii) →(Q) ; (iv) →(PRS)

MP0117

76. $\sqrt{\nu}$ versus Z graph for characteristic X–rays is as shown in figure. Match the following (assume screening constant for K_α and K_β is same and for L_α & L_β is same) :-



	Column-I		Column-II
(A)	Line – 1	(P)	L_α
(B)	Line – 2	(Q)	L_β
(C)	Line – 3	(R)	K_α
(D)	Line – 4	(S)	K_β
		(T)	Both K_α and L_β

MP0118

EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

1. The maximum kinetic energy of photo-electron liberated from the surface of lithium (work function = 2.35 eV) by electromagnetic radiation whose electric component varies with time as $E = a[1 + \cos(2\pi f_1 t)] \cos(2\pi f_2 t)$ where 'a' is a constant, $f_1 = 3.6 \times 10^{15}$ Hz and $f_2 = 1.2 \times 10^{15}$ Hz is [Take : $h = 6.6 \times 10^{-34}$ J-s]

(A) 2. 6 eV (B) 7.55 eV (C) 12.5 eV (D) 17.45 eV

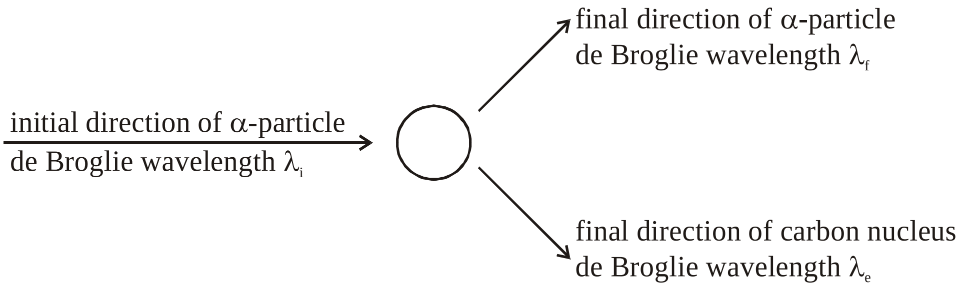
MP0119

2. A photocell in the saturation mode is irradiated by light of wavelength $\lambda = 6600$ Å. The corresponding spectral sensitivity of the cell is $s_\lambda = 4.8$ mA/W. Find the yield of photoelectrons, i.e. the number of photoelectrons produced by each incident photon. [Take : $h = 6.6 \times 10^{-34}$ J-s]

(A) 9×10^{-2} (B) 9×10^{-4} (C) 9×10^{-3} (D) 9

MP0120

3. An α-particle having a de Broglie wavelength λ_i collides with a stationary carbon nucleus. The α-particle moves off in a different direction as shown below.



After the collision, the de Broglie wavelength of the α-particle and the carbon nucleus are λ_f and λ_e respectively. Which of the following relations about de Broglie wavelength is correct ?

(A) $\lambda_i < \lambda_f$ (B) $\lambda_i > \lambda_f$ (C) $\lambda_f = \lambda_e$ (D) $\lambda_i = \lambda_e$

MP0121

4. Choose the correct statement(s) for hydrogen and deuterium atoms (considering motion of nucleus)

(A) The radius of first Bohr orbit of deuterium is less than that of hydrogen

(B) The speed of electron in the first Bohr orbit of deuterium is more than that of hydrogen.

(C) The wavelength of first Balmer line of deuterium is more than that of hydrogen

(D) The angular momentum of electron in the first Bohr orbit of deuterium is more than that of hydrogen.

MP0122

5. Apply Bohr's atomic model to a lithium atom. Assuming that its two K-shell electrons are too close to nucleus such that nucleus and K-shell electron act as a nucleus of effective positive charge equivalent to electron. The ionization energy of its outermost electron is:-

(A) 30.6 eV (B) 3.4 eV (C) 32.4 eV (D) 13.6 eV

MP0123

6. The attractive potential for an atom is given by $v = v_0 \, ln\left(r / r_0\right)$, v_0 and r_0 are constant and r is the radius of the orbit. The radius r of the n^{th} Bohr's orbit depends upon principal quantum number n as:

[JEE' 2003 (Scr)]

(A) $r \propto n$ (B) $r \propto 1/n^2$ (C) $r \propto n^2$ (D) $r \propto 1/n$

MP0124

7. A force of attraction between the positively charged nucleus and the negatively charged electron in the hydrogen atom is given by $F = \frac{ke^2}{r^2}$ where k is the constant. The electron, initially moving in a circle of radius R_1 about the nucleus, jumps suddenly into a circular orbit of radius R_2. The total energy of the atom decreased in this process is :-

(A) $ke^2\left[\frac{1}{R_1}+\frac{1}{R_2}\right]$ (B) $\frac{ke^2}{2}\left[\frac{1}{R_2}-\frac{1}{R_1}\right]$ (C) $ke^2\left[\frac{R_1R_2}{R_2-R_1}\right]$ (D) $ke^2\left[\frac{R_1R_2}{R_1+R_2}\right]$

MP0125

8. A stationary hydrogen atom of mass M emits a photon corresponding to the longest wavelength of Balmer series. The recoil velocity acquired by the atom is (R = Rydberg constant and h = plank's constant)

(A) $\frac{Rh}{M}$ (B) $\frac{Rh}{4M}$ (C) $\frac{3}{4}\frac{Rh}{M}$ (D) $\frac{5}{36}\frac{Rh}{M}$

MP0126

9. Let the potential energy of a hydrogen atom in the ground state be zero. Then its energy in the first excited state will be :

(A) 10.2eV (B) 13.6eV (C) 23.8eV (D) 27.2 eV

MP0127

10. Hydrogen atoms in ground state are excited by monochromatic radiation of wavelength 975 Å. The number of liens in the resulting spectrum will be :

(A) 3 (B) 4 (C) 6 (D) 10

MP0128

11. The relation between λ_1 : wavelength of series limit of lyman λ_2 : the wavelength of the series limit of Balmer series and λ_3 : the wavelength of first line of lyman series is

(A) $\lambda_1 = \lambda_2 + \lambda_3$ (B) $\lambda_3 = \lambda_1 + \lambda_2$ (C) $\lambda_2 = \lambda_3 - \lambda_1$ (D) none

MP0129

12. A hydrogen like gas atoms absorb radiations of wavelength λ_0 and consequently emit radiations of 6 difference wavelengths of which, three wavelengths are shorter than λ_0. Choose the correct alternative(s).

(A) The final excited state of the atoms is n = 3.

(B) The final excited state of the atoms is n = 4.

(C) The initial state of the atoms is n =1.

(D) The initial state of the atoms is n =3.

MP0130

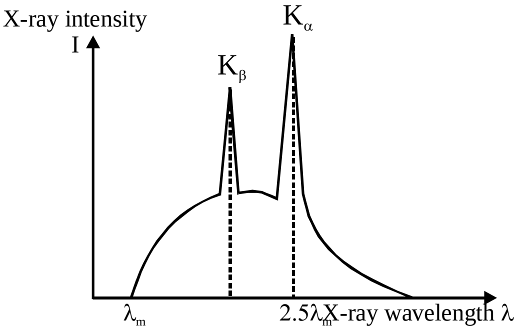
13. According to the Bohr theory of the hydrogen atom, electron starting in the 4th energy level and eventually ending in the ground state could produce a total of how many lines in the hydrogen spectra?

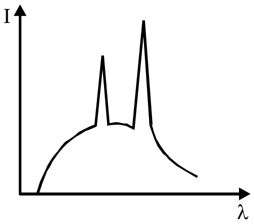
(A) 7 (B) 6 (C) 5 (D) 4

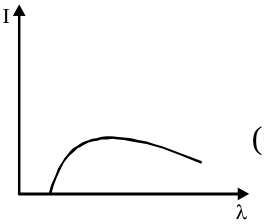
(E) 3

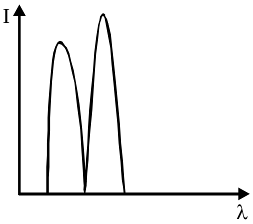
MP0131

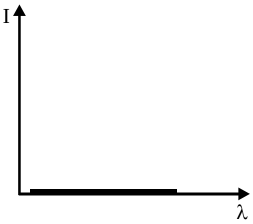
14. When an electron accelerated by potential difference U is bombarded on a specific metal the emitted X-ray spectrum obtained is shown in adjoining graph. If the potential difference is reduced to U/3, the correct spectrum is :-



(A)


(B)


(C)


(D)


MP0132

15. In an X-ray experiment target is made up of copper (Z = 29) having some impurity. The K_α line of copper have wavelength λ_0. It was observed that another K_α line due to impurity have wavelength

$\frac{784}{625}\lambda_0$. The atomic number of the impurity element is

(A) 22 (B) 23 (C) 24 (D) 26

MP0133

MULTIPLE CORRECT TYPE QUESTIONS

16. In a photoelectric effect experiment, if f is the frequency of radiations incident on the metal surface and I is the intensity of the incident radiations, then choose the correct statement(s).

(A) If f is increased keeping I and work function constant then maximum kinetic energy of photoelectron increases.

(B) If distance between cathode and anode is increased stopping potential increases

(C) If I is increased keeping f and work function constant then stopping potential remains same and saturation current increases.

(D) Work function is decreased keeping f and I constant then stopping potential increases

MP0134

17. When a monochromatic point-source of light is at a distance of 0.2 m from a small photoelectric cell, the stopping potential and the saturation current are respectively 0.6 volt and 18.0 mA. If the same source is placed 0.6 m away from the photoelectric cell, then :-

(A) The stopping potential will be 0.2 volt
(B) The stopping potential will be 1.8 volt
(C) The saturation current will be 6.0 mA
(D) The saturation current will be 2.0 mA

MP0135

18. The collector of the photocell (in photoelectric experiment) is made of tungsten while the emitter is of Platinum having work function of 10 eV. Monochromatic radiation of wavelength 124 Å & power 100 watt is incident on emitter which emits photo electrons with a quantum efficiency of 1%. The accelerating voltage across the photocell is of 10,000 volts (Use : hc = 12400eV Å)

(A) The power supplied by the accelerating voltage source is 100 watt

(B) The minimum wavelength of radiation coming from the tungsten target (collector) is 1.23 Å

(C) The power supplied by the accelerating voltage source is 10 watt

(D) The minimum wavelength of radiation coming from the tungsten target (collector) is 2.23 Å

MP0136

19. Light of wavelength λ_1 & λ_2 are falling on two metal surface A & B. For wavelength λ_1 electron ejected from both the surfaces and for wavelength λ_2 electron ejected from only surface B. On the basis of these facts which one of the following is a false statement

(A) more energy is required for ejection of electron from metal 'A'

(B) $\lambda_1 > \lambda_2$

(C) threshold wavelength for A is greater than B

(D) energy of electron ejected from metal A will be greater than electron ejected from metal B for wavelength λ

MP0137

20. The accelerating potential (V) applied between a photocathode and the respective anode is such that the fastest photoelectron can fly only one fourth of the distance between the cathode and the anode. If the distance between photocathode and anode is reduced to (1/4)th of the original value while maintaining the accelerating potential constant, then

(A) The fastest electron will reach the anode

(B) The fastest electron will reach upto one fourth of the new distance between the cathode and the anode

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(C) Kinetic energy of the fastest photoelectron emitted from photocathode will not change due to change in plate separation (keeping V constant)

(D) Kinetic energy of the fastest photoelectron will increase due to decrease in plate separation (keeping V constant)

MP0138

21. For the electron in the n^{th} orbit of hydrogen atom. Under the assumption of the Bohr's atomic model choose the **CORRECT** option(s) (Here n is the principal quantum number) :-

(A) Frequency of the electron is inversely proportional to n^3

(B) The magnitude of potential energy of the electron in an orbit is greater than its kinetic energy

(C) Magnetic induction at the nucleus produced due to the motion of electron in the n^{th} orbit is proportional to n^5

(D) Magnetic moment produced due to the motion of electron in the n^{th} orbit is proportional to n

MP0139

22. Energy liberated in the de-excitation of hydrogen atom from 3rd level to 1st level falls on a photo-cathode. Later when the same photocathode is exposed to a spectrum of some unknown hydrogen like gas, excited to 2nd energy level, it is found that the de-Broglie wavelength of the fastest photoelectrons, now ejected has decreased by a factor of 3. For this new gas, difference of energies of 2nd Lyman line and 1st Balmer line is found to be 3 times the ionization potential of the hydrogen atom. Select the correct statement(s) :

(A) The gas is lithium

(B) The gas is helium

(C) The work function of photocathode is 8.5 eV

(D) The work function of photocathode is 5.5 eV

MP0140

23. An electron of the kinetic energy 10eV collides with a hydrogen atom in 1st excited state. Assuming loss of kinetic energy in the collision to be quantized, the collision :

(A) may be perfectly inelastic (B) may be inelastic

(C) may be elastic (D) must be inelastic

MP0141

24. An X-ray tube has three main controls.

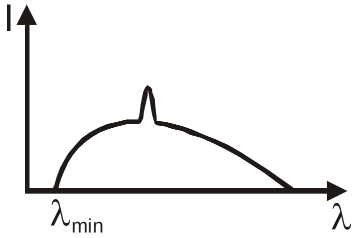
(i) the target material (its atomic number Z)

(ii) the filament current (I_f)

(iii) the accelerating voltage (V)

Figure shows a typical intensity distribution against wavelength.

Which of the following is **CORRECT**?



(A) The limit λ_{min} is proportional to V^{-1}

(B) The sharp peak shifts to the right as Z is increased

(C) The penetrating power of X-ray increases if V is increased

(D) The intensity everywhere increases if filament current I_f is increased

MP0142

COMPREHENSION TYPE QUESTIONS

Paragraph for Question 25 to 27

A mercury arc lamp provides 0.1 watt of ultra-violet radiation at a wavelength of $\lambda = 2537$ Å only. The photo tube (cathode of photo electric device) consists of potassium and has an effective area of 4 cm^2. The cathode is located at a distance of 1m from the radiation source. The work function for potassium is $\phi_0 = 2.22$ eV.

25. According to classical theory, the radiation from arc lamp spreads out uniformly in space as spherical wave. What time of exposure to the radiation should be required for a potassium atom (radius 2Å) in the cathode to accumulate sufficient energy to eject a photo-electron ?

(A) 352 second (B) 176 second (C) 704 seconds (D) No time lag

MP0143

26. To what saturation current does the flux of photons at the cathode corresponds if the photo conversion efficiency is 5%.

(A) 32.5 nA (B) 10.15 nA (C) 65 nA (D) 3.25 nA

MP0143

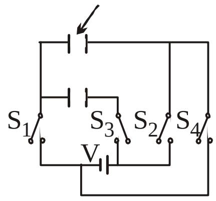
27. What is the cut off potential V_0 ?

(A) 26.9 V (B) 2.69 V (C) 1.35 V (D) 5.33 V

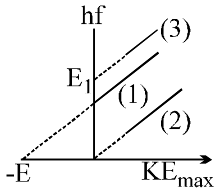
MP0143

Paragraph for question nos. 28 to 30

The circuit shown is placed in vacuum. Both the capacitors are identical and they have the same capacitance C. Light is incident on the left plate of the upper capacitor. When all the switches are open then the hf versus KE$_{max}$ is shown by the straight line (A). In all the cases, we are measuring the KE$_{max}$ when the electron reaches the opposite plate.



When only S_1 the switches and S_2 are closed, the graph becomes (B).
When only S_3 and S_4 are closed then the graph becomes (C).



28. What is the work function of the cathode?

(A) E (B) E_1 (C) $E + E_1$ (D) none of these

MP0144

29. What is the value of eV?

(A) E (B) E_1 (C) $E + E_1$ (D) none of these

MP0144

30. What is the value of E_1?

(A) 3E (B) 3E/2 (C) E/2 (D) none of these

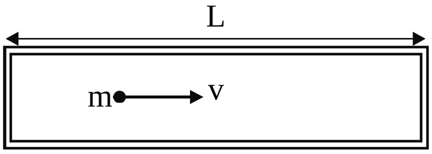
MP0144

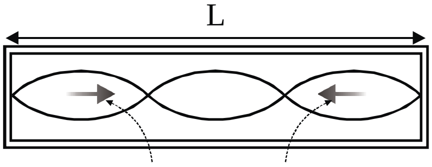
Paragraph for Question. 31 to 33

While conducting his doctoral research in theoretical physics and with no experimental evidence to go on, de Broglie reasoned by analogy with Einstein's equation E = hf and with some of the ideas of his theory of relativity. The details need not concern us, but they led de Broglie to postulate that if a material particle of momentum p = mv has a wave-like nature, then its wavelength must be given by

$\lambda = \frac{h}{p} = \frac{h}{mv}$ where h is Planck's constant. This is called the de-Broglie wavelength.

de-Broglie considered a matter wave to be a traveling wave. But suppose that a "particle" of matter is confined to a small region of space and cannot travel. How do the wave-like properties manifest themselves? This is the problem of "a particle in a box." Figure shows a particle of mass m moving in one dimension as it bounces back and forth with speed v between the ends of a box of length L. We'll call this a one-dimensional box; its width isn't relevant. A particle in a box creates a standing de Broglie wave as it reflects back and forth.





Matter waves travel in both directions.

31. What should be de-Broglie wavelength of confined particle in the box [here $n \in N$]

(A) $\frac{L}{2n}$ (B) $\frac{2L}{n}$ (C) $\frac{L}{n}$ (D) nL

MP0145

32. Confined particle's energy is given by

(A) $\frac{n^2h^2}{2mL^2}$ (B) $\frac{2n^2h^2}{mL^2}$ (C) $\frac{n^2h^2}{8mL^2}$ (D) $\frac{n^2h^2}{4mL^2}$

MP0145

33. Consider an oil drop from Millikan's oil drop experiment having diameter 1 μm confined between the plates separated by 10 μm. Density of oil is 900 kg/m^3. What is minimum energy of such an oil drop? [Given : h = 6.63 × 10^{-34} Js]

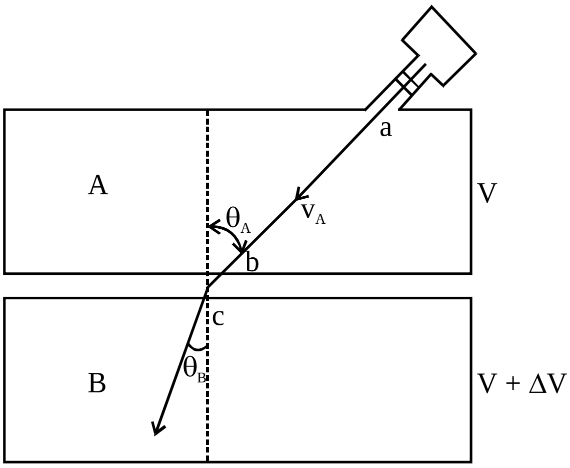
(A) 2.4 × 10^{-42} J (B) 1.2 × 10^{-42} J (C) 3.6 × 10^{-42} J (D) 4.8 × 10^{-42} J

MP0145

Paragraph for Question Nos. 34 to 36

Let a pencil of electrons from a suitable gun G enter through orifice in an enclosed metal box A, which has potential V relative to filament of the gun. Let these electrons emerge from A through orifice b and enter through c to another box B, which is maintained at a potential V + ΔV. The electric field between the two boxes changes the component of velocity of the electrons perpendicular to the adjacent surface, and the electrons enter B with a change in their direction of motion. Let v_A and v_B be the velocities of the electrons in A and B, respectively, and θ_A, θ_B the angles between these directions and the normal to the box faces at b and c. Since the electric field does not change the horizontal component of velocity,

$$v_A \sin\theta_A = v_B \sin\theta_B \qquad \frac{\sin\theta_A}{\sin\theta_B} = \frac{v_B}{v_A}$$



The "refraction" of a pencil of electrons

Now if we dealing with light waves undergoing refraction, or any other kind of wave, the relation would be

$$\frac{\sin\theta_A}{\sin\theta_B} = \mu = \frac{u_A}{u_B}$$

where μ is the relative refractive index of the two media and u_A, u_B are the corresponding velocities of light wave. Comparison of the last two equations gives the result $\frac{u_A}{u_B} = \frac{v_B}{v_A}$. We may conclude that if matter waves follow the electron along its path, the wave speed u is inversely proportional to the speed v of the electron, or $u = \frac{b}{v}$.

34. If we define the frequency of matter waves as $f = \frac{u}{\lambda}$, the

(A) Frequency of matter waves in medium A is more than that in medium B
(B) Frequency of matter waves in medium A is less than that in medium B
(C) Frequency of matter waves in medium A is same as that in medium B
(D) Cannot be predicted

MP0146

35. Suppose $V_A = 20$ volt and $V_B = 15$ volt. Choose the **CORRECT** statement :-
(A) The speed of electrons as well as speed of matter waves inside box B is more
(B) The speed of electrons as well as speed of matter waves inside box B is less
(C) The speed of electrons inside the box B is more, but speed of matter waves in box B is less
(D) The speed of electrons inside the box B is less, but speed of matter waves in box B is more

MP0146

36. The refractive index for matter waves can be defined as $\frac{c}{u}$ where c is some constant. So refractive index for matter waves

(A) is inversely proportional to λ
(B) is independent of λ
(C) is directly proportional of λ
(D) is proportional to $\sqrt{\lambda}$

MP0146

EXERCISE (JM)

1. Suppose an electron is attracted towards the origin by a force $\frac{k}{r}$ where 'k' is a cosntant and 'r' is the distance of the electron from the origin. By applying Bohr model to this sytem, the radius of the nth orbital of the electron is found to be 'r_n' and the kinetic energy of the electron to be 'T_n'. Then which of the following is true? **[AIEEE - 2008]**

(1) $T_n \propto \frac{1}{n^2}, r_n \propto n^2$

(2) T_n independent of n, $r_n \propto n$

(3) $T_n \propto \frac{1}{n}, r_n \propto n$

(4) $T_n \propto \frac{1}{n}, r_n \propto n^2$

MP0147

2. The transistion from the state n = 4 to n = 3 in a hydrogen like atom results in ultraviolet radiation. Infrared radiation will be obtained in the transition from :- **[AIEEE - 2009]**

(1) $4 \to 2$ (2) $5 \to 4$ (3) $2 \to 1$ (4) $3 \to 2$

MP0148

3. The surface of a metal is illuminated with the light of 400 nm. The kinetic energy of the ejected photoelectrons was found to be 1.68 eV. The work function of the metal is : (hc = 1240 eV-nm) **[AIEEE - 2009]**

(1) 1.51 eV (2) 1.68 eV (3) 3.09 eV (4) 1.41 eV

MP0149

Directions : Question number 4 contain Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best discribes the two statements.

4. Statement-1 : When ultraviolet light is incident on a photocell, its stopping potential is V_0 and the maximum kinetic energy of the photoelectrons is K_{max}. When the ultraviolet light is replaced by X-rays, both V_0 and K_{max} increase.

Statement-2 : Photoelectrons are emitted with speeds ranging from zero to a maximum value because of the range of frequencies present in the incident light. **[AIEEE - 2010]**

(1) Statement–1 is true, Statement–2 is false

(2) Statement–1 is true, Statement–2 is true; Statement–2 is the correct explanation of Statement–1

(3) Statement–1 is true, Statement–2 is true; Statement–2 is not the correct explanation of Statement–1

(4) Statement–1 is false, Statement–2 is true

MP0150

5. If a source of power 4kW produces 10^{20} photons/second, the radiation belongs to apart of the spectrum called :- **[AIEEE - 2010]**

(1) γ-rays (2) X-rays (3) ultraviolet rays (4) microwaves

MP0151

6. Energy required for the electron excitation in Li^{++} from the first to the third Bohr orbit is:-

[AIEEE-2011]

(1) 108.8 eV (2) 122.4 eV (3) 12.1 eV (4) 36.3 eV

MP0152

7. This question has Statememtn-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement–1 : A metallic surface is irradiated by a monochromatic light of frequency $v > v_0$ (the threshold frequency). The maximum kinetic energy and the stopping potential are K_{max} and V_0 respectively. If the frequency incident on the surface is doubled, both the K_{max} and V_0 are also boubled.

Statement-2 : The maximum kinetic energy and the stopping potential of photoelectrons emitted from a surface are linearly dependent on the frequency of incident light. **[AIEEE-2011]**

(1) Statement–1 is true, Statement–2 is true, Statement–2 is not the correct explanationof Statement–1

(2) Statement–1 is false, Statement–2 is true

(3) Statement–1 is true, Statement–2 is false

(4) Statement–1 is true, Statement–2 is true, Statement–2 is the correct explanation of Statement–1

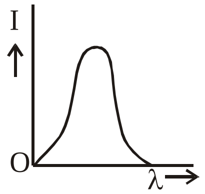
MP0153

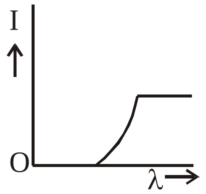
8. Hydrogen atom is excited from ground state to another state with principal quantum number equal to 4. Then the number of spectral lines in the emission spectra will be :- **[AIEEE-2012]**

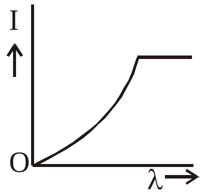
(1) 6 (2) 2 (3) 3 (4) 5

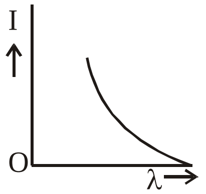
MP0154

9. The anode voltage of photocell is kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current I of the photocell varies as follows : **[AIEEE-2013]**

(1)


(2)


(3)


(4)


MP0155

10. In a hydrogen like atom electron makes transition from an energy level with quantum number n to another with quantum number (n–1). If n >> 1, the frequency of radiation emitted is proportional to :

[JEE Main-2013]

(1) $\frac{1}{n}$ (2) $\frac{1}{n^2}$ (3) $\frac{1}{n^{3/2}}$ (4) $\frac{1}{n^3}$

MP0156

11. As an electron makes a transition from an excited state to the ground state of a hydrogen - like atom/ion : **[JEE Main-2015]**

(1) kinetic energy decreases, potential energy increases but total energy remains same

(2) kinetic energy and total energy decrease but potential energy increases

(3) its kinetic energy increases but potential energy and total energy decreases

(4) kinetic energy, potential energy and total energy decrease

MP0157

12. Match List-I (Fundament Experiment) with List-II (its conclusion) and select the correct option from the choices given below the list : **[JEE Main-2015]**

	List-I		**List-II**
(A)	Franck-Hertz Experiment.	(i)	Particle nature of light
(B)	Photo-electric experiment	(iii)	Discrete energy levels of atom
(C)	Davison-Germer Experiment	(iiii)	Wave nature of electroc
		(iv)	Structure of atom

(1) A-ii, B-i, C-iii

(2) A-iv, B-iii, C-ii

(3) A-i, B-iv, C-iii

(4) A-ii, B-iv, C-iii

MP0158

13. Radiation of wavelength λ, is incident on a photocell. The fastest emitted electron has speed v. If the wavelength of changed to $\frac{3\lambda}{4}$, the speed of the fastest emitted electron will be :-**[JEE Main-2016]**

(1) $= v\left(\frac{3}{5}\right)^{1/2}$

(2) $> v\left(\frac{4}{3}\right)^{1/2}$

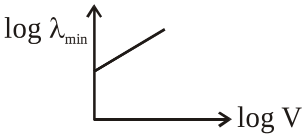
(3) $< v\left(\frac{4}{3}\right)^{1/2}$

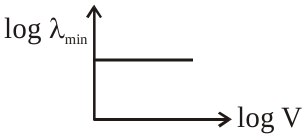
(4) $= v\left(\frac{4}{3}\right)^{1/2}$

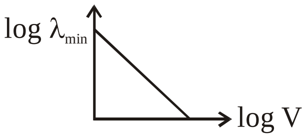
MP0159

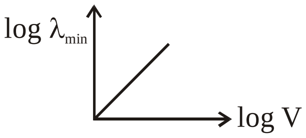
14. An electron beam is accelerated by a potential difference V to hit a metallic target to produce X-rays. It produces continuous as well as characteristic X-rays.If λ_{min} is the smallest possible wavelength of X-ray in the spectrum, the variation of log λ_{min} with log V is correctly represented in :

[JEE Main-2017]

(1) 

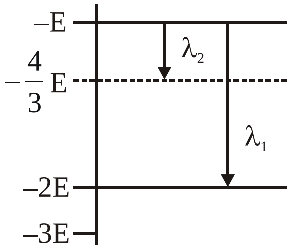
(2) 

(3) 

(4) 

MP0160

15. Some energy levels of a molecule are shown in the figure. The ratio of the wavelengths $r = \lambda_1/\lambda_2$, is given by : **[JEE Main-2017]**



(1) $r=\frac{3}{4}$ (2) $r=\frac{1}{3}$ (3) $r=\frac{4}{3}$ (4) $r=\frac{2}{3}$

MP0161

16. A particle A of mass m and initial velocity v collides with a particle B of mass $\frac{m}{2}$ which is at rest. The collision is head on, and elastic. The ratio of the de–Broglie wavelengths λ_A to λ_B after the collision is: **[JEE Main-2017]**

(1) $\frac{\lambda_A}{\lambda_B}=\frac{2}{3}$ (2) $\frac{\lambda_A}{\lambda_B}=\frac{1}{2}$ (3) $\frac{\lambda_A}{\lambda_B}=\frac{1}{3}$ (4) $\frac{\lambda_A}{\lambda_B}=2$

MP0162

17. If the series limit frequency of the Lyman series is v_L, then the series limit frequency of the Pfund series is : **[JEE Main-2018]**

(1) $16\ v_L$ (2) $v_L/16$ (3) $v_L/25$ (4) $25\ v_L$

MP0163

EXERCISE (JA)

1. Which one of the following statements is **WRONG** in the context of X-rays generated from a X-ray tube ? **[JEE 2008]**

(A) Wavelength of characteristic X-rays decreases when the atomic number of the target increases

(B) Cut-off wavelength of the continuous X-rays depends on the atomic number of the target

(C) Intensity of the characteristic X-rays depends on the electrical power given to the X-rays tube

(D) Cut-off wavelength of the continuous X-rays depends on the energy of the electrons in the X-ray tube

MP0164

Paragraph for Question Nos. 2 to 4

In a mixture of H – He^+ gas (He^+ is singly ionized He atom), H atoms and He^+ ions are excited to their respective first excited states. Subsequently, H atoms transfer their total excitation energy to He^+ ions (by collisions). Assume that the Bohr model of atom is exactly valid. **[JEE 2008]**

2. The quantum number n of the state finally populated in He^+ ions is

(A) 2 (B) 3 (C) 4 (D) 5

MP0165

3. The wavelength of light emitted in the visible region by He^+ ions after collisions with H atoms is

(A) 6.5×10^{-7} m (B) 5.6×10^{-7} m (C) 4.8×10^{-7} m (D) 4.0×10^{-7} m

MP0165

4. The ratio of the kinetic energy of the n = 2 electron for the H atom to that of He^+ ion is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 1 (D) 2

MP0165

Paragraph for Question Nos. 5 to 7

When a particle is restricted to move along x-axis between x = 0 and x = a, where a is of nanometer dimension, its energy can take only certain specific values. The allowed energies of the particle moving in such a restricted region, correspond to the formation of standing waves with nodes at its ends x = 0 and x = a. The wavelength of this standing wave is related to the linear momentum p of the particle according to the de Broglie relation. The energy of the particle of mass m is related to its linear momentum as $E = \frac{p^2}{2m}$. Thus, the energy of the particle can be denoted by a quantum number 'n' taking values 1,2,3, ... (n = l, called the ground state) corresponding to the number of loops in the standing wave. Use the model described above to answer the following three questions for a particle moving in the line x = 0 to x = a. Take $h = 6.6 \times 10^{-34}$ Js and $e = 1.6 \times 10^{-19}$ C. **[JEE-2009]**

5. The allowed energy for the particle for a particular value of n is proportional to

(A) a^{-2} (B) $a^{-3/2}$ (C) a^{-1} (D) a^2

MP0166

6. If the mass of the particle is $m = 1.0 \times 10^{-30}$ kg and a = 6.6 nm, the energy of the particle in its ground state is closest to :

(A) 0.8 meV (B) 8 meV (C) 80 meV (D) 800 meV

MP0166

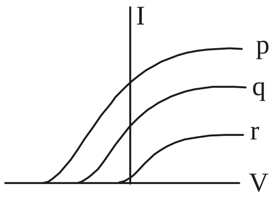
7. The speed of the particle, that can take discrete values, is proportional to

(A) $n^{-3/2}$ (B) n^{-1} (C) $n^{1/2}$ (D) n

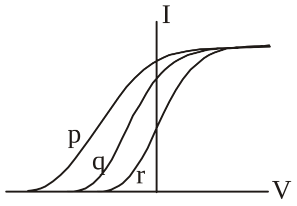
MP0166

8. Photoelectric effect experiments are performed using three different metal plates p, q and r having work functions $\phi_p = 2.0$ eV, $\phi_q = 2.5$ eV and $\phi_r = 3.0$ eV, respectively. A light beam containing wavelengths of 550 nm, 450 nm and 350 nm with equal intensities illuminates each of the plates. The correct I-V graph for the experiment is : **[JEE-2009]**

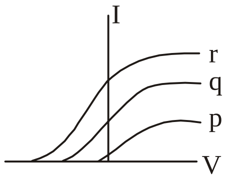
(A)



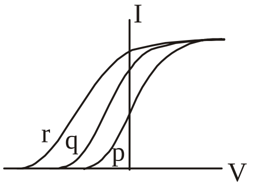
(B)



(C)



(D)



MP0167

9. An α–particle and a proton are accelerated from rest by a potential difference of 100 V. After this, their de Broglie wavelengths are λ_α and λ_p respectively. The ratio $\frac{\lambda_p}{\lambda_\alpha}$, to the nearest integer, is

[JEE 2010]

MP0168

Paragraph for Question Nos. 10 to 12

The key feature of Bohr's theory of spectrum of hydrogen atom is the quantization of angular momentum when an electron is revolving around a proton. We will extend this to a general rotational motion to find quantized rotational energy of a diatomic molecule assuming it to be rigid. The rule to be applied is Bohr's quantization condition. **[JEE 2010]**

10. A diatomic molecule has moment of inertia I. By Bohr's quantization condition its rotational energy in the nth level (n = 0 is not allowed) is

(A) $\frac{1}{n^2}\left(\frac{h^2}{8\pi^2 I}\right)$ (B) $\frac{1}{n}\left(\frac{h^2}{8\pi^2 I}\right)$ (C) $n\left(\frac{h^2}{8\pi^2 I}\right)$ (D) $n^2\left(\frac{h^2}{8\pi^2 I}\right)$

MP0169

11. It is found that the excitation frequency from ground to the first excited state of rotation for the CO molecule is close to $\frac{4}{\pi}\times 10^{11}$ Hz. Then the moment of inertia of CO molecule about its center of mass is close to [Take h = $2\pi \times 10^{-34}$ Js)

(A) 2.76×10^{-46} kg m^2 (B) 1.87×10^{-46} kg m^2

(C) 4.67×10^{-47} kg m^2 (D) 1.17×10^{-47} kg m^2

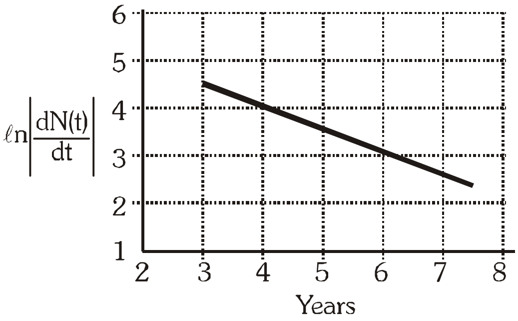
MP0169

12. In a CO molecule, the distance between C (mass = 12 a.m.u.) and O (mass = 16 a.m.u.), where 1 a.m.u. = $\frac{5}{3}\times 10^{-27} kg$, is close to

(A) 2.4×10^{-10}m (B) 1.9×10^{-10}m (C) 1.3×10^{-10}m (D) 4.4×10^{-11}m

MP0169

13. To determine the half life of a radioactive element, a student plots a graph of $\ell n\left|\frac{dN(t)}{dt}\right|$ versus t. Here $\frac{dN(t)}{dt}$ is the rate of radioactive decay at time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 years, the value of p is **[JEE 2010]**



MP0170

14. The wavelength of the first spectral line in the Balmer series of hydrogen atom is 6561 Å. The wavelength of the second spectral line in the Balmer series of singly-ionized helium atom is **[JEE 2011]**

(A) 1215 Å (B) 1640Å (C) 2430Å (D) 4687Å

MP0171

15. A silver sphere of radius 1 cm and work function 4.7 eV is suspended from an insulating thread in free-space. It is under continuous illumination of 200 nm wavelength light. As photoelectrons are emitted, the sphere gets charged and acquires a potential. The maximum number of photoelectrons emitted from the sphere is A × 10^z (where 1 < A < 10). The value of 'Z' is **[JEE 2011]**

MP0172

16. A proton is fired from very far away towards a nucleus with charge Q = 120 e, where e is the electronic charge. It makes a closest approach of 10 fm to the nucleus. The de Broglie wavelength (in units of fm) of the proton at its start is (Take : The proton mass, $m_p = (5/3) \times 10^{-27}$ kg; $h/e = 4.2 \times 10^{-15}$ J.s/C; $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ m/F; 1 fm = 10^{-15} m) **[JEE 2012]**

MP0173

17. The work functions of Silver and sodium are 4.6 and 2.3 eV, repetitively. The ratio of the slope of the stopping potential versus frequency plot for Silver to that of Sodium is. **[JEE Advanced-2013]**

MP0174

18. The radius of the orbit of an electron in a Hydrogen-like atom is 4.5 a_0, where a_0 is the Bohr radius. Its orbital angular momentum is $\frac{3h}{2\pi}$. It is given that h is Planck constant and R is Rydberg constant. The possible wavelength (s), when the atom de-excites, is (are) :- **[JEE Advanced-2013]**

(A) $\frac{9}{32R}$ (B) $\frac{9}{16R}$ (C) $\frac{9}{5R}$ (D) $\frac{4}{3R}$

MP0175

19. Consider a hydrogen atom with its electron in the nth orbital. An electromagnetic radiation of wavelength 90 nm is used to ionize the atom. If the kinetic energy of the ejected electron is 10.4 eV, then the value of n is (hc = 1242 eV nm). **[JEE Advanced-2015]**

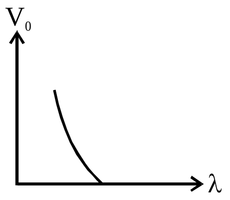
MP0176

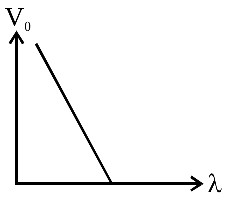
20. Planck's constant h, speed of light c and gravitational constant G are used to form a unit of length L and a unit of mass M. Then the correct option(s) is(are) :- **[JEE Advanced-2015]**

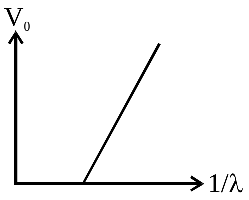
(A) $M \propto \sqrt{c}$ (B) $M \propto \sqrt{G}$ (C) $L \propto \sqrt{h}$ (D) $L \propto \sqrt{G}$

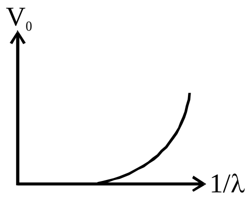
MP0177

21. For photo-electric effect with incident photon wavelength λ, the stopping potential is V_0. Identify the correct variation(s) of V_0 with λ and 1/λ. **[JEE Advanced-2015]**

(A)


(B)


(C)


(D)


MP0178

22. An electron in an excited state of Li^{2+} ion has angular momentum 3h/2π. The de Broglie wavelength of the electron in this state is $p\pi a_0$ (where a_0 is the Bohr radius). The value of p is **[JEE Advanced-2015]**

MP0179

23. In a historical experiment to determine Planck's constant, a metal surface was irradiated with light of different wavelengths. The emitted photoelectron energies were measured by applying a stopping potential. The relevant data for the wavelength (λ) of incident light and the corresponding stopping potential (V_0) are given below: **[JEE Advanced-2016]**

$\lambda(\mu m)$	$V_0(\text{Volt})$
0.3	2.0
0.4	1.0
0.5	0.4

Given that $c = 3 \times 10^8$ ms^{-1} and $e = 1.6 \times 10^{-19}$C, Planck's constant (in units of J s) found from such an experiment is :

(A) 6.0×10^{-34} (B) 6.4×10^{-34} (C) 6.6×10^{-34} (D) 6.8×10^{-34}

MP0180

24. Highly excited states for hydrogen like atoms (also called Rydberg states) with nuclear charge Ze are defined by their principal quantum number n, where n >> 1. Which of the following statement(s) is (are) true? **[JEE Advanced-2016]**

(A) Relative change in the radii of two consecutive orbitals does not depend on Z

(B) Relative change in the radii of two consecutive oribitals varies as 1/n

(C) Relative change in the energy of two consecutive orbitals varies as $1/n^3$

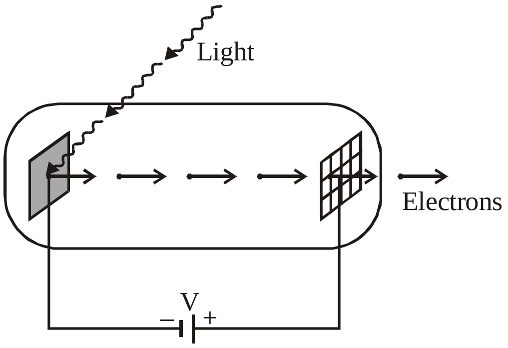
(D) Relative change in the angular momenta of two consecutive orbitals varies as 1/n

MP0181

25. A hydrogen atom in its ground state is irradiated by light of wavelength 970 Å. Taking $hc/e = 1.237 \times 10^{-6}$ eV m and the ground state energy of hydrogen atom as –13.6 eV, the number of lines present in the emission spectrum is **[JEE Advanced-2016]**

MP0182

26. Light of wavelength λ_{ph} falls on a cathode plate inside a vacuum tube as shown in the figure. The work function of the cathode surface is ϕ and the anode is a wire mesh of conducting material kept at a distance d from the cathode. A potential difference V is maintained between the electrodes. If the minimum de Broglie wavelength of the electrons passing through the anode is λ_e, which of the following statement(s) is(are) true ? **[JEE Advanced-2016]**



(A) For large potential difference ($V >> \phi/e$), λ_e is approximately halved if V is made four times

(B) λ_e increases at the same rate as λ_{ph} for $\lambda_{ph} < hc/\phi$

(C) λ_e is approximately halved, if d is doubled

(D) λ_e decreases with increase in ϕ and λ_{ph}

MP0183

27. An electron in a hydrogen atom undergoes a transition from an orbit with quantum number n_i to another with quantum number n_f. V_i and V_f are respectively the initial and final potential energies of the electron. If $\frac{V_i}{V_f} = 6.25$, then the smallest possible n_f is. **[JEE Advanced-2017]**

MP0184

28. A photoelectric material having work-function ϕ_0 is illuminated with light of wavelength λ $\left(\lambda < \frac{hc}{\phi_0}\right)$.

The fastest photoelectron has a de-Broglie wavelength λ_d. A change in wavelength of the incident light by $\Delta\lambda$ results in a change $\Delta\lambda_d$ in λ_d. Then the ratio $\Delta\lambda_d/\Delta\lambda$ is proportional to

[JEE Advanced-2017]

(A) λ_d^3/λ^2 (B) λ_d^3/λ (C) λ_d^2/λ^2 (D) λ_d/λ

MP0185

29. In a photoelectric experiment a parallel beam of monochromatic light with power of 200 W is incident on a perfectly absorbing cathode of work function 6.25 eV. The frequency of light is just above the threshold frequency so that the photoelectrons are emitted with negligible kinetic energy. Assume that the photoelectron emission efficinecy is 100% A potential difference of 500 V is applied between the cathode and the anode. All the emitted electrons are incident normally on the anode and are absorbed. The anode experiences a force $F = n \times 10^{-4}$ N due to the impact of the electrons. The value of n is........ Mass of the electron $m_e = 9 \times 10^{-31}$ kg and $1.0 \text{ eV} = 1.6 \times 10^{-19}$ J.? **[JEE Advanced-2018]**

MP0186

30. Consider a hydrogen-like ionized atom with atomic number Z with a single electron. In the emission spectrum of this atom, the photon emitted in the n = 2 to n = 1 transition has energy 74.8 eV higher than the photon emitted in the n = 3 to n = 2 transition. The ionization energy of the hydrogen atom is 13.6 eV. The value of Z is.............. . **[JEE Advanced-2018]**

MP0187

31. A free hydrogen atom after absorbing a photon of wavelength λ_a gets excited from the state n = 1 to the state n = 4. Immediately after that the electron jumps to n = m state by emitting a photon of wavelength λ_e. Let the change in momentum of atom due to the absorption and the emission are Δp_a and Δp_e , respectively. If $\lambda_a/\lambda_e = \frac{1}{5}$. Which of the option(s) is/are correct ?

[Use hc = 1242 eV nm; 1 nm = 10^{-9} m, h and c are Planck's constant and speed of light, respectively]

[JEE Advanced-2019]

(1) $\lambda_e = 418$ nm

(2) The ratio of kinetic energy of the electron in the state n = m to the state n = 1 is $\frac{1}{4}$

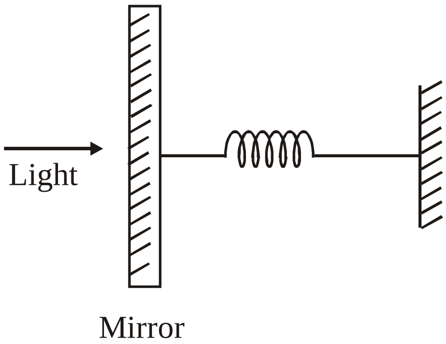
(3) m = 2

(4) $\Delta p_a/\Delta p_e = \frac{1}{2}$

MP0188

32. A perfectly reflecting mirror of mass M mounted on a spring constitutes a spring-mass system of angular frequency Ω such that $\frac{4\pi M\Omega}{h} = 10^{24}\text{m}^{-2}$ with h as Planck's constant. N photons of wavelength $\lambda = 8\pi \times 10^{-6}$m strike the mirror simultaneously at normal incidence such that the mirror gets displaced by 1μm. If the value of N is $x \times 10^{12}$, then the value of x is________.

[Consider the spring as massless] **[JEE Advanced-2019]**



MP0189

CBSE PREVIOUS YEAR'S QUESTIONS

1. Two metals A and B have work functions 4eV and 10 eV respectively. Which metal has lower threshold wavelength'? **[1; CBSE-2004]**

2. Red light, however bright it is, cannot produce the emission of electrons from a clean zinc surface. But even weak ultraviolet radiation can do so. Why?

X-rays of wavelength 'λ' fell on photosensitive surface, emitting electrons. Assuming X-rays of wavelength 'λ' fall on a photo sensitive Surface, emitting be neglected, prove that the de Broglie wavelength of electrons emitted will be $\sqrt{\frac{h\lambda}{2mc}}$

3. Define the terms: 'half-life period' and 'decay constant of a radioactive sample. Derive the relation between these terms. **[3; CBSE-2004]**

4. When a deuteron of mass 20141 u and negligible kinetic energy is absorbed by a lithium $\left({}^{6}_{3}\text{Li}\right)$ nucleus of mass 6.0155 u, the compound nucleus disintegrates spontaneously into two alpha particles, each of mass 4.0026 u. Calculate the energy in joules carried by each alpha particle (lu – 1.66 × 10^{27} kg). **[3; CBSE-2004]**

5. Ultraviolet light is incident on two photosensitive materials having work functions W_1 and W_2 ($W_1 > W_2$) in which case will the kinetic energy of the emitted electrons be greater? Why? **[1;CBSE-2005]**

6. Mention the significance of Davisson-Germer experiment An α particle and a proton are accelerated from rest through the same potential difference V. Find the ratio of de-Broglie wavelengths associated with them. **[3;CBSE-2005]**

7. (a) Draw the energy level diagram showing the emission of β-particles followed by γ-rays by a ${}^{60}_{27}\text{Co}$ nucleus.

(b) Plot the distribution of kinetic energy of β particles and state why the energy spectrum is continuous. **[3; CBSE-200S]**

8. A radioactive sample contains 2.2 mg of pure ${}^{11}_{6}\text{C}$ which has half-life period of 1224 seconds. Calculate

(i) The number of atoms present initially.

(ii) The activity when 5μg of the sample will be left. **[3; CBSE-2005]**

9. De-Broglie wavelength associated with an electron accelerated through a potential difference V is λ. What will be its wavelength when the accelerating potential is increased to 4 V? **[1; CBSE-2006]**

10. Sketch a graph between frequency of incident radiations and stopping potential for a given photosensitive material. What information can be obtained from the value of the intercept on the potential axis?

A source of light of frequency greater than the threshold frequency is placed at a distance of 1 m from the cathode of a photo-cell. The stopping potential is found to be V. If the distance of the light source from the cathode is reduced, explain giving reasons, what change will you observe in the

(i) photoelectric current,

(ii) stopping potential. **[3; CBSE-2006]**

11. Define the terms half-life period and decay constant of a radioactive substance. Write their S.I. units. Establish the relationship between the two. **[3; CBSE-2006]**

12. A neutron is absorbed by a $^{6}_{3}\mathrm{Li}$ nucleus with the subsequent emission of an alpha particle.

(i) Write the corresponding nuclear reaction.
(ii) Calculate the energy released, in MeV, in this reaction.

Given mass $^{6}_{3}\mathrm{Li}$ = 6.015126 u; mass (neutron) = 1.0086654 u; mass (alpha particle) = 4.0026044 u and mass (triton) = 3.0100000u. Take 1u = 931 MeV/c^2. **[3; CBSE-2006]**

13. Ultraviolet radiations of different frequencies v_1 and v_2 are incident on two photosensitive materials having work functions W_1 and W_2 ($W_1 > W_2$) respectively. The kinetic energy of the emitted electrons is same in both the cases. Which one of the two radiations will be of higher frequency ?

[1; CBSE-2007]

14. Define the term 'activity' of radionuclide. Write its SI unit. **[1; CBSE-2007]**

15. Draw a graph showing the variation of potential energy between a pair of nucleons as a function of their separation. Indicate the regions in which the nuclear force is (i) attractive, (ii) repulsive.

[2; CBSE-2007]

16. Draw a schematic diagram of the experimental arrangement used by Davisson and Germer to establish the wave nature of electrons. Explain briefly how the de-Broglie relation was experimentally verified in case of electrons. **[3; CBSE-2007]**

17. Draw the graph to show variation of binding energy per nucleon with mass number of different atomic nuclei. Calculate binding energy/nucleon of $^{40}_{20}\mathrm{Ca}$ nucleus.

Given:

mass of $^{40}_{20}\mathrm{Ca}$ = 39.962589 u

mass of $^{40}_{20}\mathrm{Ca}$ proton = 1.007825 u

mass of neutron – 1.008665 u and 1u = 931 MeV/C^2 **[3; CBSE-2007]**

18. An electron and alpha particle have the same de-Broglie wavelength associated with them How are their kinetic energies related to each other? **[1; CBSE-2008]**

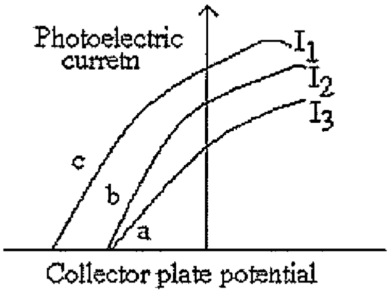
19. A nucleus $^{23}_{10}\mathrm{Ne}$ undergoes decay and becomes $^{23}_{11}\mathrm{Na}$. Calculate the maximum kinetic energy of electrons emitted assuming that the daughter nucleus and anti-neutrino carry negligible kinetic energy.

$$\left\{\begin{matrix}\text{Mass of } ^{23}_{10}\mathrm{Ne} = 22.994455\mathrm{u} \\ \text{Mass of } ^{23}_{11}\mathrm{Na} = 22.989770\mathrm{u} \\ 1\mathrm{u} = 931.5\mathrm{MeV/c^2}\end{matrix}\right\}$$

[2; CBSE-2008]

20. An electromagnetic wave of wavelength λ is incident on a photosensitive surface of negligible work function. If the photo-electrons emitted from this surface have the de-Broglie wavelength λ_1 prove that $\lambda = \left(\frac{2mc}{h}\right)\lambda_1^2$ **[3; CBSE-2008]**

21. The figure shows a plot of three curves a, b, c showing the variation of photocurrent vs collector plate potential for three different intensities I_1, I_2 and I_3 having frequencies v_1, v_2 and v_3 respectively incident on a photosensitive surface. Point out the two curves of which the incident radiations have same frequency but different intensities. **[1; CBSE-2009]**



22. Two nuclei have mass numbers in the ratio 1 :3. What is the ratio of their nuclear densities? **[1; CBSE-2009]**

23. A radioactive nucleus 'A' Undergoes a series of decays according to the following scheme:

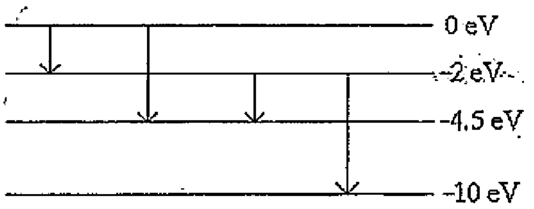
$$A \xrightarrow{\alpha} A_1 \xrightarrow{\beta} A_2 \xrightarrow{\alpha} A_3 \xrightarrow{\gamma} A_4$$

The mass number and atomic number of A4 are 172 and 69 respectively. What are these numbers for A? **[2; CBSE-2009]**

24. An electron and a proton are accelerated through the potential. Which one of the two has (i) greater . value of de-Broglie wavelength associated with it and (ii) less momentum? Justify your answer. **[3; CBSE-2009]**

25. (a) The energy levels of an atom are as shown below. Which of them will result in the transition of a photon of wavelength 275 nm?

(h) Which transition corresponds to emission of radiation of maximum wavelength? **[3; CBSE-2009]**



26. Define ionisation energy. What is its value for a hydrogen atom ? **[1; CBSE-2010]**

27. An α-particle and a proton are accelerated from rest by the same potential. Find the ratio of their de-Broglie wavelengths. **[2;CBSE-2010]**

28. Write Einstein's photoelectric equation. State clearly the three salient features observe in photoelectric effect, which can be explained on the basis of the above equation. **[2; CBSE-2010]**

29. Draw a plot of potential energy of a pair of nucleons as a function of their separation. Write two important conclusions which you can draw regarding the nature of nuclear forces. **[2; CBSE-2010]**

30. Draw a plot of the binding energy per nucleon as a function of mass number for a large number of nuclei, $2 \leq A \leq 240$. How do you explain the constancy of binding energy per nucleon in the range $30 < A < 170$ using the property that nuclear force is short-ranged ? **[2; CBSE-2010]**

31. (a) Write symbolically the β^- decay process of ${}^{32}_{15}P$.

(b) Derive an expression for the average life of a radionuclide. Give its relationship with the half-life. **[3; CBSE-2010]**

32. Write any two characteristic properties of nuclear force. **[1; CBSE-2011]**

33. Define the term 'stopping potential' in relation to photoelectric effect. **[1; CBSE-2011]**

34. Using the curve for the binding energy per nucleon as a function of mass number A, state clearly how the release in energy in the processes of nuclear fission and nuclear fusion can be explained. **[2; CBSE-2011]**

35. Draw a plot showing the variation of photoelectric current with collector plate potential for two different frequencies, $v_1 > v_2$, of incident radiation having the same intensity. In which case will the stopping potential be higher? Justify your answer. **[3; CBSE-2011]**

36. (a) Using de-Broglie's hypothesis, explain with the help of a suitable diagram, Bohr's second postulate of quantization of energy levels in a hydrogen atom,

(b) The ground state energy of hydrogen atom is –13.6eV. What are the kinetic and potential energies of the electron in this state? **[3; CBSE-2011]**

37. Define the terms (i) 'cut-off Voltage' and (ii) 'threshold frequency' in relation to the phenomenon of photoelectric effect

Using Eintein's photoelectric equation shows how the cut-off voltage and threshold frequency for a given photosensitive material can be determined with the help of a suitable plot/graph. **[3; CBSE-2012]**

38. Draw a plot of potential energy of a pair of nucleons as a function of their separations. Mark the regions where the nuclear is (i) attractive and (ii) repulsive. Write any two characteristic features of nuclear forces. **[3; CBSE-2012]**

39. In a Geiger-Marsden experiment, calculate the distance of closest approach to the nucleus of Z = 80, when a α-particle of 8Mev energy impinges on it before it comes momentarily to rest and reverses its direction.

How will the distance of closest approach be affected when the kinetic energy of the α-particle is doubles?

OR

The ground state energy of hydrogen atom is –13.6 eV. If an electron make a transition from the energy level –0.85 eV to –3.4 eV, calculate spectrum does his wavelength belong?**[3; CBSE-2012]**

40. Define the activity of a given radioactive substance. Write its S.I. unit. **[CBSE-2013]**

41. Write the expression for the deBroglie wavelength associated with a charged particle having charge 'q' and mass 'm', when it is accelerated by a potential V. **[CBSE-2013]**

42. Write Einstein's photoelectric equation and point out any two characteristic properties of photons on which this equation is based. Briefly explain the three observed features which can be explained' by this equation. **[CBSE-2013]**

43. Using Bohr's postulates, derive the expression for the frequency of radiation emitted when electron in hydrogen atom undergoes transition from higher energy state (quantum number n,) to the lower state, (n_f). When electron in hydrogen atom jumps from energy state $n_i = 4$ to $n_f = 3,2,1$, identify the spectral series to which the emission lines belong. **[CBSE-2013]**

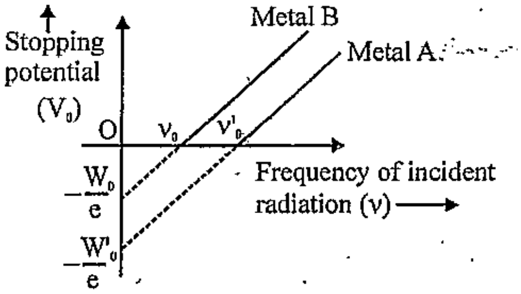
OR

(a) Draw the plot of binding energy per nucleon (BE/A) as a function of mass number A. Write two important conclusions that can be drawn regarding the nature of nuclear force.

(b) Use this graph to explain the release of energy in both the processes of nuclear fusion and fission,

(c) Write the basic nuclear process of neutron undergoing β-decay. Why is the detection of neutrinos found very difficult?

44. The graph shows the variation of stopping potential with frequency of incident radiation for two photosensitive metals A and B. Which one of the two has higher value of work-function? Justify your answer. **[CBSE-2014]**



45. Why is it found experimentally difficult to detect neutrinos in nuclear β-decay ? **[CBSE-2014]**

46. Using Rutherford model of the atom, derive the expression for the total energy of the electron in hydrogen atom. What is the significance of total negative energy possessed by the electron? **[CBSE-2014]**

OR

Using Bohr's postulates of the atomic model, derive the expression for radius of n^{th} electron orbit. Hence obtain the expression for Bohr's radius. **[CBSE-2014]**

47. When the electron orbiting in hydrogen atom in its ground state moves to the third excited state, show how the de Broglie wavelength associated with it would be affected. **[CBSE-2015]**

48. Define the terms 'stopping potential' and 'threshold frequency' in relation to photoelectric effect. How does one determine these physical quantities using einstein's equation ? **[CBSE-2015]**

49. A proton and an α particle are accelerated through the same potential difference. Which one of the two has (i) greater de-Broglie wavelength, and (ii) less kinetic energy ? Justify your answer. **[2 ; CBSE-2016]**

50. When is H_α line in the emission spectrum of hydrogen atom obtained ? Calculate the frequency of the photon emitted during this transition. **[2 ; CBSE-2016]**

OR

Calculate the wavelength of radiation emitted when electron in a hydrogen atom jumps from $n = \infty$ to $n = 1$. **[2 ; CBSE-2016]**

51. State two important properties of photon which are used to write Einstein's photoelectric equation. Define (i) stopping potential and (ii) threshold frequency, using Einstein's equation and drawing necessary plot between relevant quantities. **[3 ; CBSE-2016]**

52. (a) Derive the mathematical expression for law of radioactive decay for a sample of a radioactive nucleus. **[3 ; CBSE-2016]**

(b) How is the mean life of a given radioactive nucleus related to the decay constant ?

53. A 12·5 eV electron beam is used to excite a gaseous hydrogen atom at room temperature. Determine the wavelengths and the corresponding series of the lines emitted. **[CBSE-2017]**

54. Using photon picture of light, show how Einstein's photoelectric equation can be established. Write two features of photoelectric effect which cannot be explained by wave theory. **[CBSE-2017]**

55. Draw graphs showing variation of photoelectric current with applied voltage for two incident radiations of equal frequency and different intensities. Mark the graph for the radiation of higher intensity. **[CBSE-2018]**

56. Four nuclei of an element undergo fusion to form a heavier nucleus, with release of energy. Which of the two – the parent or the daughter necleus – would have higher binding energy per nucleon? **[CBSE-2018]**

57. If light of wavelength 412.5 nm is incident on each of the metals given below, which ones will show photoelectric emission and why? **[CBSE-2018]**

Metal	Work Function (eV)
Na	1.92
K	2.15
Ca	3.20
Mo	4.17

58. (a) State Bohr's psotulate to define stable orbits in hydrogen atom. How does de-Broglie's hypothesis explain the stability of these orbits?

(b) A hydrogen atom initially in the grond state absorbs a photon which excites it to the n = 4 level. Estimate the frequency of the photon. **[CBSE-2018]**

59. (a) Explain the processes of nuclear fission and nuclear fusion by using the plot of binding energy per nucleon (BE/A) versus the mass number A.

(b) A radioactive isotope has a half-lift of 10 years. How long will it take for the activity to resuce to 3.125%? **[CBSE-2018]**

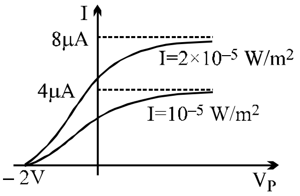
ANSWER KEY

EXERCISE (S-1)

1. Ans. 6.59×10^{-34} Js

2. Ans. (a) 3.38×10^{-19} J = 2.11 eV (b) 3.0×10^{20} photons/s

3. Ans.



4. Ans. Use $eV_0 = h\nu - \phi_0$ for both sources. From the data on the first source, ϕ_0 = 1.40 eV. Use the value to obtain for the second suorce V_0 = 1.50 V.

5. Ans. (a) 3.14×10^{-19} J , 1.05×10^{-27} kg m/s (b) 3×10^{16} photons/s (c) 0.63 m/s

6. Ans. (a) 4.04×10^{-24} kg ms^{-1} (b) 0.164 nm

7. Ans. (a) 6.95×10^{-25} J = 4.34 μeV (b) 3.78×10^{-28} J = 0.236 neV

8. Ans. (a) 6.686×10^{-21} J = 4.174×10^{-2} eV (b) 0.145 nm

9. Ans. $\sqrt{2}$

10. Ans. 4

11. Ans. 0.55 eV

12. Ans. 7

13. Ans. 2

14. Ans. 820 nm.

15. Ans. Lyman series: 103 nm and 122 nm; Balmer series: 656 nm.

16. Ans. (a) +3.4 eV; (b) –6.8 eV; (c) potential energy

17. Ans. 2.56×10^{-13} m; –2.8 keV

18. Ans. 8

19. Ans. n = 24

20. Ans. 2

21. Ans. Use $\lambda = (hc/E)$ with $E = 5.1 \times 1.602 \times 10^{-10}$ J to get $\lambda = 2.43 \times 10^{-16}$ m.

22. Ans. 6210 eV

23. Ans. 400

EXERCISE (S-2)

1. Ans. 5

2. Ans. 1

3. Ans. 1.1×10^{12}

4. Ans. (a) 10^5 s^{-1}; (b) 286.18; (d) 111 s

5. Ans. $\lambda = \dfrac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}}$

6. Ans. KE ≅ 151 eV, d_{least} = 0.5 Å

7. Ans. (i) 2 ; (ii) 13.6 eV ; (iii) 12.75 eV, 0.66 eV

8. Ans. 6.8 eV, 5×10^{15} Hz

9. Ans. (i) Allowed values of energy of neutron = 6.36 eV and 0.312 eV ; Allowed values of energy of He atom = 17.84 eV and 16.328 eV, (ii) 18.23×10^{14} Hz , 9.846×10^{15} Hz, 11.6×10^{15} Hz

10. Ans. 7.4 eV, 4.7 Volts

11. Ans. 6

12. Ans. 255

13. Ans. 3

14. Ans. 159

15. Ans. 255

16. Ans. 2

17. Ans. 98.25 eV

18. Ans. (i) 0.62Å, (ii) No, (iii) Yes 0.8 Å

19. Ans. 41

EXERCISE (O-1)

1. Ans. (A) 2. Ans. (A) 3. Ans. (B) 4. Ans. (B) 5. Ans. (B) 6. Ans. (C)

7. Ans. (A) 8. Ans. (A) 9. Ans. (D) 10. Ans. (B) 11. Ans. (B) 12. Ans. (A)

13. Ans. (B) 14. Ans. (B) 15. Ans. (D) 16. Ans. (D) 17. Ans. (D) 18. Ans. (A)

19. Ans. (D) 20. Ans. (A) 21. Ans. (C) 22. Ans. (D) 23. Ans. (D) 24. Ans. (B)

25. Ans. (A) 26. Ans. (A) 27. Ans. (A) 28. Ans. (A) 29. Ans. (A) 30. Ans. (A)

31. Ans. (A) 32. Ans. (D) 33. Ans. (B) 34. Ans. (A) 35. Ans. (B) 36. Ans. (B)

37. Ans. (D) 38. Ans. (B) 39. Ans. (A) 40. Ans. (A) 41. Ans. (D) 42. Ans. (B)

43. Ans. (C) 44. Ans. (D) 45. Ans. (D) 46. Ans. (D) 47. Ans. (C) 48. Ans. (D)

49. Ans. (C) 50. Ans. (A) 51. Ans. (C) 52. Ans. (A,B,C) 53. Ans. (A,B,D)

54. Ans. (A,C) 55. Ans. (A,B,C) 56. Ans. (B,C,D) 57. Ans. (A,D)

58. Ans. (A,B,C,D) 59. Ans. (A,B,D) 60. Ans. (A,B,C,D) 61. Ans. (B, D)

62. Ans. (A,C,D) 63. Ans. (A,B,C) 64. Ans. (A,C,D) 65. Ans. (A,C,D)

66. Ans. (B) 67. Ans. (A,C) 68. Ans. (A,C,D) 69. Ans. (A,B,C,D)

70. Ans. (A, B, C) 71. Ans. (A, C) 72. Ans. (A) P,R,S (B) P,Q (C) P,Q (D) R,S

73. Ans. (A)-QR, (B)-PS, (C)-QS, (D)-PR

74. Ans. (A) $\rightarrow$(P,R) ; (B) $\rightarrow$(Q,S) ; (C) $\rightarrow$(Q, S) ; (D)$\rightarrow$(P,R) 75. Ans. (B)

76. Ans. (A)$\rightarrow$ (S); (B) $\rightarrow$ (R); (C) $\rightarrow$ (Q); (D) $\rightarrow$ (P)

EXERCISE (O-2)

1. Ans. (D) 2. Ans. (C) 3. Ans. (A) 4. Ans. (B,D) 5. Ans. (B) 6. Ans. (A)

7. Ans. (B) 8. Ans. (D) 9. Ans. (C) 10. Ans. (C) 11. Ans. (D) 12. Ans. (B)

13. Ans. (B) 14. Ans. (B) 15. Ans. (D) 16. Ans. (A,C,D) 17. Ans. (D)

18. Ans. (A,B) 19. Ans. (B,C,D) 20. Ans. (B, C) 21. Ans. (A, B, D)

22. Ans. (B, C) 23. Ans. (B,C) 24. Ans. (A,C,D) 25. Ans. (A)

26. Ans. (A) 27. Ans. (B) 28. Ans. (A) 29. Ans. (A) 30. Ans. (B) 31. Ans. (B)

32. Ans. (C) 33. Ans. (B) 34. Ans. (C) 35. Ans. (D) 36. Ans. (A)

EXERCISE (JM)

1. Ans. (2) 2. Ans. (2) 3. Ans. (4) 4. Ans. (1) 5. Ans. (2) 6. Ans. (1)

7. Ans. (2) 8. Ans. (1) 9. Ans. (4) 10. Ans. (4) 11. Ans. (3) 12. Ans. (1)

13. Ans. (2) 14. Ans. (3) 15. Ans. (2) 16. Ans. (4) 17. Ans. (3)

EXERCISE (JA)

1. Ans. (B) 2. Ans. (C) 3. Ans. (C) 4. Ans. (A) 5. Ans. (A) 6. Ans. (B)

7. Ans. (D) 8. Ans. (A) 9. Ans. 3 10. Ans. (D) 11. Ans. (B) 12. Ans. (C)

13. Ans. 8 14. Ans. (A) 15. Ans. 7 16. Ans. 7 17. Ans. 1 18. Ans. (A, C)

19. Ans. 2 20. Ans. (A), (C), (D) 21. Ans. (A,C) 22. Ans. 2 23. Ans. (B)

24. Ans. (A, B, D) 25. Ans. 6 26. Ans. (A) 27. Ans. 5 28. Ans. (A)

29. Ans. 24 [23.60, 24.40] 30. Ans. 3 [3,3] 31. Ans. (2,3) 32. Ans. (1.00)

Chapter Contents

MODERN PHYSICS-2

MODERN PHYSICS-2

KEY CONCEPT

NUCLEAR PHYSICS

ATOMIC NUCLEUS

The atomic nucleus consists of two types of elementary particles, viz. protons and neutrons. These particles are called nucleons. The proton (denoted by p) has a charge +e and a mass $m_p = 1.6726 \times 10^{-27}$ kg, which is approximately 1840 times larger than the electron mass. The proton is the nucleus of the simplest atom with Z=1, viz the hydrogen atom.

The neutron (denoted by n) is an electrically neutral particle (its charge is zero). The neutron mass is 1.6749×10^{-27} kg. The fact that the mass of a neutron exceeds the mass of a proton by about 2.5 times the electronic masses is of essential importance. It follows from this that the neutron in free state (outside the nucleus) is unstable (radioactive). With half life equal to 12 min, the neutron spontaneously transforms into a proton by emitting an electron (e^-) and a particle called the antineutrino $(\bar{\nu})$.

This process can be schematically written as follows : ${}_0n^1 \rightarrow {}_1p^1 + {}_{-1}e^0 + \bar{\nu}$

The most important characteristics of the nucleus are the charge number Z (coinciding with atomic number of the element) and mass number A. The charge number Z is equal to the number of protons in the nucleus, and hence it determines the nuclear charge equal to Ze. The mass number A is equal to the number of nucleons in the nucleus (i.e., to the total number of protons and neutrons). Nuclei are symbolically designated as X_Z^A or ${}_ZX^A$ where X stands for the symbol of a chemical element.

For example, the nucleus of the oxygen atom is symbolically written as O_8^{16} or ${}_8O^{16}$.

The shape of nucleus is approximately spherical and its radius is approximately related to the mass number by

$R = 1.2\ A^{1/3} \times 10^{-15}\ m = 1.2 \times 10^{-15} \times A^{1/3}\ m$

Most of the chemical elements have several types of atoms differing in the number of neutrons in their nuclei. These varieties are called isotopes. For example carbon has three isotopes ${}_6C^{12}$, ${}_6C^{13}$, ${}_6C^{14}$. In addition to stable isotopes, there also exist unstable (radioactive) isotopes. Atomic masses are specified in terms of the atomic mass unit or unified mass unit (u). The mass of a neutral atom of the carbon ${}_6C^{12}$ is defined to be exactly 12 u.

$1u = 1.66056 \times 10^{-27}$ kg = 931.5 MeV.

BINDING ENERGY

The rest mass of the nucleus is smaller than the sum of the rest masses of nucleons constituting it. This is due to the fact that when nucleons combine to form a nucleus, some energy (binding energy) is liberated. The binding energy is equal to the work that must be done to split the nucleus into the particles constituting it.

The difference between the total mass of the nucleons and mass of the nucleus is called the mass defect of the nucleus represented by $\Delta m = [Zm_p + (A-Z)m_n] - m_{nuc}$

Multiplying the mass defect by the square of the velocity of light, we can find the binding energy of the nucleus.

$$BE = \Delta mc^2 = [(Zm_p + (A-Z)m_n) - m_{nuc}]c^2$$

If the masses are taken in atomic mass unit, the binding energy is given by

$$BE = [(Zm_p + (A-Z)m_n) - m_{nuc}]\ 931.5\ \text{MeV}$$

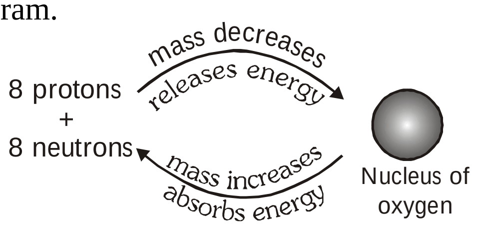
Let us take example of oxygen nucleus. It contains 8 protons and 8 neutrons.

We can discuss concept of binding energy by following diagram.

$8m_p + 8m_n >$ mass of nucleus of oxygen

For nucleus we apply mass energy conservation,

$$8m_p + 8m_n = \text{mass of nucleus} + \frac{B.E.}{c^2}$$



For general nucleus ${}^{A}_{Z}X$, mass defect = difference between total mass of nucleons and mass of the nucleus

$$\Delta m = [Zm_p + (A-Z)m_n] - M$$

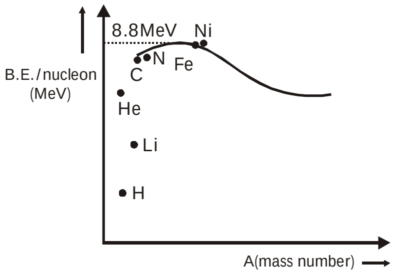
$$B.E. = \Delta mc^2\ (\text{joules}) = (\Delta m)_{\text{in amu}} \times\ 931.5\ \text{MeV}$$

Binding Energy per Nucleon

Stability of a nucleus does not depend upon binding energy of a nucleus but it depends upon binding energy per nucleon

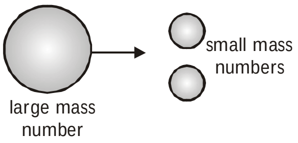
$$\text{B.E./nucleon} = \frac{\text{B.E.}}{\text{mass number}}$$

$$\text{Stability} \propto \frac{\text{B.E.}}{A}$$

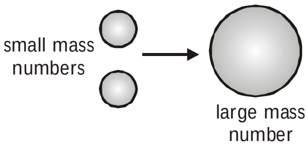


(i) B.E./A is maximum for A =62 (Ni), It is 8.79460 ± 0.00003 MeV/nucleon, means most stable nuclei are in the region of A = 62.

(ii) Heavy nuclei achieve stability by breaking into two smaller nuclei and this reaction is called fission reaction.



(iii) Nuclei achieve stability by combining and resulting into heavy nucleus and this reaction is called fusion reaction.



(iv) In both reactions products are more stable in comparison to reactants and Q value is positive.

NUCLEAR COLLISIONS

We can represent a nuclear collision or reaction by the following notation, which means X (a,b) Y

$$\underset{\text{(bombarding particle)}}{a} \quad + \quad \underset{\text{(at rest)}}{X} \quad \rightarrow Y + b$$

We can apply :

(i) Conservation of momentum (ii) Conservation of charge (iii) Conservation of mass–energy

For any nuclear reaction $\underset{K_1}{a} + \underset{K_2}{X} \rightarrow \underset{K_3}{Y} + \underset{K_4}{b}$

By mass energy conservation

(i) $K_1 + K_2 + (m_\alpha + m_x)c^2 = K_3 + K_4 + (m_Y + m_b)c^2$

(ii) Energy released in any nuclear reaction or collision is called Q value of the reaction

(iii) $Q = (K_3 + K_4) - (K_1 + K_2) = \Sigma K_P - \Sigma K_R = (\Sigma m_R - \Sigma m_P)c^2$

(iv) If Q is positive, energy is released and products are more stable in comparison to reactants.

(v) If Q is negative, energy is absorbed and products are less stable in comparison to reactants.

$Q = \Sigma(\text{B.E.})_{\text{product}} - \Sigma(\text{B.E.})_{\text{reactants}}$

Ex. Let us find the Q value of fusion reaction

$^4He + {}^4He \rightarrow {}^8Be$, if $\frac{\text{B.E.}}{A}$ of He = X and $\frac{\text{B.E.}}{A}$ of Be = Y $\Rightarrow Q = 8Y - 8X$

Q value for α decay $\quad {}_ZX^A \rightarrow {}_{Z-2}Y^{A-4} + {}_2He^4 \Rightarrow Q = K_\alpha + K_Y$ ....(i)

Momentum conservation, $p_Y = p_\alpha$ ...(ii)

$$K_\alpha = \frac{p^2}{2 \times m \times 4} \qquad K_Y = \frac{p^2}{2m(A-4)} = \frac{4K_\alpha}{A-4}$$

$$Q = K_\alpha + \frac{4K_\alpha}{A-4} = \frac{A}{A-4}K_\alpha \qquad K_\alpha = \frac{A-4}{A}Q$$

For α decay A > 210 which means maximum part of released energy is associated with K.E. of α. If Q is negative, the reaction is endoergic. The minimum amount of energy that a bombarding particle must have in order to initiate an endoergic reaction is called Threshold energy E_{th},

given by $E_{th} = -Q\left(\frac{m_1}{m_2} + 1\right)$ where m_1 = mass of the projectile.

E_{th} = minimum kinetic energy of the projectile to initiate the nuclear reaction

m_2 = mass of the target

Ex. How much energy must a bombarding proton possess to cause the reaction ${}_3Li^7 + {}_1H^1 \rightarrow {}_4Be^7 + {}_0n^1$ (Mass of ${}_3Li^7$ atom is 7.01600, mass of ${}_1H^1$ atom is 1.0783, mass of ${}_4Be^7$ atom is 7.01693)

Sol. Since the mass of an atom includes the masses of the atomic electrons, the appropriate number of electron masses must be subtracted from the given values.

Reactants : Total mass = $(7.01600 - 3\,m_e) + (1.0783 - 1\,m_e) = 8.0943 - 4m_e$

Products : Total mass = $(7.01693 - 4m_e) + 1.0087 = 8.02563 - 4m_e$

The energy is supplied as kinetic energy of the bombarding proton. The incident proton must have more than this energy because the system must possess some kinetic energy even after the reaction, so that momentum is conserved with momentum conservation taken into account, the minimum kinetic energy that the incident particle must possess can be found with the formula.

where, $Q = -[(8.02563 - 4m_e) - (8.0943 - 4\ m_e)]\ 931.5\ \text{MeV} = -\ 63.96\ \text{MeV}$

$$E_{th} = -\left(1+\frac{m}{M}\right) Q = -\left(1+\frac{1}{7}\right)(-63.96) = 73.1\ \text{MeV}$$

NUCLEAR FISSION

In 1938 by Hahn and Strassmann. By attack of a particle splitting of a heavy nucleus (A > 230) into two or more lighter nuclei. In this process certain mass disappears which is obtained in the form of energy (enormous amount)

$$A + p \rightarrow \text{excited nucleus} \rightarrow B + C + Q$$

Hahn and Strassmann done the first fision of nucleus of U^{235}).

When U^{235} is bombarded by a neutron it splits into two fragments and 2 or 3 secondary neutrons and releases about 190 MeV (≃ 200 MeV) energy per fission (or from single nucleus)

Fragments are uncertain but each time energy released is almost same.

Possible reactions are

$$U^{235} + {}_0n^1 \rightarrow Ba + Kr + 3{}_0n^1 + 200\ \text{MeV or } U^{235} + {}_0n^1 \rightarrow Xe + Sr + 2{}_0n^1 + 200\ \text{MeV}$$

and many other reactions are possible.

- The average number of secondary neutrons is 2.5.
- Nuclear fission can be explained by using "liquid drop model" also.
- The mass defect Δm is about 0.1% of mass of fissioned nucleus
- About 93% of released energy (Q) is appear in the form of kinetic energies of products and about 7% part in the form of γ – rays.

NUCLEAR CHAIN REACTION :

The equation of fission of U^{235} is $U^{235} + {}_0n^1 \rightarrow Ba + Kr + 3{}_0n^1 + Q$.

These three secondary neutrons produced in the reaction may cause of fission of three more U^{235} and give 9 neutrons, which in turn, may cause of nine more fission of U^{235} and so on.

Thus a continuous 'Nuclear Chain reaction' would start.

If there is no control on chain reaction then in a short time ($\simeq 10^{-6}$ sec.) a huge amount of energy will be released. (This is the principle of 'Atom bomb'). If chain is controlled then produced energy can be used for peaceful purposes. For example nuclear reactor (Based on fission) are generating electricity.

NATURAL URANIUM :

It is mixture of U^{235} (0.7%) and U^{238} (99.3%).

U^{235} is easily fissionable, by slow neutron (or thermal neutrons) having K.E. of the order of 0.03 eV.

But U^{238} is fissionable with fast neutrons.

Note : Chain reaction in natural uranium can't occur

To improve the quality, percentage of U^{235} is increased to 3%.

The proposed uranium is called 'Enriched Uranium' (97% U^{238} and 3% U^{235})

LOSSES OF SECONDARY NEUTRONS :

Leakage of neutrons from the system : Due to their maximum K.E. some neutrons escape from the system.

Absorption of neutrons by U^{238} : Which is not fissionable by these secondary neutrons.

CRITICAL SIZE (OR MASS) :

In order to sustain chain reaction in a sample of enriched uranium, it is required that the number of lost neutrons should be much smaller than the number of neutrons produced in a fission process. For it the size of uranium block should be equal or greater than a certain size called **critical size**.

REPRODUCTION FACTOR :

$$(K) = \frac{\text{rate of production of neutrons}}{\text{rate of loss of neutrons}}$$

(i) If size of Uranium used is 'Critical' then $K = 1$ and the chain reaction will be steady or sustained (As in nuclear reaction)

(ii) If size of Uranium used is 'Super critical' then $K > 1$ and chain reaction will accelerate resulting in a explosion (As in atom bomb)

(iii) If size of Uranium used is 'Sub Critical' then $K < 1$ and chain reaction will retard and will stop.

NUCLEAR REACTOR (K = 1) : Credit → To Enricho Fermi

Construction :

- **Nuclear Fuel :** Commonly used are U^{235} , Pu^{239} .Pu^{239} is the **best**. Its critical size is less than critical size of U^{235}.

  But Pu^{239} is not naturally available and U^{235} is used in most of the reactors.
- **Moderator** : Its function is to slow down the fast secondary neutrons. Because only slow neutrons can bring the fission of U^{235}. The moderator should be light and it should not absorb the neutrons. Commonly, Heavy water (D_2O, molecular weight 20 gm.) Graphite etc. are used. These are rich of protons. Neutrons collide with the protons and interchange their energy. Thus neutrons get slow down.
- **Control rods** :They have the ability to capture the slow neutrons and can control the chain reaction at any stage. Boron and Cadmium are best absorber of neutrons.
- **Coolant** : A substance which absorb the produced heat and transfers it to water for further use. Generally coolant is water at high pressure

FAST BREADER REACTORS

The atomic reactor in which fresh fissionable fuel (Pu^{239}) is produced along with energy. The amount of produced fuel (Pu^{239}) is more than consumed fuel (U^{235})

- **Fuel :** Natural Uranium.
- **Process**: During fission of U^{235}, energy and secondary neutrons are produced. These secondary neutrons are absorbed by U^{238} and U^{239} is formed. This U^{239} converts into Pu^{239} after two beta decay. This Pu^{239} can be separated, its half life is 2400 years.

$${}_{92}U^{238} + {}_{0}n^{1} \longrightarrow {}_{92}U^{239} \xrightarrow{2\beta^-} {}_{94}Pu^{239} \text{ (best fuel of fission)}$$

This Pu^{239} can be used in nuclear weapons because of its small critical size than U^{235}.

- **Moderator :** Are not used in these reactors.
- **Coolant :** Liquid sodium

NUCLEAR FUSION :

It is the phenomenon of fusing two or more lighter nuclei to form a single heavy nucleus.

$$A + B \rightarrow C + Q \text{ (Energy)}$$

The product (C) is more stable then reactants (A and B) and $m_c < (m_a + m_b)$

and mass defect $\Delta m = [(m_a + m_b) - m_c]$ amu

Energy released is $E = \Delta m$ 931 MeV

The total binding energy and binding energy per nucleon C both are more than of A and B.

$$\Delta E = E_c - (E_a + E_b)$$

Fusion of four hydrogen nuclei into helium nucleus :

$$4({}_1H^1) \rightarrow {}_2He^4 + 2\ {}_+\beta^0 + 2\,\nu + 26 \text{ MeV}$$

- Energy released per fission >> Energy released per fusion
- Energy per nucleon in fission $\left[= \frac{200}{235} \simeq 0.85\text{MeV}\right]$ << energy per nucleon in fusion $\left[= \frac{24}{4} \simeq 6\text{MeV}\right]$

REQUIRED CONDITION FOR NUCLEAR FUSION

- **High temperature** :

  Which provide kinetic energy to nuclei to overcome the repulsive electrostatic force between them.
- **High Pressure (or density)** :

  Which ensure frequent collision and increases the probability of fusion. The required temperature and pressure at earth (lab) are not possible. These condition exist in the sun and in many other stars. The source of energy in the sun is nuclear fusion, where hydrogen is in plasma state and there protons fuse to form helium nuclei.

HYDROGEN BOMB

It is based on nuclear fusion and produces more energy than an atom bomb.

Pair production	**Pair Annihilation**
A γ–photon of energy more than 1.02 MeV,when interact with a nucleus produces pair of electron (e^-) and positron (e^+). The energy equivalent to rest mass of e^- (or e^+)=0.51 MeV. The energy equivalent to rest mass of pair ($e^- + e^+$) = 1.02 MeV. For pair production Energy of photon $\geq$ 1.02 MeV. If energy of photon is more than 1.02 MeV, the extra energy (E–1.02) MeV devides approximately in equal amount to each particle as the kinetic energy or $(KE)_{e- \text{ or } e+} = \left[\frac{E_{Ph} - 1.02}{2}\right]$MeV If E < 1.02 MeV, pair will not produce.	When electron and positron combines they annihilates to each other and only energy is released in the form of two gama photons.If the energy of electron and positron are negligible then energy of each γ–photon is 0.5/MeV

Ex. In a nuclear reactor, fission is produced in 1 g for U^{235} (235.0439) in 24 hours by slow neutrons (1.0087 u). Assume that $_{35}Kr^{92}$ (91.8973 u) and $_{56}Ba^{141}$ (140.9139 amu) are produced in all reactions and no energy is lost.

(i) Write the complete reaction (ii) Calculate the total energy produced in kilowatt hour.

Given 1u = 931 MeV.

Sol. The nuclear fission reaction is $_{92}U^{235} + {_0}n^1 \rightarrow {_{56}}Ba^{141} + {_{36}}Kr^{92} + 3{_0}n^1$

Mass defect $\Delta m = [(m_u + m_n) - (m_{Ba} + m_{Kr} + 3m_n)] = 256.0526 - 235.8373 = 0.2153$ u

Energy released Q = 0.2153 × 931 = 200 MeV. Number of atoms in 1 g = $\frac{6.02\times10^{23}}{235} = 2.56 \times 10^{21}$

Energy released in fission of 1 g of U^{235} is $E = 200 \times 2.56 \times 10^{21} = 5.12 \times 10^{23}$ MeV

$= 5.12 \times 10^{23} \times 1.6 \times 10^{-13} = 8.2 \times 10^{10}$ J

$= \frac{8.2\times10^{10}}{3.6\times10^{6}}$ kWh $= 2.28 \times 10^4$ kWh

Ex. It is proposed to use the nuclear fusion reaction : $_1H^2 + {_1}H^2 \rightarrow {_2}He^4$ in a nuclear reactor of 200 MW rating. If the energy from above reaction is used with at 25% efficiency in the reactor, how many grams of deuterium will be needed per day. (Mass of $_1H^2$ is 2.0141 u and mass of $_2He^4$ is 4.0026 u)

Sol. Energy released in the nuclear fusion is $Q = \Delta mc^2 = \Delta m(931)$ MeV (where Δm is in amu)

$Q = (2 \times 2.0141 - 4.0026) \times 931$ MeV $= 23.834$ MeV $= 23.834 \times 10^6$ eV

Since efficiency of reactor is 25%

So effective energy used $= \frac{25}{100}\times 23.834\times10^{6}\times1.6\times10^{-19}$ J $= 9.534 \times 10^{-13}$ J

Since the two deuterium nuclei are involved in a fusion reaction,

therefore, energy released per deuterium is $\frac{9.534\times10^{-13}}{2}$.

For 200 MW power per day, number of deuterium nuclei required

$$= \frac{200\times10^{6}\times86400}{\frac{9.534\times10^{-23}}{2}} = 3.624 \times 10^{25}$$

Since 2g of deuterium constitute 6×10^{23} nuclei, therefore amount of deuterium required is

$$= \frac{2\times3.624\times10^{25}}{6\times10^{23}} = 120.83 \text{ g/day}$$

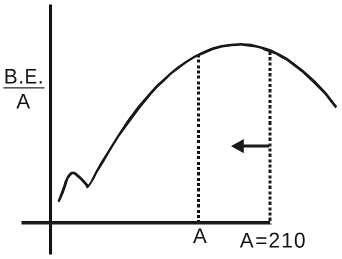
RADIOACTIVITY

The process of spontaneous disintegration shown by some unstable atomic nuclei is known as natural radioactivity. This property is associated with the emission of certain types of penetrating radiations, called radioactive rays, or Becquerel rays (α, β, γ–rays). The elements or compounds, whose atoms disintegrate and emit radiations are called radioactive elements. Radioactivity is a continuous, irreversible nuclear phenomenon.

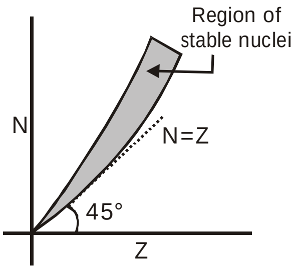
Radioactive Decays

Generally, there are three types of radioactive decays (i) α decay (ii) β^- and β^+ decay (iii) γ decay

- **α decay :** In α decay, the unstable nucleus emits an α particle. By emitting α particle, the nucleus decreases it's mass energy number and move towards stability. Nucleus having A>210 shows α decay.By releasing α particle, it can attain higher stability and Q value is positive.



- **β decay :** In beta decay (N/Z) ratio of nucleus is changed. This decay is shown by unstable nuclei. In beta decay, either a neutron is converted into proton or proton is converted into neutron. For better understanding we discuss N/Z graph. There are two type of unstable nuclides



  - **A type**

    For A type nuclides $(N/Z)_A > (N/Z)$ stable

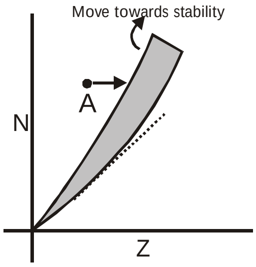
    To achieve stability, it increases Z by conversion of neutron into proton

    $${}_0n^1 \rightarrow {}_1p^1 + e^{-1} + \bar{\nu},\ {}_ZX^A \rightarrow {}_{Z+1}Y^A + \underset{(\beta\text{ particle})}{e^{-1}} + \bar{\nu}$$

    This decay is called β^{-1} decay.

    Kinetic energy available for e^{-1} and $\bar{\nu}$ is, $Q = K_\beta + K_{\bar{\nu}}$

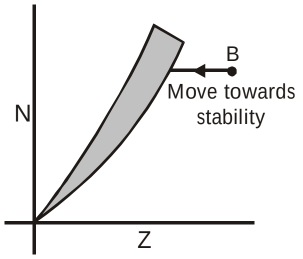
    K.E. of β satisfies the condition $0 < K_\beta < Q$



  - **B type**

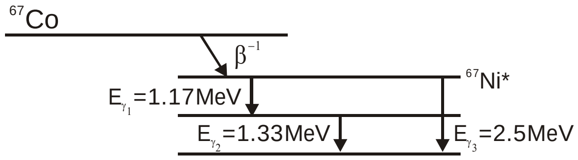
    For B type nuclides $(N/Z)_B < (N/Z)$ stable

    To achieve stability it decreases Z by the conversion of a proton into neutron. That is, $p \rightarrow n + \underset{(\text{positron})}{e^+} + \underset{(\text{neutrino})}{\nu}$, ${}_ZX^A \rightarrow {}_{Z-1}Y^A + \underset{(\beta^+\text{particle})}{e^+} + \nu$



- **γ decay :** When an α or β decay takes place, the daughter nucleus is usually in higher energy state, such a nucleus comes to ground state by emitting a photon or photons.

  Order of energy of γ photon is 100 KeV



e.g. ${}^{67}_{27}Co \rightarrow \underset{(\text{higher energy state})}{{}^{67}_{28}Ni^*} + \beta^- + \bar{\nu}$, ${}^{67}_{28}Ni^* \rightarrow {}^{67}_{28}Ni + \gamma$ photon

Properties of α, β and γ rays

Features	**α–particles**	**β–particles**	**γ–rays**
Identity	Helium nucleus or doubly ionised helium atom (${}_2He^4$)	Fast moving electrons (${}_{-}\beta^0$ or β^-)	Electromagnetic wave (photons)
Charge Mass	Twice of proton (+2e) ≈ $4m_p$ m_p–mass of proton	Electronic (– e) (rest mass of β) = (rest mass of electron)	Neutral rest mass = 0
Speed	1.4×10^7 m/s. to 2.2×10^7 m/s. (Only certain value between this range). Their speed depends on nature of the nucleus. So that it is a characteristic speed.	1% of c to 99% of c (All possible values between this range) β–particles come out with different speeds from the same type of nucleus. So that it can not be a characteristic speed.	Only $c = 3 \times 10^8$ m/s γ–photons come out with same speed from all types of nucleus. So, can not be a characteristic speed.
K.E.	≈ MeV	≈ MeV	≈ MeV
Energy spectrum	Line and discrete (or linear)	Continuous (or linear)	Line and discrete
Ionization power (α>β>γ)	10,000 times of γ–rays	100 times of γ–rays (or $\frac{1}{100}$ times of α)	1 (or $\frac{1}{100}$ times of β)
Penetration power (γ>β>α)	$\frac{1}{10000}$ times of γ–rays	$\frac{1}{100}$ times of γ–rays (100 times of α)	1(100 times of β)
Effect of electric or magnetic field	Deflection	Deflection (More than α)	No deflection
Explanation of emission	By Tunnel effect (or quantum mechanics)	By weak nuclear interactions	With the help of energy levels in nucleus

Laws of Radioactive Decay

1. The radioactive decay is a spontaneous process with the emission of α, β and γ rays. It is not influenced by external conditions such as temperature, pressure, electric and magnetic fields.

2. The rate of disintegration is directly proportional to the number of radioactive atoms present at that time i.e., rate of decay $\propto$ number of nuclei.

Rate of decay = λ (number of nuclei) i.e. $\frac{dN}{dt} = -\lambda N$

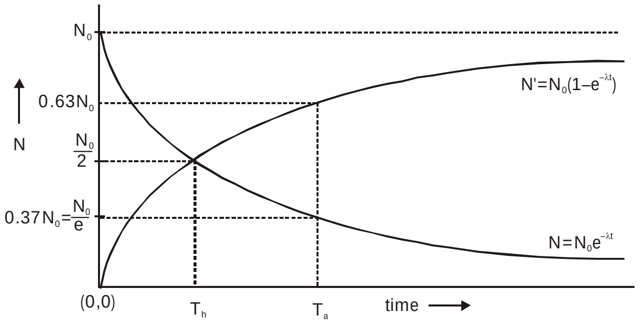
where λ is called the decay constant. This equation may be expressed in the form $\frac{dN}{N} = -\lambda dt$.

$$\int_{N_0}^{N}\frac{dN}{N} = -\lambda\int_0^t dt \Rightarrow \ell n\left(\frac{N}{N_0}\right) = -\lambda t$$

where N_0 is the number of parent nuclei at t=0. The number that survives at time t is therefore

$N=N_0e^{-\lambda t}$ and $t = \frac{2.303}{\lambda}\log_{10}\left(\frac{N_0}{N_t}\right)$ this function is plotted in figure.

Graph : Time versus N (or N')



- **Half life (T_h)** : It is the time during which number of active nuclei reduce to half of initial value.

If at t = 0 no. of active nuclei N_0 then at t = T_h number of active nuclei will be $\frac{N_0}{2}$

From decay equation $N = N_0e^{-\lambda t}$

$$\frac{N_0}{2} = N_0e^{-\lambda T}n \Rightarrow T_h = \frac{\ell n(2)}{\lambda} = \frac{0.693}{\lambda} \approx \frac{0.7}{\lambda}$$

- **Mean or Average Life (T_a)** : It is the average of age of all active nuclei i.e.

$$T_a = \frac{\text{sum of times of existance of all nuclei in a sample}}{\text{initial number of active nuclei in that sample}} = \frac{1}{\lambda}$$

(i) At t = 0, number of active nuclei = N_0 then number of active nuclei at

t = T_a is $N = N_0e^{-\lambda T_a} = N_0e^{-1} = \frac{N_0}{e} = 0.37N_0 = 37\%$ of N_0

(ii) Number nuclei which have been disintegrated within duration T_a is
$N' = N_0 - N = N_0 - 0.37\ N_0 = 0.63\ N_0 = 63\%$ of N_0

- $T_a = \frac{1}{\lambda} = \frac{T_h}{\ell n(2)} = \frac{T_h}{0.693} = 1.44\ T_h$
- Within duration $T_h \Rightarrow$ 50% of N_0 decayed and 50% of N_0 remains active
- Within duration $T_a \Rightarrow$ 63% of N_0 decayed and 37% of N_0 remains active

ACTIVITY OF A SAMPLE (A OR R) (OR DECAY RATE)

It is the rate of decay of a radioactive sample $R = -\frac{dN}{dt} = N\lambda$ or $R = R_0e^{-\lambda t}$

- Activity of a sample at any instant depends upon number of active nuclei at that instant.
  $R \propto N$ (or active mass) , $R \propto m$
- R also decreases exponentially w.r.t. time same as the number of active nuclei decreases.

- R is not a constant with N, m and time while λ, T_h and T_a are constant
- At t = 0, R = R_0 then at t = T_h $\Rightarrow R = \frac{R_0}{2}$ and at t = $T_a \Rightarrow$ $R = \frac{R_0}{e}$ or $0.37\ R_0$
- Similarly active mass of radioactive sample decreases exponentially. $m = m_0e^{-\lambda t}$
- Activity of m gm active sample (molecular weight M_w) is $R = \lambda N = \frac{0.693}{T_h}\left[\frac{N_{AV}}{M_W}\right]m$

here N_{AV} = Avogadro number = 6.023×10^{23}
SI UNIT of R : 1 becquerel (1 Bq)= 1 decay/sec
Other Unit is curie : 1 Ci = 3.70×10^{10} decays/sec
1 Rutherford : (1 Rd) =10^6 decays/s
Specific activity : Activity of 1 gm sample of radioactive substance. Its unit is Ci/gm
e.g. specific activity of radium (226) is 1 Ci/gm.

Ex. The half–life of cobalt–60 is 5.25 yrs. After how long does its activity reduce to about one eight of its original value?

Sol The activity is proportional to the number of undecayed atoms: In each half–life, the remaining sample decays to half of its initial value. Since $\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)=\frac{1}{8}$, therefore, three half–lives or 15.75 years are required for the sample to decay to 1/8th its original strength.

Ex. A count rate meter is used to measure the activity of a given sample. At one instant the meter shows 4750 counts per minute. Five minutes later it shows 2700 counts per minute.
(i) Find the decay constant. (ii) Also, find the half–life of the sample.

Sol. Initial activity $A_i = \left.\frac{-dN}{dt}\right|_{t=0} = \lambda N_0 = 4750$ ...(i) Final activity $A_f = \left.\frac{-dN}{dt}\right|_{t=5} = \lambda N = 2700$ ...(ii)

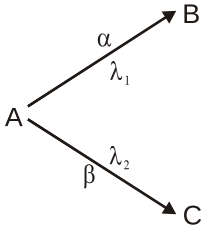
Dividing (i) by (ii), we get $\frac{4750}{2700} = \frac{N_0}{N_t}$

The decay constant is given by $\lambda = \frac{2.303}{t}\log\frac{N_0}{N_t} = \frac{2.303}{5}\log\frac{4750}{2700} = 0.113\ \text{min}^{-1}$

Half–life of the sample is $T = \frac{0.693}{\lambda} = \frac{0.693}{0.113}$ = 6.14 min

- **Parallel radioactive disintegration**

Let initial number of nuclei of A is N_0 then at any time number of nuclei of A, B & C are given by $N_0 = N_A + N_B + N_C \Rightarrow \frac{dN_A}{dt} = \frac{-d}{dt}(N_B + N_C)$



A disintegrates into B and C by emitting α, β particle.

Now, $\frac{dN_B}{dt} = -\lambda_1 N_A$ and $\frac{dN_C}{dt} = -\lambda_2 N_A \Rightarrow \frac{d}{dt}(N_B + N_C) = -(\lambda_1 + \lambda_2)N_A$

$\Rightarrow \frac{dN_A}{dt} = -(\lambda_1 + \lambda_2)N_A \Rightarrow \lambda_{eff} = \lambda_1 + \lambda_2 \Rightarrow t_{eff} = \frac{t_1t_2}{t_1 + t_2}$

Ex. The mean lives of a radioactive substances are 1620 and 405 years for α–emission and β–emission respectively. Find out the time during which three fourth of a sample will decay if it is decaying both by α–emission and β–emission simultaneously.

Sol. When a substance decays by α and β emission simultaneously, the average rate of disintegration λ_{av} is given by

$\lambda_{av} = \lambda_\alpha + \lambda_\beta$ when λ_α = disintegration constant for α–emission only λ_β = disintegration constant for β–emission only

Mean life is given by $T_m = \frac{1}{\lambda}$, $\lambda_{av} = \lambda_\alpha + \lambda_\beta \Rightarrow \frac{1}{T_m} = \frac{1}{T_\alpha} + \frac{1}{T_\beta} = \frac{1}{1620} + \frac{1}{405} = \frac{1}{324}$

$\lambda_{av} \times t = 2.303 \log \frac{N_0}{N_t}$, $\frac{1}{324} t = 2.303 \log \frac{100}{25} \Rightarrow t = 2.303 \times 324 \log 4 = 449$ years.

Ex. A radioactive decay is given by $A \xrightarrow[t_{1/2}=8\text{ yrs}]{} B$

Only A is present at t=0. Find the time at which if we are able to pick one atom out of the sample, then probability of getting B is 15 times of getting A.

Sol.

$$\begin{array}{llccc} & & A & \longrightarrow & B \\ \text{at} & t=0 & N_0 & & 0 \\ \text{at} & t=t & N & & N_0 - N \end{array}$$

Probability of getting A, $P_A = \frac{N}{N_0}$

Probability of getting B, $P_B = \frac{N_0 - N}{N_0} \Rightarrow P_B = 15\, P_A \Rightarrow \frac{N_0 - N}{N_0} = 15 \frac{N}{N_0} \Rightarrow N_0 = 16N \Rightarrow N = \frac{N_0}{16}$

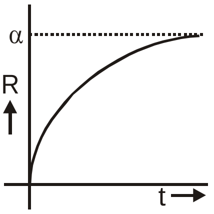
Remaining nuclei are $\frac{1}{16}$ th of initial nuclei, hence required time t = 4 half lives = 32 years

Radioactive Disintegration with Successive Production

$$\xrightarrow[(\alpha=\text{rate of production})]{\alpha} A \xrightarrow{\lambda} B$$

$\frac{dN_A}{dt} = \alpha = \lambda N_A$ ....(i)

when N_A in maximum $\frac{dN_A}{dt} = 0 = \alpha - \lambda N_A = 0$, N_A max $= \frac{\alpha}{\lambda} = \frac{\text{rate of production}}{\lambda}$



By equation (i) $\int_0^t \frac{dN_A}{\alpha - \lambda N_A} = \int_0^t dt$, Number of nuclei is $N_A = \frac{\alpha}{\lambda}\left(1 - e^{-\lambda t}\right)$

Ex. $\xrightarrow{10^{21}\text{ per sec}} A \xrightarrow{\lambda=\frac{1}{30}} B$

A shows radioactive disintegration and it is continuously produced at the rate of 10^{21} per sec. Find maximum number of nuclei of A.

Sol. At maximum, $r_{\text{production}} = r_{\text{decay}} \Rightarrow 10^{21} = \frac{1}{30}N \Rightarrow N=30 \times 10^{21}$

Soddy and Fajan's Group Displacement Laws :

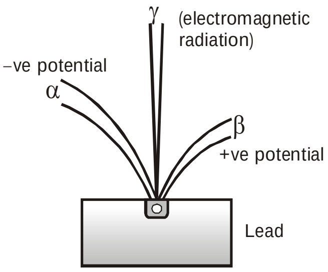
(i) **α–decay :** The emission of one α–particle reduces the mass number by 4 units and atomic number by 2 units. If parent and daughter nuclei are represented by symbols X and Y respectively then, ${}_{Z}X^{A} \rightarrow {}_{Z-2}Y^{A-4} + {}_{2}He^{4}(\alpha)$

(ii) **β–decay :** Beta particles are said to be fast moving electrons coming from the nucleus of a radioactive substance. Does it mean that a nucleus contains electrons? No, it is an established fact that nucleus does not contain any electrons. When a nucleus emits a beta particle, one of its neutrons breaks into a proton, an electron (i.e., β–particle) and an antineutrino $n \rightarrow p + e + \bar{\nu}$

where n= neutron p = proton e = β–particle

Thus emission of a beta particle is caused by the decay of a neutron into a proton. The daughter nucleus thus has an atomic number greater than one (due to one new proton in the nucleus) but same mass number as that of parent nucleus. Therefore, representing the parent and daughter nucleus by symbols X and Y respectively, we have ${}_{Z}X^{A} \rightarrow {}_{Z+1}Y^{A} + \beta + \bar{\nu}$

(iii) **γ–decay :** When parent atoms emit gamma rays, no charge is involved as these are neutral rays. Thus there is no effect on the atomic number and mass number of the parent nucleus. However the emission of γ–rays represents energy. Hence the emission of these rays changes the nucleus from an excited (high energy) state to a less excited (lower energy) state.



EXERCISE (S-1)

Nuclear Physics :

1. Suppose that the Sun consists entirely of hydrogen atom and releases the energy by the nuclear reaction, $4\,{}_1^1H \longrightarrow {}_2^4He$ with 26 MeV of energy released. If the total output power of the Sun is assumed to remain constant at 3.9×10^{26} W, find the time it will take to burn all the hydrogen. Take the mass of the Sun as 1.7×10^{30} kg.

MP0190

2. When two deuterons (${}_1H^2$) fuse to from a helium nucleus ${}_2He^4$, 23.6 MeV energy is released. Find the binding energy of helium if it is 1.1 MeV for each nucleon of deuterium.

MP0191

3. Highly energetic electrons are bombarded on a target of an element containing 30 neutrons. The ratio of radii of nucleus to that of helium nucleus is $(14)^{1/3}$. Find

(a) atomic number of the nucleus

(b) the frequency of K_α line of the X-ray produced. ($R = 1.1\times 10^7$ m^{-1} and $c = 3 \times 10^8$ m/s)

[JEE 2005]

MP0192

Radioactivity :

4. The kinetic energy of an α – particle which flies out of the nucleus of a Ra^{226} atom in radioactive disintegration is 4.78 MeV. Find the total energy evolved during the escape of the α – particle.

MP0193

5. U^{238} and U^{235} occur in nature in an atomic ratio 140 : 1. Assuming that at the time of earth's formation the two isotopes were present in equal amounts. Calculate the age of the earth.

(Half life of $u^{238} = 4.5 \times 10^9$ yrs & that of $U^{235} = 7.13 \times 10^8$ yrs)

MP0194

6. At t = 0, a sample is placed in a reactor. An unstable nuclide is produced at a constant rate R in the sample by neutron absorption. This nuclide β^- decays with half life τ. Find the time required to produce 80% of the equilibrium quantity of this unstable nuclide.

MP0195

EXERCISE (S-2)

1. An experiment is done to determine the half–life of radioactive substance that emits one β–particle for each decay process. Measurement show that 8.4 β are emitted each second by 2.5 mg of the substance. The atomic weight of the substance is 230. Find the half life of the substance.

MP0196

2. A wooden piece of great antiquity weighs 50 gm and shows C^{14} activity of 320 disintegrations per minute. Estimate the length of the time which has elapsed since this wood was part of living tree, assuming that living plants show a C^{14} activity of 12 disintegrations per minute per gm. The half life of C^{14} is 5730 yrs.

MP0197

3. A body of mass m_0 is placed on a smooth horizontal surface. The mass of the body is decreasing exponentially with disintegration constant λ. Assuming that the mass is ejected backward with a relative velocity u. Initially the body was at rest. Find the velocity of body after time t.

MP0198

4. A radionuclide with disintegration constant λ is produced in a reactor at a constant rate α nuclei per sec. During each decay energy E_0 is released. 20% of this energy is utilised in increasing the temperature of water. Find the increase in temperature of m mass of water in time t. Specific heat of water is S. Assume that there is no loss of energy through water surface.

MP0199

5. The element Curium $^{248}_{96}\text{Cm}$ has a mean life of 10^{13} seconds. Its primary decay modes are spontaneous fission and α decay, the former with a probability of 8% and the latter with a probability of 92%. Each fission releases 200 MeV of energy. The masses involved in α decay are as follows

$^{248}_{96}\text{Cm} = 248.072220\text{u},\ ^{244}_{94}\text{Pu} = 244.064100\text{u}\ \&\ ^{4}_{2}\text{He} = 4.002603\text{u}.$

Calculate the power output from a sample of 10^{20} Cm atoms. (l u = 931 MeV/c^2)

MP0200

6. A small quantity of solution containing ^{24}Na radionuclide (half life 15 hours) of activity 1.0 microcurie is injected into the blood of a person. A sample of the blood of volume 1 cm^3 taken after 5 hours shows an activity of 296 distintegration per minute. Determine the total volume of blood in the body of the person. Assume that the radioactive solution mixes uniformly in the blood of the person.
(1 Curie = 3.7 × 10^{10} disintegrations per second)

MP0201

7. The positron is a fundamental particle with the same mass as that of the electron and with a charge equal to that of an electron but of opposite sign. When a positron and an electron collide, they may annihilate each other. The energy corresponding to their mass appears in two photons of equal energy. Find the wavelength of the radiation emitted. [Take : mass of electron = $(0.5/C^2)$MeV and hC = 1.2×10^{-12} MeV.m where h is the Plank's constant and C is the velocity of light in air]

MP0202

8. A π^+ meson of negligible initial velocity decays to a μ^+ (muon) and a neutrino. With what kinetic energy (in eV) does the muon move? (The rest mass of neutrino can be considered zero. The rest mass of the π^+ meson is 150 MeV and the rest mass of the muon is 100 MeV.) Take neutrino to behave like a photon. Take $\sqrt{2}$ = 1.41.

MP0203

9. A radioactive decay counter is switched on at t = 0. A β - active sample is present near the counter. The counter registers the number of β-particles emitted by the sample. The counter registers 1×10^5 β-particles at t = 36 s and 1.11×10^5 β-particles at t = 108 s. Find $T_{1/2}$ of this sample

MP0204

10. A radioactive sample emits n β-particles in 2 sec. In next 2 sec it emits 0.75 n β-particles, what is the mean life of the sample? **[JEE 2003]**

MP0205

11. The age of a rock containing lead and uranium is equal to 1.5×10^9 yrs. The uranium is decaying into lead with half life equal to 4.5×10^9 yrs. Find the ratio of lead to uranium present in the rock, assuming initially no lead was present in the rock. (Given $2^{1/3} = 1.259$). **[JEE 2004]**

MP0206

EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

Nuclear Physics :

1. In Rutherford's famous gold foil scattering experiment, he found that most alpha particles would pass through the foil undeflected. Which one of the following nuclear properties can be inferred from this observation?

(A) The nucleus must have a positive charge

(B) Most of the mass of an atom is in the nucleus

(C) The nucleus contains both protons neutrons

(D) The diameter of the nucleus is small compared to the diameter of the atom

(E) None of these

MP0207

2. Let u be denote one atomic mass unit. One atom of an element of mass number A has mass exactly equal to Au

(A) for any value of A (B) only for A = 1

(C) only for A = 12 (D) for any value of A provided the atom is stable

MP0208

3. If radius of the $^{27}_{13}\text{Al}$ nucleus is estimated to be 3.6 fermi, then the radius of $^{125}_{52}\text{Te}$ nucleus be nearly-

(A) 6 fermi (B) 8 fermi (C) 4 fermi (D) 5 fermi

[AIEEE - 2005]

MP0209

4. The surface area of a nucleus varies with mass number A as

(A) $A^{2/3}$ (B) $A^{1/3}$ (C) A (D) None

MP0210

5. A nucleus disintegrates into two nuclear parts which have their velocities in the ratio 2 : 1 The ratio of their nuclear sizes will be-

(A) $2^{1/3} : 1$ (B) $1 : 3^{1/2}$ (C) $3^{1/2} : 1$ (D) $1 : 2^{1/3}$

[AIEEE - 2004]

MP0211

6. The binding energy per nucleon for C^{12} is 7.68 MeV and that for C^{13} is 7.5 MeV. The energy required to remove a neutron from C^{13} is

(A) 5.34 MeV (B) 5.5 MeV (C) 9.5 MeV (D) 9.34 MeV

MP0212

7. The following nuclear reaction is an example of ${}^{12}_{6}C + {}^{4}_{2}H \rightarrow {}^{16}_{8}O$ + energy

(A) fission (B) fusion (C) alpha decay (D) beta decay

MP0213

8. Fast neutrons may most easily be slowed down by which one of the following methods?

(A) passing them through a substance rich in hydrogen

(B) allowing them to collide elastically with heavy nuclei

(C) using lead shielding

(D) passing them through an increasing potential gradient space

MP0214

9. A nuclear transformation is denoted by X(n, α) $\rightarrow {}^{7}_{3}Li$. Which of the following is the nucleus of element X ?

(A) ${}^{12}_{6}C$ (B) ${}^{10}_{5}B$ (C) ${}^{9}_{5}B$ (D) ${}^{11}_{4}Be$

[AIEEE - 2005]

MP0215

10. A certain radioactive nuclide of mass number m_x disintegrates, with the emission of an electron and γ radiation only, to give second nuclide of mass number m_y. Which one of the following equation correctly relates m_x and m_y?

(A) $m_y = m_x + 1$ (B) $m_y = m_x - 2$ (C) $m_y = m_x - 1$ (D) $m_y = m_x$

MP0216

11. A nucleus with Z = 92 emits the following in a sequence : $\alpha, \alpha, \beta^-, \beta^-, \alpha, \alpha, \alpha, \alpha, \beta^-, \beta^-, \alpha, \beta^+, \beta^+, \alpha$. The Z of the resulting nucleus is-

(A) 76 (B) 78 (C) 82 (D) 74

[AIEEE - 2003]

MP0217

12. In an α-decay the Kinetic energy of α particle is 48 MeV and Q-value of the reaction is 50 MeV. The mass number of the mother nucleus is: (Assume that daughter nucleus is in ground state)

(A) 96 (B) 100 (C) 104 (D) none of these

MP0218

13. When U^{238} nucleus originally at rest, decays by emitting an alpha particle having a speed u, the recoil speed of the residual nucleus is-

(A) $\frac{4u}{238}$ (B) $-\frac{4u}{234}$ (C) $\frac{4u}{234}$ (D) $-\frac{4u}{238}$

[AIEEE - 2003]

MP0219

14. If a star converts all of its Helium into oxygen nucleus, find the amount of energy released per nucleus of oxygen. O = 15.9994 amu and He = 4.0026 amu

(A) 7.26 MeV (B) 7 MeV (C) 10.24 MeV (D) 5.12 MeV

[JEE' 2005 (Scr)]

MP0220

15. The nucleus of element X (A = 220) undergoes α-decay. If Q-value of the reaction is 5.5 MeV, then the kinetic energy of α-particle is :

(A) 5.4 MeV (B) 10.8 MeV (C) 2.7 MeV (D) None

[JEE 2003 (Scr)]

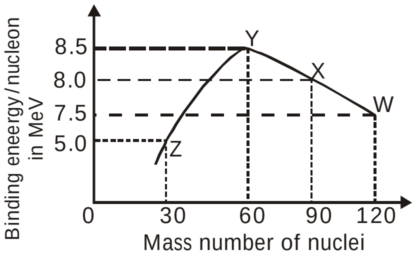
MP0221

16. In the uranium radioactive series the initial nucleus is $_{92}U^{238}$, and the final nucleus is $_{82}Pb^{206}$. When the uranium nucleus decays to lead, the number of α – particles emitted is.. and the number of β-particles emitted...

(A) 6, 8 (B) 8, 6 (C) 16, 6 (D) 32, 12

MP0222

17. Binding energy per nucleon versus mass number curve for nuclei is shown in figure. W, X, Y and Z are four nuclei indicated on the curve. The process that would release energy is :



(A) Y → 2Z (B) W → X + Z (C) W → 2Y (D) X → Y + Z

MP0223

18. An alpha nucleus of energy $\frac{1}{2}mv^2$ bombards a heavy nuclear target of charge Ze. Then the distance of closest approach for the alpha nucleus will be proportional to-

(A) v^2 (B) 1/m (C) $1/v^4$ (D) 1/Ze

[AIEEE - 2006]

MP0224

19. If the binding energy per nucleon in $^{7}_{3}Li$ and $^{4}_{2}He$ nuclei are 5.60 MeV and 7.06 MeV respectively, then in the reaction : $p + {}^{7}_{3}Li \rightarrow 2\,{}^{4}_{2}He$ energy of proton must be-

(A) 28.24 MeV (B) 17.28 MeV (C) 1.46 MeV (D) 39.2 MeV

[AIEEE - 2006]

MP0225

20. If M_0 is the mass of an oxygen isotope $_{8}O^{17}$, M_p and M_n are the masses of a proton and a neutron, respectively, the nuclear binding energy of the isotope is-

(A) $(M_o - 8M_p)c^2$ (B) $(M_o - 8Mp - 9M_n)c^2$

(C) M_oc^2 (D) $(M_o - 17M_n)c^2$

[AIEEE - 2007]

MP0226

21. In the options given below, let E denote the rest mass energy of a nucleus and *n* a neutron. The correct option is :-

(A) $E\left({}^{236}_{92}U\right) > E\left({}^{137}_{53}I\right) + E\left({}^{97}_{39}Y\right) + 2E(n)$ (B) $E\left({}^{236}_{92}U\right) < E\left({}^{137}_{53}I\right) + E\left({}^{97}_{39}Y\right) + 2E(n)$

(C) $E\left({}^{236}_{92}U\right) < E\left({}^{140}_{56}Ba\right) + E\left({}^{94}_{36}Kr\right) + 2E(n)$ (D) $E\left({}^{236}_{92}U\right) = E\left({}^{140}_{56}Ba\right) + E\left({}^{94}_{36}Kr\right) + 2E(n)$

[JEE 2007]

MP0227

Radoactivity :

22. The half–life of ^{131}I is 8 days. Given a sample of ^{131}I at time t = 0, we can assert that :

(A) no nucleus will decay before t = 4 days (B) no nucleus will decay before t = 8 days

(C) all nuclei will decay before t = 16 days (D) a given nucleus may decay at any time after t = 0

MP0228

23. Activity of a radioactive substance is R_1 at time t_1 and R_2 at time $t_2 (t_2 > t_1)$. Then the ratio $\frac{R_2}{R_1}$ is:

(A) $\frac{t_2}{t_1}$ (B) $e^{-\lambda(t_1+t_2)}$ (C) $e\left(\frac{t_1-t_2}{\lambda}\right)$ (D) $e^{\lambda(t_1-t_2)}$

MP0229

24. A particular nucleus in a large population of identical radioactive nuclei did survive 5 half lives of that isotope. Then the probability that this surviving nucleus will survive the next half life :

(A) $\frac{1}{32}$ (B) $\frac{1}{5}$ (C) $\frac{1}{2}$ (D) $\frac{1}{10}$

MP0230

25. A certain radio active substance has a half life of 5 years. Thus for a nucleus in a sample of the element, the probability of decay in ten years is

(A) 50% (B) 75% (C) 100% (D) 60%

MP0231

26. The activity of a sample reduces from A_0 to $A_0/\sqrt{3}$ in one hour. The activity after 3 hours more will be :-

(A) $\frac{A_0}{3\sqrt{3}}$ (B) $\frac{A_0}{9}$ (C) $\frac{A_0}{9\sqrt{3}}$ (D) $\frac{A_0}{27}$

MP0232

27. Half life of radium is 1620 years. How many radium nuclei decay in 5 hours in 5 gm radium? (Atomic weight of radium = 223)

(A) 9.1×10^{12} (B) 3.23×10^{15} (C) 1.72×10^{20} (D) 3.3×10^{17}

MP0233

28. The activity of a sample of radioactive material is A_1 at time t_1 and A_2 at time t_2 ($t_2 > t_1$). Its mean life is T.

(A) $A_1 t_1 = A_2 t_2$
(B) $\frac{A_1 - A_2}{t_2 - t_1}$ = constant
(C) $A_2 = A_1 e^{(t_1 - t_2)/T}$
(D) $A_2 = A_1 e^{(t_1/Tt_2)}$

MP0234

29. The decay constant of the end product of a radioactive series is
(A) zero
(B) infinite
(C) finite (non zero)
(D) depends on the end product.

MP0235

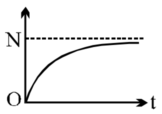
30. A radioactive substance is dissolved in a liquid and the solution is heated. The activity of the solution
(A) is smaller than that of element
(B) is greater than that of element
(C) is equal to that of element
(D) will be smaller or greater depending upon whether the solution is weak or concentrated.

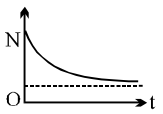
MP0236

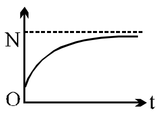
31. In a certain nuclear reactor, a radioactive nucleus is being produced at a constant rate = 1000 /s. The mean life of the radionuclide is 40 minutes. At steady state, the number of radionuclide will be
(A) 4×10^4
(B) 24×10^4
(C) 24×10^5
(D) 24×10^6

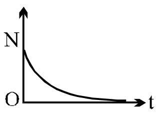
MP0237

32. In the above question, if there were 20×10^5 radionuclide at t = 0, then the graph of N v/s t is

(A) 

(B) 

(C) 

(D) 

MP0238

33. Which of the following cannot be emitted by radioactive substances during their decay ?
(A) Protons
(B) Neutrinos
(C) Helium nuclei
(D) Electrons

[AIEEE - 2003]
MP0239

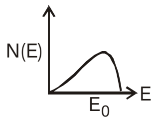
34. A 280 days old radioactive substance shows an activity of 6000 dps, 140 days later it's activity becomes 3000dps. What was its initial activity.
(A) 20000 dps
(B) 24000 dps
(C) 12000 dps
(D) 6000 dps

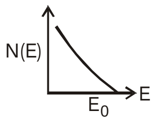
[JEE 2004 (Scr)]
MP0240

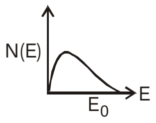
35. When $_3Li^7$ nuclei are bombarded by protons and the resultant nuclei are $_4Be^8$, the emitted particles will be-
(A) alpha particles
(B) beta particles
(C) gamma photons
(D) neutrons

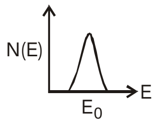
[AIEEE - 2006]
MP0241

36. The energy spectrum of β-particles [number N(E) as a function of β-energy E] emitted from a radioactive source is- **[AIEEE - 2006]**

(A) 

(B) 

(C) 

(D) 

MP0242

37. In gamma ray emission from a nucleus

(A) both the neutron number and the proton number change

(B) there is no change in the proton number and the neutron number

(C) only the neutron number changes

(D) only the proton number changes

[AIEEE-2007]

MP0243

38. The half-life period of a radioactive element X is same as the mean life time of another radioactive element Y. Initially they have the same number of atoms. Then-

(A) X will decay faster than Y

(B) Y will decay faster than X

(C) Y and X have same decay rate initially

(D) X and Y decay at same rate always

[AIEEE - 2007]

MP0244

39. Given a sample of Radium-226 having half-life of 4 days. Find the probability, a nucleus disintegrates within 2 half lives.

(A) 1 (B) 1/2 (C) 3/4 (D) 1/4

[JEE 2006]

MP0245

MATRIX MATCH TYPE QUESTION

40. Match the following Columns :-

	Column-I		**Column-II**
(A)	Nuclear fusion	(P)	Converts some matter into energy
(B)	Nuclear fission	(Q)	Generally occurs for nuclei with low atomic number
(C)	β–decay	(R)	Generally occurs for nuclei with higher atomic number
(D)	Exothermic nuclear reaction	(S)	Essentially proceeds by weak nuclear forces

[JEE 2006]

MP0246

41. Some laws/processes are given in **Column I**. Match these with the physical phenomena given in **Column II** and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the ORS.

Column-I		**Column-II**
(A) Transition between two atomic energy levels	(P)	Characteristic X-rays
(B) Electron emission from a material	(Q)	Photoelectric effect
(C) Mosley's law	(R)	Hydrogen spectrum
(D) Change of photon energy into kinetic energy of electrons	(S)	β-decay

[JEE 2007]

MP0247

EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

1. The binding energies of nuclei X and Y are E_1 and E_2 respectively. Two atoms of X fuse to give one atom of Y and an energy Q is released. Then:

(A) $Q = 2E_1 – E_2$ (B) $Q = E_2 – 2E_1$ (C) $Q = 2E_1 + E_2$ (D) $Q = 2E_2 + E_1$

MP0248

2. If each fission in a U^{235} nucleus releases 200 MeV, how many fissions must occurs per second to produce a power of 1 KW

(A) 1.325×10^{13} (B) 3.125×10^{13} (C) 1.235×10^{13} (D) 2.135×10^{13}

MP0249

3. The binding energies of the atom of elements A & B are E_a & E_b respectively. Three atoms of the element B fuse to give one atom of element A. This fusion process is accompanied by release of energy e. Then E_a, E_b are related to each other as

(A) $E_a + e = 3E_b$ (B) $E_a = 3E_b$ (C) $E_a - e = 3E_b$ (D) $E_a + 3E_b + e = 0$

MP0250

4. The rest mass of the deuteron, ${}_1^2H$, is equivalent to an energy of 1876 MeV, the rest mass of a proton is equivalent to 939 MeV and that of a neutron to 940 MeV. A deuteron may disintegrate to a proton and a neutron if it :

(A) emits a γ - ray photon of energy 2 MeV (B) captures a γ - ray photon of energy 2 MeV

(C) emits a γ - ray photon of energy 3 MeV (D) captures a γ - ray photon of energy 3 MeV

MP0251

5. The number of α and β^- emitted during the radioactive decay chain starting from ${}_{88}^{226}Ra$ and ending at ${}_{82}^{206}Pb$ is

(A) 3α & $6\beta^-$ (B) 4α & $5\beta^-$ (C) 5α & $4\beta^-$ (D) 6α & $6\beta^-$

MP0252

6. In a radioactive element the fraction of initial amount remaining after its mean life time is

(A) $1 - \frac{1}{e}$ (B) $\frac{1}{e^2}$ (C) $\frac{1}{e}$ (D) $1 - \frac{1}{e^2}$

MP0253

7. Two radioactive material A_1 and A_2 have decay constants of $10\lambda_0$ and λ_0. If initially they have same number of nuclei, the ratio of number of their undecayed nuclei will be (1/e) after a time

(A) $\frac{1}{\lambda_0}$ (B) $\frac{1}{9\lambda_0}$ (C) $\frac{1}{10\lambda_0}$ (D) 1

MP0254

8. 90% of a radioactive sample is left undecayed after time t has elapsed. What percentage of the initial sample will decay in a total time 2t :

(A) 20% (B) 19% (C) 40% (D) 38%

MP0255

9. A radioactive material of half-life T was produced in a nuclear reactor at different instants, the quantity produced second time was twice of that produced first time. If now their present activities are A_1 and A_2 respectively then their age difference equals:

(A) $\frac{T}{\ln 2}\left|\ln\frac{A_1}{A_2}\right|$ (B) $T\left|\ln\frac{A_1}{A_2}\right|$ (C) $\frac{T}{\ln 2}\left|\ln\frac{A_2}{2A_1}\right|$ (D) $T\left|\ln\frac{A_2}{2A_1}\right|$

MP0256

10. The half-life of substance X is 45 years, and it decomposes to substance Y. A sample from a meteorite was taken which contained 2% of X and 14% of Y by quantity of substance. If substance Y is not normally found on a meteorite, what is the approximate age of the meteorite?

(A) 270 years (B) 135 years (C) 90 years (D) 45 years

MP0257

11. At time t = 0, N_1 nuclei of decay constant λ_1 & N_2 nuclei of decay constant λ_2 are mixed . The decay rate of the mixture is :

(A) $N_1N_2e^{-(\lambda_1+\lambda_2)t}$

(B) $+\left(\frac{N_1}{N_2}\right)e^{-(\lambda_1-\lambda_2)t}$

(C) $+\left(N_1\lambda_1e^{-\lambda_1 t}+N_2\lambda_2e^{-\lambda_2 t}\right)$

(D) $+\,N_1\lambda_1\,N_2\lambda_2\,e^{-(\lambda_1+\lambda_2)t}$

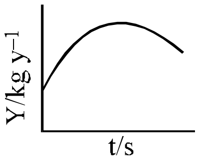
MP0258

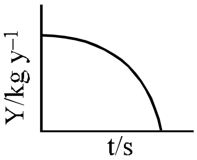
12. A radioactive nuclide can decay simultaneously by two different processes which have decay constants λ_1 and λ_2. The effective decay constant of the nuclide is λ, then :

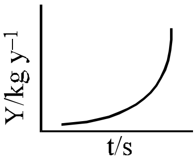
(A) $\lambda = \lambda_1 + \lambda_2$ (B) $\lambda = 1/2(\lambda_1 + \lambda_1)$ (C) $\frac{1}{\lambda}=\frac{1}{\lambda_1}+\frac{1}{\lambda_2}$ (D) $\lambda = \sqrt{\lambda_1\lambda_2}$

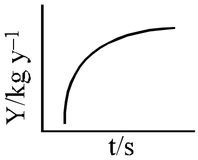
MP0259

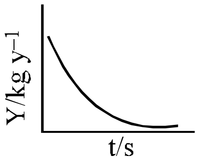
13. The radioactive nucleus of an element X decays to a stable nucleus of element Y. A graph of the rate of formation of Y against time would look like

(A) 

(B) 

(C) 

(D) 

(E) 

MP0260

14. The half life of a neutron is 800 sec. 10^8 neutrons at a certain instant are projected from one space station towards another space station, situated 3200 km away, with a velocity 2000 m/s. Their velocity remains constant during the journey. How many neutrons reach the other station?

(A) 50×10^6 (B) 25×10^6 (C) 80×10^5 (D) 25×10^5

MP0261

15. A radioactive source in the form of a metal sphere of diameter 3.2×10^{-3} m emits β-particle at a constant rate of 6.25×10^{10} particle/sec. The source is electrically insulated and all the β-particle are emitted from the surface. The potential of the sphere will rise to 1 V in time

(A) 180 μ sec (B) 90 μ sec (C) 18 μ sec (D) 9 μ sec

MP0262

16. In the nuclear fusion reaction,

$$^{2}_{1}H + ^{3}_{1}H \rightarrow ^{4}_{2}He + n$$

given that the repulsive potential energy between the two nuclei is 7.7×10^{-14} J, the temperature at which the gases must be heated to initiate the reaction is nearly [Boltzmann's constant $k = 1.38 \times 10^{-23}$ J/K]-

(A) 10^7 K (B) 10^5 K (C) 10^3 K (D) 10^9 K

[AIEEE - 2003]

MP0263

17. The binding energy per nucleon of deuteron $(^{2}_{1}H)$ and helium nucleus $(^{4}_{2}He)$ is 1.1 MeV and 7 MeV respectively. If two deuteron nuclei react to form a single helium nucleus, then the energy released is-

(A) 13.9 MeV (B) 26.9 MeV (C) 23.6 MeV (D) 19.2 MeV

[AIEEE - 2004]

MP0264

MULTIPLE CORRECT TYPE QUESTIONS

18. When a nucleus with atomic number Z and mass number A undergoes a radioactive decay process:

(A) both Z and A will decrease, if the process is α decay

(B) Z will decrease but A will not change, if the process is β^+ decay

(C) Z will decrease but A will not change, if the process is β^- decay

(D) Z and A will remain unchanged, if the process is γ decay.

MP0265

19. When the atomic number A of the nucleus increases

(A) initially the neutron–proton ratio is constant = 1

(B) initially neutron–proton ratio increases and later decreases

(C) initially binding energy per nucleon increases and later decreases

(D) the binding energy per nucleon increases when the neutron–proton ratio increases.

MP0266

20. Let m_p be the mass of a proton, m_n the mass of a neutron, M_1 the mass of a $^{20}_{10}Ne$ nucleus and M_2 the mass of a $^{40}_{20}Ca$ nucleus. Then

(A) $M_2 = 2M_1$ (B) $M_2 > 2M_1$ (C) $M_2 < 2M_1$ (D) $M_1 < 10(m_n + m_p)$

MP0267

21. The decay constant of a radio active substance is 0.173 $(\text{years})^{-1}$. Therefore :

(A) Nearly 63% of the radioactive substance will decay in (1/0.173) year.

(B) half life of the radio active substance is (1/0.173) year.

(C) one -forth of the radioactive substance will be left after nearly 8 years.

(D) all the above statements are true.

MP0268

22. Which of the following statement(s) is/are correct?

(A) The rest mass of a stable nucleus is less than the sum of the rest masses of its separated nucleons.

(B) The rest mass of a stable nucleus is greater than the sum of the rest masses of its separated nucleons.

(C) In nuclear fusion, energy is released by fusion of two nuclei of medium mass (approx. 100 amu)

(D) In nuclear fission, energy is released by fragmentation of a very heavy nucleus.

MP0269

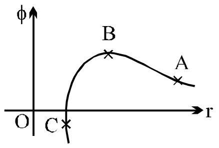
23. The graph shown by the side shows the variation of potential energy ϕ of a proton with its distance 'r' from a fixed sodium nucleus, as it approaches the nucleus, placed at origin O. Then the portion.

(A) AB indicates nuclear repulsion

(B) AB indicates electrostatic repulsion

(C) BC indicates nuclear attraction

(D) BC represents electrostatic interaction



MP0270

24. The instability of the nucleus can be due to various causes. An unstable nucleus emits radiations if possible to transform into less unstable state. Then the cause and the result can be

(A) a nucleus of excess nucleons is α active

(B) an excited nucleus of excess protons is β^- active

(C) an excited nucleus of excess protons is β^+ active

(D) an nucleus of excess neutrons is β^- active

MP0271

25. In β-decay, the Q-value of the process is E. Then

(A) K.E. of a β-particle cannot exceed E.

(B) K.E. of anti neutrino emitted lies between Zero and E.

(C) N/Z ratio of the nucleus is altered.

(D) Mass number (A) of the nucleus is altered.

MP0272

26. Consider the following nuclear reactions and select the correct statements from the options that follow.

Reaction I : $n \rightarrow p + e^- + \bar{v}$ **Reaction II :** $p \rightarrow n + e^+ + v$

(A) Free neutron is unstable, therefore reaction I is possible

(B) Free proton is stable, therefore reaction II is not possible

(C) Inside a nucleus, both decays (reaction I and II) are possible

(D) Inside a nucleus, reaction I is not possible but reaction II is possible.

MP0273

27. When the nucleus of an electrically neutral atom undergoes a radioactive decay process, it will remain neutral after the decay if the process is :

(A) α decay (B) β^- decay (C) γ decay (D) K-capture

MP0274

MATRIX MATCH TYPE QUESTION

28. In the following, column I lists some physical quantities & the column II gives approx. energy values associated with some of them. Choose the appropriate value of energy from column II for each of the physical quantities in column I and write the corresponding letter A, B, C etc. against the number (i), (ii), (iii), etc. of the physical quantity in the answer book. In your answer, the sequence of column I should be maintained.

	Column I		**Column II**
(i)	Energy of thermal neutrons	(A)	0.025 eV
(ii)	Energy of X-rays	(B)	0.5 eV
(iii)	Binding energy per nucleon	(C)	3 eV
(iv)	Photoelectric threshold of metal	(D)	20 eV
		(E)	10 keV
		(F)	8 MeV

MP0275

EXERCISE - JM

1. This question contains Statement-1 and Statement-2. Out of the four choices given after the statements, choose the one that best describes the two statements.

Statement-1 : Energy is released when heavy nuclei undergo fission or light nuclei undergo fusion.

Statement-2 : For heavy nuclei, binding energy per nucleon increases with increasing Z while for light nuclei it decreases with increasing Z.

(1) Statement-1 is false, Statement-2 is true.

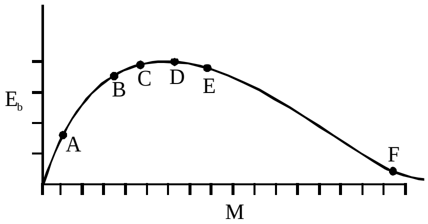
(2) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(3) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

(4) Statement-1 is true, Statement-2 is false.

[AIEEE - 2008]
MP0276

2.



The above is a plot of binding energy per nucleon E_b, against the nuclear mass M; A, B, C, D, E, F correspond to different nuclei. Consider four reactions :

(i) $A + B \rightarrow C + \varepsilon$ (ii) $C \rightarrow A + B + \varepsilon$
(iii) $D + E \rightarrow F + \varepsilon$ (iv) $F \rightarrow D + E + \varepsilon$

where ε is the energy released ? In which reactions is ε positive ?

(i) $A + B \rightarrow C + \varepsilon$ (ii) $C \rightarrow A + B + \varepsilon$ (iii) $D + E \rightarrow F + \varepsilon$ (iv) $F \rightarrow D + E + \varepsilon$

(1) (ii) and (iv) (2) (ii) and (iii) (3) (i) and (iv) (4) (i) and (iii)

[AIEEE - 2008]
MP0277

Directions : Questions number 3 – 4 are based on the following paragraph.

A nucleus of mass M + Δm is at rest and decays into two daughter nuclei of equal mass $\frac{M}{2}$ each. Speed of light is c.

3. The speed of daughter nuclei is :-

(1) $c\sqrt{\frac{\Delta m}{M+\Delta m}}$ (2) $c\frac{\Delta m}{M+\Delta m}$ (3) $c\sqrt{\frac{2\Delta m}{M}}$ (4) $c\sqrt{\frac{\Delta m}{M}}$

[AIEEE-2010]
MP0278

4. The binding energy per nucleon for the parent nucleus is E_1 an that for the daughter nuclei is E_2. Then:-

(1) $E_1 = 2E_2$ (2) $E_2 = 2E_1$ (3) $E_1 > E_2$ (4) $E_2 > E_1$

[AIEEE - 2010]
MP0279

5. A radioactive nucleus (initial mass number A and atomic number Z) emits 3 α-particles and 2 positrons. The ratio of number of neutrons to that of protons in the final nucleus will be:-

(1) $\frac{A-Z-4}{Z-2}$ (2) $\frac{A-Z-8}{Z-4}$ (3) $\frac{A-Z-4}{Z-8}$ (4) $\frac{A-Z-12}{Z-4}$

[AIEEE - 2010]
MP0280

6. The half life of a radioactive substance is 20 minutes. The approximate time interval $(t_2 - t_1)$ between the time t_2 when $\frac{2}{3}$ of it has decayed and time t_1 when $\frac{1}{3}$ of it had decayed is :-

(1) 20 min (2) 28 min (3) 7 min (4) 14 min

[AIEEE - 2011]
MP0281

7. After absorbing a slowly moving neutron of mass m_N (momentum ~0) a nucleus of mass M breaks into two nuclei of masses m_1 and $5m_1(6m_1 = M + m_N)$, respectively. If the de Broglie wavelength of the nucleus with mass m_1 is λ, then de Broglie wavelength of the other nucleus will be:-

(1) 25 λ (2) 5λ (3) $\frac{\lambda}{5}$ (4) λ

[AIEEE - 2011]
MP0282

8. **Statement-1:** A nucleus having energy E_1 decays be β^- emission to daughter nucleus having energy E_2, but the β^- rays are emitted with a continuous energy spectrum having end point energy $E_1–E_2$.

Statement-1: To conserve energy and momentum in β-decay at least three particles must take part in the transformation.

(1) Statement-1 is incorrect, statement-2 is correct
(2) Statement-1 is correct, statement-2 is incorrect
(3) Statement-1 is correct, statement-2 correct; statement-2 is the correct explanation of statement-1
(4) Statement-1 is correct, statement-2 is correct; statement -2 is not the correct explanation of statement-1.

[AIEEE - 2011]
MP0283

9. Assume that a neutron breaks into a proton and an electron. The energy released during this process is :

(Mass of neutron = 1.6725×10^{-27} kg

Mass of proton = 1.6725×10^{-27} kg

Mass of electron = 9×10^{-31} kg)

(1) 5.4 MeV (2) 0.73 MeV (3) 7.10 MeV (4) 6.30 MeV

[AIEEE - 2012]

MP0284

10. Hydrogen ($_1H^1$), Deuterium ($_1H^2$), singly ionised Helium ($_2He^4$)$^+$ and doubly ionised lithium ($_3Li^6$)$^{++}$ all have one electron around the nucleus. Consider an electron transition from n = 2 to n = 1. If the wave lengths of emitted radiation are λ_1, λ_2, λ_3 and λ_4 respectively then approximately which one of the following is **correct** ?

(1) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$ (2) $\lambda_1 = 2\lambda_2 = 3\lambda_3 = 4\lambda_4$

(3) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$ (4) $\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$

[JEE-Main-2014]

MP0285

11. The radiation corresponding to 3 → 2 transition of hydrogen atom falls on a metal surface to produce photoelectrons. These electrons are made to enter a magnetic field of 3×10^{-4}T. If the radius of the largest circular path followed by these electrons is 10.0 mm, the work function of the metal is close to:-

(1) 0.8 eV (2) 1.6 eV (3) 1.8 eV (4) 1.1 eV

[JEE-Main-2014]

MP0286

12. Half-lives of two radioactive elements A and B are 20 minutes and 40 minutes, respectively. Initially, the samples have equal number of nuclei. After 80 minutes , the ratio of decayed numbers of A and B nuclei will be :-

(1) 5 : 4 (2) 1 : 16 (3) 4 : 1 (4) 1 : 4

[JEE-Main-2016]

MP0287

13. A radioactive nucleus A with a half life T, decays into a nucleus B. At t = 0, there is no nucleus B. At sometime t, the ratio of the number of B to that of A is 0.3. Then, t is given by :

(1) $t = T \log(1.3)$ (2) $t = \dfrac{T}{\log(1.3)}$ (3) $t = \dfrac{T}{2}\dfrac{\log 2}{\log 1.3}$ (4) $t = T\dfrac{\log 1.3}{\log 2}$

[JEE-Main-2017]

MP0288

14. An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let λ_n, λ_g be the de Broglie wavelength of the electron in the nth state and the ground state respectively. Let Λ_n be the wavelength of the emitted photon in the transition from the nth state to the ground state. For large n, (A, B are constants)

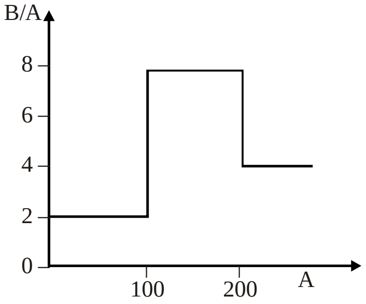
(1) $\Lambda_n \approx A + B\lambda_n$ (2) $\Lambda_n^2 \approx A + B\lambda_n^2$ (3) $\Lambda_n^2 \approx \lambda$ (4) $\Lambda_n \approx A + \dfrac{B}{\lambda_n^2}$

[JEE-Main-2018]

MP0289

EXERCISE - JA

1. Assume that the nuclear binding energy per nucleon (B/A) versus mass number (A) is as shown in the figure. Use this plot to choose the correct choice(s) given below : **[JEE 2008]**



(A) Fusion of two nuclei with mass numbers lying in the range of 1 < A < 50 will release energy

(B) Fusion of two nuclei with mass numbers lying in the range of 51 < A < 100 will release energy

(C) Fission of a nucleus lying in the mass range of 100 < A < 200 will release energy when broken into two equal fragments

(D) Fission of a nucleus lying in the mass range of 200 < A < 260 will release energy when broken into two equal fragments

MP0290

2. A radioactive sample S1 having an activity of 5μCi has twice the number of nuclei as another sample S2 which has an activity of 10μCi. The half lives of S1 and S2 can be : **[JEE 2008]**

(A) 20 years and 5 years, respectively (B) 20 years and 10 years, respectively

(C) 10 years each (D) 5 years each

MP0291

Paragraph for Question Nos. 3 to 5

Scientists are working hard to develop nuclear fusion reactor. Nuclei of heavy hydrogen, ^2_1H, known as deuteron and denoted by D, can be thought of as a candidate for fusion reactor. The D-D reaction is $^2_1H + ^2_1H \rightarrow ^3_2He + n + \text{energy}$. In the core of fusion reactor, a gas of heavy hydrogen is fully ionized into deuteron nuclei and electrons. This collection of ^2_1H nuclei and electrons is known as plasma. The nuclei move randomly in the reactor core and occasionally come close enough for nuclear fusion to take place. Usually, the temperatures in the reactor core are too high and no material wall can be used to confine the plasma. Special techniques are used which confine the plasma for a time t_0 before the particles fly away from the core. If n is the density (number/volume) of deuterons, the product nt_0 is called Lawson number. In one of the criteria, a reactor is termed successful if Lawson number is greater than 5×10^{14} s/cm^3. It may be helpful to use the following :

Boltzmann constant $k = 8.6 \times 10^{-5}$ eV/K ; $\dfrac{e^2}{4\pi\varepsilon_0} = 1.44 \times 10^{-9}$ eVm.

3. In the core of nuclear fusion reactor, the gas becomes plasma because of **[JEE-2009]**

(A) strong nuclear force acting between the deuterons

(B) Coulomb force acting between the deuterons

(C) Coulomb force acting between deuteron-electron pairs

(D) the high temperature maintained inside the reactor core

MP0292

4. Assume that two deuteron nuclei in the core of fusion reactor at temperature T are moving towards each other, each with kinetic energy 1.5 kT, when the separation between them is large enough to neglect Coulomb potential energy. Also neglect any interaction from other particles in the core. The minimum temperature T required for them to reach a separation of 4×10^{-15} m is in the range

(A) 1.0×10^9 K < T < 2.0×10^9 K (B) 2.0×10^9 K < T < 3.0×10^9 K

(C) 3.0×10^9 K < T < 4.0×10^9 K (D) 4.0×10^9 K < T < 5.0×10^9 K

[JEE-2009]

MP0292

5. Results of calculations for four different designs of a fusion reactor using D-D reaction are given below. Which of these is most promising based on Lawson criterion?

(A) deuteron density = 2.0×10^{12} cm^{-3} , confinement time = 5.0×10^{-3} s

(B) deuteron density = 8.0×10^{14} cm^{-3} , confinement time = 9.0×10^{-1} s

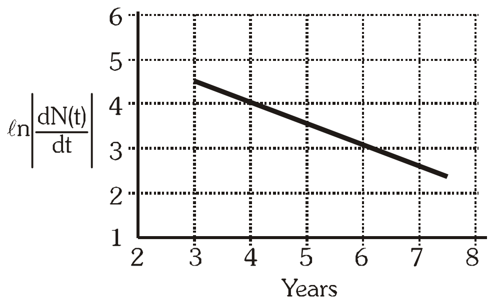
(C) deuteron density = 4.0×10^{23} cm^{-3}, confinement time = 1.0×10^{-11} s

(D) deuteron density = 1.0×10^{24} cm^{-3} , confinement time = $4.0 \text{ x } 10^{-12}$ s

[JEE-2009]

MP0292

6. To determine the half life of a radioactive element, a student plots a graph of $\ell n\left|\frac{dN(t)}{dt}\right|$ versus t. Here $\frac{dN(t)}{dt}$ is the rate of radioactive decay at time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 years, the value of p is **[JEE-2010]**



MP0293

7. The activity of a freshly prepared radioactive sample is 10^{10} disintegrations per second, whose mean life is 10^9 s. The mass of an atom of this radioisotope is 10^{-25} kg. The mass (in mg) of the radioactive sample is **[JEE-2011]**

MP0294

Paragraph for Questions 8 and 9

The β-decay process, discovered around 1900, is basically the decay of a neutron (n). In the laboratory, a proton (p) and an electron (e^-) are observed as the decay products of the neutron. Therefore, considering the decay of a neutron as a two-body decay process, it was predicted theoretically that the kinetic energy of the electron should be a constant. But experimentally, it was observed that the electron kinetic energy has a continuous spectrum. Considering a three-body decay process, i.e. n→ p + e^- + $\vec{v}_e$, around 1930, Pauli explained the observed electron energy spectrum. Assuming the anti-neutrino $\left(\vec{v}_e\right)$ to be massless and possessing negligible energy, and the neutron to be at rest, momentum and energy conservation principles are applied. From this calculation, the maximum kinetic energy of the electron is 0.8×10^6 eV. The kinetic energy carried by the proton is only the recoil energy.

8. If the anti-neutrino had a mass of 3 eV/c^2 (where c is the speed of light) instead of zero mass, what should be the range of the kinetic energy, K, of the electron?

(A) $0 \le K \le 0.8 \times 10^6 eV$ (B) $3.0eV \le K \le 0.8 \times 10^6 eV$

(C) $3.0eV \le K < 0.8 \times 10^6 eV$ (D) $0 \le K < 0.8 \times 10^6 eV$

[JEE 2012]

MP0295

9. What is the maximum energy of the anti-neutrino?

(A) zero (B) much less than 0.8×10^6 eV

(C) Nearly 0.8×10^6 eV (D) Much larger than 0.8×10^6 eV

[JEE 2012]

MP0295

10. A freshly prepared sample of a radioisotope of half-life 1386 s has activity 10^3 distintegrations per second. Given that $\ell n\ 2 = 0.693$, the fraction of the initial number of nuclei (expressed in nearest integer percentage) that will decay in the first 80 s after preparation of the sample is.

[JEE Advance-2013]

MP0296

Paragraph for Questions 11 and 12

The mass of a nucleus ${}^{A}_{Z}X$ is less than the sum of the masses of (A – Z) number of neutrons and Z number of protons in the nucleus. The energy equivalent to the corresponding mass difference is known as the binding energy of the nucleus. A heavy nucleus of mass M can break into two light nuclei of masses m_1 and m_2 only if $(m_1 + m_2) < M$. Also two light nuclei of masses m_3 and m_4 can undergo complete fusion and form a heavy nucleus of mass M' only if $(m_3 + m_4) > M'$. The masses of some neutral atoms are given in the table below :- **[JEE Advance-2013]**

$^{1}_{1}H$	1.007825 u	$^{2}_{1}H$	2.014102 u	$^{3}_{1}H$	3.016050 u	$^{4}_{2}He$	4.002603 u
$^{6}_{3}Li$	6.015123 u	$^{7}_{3}Li$	7.016004 u	$^{70}_{30}Zn$	69.925325 u	$^{82}_{34}Se$	81.916709 u
$^{152}_{64}Gd$	151.919803 u	$^{206}_{82}Pb$	205.974455 u	$^{209}_{83}Bi$	208.980388 u	$^{210}_{84}Po$	209.982876 u

(1u = 932 MeV/c^2)

11. The kinetic energy (in keV) of the alpha particle, when the nucleus $^{210}_{84}Po$ at rest undergoes alpha decay, is :-

(A) 5319 (B) 5422 (C) 5707 (D) 5818

MP0297

12. The correct statement is :-

(A) The nucleus $^{6}_{3}Li$ can emit an alpha particle

(B) The nucleus $^{210}_{84}Po$ can emit a proton

(C) Deuteron and alpha particle can undergo complete fusion

(D) The nuclei $^{70}_{30}Zn$ and $^{82}_{34}Se$ can undergo complete fusion

MP0297

13. Match List I of the nuclear processes with List II containing parent nucleus and one of the end products of each process and then select the correct answer using the codes given below the lists:

[JEE Advance-2013]

	List I		**List II**
P.	Alpha decay	1.	$^{15}_{8}O \to ^{15}_{7}N + ...$
Q.	β^{+} decay	2.	$^{238}_{92}U \to ^{234}_{90}Th + ...$
R.	Fission	3.	$^{185}_{83}Bi \to ^{184}_{82}Pb + ...$
S.	Proton emission	4.	$^{239}_{94}Pu \to ^{140}_{57}La + ...$

Codes :

	P	Q	R	S
(A)	4	2	1	3
(B)	1	3	2	4
(C)	2	1	4	3
(D)	4	3	2	1

MP0298

14. If λ_{Cu} is the wavelength of K_{α} X-ray line of copper (atomic number 29) and λ_{Mo} is the wavelength of the K_{α} X-ray line of molybdenum (atomic number 42), then the ratio $\frac{\lambda_{Cu}}{\lambda_{Mo}}$ is close to :-

(A) 1.99 (B) 2.14 (C) 0.50 (D) 0.48

[JEE Advance-2014]

MP0299

15. A metal surface is illuminated by light of two different wavelength 248 nm and 310 nm. The maximum speeds of the photoelectrons corresponding to these wavelengths are u_1 and u_2, respectively. If the ratio $u_1 : u_2 = 2 : 1$ and hc = 1240 eV nm, the work function of the metal is nearly :-

(A) 3.7 eV (B) 3.2 eV (C) 2.8 eV (D) 2.5 eV

[JEE Advance-2014]

MP0300

16. A nuclear power plant supplying electrical power to a village uses a radioactive material of half life T years as the fuel. The amount of fuel at the beginning is such that the total power requirement of the village is 12.5% of the electrical power available from the plant at that time. If the plant is able to meet the total power needs of the village for a maximum period of nT years, then the value of n is.

[JEE Advance-2015]

MP0301

17. Match the nuclear processes given in column-I with the appropriate option(s) in column-II.

[JEE Advance-2015]

	Column-I		**Column-II**
(A)	Nuclear fusion	(P)	Absorption of thermal neutrons by ${}^{235}_{92}U$
(B)	Fission in a nuclear reactor	(Q)	${}^{60}_{27}Co$ nucleus
(C)	β-decay	(R)	Energy production in stars via hydrogen conversion to helium
(D)	γ-ray emission	(S)	Heavy water
		(T)	Neutrino emission

MP0302

18. For a radioactive material, its activity A and rate of change of its activity R are defined as $A = -\frac{dN}{dt}$ and $R = -\frac{dA}{dt}$, where N(t) is the number of nuclei at time t. Two radioactive sources P (mean life τ) and Q (mean life 2τ) have the same activity at t = 0. Their rates of change of activities at t = 2τ are R_P and R_Q, respectively. If $\frac{R_P}{R_Q} = \frac{n}{e}$, then the value of n is :

[JEE Advance-2015]

MP0303

19. A fission reaction is given by ${}^{236}_{92}U \rightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr + x + y$, where x and y are two particles. Considering ${}^{236}_{92}U$ to be at rest, the kinetic energies of the products are denoted by $K_{Xe}, K_{sr}, K_x(2MeV)$ and K_y(2MeV), respectively. Let the binding energies per nucleon of ${}^{236}_{92}U$, ${}^{140}_{54}Xe$ and ${}^{94}_{38}Sr$ be 7.5 MeV, 8.5 MeV and 8.5 MeV, respectively. Considering different conservation laws, the correct options(s) is (are) :- **[JEE Advance-2015]**

(A) x = n, y = n, K_{sr} = 129 MeV, K_{xe} = 86 MeV

(B) x = p, y = e^-, K_{Sr} = 129 MeV, K_{xe} = 86 MeV

(C) x = p, y = n, K_{Sr} = 129 MeV, K_{xe} = 86 MeV

(D) x = n, y = n, K_{Sr} = 86 MeV, K_{xe} = 129 MeV

MP0304

20. The isotope ${}^{12}_{5}B$ having a mass 12.014 u undergoes β-decay to ${}^{12}_{6}C$. ${}^{12}_{6}C$ has an excited state of the nucleus (${}^{12}_{6}C^*$) at 4.041 MeV above its ground state. If ${}^{12}_{5}B$ decays to ${}^{12}_{6}C^*$, the maximum kinetic energy of the β-particle in units of MeV is (1u = 931.5 MeV/c^2, where c is the speed of light in vacuum). **[JEE Advance-2016]**

MP0305

21. The electrostatic energy of Z protons uniformly distributed throughout a spherical nucleus of radius R is given by $E = \frac{3}{5}\frac{Z(Z-1)e^2}{4\pi\varepsilon_0 R}$ **[JEE Advance-2016]**

The measured masses of the neutron, ${}^{1}_{1}H$, ${}^{15}_{7}N$ and ${}^{15}_{8}O$ are 1.008665 u, 1.007825 u, 15.000109 u and 15.003065 u respectively. Given that the radii of both the ${}^{15}_{7}N$ and ${}^{15}_{8}O$ nuclei are same, 1u = 931.5 MeV/c^2 (c is the speed of light) and $\frac{e^2}{(4\pi\varepsilon_0)}$ = 1.44 MeV fm. Assuming that the difference between the binding energies of ${}^{15}_{7}N$ and ${}^{15}_{8}O$ is purely due to the electrostatic energy, the radius of either of the nuclei is (1fm = 10^{-15}m)

(A) 2.85 fm (B) 3.03 fm (C) 3.42 fm (D) 3.80 fm

MP0306

22. An accident in a nuclear laboratory resulted in deposition of a certain amount of radioactive material of half-life 18 days inside the laboratory. Tests revealed that the radiation was 64 times more than the permissible level required for safe operation of the laboratory. What is the minimum number of days after which the laboratory can be considered safe for use? **[JEE Advance-2016]**

(A) 64 (B) 90 (C) 108 (D) 120

MP0307

23. ${}^{131}I$ is an isotope of Iodine that β decays to an isotope of Xenon with a half-life of 8 days. A small amount of a serum labelled with ${}^{131}I$ is injected into the blood of a person. The activity of the amount of ${}^{131}I$ injected was 2.4×10^5 Becquerel (Bq). It is known that the injected serum will get distributed uniformly in the blood stream in less than half an hour. After 11.5 hours, 2.5 ml of blood is drawn from the person's body, and gives an activity of 115 Bq. The total volume of blood in the person's body, in liters is approximately (you may use $e^x \approx 1 + x$ for $|x| << 1$ and ln 2 ≈ 0.7).

[JEE Advance-2017]

MP0308

24. In a radioactive decay chain, ${}^{232}_{90}Th$ nucleus decays to ${}^{212}_{82}Pb$ nucleus. Let N_α and N_β be the number of α and β^- particles, respectively, emitted in this decay process. Which of the following statements is (are) true ?

(A) $N_\alpha = 5$ (B) $N_\alpha = 6$ (C) $N_\beta = 2$ (D) $N_\beta = 4$

[JEE Advance-2018]

MP0309

25. Suppose a ${}^{226}_{88}Ra$ nucleus at rest and in ground state undergoes α-decay to a ${}^{222}_{86}Rn$ nucleus in its excited state. The kinetic energy of the emitted α particle is found to be 4.44 MeV. ${}^{222}_{86}Rn$ nucleus then goes to its ground state by γ-decay. The energy of the emitted γ-photon is _______ keV,

[Given: atomic mass of ${}^{226}_{88}Ra = 226.005u$, atomic mass of ${}^{222}_{86}Rn = 222.000u$, atomic mass of α particle = 4.000u, 1u = 931 MeV/c^2, c is speed of the light] **[JEE Advanced-2019]**

MP0310

ANSWER KEY

EXERCISE (S-1)

1. Ans. $8/3\times10^{18}$ sec

2. Ans. 28 MeV

3. Ans. $\nu = 1.546 \times 10^{18}$ Hz; Z = 26

4. Ans. 4.87 MeV

5. Ans. 6.04×10^{9} yrs

6. Ans. $t = \left(\frac{\ln 5}{\ln 2}\right)\tau$

EXERCISE (S-2)

1. Ans. 1.7×10^{10} years

2. Ans. 5196 yrs

3. Ans. $v = u\lambda t$

4. Ans. $\Delta T = \dfrac{0.2E_0\left[\alpha t-\frac{\alpha}{\lambda}(1-e^{-\lambda t})\right]}{mS}$

5. Ans. $\cong 33.298\ \mu W$

6. Ans. 6 litre

7. Ans. 2.4×10^{-12} m

8. Ans. 9.00×10^{6}

9. Ans. ($T_{1/2}$ = 10.8 sec)

10. Ans. $1.75n = N_0(1 - e^{-4\lambda})$, 6.95 sec, $\dfrac{2}{\ln\left(\frac{4}{3}\right)}$

11. Ans. 0.259

EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

1. Ans. (D)
2. Ans. (C)
3. Ans. (A)
4. Ans. (A)
5. Ans. (D)
6. Ans. (A)
7. Ans. (B)
8. Ans. (A)
9. Ans. (B)
10. Ans. (D)
11. Ans. (B)
12. Ans. (B)
13. Ans. (C)
14. Ans. (C)
15. Ans. (A)
16. Ans. (B)
17. Ans. (C)
18. Ans. (B)
19. Ans. (B)
20. Ans. (B)
21. Ans. (A)
22. Ans. (D)
23. Ans. (D)
24. Ans. (C)
25. Ans. (B)
26. Ans. (B)
27. Ans. (B)
28. Ans. (C)
29. Ans. (A)
30. Ans. (C)
31. Ans. (C)
32. Ans. (C)
33. Ans. (A)
34. Ans. (B)
35. Ans. (C)
36. Ans. (C)
37. Ans. (B)
38. Ans. (B)
39. Ans. (C)

MATRIX MATCH TYPE QUESTION

40. Ans. (A) P, Q; (B) P, R; (C) S, P; (D) P, Q, R

41. Ans. (A) R, P; (B) Q, S; (C) P; (D) Q

EXERCISE (O-2)

1. Ans. (B)
2. Ans. (B)
3. Ans. (C)
4. Ans. (D)
5. Ans. (C)
6. Ans. (C)
7. Ans. (B)
8. Ans. (B)
9. Ans. (C)
10. Ans. (B)
11. Ans. (C)
12. Ans. (A)
13. Ans. (E)
14. Ans. (B)
15. Ans. (C)
16. Ans. (D)
17. Ans. (C)
18. Ans. (A, B, D)
19. Ans. (A, C)
20. Ans. (C, D)
21. Ans. (A, C)
22. Ans. (A, D)
23. Ans. (B, C)
24. Ans. (A, C, D)
25. Ans. (A, B, C)
26. Ans. (A, B, C)
27. Ans. (C, D)

28. Ans. (i) A, (ii) E (iii) F (iv) C

EXERCISE - JM

1. Ans. (4)
2. Ans. (3)
3. Ans. (3)
4. Ans. (4)
5. Ans. (3)
6. Ans. (1)
7. Ans. (4)
8. Ans. (3)
9. Ans. (Bonus)
10. Ans. (1)
11. Ans. (4)
12. Ans. (1)
13. Ans. (4)
14. Ans. (4)

EXERCISE - JA

1. Ans. (B, D)
2. Ans. (A)
3. Ans. (D)
4. Ans. (A)
5. Ans. (B)
6. Ans. (8)
7. Ans. (1)
8. Ans. (D)
9. Ans. (C)
10. Ans. 4
11. Ans. (A)
12. Ans. (C)
13. Ans. (C)
14. Ans. (B)
15. Ans. (A)
16. Ans. 3
17. Ans. (A)-R or R,T; (B)-P & S; (C)-Q & T; (D)-R
18. Ans. 2
19. Ans. (A)
20. Ans. 9
21. Ans. (C)
22. Ans. (C)
23. Ans. 5
24. Ans. (A,C)
25. Ans. (135.00)

Important Notes

ALLEN
CAREER INSTITUTE
KOTA (RAJASTHAN)

Chapter Contents

SEMICONDUCTOR

KEY CONCEPTS

ENERGY BANDS IN SOLIDS

Based on Pauli's exclusion principle

In an isolated atom electrons present in energy level but in solid, atoms are not isolated, there is interaction among each other, due to this energy level splited into different energy levels. Quantity of these different energy levels depends on the quantity of interacting atoms. Splitting of sharp and closely compact energy levels result into energy band. This is discrete in nature. Order of energy levels in a band is 10^{23} and their energy difference = 10^{-23} eV.

Energy Band

Range of energy possessed by electron in a solid is known as energy band.

Valence Band (VB)

Range of energies possessed by valence electron is known as valence band.

(a) Have bonded electron.

(b) No flow of current due to such electron.

(c) Always fulfill by electron.

Conduction Band (CB)

Range of energies possessed by free electron is known as conduction band.

(a) It has conducting electrons.

(b) Current flows due to such electrons.

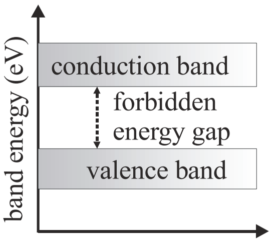
(c) If conduction band is fully empty then current conduction is not possible.

(d) Electrons may exist or not in it.

Forbidden Energy gap (FEG) (ΔEg)

$$\Delta E_g = (C\,B)_{min} - (V\,B)_{max}$$

Energy gap between conduction band and valence band, where no free electron can exist.



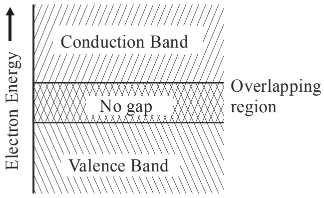
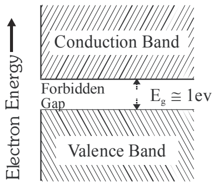
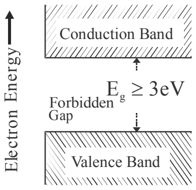
- Width of forbidden energy gap depends upon the nature of substance.
- Width is more, then valence electrons are strongly attached with nucleus
- Width of forbidden energy gap is represented in eV.
- As temperature increases forbidden energy gap decreases (very slightly).

CLASSIFICATION OF CONDUCTORS, INSULATORS AND SEMICONDUCTOR : -

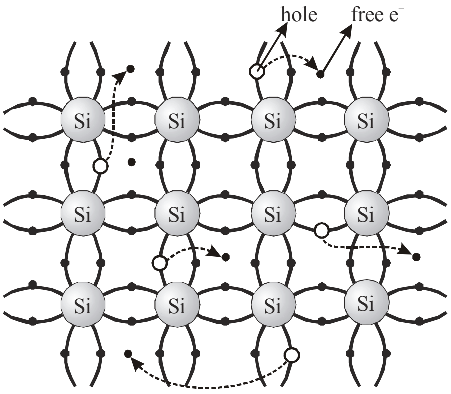
On the basis of the relative values of electrical conductivity and energy bands the solids are broadly classified into three categories

(i) Conductor (ii) Semiconductor (iii) Insulator

Comparison between conductor, semiconductor and insulator :

Properties	**Conductor**	**Semiconductor**	**Insulator**
Resistivity	$10^{-2} - 10^{-8}\ \Omega m$	$10^{-5} - 10^{6}\ \Omega m$	$10^{11} - 10^{19}\ \Omega m$
Conductivity	$10^{2} - 10^{8}$ mho/m	$10^{5} - 10^{-6}$ mho/m	$10^{-11} - 10^{-19}$ mho/m
Temp. Coefficient of resistance (α)	Positive	Negative	Negative
Current	Due to free electrons	Due to electrons and holes	No current
Energy band diagram	 Conductor	 Semi conductor	 Insulator
Forbidden energy gap	$\cong$ 0eV	$\cong$ 1eV	$\geq$ 3eV
Example :	Pt, Al, Cu, Ag	Ge, Si, GaAs, GaF$_2$	Wood, plastic, Diamond, Mica

CONCEPT OF "HOLES" IN SEMICONDUCTORS



Due to external energy (temp. or radiation) when electron goes from valence band to conduction band (i.e. bonded electrons becomes free) a vacancy of free e$^-$ creats in valence band,

which has same charge as electron but positive. This positively charged vaccancy is termed as hole and shown in figure.

- It is deficiency of electron in VB.
- It acts as positive charge carrier.
- It's effective mass is more than electron.
- It's mobility is less than electron.

Note : Hole acts as virtual charge carrier, although it has no physical significance.

- Number of electrons reaching from VB to CB at temperature T kelvin

$$\boxed{n = A\,T^{3/2}\,e^{-\frac{E_g}{2kT}} = AT^{3/2}\exp\left[-\frac{E_g}{2kT}\right]}$$

where

k = Boltzmann constant = 1.38×10^{-23} J/K

T = absolute temperature

A = constant

E_g = energy gap between CB and VB

- In silicon at room temperature out of 10^{12} Si atoms only one electron goes from VB to CB.
- In germanium at room temperature out of 10^{9} Ge atoms only one electron goes from VB to CB.

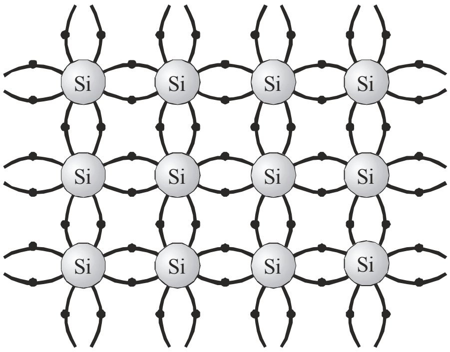
EFFECT OF TEMPERATURE ON SEMICONDUCTOR

At absolute zero kelvin temperature

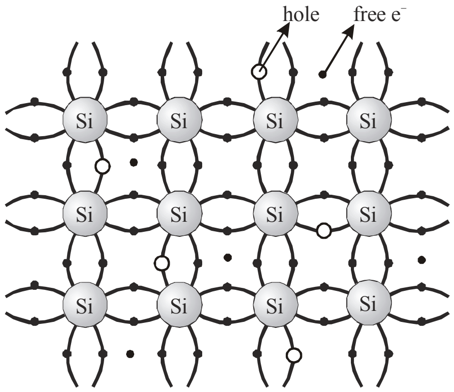
At this temperautre covalent bonds are very strong and there are no free electrons and semiconductor behaves as perfect insulator.

Above absolute temperature

With increase in temperature few valence electrons jump into conduction band and hence it behaves as poor conductor.



at 0 K
Valence band fully filled
Conduction band fully empty



at high temperature
Valence band partially empty
Conduction band partially filled

EFFECT OF IMPURITY IN SEMICONDUCTOR

Doping is a method of addition of "desirable" impurity atoms to pure semiconductor to increase conductivity of semiconductor. or

Doping is a process of deliberate addition of a desirable impurity atoms to a pure semiconductor to modify its properties in controlled manner.

Added impurity atoms are called dopants.

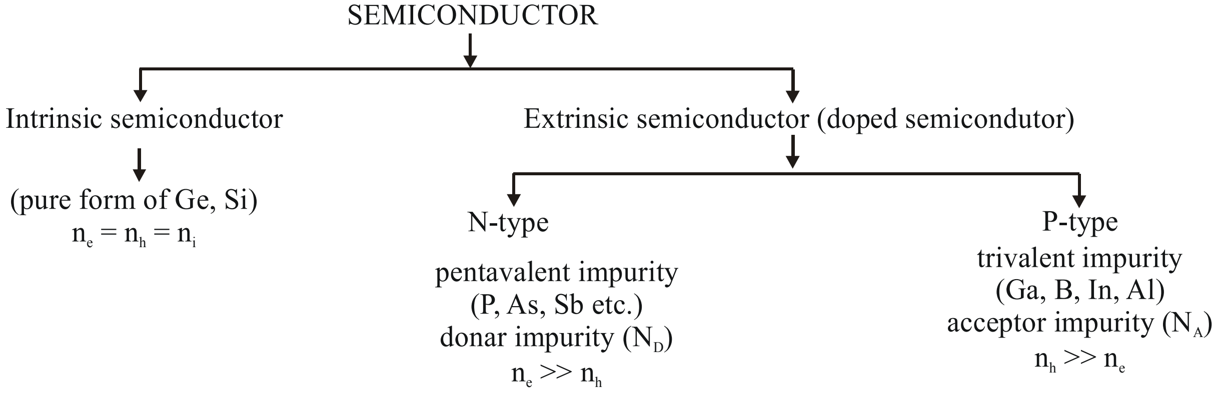
The impurity added may be ≈ 1 part per million (ppm).

- The dopant atom should take the position of semiconductor atom in the lattice.
- The presence of the dopant atom should not distort the crystal lattice.
- The size of the dopant atom should be almost the same as that of the crystal atom.
- The concentration of dopant atoms should not be large (not more than 1% of the crystal atom).

It is to be noted that the doping of a semiconductor increases its electrical conductivity to a great extent.

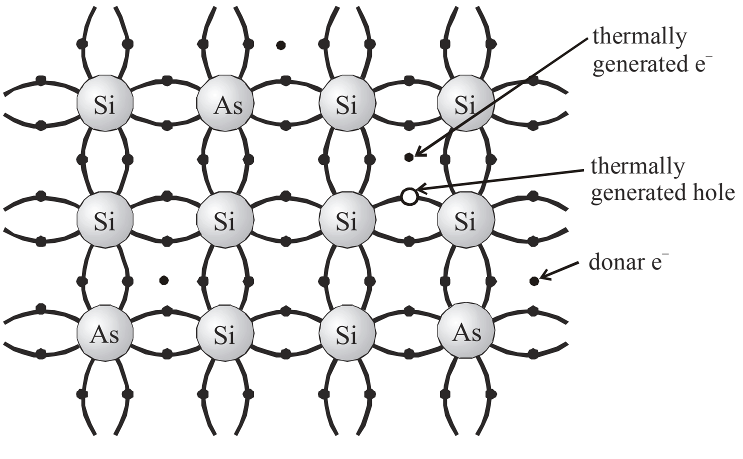
- The concentration of dopant atoms be very low, doping ratio is vary from impure : pure :: $1 : 10^6$ to $1 : 10^{10}$ In general it is $1 : 10^8$
- There are two main method of doping.
  (i) Alloy method (ii) Diffusion method (The best)
- The size of dopant atom (impurity) should be almost the same as that of crystal atom. So that crystalline structure of solid remain unchanged.

CLASSIFICATION OF SEMICONDUCTOR



N type semiconductor

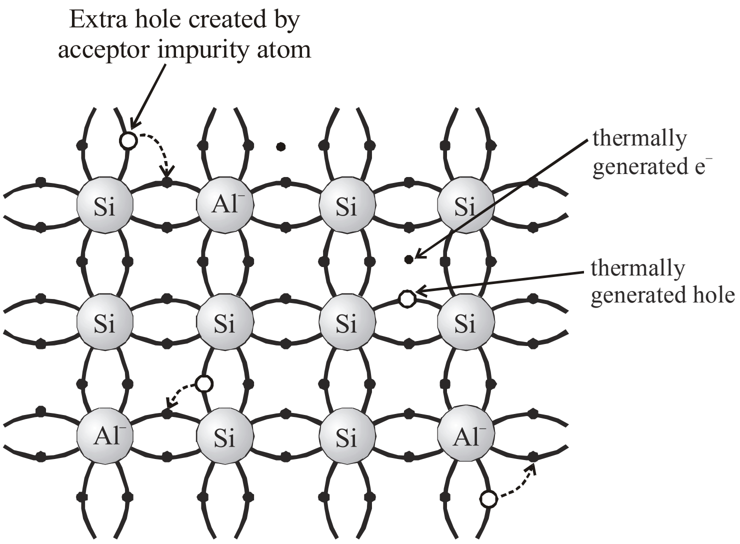
When a pure semiconductor (Si or Ge) is doped by pentavalent impurity (P, As, Sb, Bi) then four electrons out of the five valence electrons of impurity take part, in covalent bonding, with four silicon atoms surrounding it and the fifth electron is set free. These impurity atoms which donate free e^- for conduction are called as Donar impurity (N_D). Due to donar impurity free e^- increases very much so it is called as "N" type semiconductor. By donating e^- impurity atoms get positive charge and hence known as "Immobile Donar positive Ion". In N-type semiconductor free e^- are called as "majority" charge carriers and "holes" are called as "minority" charge carriers.



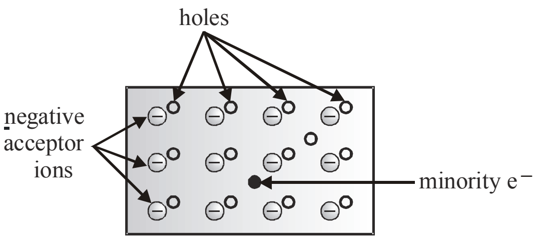
N-type semiconducting crystal

P type semiconductor

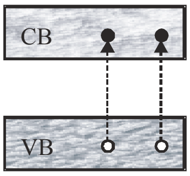
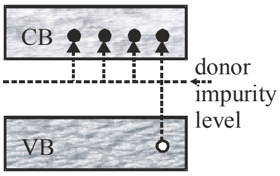
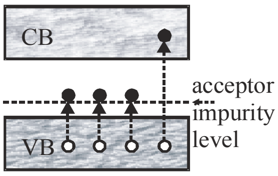
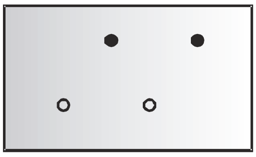
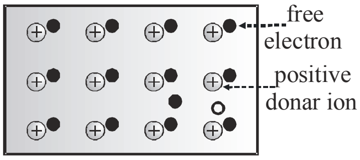
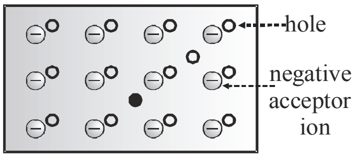
When a pure semiconductor (Si or Ge) is doped by trivalent impurity (B, Al, In, Ga) then outer most three electrons of the valence band of impurity take part, in covalent bonding with four silicon atoms surrounding it and except one electron from semiconductor and make hole in semiconductor. These impurity atoms which accept bonded e^- from valance band are called as Acceptor impurity (N_A). Here holes increases very much so it is called as "P" type semiconductor and impurity ions known as "Immobile Acceptor negative Ion". In P-type semiconductor free e^- are called as minority charge carries and holes are called as majority charge carriers.



P-type semiconducting crystal



P-type semiconductor

	Intrinsic Semiconductor	**N-type (Pentavalent impurity)**	**P-type(Trivalent impurity)**
1.			
2.			
3.	Current due to electron and hole	Mainly due to electrons	Mainly due to holes
4.	$n_e = n_h = n_i$	$n_h << n_e$ ($N_D \simeq n_e$)	$n_h >> n_e$ ($N_A \simeq n_h$)
5.	$I = I_e + I_h$	$I \simeq I_e$	$I \simeq I_h$
6.	Entirely neutral	Entirely neutral	Entirely neutral
7.	Quantity of electrons and holes are equal	Majority - Electrons Minority - Holes	Majority - Holes Minority - Electrons

Mass action Law

In semiconductors due to thermal effect, generation of free e$^-$ and hole takes place.

Apart from the process of generation, recombination also occurs simultaneously, in which free e$^-$ further recombine with hole.

At equilibrium rate of generation of charge carries is equal to rate of recombination of charge carrier. The recombination occurs due to e^- colliding with a hole, larger value of n_e or n_h, higher is the probability of their recombination.

Hence for a given semiconductor rate of recombination $\propto n_e \times n_h$

so rate of recombination = R $n_e \times n_h$ R = recombination coefficient,

The value of R remains constant for a solid, according to the law of thermodynamics until crystalline lattice structure remains same.

For intrinsic semiconductor $n_e = n_h = n_i$

so rate of recombination = R n_i^2

$$R\, n_e \times n_h = R\, n_i^2 \quad \Rightarrow \quad n_i^2 = n_e \times n_h$$

Under thermal equilibrium, the product of the concentration 'n_e' of free electrons and the concentration n_h of holes is a constant and it is independent of the amount of doping by acceptor and donor impurities.

Thus from mass action law $\boxed{n_e \times n_h = n_1^2}$

Electron-hole Recombination :

It is necessarly to complete a band that electron is shared from neighbouring atoms or it may also be received from conduction band. In the second case electron recombines with the hole of valnce band. This process is known as electron-hole recombination.

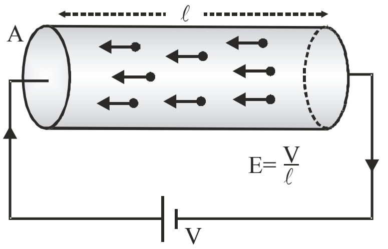
The breaking of bands or generation of electron-hole pairs, and completion of bands due to recombination is taking place continuously.

At equilibrium, the rate of generation becomes equal to the rate of recombination, giving a fixed number of free electrons and holes.

RESISTIVITY AND CONDUCTIVITY OF SEMICONDUCTOR

Conduction in conductor

Relation between current (I) and drift velocity (v_d)



$I = neAv_d$ n = number of electron in unit volume

A= cross sectional area

current density $J = \frac{I}{A}$ amp/m^2 = ne v_d drift velocity of electron $v_d = \mu E$

$J = ne\,\mu E$ $J = \sigma E$

Conductivity $\sigma = ne\mu = 1/\rho$ ρ = Resistivity

Mobility $\mu = \frac{v_d}{E}$

Conduction in Semiconductor

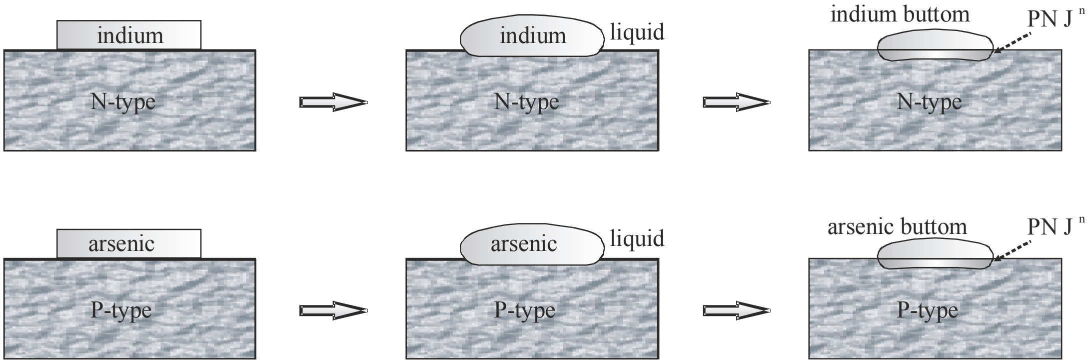
Intrinsic semiconductor	**P - type**	**N - type**
$n_e = n_h$	$n_h >> n_e$	$n_e >> n_h$
$J = ne\,[v_e + v_h]$	$J \cong e\,n_h\,v_h$	$J \cong e\,n_e\,v_e$
$\sigma = \frac{1}{\rho} = en\,[\mu_e + \mu_h]$	$\sigma = \frac{1}{\rho} \cong e\,n_h\,\mu_h$	$\sigma = \frac{1}{\rho} \cong e\,n_e\,\mu_e$

P - N JUNCTION

Techniques for making P-N junction

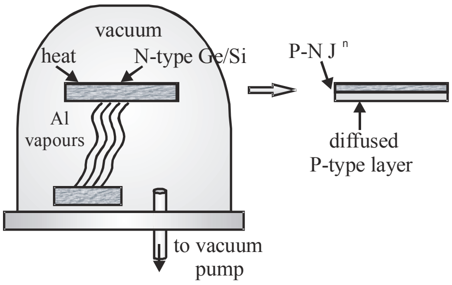
(i) Alloy Method or Alloy Junction

Here a small piece of III group impurity like indium is placed over n–Ge or n–Si and melted as shown in figure ultimetely P – N junction form.



(ii) Diffusion Junction :-

A heated P–type semiconductor is kept in pentavalent impurity vapours which diffuse into P–type semiconductor as shown and make P–N junction.

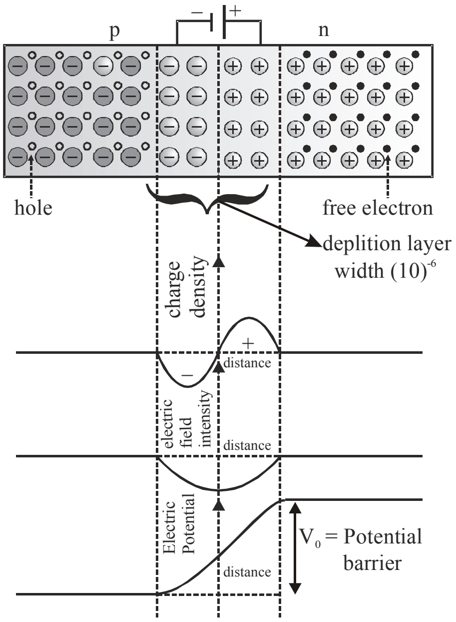


(iii) Vapour deposited junction or epitaxial junction :

If we want to grow a layer of n–Si or p–Si then p–Si wafer is kept in an atmosphere of Silane (a silicon compound which dissociates into Si at high temperatures) plus phosphorous vapours. On craking of silane at high temperature a fresh layer on n–Si grows on p–Si giving the "P–N junction". Since this junction growth is layer by so it is also referred as layer growth or epitaxial junction formation of P–N junction.

Description of P-N Junction without applied voltage or bias

Given diagram shows a P–N junction immediately after it is formed.



P region has mobile majority holes and immobile negatively charged impurity ions.

N region has mobile majority free electrons and immobile positively charged impurity ions.

Due to concentration difference diffusion of holes starts from P to N side and diffusion of e^- s starts N to P side.

Due to this a layer of only positive (in N side) and negative (in P–side) started to form which generate an electric field (N to P side) which oppose diffusion process, during diffusion magnitude of electric field increases due to this diffusion it gradually decreased and ultimately stope.

The layer of immobile positive and negative ions, which have no free electrons and holes called as **depletion layer** as shown in diagram.

- **Width of depletion layer $\cong$ 10^{-6} m**

  (a) As doping increases depletion layer decreases

  (b) As temperature is increased depletion layer also increases.

  (c) P-N junction → nonohmic, due to nonlinear relation between I and V.

- **Potential Barrier or contact potential**

  Ge $\longrightarrow$ 0.3 V Si $\longrightarrow$ 0.7 V

- Electric field, produce due to potential barrier $E = \frac{V}{d} = \frac{0.5}{10^{-6}} \Rightarrow E \cong 10^5$ V/m

  This field prevents the respective majority carrier from crossing barrier region

DIFFUSION AND DRIFT CURRENT

(1) Diffusion current – P to N side (2) Drift current – N to P side

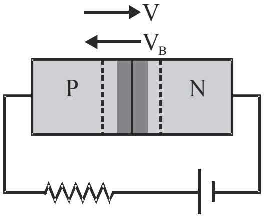
If there is no biasing diffusion current = drift current

So total current is zero

BEHAVIOUR OF P–N JUNCTION WITH AN EXTERNAL VOLTAGE APPLIED OR BIAS

Forward Bias

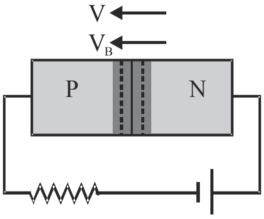
If we apply a voltage "V" such that P–side is positive and N–side is negative as shown in diagram.



The applied voltage is opposite to the junction barrier potential.Due to this effective potential barrier decreases, junction width also decreases, so more majority carriers will be allowed to flow across junction. It means the current flow in principally due to majority charge carriers and it is in the order of mA called as forward Bias.

Reverse Bias

If we apply a voltage "V" such that P–side is negative and N–side is positive as shown in diagram. The applied voltage is in same direction as the junction barrier potential. Due to this effective potential barrier increase junction, width also increases, so no majority carriers will be allowed to flow across junction.

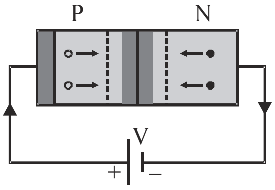
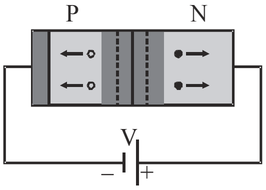
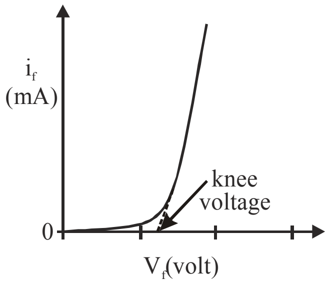
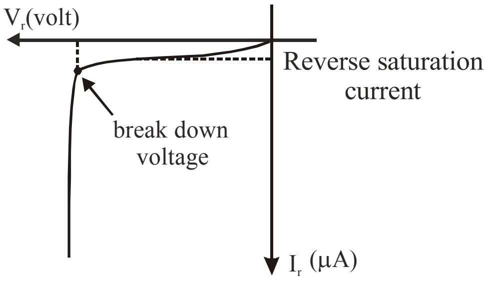


Only minority carriers will drifted. It means the current flow in principally due to minority charge carriers and is very small (in the order of μA). This bias is called as reversed Bias.

- In reverse bias, the current is very small and nearly constant with bias (termed as reverse saturation current). However interesting behaviour results in some special cases if the reverse bias is increased further beyond a certain limit, above particular high voltage breakdown of depletion layer started.
- Breakdown of a diode is of following two types :

(i) Zener breakdown (ii) Avalanche breakdown

Comparison between Forward Bias and Reverse Bias

Forward Bias	Reverse Bias
P → positive N → negative	P → negative N → positive
	
1. Potential Barrier reduces	1. Potential Barrier increases.
2. Width of depletion layer decreases	2. Width of depletion layer increases.
3. P-N jn. provide very small resistance	3. P-N jn. provide high resistance
4 Forward current flows in the circuit	4. Very small current flows.
5. Order of forward current is milli ampere.	5. Order of current is micro ampere for Ge or Neno ampere for Si.
6. Current flows mainly due to majority carriers.	6. Current flows mainly due to minority carriers.
7. Forward characteristic curves.	7. Reverse characteristic curve
	
8. Forward resistance $R_f = \frac{\Delta V_f}{\Delta I_f} \cong 100\Omega$	8. Reverse resistance $R_r = \frac{\Delta V_r}{\Delta I_r} \cong 10^6\Omega$
9. Order of knee or cut in voltage Ge → 0.3 V Si → 0.7 V	9. Breakdown voltage Ge → 25 V Si → 35 V

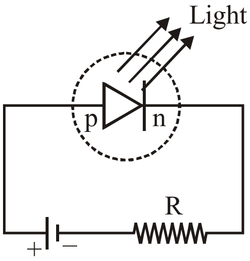
Special point : Generally $\frac{R_r}{R_f} = 10^3 : 1$ for Ge $\frac{R_r}{R_f} = 10^4 : 1$ for Si

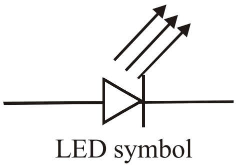
Light Emitting Diode (LED):

A light emitting diode is simply a forward biased p-n junction which emits spontaneous light radiation. When forward bias is applied, the electron and holes at the junction recombine and energy released is emitted in the form of light. for visible radiation phosphorus doped GaAs is commonly used. The advantages of LEDs are:

(i) Low operational voltage and less power.
(ii) Fast action with no warm up time.
(iii) Emitted light is nearly monochromatic.
(iv) They have long life.

I-V characterisitics of LED are similar to that of Si junction diode but the threshold voltages are much higher and slightly different for each colour. The reverse breakdown voltages of LED's are very low, about 5 V.

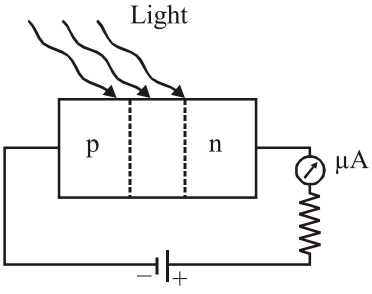



LED symbol

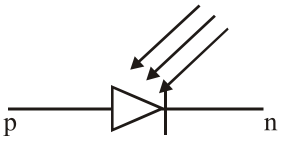
Photodiode:

It is a reversed-biased p-n junction, illuminated by radiation. When p-n junction is reversed biased with no current, a very small reverse saturated current flows across the junction called the dark current. When the junction is illuminated with light, electron-hole pairs are created at the junction, due to which additional current begins to flow across the junction; the current is solely due to minority charge carriers.

(1) A photodiode is used in reverse bias, although in forward bias current is more than current in reverse bias because in reverse bias it is easier to observe change in current with change in light intensity.
(2) Photodiode is used to measure light intensity because reverse current increases with increase of intensity of light.

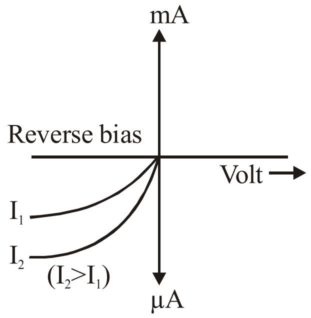


(a)



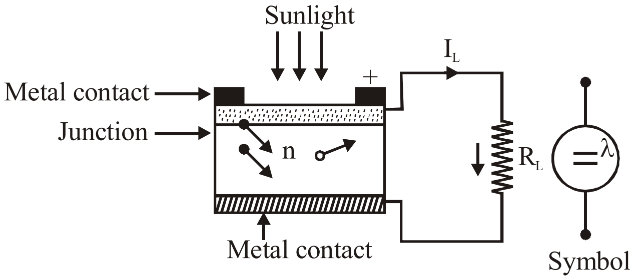
Symbol
(b)

The characteristic curves of a photodiode for two different illurninatios I_1 and I_2 ($I_2 > I_1$) are shown in figure.

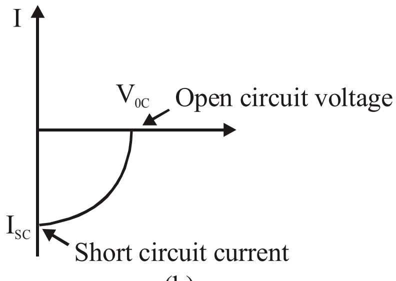


Solar Cell

A solar cell is a junction diode which converts tight energy'into electrical energy. A p-n junction solar cell consists of a large junction with no external biasing.



Construction and working
(a)



(b)

The surface layer of p-region is made very thin so that the incident photons may easily penetrate to reach the junction which is the active region. In an operation in the photovoltaic mode (i.e., generation of voltage due to bombardment of optical photons); the materials suitable for photocells are silicon (Si), gallium arsenide (GaAs), cadmium sulphide (CdS) and cadmium selenide (CdSe).

Working:

When photons of energy greater than band gap energy ($h\nu > E_g$) are made incident on the junction, electron-hole pairs are created which move in opposite directions due to junction field. These are collected at two sides of junction, thus producing photo-voltage; this gives rise to photocurrent. The characteristic curve of solar cell is shown in fig. Solar cells are used in satellites to recharge their batteries.

11. REVERSE BREAKDOWN

If the reverse bias voltage is made too high, the current through the PN junction increases rapidly at Y_z. The voltage at which this happens is called **breakdown voltage** or **Zener voltage.**

There two mechanism which causes this breakdown. One is called avalanche breakdown and other is called **Zener breakdown.**

Zener breakdown: When reverse bias is increased the electric field at then junction also increases. At some stage the electric field becomes so high that it breaks the covalent bonds creating electron, hole pairs, thus a large number of carriers are generated. This causes a large current to flow. This machanism is know as Zener breakdown.

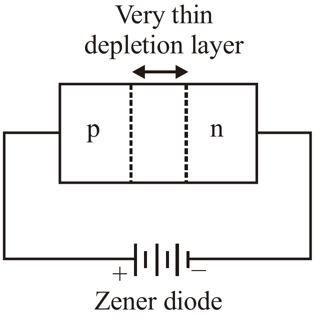
Avalanche breakdown : At high reverse voltage, due to high electric field, the rniniority charge carriers, while crossing the junction acquires very high velocities. These by collision breaks down the covalent bonds, generating more carriers. A chain reaction is established, giving rise to high current. This mechanism is called **avalanche breakdown..**

Zener Diode:

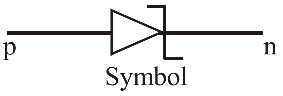
A zener diode is a specially designed heavily doped p-n junction, having a very thin depletion layer and having a very sharp breakdown voltage. It is always operated in breakdown region. Its breakdown voltage V_z is less than 6 V.

Zener Diode as a Voltage Regulator:

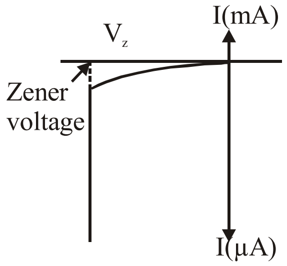
Zener diode may be used as a voltage regulator. The circuit of zener-diode is shown in figure.



(a)



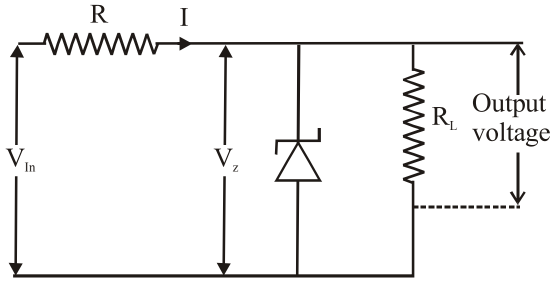
(b)



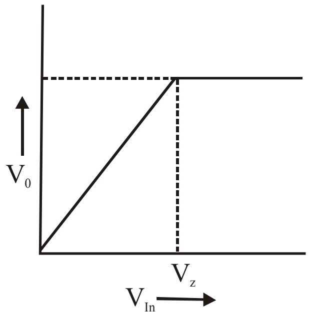
(c) characteristics

In breakdown region the equation: $V_0 = V_z = V_{in} - RI$

Clearly, when the input voltage exceeds zener voltage to keep the voltage regularity, the extra input voltage appears across series resistance R. The voltage regulation curve is shown in figure.



(d)



(e)

Zener Break down

Where covalent bonds of depletion layer, its self break, due to high electric field of very high Reverse bias voltage.

very-very high reverse bias voltage.

This phenomena predominant

(i) At lower voltage after "break down"

(ii) In P – N having "High doping"

(iii) P – N Jn. having thin depletion layer

Here P – N not damage paramanently

"In D.C voltage stablizer zener phenomenan is used".

Avalanche Break down

Here covalent bonds of depletion layers are broken by collision of "Minorities" which aquire high kinetic energy from high electric field of

This phenomena predominant

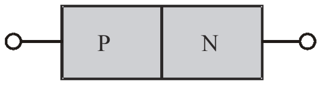
(i) At high voltage after breakdown

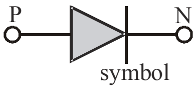
(ii) In P – N having "Low doping"

(iii) P – N Jn. having thick depletion layer

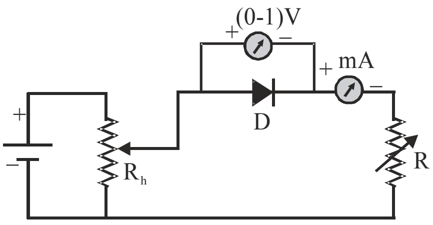
Here P – N damage peramanentaly due to "Heating effect" due to abruptly increment of minorities during repeatative collisoins.

CHARACTERISTIC CURVE OF P-N JUNCTION DIODE

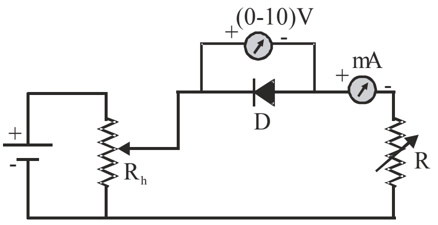




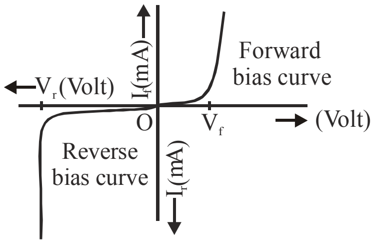
symbol



Forward bias



Reverse bias



In forward bias when voltage is increased from 0V is steps and corresponding value of current is measured, the curve comes as OB of figure. We may note that current increase very sharply after a certain voltage knee voltage. At this voltage, barrier potential is completely eliminated and diode offers a low resistance.

In reverse bias a microammeter has been used as current is very very small. When reverse voltage is increased from 0V and corresponding values of current measured the plot comes as OCD. We may note that reverse current is almost constant hence called reverse saturation current. It implies that diode resistance is very high. As reverse voltage reaches value V_B, called breakdown voltage, current increases very sharply.

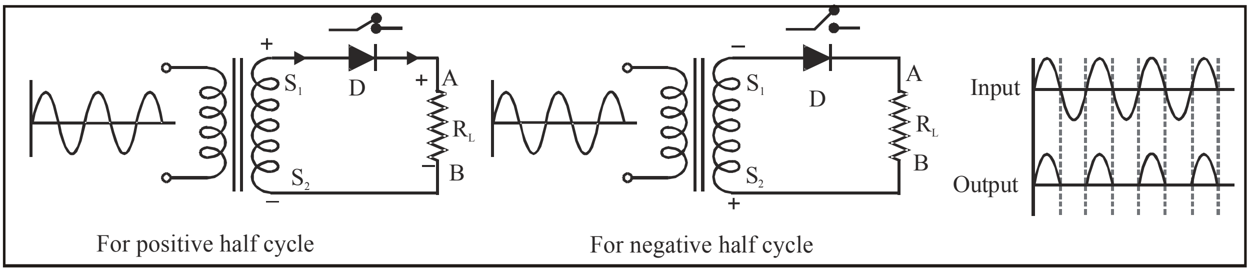
For Ideal Diode

in forward bias	in reverse bias
 $R_f = 0$ 'ON' switch	 $R_r = \infty$ 'OFF' switch

RECTIFIER

It is device which is used for converting alternating current into direct current.

Half wave rectifier



For positive half cycle For negative half cycle

During the first half (positive) of the input signal, let S_1 is at positive and S_2 is at negative potential. So, the PN junction diode D is forward biased. The current flows through the load resistance R_L and output voltage is obtained.

During the second half (negative) of the input signal, S_1 and S_2 would be negative and positive respectively. The PN junction diode will be reversed biased. In this case, practically no current would flow through the load resistance. So, there will be no output voltage.

Thus, corresponding to an alternating input signal, we get a unidirectional pulsating output as shown.

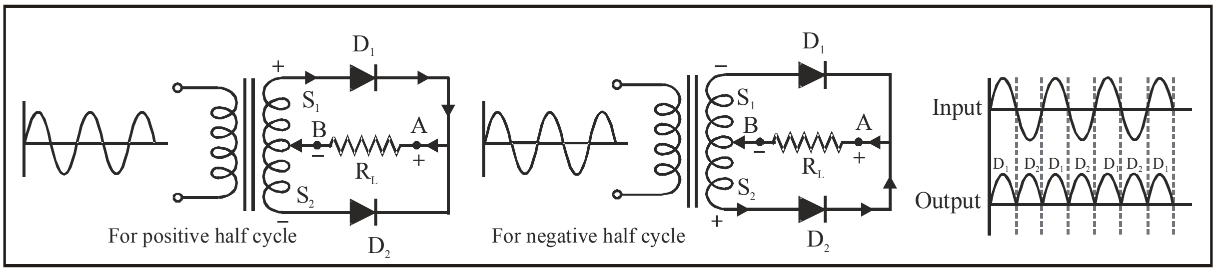
Peak inverse voltage (PIV)

In half wave rectifier PIV = maximum voltage across secondary coil of transformer (V_s)

$$= \text{Peak value of output } (V_m)$$

Full wave rectifier

When the diode rectifies the whole of the AC wave, it is called full wave rectifier. Figure shows the experiemental arrangement for using diode as full wave rectifier. The alternating signal is fed to the primary a transformer. The output signal appears across the load resistance R_L.



For positive half cycle For negative half cycle

During the positive half of the input signal :

Let S_1 positive and S_2 negative.

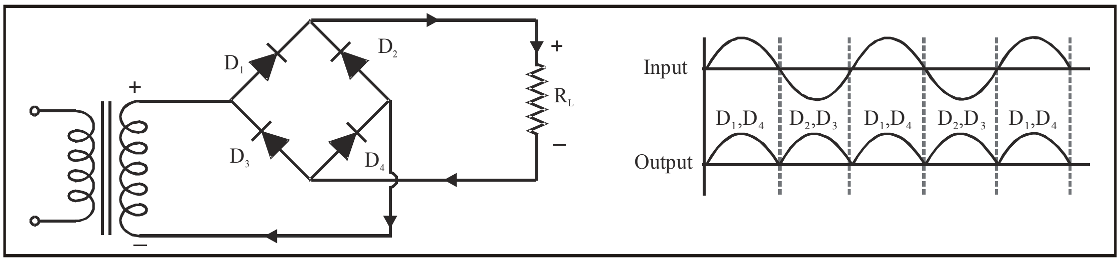
In this case diode D_1 is forward biased and D_2 is reverse biased. So only D_1 conducts and hence the flow of current in the load resistance R_L is from A to B.

During the negative half of the input signal :

Now S_1 is negative and S_2 is positive. So D_1 is reverse-biased and D_2 is forward biased. So only D_2 conducts and hence the current flows through the load resistance R_L from A to B.

It is clear that whether the input signal is positive or negative, the current always flows through the load resistance in the same direction and full wave rectification is obtained.

Bridge Rectifier



TRANSISTOR

Inventor William Bradford Shockley, John Bardeen and Walter Houser Brattain.

Transistor is a three terminal device which transfers a signal from low resistance circuit to high resistance circuit.

It is formed when a thin layer of one type of extrinsic semiconductor (P or N type) is sandwitched between two thick layers of other type of extrinsic semiconductor.

Each transistor have three terminals which are :-

(i) Emitter

(ii) Base

(iii) Collector

Emitter

It is the left most part of the transistor. It emits the majority carrier towards base. It is highly doped and medium in size.

Base

It is the middle part of transistor which is sandwitched by emitter (E) and collector (C). It is lightly doped and very thin in size.

Collector

It is right part of the transistor which collect the majority carriers emitted by emitter. It has large size and moderate doping.

There are two semiconducting PN-junctions in a transistor

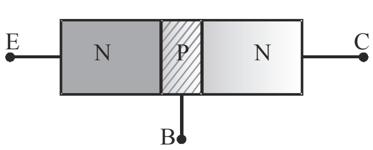
(i) The junction between emitter and base is known as emitter-base junction **(J_{EB})**.

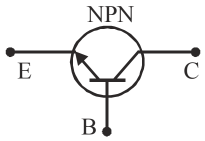
(ii) The junction between base and collecter is known as base-collector junction **(J_{CB})**.

TRANSISTOR ARE OF TWO TYPES

N-P-N Transistor

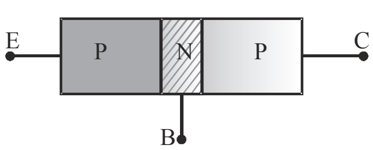
If a thin layer of P-type semiconductor is sandwitched between two thick layers of N-type semiconductor is known as NPN transistor.

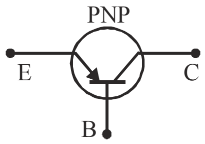




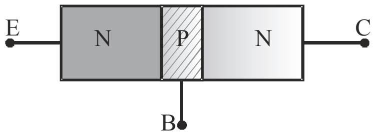
P-N-P Transistor

If a thin layer of N-type of semiconductor is sandwitched between two thick layer of P-type semiconductor is known as PNP transistor.





- Transistor have two P-N Junction J_{EB} and J_{CB}, therefore it can be biased in four following ways as given below:



Emitter-Base	Collector-Base	Region of working
Forward biased	Reverse biased	Active
Reverse biased	Forward biased	Inverse Active
Reverse biased	Reverse biased	Cut off
Forward biased	Forward biased	Saturation

- Comparsion between E, B and C

Emitter	Medium size	High dopping
Base	Smallest size	Low dopping
Collector	Largest size	Medium dopping

- The collector region is made physically larger than the emitter. Because collector has to dissipiate much greater power.

- Transistor all mostly work in active region in electronic devices & transistor work as an amplifier in Active region only.

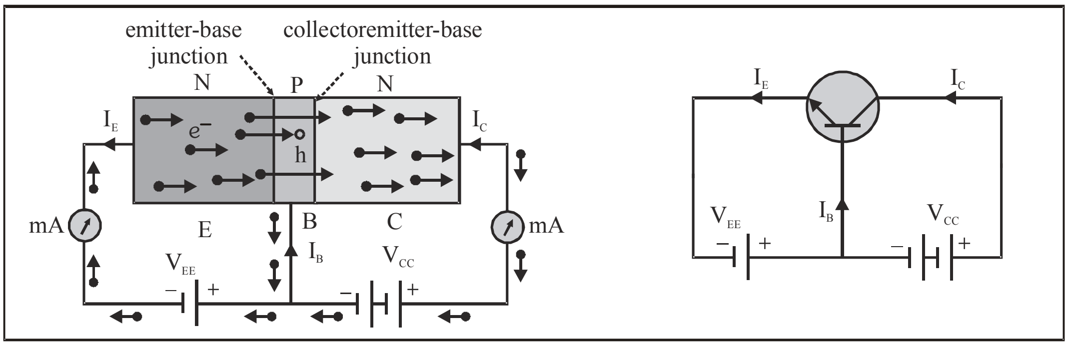
- Transistor i.e. It is a short form of two words "Transfer resistors". Signal is introduced at low resistance circuit and out put is taken at high resistance circuit.

- Base is lightly doped. Otherwise the most of the charge carrier from the emitter recombine in base region and not reaches at collector.

- Transistor is a current operated device i.e. the action of transistor is controlled by the motion of charge carriers. i.e. current.

WORKING OF NPN TRANSISTOR

The emitter Base junction is forward bias and collector base junction is reversed biased of n-p-n transistor in circuit (A) and symbolic representation is shown in Figure.



When emitter base junction is forward bias, electrons (majority carriers) in emitter are repelled toward base.

The barrier of emitter base junction is reduced and the electron enter the base, about 5% of these electron recombine with hole in base region result in small current (I_b).

The remaining electron ($\approx$95%) enter the collector region because they are attracted towards the positive terminal of battery.

For each electron entering the positive terminal of the battery is connected with collector base junction an electron from negative terminal of the battery connected with emitter base junction enters the region.

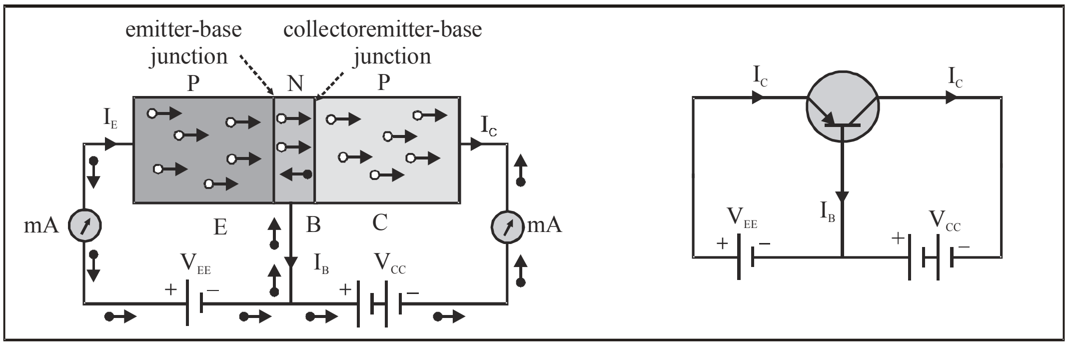
The emitter current (I_e) is more than the collector (I_c).

The base current is the difference between I_e and I_c and proportional to the number of electron hole recombination in the base.

$$I_e = I_b + I_c$$

WORKING OF PNP TRANSISTOR

When emitter-base junction is forward biased holes (majority carriers) in the emitter are repelled towards the base and diffuse through the emitter base junction. The barrier potential of emitter-base junction decreases and hole enter the n-region (i.e. base). A small number of holes ($\approx$5%) combine with electron of base-region resulting small current (I_b). The remaining hole ($\approx$95%) enter into the collector region because they are attracted towards negative terminal of the battery connected with the collector-base junction. These hole constitute the collector current (I_c).



As one hole reaches the collector, it is neutralized by the battery. As soon as one electron and a hole is neutralized in collector a covalent bond is broken in emitter region. The electron hole pair is produced. The released electron enter the positive terminal of battery and hole more towards the collector.

Basic Transistor Circuit Configurations :-

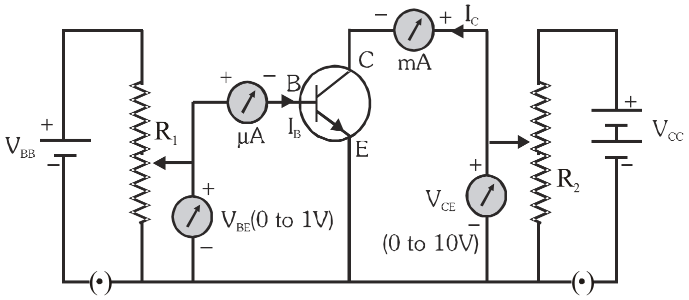
To study about the characterstics of transistor we have to make a circuit [In which four terminals are required]. But the transistor have three terminals, so one of the terminal of transistor is made common in input and output both. Thus, we have three possible configuration of transistor circuit.

(i) Common base (ii) Common emitter (iii) Common collector

In these three common emitter is widely used and common collector is rarely used.

Common emitter characterstics of a transistor

Circuit Diagram :



Circuit diagram for characteristic curve of n-p-n transistor in CE mode

Input characterstics

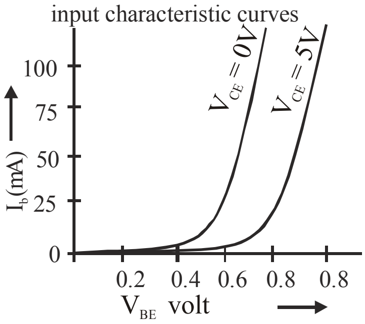
The variation of base current (I_b) (input) with base emitter voltage (V_{EB}) at constant-emitter voltage (V_{CE}) is called input characterstic.

(i) Keep the collector-emitter voltage (V_{CE}) constant (say V_{CE} = 1V)

(ii) Now change emitter base voltage by R_1 and note the corresponding value of base current (I_b).

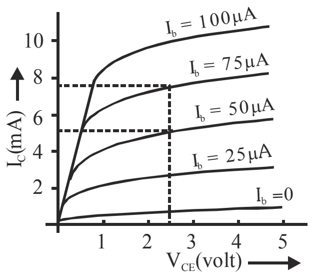
(iii) Plot the graph between V_{EB} and I_b.

(iv) A set of such curves can be plotted at different (V_{CE} = 2V)



Output characterstics

The variation of collector current I_c (output) with collector-emitter voltage (V_{CE}) at constant base current (I_b) is called output characterstic.



(i) Keep the base current (I_b) constant (say $I_b = 10\mu A$)

(ii)Now change the collector-emitter voltage (V_{CE}) using variable resistance R_2 and note the corresponding values of collector current (I_c).

(iii) Plot the graph between (V_{CE} versus I_c)

(iv) A set of such curves can be plotted at different fixed values of base current (say 0, 20 μA, 30 μA etc.)

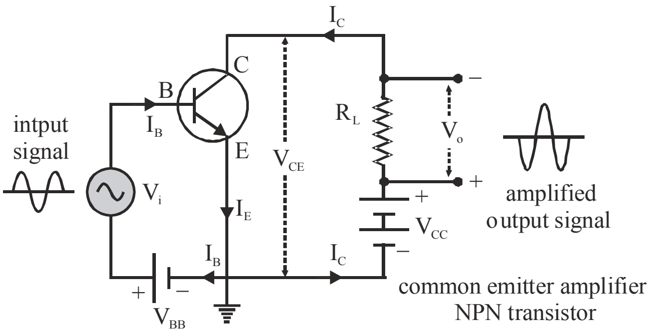
TRANSISTOR AS AN AMPLIFIER

The process of increasing the amplitude of input signal without distorting its wave shape and without changing its frequency is known as amplification.

A device which increases the amplitude of the input signal is called amplifier.

A transistor can be used as an amplifier in active state.

A basic circuit of a common emitter transistor amplifier is shown.



common emitter amplifier
NPN transistor

LOGIC GATES

INTRODUCTION :

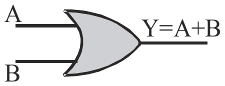
- A logic gate is a digital circuit which is based on certain logical relationship between the input and the output voltages of the circuit.
- The logic gates are built using the semiconductor diodes and transistors.
- Each logic gate is represented by its characteristic symbol.
- The operation of a logic gate is indicated in a table, known as truth table. This table contains all possible combinations of inputs and the corresponding outputs.
- A logic gate is also represented by a Boolean algebraic expression. Boolean algebra is a method of writing logical equations showing how an output depends upon the combination of inputs. Boolean algebra was invented by George Boole.

BASIC LOGIC GATES

There are three basic logic gates. They are (1) OR gate(2) AND gate, and (3) NOT gate

- **The OR gate :-** The output of an OR gate attains the state 1 if one or more inputs attain the state 1.

  Logic symbol of OR gate



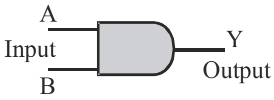
The Boolean expression of OR gate is Y = A + B, read as Y equals A 'OR' B.

Truth table of a two-input OR gate

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

- **The AND gate :-** The output of an AND gate attains the state 1 if and only if all the inputs are in state 1.

  Logic symbol of AND gate

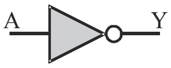


The Boolean expression of AND gate is Y = A.B
It is read as Y equals A 'AND' B

Truth table of a two-input AND gate

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

- **The NOT gate :** The output of a NOT gate attains the state 1 if and only if the input does not attain the state 1.

  Logic symbol of NOT gate 

  The Boolean expression is $Y = \overline{A}$, read as Y equals NOT A.

  Truth table of NOT gate

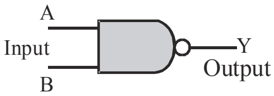
A	Y
0	1
1	0

COMBINATION OF GATES :

The three basis gates (OR, AND and NOT) when connected in various combinations give us logic gates such as NAND, NOR gates, which are the universal building blocks of digital circuits.

- **The NAND gate :**

  Logic symbol of NAND gate



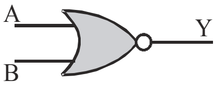
The Boolean expression of NAND gate is $Y = \overline{A.B}$

Truth table of a NAND gate

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

- **The NOR gate :**

  Logic symbol of NOR gate



The Boolean expression of NOR gate is $Y = \overline{A + B}$

Truth table of a NOR gate

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

UNIVERSAL GATES :

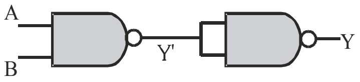
The NAND or NOR gate is the universal building block of all digital circuits. Repeated use of NAND gates (or NOR gates) gives other gates. Therefore, any digital system can be achieved entirely from NAND or NOR gates. We shall show how the repeated use of NAND (and NOR) gates will gives us different gates.

- **The NOT gate from a NAND gate :-** When all the inputs of a NAND gate are connected together, as shown in the figure, we obtain a NOT gate

Truth table of a single input NAND gate

A	B= (A)	Y
0	0	1
1	1	0

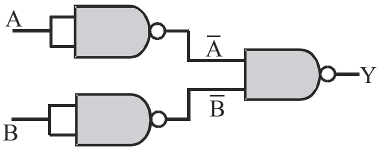
- **The AND gate from a NAND gates :-** If a NAND gate is followed by a NOT gate (i.e., a single input NAND gate), the resulting circuit is an AND gate as shown in figure and truth table given show how an AND gate has been obtained from NAND gates.



Truth table

A	B	Y'	Y
0	0	1	0
0	1	1	0
1	0	1	0
1	1	0	1

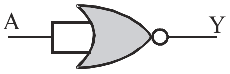
- **The OR gate from NAND gates :-** If we invert the inputs A and B and then apply them to the NAND gate, the resulting circuit is an OR gate.



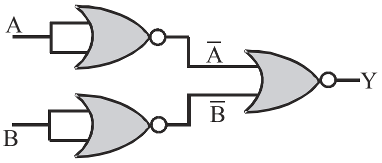
Truth table

A	B	$\overline{A}$	$\overline{B}$	Y
0	0	1	1	0
0	1	1	0	1
1	0	0	1	1
1	1	0	0	1

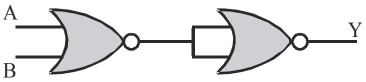
- **The NOT gate from NOR gates :-** When all the inputs of a NOR gate are connected together as shown in the figure, we obtain a NOT gate



- **The AND gate from NOR gates :-** If we invert the inputs A and B and then apply them to the NOR gate, the resulting circuit is an AND gate.



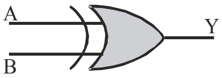
- **The OR gate from NOR gate :-** If a NOR gate is followed by a single input NOR gate (NOT gate), the resulting circuit is an OR gate.



XOR AND XNOR GATES :

- **The Exclusive - OR gate (XOR gate):-** The output of a two-input XOR gate attains the state 1 if one and only one input attains the state 1.

  Logic symbol of XOR gate



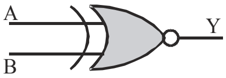
  The Boolean expression of XOR gate is $Y = A.\overline{B} + \overline{A}.B$ or Y = A ⊕B

  Truth table of a XOR gate

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

- **Exclusive - NOR gate (XNOR gate):-** The output is in state 1 when its both inputs are the same that is, both 0 or both 1.

  Logic symbol of XNOR gate

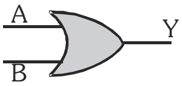
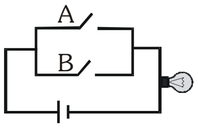
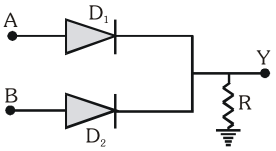


  The Boolean expression of XNOR gate is $Y = A.B + \overline{A}.\overline{B}$ or $Y = \overline{A \oplus B}$ or A ⊙ B

  Truth table of a XNOR gate

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

SUMMARY OF LOGIC GATES

<table>
<tr><th>Names</th><th>Symbol</th><th>Boolean Expression</th><th>Truth table</th><th>Electrical analogue</th><th>Circuit diagram (Practical Realisation)</th></tr>
<tr><td>OR</td><td>
</td><td>$Y = A + B$</td><td><table><tr><th>A</th><th>B</th><th>Y</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table></td><td>
</td><td>
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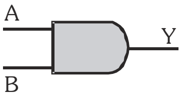
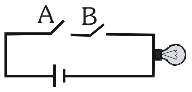
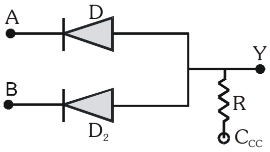
<tr><td>AND</td><td>
</td><td>$Y = A. B$</td><td><table><tr><th>A</th><th>B</th><th>Y</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table></td><td>
</td><td>
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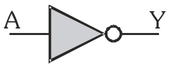
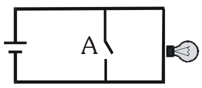
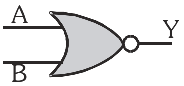
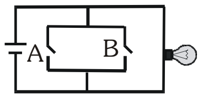
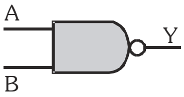
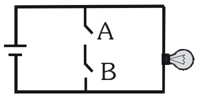
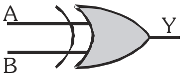
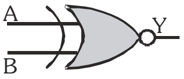
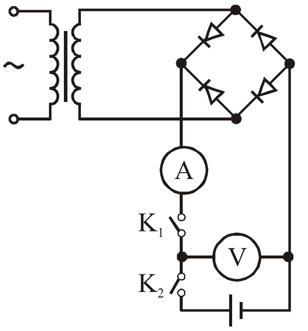
<tr><td>NOT
Inverter</td><td>
</td><td>$Y = \bar{A}$</td><td><table><tr><th>A</th><th>Y</th></tr><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>0</td></tr></table></td><td>
</td><td></td></tr>
<tr><td>NOR
(OR
+NOT)</td><td>
</td><td>$Y = \overline{A + B}$</td><td><table><tr><th>A</th><th>B</th><th>Y</th></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table></td><td>
</td><td></td></tr>
<tr><td>NAND
(AND+NOT)</td><td>
</td><td>$Y = \overline{A.B}$</td><td><table><tr><th>A</th><th>B</th><th>Y</th></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table></td><td>
</td><td></td></tr>
<tr><td>XOR
(Exclusive
OR)</td><td>
</td><td>$Y = A \oplus B$
or
$Y = \bar{A}.B + A\bar{B}$</td><td><table><tr><th>A</th><th>B</th><th>Y</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table></td><td></td><td></td></tr>
<tr><td>XNOR
(Exclusive
NOR)</td><td>
</td><td>$Y = A \odot B$
or
$Y = A.B + \bar{A}.\bar{B}$
or
$Y = \overline{A \oplus B}$</td><td><table><tr><th>A</th><th>B</th><th>Y</th></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table></td><td></td><td></td></tr>
</table>

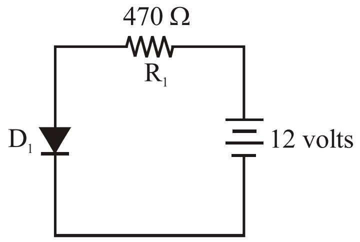
EXERCISE-S

1. The battery is charged from full wave rectifier fed by a sinusoidal voltage (see figure). Ideal diodes, ammeter and voltmeter show the time-average value. At idle with only key K_1 closed, voltmeter shows 12 V, and current is then absent, ie, reading of ammeter is 0. If only the key K_2 is closed, the voltmeter shows battery voltage at 12.3 V. During charging, when the K_2 and K_1 are closed, the voltmeter shows 12.8 V and ammeter shows 5 A. Find the internal resistance of battery.



SC0001

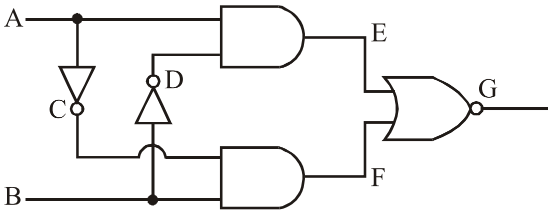
2. Complete the following table of values for this diode circuit, assuming a typical forward voltage drop of 0.65 volts for the diode:



	R_1	D_1	Total
V			12 V
I			
R	470 Ω		
P			

SC0002

3. Write a truth table for the circuit in figure, including the states at C, D, E, F and G.



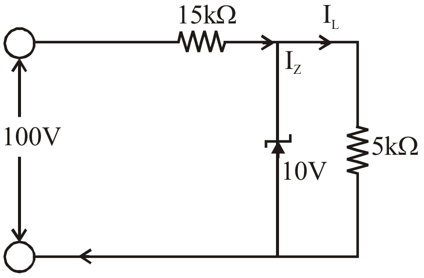
SC0003

4. For the circuit shown in figure, find

i) the output voltage

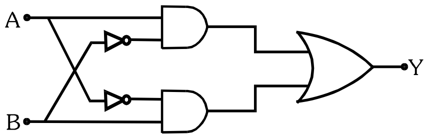
ii) the voltage drop across series resistance

iii) the current through zener diode.



SC0004

5. Write down the actual logic operation carried by the following circuit. Explain your answer.



SC0005

EXERCISE-O

SINGLE CORRECT TYPE QUESTIONS

1. An electric field is applied to a semi-conductor. Let the number of charge carriers be n and the average drift speed be v. If the temperature is increased

(A) Both n and v will increase (B) n will increase but v will decrease

(C) v will increase but n will decrease (D) Both n and v will decrease

SC0006

2. The manifestation of band structure in solids is due to - **[AIEEE-2004]**

(A) Bohr's correspondence principle (B) Pauli's exclusion principle

(C) Heisenberg's uncertainty principle (D) Boltzmann's law

SC0007

3. P-type semiconductor is formed when

A. As impurity is mixed in Si B. Al impurity is mixed in Si

C. B impurity is mixed in Ge D. P impurity is mixed in Ge

(A) A and C (B) A and D (C) B and C (D) B and D

SC0008

4. In extrinsic semiconductors -

(A) The conduction band and valence band overlap

(B) The gap between conduction band and valence band is more than 16 eV

(C) The gap between conduction band and valence band is near about 1 eV

(D) The gap between conduction band and valence band will be 100 eV and more

SC0009

5. A strip of copper and another of germanium are cooled from room temperature to 80 K. The resistance of - **[AIEEE-2004]**

(A) copper strip increases and that of germanium decreases

(B) copper strip decreases and that of germanium increases

(C) each of these increases

(D) each of these decreases

SC0010

6. In a P-N junction diode not connected to any circuit -

(A) the potential in the same everywhere

(B) the P-type side is at a higher potential then the N-type side

(C) there is an electric field at the junction directed from the N-type side to the P-type side

(D) there is an electric field at the junction directed from the P-type to the N-type side

SC0011

7. When p-n junction diode is forward biased, then. **[AIEEE-2004]**

(A) both the depletion region and barrier height are reduced

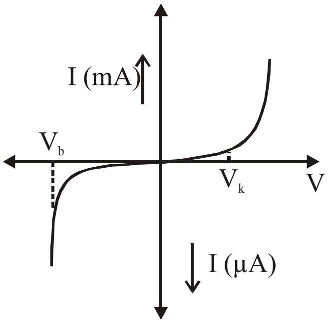
(B) the depletion region is widened and barrier height is reduced

(C) the depletion region is reduced and barrier height is increased

(D) both the depletion region and barrier height are increased

SC0012

8. The V-I characteristic for a p-n junction diode is plotted as shown in the figure. From the plot we can conclude that [$V_b \rightarrow$ breakdown voltage, $V_k \rightarrow$ knee voltage]



(A) the forward bias resistance of diode is very high; almost infinity for small values of V and after a certain value it becomes very low

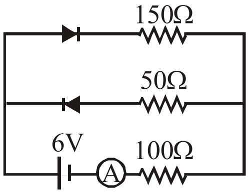
(B) the reverse bias resistance of diode is very high in the beginning upto breakdown voltage is not achieved

(C) both forward and reverse bias resistances are same for all voltages

(D) both (A) and (B) are correct

SC0013

9. In the below given arrangement determine the ammeter reading, if each diodes have a forward resistance of 50 Ω and infinite backward resistance -

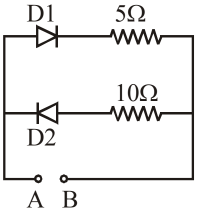


(A) zero (B) 0.02 (C) 0.03 (D) 0.036

SC0014

10. A 2V battery is connected across AB as shown in the figure. The value of the current supplied by the battery when in one case battery's positive terminal is connected to A and in other case when positive terminal of battery is connected to B will respectively be :-

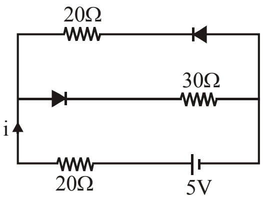
(JEE-Main Online-2015)



(A) 0.1 A and 0.2 A (B) 0.4 A and 0.2 A (C) 0.2 A and 0.4 A (D) 0.2 A and 0.1 A

SC0015

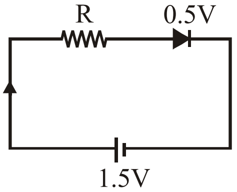
11. Current in the circuit will be -



(A) $\frac{5}{40}$A (B) $\frac{1}{10}$A (C) $\frac{5}{10}$A (D) $\frac{5}{20}$A

SC0016

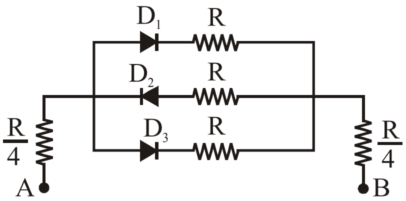
12. The diode used in the circuit shown in the figure has a constant voltage drop of 0.5 V at all currents and a maximum power rating of 100 milliwatts. What should be the value of the resistor R connected in series with the diode for obtaining maximum current -



(A) 1.5 Ω (B) 5 Ω (C) 6.67 Ω (D) 200 Ω

SC0017

13. In the following circuits PN-junction diodes D_1, D_2 and D_3 are ideal for the following potential of A and B, the correct increasing order of resistance between A and B will be -

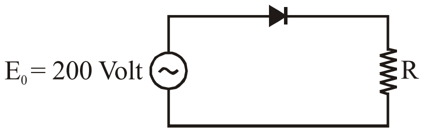


(i) –10V, –5V, (ii) –5V, –10V (iii) –4V, –12V

(A) (i) < (ii) < (iii) (B) (iii) < (ii) < (i) (C) (ii) = (iii) < (i) (D) (i) = (iii) < (ii)

SC0018

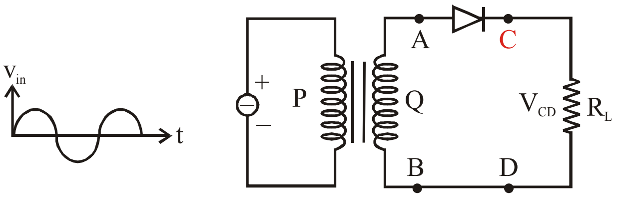
14. A sinusoidal voltage of peak value 200 volt is connected to a diode and resistor R in the circuit shown so that half wave rectification occurs. If the forward resistance of the diode is negligible compared to R then rms voltage (in volt) across R is approximately -

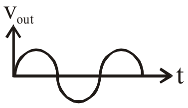


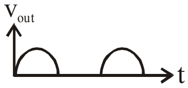
(A) 200 (B) 100 (C) $\frac{200}{\sqrt{2}}$ (D) 280

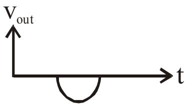
SC0019

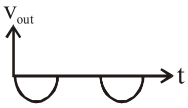
15. In the half-wave rectifier circuit shown. Which one of the following wave forms is true for V_{CD}, if the input is as shown?



(A)


(B)


(C)


(D)


SC0020

16. Zener diode is a p-n junction which has -

(A) p-end heavily doped, n-end lightly doped

(B) n-end heavily doped, p-end lightly doped

(C) both p and n-ends heavily doped

(D) both p and n-ends lightly doped

SC0021

17. Zener diode has both p and n-ends heavily doped so that -

(A) it has small thickness of depletion region

(B) it has large thickness of depletion region due to large recombination

(C) it has large reverse bias voltage

(D) it has weak reverse current when reverse biased

SC0022

18. Most important use of zener diode is to have –

(A) constant voltage across applied load

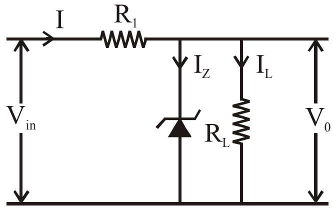
(B) any desired current at constant voltage

(C) a p-n junction working under constant regulated voltage conditions

(D) a p-n junction to operate at high voltages

SC0023

19. In given figure when input voltage increases,



(A) the current through R_S, R_L and zener increases

(B) the current through R_S increases, zener increases but through R_L remains constant

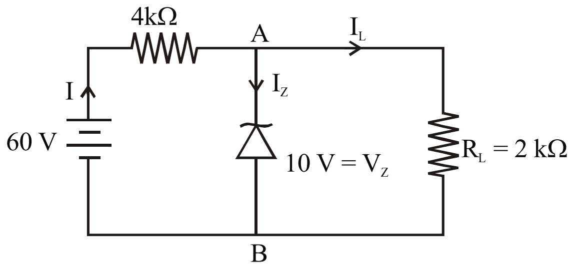
(C) the current through R_S increases, through zener decreases, R_L increases

(D) the current through R_S increases, through zener remains constant but R_L increases

SC0024

20. A Zener diode is connected to a battery and a load as shown below. The currents I, I_Z and I_L are respectively.

(JEE-Main Online-2014)



(A) 12.5 mA, 5 mA, 7.5 mA

(B) 15 mA, 7.5 mA, 7.5 mA

(C) 12.5 mA, 7.5 mA, 5 mA

(D) 15 mA, 5 mA, 10 mA

SC0025

21. A transistor is used in common emitter mode as an amplifier, then:
(1) the base emitter junction is forward biased
(2) the base emitter junction is reverse biased
(3) the input signal is connected in series with the voltage applied to bias the base emitter junction
(4) the Input signal is connected is series with the voltage applied to bias the base collector junction which is correct-
(A) 1, 2, 3 (B) 1, 2, 3, 4 (C) 1, 3 (D) 2, 3, 4

SC0026

22. In a n-p-n transistor circuit the collector current is 10 mA. If 90% of the electron emitted reach the collector then -
(A) the emitter current will be 9 mA (B) the base current will be 9 mA
(C) the emitter current will be 11 mA (D) the base current will be -1 mA

SC0027

23. n-p-n transistors are preferred to p-n-p transistors because -
(A) they have low cost
(B) they have low dissipation energy
(C) they are capable of handling large power
(D) electrons have high mobility than holes.

SC0028

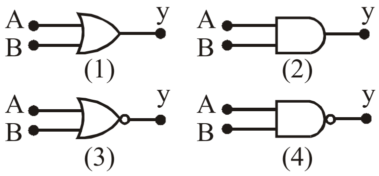
24. When npn transistor is used as an amplifier **[AIEEE-2004]**
(A) electrons move from collector to base
(B) holes move from emitter to base .
(C) electrons move from base to collector
(D) holes move from base to emitter

SC0029

25. For a transistor, α is 0.8. The transistor is in CE configuration. The change in the collector current when the base current changes by 6 mA is :-
(A) 6 mA (B) 4.8mA (C) 24mA (D) 8mA

SC0030

26. Given below are four logic gate symbol (figure). Those for OR, NOR and NAND are respectively



(A) 1,4,3 (B) 4,1,2 (C) 1,3,4 (D) 4,2,1

SC0031

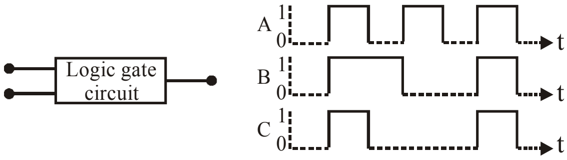
27. The following truth table corresponds to the logic gate -

A	0	0	1	1
B	0	1	0	1
X	0	1	1	1

(A) NAND (B) OR (C) AND (D) XOR

SC0032

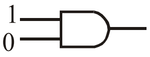
28. The following figure shows a logic gate circuit with two input A and B output C. The voltage waveforms of A, B and C are as shown in second figure below. The logic gate is :

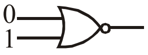


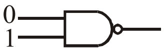
(A) OR gate (C) NAND gate (B) AND gate (D) NOR gate

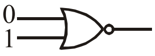
SC0033

29. Which of the following gates will have an output of 1 -

(A) 

(B) 

(C) 

(D) 

SC0034

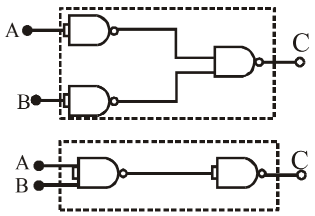
30. The combination of 'NAND' gates shown here under (figure) are equivalent to -

(A) An OR gate and an AND gate respectively

(B) An AND gate and a NOT gate respectively

(C) An AND gate and an OR gate respectively

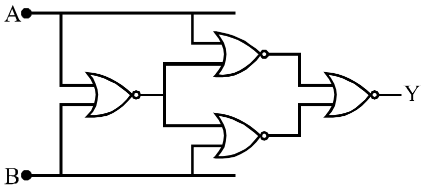
(D) An OR gate and a NOT gate respectively



SC0035

31. A system of four gates is set up as shown.The 'truth table' corresponding to this system is :-

(JEE-Main Online-2013)



(A)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

(B)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

(C)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(D)

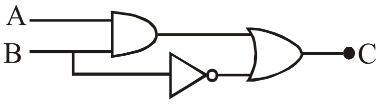
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

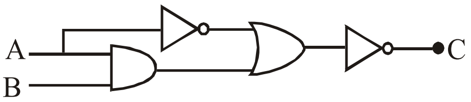
SC0036

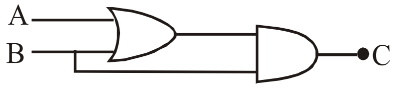
32. Which of the following circuits correctly represents the following truth table ?

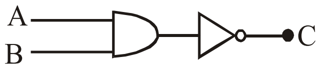
(JEE-Main Online-2013)

A	B	C
0	0	0
0	1	0
1	0	1
1	1	0

(A) 

(B) 

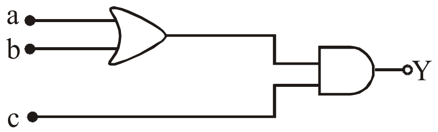
(C) 

(D) 

SC0037

33. To get an output of 1 from the circuit shown in figure the input must be :-

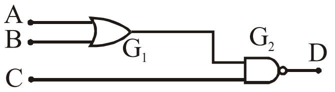
(JEE-Main Online-2016)



(A) a = 1, b = 0, c = 1
(B) a = 1, b = 0, c = 0
(C) a = 0, b = 1, c = 0
(D) a = 0, b = 0, c = 1

SC0038

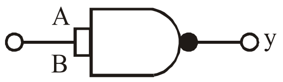
34. For the given combination of gates, if the logic states of inputs A, B, C are as follows A = B = C = 0 and A = B = 1, C = 0 then the logic states of output D are -



(A) 0, 0
(B) 0, 1
(C) 1, 0
(D) 1,1

SC0039

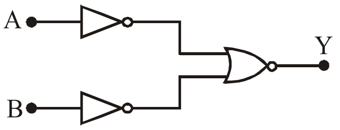
35. This symbol represents



(A) NOT gate
(B) OR
(C) AND gate
(D) NOR gate

SC0040

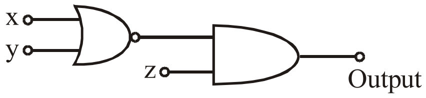
36. Which logic gate is represented by the following combination of logic gates -



(A) OR
(B) NAND
(C) AND
(D) NOR

SC0041

37. In which of the following cases, we would obtain an output of one.



(A) x = 1, y = 1, z = 1
(B) x = 1, y = 1, z = 0
(C) x = 0, y = 1, z = 1
(D) x = 0, y = 0, z = 1

SC0042

38. In a full wave rectifier circuit operating from 50 Hz mains frequency, the fundamental frequency in the ripple would be :- **[AIEEE-2005]**

(A) 50 Hz
(B) 25Hz
(C) 100 Hz
(D) 70.7 Hz

SC0043

39. The electrical conductivity of a semiconductor increases when electromagnetic radiation of wavelength shorter than 2480 nm is incident on it. The band gap (in eV) for the semiconductor is

[AIEEE-2005]

(A) 1.1 eV
(B) 2.5 eV
(C) 0.5 eV
(D) 0.7 eV

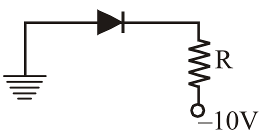
SC0044

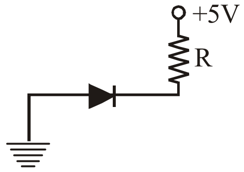
40. If the ratio of the concentration of electrons to that of holes in a semiconductor is $\frac{7}{5}$ and the ratio of currents is $\frac{7}{4}$ then what is the ratio of their drift velocities - **[AIEEE-2006]**

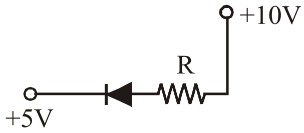
(A) $\frac{5}{4}$ (B) $\frac{4}{7}$ (C) $\frac{5}{6}$ (D) $\frac{4}{5}$

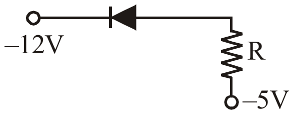
SC0045

41. In the following, which one of the diodes is reverse biased ; **[AIEEE-2006]**

(A)


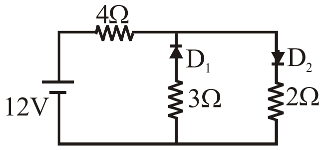
(B)


(C)


(D)


SC0046

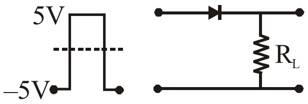
42. The circuit has two oppositely connected ideal diodes in parallel. What is the current flowing in the circuit ? **[AIEEE-2006]**

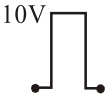


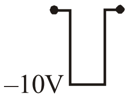
(A) 2.31 A (B) 1.33 A (C) 1.71 A (D) 2.00 A

SC0047

43. If in a p-n junction diode, a square input signal of 10V is applied as shown. Then the output signal across R_L will be :- **[AIEEE-2007]**



(A)


(B)


(C)


(D)


SC0048

44. Carbon, silicon and germanium have four valence electrons each. At room temperature which one of the following statements is most appropriate ? **[AIEEE-2007]**

(A) The number of free conduction electrons is significant in C but small in Si and Ge

(B) The number of free conduction electrons is negligibly small in all the three

(C) The number of free electrons for conduction is significant in all the three

(D) The number of free electrons for conduction is significant only in Si and Ge but small in C

SC0049

MULTIPLE CORRECT TYPE QUESTIONS

45. Which of the following statements is correct

(A) Resistance of semiconductor decreases with increase in temperature

(B) In an electric field, displacement of holes is opposite to the displacement of electrons

(C) Resistance of a conductor decreases with the increase in temperature

(D) n-type semiconductors are neutral

SC0050

46. Pick out the correct statement the reverse current in p-n junction diode

(A) can be minimum and constant

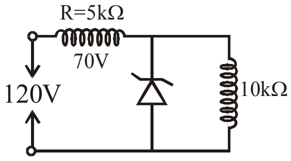
(B) remains constant even after the breakdown voltage

(C) becomes infinity at breakdown

(D) reverse current is controlled by external resistance

SC0051

47. For the circuit shown in the figure:



(A) current through zener diode is 4 mA
(B) current through zener diode is 9 mA
(C) the output voltage is 50 V
(D) the output voltage is 40 V

SC0052

48. Which of the following devices are heavily doped p-n junction

(A) Photo diode
(B) Light emitting diode
(C) Solar cell
(D) Zener mode

SC0053

49. Which of the following statements is correct for proper working of zener diode ?

(A) Reverse bias voltage should be less than or equal to zener breakdown voltage

(B) Reverse bias voltage applied must be greater than zener breakdown voltage.

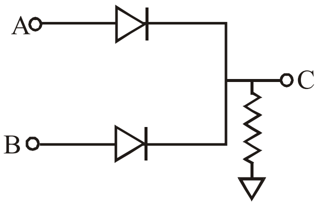
(C) Zener is to be reverse biased for zener action

(D) For given zener diode there can be different zener breakdown voltages

SC0054

EXERCISE-JM

1. In the circuit below, A and B represent two inputs and C represents the output. **[AIEEE-2008]**

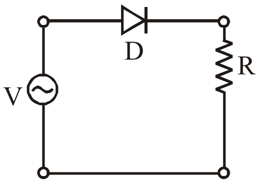


The circuit represents

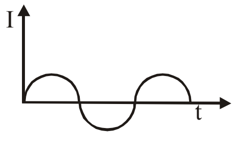
(1) AND gate (2) NAND gate (3) OR gate (4) NOR gate

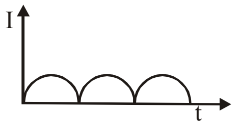
SC0055

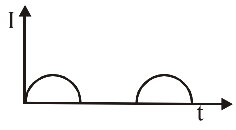
2. An p-n junction (D) shown in the figure can act as a rectifier. An alternating current source (V) is connected in the circuit.

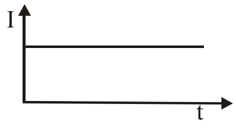


The current (I) in the resistor R can be shown by: **[AIEEE-2009]**

(1)


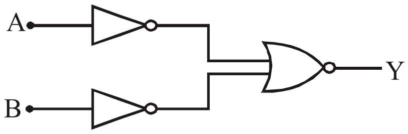
(2)


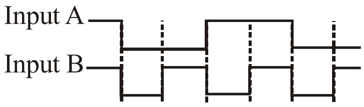
(3)


(4)


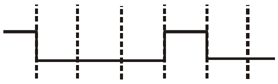
SC0056

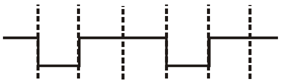
3. The logic circuit shown below has the input wave forms 'A' and 'B' as shown. Pick out the correct output waveform. **[AIEEE-2009]**

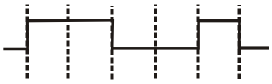


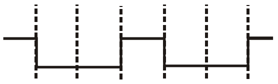


Output is -

(1)


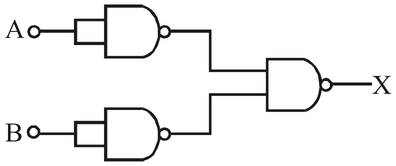
(2)


(3)


(4)


SC0057

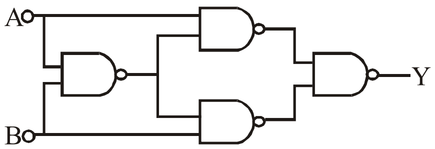
4. The combination of gates shown below yields. **[AIEEE-2010]**



(1) XOR gate (2) NAND gate (3) OR gate (4) NOT gate

SC0058

5. Truth table for system of four NAND gates as shown in figure is: **[AIEEE-2012]**



(1)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

(2)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

(3)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

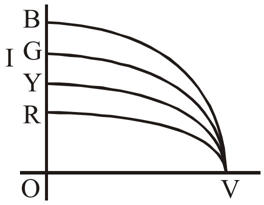
(4)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

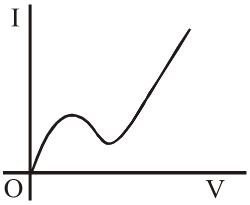
SC0059

6. The I-V characteristic of an LED is **[JEE Main 2013]**

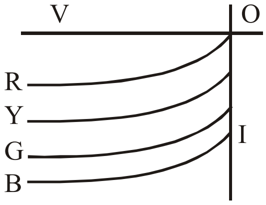
(1)



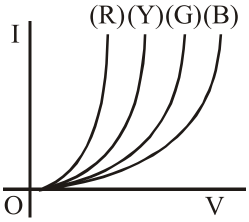
(2)



(3)



(4)

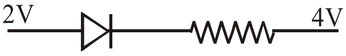


SC0060

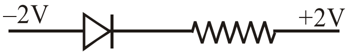
7. The forward biased diode connection is: **[JEE Main-2014]**

(1)

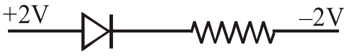
(2)



(3)



(4)



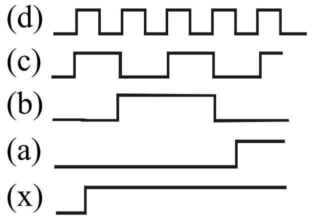
SC0061

8. For a common emitter configuration, if α and β have their usual meanings, the **incorrect** relationship between α and β is **[JEE Main-2016]**

(1) $\alpha = \frac{\beta^2}{1+\beta^2}$ (2) $\frac{1}{\alpha} = \frac{1}{\beta} + 1$ (3) $\alpha = \frac{\beta}{1-\beta}$ (4) $\alpha = \frac{\beta}{1+\beta}$

SC0062

9. If a, b, c, d are inputs to a gate and x is its output, then as per the following time graph, the gate is **[JEE Main-2016]**



(1) NAND (2) NOT (3) AND (4) OR

SC0063

10. The temperature dependence of resistances of Cu and undoped Si in the temperature range 300-400K, is best described by :- **[JEE Main-2016]**

(1) Linear decrease for Cu, linear decrease for Si.

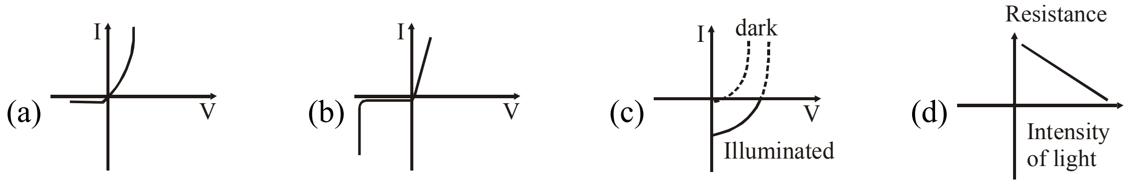
(2) Linear increase for Cu, linear increase for Si.

(3) Linear increase for Cu, exponential increase for Si

(4) Linear increase for Cu, exponential decrease for Si

SC0064

11. Identify the semiconductor devices whose characteristics are given below, in the order (a), (b), (c), (d):- **[JEE Main-2016]**



(1) Zener diode, Solar cell, Simple diode, Light dependent resistance

(2) Simple diode, Zener diode, Solar cell, Light dependent resistance

(3) Zener diode, Simple diode, Light dependent resistance, Solar cell

(4) Solar cell, Light dependent resistance, Zener diode, Simple diode

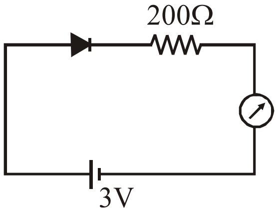
SC0065

12. In a common emitter amplifier circuit using an n-p-n transistor, the phase difference between the input and the output voltages will be : **[JEE Main-2017]**

(1) 135° (2) 180° (3) 45° (4) 90°

SC0066

13. The reading of the ammeter for a silicon diode in the given circuit is :- **[JEE Main-2018]**



(1) 15 mA (2) 11.5 mA (3) 13.5 mA (4) 0

SC0067

CBSE Previous Year's Questions

1. Draw the voltage current characteristic of a zener diode. **[1; CBSE-2004]**

2. With the help of a labelled circuit diagram, explain how an n-p-n transistor can be used as an amplifier in common-emitter configuration. Explain how the input and output voltages are out of phase by 180° for a common-emitter transistor amplifier. **[5; CBSE-2004]**

3. How does the resistivity of (i) a conductor and (ii) a semiconductor vary with temperature? Give reason for each case. **[2; CBSE-2005]**

4. On the basis of the energy band diagrams. Distinguish between metals, insulators and semiconductors. **[3; CBSE-2005]**

5. (a) With the help of a circuit diagram explain the working of transistor as oscillator.

(b) Draw a circuit diagram for a two input OR gate and explain its working with the help of input, output waveforms. **[5; CBSE-2005]**

6. (a) Explain briefly with the help of a circuit diagram, how V -1 characteristics of a p-n junction diode are obtained in (i) forward bias, and (ii) reverse bias.

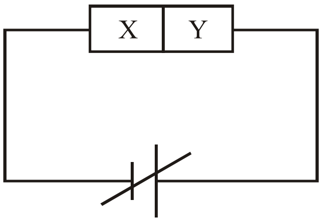
(b) A photodiode is fabricated from a semiconductor with a band gap of 2.8 eV. Can it detect wave length of 6000 nm? Justify. **[5; CBSE-2005]**

7. Explain (i) forward biasing, (ii) reverse biasing of a P-N junction diode. With the help of a circuit diagram, explain the use of this device as a half - wave rectifier. **[3; CBSE-2006]**

8. What are energy bands? How are these formed? Distinguish between a conductor, an insulator and a semiconductor on the basis of energy band diagram. **[5; CBSE-2006]**

9. Explain the function of base region of a transistor. Why is this region made thin and lightly doped? Draw a circuit diagram to study the input and output characteristics of n-p-n transistor in a common emitter (CE) configuration. Show these characteristics graphically. Explain how current amplification factor of the transistor is calculated using output characteristics. **[5; CBSE-2006]**

10. Two semiconductor materials X and Y shown in the given figure, are made by doping germanium crystal with indum and arsenic respectively. The two are joined end to end and connected to a battery as shown. **[2; CBSE-2007]**



(i) Will the junction be forward biased or reverse biased ?

(ii) Sketch a V-I graph for this arrangement.

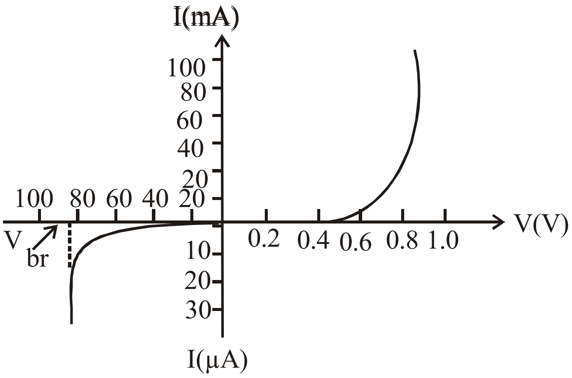
11. Draw the circuit diagram of a common emitter amplifier using n-p-n transistor. What is the phase difference between the input signal and output voltage ? State two reasons why a common emitter amplifier is preferred to a common base amplifier. **[3; CBSE-2007]**

12. Explain the formation of energy band in solids. Draw energy band diagram for (i) a conductor, (ii) an intrinsic semiconductor. **[3; CBSE-2007]**

13. Write the acronym LASER in expanded form. State any four reasons for preferring diode lasers as light sources for optical communication links. **[3; CBSE-2007]**

14. Distinguish between an intrinsic semiconductor and P-type semiconductor. Give reason, why, a P-type semiconductor crystal is electrically neutral, although $n_h >> n_e$? **[2; CBSE-2008]**

15. The figure below shows the V-I characteristic of a semiconductor diode.

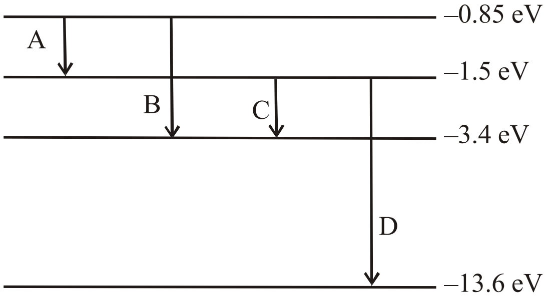


(i) Identify the semiconductor diode used.

(ii) Draw the circuit diagram to obtain the given characteristic of this device.

(iii) Briefly explain how this diode can be used as a voltage regulator. **[3; CBSE-2008]**

16. The energy level diagram of an element is given below. Identify, by doing necessary calculations, which transition corresponds to emission of a spectral line of wavelength 102.7 nm.



[3; CBSE-2008]

17. (i) Draw a circuit diagram to study the input and output characteristics of an n-p-n transistor in its common emitter configuration. Draw the typical input and output characteristics.

(ii) Explain, with the help of a circuit diagram, the working of n -p-n transistor as a common emitter amplifier. **[5; CBSE-2009]**

18. How is a zener diode fabricated so as to make it a special purpose diode? Draw I -V characteristics of zener diode and explain the significance of break down voltage. Explain briefly, with the help of a circuit diagram, how a-p-n junction diode work **[5; CBSE-2009]**

19. (a) Draw the circuit diagram of a p-n junction diode in (i) forward bias, (ii) reverse bias. How are these circuits used to study the V-I characteristics of a silicon diode ? Draw the typical V-I characteristics.

(b) What is a light emitting diode (LED)? Mention two important advantages of LEDs over conventional lamps. **[5; CBSE-2010]**

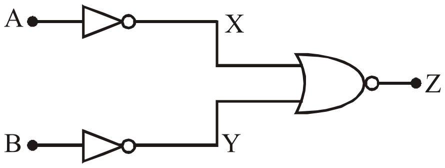
20. (a) Draw the circuit arrangement for studying the input and output characteristics of an n-p-n transistor in CE configuration. With the help of these characteristics define (i) input resistance, (ii) current amplification factor.

(b) Describe briefly with the help of a circuit diagram how an n-p-n transistor is used to produce self-sustained oscillations. **[5; CBSE-2010]**

21. What happens to the width of depletion layer of a p-n junction when it is (i) forward biased, (ii) reverse biased? **[1; CBSE-2011]**

22. Draw a labelled diagram of a full wave rectifier circuit. State its working principle. Show the input-output waveforms. **[3; CBSE-2011]**

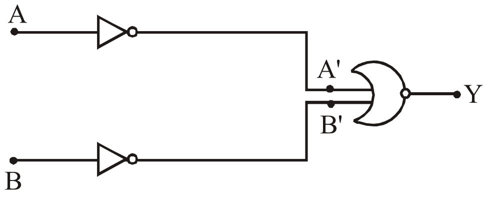
23. You are given a circuit below. Write its truth table, hence, identify the logic operation carried out by this circuit. Draw the logic symbol of the gate it corresponds to. **[3; CBSE-2011]**



24. Describe briefly with the help of a circuit diagram, how the flow of current carries in a p-n-p transistor is regulated with emitter-base junction forward and base-collector junction reverse biased. **[2;CBSE-2012]**

25. (a) Describe briefly, with the help of a diagram, the role of the two important processes involved in the formation of a p-n junction

(b) Name the device which is used as a voltage regulator. Draw the necessary circuit diagram and explain its working



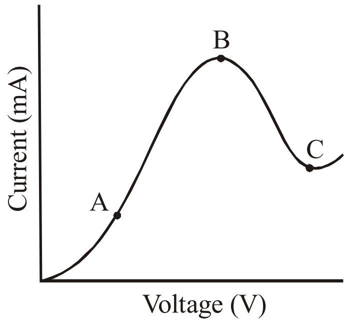
OR

(a) Explain briefly the principle on which a transistor-amplifier works as an oscillator. Draw the necessary circuit diagram and explain its working.

(b) Identify the equivalent gate for the following circuit and write its truth table.**[5; CBSE-2012]**

26. The graph shown in the figure represents a plot of current versus voltage for a given semiconductor. Identity the region, if any, over which the semiconductor has a negative resistance.

[3; CBSE-2013]



27. Draw typical output characteristics of an n-p-n transistor in CE configuration. Show how these characteristics can be used to determine output resistance. **[5; CBSE-2013]**

OR

28. Draw V -1 characteristics of a-p-n junction diode. Answer the following questions, giving reasons:
(i) Why is the current under reverse bias almost independent of the applied potential upto a critical voltage?
(ii) Why does the reverse current show a sudden increase at the critical voltage ?
Name any semiconductor device which operates under the reverse bias in the breakdown region.

[5; CBSE-2013]

29. Draw a circuit diagram of n-p-n transistor amplifier in CE configuration. Under what condition does the transistor acts as an amplifier ? **[2; CBSE-2014]**

30. Explain, with the help of a circuit diagram, the working of a p-n junction diode as a half-wave rectifier. **[2; CBSE-2014]**

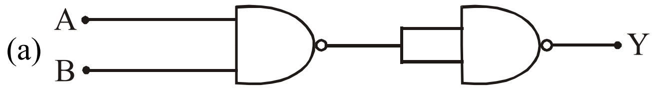
31. Write any two distinguishing features between conductors, semiconductors and insulators on the basis of energy band diagrams. **[3; CBSE-2014]**

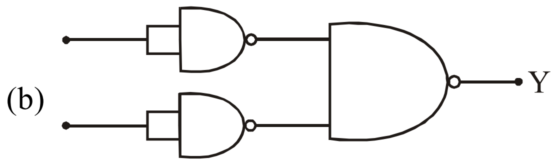
32. With the help of a circuit diagram, explain the working of a junction diode as a full wave rectifier. Draw its input and output waveforms. Which characteristic property makes the junction diode suitable for rectification ? **[3; CBSE-2015]**

33. The outputs of two NOT gates are fed to a NOR gate. Draw the logic circuit of the combination of gates. Write its truth table. Identify the gate equivalent to this circuit. **[3; CBSE-2015]**

OR

You are given two circuits (a) and (b) as shown in the figures, which consist of NAND gates. Identify the ligic operation carried out by the two. Write the truth tables for each. Identify the gates equivalent to the two circuits.





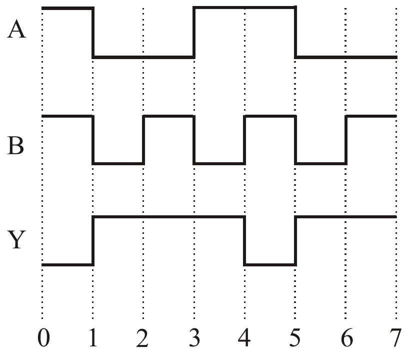
[3; CBSE-2015]

34. (i) Name two important processes that occur during the formation of a pn junction.

(ii) Draw the circuit diagram of a full wave rectifier along with the inpur and output waveforms. Briefly explain how the output voltage/current is unidirectional. **[CBSE-2016]**

35. (i) Distinguish between a conductor and a semi conductor on the basis of energy band diagram.

(ii) The following figure shows the input waveforms (A, B) and the output waveform (Y) of a gate. Identify the gate, write its truth table and draw its logic symbol. **[CBSE-2016]**

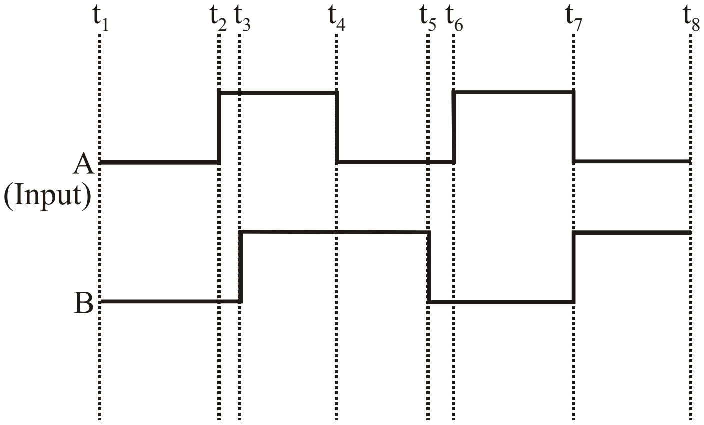


36. (a) In the following diagram, is the junction diode forward biased or reverse biased ? **[CBSE-2017]**

(b) Draw the circuit diagram of a full wave rectifier and state how it works.

37. (a) Write the functions of the three segments of a transistor. **[CBSE-2017]**

(b) The figure shows the input waveforms A and B for 'AND' gate. Draw the output waveform and write the truth table for this logic gate.



38. (a) A student wants to use two p-n junction diodes to convert alternating current into direct current. Draw the labelled circuit diagram she would use and explain how it works.

(b) Give the truth table and circuit symbol for NAND gate. **[CBSE-2018]**

39. Draw the typical input and output characteristics of an n-p-n transistor in CE configuration. Show how these characteristics can be used to determine (a) the input resistance (r_i) and (b) current amplification factor (β). **[CBSE-2018]**

ANSWER KEY

EXERCISE-S

1. Ans. 0.1 Ω

2. Ans.

	R_1	D_1	Total
V	11.35V	0.65V	12V
I	24.15 mA	24.15 mA	24.15 mA
R	470 Ω		
P	274.1 mW	15.7 mW	289.8 mW

3. Ans.

A	B	C	D	E	F	G
0	0	1	1	0	0	1
0	1	1	0	0	1	0
1	0	0	1	1	0	0
1	1	0	0	0	0	1

4. Ans. (i) 10 V (ii) 90 V (iii) 4 mA

EXERCISE-O

SINGLE CORRECT TYPE QUESTIONS

1. Ans. (B) **2. Ans. (B)** **3. Ans. (C)** **4. Ans. (C)** **5. Ans. (B)** **6. Ans. (C)**
7. Ans. (A) **8. Ans. (D)** **9. Ans. (B)** **10. Ans. (B)** **11. Ans. (B)** **12. Ans. (B)**
13. Ans. (C) **14. Ans. (B)** **15. Ans. (B)** **16. Ans. (C)** **17. Ans. (A)** **18. Ans. (A)**
19. Ans. (B) **20. Ans. (C)** **21. Ans. (C)** **22. Ans. (C)** **23. Ans. (D)** **24. Ans. (C)**
25. Ans. (C) **26. Ans. (C)** **27. Ans. (B)** **28. Ans. (C)** **29. Ans. (C)** **30. Ans. (A)**
31. Ans. (D) **32. Ans. (B)** **33. Ans. (A)** **34. Ans. (D)** **35. Ans. (A)** **36. Ans. (C)**
37. Ans. (D) **38. Ans. (C)** **39. Ans. (C)** **40. Ans. (A)** **41. Ans. (B)** **42. Ans. (D)**
43. Ans. (D) **44. Ans. (D)**

MULTIPLE CORRECT TYPE QUESTIONS

45. Ans. (A,B,D) **46. Ans. (A,C,D)** **47. Ans. (B,C)** **48. Ans. (B,D)** **49. Ans. (B,C)**

EXERCISE-JM

1. Ans. (3) **2. Ans. (3)** **3. Ans. (1)** **4. Ans. (3)** **5. Ans. (3)** **6. Ans. (4)**
7. Ans. (4) **8. Ans. (1 or 3)** **9. Ans. (4)** **10. Ans. (4)** **11. Ans. (2)** **12. Ans. (2)**
13. Ans. (2)

KEY CONCEPTS

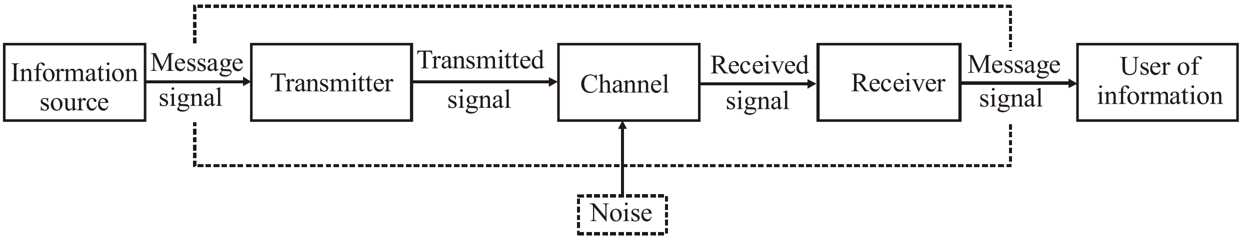
1. INTRODUCTION

Communication means transmission of information. Everyone experiences the need to impart or receive information continuously in the surrounding and for this , we speak, listen, send message by a messenger, use coded signalling methods through smoke or flags or beating of drum etc. and these days we are using telephones, TV, radio, satellite communication etc. The aim of this chapter is to introduce the concepts of communication namely the mode of communication, the need for modulation, production and detection of amplitude modulation.

Elements of a Communication System :

Every communication system has three essential elements-

(i) transmitter (ii) medium/channel (iii) receiver



Transmitter converts the message signal into an electric signal and transmits through channel. The receiver receives the transmitted signal and reconstructs the original message signal to the end user. There are two basic modes of communication: (i) point-to-point and (ii) broadcast.

In point-to-point communication mode, communication takes place over a link between a single transmitter and a receiver as in telephony. In the broadcast mode, there are a large number of receivers corresponding to a single transmitter. Radio and television are most common examples of braoadcast mode of communication. However the communication system can be classified as follows :

Types of Communication Systems

On the basis of nature of Information
- (1) Speech Transmission (Radio)
- (2) Picture Transmission (TV)
- (3) Fascimile Transmission (FAX)
- (4) Data Transmission (Internet)

On the basis of signal transmitted
- (1) Analog
- (2) Digital

On the basis of transmission Channel
- (1) Line communication
  - (a) Two wire Transmission line
  - (b) Coaxial cable
  - (c) Optical fibre
- (2) Space communication

On the basis of type of modulation
- (1) Continuous wave modulation
  - (a) Amplitude
  - (b) Frequency
  - (c) phase
- (2) Pulse Modulation
  - (a) PAM
  - (b) PTM — PPM, PWM
  - (c) PCM

Basic terminology Used in Electronic Communication systems :

(i) Transducer. Transducer is the device that converts one form of energy into another. Microphone, photo detectors and piezoelectric sensors are types of transducer. They convert information into electrical signal.

(ii) Signal Signal is the information converted in electrical form. Signals can be analog or digital. Sound and picture signals inTV are analog.

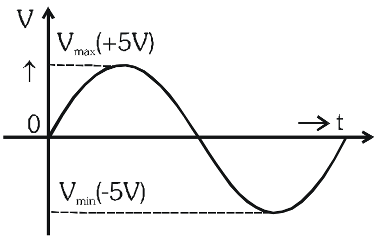
It is defined as a single–valued function of time which has a unique value at every instant of time.

(1) Analog Signal :–

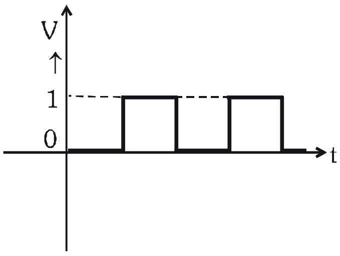
A continuously varying signal (Voltage or Current) is called an analog signal. A decimal number with system base 10 is used to deal with analog signal.

(2) Digital Signal :–

A signal that can have only discrete stepwise values is called a digital signal. A binary number system with base 2 is used to deal with digital signals.



An Analog Signal



An Digital Signal

(iii) Noise : There are unwanted signals that tend to disturb the transmission and processing of message signals. The source of noise can be inside or outside the system.

(iv) Transmitter : A transmitter processes the incoming message signal to make it suitable for transmission through a channel and subsequent reception.

(v) Receiver : A receiver extracts the desired message signals from the received signals at the channel output.

(vi) Attenuation : It is the loss of strength of a signals while propagating through a medium. It is like damping of oscillations.

(vii) Amplification : It is the process of increasing the amplitude (and therefore the strength) of a signal using an electronic circuit called the amplifier. Amplification is absolutely necessary to compensate for the attenuation of the signal in communication systems.

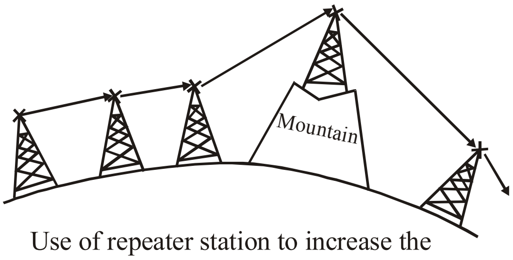
(viii) Range : It is the largest distance between the source and the destination upto which the signal gets received with sufficient strength.

(ix) Bandwidth : It is the frequency range over which an equipment operates or the portion of the spectrum occupied by the signal.

(x) Modulation : The original low frequency message/information signal connot be transmitted to long distances. So, at the transmitter end, information contained in the low frequency message signal is superimposed on a high frequency wave, which acts as a carrier of the information. This process is known as modulation.

(xi) Demodulation : The process of retrieval of original information from the carrier wave at the receiver end is termed as demodulation. This process is the reverse of modulation.

(xii) Repeater : A repeater acts as a receiver and a transmitter. A repeater picks up the signal which is coming from the transmitter, amplifies and retransmits it with a change in carrier frequency. Repeaters are necessary to extend the range of a communication system as shown in figure A communication satellite is basically a repeater station in space.



Use of repeater station to increase the range of communication

BANDWIDTH

Bandwidth of signals :

Different signals used in a comminication system such as voice, music, picture, computer data etc. all have different ranges of frequency. The difference of maximum and minimum frequency in the range of each signal is called bandwidth of that signal.

Bandwidth can be of message signal as well as of transmission medium.

(i) Bandwidth for analog signals

Bandwdith for some analog sinals are listed below :

Signal	Frequency range	Bandwidth required
Speech	300-3100 Hz	3100-300 =2800 Hz
Music	High frequencies produced by musical instrument Audible range =20 Hz - 20 kHz	20 kHz
Picture	-	4.2 MHz
TV	Contains both voice and picture	6 MHz

(ii) Bandwidth for digital signal

Basically digital signals are rectanglar waves and these can be splitted into a superposition of sinusoidal waves of frequencies ν_0, $2\nu_0$, $3\nu_0$, $4\nu_0$, $n\nu_0$, where n is an integer extending to infinity. This implies that the infinite band width is required to reproduce the rectangular waves. However, for practical purposes, higher harmonics are neglected for limiting the bandwidth

Band width of Transmission Medium

Different types of transmission media offer different band width in which some of are listed below

Frequency Bands

	Service	Frequency range	Remarks
1	Wire (most common : Coaxial Cable)	750 MHz (Bandwidth)	Normally operated below 18 GHz
2	Free space (radio waves)	Few hundred kHz to GHz	
	(i) Standard AM broadcast	540kHz -1600 kHz	
	(ii) FM	88-108 MHz	
	(iii) Television	54-72 MHz 76-88 MHz 174-216 MHz 420-890 MHz	VHF (Very) high frequencies) TV UHF (Ultra hight frequency) TV
	(iv) Cellular mobile radio	896-901 MHz 840-935 MHz	Mobile to base Station Base station to mobile
	(v) Satellite Communication	5.925-6.425 GHz 3.7 - 4.2 GHz	Uplinking Downlinking
3	Optical communication using fibres	1THz-1000 THz (microwaves-ultra violet)	One single optical fibre offers bandwidth > 100 GHz

Propagation of Electromagnetic Waves :

In case of radio waves communication, an antenna at the transmitter radiates the electromagnetic waves (em waves). The em waves travel through the space and reach the receiving antenna at the other end. As the em wave travels away from the transmitter, their strength keeps on decreasing. Many factors influence the propagation of em waves including the path they follow. The composition of the earth's atmosphere also plays a vital role in the propagation of em waves, as summarised below.

Table 4 Layers of atmosphere and their interaction with the propagating em waves

Atmospheric stratum (layer)	Height over earth's surface (approx)	Exists during	Frequencies most likely affected
1. Troposphere	10 km	Day and night	VHF (upto several GHz)
2. Ionosphere			
(i) D (part of stratosphere)	65-75 km	Day only	Reflects LF, absorbs MF & HF to some degree
(ii) E (part of stratosphere)	100 km	Day only	Helps surface waves, reflects HF
(iii) F_1 (Part of Mesosphere)	170-190 km	Daytime, merges with F_2 at night	Partially absorbs HF waves yet allowing them to reach F_2
(iv) F_2 (Thermosphere)	300 km at night, 250-400 km during daytime	Day and night	Efficiently reflects HF waves particularly at night

Ground Wave Propagation :

(a) The radio waves which travel through atmosphere following the surface of earth are known as ground waves or surface waves and their propagation is called ground wave propagation or surface wave propagation.

(b) The ground wave transmission becomes weaker with increase in frequency because more absorption of ground waves takes place at higher frequency during propagation through atmosphere.

(c) The ground wave propagation is suitabel for low and medium frequency i.e. upto 2 or 3 MHz only.

(d) The ground wave propagation is generally used for local band broadcasting and is commonly called medium wave.

(e) The maximum range of ground or surface wave propagation depends on two factors :
(i) The frequency of the radio waves and
(ii) Power of the transmitter

Sky Wave Propagation :

(a) The sky waves are the radio waves of frequency between 2 MHz to 30 MHz.

(b) The ionoopheric layer acts as a reflector for a certain range of frequencies (3 to 30 MHz). Electromagnetic waves of frequencies higher than 30 MHz penetrate the ionosphere and escape.

(c) The highest frequency of radiowaves which when sent straight (i.e. normally) towards the layer of ionosphere gets refelcted from ionosphere and returns to the earth is called critical frequency.. If is given by $f_c = 9 (N_{max})^{1/2}$, where N is the number density of electron/m^3.

Space wave propagation :

(a) The space waves are the radiowaves of very high frequency (i.e. between 30 MHz. to 300 MHz or more).

(b) the space waves can travel through atmosphere from transmitter antenna to receiver antenna either derectly or after reflection from ground in the earth's troposphere region. That is why the space wave propagation is also called as tropospherical propagation or line of sight propagation.

(c) The range of communication of space wave propagation can be increased by increasing the heights of transmitting and receiving antenna.

(d) If the transmitting antenna is at a height h_T, then you can show that the distance to the horizontal d_T is given as $\boxed{d_T = \sqrt{2Rh_T}}$, where R is the radius of the earth (approximately 6400 km). d_T is also called the radio maximum line-of sight distance d_m between the two antennas having heights h_T and h_R above the earth is given by :

$$\boxed{d_M = \sqrt{2Rh_T} + \sqrt{2Rh_R}}$$

where h_R is the height of receiving antenna.

Modulation

- It is a process by which any electrical signal called input / baseband or modulating signal, is mounted onto another signal of high frequency which is known as carrier signal.
- It is defined as the process by which some characteristic (called parameter) of carrier signal is varied in accordance with the instantaneous value of the baseband signal.

* The signal which results from this process is known as modulated signal.

Need for Modulation :

(i) To aviod interference :

If many modulating signals travel directly through the same transmission channel, they will interfere with each other and result in distortion.

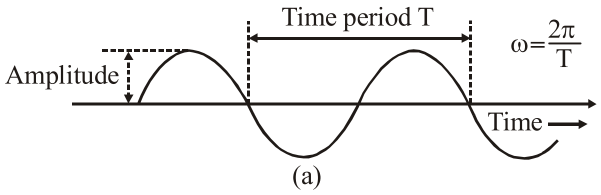
(ii) To design antennas of practicable size :

The minimum height of antenna (not of antenna tower) should be $\lambda/4$ where λ is wavelength of modulating signal. This minimum size becomes impracticable because the frequency of the modulating signal can be upto 5 kHz which corresponds to a wavelength of $3 \times 10^8/5 \times 10^3 = 60$ km. This will require an antenna of the minimum height of $\lambda/4 = 15$ km. This size of an antenna is not practical.

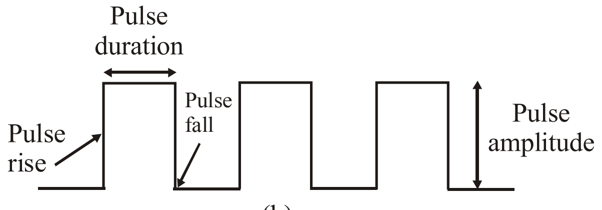
(iii) Effective Power Radiated by an Antenna :

A theoretical study of radiation from a linear antenna (length ℓ) shows that the power radiated is proportional to $(\text{frequency})^2$ i.e. $(\ell/\lambda)^2$. For a good transmission, we need high powers and hence this also points out to the need of using high frequency transmission.

The above discussion suggests that there is a need for translating the original low frequency baseband message signal into high frequency wave before transmission. In doing so, we take the help of a high frequency signal, which we already know now, is known as the carrier wave, and a process known as modulation which attaches information to it. The carrier wave may be continuous (sinusoidal) or in the form of pulses, as shown in figure



(a)



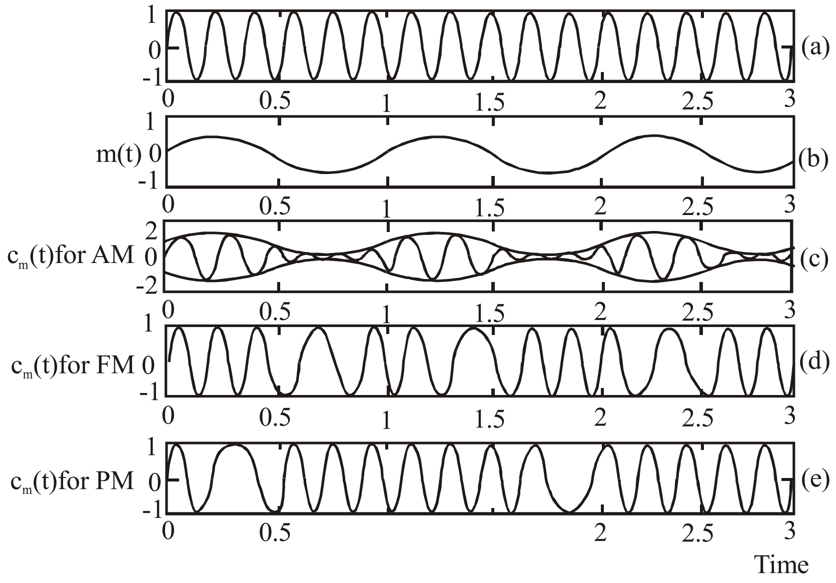
(b)

Carrier wave : Sinusoidal

A sinusoidal carrier wave can be represented as

$c(t) = A_C \sin(\omega_c t + \phi)$

where c(t) is the signal strength (voltage or current), A_C is the amplitude, ω_c ($= 2\pi f_c$) is the angular frequency and ϕ is the initial phase of the carrier wave. Thus, modulation can be affected by varying, any of three parameters, viz A_c, ω_c and ϕ, of the carrier wave can as per the parameter of the message or information signal. This results in three types of modulation : (i) Amplitude modulation (AM) (ii) Frequency modulation (FM) and (iii) Phase modulation (PM), as shown in figure.



Modulation of a carrier wave : (a) a sinusoidal carrier wave (b) a modulating signal : (c) amplitude modulatin : (d) frequency modulation : and (e) phase modulation

Carrier Wave Pulses :

Similarly, the significant characteristics of a pulse are : Pulse Amplitude, Pulse duration or pulse Width, and pulse Position (denoting the time of rise or fall of the pulse amplitude) Hence, different types of pulse modulation are (a) pulse amplitude modulation (PAM), (b) Pulse duration modulation (PDM) or pulse width modulation (PWM), and (c) Pulse position modulation (PPM).

Ex.1 A separate high freq. wave (i.e. carrier wave) is needed in modulation why ?

Ans. This is because we cannot change any of the characteristics (amplitude, frequency or phase) of the audio signal as this would change the message to be communicated. So keeping the audio signal same, the amplitude of freq. or phase of the high freq. carrier wave is modified in accordance with the modulating (i.e. audio signal) signal.

Amplitude Modulation :

In amplitude modulation the amplitude of the carrier is varied in accordance with the information signals. Let $c(t) = A_c \sin\omega_c t$ represent carrier wave and $m(t) = A_m \sin \omega_m t$ represent the message or the modulating signal where $\omega_m = 2\pi f_m$ is the angular frequency of the message signal. The modulated signal $c_m(t)$ can be written as

$$c_m(t) = (A_c + A_m \sin \omega_m t) \sin \omega_c t$$

$$= A_c\left(1+\frac{A_m}{A_c}\sin\omega_m t\right)\sin\omega_c t \qquad .....(1)$$

Note that the modulated signal now contains the message signal & it can be written as :

$$c_m(t) = A_c \sin\omega_c t + \mu A_c \sin\omega_m t \sin\omega_c t \qquad ......(2)$$

Here **$\mu = A_m/A_c$ is the modulation index**

Often, μ is expressed in percentage and is called the percentage modulation. Importance of modulation index is that it determine the quality of the transmitted signal. When modulation index is small, variation in carrier amplitude will be small. Therefore, audio signal being transmitted will be weak. As the modulation index increases, the audio signal on reception becomes clearer. If the modulation index becomes greater than one, the signal gets lost partially.

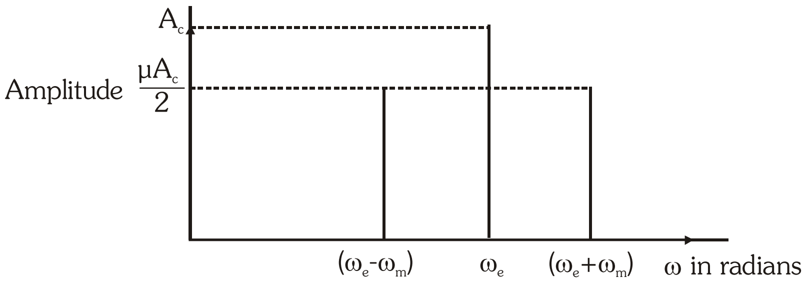
If a carrier wave is modulated by several sine waves the total modulated index m_t is given by

$$m_t = \sqrt{m_1^2 + m_2^2 + m_3^2 + ...}$$

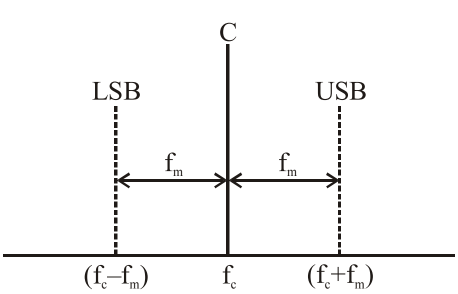
Using the trignomatric relation sinA sinB = ½ (cos (A – B) – cos (A + B), we can write c_m (t) of eq. (15.4)as

$$c_m(t) = A_c \sin\omega_c t + \frac{\mu A_c}{2}\cos(\omega_c - \omega_m)\,t - \frac{\mu A_c}{2}\cos(\omega_c + \omega_m)\,t \qquad ......(3)$$

Here $\omega_c - \omega_m$ and $\omega_c + \omega_m$ are respectively called the called the lower side and upper side frequencies. The modulated signal now consists of the carrier wave of frequency ω_c plus two sinusoidal waves each with a frequency slightly different from, know as side bands. The frequency spectrum of the amplitude modulated signal is shown in figure :



As long as the broadcast frequencies (carrier waves) are sufficiently spaced out so that sidebands do not overlap, different stations can operate without interfering with each other.



$f_{SB} = f_c \pm f_m$

$\therefore$ Frequency of lower side band is

$f_{LSB} = f_c - f_m$ ... (4)

and frequency of upper side band is

$f_{USB} = f_c + f_m$

Band width of amplitude modulated wave is

$= f_{USB} - f_{LSB}$

$= (f_f + f_m) - (f_c - f_m) = 2f_m$ .... (5)

Band width = twice the frequency of the modulating signal

Phase modulation & frequency modulation are 2 different types of angular modulation.

Power in AM waves : Power dissipated in any circuit $P = \frac{V_{rms}^2}{R}$

Hence : (i) Carrier power $P_c = \frac{\left(\frac{E_c}{\sqrt{2}}\right)^2}{R} = \frac{E_c^2}{2R}$

(ii) Total power of side bands $P_{sb} = \frac{\left(\frac{m_a E_c}{2\sqrt{2}}\right)^2}{R} \times 2 = \frac{m_a^2 E_c^2}{4R}$

(iii) Total power of AM wave $P_{Total} = P_c + P_{ab} = \frac{E_c^2}{2R}\left(1 + \frac{m_a^2}{2}\right)$

(iv) $\frac{P_t}{P_c} = \left(1 + \frac{m_a^2}{2}\right)$ and $\frac{P_{ab}}{P_t} = \frac{m_a^2/2}{\left(1 + \frac{m_a^2}{2}\right)}$

(v) Maximum power in the AM (without distortion) will occur when $m_a = 1$

i.e., $P_t = 1.5\ P = 3P_{ab}$

(vi) If I_c = Unmodulated current and I_t = total or modulated current

$\Rightarrow \frac{P_t}{P_c} = \frac{I_t^2}{I_c^2} \Rightarrow \frac{I_t}{I_c} = \sqrt{\left(1 + \frac{m_a^2}{2}\right)}$

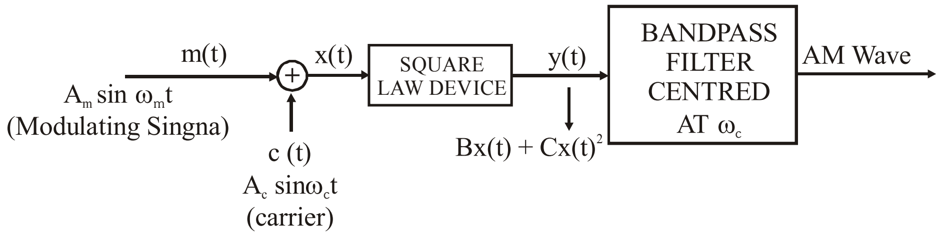
Ex.2 A message signal of frequency 10 kHz and peak voltage of 10 volts is used to modulate a carrier of frequency 1 Mhz and peak voltage of 20 volts. Determine (a) modulation index, (b) the side bands produced.

Sol. (a) Modulation index = 10/20 = 0.5

(b) The side bands are at (1000 + 10 kHz) = 1010 kHz and (1000–10 kHz) = 990 kHz.

Production of Amplitude modulated Wave :

Ampitude modulation can be produced by a veriety of methods. A conceptually simple method is shown in the block diagram of figure.



Here the modulating signal $A_m \sin \omega_m t$ is added to the carrier signal $A_c \sin \omega_c t$ to produce the signal x (t). This signal $x(t) = A_m \sin \omega_m t + A_c \sin\omega_c t$ is passed through a square law device which is a non-linear device which produces an output

$$y(t) = B\,x(t) + Cx^2(t) \qquad ......(4)$$

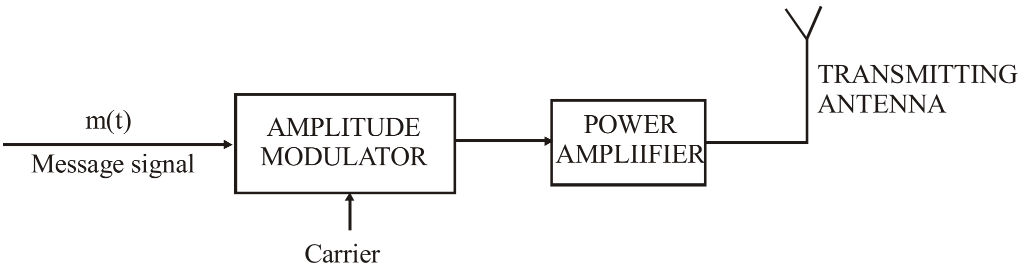
where B and C are constants. Thus,

$$y(t) = BA_m \sin\omega_m t + BA_c \sin \omega_c t$$
$$+ C\,[A_m{}^2 \sin^2 \omega_m t + A_c{}^2 \sin^2 \omega_c t + 2A_m A_c \sin\omega_m t \sin\omega_c t] \qquad ......(5)$$
$$= BA_m \sin\omega_m t + BA_c \sin \omega_c t$$
$$\frac{CA_m^2}{2} + A_c^2 - \frac{CA_m^2}{2}\cos 2\omega_m t - \frac{CA_c^2}{2}\cos 2\omega_c t$$
$$+ CA_m A_c \cos(\omega_c - \omega_m)t - CA_m A_c \cos(\omega_c + \omega_m)t \qquad ......(6)$$

where the trigonometric relations $\sin^2 A = (1 - \cos 2A)/2$ and the relation for sinA sinB mentioned earlier are used.

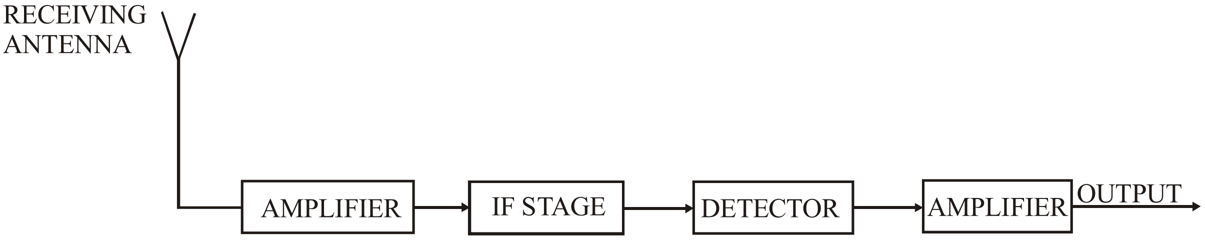
In equation (6), there is a dc term C/2 $(A_m^2 + A_c^2)$ and sinusoids of frequencies ω_m, $2\omega_m$, $\omega_c - \omega_m$ and $\omega_c + \omega_m$. The output of the band pass filter therefore is of the same form as equation (3) and is therefore an AM wave.

It is to be mentioned that the modulated signal connot be transmitted as such. The modulator is to be followed by a power amplifier which provides the necesary power and then the modulated signal is fed to an antenna of appropriate size for radiation as shown in figure.

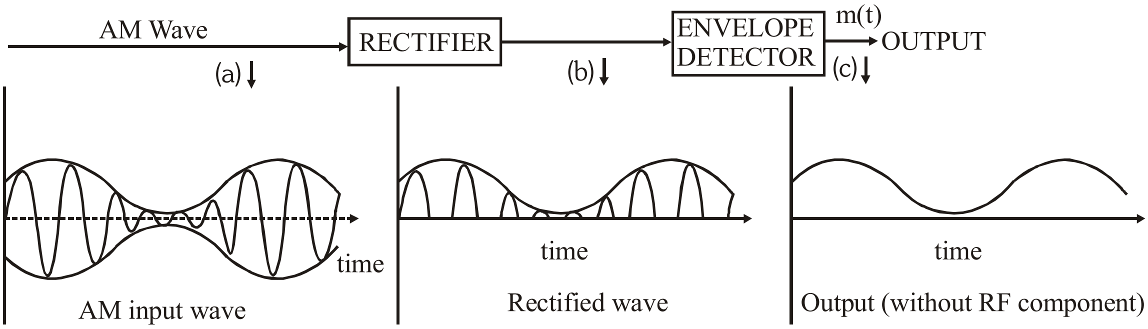


Detection of Amplitude Modulated Wave :

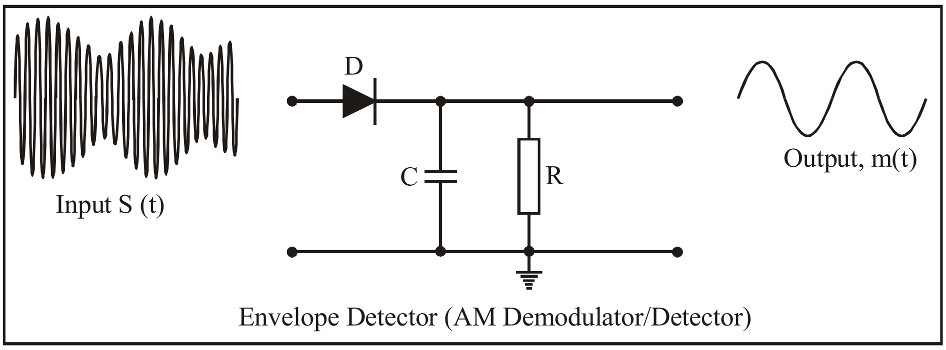
The transmitted message gets attenuated in propagating through the channel. The receiving antenna is therefore to be followed by an amplifier and a detector. In addition, to facilitate further processing, the carrier frequency is usually changed to a lower frequency by what is called an intermediate frequency (IF) stage preceding the detection. The detected signal may not be strong enough to be made use of and hence is required to be amplified. A block diagram of a typical receiver is shown in figure.



Detection is the process of recovering the modulating signal from the modulated carrier wave. We just saw that the modulated carrier wave contains the frequencies ω_c and $\omega_c \pm \omega_m$. In order to obtain the original message signal m(t) of angular frequency ω_m, a simple method is shown in the from of a block diagram in figure.



The modulated signal of the form given in (a) of above figure is passed through a rectifier to produce the output shown in (b). This envelope of signal (b) is the message signal In order to retrieve m(t), the signal is passed through an envelope detector (which may consist of a simple RC circuit).



Envelope Detector (AM Demodulator/Detector)

This is essentially just a half wave rectifier which charges a capacitor to a voltage to the peak voltage of the incoming AM waveform, S(t). When the input wave's amplitude increases, the capacitor voltage in increased via the rectifying diode. When the input's amplitude falls, the capacitor voltage is reduced by being discharged by a 'bleed' resistor, R. The main advantage of this form of AM Demodulator is that it is very simple and cheap!

The circuit relies upon the behaviour of the diode-allowing current through when the input is +ve with respect to the capacitor voltage, hence 'topping up' the capacitor voltage to the peak level, but blocking any current from flowing back out through the diode when the input voltage is below the capacitor voltage.

Consider what happens when we have a carrier frequency, f_c, and use an envelope detector whose time constant, $\tau = (RC)$. The time between successive peaks of the carrier will be $T = \frac{1}{f_c}$

Each peak will charge the capacitor to some voltage, V_{peak}, which is proportional to the modulated amplitude of the AM wave. Between each peak and the next the capacitor voltage will therefore be discharged to

$$V'_{peak} = V_{peak}\ \mathrm{Exp}\ \{-T/\tau\}$$

which, provided that $T << \tau$, is approximately the same as

$$V'_{peak} \approx V_{peak}\ [1 - T/\tau]$$

The peak-to-peak size of the ripple, ΔV, will therefore be

$$\Delta V \approx \frac{V_{peak} T}{\tau} = \frac{V_{peak}}{f_c \tau}$$

A sudden, large reduction in the amplitude of the input AM wave means that capacitor charge isn't being 'topped up' by each cycle peak. The capacitor voltage therefore falls exponentially until it reaches the new, smaller, peak value. In practice the modulating signal is normally restricted to a specific frequency range. This limits the maximum rate of fall of the AM wave's amplitude. We can therefore hope to avoid negative peak clipping by arranging that the detector's time constant $\tau << t_m$ where

$$t_m = 1/f_m$$

and f_m is the highest modulation frequency used in a given situation.

The above implies that we can avoid negative peak clipping by choosing a small value of τ. However, to minimise ripple we want to make τ as large as possible. In practice we should therefore choose a value

$$1/f_m >> \tau >> 1/f_c$$

to minimise the signal distortions caused by these effects. This is clearly only possible if the modulation frequency $f_m << f_c$. Envelope detector only work satisfactorily when we ensure this inequality is true. With the modulation index and the resistor the capacitor can be computed by

$$C \leq \frac{\sqrt{\left(\frac{1}{m}\right)^2 - 1}}{2\pi R f_m (\max)}$$

EXERCISE-S

1. Is it necessary for a transmitting antenna to be at the same height as that of the receiving antenna for 'line-of-sight communication? A TV transmitting antenna is 81m tall. How much service area can it cover if the receiving antenna is at the ground level?

PC0001

2. A carrier wave of peak voltage 12 V is used to transmit a message signal. What should be the peak 'voltage of the modulating signal in order to have a modulation index of 75%?

PC0002

3. For an amplitude modulated wave, the maximum amplitude is found to be 10V while the minimum amplitude is found to be 2V. Determine the modulation index μ.

PC0003

4. If the sum of the heights of transmitting and receiving antennas in line of sight of communication is fixed at h, show that the range is maximum when the two antennas have a height h/2 each.

PC0004

EXERCISE-O

SINGLE CORRECT TYPE QUESTIONS

1. If a carrier wave of 1000 kHz is used to carry the signal, the length of transmitting antenna will be equal to :-

(A) 3 m (B) 30m (C) 300 m (D) 3000 m

PC0005

2. The maximum range of ground or surface wave propagation depends on :-

(A) the frequency of the radio waves only (B) power of the transmitter only

(C) both of them (D) none of them

PC0006

3. For television broadcasting, the frequency employed is normally :-

(A) 30-300MHz (B) 30-300 GHz (C) 30-300KHz (D) 30-3OOHz-

PC0007

4. The audio signal :-

(A) can be sent directly over the air for large distance

(B) cannot be sent directly over the air for large distance

(C) possess very high frequency

(D) none of the above

PC0008

5. For a carrier frequency of 100 kHz and a modulating frequency of 5 kHz what is the width of AM transmission-

(A) 5 kHz (B) 10kHz (C) 20 kHz (D) 200 KHz

PC0009

6. Three waves A, B and C of frequencies 1600 kHz, 5 MHz and 60 MHz, respectively are to be transmitted from one place to another. Which of the following is the most appropriate mode of communication :

(A) A is transmitted via space wave while B and C are transmitted via sky wave.

(B) A is transmitted via ground wave, B via sky wave and C via space wave.

(C) B and C are transmitted via ground wave while A is transmitted via sky wave.

(D) B is transmitted via ground wave we while A and C are transmitted via space wave.

PC0010

7. A speech signal of 3kHz is used to modulate a carrier signal of frequency 1 MHz, using amplitude modulation. The frequencies of the side bands will be

(A) 1.003 MHz and 0.997 MHz. (B) 3001 kHz and 2997 kHz.

(C) 1003 kHz and 1000 kHz. (D) 1 MHz and 0.997 MHz.

PC0011

8. A message signal of frequency ω_m is superposed on a carrier wave of frequency ω_e to get an amplitude mediated wave (AM) The frequency of the AM wave will be

(A) ω_m (B) ω_e (C) $\frac{\omega_e + \omega_m}{2}$ (D) $\frac{\omega_e - \omega_m}{2}$

PC0012

9. A basic communication system consists of

(A) transmitter (B) information source. (C) user of information.

(D) channel (E) receiver.

Choose the correct sequence in which these are arranged in a basic communication system :-

(A) ABCDE (B) BADEC (C) BDACE (D) BEADC

PC0013

10. **Statement 1 :** Skywave can not be observed on moon.

Statement 2 : Atmosphere of variable refractive index is require for propagation of skywave.

(A) Both Statement-1 and Statement-2 are true, and Statement-2 is the correct explanation of Statement-1

(B) Both Statement-1 and Statement-2 are true but Statement-2 is not the correct explanation of Statement- 1.

(C) Statement-1 is true but Statement-2 is false.

(D) Statement-1 is false but Statement-2 is true.

PC0014

11. **Statement 1 :** Ground wave communication is effective only allow frequencies in the range 500kHz to about 1500 kHz.

Statement 2 : The decrease in the intensity of the signal due to absorption by the earth and its atmosphere is higher for higher frequencies.

(A) Both Statement-1 and Statement-2 are true, and Statement-2 is the correct explanation of Statement-1.

(B) Both Statement-1 and Statement-2 are true but Statement-2 is not the correct explanation of Statement-I .

(C) Statement-1 is true but Statement-2 is false.

(D) Statement-1 is false but Statement-2 is true.

PC0015

MULTIPLE CORRECT TYPE QUESTIONS

12. An audio signal of I5kHz frequency cannot be transmitted over long distances without modulation because

(A) the size of the required antenna would be at least 5 km which is not convenient.

(B) the audio signal cannot be transmitted through skywaves.

(C) the size of the required antenna would be at least 20 km, which is not convenient.

(D) effective power transmitted would be very low, if the size of the antenna is less than 5 km.

PC0016

13. Audio sine waves of 3 kHz frequency are used to amplitude modulate a carrier signal of 1.5 MHz. Which of the following statements are true?

(A) The side band frequencies are 1506 kHz and 1494 kHz.

(B) The bandwidth required for amplitude modulation is 6kHz.

(C) The bandwidth required for amplitude modulation is 3 MHz.

(D) The side band frequencies are 1503 kHz and 1497 kHz.

PC0017

EXERCISE-JM

1. This questions has Statement-1 and Statement-2. Of the four choice given after the statements, choose the one that best describes the two statements.

Statement-1 : Sky wave signals are used for long distance radio communication. These signals are in general, less stable than ground wave signals. **[AIEEE-2011]**

Statement-2 : The state of ionosphere varies from hour to hour, day to day and season to season.

(1) Statement-1 is true, Statement-2 is true, Statement-2 is the correct explanation of statement-1
(2) Statement-1 is true, Statement-2 is true, Statrment-2 is not the correct explanation of Statement-1
(3) Statement-1 is false, Statement-2 is true
(4) Statement-1 is true, Statement-2 is false

PC0018

2. A radar has a power of 1 kW and is operating at a frequency of 10 GHz. It is located on a mountain top of height 500 m. The maximum distance up to which it can detect object located on the surface of the earth (Radius of earth = 6.4×10^6 m) is : **[AIEEE-2012]**

(1) 40 km (2) 64 (3) 80 km (4) 16km

PC0019

3. A diode detector is used to detect an amplitude modulated wave of 60% modulation by using a condenser of capacity 250 pico farad in parallel with a load resistance 100 kilo ohm. Find the maximum modulated frequency which could be detected by it. **[JEE Main-2013]**

(1) 10.62kHz (2) 5.31 MHz (3) 5.31 kHz (4) 10.62MHz

PC0020

4. A single of 5 kHz frequency is amplitude modulated on a carrier wave of frequency 2 MHz. The frequencies of the resultant signal is/are - **[JEE Main-2015]**

(1) 2005 kHz, 2000 kHz and 1995 kHz (2) 2000 kHz and 1995 kHz
(3) 2 MHz only (4) 2005 kHz and 1995 kHz

PC0021

5. Choose the correct statement : **[JEE Main-2016]**

(1) In frequency modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the frequency of the audio signal.
(2) In amplitude modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.
(3) In amplitude modulation the frequency of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.
(4) In frequency modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

PC0022

6. In amplitude modulation, sinusoidal carrier frequency used is denoted by ω_c and the signal frequency is denoted by ω_m. The bandwidth ($\Delta\omega_m$) of the signal is such that $\Delta\omega_m << \omega_c$. Which of the following frequencies is not contained in the modulated wave ? **[JEE Main-2017]**

(1) $\omega_m + \omega_c$ (2) $\omega_c - \omega_m$ (3) ω_m (4) ω_c

PC0023

7. A telephonic communication service is working at carrier frequency of 10 GHz. Only 10% of it is utilized for transmission. How many telephonic channels can be transmitted simultaneously if each channel requires a bandwidth of 5 kHz ? **[JEE Main-2018]**

(1) 2×10^4 (2) 2×10^5 (3) 2×10^6 (4) 2×10^3

PC0024

CBSE PREVIOUS YEAR'S QUESTIONS

1. A.T.V. tower has a height of 400 m at a given place. Calculate as coverage range, if the radius of the earth is 6400 km. **[2; CBSE-2004]**

2. Why is shortwave bind used for long distance radio broadcast? **[1; CBSE-2004]**

3. Distinguish between analog and digital communication Write any two modulation techniques employed for the digital data. Describe briefly any one of the techniques used. **[3; CBSE-2005]**

4. A ground receiver stations receiving a signal at (a) 5MHz and (b) 100 MHz, transmitted from a ground transmitter at a height of 300 m located at a distance of 100 km. Identify whether it is coming via space wave or sky wave propagation or satellite transponder. (Given the value of radius of the earth is 6400 km and maximum electron density, $N_{max} = 1012\ m^{-3}$).**[3; CBSE-2005]**

5. Consider an optical communication system operating at $\lambda - 800$ nm. Suppose, only 1% of the optical source frequency is the available channel band-width for optical communication. How many channels can be accommodated for transmitting **[3; CBSE-2006]**

(a) audio-signals requiring a band-width of 8 kHz,

(b) video TV signals requiring an approximate band-width of 4.5 MHz? Support your answer with suitable calculations.

6. Distinguish between frequency modulation and amplitude modulation. Why is an FM signal less susceptible to noise than an AM signal? **[3; CBSE-2006]**

7. Give any one difference between FAX and e-mail systems of communication. **[1; CBSE-2006]**

8. What is modulation? Explain the need of modulating a low frequency information signal. With the help of diagrams, differentiate between PAM and PDM. **[3; CBSE-2007]**

9. What should be the length of dipole antenna for a carrier wave of frequency 6×10^8 Hz? **[1; CBSE-2007]**

10. A transmitting antenna at the top of a tower has a height of 36m and the height of the receiving antenna is 49m. What is the maximum distance between them for satisfactory communication in the LOS mode? (Radius of earth = 6400km). **[3; CBSE-2008]**

11. What is meant by term 'modulation'? Draw a block diagram of a simple modulator for obtaining an AM signal. **[2; CBSE-2009]**

12. Why is high frequency carrier waves used for transmission? **[2; CBSE-2009]**

13. By what percentage will the transmission range of a TV tower be affected when the height of tower is increased by 21%? **[12; CBSE-2009]**

14. Write two factors justifying the need of modulating a signal. A carrier wave of peak voltage 12 V is used to transmit a message signal. What should be the peak voltage of the modulating signal in order to have a modulation index of 75% ? **[12; CBSE-2010]**

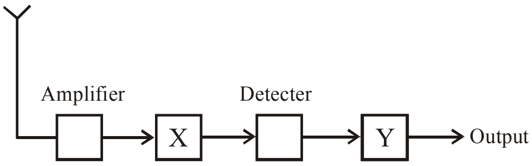
15. Which mode of propagation is used by short wave broadcast services having frequency range from a few MHz upto 30 MHz ? Explain diagrammatically how long distance communication can be achieved by this mode. Why is there an upper limit to frequency of waves used in this mode ? **[3; CBSE-2010]**

16. What is sky wave communication ? Why is this mode of propagation restricted to the frequencies only upto few MHz? **[2; CBSE-2011]**

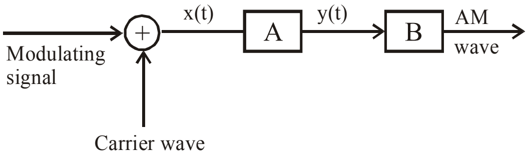
17. Write briefly any two factors which demonstrate the need for modulating a signal. Draw a suitable diagram to show amplitude modulation using a sinusoidal signal as the modulating signal. **[3; CBSE-2011]**

18. Mention three different modes of propagation used in communication system. Explain with the help of a diagram how long distance communication can be achieved by ionospheric reflection of radio waves. **[3; CBSE-2012]**

19. In the given block diagram of a receiver, identify the boxes labeled as X and Y and write their functions. **[2; CBSE-2012]**



20. In the block diagram of a simple modulator for obtaining an AM signal, shown in the figure, identify the boxes A and B. Write their functions. **[12; CBSE-2013]**



21. Name the type of waves which are used for line of sight (LOS) communication. What is the range of their frequencies?

A transmitting antenna at the top of a tower has a height of 20 m and the height of the receiving antenna is 45 m. Calculate the maximum distance between them for satisfactory communication in LOS mode. (Radius of the Earth = 6.4×10^6m) **[3; CBSE-2013]**

22. Write the functions of the following in communication systems.

(i) Transmitter (ii) Modulator **[2; CBSE-2014]**

23. Write two basic modes of communication Explain the process of amplitude modulation. Draw a schematic sketch showing how amplitude modulated signal is obtained by superposing a modulating signal over a sinusoidal carrier wave. **[3; CBSE-2014]**

24. What is the function of a band pass filter used in a modulator for obtaining AM signal ? **[1; CBSE-2015]**

25. Differentiate between amplitude modulated (AM) and frequency modulated (FM) waves by drawing suitable diagrams. Why is FM signal preferred over AM signal ? **[2; CBSE-2015]**

26. Name the three different modes of propagation in a communication system. **[3; CBSE-2015]**

State briefly why do the electromagnetic waves with frequency range from a few MHz upto 30 MHz can reflect back to the earth. What happens when the frequency range exceeds this limit ?

27. A signal of 5 kHz frequency is amplitude modulated on a carrier wave of frequency 2 MHz. What are the frequencies of the side bands produced ? **[CBSE-2016]**

28. Why is base band signal not transmitted directly ? Give any two reasons. **[CBSE-2016]**

29. What is space wave propagation? State the factors which limit its range of propagation. Derive an expression for the maximum line of sight distance between two antennas for space wave propagation. **[CBSE-2016]**

30. (a) How is amplitude modulation achieved ?

(b) The frequencies of two side bands in an AM wave are 640 kHz and 660 kHz respectively. Find the frequencies of carrier and modulating signal. What is the bandwidth required for amplitude modulation ? **[CBSE-2017]**

31. Draw a block diagram of a generalized communication system. Write the functions of each of the following: **[CBSE-2017]**

(a) Transmitter
(b) Channel
(c) Receiver

32. Which modes of propagation is used by short wave broadchast services? **[CBSE-2018]**

33. A carrier wave of peak voltage 15 V is used to transmit a message signal. Find the peak voltage of the modulating signal in order to have a modulation index of 60%. **[CBSE-2018]**

34. (a) Give three reasons why modulation of a message signal is necessary for long distance transmission.

(b) Show graphically an audio signal, a carrier wave and an amplitude modulated wave.

[CBSE-2018]

ANSWER KEY

EXERCISE-S

1. **Ans.** No. Service area will be A = $\pi d_7^2 = \frac{22}{7} \times 162 \times 6.4 \times 10^6 = 3258$ km^2.

2. **Ans.** $\mu = 0.75 = \frac{A_m}{A_c}$; $A_m = 0.75 \times 12 = 9V$

3. **Ans.** Since the AM wave is given by $(A_c + A_m)$ sin ω_m t) cos ω_m t, the maximum amplitude is M1 = $A_c + A_m$. Hence the modulation index is

$$m = \frac{A_m}{A_c} = \frac{8}{12} = \frac{2}{3}$$

with $M_2 = 0$, clearly, m = 1 irrespective of M_1

EXERCISE-O

SINGLE CORRECT TYPE QUESTIONS

1. Ans. (C)
2. Ans. (C)
3. Ans. (A)
4. Ans. (B)
5. Ans. (B)
6. Ans. (B)
7. Ans. (A)
8. Ans. (B)
9. Ans. (B)
10. Ans. (A)
11. Ans. (A)

MULTIPLE CORRECT TYPE QUESTIONS

12. Ans. (A,B,D)
13. Ans. (B,D)

EXERCISE-JM

1. Ans. (1)
2. Ans. (3)
3. Ans. (3)
4. Ans. (1)
5. Ans. (2)
6. Ans. (3)
7. Ans. (2)